BCK-lattices edit

Abbreviation: BCKlat

Definition 1. A *BCK-lattice* is a structure $\mathbf{A} = \langle A, \vee, \wedge, \rightarrow, 1\rangle$ of type $\langle 2, 2, 2, 0\rangle$ such that

$\langle A, \vee, \rightarrow, 1\rangle$ is a BCK-join-semilattice

$\langle A, \wedge, \rightarrow, 1\rangle$ is a BCK-meet-semilattice

Remark: $x \leq y \iff x \rightarrow y = 1$ is a partial order, with 1 as greatest element, and $\vee$, $\wedge$ are a join and meet for this order. [1]

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be BCK-lattices. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \rightarrow B$ that is a homomorphism:

$h(x \vee y) = h(x) \vee h(y)$, $h(x \wedge y) = h(x) \wedge h(y)$, $h(x \rightarrow y) = h(x) \rightarrow h(y)$ and $h(1) = 1$.

Basic Results.

Examples.

1.

Properties. (description)

Classtype	variety
Equational theory	
Quasiequational theory	
First-order theory	
Locally finite	
Residual size	
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes $n = 2$
Congruence regular	
Congruence uniform	
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n) =$ number of members of size n.

$$
\begin{aligned}
f(1) &= 1 \\
f(2) &= \\
f(3) &= \\
f(4) &= \\
f(5) &= \\
f(6) &=
\end{aligned}
$$

Subclasses.

Heyting algebras

Superclasses.

BCK-join-semilattices

BCK-meet-semilattices

References

[1] Pawel M. Idziak, *Lattice operation in BCK-algebras*, Math. Japon., **29**, 1984, 839–846 MRreview