

ANA axioms

ANA001-0.ax Analysis (limits) axioms for continuous functions

$x + n_0 = x$ cnf(right_identity, axiom)
 $n_0 + x = x$ cnf(left_identity, axiom)
 $\neg x < x$ cnf(reflexivity_of_less_than, axiom)
 $(x < y \text{ and } y < z) \Rightarrow x < z$ cnf(transitivity_of_less_than, axiom)
 $(n_0 < x \text{ and } n_0 < y) \Rightarrow n_0 < \min(x, y)$ cnf(axiom_21, axiom)
 $(n_0 < x \text{ and } n_0 < y) \Rightarrow \min(x, y) < x$ cnf(axiom_22, axiom)
 $(n_0 < x \text{ and } n_0 < y) \Rightarrow \min(x, y) < y$ cnf(axiom_23, axiom)
 $(x < \text{half}(xa) \text{ and } y < \text{half}(xa)) \Rightarrow x + y < xa$ cnf(axiom_3, axiom)
 $|x| + |y| < xa \Rightarrow |x + y| < xa$ cnf(c17, axiom)
 $(x + y) + z = x + (y + z)$ cnf(axiom_5, axiom)
 $x + y = y + x$ cnf(axiom_61, axiom)
 $n_0 < xa \Rightarrow n_0 < \text{half}(xa)$ cnf(axiom_62, axiom)
 $n_0 < xa \Rightarrow n_0 < \text{half}(xa)$ cnf(axiom_7, axiom)
 $-(x + y) = -x - y$ cnf(axiom_8, axiom)

ANA002-0.ax Analysis (limits) axioms for continuous functions

$|x + y| \leq |x| + |y|$ cnf(absolute_sum_less_or_equal_sum_of_absolutes_1, axiom)
 $|x| + |y| \leq z \Rightarrow |x + y| \leq z$ cnf(absolute_sum_less_or_equal_sum_of_absolutes_2, axiom)
 $x \leq y \Rightarrow \min(x, y) = x$ cnf(minimum_1, axiom)
 $\min(x, y) \leq x$ cnf(minimum_2, axiom)
 $z \leq \min(x, y) \Rightarrow z \leq x$ cnf(minimum_3, axiom)
 $x \leq y \Rightarrow x \leq \min(x, y)$ cnf(minimum_4, axiom)
 $y \leq x \Rightarrow \min(x, y) = y$ cnf(minimum_5, axiom)
 $\min(x, y) \leq y$ cnf(minimum_6, axiom)
 $z \leq \min(x, y) \Rightarrow z \leq y$ cnf(minimum_7, axiom)
 $y \leq x \Rightarrow y \leq \min(x, y)$ cnf(minimum_8, axiom)
 $\min(x, y) \leq n_0 \Rightarrow (x \leq n_0 \text{ or } y \leq n_0)$ cnf(minimum_9, axiom)
 $\text{half}(x) + \text{half}(x) = x$ cnf(half_plus_half_is_whole, axiom)
 $\text{half}(x) + \text{half}(x) \leq x$ cnf(half_plus_half_less_or_equal_whole, axiom)
 $x \leq \text{half}(x) + \text{half}(x)$ cnf(whole_less_or_equal_half_plus_half, axiom)
 $(x \leq \text{half}(z) \text{ and } y \leq \text{half}(z)) \Rightarrow x + y \leq z$ cnf(less_or_equal_sum_of_halves, axiom)
 $\text{half}(x) \leq n_0 \Rightarrow x \leq n_0$ cnf(zero_and_half, axiom)
 $x \leq y \Rightarrow x + z \leq y + z$ cnf(add_to_both_sides_of_less_equal_1, axiom)
 $(x \leq y \text{ and } z \leq w) \Rightarrow x + z \leq y + w$ cnf(add_to_both_sides_of_less_equal_2, axiom)
 $x \leq y \text{ or } y \leq x$ cnf(commutativity_of_less_or_equal, axiom)
 $(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z$ cnf(transitivity_of_less_or_equal, axiom)
 $x + y = y + x$ cnf(commutativity_of_add, axiom)
 $x + y \leq y + x$ cnf(commutativity_of_add_for_less_or_equal, axiom)
 $(x + y) + z = x + (y + z)$ cnf(associativity_of_add, axiom)
 $(x + y) + z \leq x + (y + z)$ cnf(associativity_of_add_for_less_or_equal_1, axiom)
 $x + (y + z) \leq (x + y) + z$ cnf(associativity_of_add_for_less_or_equal_2, axiom)
 $x = y \Rightarrow x \leq y$ cnf(equal_implies_less_or_equal, axiom)

ANA problems

ANA001-1.p Attaining minimum (or maximum) value

A continuous function f in a closed real interval $[a, b]$ attains its minimum (or maximum) in this interval.

$x \leq x$ cnf(refelxivity, axiom)
 $x \leq y \text{ or } y \leq x$ cnf(totality, axiom)
 $(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z$ cnf(transitivity, axiom)
 $(x \leq y \text{ and } y \leq x) \Rightarrow f(x) \leq f(y)$ cnf(function, hypothesis)
 $(\text{lower} \leq x \text{ and } x \leq \text{upper}) \Rightarrow x \in [\text{lower}, \text{upper}]$ cnf(in_interval, hypothesis)
 $a \leq \text{extreme_point}$ cnf(interval_1, hypothesis)
 $\text{extreme_point} \leq b$ cnf(interval_2, hypothesis)
 $x \in [a, \text{extreme_point}] \Rightarrow f(\text{extreme_point}) \leq f(x)$ cnf(below_extreme_point, hypothesis)
 $x \in [a, b] \Rightarrow (x \leq \text{extreme_point} \text{ or } a \leq q(x))$ cnf(q_function_1, hypothesis)

$(x \in [a, b] \text{ and } f(x) \leq q(x)) \Rightarrow x \leq \text{extreme_point} \quad \text{cnf}(q_function_2, \text{hypothesis})$
 $x \in [a, b] \Rightarrow (x \leq \text{extreme_point} \text{ or } q(x) \leq x) \quad \text{cnf}(q_function_3, \text{hypothesis})$
 $x \in [a, b] \Rightarrow a \leq h(x) \quad \text{cnf}(h_function_1, \text{hypothesis})$
 $x \in [a, b] \Rightarrow h(x) \leq b \quad \text{cnf}(h_function_2, \text{hypothesis})$
 $x \in [a, b] \Rightarrow f(h(x)) \leq f(x) \quad \text{cnf}(h_function_3, \text{hypothesis})$
 $(x \in [a, b] \text{ and } y \in [a, b] \text{ and } f(y) \leq f(x)) \Rightarrow h(x) \leq y \quad \text{cnf}(h_function_4, \text{hypothesis})$
 $x \in [a, b] \Rightarrow a \leq k(x) \quad \text{cnf}(k_function_1, \text{hypothesis})$
 $x \in [a, b] \Rightarrow k(x) \leq b \quad \text{cnf}(k_function_2, \text{hypothesis})$
 $f(x) \leq f(k(x)) \Rightarrow \neg x \in [a, b] \quad \text{cnf}(\text{prove_something}, \text{negated_conjecture})$

ANA002-1.p Intermediate value theorem

If a function f is continuous in a real closed interval $[a, b]$, where $f(a) \leq 0$ and $0 \leq f(b)$, then there exists X such that $f(X) = 0$.

