On Categorical Relationship among various Fuzzy Topological Systems, Fuzzy Topological Spaces and related Algebraic Structures

BLAST 2013

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- For each $\mathbb{A} - object$ A, a morphism $id_A : A \longrightarrow A$ called $\mathbb{A} - identity$ on A.

cont.

Definition

- A composition law associating with each $\mathbb{A} - morphism$ $f : A \longrightarrow B$ and each $\mathbb{A} - morphism\ g : B \longrightarrow C$ an A $- morphism\ g \circ f : A \longrightarrow C$, called the composite of f and g, subject to the following conditions

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- A composition law associating with each $\mathbb{A} - \textit{morphism}$ $f : A \longrightarrow B$ and each $\mathbb{A} - \textit{morphism}\ g : B \longrightarrow C$ an $\mathbb{A} - \textit{morphism}\ g \circ f : A \longrightarrow C$, called the composite of f and g, subject to the following conditions
- composition is associative.
- $\mathbb{A} - \textit{identities}$ act as identities with respect to composition.

Functors

Definition

If $\mathbb{A}$ and $\mathbb{B}$ are categories, then the functor F from $\mathbb{A}$ to $\mathbb{B}$ is a function that assigns to each $\mathbb{A} - object\ A$ a $\mathbb{B} - object\ F(A)$, and to each $\mathbb{A} - morphism\ f : A \longrightarrow A'$ a $\mathbb{B} - morphism$ $F(f) : F(A) \longrightarrow F(A')$ in such a way that

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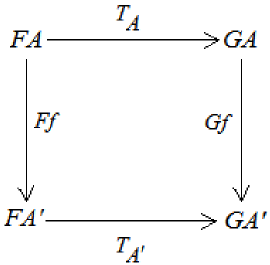
- $F-$ preserves compositions i.e. $F(f \circ g) = F(f) \circ F(g)$ whenever $f \circ g$ is defined and
- $F-$ preserve identity morphisms i.e. $F(id_A) = id_{F(A)}$ for each $\mathbb{A} - object$ A.

Natural Transformation

Definition

Let $F, G : \mathbb{A} \longrightarrow \mathbb{B}$ be functors. A natural transformation T from F to G (denoted by $T : F \longrightarrow G$) is a function that assigns to each $\mathbb{A} - object\ A$ a $\mathbb{B} - morphism\ T_A : FA \longrightarrow GA$ in such a way that the following naturality condition holds: for each $\mathbb{A} - morphism\ f : A \longrightarrow A'$, the square

cont.



commutes.

G-universal arrow

Definition

A G-structured arrow (g, A) with domain B is called G-universal for B provided that for each G-structured arrow (g', A') with domain B there exists a unique $\mathbb{A} - morphism$ $\hat{f} : A \longrightarrow A'$ with $g' = G(\hat{f}) \circ g$.

G-couniversal arrow

Definition

A G-costructured arrow (A, g) with codomain B is called G-couniversal for B provided that for each G-costructured arrow (A', g') with codomain B there exists a unique $\mathbb{A} -$ *morphism* $\hat{f} : A' \longrightarrow A$ with $g' = g \circ G(\hat{f})$.

Right Adjoint

Definition

A functor $G : \mathbb{A} \longrightarrow \mathbb{B}$ is said to be right adjoint provided that for every $\mathbb{B} - object\ B$ there exists a G-universal arrow with domain B.

Left Adjoint

Definition

A functor $G : \mathbb{A} \longrightarrow \mathbb{B}$ is said to be left adjoint provided that for every $\mathbb{B}-$ *object* B there exists a G-couniversal arrow with codomain B.

Top Sys

Topological System

A Topological system is a triple $(X, \models, A)$ where X is a set, A is a frame and $\models$, is a relation $\models \subseteq X \times A$, matches the logic of finite observations. Formally,

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- If S is any subset of A, then $x \models \bigvee S \Leftrightarrow x \models a$ for some $a \in S$.

