

Action algebras edit

Abbreviation: Act

Definition 1. An *action algebra* is a structure $\mathbf{A} = \langle A, \vee, \perp, \cdot, 1, *, \backslash, / \rangle$ of type $\langle 2, 0, 2, 0, 1, 2, 2 \rangle$ such that

$\langle A, \vee, \perp, \cdot, 1, * \rangle$ is a Kleene algebras

\backslash is the left residual of \cdot : $y \leq x \backslash z \iff xy \leq z$

$/$ is the right residual of \cdot : $x \leq z/y \iff xy \leq z$

Remark:

Morphisms. Let \mathbf{A} and \mathbf{B} be action algebras. A morphism from \mathbf{A} to \mathbf{B} is a function $h : A \rightarrow B$ that is a homomorphism: $h(x \vee y) = h(x) \vee h(y)$, $h(x \cdot y) = h(x) \cdot h(y)$, $h(x \backslash y) = h(x) \backslash h(y)$, $h(x/y) = h(x)/h(y)$, $h(x^*) = h(x)^*$, $h(\perp) = \perp$ and $h(1) = 1$.

Basic Results.

Examples.

1.

Properties. (description)

Classtype	variety [1]
Equational theory	
Quasiequational theory	undecidable
First-order theory	undecidable
Locally finite	no
Residual size	unbounded
Congruence distributive	yes [2]
Congruence modular	yes
Congruence n-permutable	yes, $n = 4$ [2]
Congruence regular	
Congruence uniform	
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n)$ = number of members of size n .

$$\begin{aligned}f(1) &= 1 \\f(2) &= 1 \\f(3) &= 3 \\f(4) &= 20 \\f(5) &= 149 \\f(6) &= 1488\end{aligned}$$

Subclasses.

Action lattices

Superclasses.

Kleene algebras

Residuated join-semilattices

REFERENCES

- [1] Vaughan Pratt, *Action logic and pure induction*, “Logics in AI (Amsterdam, 1990)”, Lecture Notes in Comput. Sci., 478, 1991, 97–120, 92d:03016
- [2] C.J. van Alten and J.G. Raftery, *Embedding Theorems and Rule Separation in Logics without Weakening*, Studia Logica, 2004, ...–..., preprint