$(\text{lower} \leq x \text{ and } x \leq \text{upper}) \Rightarrow x \in [\text{lower}, \text{upper}] \quad \text{cnf}(\text{in_interval}, \text{axiom})$
 $x \leq x \quad \text{cnf}(\text{reflexivity_of_less}, \text{axiom})$
 $x \leq y \text{ or } y \leq x \quad \text{cnf}(\text{totality_of_less}, \text{axiom})$
 $(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z \quad \text{cnf}(\text{transitivity_of_less}, \text{axiom})$
 $x \leq q(y, x) \Rightarrow x \leq y \quad \text{cnf}(\text{interpolation}_1, \text{axiom})$
 $q(x, y) \leq x \Rightarrow y \leq x \quad \text{cnf}(\text{interpolation}_2, \text{axiom})$
 $(x \leq h(x) \text{ and } x \in [a, b]) \Rightarrow f(x) \leq n_0 \quad \text{cnf}(\text{continuity}_1, \text{axiom})$
 $(y \leq x \text{ and } f(y) \leq n_0 \text{ and } x \in [a, b]) \Rightarrow (f(x) \leq n_0 \text{ or } y \leq h(x)) \quad \text{cnf}(\text{continuity}_2, \text{axiom})$
 $(k(x) \leq x \text{ and } x \in [a, b]) \Rightarrow n_0 \leq f(x) \quad \text{cnf}(\text{continuity}_3, \text{axiom})$
 $(x \leq y \text{ and } n_0 \leq f(y) \text{ and } x \in [a, b]) \Rightarrow (n_0 \leq f(x) \text{ or } k(x) \leq y) \quad \text{cnf}(\text{continuity}_4, \text{axiom})$
 $(x \leq b \text{ and } f(x) \leq n_0) \Rightarrow x \leq l \quad \text{cnf}(\text{crossover}_1, \text{axiom})$
 $g(x) \leq b \text{ or } l \leq x \quad \text{cnf}(\text{crossover}_2 \text{ and } g_function_1, \text{axiom})$
 $f(g(x)) \leq n_0 \text{ or } l \leq x \quad \text{cnf}(\text{crossover}_3 \text{ and } g_function_2, \text{axiom})$
 $g(x) \leq x \Rightarrow l \leq x \quad \text{cnf}(\text{crossover}_4 \text{ and } g_function_3, \text{axiom})$
 $a \leq b \quad \text{cnf}(\text{the_interval}, \text{hypothesis})$
 $f(a) \leq n_0 \quad \text{cnf}(\text{lower_mapping}, \text{hypothesis})$
 $n_0 \leq f(b) \quad \text{cnf}(\text{upper_mapping}, \text{hypothesis})$
 $\neg n_0 \in [f(x), f(x)] \quad \text{cnf}(\text{prove_there_is_x_which_crosses}, \text{negated_conjecture})$

ANA002-2.p Intermediate value theorem

If a function f is continuous in a real closed interval $[a, b]$, where $f(a) \leq 0$ and $0 \leq f(b)$, then there exists X such that $f(X) = 0$.

$(\text{lower} \leq x \text{ and } x \leq \text{upper}) \Rightarrow x \in [\text{lower}, \text{upper}] \quad \text{cnf}(\text{in_interval}, \text{axiom})$
 $x \leq x \quad \text{cnf}(\text{reflexivity_of_less}, \text{axiom})$
 $x \leq y \text{ or } y \leq x \quad \text{cnf}(\text{totality_of_less}, \text{axiom})$
 $(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z \quad \text{cnf}(\text{transitivity_of_less}, \text{axiom})$
 $x \leq q(y, x) \Rightarrow x \leq y \quad \text{cnf}(\text{interpolation}_1, \text{axiom})$
 $q(x, y) \leq x \Rightarrow y \leq x \quad \text{cnf}(\text{interpolation}_2, \text{axiom})$
 $(x \leq h(x) \text{ and } x \in [a, b]) \Rightarrow f(x) \leq n_0 \quad \text{cnf}(\text{continuity}_1, \text{axiom})$
 $(y \leq x \text{ and } f(y) \leq n_0 \text{ and } x \in [a, b]) \Rightarrow (f(x) \leq n_0 \text{ or } y \leq h(x)) \quad \text{cnf}(\text{continuity}_2, \text{axiom})$
 $(k(x) \leq x \text{ and } x \in [a, b]) \Rightarrow n_0 \leq f(x) \quad \text{cnf}(\text{continuity}_3, \text{axiom})$
 $(x \leq y \text{ and } n_0 \leq f(y) \text{ and } x \in [a, b]) \Rightarrow (n_0 \leq f(x) \text{ or } k(x) \leq y) \quad \text{cnf}(\text{continuity}_4, \text{axiom})$
 $(x \leq b \text{ and } f(x) \leq n_0) \Rightarrow x \leq l \quad \text{cnf}(\text{crossover}_1, \text{axiom})$
 $g(x) \leq b \text{ or } l \leq x \quad \text{cnf}(\text{crossover}_2 \text{ and } g_function_1, \text{axiom})$
 $f(g(x)) \leq n_0 \text{ or } l \leq x \quad \text{cnf}(\text{crossover}_3 \text{ and } g_function_2, \text{axiom})$
 $g(x) \leq x \Rightarrow l \leq x \quad \text{cnf}(\text{crossover}_4 \text{ and } g_function_3, \text{axiom})$
 $a \leq b \quad \text{cnf}(\text{the_interval}, \text{hypothesis})$
 $f(a) \leq n_0 \quad \text{cnf}(\text{lower_mapping}, \text{hypothesis})$
 $n_0 \leq f(b) \quad \text{cnf}(\text{upper_mapping}, \text{hypothesis})$
 $\neg n_0 \in [f(l), f(l)] \quad \text{cnf}(\text{prove_there_is_x_which_crosses}, \text{negated_conjecture})$

ANA002-3.p Intermediate value theorem

If a function f is continuous in a real closed interval $[a, b]$, where $f(a) \leq 0$ and $0 \leq f(b)$, then there exists X such that $f(X) = 0$.

$x \leq x \quad \text{cnf}(\text{reflexivity_of_less}, \text{axiom})$
 $x \leq y \text{ or } y \leq x \quad \text{cnf}(\text{totality_of_less}, \text{axiom})$
 $(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z \quad \text{cnf}(\text{transitivity_of_less}, \text{axiom})$
 $x \leq q(y, x) \Rightarrow x \leq y \quad \text{cnf}(\text{interpolation}_1, \text{axiom})$

$q(x, y) \leq x \Rightarrow y \leq x$ cnf(interpolation₂, axiom)
 $(x \leq h(x) \text{ and } a \leq x \text{ and } x \leq b) \Rightarrow f(x) \leq n_0$ cnf(continuity₁, axiom)
 $(y \leq x \text{ and } f(y) \leq n_0 \text{ and } a \leq x \text{ and } x \leq b) \Rightarrow (f(x) \leq n_0 \text{ or } y \leq h(x))$ cnf(continuity₂, axiom)
 $(k(x) \leq x \text{ and } a \leq x \text{ and } x \leq b) \Rightarrow n_0 \leq f(x)$ cnf(continuity₃, axiom)
 $(x \leq y \text{ and } n_0 \leq f(y) \text{ and } a \leq x \text{ and } x \leq b) \Rightarrow (n_0 \leq f(x) \text{ or } k(x) \leq y)$ cnf(continuity₄, axiom)
 $(x \leq b \text{ and } f(x) \leq n_0) \Rightarrow x \leq l$ cnf(crossover₁, axiom)
 $g(x) \leq b \text{ or } l \leq x$ cnf(crossover2_and_g_function₁, axiom)
 $f(g(x)) \leq n_0 \text{ or } l \leq x$ cnf(crossover3_and_g_function₂, axiom)
 $g(x) \leq x \Rightarrow l \leq x$ cnf(crossover4_and_g_function₃, axiom)
 $a \leq b$ cnf(the_interval, hypothesis)
 $f(a) \leq n_0$ cnf(lower_mapping, hypothesis)
 $n_0 \leq f(b)$ cnf(upper_mapping, hypothesis)
 $f(x) \leq n_0 \Rightarrow \neg n_0 \leq f(x)$ cnf(prove_there_is_x_which_crosses, negated_conjecture)

ANA002-4.p Intermediate value theorem

If a function f is continuous in a real closed interval $[a, b]$, where $f(a) \leq 0$ and $0 \leq f(b)$, then there exists X such that $f(X) = 0$.