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Continuous map

Let $D = (X, \models, A)$ and $E = (Y, \models', B)$ be topological systems. A continuous map $f : D \longrightarrow E$ is a pair (f_1, f_2) where,

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- $f_2 : B \longrightarrow A$ is a frame homomorphism and
- $x \models f_2(x)$ iff $f_1(x) \models' b$, for all $x \in X$ and $b \in B$.

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$$g_1 \circ f_1 : X \longrightarrow Z$$
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Top Sys

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Topological systems together with continuous maps form the category Top Sys.

Top

Top

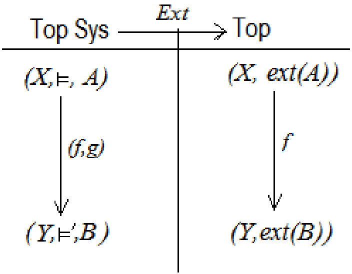
Topological spaces together with continuous maps form the category Top.

Frm

Frm

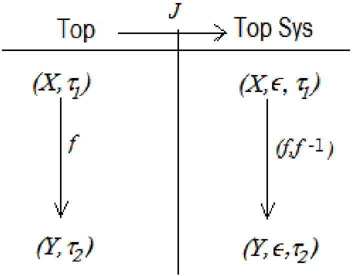
Frames together with frame homomorphisms form the category Frm.

Ext

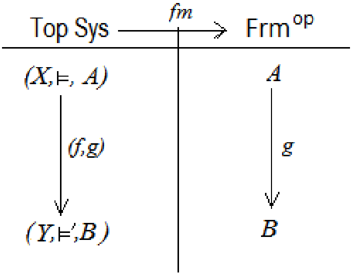


where $ext(a) = \{x \mid x \models a\}$ and $ext(A) = \{ext(a)\}_{a \in A}$.

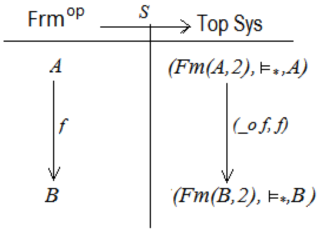
J



fm



S



where $Fm(A,2) = \{frame\ homomorphism : A \longrightarrow 2\}$ and $x \models_* a$ iff $x(a) = \top$.

Results

1. *Ext* is the right adjoint to the functor J.

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3. $Ext \circ S$ is the right adjoint to the functor $fm \circ J$.

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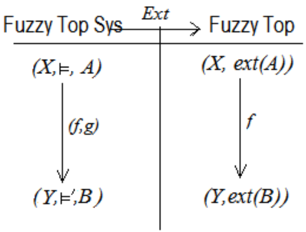

where $ext(a) : X \longrightarrow [0,1]$ s.t. $ext(a)(x) = gr(x \models a)$ and $ext(A) = \{ext(a)\}_{a \in A}$.

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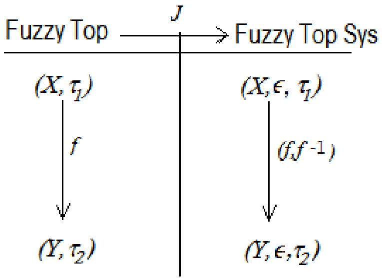

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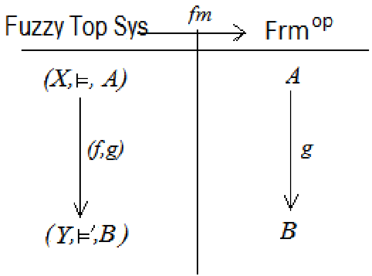

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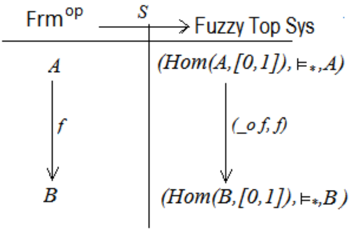

where $Hom(A,[0,1]) = \{frame\ homomorphism\ v : A \longrightarrow [0,1]\}$ and $gr(v \models_* a) = v(a)$.

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Definition of Fuzzy Topological System given by Apostolos and Paiva

A fuzzy topological system is an object (U, X, α) of $Dial_I(Set)$ such that X is a frame and α satisfies the following conditions:

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- $\alpha(u, \top) = 1$ and $\alpha(u, \bot) = 0$ for all $u \in U$.

