Cancellative commutative monoids edit

Abbreviation: CanCMon

Definition 1. A *cancellative commutative monoid* is a cancellative monoid $\mathbf{M} = \langle M, \cdot, e\rangle$ such that

$\cdot$ is commutative: $x \cdot y = y \cdot x$

Morphisms. Let $\mathbf{M}$ and $\mathbf{N}$ be cancellative commutative monoids. A morphism from $\mathbf{M}$ to $\mathbf{N}$ is a function $h : M \to N$ that is a homomorphism:

$h(x \cdot y) = h(x) \cdot h(y)$, $h(e) = e$

Basic Results. All commutative free monoids are cancellative.

All finite commutative (left or right) cancellative monoids are reducts of abelian groups.

Examples.

1. $\langle \mathbb{N}, +, 0\rangle$, the natural numbers, with addition and zero.

Properties. (description)

Classtype	quasivariety
Equational theory	
Quasiequational theory	
First-order theory	undecidable
Locally finite	no
Residual size	unbounded
Congruence distributive	no
Congruence modular	
Congruence n-permutable	
Congruence regular	
Congruence uniform	
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n) =$ number of members of size n.

$$\begin{aligned} f(1) &= 1 \\ f(2) &= 1 \\ f(3) &= 1 \\ f(4) &= 2 \\ f(5) &= 1 \\ f(6) &= 1 \\ f(7) &= 1 \end{aligned}$$

Subclasses.

Abelian groups

Cancellative commutative residuated lattices

Superclasses.

Cancellative commutative semigroups

Cancellative monoids

Commutative monoids

References

[1]