$x \leq x$ cnf(reflexivity_of_less, axiom)
 $x \leq y \text{ or } y \leq x$ cnf(totality_of_less, axiom)
 $(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z$ cnf(transitivity_of_less, axiom)
 $x \leq q(y, x) \Rightarrow x \leq y$ cnf(interpolation₁, axiom)
 $q(x, y) \leq x \Rightarrow y \leq x$ cnf(interpolation₂, axiom)
 $(x \leq h(x) \text{ and } a \leq x \text{ and } x \leq b) \Rightarrow f(x) \leq n_0$ cnf(continuity₁, axiom)
 $(y \leq x \text{ and } f(y) \leq n_0 \text{ and } a \leq x \text{ and } x \leq b) \Rightarrow (f(x) \leq n_0 \text{ or } y \leq h(x))$ cnf(continuity₂, axiom)
 $(k(x) \leq x \text{ and } a \leq x \text{ and } x \leq b) \Rightarrow n_0 \leq f(x)$ cnf(continuity₃, axiom)
 $(x \leq y \text{ and } n_0 \leq f(y) \text{ and } a \leq x \text{ and } x \leq b) \Rightarrow (n_0 \leq f(x) \text{ or } k(x) \leq y)$ cnf(continuity₄, axiom)
 $(x \leq b \text{ and } f(x) \leq n_0) \Rightarrow x \leq l$ cnf(crossover₁, axiom)
 $g(x) \leq b \text{ or } l \leq x$ cnf(crossover2_and_g_function₁, axiom)
 $f(g(x)) \leq n_0 \text{ or } l \leq x$ cnf(crossover3_and_g_function₂, axiom)
 $g(x) \leq x \Rightarrow l \leq x$ cnf(crossover4_and_g_function₃, axiom)
 $a \leq b$ cnf(the_interval, hypothesis)
 $f(a) \leq n_0$ cnf(lower_mapping, hypothesis)
 $n_0 \leq f(b)$ cnf(upper_mapping, hypothesis)
 $f(l) \leq n_0 \Rightarrow \neg n_0 \leq f(l)$ cnf(prove_there_is_x_which_crosses, negated_conjecture)

ANA003-1.p Lemma 1 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

include('Axioms/ANA001-0.ax')

$n_0 < x \Rightarrow n_0 < fp_{31}(x)$ cnf(c₁₀, negated_conjecture)
 $(n_0 < x \text{ and } |y + -a| < fp_{31}(x)) \Rightarrow |f(y) + -l_1| < x$ cnf(c₁₁, negated_conjecture)
 $n_0 < x \Rightarrow n_0 < fp_{32}(x)$ cnf(c₁₂, negated_conjecture)
 $(n_0 < x \text{ and } |y + -a| < fp_{32}(x)) \Rightarrow |g(y) + -l_2| < x$ cnf(c₁₃, negated_conjecture)
 $n_0 < b$ cnf(c₁₄, negated_conjecture)
 $n_0 < x \Rightarrow |fp_{33}(x) + -a| < x$ cnf(c₁₅, negated_conjecture)
 $n_0 < x \Rightarrow \neg |f(fp_{33}(x)) + -l_1| + |g(fp_{33}(x)) + -l_2| < b$ cnf(c₁₆, negated_conjecture)

ANA003-2.p Lemma 1 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

$x + n_0 = x$ cnf(right_identity, axiom)
 $n_0 + x = x$ cnf(left_identity, axiom)
 $\neg x < x$ cnf(reflexivity_of_less_than, axiom)
 $(x < y \text{ and } y < z) \Rightarrow x < z$ cnf(transitivity_of_less_than, axiom)
 $(n_0 < x \text{ and } n_0 < y) \Rightarrow n_0 < \min(x, y)$ cnf(axiom_2₁, axiom)
 $(n_0 < x \text{ and } n_0 < y) \Rightarrow \min(x, y) < x$ cnf(axiom_2₂, axiom)
 $(n_0 < x \text{ and } n_0 < y) \Rightarrow \min(x, y) < y$ cnf(axiom_2₃, axiom)
 $(x < \text{half}(xa) \text{ and } y < \text{half}(xa)) \Rightarrow x + y < xa$ cnf(axiom_3, axiom)
 $|x| + |y| < xa \Rightarrow |x + y| < xa$ cnf(c₁₇, axiom)
 $n_0 < xa \Rightarrow n_0 < \text{half}(xa)$ cnf(axiom_7, axiom)
 $n_0 < x \Rightarrow n_0 < fp_{31}(x)$ cnf(c₁₀, negated_conjecture)
 $(n_0 < x \text{ and } |y + -a| < fp_{31}(x)) \Rightarrow |f(y) + -l_1| < x$ cnf(c₁₁, negated_conjecture)
 $n_0 < x \Rightarrow n_0 < fp_{32}(x)$ cnf(c₁₂, negated_conjecture)

$(n_0 < x \text{ and } |y + -a| < \text{fp}_{32}(x)) \Rightarrow |g(y) + -l_2| < x \quad \text{cnf}(c_{13}, \text{negated_conjecture})$
 $n_0 < b \quad \text{cnf}(c_{14}, \text{negated_conjecture})$
 $n_0 < x \Rightarrow |\text{fp}_{33}(x) + -a| < x \quad \text{cnf}(c_{15}, \text{negated_conjecture})$
 $n_0 < x \Rightarrow \neg |f(\text{fp}_{33}(x)) + -l_1| + |g(\text{fp}_{33}(x)) + -l_2| < b \quad \text{cnf}(c_{16}, \text{negated_conjecture})$

ANA003-3.p Lemma 1 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

include('Axioms/ANA002-0.ax')

$\text{delta}_1(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0 \quad \text{cnf}(\text{clause}_1, \text{hypothesis})$

$\text{delta}_2(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0 \quad \text{cnf}(\text{clause}_2, \text{hypothesis})$

$|z + -a_real_number| \leq \text{delta}_1(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |f(z) + -f(a_real_number)| \leq \varepsilon) \quad \text{cnf}(\text{clause}_3, \text{hypothesis})$

$|z + -a_real_number| \leq \text{delta}_2(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |g(z) + -g(a_real_number)| \leq \varepsilon) \quad \text{cnf}(\text{clause}_4, \text{hypothesis})$

$\neg \text{epsilon}_0 \leq n_0 \quad \text{cnf}(\text{clause}_5, \text{hypothesis})$

$\delta \leq n_0 \text{ or } |xs(\delta) + -a_real_number| \leq \delta \quad \text{cnf}(\text{clause}_6, \text{hypothesis})$

$|f(xs(\delta)) + -f(a_real_number)| + |g(xs(\delta)) + -g(a_real_number)| \leq \text{epsilon}_0 \Rightarrow \delta \leq n_0 \quad \text{cnf}(\text{clause_7}_2, \text{negated_conjecture})$

ANA003-4.p Lemma 1 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

$z \leq \min(x, y) \Rightarrow z \leq x \quad \text{cnf}(\text{minimum}_3, \text{axiom})$

$z \leq \min(x, y) \Rightarrow z \leq y \quad \text{cnf}(\text{minimum}_7, \text{axiom})$

$\min(x, y) \leq n_0 \Rightarrow (x \leq n_0 \text{ or } y \leq n_0) \quad \text{cnf}(\text{minimum}_9, \text{axiom})$

$(x \leq \text{half}(z) \text{ and } y \leq \text{half}(z)) \Rightarrow x + y \leq z \quad \text{cnf}(\text{less_or_equal_sum_of_halves}, \text{axiom})$

$\text{half}(x) \leq n_0 \Rightarrow x \leq n_0 \quad \text{cnf}(\text{zero_and_half}, \text{axiom})$

$\text{delta}_1(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0 \quad \text{cnf}(\text{clause}_1, \text{hypothesis})$

$\text{delta}_2(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0 \quad \text{cnf}(\text{clause}_2, \text{hypothesis})$

$|z + -a_real_number| \leq \text{delta}_1(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |f(z) + -f(a_real_number)| \leq \varepsilon) \quad \text{cnf}(\text{clause}_3, \text{hypothesis})$

$|z + -a_real_number| \leq \text{delta}_2(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |g(z) + -g(a_real_number)| \leq \varepsilon) \quad \text{cnf}(\text{clause}_4, \text{hypothesis})$

$\neg \text{epsilon}_0 \leq n_0 \quad \text{cnf}(\text{clause}_5, \text{hypothesis})$

$\delta \leq n_0 \text{ or } |xs(\delta) + -a_real_number| \leq \delta \quad \text{cnf}(\text{clause}_6, \text{hypothesis})$

$|f(xs(\delta)) + -f(a_real_number)| + |g(xs(\delta)) + -g(a_real_number)| \leq \text{epsilon}_0 \Rightarrow \delta \leq n_0 \quad \text{cnf}(\text{clause_7}_2, \text{negated_conjecture})$