$Ł_n^c$-algebra

An $Ł_n^c$-algebra is an MV_n algebra enriched by n constants. That is, it is an MV_n algebra $\mathcal{A} = (A, \wedge, \vee, *, \oplus, \rightarrow, ^{\perp}, 0, 1)$ in which the algebra $\bar{n}$ is embedded.

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$Ł_n^c$-homomorphism is a function between two $Ł_n^c$-algebras which preserves the operations.

$FBSy_n$

$\bar{n}$-fuzzy Boolean System

An $\bar{n}$-fuzzy Boolean System is a triple $(X, \models, A)$ where X is a set, A is an $Ł_n^c$-algebra and $\models$ is an $\bar{n}$ valued fuzzy relation from X to A such that

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1. $gr(x \models a * b) = max(0, gr(x \models a) + gr(x \models b) - 1)$

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3. $gr(x \models r) = r$ for all $r \in \bar{n}$

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Composition

Let $D = (X, \models', A)$, $E = (Y, \models'', B)$, $F = (Z, \models''', C)$. Let $(f_1, f_2) : D \longrightarrow E$ and $(g_1, g_2) : E \longrightarrow F$ be continuous maps.

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i.e $(g_1, g_2) \circ (f_1, f_2) = (g_1 \circ f_1, f_2 \circ g_2)$.

$\bar{n}$-fuzzy Boolean System

$FBSy_n$

$\bar{n}$-fuzzy Boolean Systems together with continuous functions forms the category $\bar{n}$-fuzzy Boolean System($FBSy_n$).

$\mathbf{FBS}_n$

$\bar{n}$-fuzzy Boolean Space

For an $\bar{n}$-fuzzy topological space (X, τ) is called an $\bar{n}$-fuzzy Boolean space iff (X, τ) is zero dimensional, compact and Kolmogorov.

FBS_n

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FBS_n

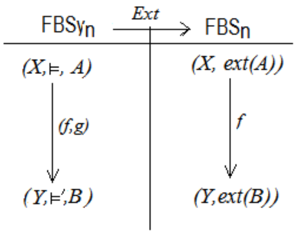
$\bar{n}$-fuzzy topological space (X, τ) together with continuous map forms the category FBS_n.

$Ł_n^c$-Alg

$Ł_n^c$-Alg

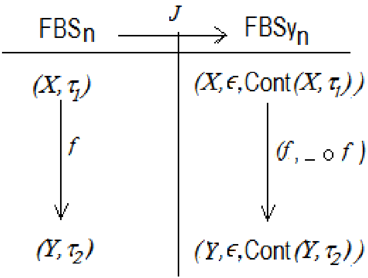
$Ł_n^c$-algebra together with $Ł_n^c$-Alg homomorphisms form the category $Ł_n^c$-Alg.

Ext

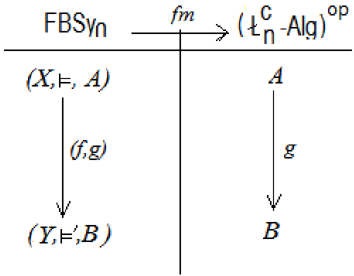


where $ext(a) : X \longrightarrow \bar{n}$ s.t. $ext(a)(x) = gr(x \models a)$ and $ext(A) = \{ext(a)\}_{a \in A}$.

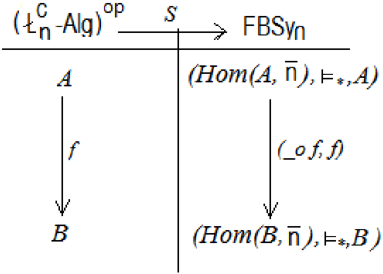
J



fm



S



where $Hom(A, \bar{n}) = \{Ł_n^c \ hom \ v : A \longrightarrow \bar{n}\}$ and $gr(v \models_* a) = v(a)$.

Results

1. *Ext* is the right adjoint to the functor J.

Results

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2. *fm* is the left adjoint to the functor S.

