Boolean monoids edit

Abbreviation: BMon

Definition 1. A *Boolean monoid* is a structure $\mathbf{A} = \langle A, \vee, 0, \wedge, 1, \neg, \cdot, e\rangle$ such that

$\langle A, \vee, 0, \wedge, 1, \neg\rangle$ is a Boolean algebra

$\langle A, \cdot, e\rangle$ is a monoids

$\cdot$ is *join-preserving* in each argument: $(x \vee y) \cdot z = (x \cdot z) \vee (y \cdot z)$ and $x \cdot (y \vee z) = (x \cdot y) \vee (x \cdot z)$

$\cdot$ is *normal* in each argument: $0 \cdot x = 0$ and $x \cdot 0 = 0$

Remark:

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be Boolean monoids. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \to B$ that is a Boolean homomorphism and preserves $\cdot$, e:

$h(x \cdot y) = h(x) \cdot h(y)$ and $h(e) = e$

Basic Results.

Examples.

1.

Properties. (description)

Classtype	variety
Equational theory	
Quasiequational theory	
First-order theory	
Locally finite	no
Residual size	unbounded
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence regular	yes
Congruence uniform	yes
Congruence extension property	yes
Definable principal congruences	no
Equationally def. pr. cong.	no
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n)$ = number of members of size n.

$$
\begin{aligned}
f(1) &= 1\\
f(2) &= 1\\
f(3) &= 0\\
f(4) &= 9\\
f(5) &= 0\\
f(6) &= 0\\
f(7) &= 0\\
f(8) &= 258
\end{aligned}
$$

Subclasses.
Sequential algebras

Superclasses.
Boolean algebras with operators

References

[1]