ANA004-1.p Lemma 2 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

include('Axioms/ANA001-0.ax')

$n_0 < x \Rightarrow n_0 < \text{fp}_{31}(x) \quad \text{cnf}(c_{10}, \text{negated_conjecture})$

$(n_0 < x \text{ and } |y + -a| < \text{fp}_{31}(x)) \Rightarrow |f(y) + -l_1| < x \quad \text{cnf}(c_{11}, \text{negated_conjecture})$

$n_0 < x \Rightarrow n_0 < \text{fp}_{32}(x) \quad \text{cnf}(c_{12}, \text{negated_conjecture})$

$(n_0 < x \text{ and } |y + -a| < \text{fp}_{32}(x)) \Rightarrow |g(y) + -l_2| < x \quad \text{cnf}(c_{13}, \text{negated_conjecture})$

$n_0 < b \quad \text{cnf}(c_{14}, \text{negated_conjecture})$

$n_0 < x \Rightarrow |\text{fp}_{33}(x) + -a| < x \quad \text{cnf}(c_{15}, \text{negated_conjecture})$

$n_0 < x \Rightarrow \neg |(f(\text{fp}_{33}(x)) + -l_1) + (g(\text{fp}_{33}(x)) + -l_2)| < b \quad \text{cnf}(c_{16}, \text{negated_conjecture})$

ANA004-2.p Lemma 2 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

$x + n_0 = x \quad \text{cnf}(\text{right_identity}, \text{axiom})$

$n_0 + x = x \quad \text{cnf}(\text{left_identity}, \text{axiom})$

$\neg x < x \quad \text{cnf}(\text{reflexivity_of_less_than}, \text{axiom})$

$(x < y \text{ and } y < z) \Rightarrow x < z \quad \text{cnf}(\text{transitivity_of_less_than}, \text{axiom})$

$(n_0 < x \text{ and } n_0 < y) \Rightarrow n_0 < \min(x, y) \quad \text{cnf}(\text{axiom_2}_1, \text{axiom})$

$(n_0 < x \text{ and } n_0 < y) \Rightarrow \min(x, y) < x \quad \text{cnf}(\text{axiom_2}_2, \text{axiom})$

$(n_0 < x \text{ and } n_0 < y) \Rightarrow \min(x, y) < y \quad \text{cnf}(\text{axiom_2}_3, \text{axiom})$

$(x < \text{half}(xa) \text{ and } y < \text{half}(xa)) \Rightarrow x + y < xa \quad \text{cnf}(\text{axiom}_3, \text{axiom})$

$n_0 < xa \Rightarrow n_0 < \text{half}(xa) \quad \text{cnf}(\text{axiom}_7, \text{axiom})$

$n_0 < x \Rightarrow n_0 < \text{fp}_{31}(x) \quad \text{cnf}(c_{10}, \text{negated_conjecture})$

$(n_0 < x \text{ and } |y + -a| < \text{fp}_{31}(x)) \Rightarrow |f(y) + -l_1| < x \quad \text{cnf}(c_{11}, \text{negated_conjecture})$

$n_0 < x \Rightarrow n_0 < \text{fp}_{32}(x) \quad \text{cnf}(c_{12}, \text{negated_conjecture})$

$(n_0 < x \text{ and } |y + -a| < \text{fp}_{32}(x)) \Rightarrow |g(y) + -l_2| < x \quad \text{cnf}(c_{13}, \text{negated_conjecture})$

$n_0 < b \quad \text{cnf}(c_{14}, \text{negated_conjecture})$

$n_0 < x \Rightarrow |\text{fp}_{33}(x) + -a| < x \quad \text{cnf}(c_{15}, \text{negated_conjecture})$

$n_0 < x \Rightarrow \neg |(f(\text{fp}_{33}(x)) + -l_1) + (g(\text{fp}_{33}(x)) + -l_2)| < b \quad \text{cnf}(c_{16}, \text{negated_conjecture})$

ANA004-3.p Lemma 2 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

include('Axioms/ANA002-0.ax')

$\text{delta}_1(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0$ cnf(clause₁, hypothesis)

$\text{delta}_2(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0$ cnf(clause₂, hypothesis)

$|z + -\text{a_real_number}| \leq \text{delta}_1(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |f(z) + -f(\text{a_real_number})| \leq \varepsilon)$ cnf(clause₃, hypothesis)

$|z + -\text{a_real_number}| \leq \text{delta}_2(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |g(z) + -g(\text{a_real_number})| \leq \varepsilon)$ cnf(clause₄, hypothesis)

$-\text{epsilon}_0 \leq n_0$ cnf(clause₅, hypothesis)

$\delta \leq n_0 \text{ or } |\text{xs}(\delta) + -\text{a_real_number}| \leq \delta$ cnf(clause₆, hypothesis)

$|(f(\text{xs}(\delta)) + -f(\text{a_real_number})) + (g(\text{xs}(\delta)) + -g(\text{a_real_number}))| \leq \text{epsilon}_0 \Rightarrow \delta \leq n_0$ cnf(clause_7₁, negated_conjecture)

ANA004-4.p Lemma 2 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

$|x| + |y| \leq z \Rightarrow |x + y| \leq z$ cnf(absolute_sum_less_or_equal_sum_of_absolutes₂, axiom)

$z \leq \min(x, y) \Rightarrow z \leq x$ cnf(minimum₃, axiom)

$z \leq \min(x, y) \Rightarrow z \leq y$ cnf(minimum₇, axiom)

$\min(x, y) \leq n_0 \Rightarrow (x \leq n_0 \text{ or } y \leq n_0)$ cnf(minimum₉, axiom)

$(x \leq \text{half}(z) \text{ and } y \leq \text{half}(z)) \Rightarrow x + y \leq z$ cnf(less_or_equal_sum_of_halves, axiom)

$\text{half}(x) \leq n_0 \Rightarrow x \leq n_0$ cnf(zero_and_half, axiom)

$\text{delta}_1(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0$ cnf(clause₁, hypothesis)

$\text{delta}_2(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0$ cnf(clause₂, hypothesis)

$|z + -\text{a_real_number}| \leq \text{delta}_1(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |f(z) + -f(\text{a_real_number})| \leq \varepsilon)$ cnf(clause₃, hypothesis)

$|z + -\text{a_real_number}| \leq \text{delta}_2(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |g(z) + -g(\text{a_real_number})| \leq \varepsilon)$ cnf(clause₄, hypothesis)

$-\text{epsilon}_0 \leq n_0$ cnf(clause₅, hypothesis)

$\delta \leq n_0 \text{ or } |\text{xs}(\delta) + -\text{a_real_number}| \leq \delta$ cnf(clause₆, hypothesis)

$|(f(\text{xs}(\delta)) + -f(\text{a_real_number})) + (g(\text{xs}(\delta)) + -g(\text{a_real_number}))| \leq \text{epsilon}_0 \Rightarrow \delta \leq n_0$ cnf(clause_7₁, negated_conjecture)

ANA004-5.p Lemma 2 for the sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

$|x + y| \leq |x| + |y|$ cnf(absolute_sum_less_or_equal_sum_of_absolutes₁, axiom)

$\min(x, y) \leq x$ cnf(minimum₂, axiom)

$x \leq y \Rightarrow x \leq \min(x, y)$ cnf(minimum₄, axiom)

$\min(x, y) \leq y$ cnf(minimum₆, axiom)

$y \leq x \Rightarrow y \leq \min(x, y)$ cnf(minimum₈, axiom)

$(x \leq \text{half}(z) \text{ and } y \leq \text{half}(z)) \Rightarrow x + y \leq z$ cnf(less_or_equal_sum_of_halves, axiom)

$\text{half}(x) \leq n_0 \Rightarrow x \leq n_0$ cnf(zero_and_half, axiom)

$x \leq y \text{ or } y \leq x$ cnf(commutativity_of_less_or_equal, axiom)

$(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z$ cnf(transitivity_of_less_or_equal, axiom)

$\text{delta}_1(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0$ cnf(clause₁, hypothesis)

$\text{delta}_2(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0$ cnf(clause₂, hypothesis)