Results

1. *Ext* is the right adjoint to the functor J.
2. *fm* is the left adjoint to the functor S.
3. $Ext \circ S$ is the right adjoint to the functor $fm \circ J$.

Results

1. Ł$_n^c - Alg$ is dually equivalent to the category $FBSy_n$.

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1. $Ł_n^c - Alg$ is dually equivalent to the category $FBSy_n$.
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$\mathscr{F}$-Top Sys

$\mathscr{F}$-Topological System

A $\mathscr{F}$-topological system is a quadruple $(X, \tilde{A}, \models, P)$, where $(X, \tilde{A})$ is a non-empty fuzzy set, P is a frame and $\models$ is a $[0, 1]$- fuzzy relation from X to P such that

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1. $gr(x \models p) \in [0, 1]$

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2. $gr(x \models p) \leq \tilde{A}(x)$
3. if S is a finite subset of P, then $gr(x \models \bigwedge S) = inf\{gr(x \models s) : s \in S\}$
4. if S is any subset of P, then $gr(x \models \bigvee S) = sup\{gr(x \models s) : s \in S\}$

$\mathscr{F}$-Top Sys

Continuous map

Let $D = (X, \tilde{A}, \models, P)$ and $E = (Y, \tilde{B}, \models', Q)$ be $\mathscr{F}$-topological systems.

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Let $D = (X, \tilde{A}, \models, P)$ and $E = (Y, \tilde{B}, \models', Q)$ be $\mathscr{F}$-topological systems.A continuous map $f : D \longrightarrow E$ is a pair (f_1, f_2) where,

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1. $f_1 : (X, \tilde{A}) \longrightarrow (Y, \tilde{B})$ is a proper function from $(X, \tilde{A})$ to $(Y, \tilde{B})$.
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2. $f_2 : Q \longrightarrow P$ is a frame homomorphism and
3. $gr(x \models f_2(q)) = gr(f_1(x) \models' q)$, for all $x \in X$ and $q \in Q$.

$\mathscr{F}$-Top Sys

Identity map

Let $D = (X, \tilde{A}, \models, P)$ be a $\mathscr{F}$-topological system. The identity map $I_D : D \longrightarrow D$ is a pair (I_1, I_2) defined by

$\mathscr{F}$-Top Sys

Identity map

Let $D = (X, \tilde{A}, \models, P)$ be a $\mathscr{F}$-topological system. The identity map $I_D : D \longrightarrow D$ is a pair (I_1, I_2) defined by

$$I_1 : (X, \tilde{A}) \longrightarrow (X, \tilde{A}) \ s.t. \ I_1(x_1, x_2) = \tilde{A}(x) \ \textit{iff} \ x_1 = x_2$$
$$= 0 \qquad \textit{otherwise}$$

and $I_2 : P \longrightarrow P$ *is identity morphism of* P.

$\mathscr{F}$-Top Sys

Composition

Let $D = (X, \tilde{A}, \models', P)$, $E = (Y, \tilde{B}, \models'', Q)$, $F = (Z, \tilde{C}, \models''', R)$.
Let $(f_1, f_2) : D \longrightarrow E$ and $(g_1, g_2) : E \longrightarrow F$ be continuous maps.
The composition $(g_1, g_2) \circ (f_1, f_2) : D \longrightarrow F$ is defined by

$\mathscr{F}$-Top Sys

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$$g_1 \circ f_1 : (X, \tilde{A}) \longrightarrow (Z, \tilde{C})$$
$$f_2 \circ g_2 : R \longrightarrow P$$

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$$f_2 \circ g_2 : R \longrightarrow P$$

i.e. $(g_1, g_2) \circ (f_1, f_2) = (g_1 \circ f_1, f_2 \circ g_2)$.

$\mathscr{F}$-Top Sys

$\mathscr{F}$-Top Sys

$\mathscr{F}$-topological systems together with continuous maps form the category $\mathscr{F}$-Top Sys.

$\mathscr{F}$-Top

$\mathscr{F}$-Top

$\mathscr{F}$-topological spaces together with continuous maps form the category $\mathscr{F}$-Top.

Frm

Frm

Frames together with frame homomorphisms form the category Frm.

