Commutative BCK-algebras edit

Abbreviation: ComBCK

Definition 1. A *commutative BCK-algebra* is a structure $\mathbf{A} = \langle A, \cdot, 0\rangle$ of type $\langle 2, 0\rangle$ such that

(1): $((x \cdot y) \cdot (x \cdot z)) \cdot (z \cdot y) = 0$

(2): $x \cdot 0 = x$

(3): $0 \cdot x = 0$

(4): $x \cdot y = y \cdot x = 0 \implies x = y$

(5): $x \cdot (x \cdot y) = y \cdot (y \cdot x)$

Remark: Note that the commutativity does not refer to the operation $\cdot$, but rather to the term operation $x \wedge y = x \cdot (x \cdot y)$, which turns out to be a meet with respect to the following partial order:

$x \le y \iff x \cdot y = 0$, with 0 as least element.

Definition 2. A *commutative BCK-algebra* is a BCK-algebra $\mathbf{A} = \langle A, \cdot, 0\rangle$ such that

$x \cdot (x \cdot y) = y \cdot (y \cdot x)$

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be commutative BCK-algebras. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \to B$ that is a homomorphism:

$h(x \cdot y) = h(x) \cdot h(y)$ and $h(0) = 0$

Basic Results.

Examples.

1.

Finite Members. $f(n) =$ number of members of size n.

$f(1) =\ \ 1$
$f(2) =$
$f(3) =$
$f(4) =$
$f(5) =$
$f(6) =$

Subclasses.
Tarski algebras

Superclasses.
BCK-algebras

Properties. (description)

Classtype	variety
Equational theory	
Quasiequational theory	
First-order theory	
Locally finite	no
Residual size	unbounded
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes, $n = 3$
Congruence regular	
Congruence uniform	
Congruence extension property	
Definable principal congruences	no
Equationally def. pr. cong.	no
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

References

[1]