Boolean lattices edit

Abbreviation: BoolLat

Definition 1. A *Boolean lattice* is a bounded distributive lattice $\mathbf{L} = \langle L, \vee, 0, \wedge, 1\rangle$ such that

every element has a complement: $\exists y(x \vee y = 1$ and $x \wedge y = 0)$

Morphisms. Let $\mathbf{L}$ and $\mathbf{M}$ be bounded distributive lattices. A morphism from $\mathbf{L}$ to $\mathbf{M}$ is a function $h : L \to M$ that is a bounded lattice homomorphism:

$h(x \vee y) = h(x) \vee h(y)$, $h(x \wedge y) = h(x) \wedge h(y)$, $h(0) = 0$, $h(1) = 1$

Basic Results.

Examples.

1. $\langle mathcal P(S), \cup, \emptyset, \cap, S\rangle$, the collection of subsets of a set S, with union, empty set, intersection, and the whole set S.

Properties. (description)

Classtype	first-order
Equational theory	decidable
Quasiequational theory	decidable
First-order theory	decidable
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes
Congruence regular	yes
Congruence uniform	yes
Congruence extension property	yes
Definable principal congruences	yes
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	
Locally finite	yes
Residual size	

Finite Members. Any finite member is a power of the 2-element Boolean lattice.

Subclasses.

Boolean algebras

Superclasses.

Complemented modular lattices

Bounded distributive lattices

References

[1]