Bounded distributive lattices edit

Abbreviation: BDLat

Definition 1. A *bounded distributive lattice* is a structure $\mathbf{L} = \langle L, \vee, 0, \wedge, 1\rangle$ such that

$\langle L, \vee, \wedge\rangle$ is a distributive lattice

0 is the least element: $0 \leq x$

1 is the greatest element: $x \leq 1$

Morphisms. Let $\mathbf{L}$ and $\mathbf{M}$ be bounded distributive lattices. A morphism from $\mathbf{L}$ to $\mathbf{M}$ is a function $h : L \to M$ that is a homomorphism:

$h(x \vee y) = h(x) \vee h(y)$, $h(x \wedge y) = h(x) \wedge h(y)$, $h(0) = 0$, $h(1) = 1$

Basic Results.

Examples.

1. $\langle \mathcal{P}(S), \cup, \emptyset, \cap, S\rangle$, the collection of subsets of a set S, with union, empty set, intersection, and the whole set S.

Properties. (description)

Classtype	variety
Equational theory	decidable
Quasiequational theory	decidable
First-order theory	undecidable
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	no
Congruence regular	no
Congruence uniform	no
Congruence extension property	yes
Definable principal congruences	no
Equationally def. pr. cong.	no
Amalgamation property	yes
Strong amalgamation property	no
Epimorphisms are surjective	no
Locally finite	yes
Residual size	2

Finite Members. $f(n) =$ number of members of size n.

$f(1) = 1$	$f(6) = 5$	$f(11) = 82$	$f(16) = 1639$
$f(2) = 1$	$f(7) = 8$	$f(12) = 151$	$f(17) = 2978$
$f(3) = 1$	$f(8) = 15$	$f(13) = 269$	$f(18) = 5483$
$f(4) = 2$	$f(9) = 26$	$f(14) = 494$	$f(19) = 10006$
$f(5) = 3$	$f(10) = 47$	$f(15) = 891$	$f(20) = 18428$

Values known up to size 49 [1].

Subclasses.
Boolean algebras
Complete distributive lattices

Superclasses.
Distributive lattices
Bounded modular lattices

References

[1] Marcel Erne, Jobst Heitzig and Jürgen Reinhold, *On the number of distributive lattices*, Electron. J. Combin., **9**, 2002,Research Paper 24, 23 pp. (electronic) MRreview