$|z + -\text{a_real_number}| \leq \text{delta}_1(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |f(z) + -f(\text{a_real_number})| \leq \varepsilon)$ cnf(clause₃, hypothesis)

$|z + -\text{a_real_number}| \leq \text{delta}_2(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |g(z) + -g(\text{a_real_number})| \leq \varepsilon)$ cnf(clause₄, hypothesis)

$-\text{epsilon}_0 \leq n_0$ cnf(clause₅, hypothesis)

$\delta \leq n_0 \text{ or } |\text{xs}(\delta) + -\text{a_real_number}| \leq \delta$ cnf(clause₆, hypothesis)

$|(f(\text{xs}(\delta)) + -f(\text{a_real_number})) + (g(\text{xs}(\delta)) + -g(\text{a_real_number}))| \leq \text{epsilon}_0 \Rightarrow \delta \leq n_0$ cnf(clause_7₁, negated_conjecture)

ANA005-1.p The sum of two continuous functions is continuous

include('Axioms/ANA001-0.ax')

$n_0 < x \Rightarrow n_0 < \text{fp}_{31}(x)$ cnf(c₁₀, negated_conjecture)

$(n_0 < x \text{ and } |y + -a| < \text{fp}_{31}(x)) \Rightarrow |f(y) + -l_1| < x$ cnf(c₁₁, negated_conjecture)

$n_0 < x \Rightarrow n_0 < \text{fp}_{32}(x)$ cnf(c₁₂, negated_conjecture)

$(n_0 < x \text{ and } |y + -a| < \text{fp}_{32}(x)) \Rightarrow |g(y) + -l_2| < x$ cnf(c₁₃, negated_conjecture)

$n_0 < b$ cnf(c₁₄, negated_conjecture)

$n_0 < x \Rightarrow |\text{fp}_{33}(x) + -a| < x$ cnf(c₁₅, negated_conjecture)

$n_0 < x \Rightarrow \neg |(f(\text{fp}_{33}(x)) + g(\text{fp}_{33}(x))) + -(l_1 + l_2)| < b$ cnf(c₁₆, negated_conjecture)

ANA005-2.p The sum of two continuous functions is continuous

$x + n_0 = x$ cnf(right_identity, axiom)

$n_0 + x = x$ cnf(left_identity, axiom)

$\neg x < x$ cnf(reflexivity_of_less_than, axiom)

$(x < y \text{ and } y < z) \Rightarrow x < z$ cnf(transitivity_of_less_than, axiom)

$(n_0 < x \text{ and } n_0 < y) \Rightarrow n_0 < \min(x, y)$ cnf(axiom_2₁, axiom)

$(n_0 < x \text{ and } n_0 < y) \Rightarrow \min(x, y) < x$ cnf(axiom_22, axiom)
 $(n_0 < x \text{ and } n_0 < y) \Rightarrow \min(x, y) < y$ cnf(axiom_23, axiom)
 $(x < \text{half}(xa) \text{ and } y < \text{half}(xa)) \Rightarrow x + y < xa$ cnf(axiom_3, axiom)
 $n_0 < xa \Rightarrow n_0 < \text{half}(xa)$ cnf(axiom_7, axiom)
 $n_0 < x \Rightarrow n_0 < \text{fp}_{31}(x)$ cnf(c10, negated_conjecture)
 $(n_0 < x \text{ and } |y + -a| < \text{fp}_{31}(x)) \Rightarrow |f(y) + -l_1| < x$ cnf(c11, negated_conjecture)
 $n_0 < x \Rightarrow n_0 < \text{fp}_{32}(x)$ cnf(c12, negated_conjecture)
 $(n_0 < x \text{ and } |y + -a| < \text{fp}_{32}(x)) \Rightarrow |g(y) + -l_2| < x$ cnf(c13, negated_conjecture)
 $n_0 < b$ cnf(c14, negated_conjecture)
 $n_0 < x \Rightarrow |\text{fp}_{33}(x) + -a| < x$ cnf(c15, negated_conjecture)
 $n_0 < x \Rightarrow \neg(|f(\text{fp}_{33}(x)) + g(\text{fp}_{33}(x))| + -(l_1 + l_2)) < b$ cnf(c16, negated_conjecture)

ANA005-3.p The sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

include('Axioms/ANA002-0.ax')

$\text{delta}_1(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0$ cnf(clause_1, hypothesis)
 $\text{delta}_2(\varepsilon) \leq n_0 \Rightarrow \varepsilon \leq n_0$ cnf(clause_2, hypothesis)
 $|z + -a_real_number| \leq \text{delta}_1(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |f(z) + -f(a_real_number)| \leq \varepsilon)$ cnf(clause_3, hypothesis)
 $|z + -a_real_number| \leq \text{delta}_2(\varepsilon) \Rightarrow (\varepsilon \leq n_0 \text{ or } |g(z) + -g(a_real_number)| \leq \varepsilon)$ cnf(clause_4, hypothesis)
 $\neg \text{epsilon}_0 \leq n_0$ cnf(clause_5, hypothesis)
 $\delta \leq n_0 \text{ or } |xs(\delta) + -a_real_number| \leq \delta$ cnf(clause_6, hypothesis)
 $|f(xs(\delta)) + g(xs(\delta)) + (-f(a_real_number) + -g(a_real_number))| \leq \text{epsilon}_0 \Rightarrow \delta \leq n_0$ cnf(clause_7, negated_conjecture)

ANA005-4.p The sum of two continuous functions is continuous

A lemma formed by adding in some resolvants and taking out the corresponding clauses.

$\text{less_or_equalish}(|x + y|, |x| + |y|)$ cnf(absolute_sum_less_or_equal_sum_of_absolutes_1, axiom)
 $\text{less_or_equalish}(\min(x, y), x)$ cnf(minimum_2, axiom)
 $\text{less_or_equalish}(x, y) \Rightarrow \text{less_or_equalish}(x, \min(x, y))$ cnf(minimum_4, axiom)
 $\text{less_or_equalish}(\min(x, y), y)$ cnf(minimum_6, axiom)
 $\text{less_or_equalish}(y, x) \Rightarrow \text{less_or_equalish}(y, \min(x, y))$ cnf(minimum_8, axiom)
 $(\text{less_or_equalish}(x, \text{half}(z)) \text{ and } \text{less_or_equalish}(y, \text{half}(z))) \Rightarrow \text{less_or_equalish}(x + y, z)$ cnf(less_or_equal_sum_of_halves, axiom)
 $\text{less_or_equalish}(\text{half}(x), n_0) \Rightarrow \text{less_or_equalish}(x, n_0)$ cnf(zero_and_half, axiom)
 $\text{less_or_equalish}(x, y) \text{ or } \text{less_or_equalish}(y, x)$ cnf(commutativity_of_less_or_equal, axiom)
 $(\text{less_or_equalish}(x, y) \text{ and } \text{less_or_equalish}(y, z)) \Rightarrow \text{less_or_equalish}(x, z)$ cnf(transitivity_of_less_or_equal, axiom)
 $x + y = y + x$ cnf(commutativity_of_add, axiom)
 $(x + y) + z = x + (y + z)$ cnf(associativity_of_add, axiom)
 $x = y \Rightarrow y = x$ cnf(symmetry, axiom)
 $(x = y \text{ and } y = z) \Rightarrow x = z$ cnf(transitivity, axiom)
 $(x = z \text{ and } \text{less_or_equalish}(x, y)) \Rightarrow \text{less_or_equalish}(z, y)$ cnf(less_or_equal_substitution_1, axiom)
 $x = z \Rightarrow |x| = |z|$ cnf(absolute_substitution, axiom)
 $x = z \Rightarrow x + y = z + y$ cnf(add_substitution_1, axiom)
 $y = z \Rightarrow x + y = x + z$ cnf(add_substitution_2, axiom)
 $\text{less_or_equalish}(\text{delta}_1(\varepsilon), n_0) \Rightarrow \text{less_or_equalish}(\varepsilon, n_0)$ cnf(clause_1, hypothesis)
 $\text{less_or_equalish}(\text{delta}_2(\varepsilon), n_0) \Rightarrow \text{less_or_equalish}(\varepsilon, n_0)$ cnf(clause_2, hypothesis)
 $\text{less_or_equalish}(|z + -a_real_number|, \text{delta}_1(\varepsilon)) \Rightarrow (\text{less_or_equalish}(\varepsilon, n_0) \text{ or } \text{less_or_equalish}(|f(z) + -f(a_real_number)|, \varepsilon))$
 $\text{less_or_equalish}(|z + -a_real_number|, \text{delta}_2(\varepsilon)) \Rightarrow (\text{less_or_equalish}(\varepsilon, n_0) \text{ or } \text{less_or_equalish}(|g(z) + -g(a_real_number)|, \varepsilon))$
 $\neg \text{less_or_equalish}(\text{epsilon}_0, n_0)$ cnf(clause_5, hypothesis)
 $\text{less_or_equalish}(\delta, n_0) \text{ or } \text{less_or_equalish}(|xs(\delta) + -a_real_number|, \delta)$ cnf(clause_6, hypothesis)
 $\text{less_or_equalish}(|f(xs(\delta)) + g(xs(\delta)) + (-f(a_real_number) + -g(a_real_number))|, \text{epsilon}_0) \Rightarrow \text{less_or_equalish}(\delta, n_0)$ cnf(

