Commutative monoids edit

Abbreviation: CMon

Definition 1. A *commutative monoid* is a monoids $\mathbf{M} = \langle M, \cdot, e\rangle$ such that

$\cdot$ is commutative: $x \cdot y = y \cdot x$

Definition 2. A *commutative monoid* is a structure $\mathbf{M} = \langle M, \cdot, e\rangle$, where $\cdot$ is an infix binary operation, called the *monoid product*, and e is a constant (nullary operation), called the *identity element*, such that

$\cdot$ is commutative: $x \cdot y = y \cdot x$

$\cdot$ is associative: $(x \cdot y) \cdot z = x \cdot (y \cdot z)$

e is an identity for $\cdot$: $e \cdot x = x$

Morphisms. Let $\mathbf{M}$ and $\mathbf{N}$ be commutative monoids. A morphism from $\mathbf{M}$ to $\mathbf{N}$ is a function $h : M \to N$ that is a homomorphism:

$h(x \cdot y) = h(x) \cdot h(y)$, $h(e) = e$

Basic Results.

Examples.

1. $\langle \mathbb{N}, +, 0\rangle$, the natural numbers, with addition and zero. The finitely generated free commutative monoids are direct products of this one.

Finite Members. $f(n) =$ number of members of size n.

$f(1) =$	1
$f(2) =$	2
$f(3) =$	5
$f(4) =$	19
$f(5) =$	78
$f(6) =$	421
$f(7) =$	2637

Subclasses.
Abelian groups
Semilattices with identity

Superclasses.
Commutative semigroups
Monoids

References

[1]

Properties. (description)

Classtype	variety
Equational theory	decidable
Quasiequational theory	decidable
First-order theory	undecidable
Locally finite	no
Residual size	unbounded
Congruence distributive	no
Congruence modular	no
Congruence n-permutable	no
Congruence regular	no
Congruence uniform	no
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	no
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	