Commutative rings edit

Abbreviation: CRng

Definition 1. A *commutative ring* is a rings $\mathbf{R} = \langle R, +, -, 0, \cdot\rangle$ such that

$\cdot$ is commutative: $x \cdot y = y \cdot x$

Remark: $Idl(R) = \{allidealsofR\}$

I is an ideal if $a, b \in I \implies a + b \in I$

and $\forall r \in R\ (r \cdot I \subseteq I)$

Morphisms. Let $\mathbf{R}$ and $\mathbf{S}$ be commutative rings with identity. A morphism from $\mathbf{R}$ to $\mathbf{S}$ is a function $h : R \to S$ that is a homomorphism:

$h(x + y) = h(x) + h(y)$, $h(x \cdot y) = h(x) \cdot h(y)$

Remark: It follows that $h(0) = 0$ and $h(-x) = -h(x)$.

Basic Results. 0 is a zero for $\cdot$: $0 \cdot x = x$ and $x \cdot 0 = 0$.

Examples.

1. $\langle \mathbb{Z}, +, -, 0, \cdot\rangle$, the ring of integers with addition, subtraction, zero, and multiplication.

Properties. (description)

Classtype	variety
Equational theory	decidable
Quasiequational theory	
First-order theory	undecidable
Locally finite	no
Residual size	unbounded
Congruence distributive	no
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence regular	yes
Congruence uniform	yes
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n) =$ number of members of size n.

$f(1) =$
$f(2) =$
$f(3) =$
$f(4) =$
$f(5) =$
$f(6) =$ $4http://www.research.att.com/cgi-bin/access.cgi/as/njas/sequences/eisA.cgi?Anum =$

Subclasses.
Commutative rings with identity
Fields

Superclasses.
Rings

References

[1]