

Commutative rings with identity edit

Abbreviation: \mathbf{CRng}_1

Definition 1. A *commutative ring with identity* is a rings with identity $\mathbf{R} = \langle R, +, -, 0, \cdot, 1 \rangle$ such that \cdot is commutative: $x \cdot y = y \cdot x$

Morphisms. Let \mathbf{R} and \mathbf{S} be commutative rings with identity. A morphism from \mathbf{R} to \mathbf{S} is a function $h : R \rightarrow S$ that is a homomorphism:

$$h(x + y) = h(x) + h(y), h(x \cdot y) = h(x) \cdot h(y), h(1) = 1$$

Remark: It follows that $h(0) = 0$ and $h(-x) = -h(x)$.

Basic Results. 0 is a zero for \cdot : $0 \cdot x = x$ and $x \cdot 0 = 0$.

Examples.

1. $\langle \mathbb{Z}, +, -, 0, \cdot, 1 \rangle$, the ring of integers with addition, subtraction, zero, multiplication, and one.

Properties. (description)

Classtype	variety
Equational theory	decidable
Quasiequational theory	
First-order theory	undecidable
Locally finite	no
Residual size	unbounded
Congruence distributive	no
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence regular	yes
Congruence uniform	yes
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n)$ = number of members of size n .

$$f(1) = 1$$

$$f(2) = 1$$

$$f(3) = 1$$

$$f(4) = 4$$

$$f(5) = 1$$

$$f(6) = 1$$

Subclasses.

Boolean algebras

Integral domains

Superclasses.

Commutative rings

Rings with identity

REFERENCES

[1]