Boolean semilattices edit

Abbreviation: BSlat

Definition 1. A *Boolean semilattice* is a structure $\mathbf{A} = \langle A, \vee, 0, \wedge, 1, \neg, \cdot\rangle$ such that

$\mathbf{A}$ is in the variety generated by complex algebras of semilattices

Let $\mathbf{S} = \langle S, \cdot\rangle$ be a semilattice. The *complex algebra* of $\mathbf{S}$ is $Cm(\mathbf{S}) = \langle P(S), \cup, \emptyset, \cap, S, -, \cdot\rangle$, where $\langle P(S), \cup, \emptyset, \cap, S, -\rangle$ is the Boolean algebra of subsets of S, and

$X \cdot Y = \{x \cdot y \mid x \in X,\ y \in Y\}$.

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be Boolean semilattices. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \to B$ that is a Boolean homomorphism and preserves $\cdot$:

$h(x \cdot y) = h(x) \cdot h(y)$

Basic Results.

Examples.

1.

Properties. (description)

Classtype	variety
Finitely axiomatizable	open
Equational theory	
Quasiequational theory	
First-order theory	
Locally finite	no
Residual size	unbounded
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence regular	yes
Congruence uniform	
Congruence extension property	yes
Definable principal congruences	
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n) =$ number of members of size n.

$f(1) =$	1
$f(2) =$	1
$f(3) =$	0
$f(4) =$	5
$f(5) =$	0
$f(6) =$	0
$f(7) =$	0
$f(8) =$	≥ 97 out of 104

Some members of BSlat

Subclasses.

Variety generated by complex algebras of linear semilattices

Superclasses.

Commutative Boolean semigroups

References

[1]