Directed complete partial orders edit

Abbreviation: DCPO

Definition 1. A *directed complete partial order* is a poset $\mathbf{P} = \langle P, \leq \rangle$ such that every directed subset of P has a least upper bound: $\forall D \subseteq P \ (D \neq \emptyset \text{and} \forall x, y \in D \ \exists z \in D(x, y \leq z) \implies \exists z \in P(z = \bigvee D))$.

Morphisms. Let $\mathbf{P}$ and $\mathbf{Q}$ be directed complete partial orders. A morphism from $\mathbf{P}$ to $\mathbf{Q}$ is a function $f : P \to Q$ that is *Scott-continuous*, which means that f preserves all directed joins:

$z = \bigvee D \implies f(z) = \bigvee f[D]$

Basic Results.

Examples.

1. $\langle \mathbb{R}, \leq \rangle$, the real numbers with the standard order.

2. $\langle P(S), \subseteq \rangle$, the collection of subsets of a sets S, ordered by inclusion.

Properties. (description)

Classtype	second-order
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n) =$ number of members of size n.

$f(1) =$ 1
$f(2) =$
$f(3) =$
$f(4) =$
$f(5) =$
$f(6) =$

Subclasses.
Complete semilattices

Superclasses.
Directed partial orders

References

[1]