Commutative semigroups edit

Abbreviation: CSgrp

Definition 1. A *commutative semigroup* is a semigroups $\mathbf{S} = \langle S, \cdot \rangle$ such that

$\cdot$ is commutative: $xy = yx$

Definition 2. A *commutative semigroup* is a structure $\mathbf{S} = \langle S, \cdot \rangle$, where $\cdot$ is an infix binary operation, called the *semigroup product*, such that

$\cdot$ is associative: $(xy)z = x(yz)$

$\cdot$ is commutative: $xy = yx$

Morphisms. Let $\mathbf{S}$ and $\mathbf{T}$ be commutative semigroups. A morphism from $\mathbf{S}$ to $\mathbf{T}$ is a function $h : S \to T$ that is a homomorphism:

$h(xy) = h(x)h(y)$

Basic Results.

Examples.

1. $\langle \mathbb{N}, + \rangle$, the natural numbers, with addition.

Properties. (description)

Classtype	variety
Equational theory	decidable in polynomial time
Quasiequational theory	decidable
First-order theory	
Locally finite	no
Residual size	
Congruence distributive	no
Congruence modular	no
Congruence n-permutable	no
Congruence regular	no
Congruence uniform	no
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	no
Amalgamation property	no
Strong amalgamation property	no
Epimorphisms are surjective	no

Finite Members. $f(n)$ = number of members of size n.

$Searchforfinitecommutativesemigroupsf(1) =$	1
$f(2) =$	3
$f(3) =$	12
$f(4) =$	58
$f(5) =$	325
$f(6) =$	2143
$f(7) =$	17291

Subclasses.
Semilattices
Commutative monoids

Superclasses.
Semigroups
Partial commutative semigroups

References

[1]