ANA005-5.p The sum of two continuous functions is continuous

$\text{less_or_equalish}(|x + y|, |x| + |y|)$ cnf(absolute_sum_less_or_equal_sum_of_absolutes_1, axiom)
 $\text{less_or_equalish}(x, y) \Rightarrow \min(x, y) = x$ cnf(minimum_1, axiom)
 $\text{less_or_equalish}(y, x) \Rightarrow \min(x, y) = y$ cnf(minimum_5, axiom)
 $\text{half}(x) + \text{half}(x) = x$ cnf(half_plus_half_is_whole, axiom)
 $\text{less_or_equalish}(\text{half}(x), n_0) \Rightarrow \text{less_or_equalish}(x, n_0)$ cnf(zero_and_half, axiom)
 $\text{less_or_equalish}(x, y) \Rightarrow \text{less_or_equalish}(x + z, y + z)$ cnf(add_to_both_sides_of_less_equal_1, axiom)
 $\text{less_or_equalish}(x, y) \text{ or } \text{less_or_equalish}(y, x)$ cnf(commutativity_of_less_or_equal, axiom)
 $(\text{less_or_equalish}(x, y) \text{ and } \text{less_or_equalish}(y, z)) \Rightarrow \text{less_or_equalish}(x, z)$ cnf(transitivity_of_less_or_equal, axiom)
 $x + y = y + x$ cnf(commutativity_of_add, axiom)
 $(x + y) + z = x + (y + z)$ cnf(associativity_of_add, axiom)

$x=y \Rightarrow \text{less_or_equalish}(x,y) \quad \text{cnf}(\text{equal_implies_less_or_equal}, \text{axiom})$
 $x=y \Rightarrow y=x \quad \text{cnf}(\text{symmetry}, \text{axiom})$
 $(x=y \text{ and } y=z) \Rightarrow x=z \quad \text{cnf}(\text{transitivity}, \text{axiom})$
 $(x=z \text{ and } \text{less_or_equalish}(x,y)) \Rightarrow \text{less_or_equalish}(z,y) \quad \text{cnf}(\text{less_or_equal_substitution}_1, \text{axiom})$
 $x=z \Rightarrow |x|=|z| \quad \text{cnf}(\text{absolute_substitution}, \text{axiom})$
 $x=z \Rightarrow x+y=z+y \quad \text{cnf}(\text{add_substitution}_1, \text{axiom})$
 $y=z \Rightarrow x+y=x+z \quad \text{cnf}(\text{add_substitution}_2, \text{axiom})$
 $\text{less_or_equalish}(\text{delta}_1(\varepsilon), n_0) \Rightarrow \text{less_or_equalish}(\varepsilon, n_0) \quad \text{cnf}(\text{clause}_1, \text{hypothesis})$
 $\text{less_or_equalish}(\text{delta}_2(\varepsilon), n_0) \Rightarrow \text{less_or_equalish}(\varepsilon, n_0) \quad \text{cnf}(\text{clause}_2, \text{hypothesis})$
 $\text{less_or_equalish}(|z+-a_real_number|, \text{delta}_1(\varepsilon)) \Rightarrow (\text{less_or_equalish}(\varepsilon, n_0) \text{ or } \text{less_or_equalish}(|f(z)+-f(a_real_number)|, \varepsilon))$
 $\text{less_or_equalish}(|z+-a_real_number|, \text{delta}_2(\varepsilon)) \Rightarrow (\text{less_or_equalish}(\varepsilon, n_0) \text{ or } \text{less_or_equalish}(|g(z)+-g(a_real_number)|, \varepsilon))$
 $-\text{less_or_equalish}(\text{epsilon}_0, n_0) \quad \text{cnf}(\text{clause}_5, \text{hypothesis})$
 $\text{less_or_equalish}(\delta, n_0) \text{ or } \text{less_or_equalish}(|xs(\delta) + -a_real_number|, \delta) \quad \text{cnf}(\text{clause}_6, \text{hypothesis})$
 $\text{less_or_equalish}(|(f(xs(\delta))+g(xs(\delta)))+(-f(a_real_number))+(-g(a_real_number))|, \text{epsilon}_0) \Rightarrow \text{less_or_equalish}(\delta, n_0) \quad \text{cnf}(\text{clause}_7, \text{hypothesis})$

ANA006-1.p Analysis (limits) axioms for continuous functions
include('Axioms/ANA001-0.ax')

ANA006-2.p Analysis (limits) axioms for continuous functions
include('Axioms/ANA002-0.ax')

ANA007-1.p Problem about Big-O notation

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

$\text{class_OrderedGroup_Ocomm_monoid_mult}(t_a) \Rightarrow \text{c_SetsAndFunctions_Oelt_set_times}(c_1, v_y, t_a) = v_y \quad \text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_times}, \text{axiom})$
 $\text{class_OrderedGroup_Ocomm_monoid_add}(t_a) \Rightarrow \text{c_SetsAndFunctions_Oelt_set_plus}(c_0, v_y, t_a) = v_y \quad \text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_plus}, \text{axiom})$
 $-\text{c_lessequals}(c_HOL_Oabs(v_f(v_x(v_U))), t_b), \text{c_times}(v_U, c_HOL_Oabs(v_f(v_x(v_U))), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_0, \text{negated_conjecture})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_b) \quad \text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$

ANA007-2.p Problem about Big-O notation

$-\text{c_lessequals}(c_HOL_Oabs(v_f(v_x(v_U))), t_b), \text{c_times}(v_U, c_HOL_Oabs(v_f(v_x(v_U))), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_0, \text{negated_conjecture})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_b) \quad \text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$
 $\text{class_Orderings_Oorder}(t_a) \Rightarrow \text{c_lessequals}(v_x, v_x, t_a) \quad \text{cnf}(\text{cls_Orderings_Oorder_class_Oaxioms}_1, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_a) \Rightarrow v_c = c_times(c_1, v_c, t_a) \quad \text{cnf}(\text{cls_Ring_and_Field_Omult_cancel_right}_1, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_Orderings_Oorder}(t) \quad \text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{44}, \text{axiom})$

ANA009-1.p Problem about Big-O notation

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

$\text{class_OrderedGroup_Ocomm_monoid_mult}(t_a) \Rightarrow \text{c_SetsAndFunctions_Oelt_set_times}(c_1, v_y, t_a) = v_y \quad \text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_times}, \text{axiom})$
 $\text{class_OrderedGroup_Ocomm_monoid_add}(t_a) \Rightarrow \text{c_SetsAndFunctions_Oelt_set_plus}(c_0, v_y, t_a) = v_y \quad \text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_plus}, \text{axiom})$
 $\text{c_lessequals}(v_lb(v_U), v_f(v_U), t_b) \quad \text{cnf}(\text{cls_conjecture}_0, \text{negated_conjecture})$
 $\text{c_lessequals}(v_f(v_U), \text{c_plus}(v_lb(v_U), v_g(v_U), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_1, \text{negated_conjecture})$
 $-\text{c_lessequals}(c_0, \text{c_plus}(v_f(v_x), \text{c_uminus}(v_lb(v_x), t_b), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_2, \text{negated_conjecture})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_b) \quad \text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$

