Directoids edit

Abbreviation: Dtoid

Definition 1. A *directoid* is a structure $\mathbf{A} = \langle A, \cdot \rangle$, where $\cdot$ is an infix binary operation such that

$\cdot$ is idempotent: $x \cdot x = x$

$(x \cdot y) \cdot x = x \cdot y$

$y \cdot (x \cdot y) = x \cdot y$

$x \cdot ((x \cdot y) \cdot z) = (x \cdot y) \cdot z$

Remark:

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be directoids. A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \to B$ that is a homomorphism:

$h(xy) = h(x)h(y)$

Basic Results. The relation $x \leq y \iff x \cdot y = x$ is a partial order.

Examples.

1.

Properties. (description)

Classtype	variety
Equational theory	
Quasiequational theory	
First-order theory	
Locally finite	
residual size	unbounded
Congruence distributive	no
Congruence modular	no
Congruence n-permutable	no
Congruence regular	no
Congruence uniform	no
Congruence types	semilattice (5)
Congruence extension property	
Definable principal congruences	
Equationally def. pr. cong.	no
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

Finite Members. $f(n) =$ number of members of size n.

$f(1) = \quad 1$
$f(2) =$
$f(3) =$
$f(4) =$
$f(5) =$
$f(6) =$
$f(7) =$

Subclasses.
Semilattices

Superclasses.
Groupoids

References

[1]