Ext

Let $(X, \tilde{A}, \models, P)$ be a $\mathscr{F}$-topological system and $p \in P$. For each p, its extent in $(X, \tilde{A}, \models, P)$ is given by $ext(p) = (X, ext^*(p))$ where $ext^*(p)$ is a mapping from X to $[0, 1]$ given by $ext^*(p)(x) = gr(x \models p)$ for all $x \in X$.

Ext

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i.e. $ext^*(p) : X \longrightarrow [0,1]$ such that $ext^*(p)(x) = gr(x \models p)$ for all $x \in X$.
Also $ext(P) = \{(X, ext^*(p))\}_{p \in P} = (X, ext^* P)$ where $ext^* P = \{ext^* p\}_{p \in P}$.

Ext

Ext is a (forgetful) functor from $\mathscr{F}$-Top Sys to $\mathscr{F}$-Top defined thus.
Ext acts on the object $(X, \tilde{A}, \models', P)$ as $Ext(X, \tilde{A}, \models', P) = (X, \tilde{A}, ext(P))$ and on the morphism (f_1, f_2) as $Ext(f_1, f_2) = f_1$.

J

J is a functor from $\mathscr{F}$-Top to $\mathscr{F}$-Top Sys defined thus. J acts on the object $(X, \tilde{A}, \tau)$ as $J(X, \tilde{A}, \tau) = (X, \tilde{A}, \in, \tau)$ where $gr(x \in \tilde{T}) = \tilde{T}(x)$ for $\tilde{T} \in \tau$ and on the morphism f as $J(f) = (f, f^{-1})$.

Loc

Loc is a functor from $\mathscr{F}$-Top Sys to Frm^{op} defined thus. *Loc* acts on the object $(X, \tilde{A}, \models, P)$ as $Loc(X, \tilde{A}, \models, P) = P$ and on the morphism (f_1, f_2) as $Loc(f_1, f_2) = f_2$.

S

S is a functor from Frm^{op} to $\mathscr{F}$-Top Sys defined thus. S acts on the object P as $S(P) = (Hom(P, [0,1]), \tilde{P}, \models_*, P)$, where $Hom(P, [0,1]) = \{frame\ hom\ v : P \longrightarrow [0,1]\}$, $gr(v \models_* p) = v(p)$ and $\tilde{P}(v) = \bigvee_{p \in P} v(p)$, and on the morphism f as $S(f) = (\_ \circ f, f)$.

Results

1. *Ext* is the right adjoint to the functor J.

Results

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2. *Loc* is the left adjoint to the functor S.

Results

1. *Ext* is the right adjoint to the functor J.
2. *Loc* is the left adjoint to the functor S.
3. $Ext \circ S$ is the right adjoint to the functor $Loc \circ J$.

Future Direction

- Finding duality in more general settings.

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- Exploring the properties of many valued geometric logic.

Future Direction

- Finding duality in more general settings.
- Introducing a notion of many valued geometric logic.
- Exploring the properties of many valued geometric logic.
- Introducing some notion of fuzzy topological systems to connect existing notion of fuzzy topological spaces(in more general settings) and finding the related algebraic structures.

References

1. George E. Strecker Adamek Jiri, Horst Herrlich; Abstract and Concrete Categories, John Wiley & Sons. ISBN 0-471-60922-6, 1990.

2. Apostolos Syropoulos and Valeria de Pavia; Fuzzy topological systems, 8th Panhellenic Logic Symposium, Ioannina, Greece, July, 2011.

3. M.K. Chakraborty and P. Jana; Some Topological Systems: their Categorical Relationship with Fuzzy Topological Spaces and related Algebras, Fuzzy Sets and Systems (submitted (2012)).

Return

Refs. cont.

4. Steven J. Vickers; Topology Via Logic, volume 5, Cambridge Tracts in Theoretical Computer Science University Press, 1989.

5. L.A. Zadeh; Fuzzy sets, Information and Control, 8, 1965, pp. 338–353.

6. Yoshihiro Maruyama, Fuzzy Topology and Łukasiewicz Logics from the Viewpoint of Duality Theory, Studia Logica, 94, 2010, pp. 245–269.

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Thank You