

Boolean semigroups [edit](#)

Abbreviation: BSgrp

Definition 1. A *Boolean semigroup* is a structure $\mathbf{A} = \langle A, \vee, 0, \wedge, 1, \neg, \cdot \rangle$ such that

$\langle A, \vee, 0, \wedge, 1, \neg \rangle$ is a Boolean algebra

$\langle A, \cdot \rangle$ is a semigroups

\cdot is *join-preserving* in each argument: $(x \vee y) \cdot z = (x \cdot z) \vee (y \cdot z)$ and $x \cdot (y \vee z) = (x \cdot y) \vee (x \cdot z)$

\cdot is *normal* in each argument: $0 \cdot x = 0$ and $x \cdot 0 = 0$

Morphisms. Let \mathbf{A} and \mathbf{B} be Boolean monoids. A morphism from \mathbf{A} to \mathbf{B} is a function $h : A \rightarrow B$ that is a Boolean homomorphism and preserves \cdot :

$$h(x \cdot y) = h(x) \cdot h(y)$$

Basic Results.

Examples.

1.

Properties. (description)

Classtype	variety
Equational theory	
Quasiequational theory	
First-order theory	
Locally finite	no
Residual size	unbounded
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence regular	yes
Congruence uniform	yes
Congruence extension property	yes
Definable principal congruences	no
Equationally def. pr. cong.	no
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n)$ = number of members of size n .

$$\begin{aligned}f(1) &= 1 \\f(2) &= 2 \\f(3) &= 0 \\f(4) &= 28 \\f(5) &= 0 \\f(6) &= 0 \\f(7) &= 0 \\f(8) &= 5457\end{aligned}$$

Subclasses.

Boolean monoids

Variety generated by complex algebras of semigroups

Superclasses.

Boolean algebras with operators

REFERENCES

[1]