ANA009-2.p Problem about Big-O notation

$(\text{class_OrderedGroup_Oordered_ab_semigroup_add_imp_le}(t_a) \text{ and } \text{c_lessequals}(v_a, v_b, t_a)) \Rightarrow \text{c_lessequals}(\text{c_plus}(v_a, v_b, t_a), v_a, t_a) \quad \text{cnf}(\text{cls_OrderedGroup_Oright_mult_cancel_right}_1, \text{axiom})$
 $\text{class_OrderedGroup_Oab_group_add}(t_a) \Rightarrow \text{c_plus}(v_a, \text{c_uminus}(v_a, t_a), t_a) = c_0 \quad \text{cnf}(\text{cls_OrderedGroup_Oright_mult_cancel_right}_1, \text{axiom})$
 $\text{c_lessequals}(v_lb(v_U), v_f(v_U), t_b) \quad \text{cnf}(\text{cls_conjecture}_0, \text{negated_conjecture})$
 $-\text{c_lessequals}(c_0, \text{c_plus}(v_f(v_x), \text{c_uminus}(v_lb(v_x), t_b), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_2, \text{negated_conjecture})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_OrderedGroup_Oordered_ab_semigroup_add_imp_le}(t) \quad \text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{44}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_OrderedGroup_Oab_group_add}(t) \quad \text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{44}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_b) \quad \text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$

ANA010-1.p Problem about Big-O notation

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

$(\text{class_Orderings_Oorder}(t_a) \text{ and } \text{c_lessequals}(v_y, v_z, t_a) \text{ and } \text{c_lessequals}(v_x, v_y, t_a)) \Rightarrow \text{c_lessequals}(v_x, v_z, t_a) \quad \text{cnf}(\text{cls_Orderings_Oorder_class_Oaxioms}_1, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_a) \Rightarrow \text{c_HOL_Oabs}(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a)) \quad \text{cnf}(\text{cls_Ring_and_Field_Omult_cancel_right}_1, \text{axiom})$

```

class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)
c_lessequals(c_0, v_f(v_U), t_b)    cnf(cls_conjecture_0, negated_conjecture)
c_lessequals(v_f(v_U), c_times(v_c, v_g(v_U), t_b), t_b)    cnf(cls_conjecture_1, negated_conjecture)
¬ c_lessequals(v_f(v_x(v_U)), c_times(v_U, c_HOL_Oabs(v_g(v_x(v_U))), t_b), t_b), t_b)    cnf(cls_conjecture_2, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA010-2.p Problem about Big-O notation

```

(class_OrderedGroup_Olordered_ab_group_abs(t_a) and c_lessequals(c_0, v_y, t_a)) ⇒ c_HOL_Oabs(v_y, t_a) = v_y    cnf(c_HOL_Oabs, negated_conjecture)
(class_Orderings_Oorder(t_a) and c_lessequals(v_y, v_z, t_a) and c_lessequals(v_x, v_y, t_a)) ⇒ c_lessequals(v_x, v_z, t_a)    cnf(c_lessequals, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a))    cnf(c_HOL_Oabs, negated_conjecture)
c_lessequals(c_0, v_f(v_U), t_b)    cnf(cls_conjecture_0, negated_conjecture)
c_lessequals(v_f(v_U), c_times(v_c, v_g(v_U), t_b), t_b)    cnf(cls_conjecture_1, negated_conjecture)
¬ c_lessequals(v_f(v_x(v_U)), c_times(v_U, c_HOL_Oabs(v_g(v_x(v_U))), t_b), t_b), t_b)    cnf(cls_conjecture_2, negated_conjecture)
class_OrderedGroup_Olordered_ab_group_abs(t) ⇒ class_Orderings_Oorder(t)    cnf(clsrel_OrderedGroup_Olordered_ab_group_abs, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t) ⇒ class_OrderedGroup_Olordered_ab_group_abs(t)    cnf(clsrel_Ring_and_Field_Ordered_idom, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA012-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a))    cnf(c_HOL_Oabs, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)
v_c ≠ c_0    cnf(cls_conjecture_0, negated_conjecture)
¬ c_lessequals(c_1, c_times(v_U, c_HOL_Oabs(v_c, t_a), t_a), t_a)    cnf(cls_conjecture_1, negated_conjecture)
class_Ring_and_Field_Ordered_field(t_a)    cnf(tfree_tcs, negated_conjecture)

```

ANA012-2.p Problem about Big-O notation

```

v_c ≠ c_0    cnf(cls_conjecture_0, negated_conjecture)
¬ c_lessequals(c_1, c_times(v_U, c_HOL_Oabs(v_c, t_a), t_a), t_a)    cnf(cls_conjecture_1, negated_conjecture)
class_Ring_and_Field_Ordered_field(t_a)    cnf(tfree_tcs, negated_conjecture)
(class_OrderedGroup_Olordered_ab_group_abs(t_a) and c_HOL_Oabs(v_a, t_a) = c_0) ⇒ v_a = c_0    cnf(cls_OrderedGroup_Olordered_ab_group_abs, negated_conjecture)
class_Orderings_Oorder(t_a) ⇒ c_lessequals(v_x, v_x, t_a)    cnf(cls_Orderings_Oorder_class_Oaxioms_1_0, axiom)
class_Ring_and_Field_Ofield(t_a) ⇒ (v_a = c_0 or c_times(c_HOL_Oinverse(v_a, t_a), v_a, t_a) = c_1)    cnf(cls_Ring_and_Field_Ofield, negated_conjecture)
class_Ring_and_Field_Ordered_field(t) ⇒ class_Ring_and_Field_Ofield(t)    cnf(clsrel_Ring_and_Field_Ordered_field, negated_conjecture)
class_Ring_and_Field_Ordered_field(t) ⇒ class_OrderedGroup_Olordered_ab_group_abs(t)    cnf(clsrel_Ring_and_Field_Ordered_field, negated_conjecture)
class_Ring_and_Field_Ordered_field(t) ⇒ class_Orderings_Oorder(t)    cnf(clsrel_Ring_and_Field_Ordered_field_58, axiom)

```

ANA013-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a))    cnf(c_HOL_Oabs, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)
¬ c_lessequals(c_times(c_HOL_Oabs(v_c, t_b), c_HOL_Oabs(v_f(v_x(v_U))), t_b), t_b), c_times(v_U, c_HOL_Oabs(v_f(v_x(v_U))), t_b)    cnf(cls_conjecture_1, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA013-2.p Problem about Big-O notation

```

¬ c_lessequals(c_times(c_HOL_Oabs(v_c, t_b), c_HOL_Oabs(v_f(v_x(v_U))), t_b), t_b), c_times(v_U, c_HOL_Oabs(v_f(v_x(v_U))), t_b)    cnf(cls_conjecture_1, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)
class_Orderings_Oorder(t_a) ⇒ c_lessequals(v_x, v_x, t_a)    cnf(cls_Orderings_Oorder_class_Oaxioms_1_0, axiom)
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Orderings_Oorder(t)    cnf(clsrel_Ring_and_Field_Ordered_idom_44, axiom)

```

ANA014-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)    cnf(c_times, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a))    cnf(c_HOL_Oabs, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)

```


`class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y` `cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)`
`v_c ≠ c_0` `cnf(cls_conjecture_0, negated_conjecture)`
`¬ c_lessequals(c_HOL_Oabs(v_f(v_x(v_U))), t_a), c_times(v_U, c_times(c_HOL_Oabs(v_c, t_a), c_HOL_Oabs(v_f(v_x(v_U))), t_a), t_a)`
`class_Ring_and_Field_Ordered_field(t_a)` `cnf(tfree_tcs, negated_conjecture)`

