Cancellative residuated lattices edit

Abbreviation: CanRL

Definition 1. A *cancellative residuated lattice* is a residuated lattice $\mathbf{L} = \langle L, \vee, \wedge, \cdot, e, \backslash, /\rangle$ such that

$\cdot$ is right-cancellative: $xz = yz \implies x = y$

$\cdot$ is left-cancellative: $zx = zy \implies x = y$

Morphisms. Let $\mathbf{L}$ and $\mathbf{M}$ be cancellative residuated lattices. A morphism from $\mathbf{L}$ to $\mathbf{M}$ is a function $h : L \to M$ that is a homomorphism:

$h(x \vee y) = h(x) \vee h(y)$, $h(x \wedge y) = h(x) \wedge h(y)$ $h(x \cdot y) = h(x) \cdot h(y)$, $h(x \backslash y) = h(x) \backslash h(y)$, $h(x/y) = h(x)/h(y)$ and $h(e) = e$

Basic Results.

Examples.

1.

Properties. (description)

Classtype	variety
Equational theory	
Quasiequational theory	
First-order theory	undecidable
Locally finite	no
Residual size	unbounded
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence regular	no
Congruence e-regular	yes
Congruence uniform	no
Congruence extension property	no
Definable principal congruences	no
Equationally def. pr. cong.	no
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n) =$ number of members of size n.

$None$

Subclasses.

Cancellative commutative residuated lattices

Cancellative distributive residuated lattices

Superclasses.

Residuated lattices

References

[1]