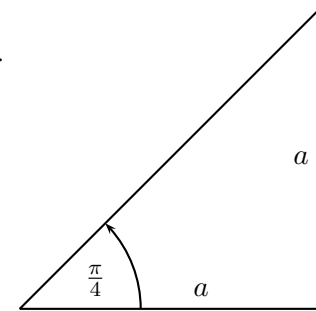


Some fascinating properties of π (check them out!)

Definition: In **any** circle the ratio $\frac{\text{circumference}}{\text{diameter}} = \frac{C}{d}$ is a **constant** called π

$$\implies C = \pi d = 2\pi r \text{ where } r = \frac{d}{2} \text{ is the radius}$$

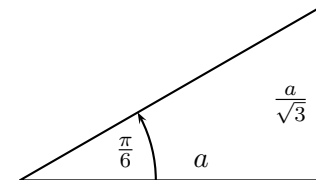
From **trigonometry** $\tan \frac{\pi}{4} = 1 \implies \frac{\pi}{4} = \tan^{-1}(1)$



From **calculus** $\tan^{-1}(x) = \int \frac{dx}{x^2+1} = \int 1 - x^2 + x^4 - x^6 + \dots dx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

$$\implies \pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \dots \text{ (Gregory-Leibniz ~1675)} = 3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679821480865132823066470938446095505822317253594081281361180964876169246206710566350220616896320306737631164132220666133013068201229714635531147991406895501407912664478203126732654404781066595964607042546556524417067252587232400021097700493796226210974$$

$$\pi = 6 \tan^{-1} \left(\frac{\sqrt{3}}{3} \right) = 2\sqrt{3} \left(1 - \frac{1}{3^{13}} + \frac{1}{3^{25}} - \frac{1}{3^{37}} + \frac{1}{3^{49}} - \dots \right) \text{ (Sharp 1699)}$$



Area of a circle of radius r is $4 \int_0^r \sqrt{r^2 - x^2} dx = \dots = \pi r^2$ (substitute $x = r \sin \theta$)

Volume of a sphere of radius r is $2 \int_0^r \pi(r^2 - x^2) dx = \dots = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$ (easy)

$$e^{i\pi} + 1 = 0 \text{ (Euler 1735)} \quad 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{49} + \frac{1}{64} + \dots = \frac{\pi^2}{6} \text{ (try it)}$$