Basic logic algebras edit

Abbreviation: BLA

Definition 1. A *basic logic algebra* or *BL-algebra* is a structure $\mathbf{A} = \langle A, \vee, 0, \wedge, 1, \cdot, \rightarrow \rangle$ such that

$\langle A, \vee, 0, \wedge, 1 \rangle$ is a bounded lattice

$\langle A, \cdot, 1 \rangle$ is a commutative monoid

$\rightarrow$ gives the residual of $\cdot$: $x \cdot y \leq z \iff y \leq x \rightarrow z$

linearity: $(x \rightarrow y) \vee (y \rightarrow x) = 1$

BL: $x \cdot (x \rightarrow y) = x \wedge y$

Remark: The BL identity implies that the lattice is distributive.

Definition 2. A *basic logic algebra* is a FLe-algebra $\mathbf{A} = \langle A, \vee, 0, \wedge, 1, \cdot, \rightarrow \rangle$ such that

linearity: $(x \rightarrow y) \vee (y \rightarrow x) = 1$

BL: $x \cdot (x \rightarrow y) = x \wedge y$

Remark: The BL identity implies that the identity element 1 is the top of the lattice.

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be basic logic algebras. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \rightarrow B$ that is a homomorphism:

$h(x \vee y) = h(x) \vee h(y)$, $h(1) = 1$, $h(x \wedge y) = h(x) \wedge h(y)$, $h(0) = 0$, $h(x \cdot y) = h(x) \cdot h(y)$, $h(x \rightarrow y) = h(x) \rightarrow h(y)$

Basic Results.

Examples.

1.

Finite Members. $f(n) =$ number of members of size n.

$$
\begin{aligned}
f(1) &= 1 \\
f(2) &= 1 \\
f(3) &= 2 \\
f(4) &= 5
\end{aligned}
$$

The number of subdirectly irreducible BL-algebras of size n is 2^{n-2}.

Subclasses.

MV-algebras

Heyting algebras

Properties. (description)

Classtype	variety
Equational theory	decidable
Quasiequational theory	
First-order theory	
Locally finite	no
Residual size	unbounded
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence e-regular	yes, $e = 1$
Congruence uniform	no
Congruence extension property	yes
Definable principal congruences	
Equationally def. pr. cong.	no
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Superclasses.

Generalized basic logic algebras

FLew-algebras

References

[1]