BCK-join-semilattices edit

Abbreviation: BCKJSlat

Definition 1. A *BCK-join-semilattice* is a structure $\mathbf{A} = \langle A, \vee, \rightarrow, 1\rangle$ of type $\langle 2, 2, 0\rangle$ such that

(1): $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$

(2): $1 \rightarrow x = x$

(3): $x \rightarrow 1 = 1$

(4): $x \rightarrow (x \vee y) = 1$

(5): $x \vee ((x \rightarrow y) \rightarrow y) = ((x \rightarrow y) \rightarrow y)$

$\vee$ is idempotent: $x \vee x = x$

$\vee$ is commutative: $x \vee y = y \vee x$

$\vee$ is associative: $(x \vee y) \vee z = x \vee (y \vee z)$

Remark: $x \leq y \iff x \rightarrow y = 1$ is a partial order, with 1 as greatest element, and $\vee$ is a join for this order. [1]

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be BCK-join-semilattices. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \rightarrow B$ that is a homomorphism:

$h(x \vee y) = h(x) \vee h(y)$, $h(x \rightarrow y) = h(x) \rightarrow h(y)$ and $h(1) = 1$

Basic Results.

Examples.

1.

Finite Members. $f(n) =$ number of members of size n.

$f(1) = \ 1$
$f(2) =$
$f(3) =$
$f(4) =$
$f(5) =$
$f(6) =$

Subclasses.
BCK-lattices

Superclasses.
BCK-algebras

References

[1] Pawel M. Idziak, *Lattice operation in BCK-algebras*, Math. Japon., **29**, 1984, 839–846 MRreview

Properties. (description)

Classtype	variety
Equational theory	
Quasiequational theory	
First-order theory	
Locally finite	
Residual size	
Congruence distributive	
Congruence modular	
Congruence n-permutable	
Congruence regular	
Congruence uniform	
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	