# Brouwerian algebras edit

**Abbreviation: BrA**

**Definition 1.** A *Brouwerian algebra* is a structure $\mathbf{A} = \langle A, \vee, \wedge, 1, \to \rangle$ such that

$\langle A, \vee, \wedge, 1\rangle$ is a distributive lattice with top

$\to$ gives the residual of $\wedge$: $x \wedge y \leq z \iff y \leq x \to z$

**Morphisms.** Let $\mathbf{A}$ and $\mathbf{B}$ be Brouwerian algebras. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \to B$ that is a homomorphism:

$h(x \vee y) = h(x) \vee h(y)$, $h(x \wedge y) = h(x) \wedge h(y)$, $h(1) = 1$, $h(x \to y) = h(x) \to h(y)$

**Definition 2.** A *Brouwerian algebra* is a BL-algebra $\mathbf{A} = \langle A, \vee, \wedge, 1, \cdot, \to \rangle$ such that

$x \wedge y = x \cdot y$

**Basic Results.**

**Examples.**

**1.**

**Properties.** (description)

| | |
|---|---|
| Equational theory | decidable |
| Quasiequational theory | decidable |
| First-order theory | undecidable |
| Locally finite | no |
| Residual size | unbounded |
| Congruence distributive | yes |
| Congruence modular | yes |
| Congruence n-permutable | yes, $n = 2$ |
| Congruence e-regular | yes, $e = 1$ |
| Congruence uniform | no |
| Congruence extension property | yes |
| Definable principal congruences | yes |
| Equationally def. pr. cong. | yes |
| Amalgamation property | yes |
| Strong amalgamation property | yes |
| Epimorphisms are surjective | yes |

**Finite Members.** $f(n) =$ number of members of size $n$.

$f(1) =$ 1
$f(2) =$ 1
$f(3) =$ 1
$f(4) =$ 2
$f(5) =$ 3
$f(6) =$ 5
$f(7) =$ 8
$f(8) =$ 15
$f(9) =$ 26
$f(10) =$ 47
$f(11) =$ 82
$f(12) =$ 151
$f(13) =$ 269
$f(14) =$ 494
$f(15) =$ 891
$f(16) =$ 1639
$f(17) =$ 2978
$f(18) =$ 5483
$f(19) =$ 10006
$f(20) =$ 18428
$Valuesknownuptosize 49 [Erne, Heitzig, Reinhold (2002)]$

**Subclasses.**
Generalized Boolean algebras
Heyting algebras

**Superclasses.**
Distributive lattices
Basic logic algebras

## References

[1]