Boolean algebras with operators edit

Abbreviation: BAO

Definition 1. A *Boolean algebra with operators* is a structure $\mathbf{A} = \langle A, \vee, 0, \wedge, 1, \neg, f_i\ (i \in I)\rangle$ such that

$\langle A, \vee, 0, \wedge, 1, \neg\rangle$ is a Boolean algebra

f_i is *join-preserving* in each argument: $f_i(\ldots, x\vee y, \ldots) = f_i(\ldots, x, \ldots)\vee f_i(\ldots, y, \ldots)$

f_i is *normal* in each argument: $f_i(\ldots, 0, \ldots) = 0$

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be Boolean algebras with operators of the same signature. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \to B$ that is a Boolean homomorphism and preserves all the operators:

$h(f_i(x_0, \ldots, x_{n-1})) = f_i(h(x_0), \ldots, h(x_{n-1}))$

Basic Results.

Examples.

1.

Properties. (description)

Classtype	variety
Equational theory	decidable
Quasiequational theory	
First-order theory	undecidable
Locally finite	no
Residual size	unbounded
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence regular	yes
Congruence uniform	yes
Congruence extension property	yes
Definable principal congruences	no
Equationally def. pr. cong.	no
Amalgamation property	yes
Strong amalgamation property	yes
Epimorphisms are surjective	yes

Subclasses.
Modal algebras
Boolean monoids

Superclasses.
Boolean algebras

References

[1]