Distributive lattices edit

Abbreviation: DLat

Definition 1. A *distributive lattice* is a lattice $\mathbf{L} = \langle L, \vee, \wedge \rangle$ such that
$\wedge$ distributes over $\vee$: $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

Definition 2. A *distributive lattice* is a lattice $\mathbf{L} = \langle L, \vee, \wedge \rangle$ such that
$\vee$ distributes over $\wedge$: $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

Definition 3. A *distributive lattice* is a lattice $\mathbf{L} = \langle L, \vee, \wedge \rangle$ such that
$(x \wedge y) \vee (x \wedge z) \vee (y \wedge z) = (x \vee y) \wedge (x \vee z) \wedge (y \vee z)$

Definition 4. A *distributive lattice* is a lattice $\mathbf{L} = \langle L, \vee, \wedge \rangle$ such that $\mathbf{L}$ has no sublattice isomorphic to the diamond $\mathbf{M}_3$ or the pentagon $\mathbf{N}_5$

Morphisms. Let $\mathbf{L}$ and $\mathbf{M}$ be distributive lattices. A morphism from $\mathbf{L}$ to $\mathbf{M}$ is a function $h : L \to M$ that is a homomorphism:
$h(x \vee y) = h(x) \vee h(y)$, $h(x \wedge y) = h(x) \wedge h(y)$

Basic Results.

Examples.

1. $\langle P(S), \cup, \cap, \subseteq \rangle$, the collection of subsets of a sets S, ordered by inclusion.

Finite Members. $f(n) =$ number of members of size n.

Properties. (description)

Classtype	variety
Equational theory	decidable
Quasiequational theory	decidable
First-order theory	undecidable
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	no
Congruence regular	no
Congruence uniform	no
Congruence extension property	yes
Definable principal congruences	no
Equationally def. pr. cong.	yes, $\langle c,d\rangle \in \mathrm{Cg}(a,b) \iff (a\wedge b)\wedge c=(a\wedge b)\wedge d$ $(a\vee b)\vee c=(a\vee b)\vee d$
Amalgamation property	yes
Strong amalgamation property	no
Epimorphisms are surjective	no
Locally finite	yes
Residual size	2

$$
\begin{aligned}
f(1) &= 1\\
f(2) &= 1\\
f(3) &= 1\\
f(4) &= 2\\
f(5) &= 3\\
f(6) &= 5\\
f(7) &= 8\\
f(8) &= 15\\
f(9) &= 26\\
f(10) &= 47\\
f(11) &= 82\\
f(12) &= 151\\
f(13) &= 269\\
f(14) &= 494\\
f(15) &= 891\\
f(16) &= 1639\\
f(17) &= 2978\\
f(18) &= 5483\\
f(19) &= 10006\\
f(20) &= 18428
\end{aligned}
$$

Values known up to size 49 [1]

Subclasses.
One-element algebras
Bounded distributive lattices
Complete distributive lattices

Superclasses.
Modular lattices
Semidistributive lattices

References

[1] M. Erné, J. Heitzig, J. Reinhold, *On the number of distributive lattices*, Electronic J. Combinatorics 9 (2002), no. 1, Research Paper 24, 23 pp.