Abelian groups edit

Abbreviation: AbGrp

Definition 1. An *abelian group* is a structure $\mathbf{G} = \langle G, +, -, 0\rangle$, where $+$ is an infix binary operation, called the *group addition*, $-$ is a prefix unary operation, called the *group negative* and 0 is a constant (nullary operation), called the *additive identity element*, such that

$+$ is commutative: $x + y = y + x$

$+$ is associative: $(x + y) + z = x + (y + z)$

0 is an additive identity for $+$: $0 + x = x$

$-$ gives an additive inverse for $+$: $-x + x = 0$

Morphisms. Let $\mathbf{G}$ and $\mathbf{H}$ be abelian groups. A morphism from $\mathbf{G}$ to $\mathbf{H}$ is a function $h : G \to H$ that is a homomorphism: $h(x + y) = h(x) + h(y)$

Remark: It follows that $h(-x) = -h(x)$, $h(0) = 0$.

Basic Results.

Examples.

1. $\langle \mathbb{Z}, +, -, 0\rangle$, the integers, with addition, unary subtraction, and zero. The variety of abelian groups is generated by this algebra.

Finite Members. $f(n) =$ number of members of size n.

$f(1) = 1$	$f(6) = 1$	$f(11) = 1$
$f(2) = 1$	$f(7) = 1$	$f(12) = 2$
$f(3) = 1$	$f(8) = 3$	$f(13) = 1$
$f(4) = 2$	$f(9) = 2$	$f(14) = 1$
$f(5) = 1$	$f(10) = 1$	$f(15) = 1$

`http://www.research.att.com/projects/OEIS?Anum=A000688`

Subclasses.
Boolean groups
Commutative rings

Superclasses.
Groups
Commutative monoids

References

[1] W. Szmielew, *Decision problem in group theory*, Library of the Tenth International Congress of Philosophy, Amsterdam, August 11–18, 1948, Vol.1, Proceedings of the Congress, 1949, 763–766 MRreview

Properties. (description)

Classtype	variety
Equational theory	decidable in polynomial time
Quasiequational theory	decidable
First-order theory	decidable [1]
Locally finite	no
Residual size	ω
Congruence distributive	no ($\mathbb{Z}_2 \times \mathbb{Z}_2$)
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$, $p(x, y, z) = x - y + z$
Congruence regular	yes, congruences are determined by subalgebras
Congruence uniform	yes
Congruence types	permutational
Congruence extension property	yes, if $K \leq H \leq G$ then $K \leq G$
Definable principal congruences	no
Equationally def. pr. cong.	no
Amalgamation property	yes
Strong amalgamation property	yes
Epimorphisms are surjective	yes