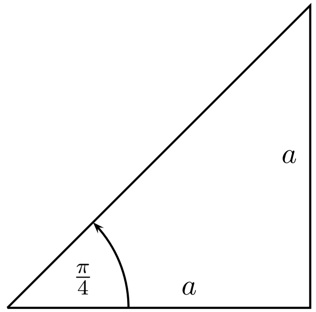
Some fascinating properties of π (check them out!)

Definition: In any circle the ratio $\frac{\text{circumference}}{\text{diameter}} = \frac{C}{d}$ is a constant called π



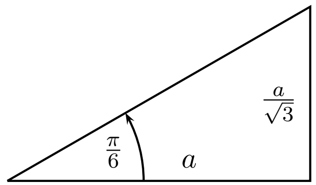
$\Longrightarrow \quad C = \pi d = 2\pi r$ where $r = \frac{d}{2}$ is the radius

From trigonometry $\tan\frac{\pi}{4} = 1 \quad \Longrightarrow \quad \frac{\pi}{4} = \tan^{-1}(1)$

From calculus $\tan^{-1}(x) = \int \frac{dx}{x^2+1} = \int 1 - x^2 + x^4 - x^6 + \cdots dx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

$\Longrightarrow \quad \pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \cdots$ (Gregory-Leibniz∼1675) $= 3.1415926535897932384626433832\ldots$

$\pi = 6\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = 2\sqrt{3}\left(1 - \frac{1}{3^1 3} + \frac{1}{3^2 5} - \frac{1}{3^3 7} + \frac{1}{3^4 9} - \cdots\right)$ (Sharp 1699)



Area of a circle of radius r is $4\int_0^r \sqrt{r^2 - x^2}dx = \ldots = \pi r^2$ (substitute $x = r\sin\theta$)

Volume of a sphere of radius r is $2\int_0^r \pi(r^2 - x^2)\,dx = \ldots = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$ (easy)

$e^{i\pi} + 1 = 0$ (Euler 1735) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{49} + \frac{1}{64} + \cdots = \frac{\pi^2}{6}$ (try it)