ANA014-2.p Problem about Big-O notation

`v_c ≠ c_0` `cnf(cls_conjecture_0, negated_conjecture)`
`¬ c_lessequals(c_HOL_Oabs(v_f(v_x(v_U))), t_a), c_times(v_U, c_times(c_HOL_Oabs(v_c, t_a), c_HOL_Oabs(v_f(v_x(v_U))), t_a), t_a)`
`class_Ring_and_Field_Ordered_field(t_a)` `cnf(tfree_tcs, negated_conjecture)`
`class_OrderedGroup_Omonoid_mult(t_a) ⇒ c_times(c_1, v_y, t_a) = v_y` `cnf(cls_OrderedGroup_Omonoid_mult_class_Oaxioms_10, axiom)`
`class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)`
`class_Orderings_Order(t_a) ⇒ c_lessequals(v_x, v_x, t_a)` `cnf(cls_Orderings_Order_class_Oaxioms_10, axiom)`
`class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)`
`class_Ring_and_Field_Ofield(t_a) ⇒ (v_a = c_0 or c_times(c_HOL_Oinverse(v_a, t_a), v_a, t_a) = c_1)` `cnf(cls_Ring_and_Field_Ofield_12, axiom)`
`class_Ring_and_Field_Ofield(t) ⇒ class_OrderedGroup_Omonoid_mult(t)` `cnf(clsrel_Ring_and_Field_Ofield_12, axiom)`
`class_Ring_and_Field_Ofield(t) ⇒ class_OrderedGroup_Osemigroup_mult(t)` `cnf(clsrel_Ring_and_Field_Ofield_21, axiom)`
`class_Ring_and_Field_Ordered_field(t) ⇒ class_Ring_and_Field_Ofield(t)` `cnf(clsrel_Ring_and_Field_Ordered_field_12, axiom)`
`class_Ring_and_Field_Ordered_field(t) ⇒ class_Ring_and_Field_Ordered_idom(t)` `cnf(clsrel_Ring_and_Field_Ordered_field_58, axiom)`
`class_Ring_and_Field_Ordered_field(t) ⇒ class_Orderings_Order(t)` `cnf(clsrel_Ring_and_Field_Ordered_field_58, axiom)`

ANA015-1.p Problem about Big-O notation

`include('Axioms/ANA003-0.ax')`
`include('Axioms/MS001-1.ax')`
`include('Axioms/MS001-0.ax')`
`class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)`
`class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)`
`(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals(v_a, v_b, t_a)`
`class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y` `cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)`
`class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y` `cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)`
`v_c ≠ c_0` `cnf(cls_conjecture_0, negated_conjecture)`
`c_lessequals(c_HOL_Oabs(v_g(v_U), t_a), c_times(v_d, c_HOL_Oabs(v_f(v_U), t_a), t_a), t_a)` `cnf(cls_conjecture_1, negated_conjecture)`
`¬ c_lessequals(c_HOL_Oabs(c_times(c_HOL_Oinverse(v_c, t_a), v_g(v_x(v_U))), t_a), t_a), c_times(v_U, c_HOL_Oabs(v_f(v_x(v_U))), t_a)`
`class_Ring_and_Field_Ordered_field(t_a)` `cnf(tfree_tcs, negated_conjecture)`

ANA015-2.p Problem about Big-O notation

`class_OrderedGroup_Olordered_ab_group_abs(t_a) ⇒ c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)` `cnf(cls_OrderedGroup_Olordered_ab_group_abs_class_Oaxioms_10, axiom)`
`class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)`
`class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)`
`(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals(v_a, v_b, t_a)`
`c_lessequals(c_HOL_Oabs(v_g(v_U), t_a), c_times(v_d, c_HOL_Oabs(v_f(v_U), t_a), t_a), t_a)` `cnf(cls_conjecture_1, negated_conjecture)`
`¬ c_lessequals(c_HOL_Oabs(c_times(c_HOL_Oinverse(v_c, t_a), v_g(v_x(v_U))), t_a), t_a), c_times(v_U, c_HOL_Oabs(v_f(v_x(v_U))), t_a)`
`class_Ring_and_Field_Ofield(t) ⇒ class_OrderedGroup_Osemigroup_mult(t)` `cnf(clsrel_Ring_and_Field_Ofield_21, axiom)`
`class_Ring_and_Field_Ordered_field(t) ⇒ class_Ring_and_Field_Ofield(t)` `cnf(clsrel_Ring_and_Field_Ordered_field_12, axiom)`
`class_Ring_and_Field_Ordered_field(t) ⇒ class_Ring_and_Field_Ordered_idom(t)` `cnf(clsrel_Ring_and_Field_Ordered_field_58, axiom)`
`class_Ring_and_Field_Ordered_field(t) ⇒ class_Ring_and_Field_Oordered_semiring(t)` `cnf(clsrel_Ring_and_Field_Ordered_field_58, axiom)`
`class_Ring_and_Field_Ordered_field(t) ⇒ class_OrderedGroup_Olordered_ab_group_abs(t)` `cnf(clsrel_Ring_and_Field_Ordered_field_58, axiom)`
`class_Ring_and_Field_Ordered_field(t_a)` `cnf(tfree_tcs, negated_conjecture)`

ANA016-1.p Problem about Big-O notation

`include('Axioms/ANA003-0.ax')`
`include('Axioms/MS001-1.ax')`
`include('Axioms/MS001-0.ax')`
`class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)`
`class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)`
`(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals(v_a, v_b, t_a)`
`class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y` `cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)`
`class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y` `cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)`
`v_c ≠ c_0` `cnf(cls_conjecture_0, negated_conjecture)`
`c_lessequals(c_HOL_Oabs(v_g(v_U), t_a), c_times(v_d, c_HOL_Oabs(v_f(v_U), t_a), t_a), t_a)` `cnf(cls_conjecture_1, negated_conjecture)`
`v_g(v_x) ≠ c_times(v_c, c_times(c_HOL_Oinverse(v_c, t_a), v_g(v_x), t_a), t_a)` `cnf(cls_conjecture_2, negated_conjecture)`
`class_Ring_and_Field_Ordered_field(t_a)` `cnf(tfree_tcs, negated_conjecture)`

ANA016-2.p Problem about Big-O notation

```

v_c ≠ c_0      cnf(cls_conjecture_0, negated_conjecture)
v_g(v_x) ≠ c_times(v_c, c_times(c_HOL_Oinverse(v_c, t_a), v_g(v_x), t_a), t_a)  cnf(cls_conjecture_2, negated_conjecture)
class_Ring_and_Field_Ordered_field(t_a)  cnf(tfree_tcs, negated_conjecture)
class_OrderedGroup_Omonoid_mult(t_a) ⇒ c_times(c_1, v_y, t_a) = v_y      cnf(cls_OrderedGroup_Omonoid_mult_class_Oax)
class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)
class_Ring_and_Field_Ofield(t_a) ⇒ (v_a = c_0 or c_times(v_a, c_HOL_Oinverse(v_a, t_a), t_a) = c_1)  cnf(cls_Ring_and_Field_Ofield)
class_Ring_and_Field_Ofield(t) ⇒ class_OrderedGroup_Omonoid_mult(t)  cnf(clsrel_Ring_and_Field_Ofield_12, axiom)
class_Ring_and_Field_Ofield(t) ⇒ class_OrderedGroup_Osemigroup_mult(t)  cnf(clsrel_Ring_and_Field_Ofield_21, axiom)
class_Ring_and_Field_Ordered_field(t) ⇒ class_Ring_and_Field_Ofield(t)  cnf(clsrel_Ring_and_Field_Ordered_field)

```

ANA017-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)
class_OrderedGroup_Oab_semigroup_mult(t_a) ⇒ c_times(v_a, v_b, t_a) = c_times(v_b, v_a, t_a)  cnf(cls_OrderedGroup_Oab_semigroup_mult)
class_OrderedGroup_Oab_semigroup_mult(t_a) ⇒ c_times(v_a, c_times(v_b, v_c, t_a), t_a) = c_times(v_b, c_times(v_a, v_c, t_a), t_a)
class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals(c_times(v_a, v_b, t_a), c_times(v_b, v_a, t_a), t_a)
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y  cnf(cls_SetsAndFunctions_Oelt_set_times)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y  cnf(cls_SetsAndFunctions_Oelt_set_plus)
c_lessequals(c_HOL_Oabs(v_b(v_U), t_b), c_times(v_ca, c_HOL_Oabs(v_f(v_U), t_b), t_b), t_b)  cnf(cls_conjecture_0, negated_conjecture)
¬ c_lessequals(c_HOL_Oabs(c_times(v_c, v_b(v_x(v_U))), t_b), t_b), c_times(v_U, c_HOL_Oabs(v_f(v_x(v_U))), t_b), t_b), t_b)  cnf(cls_conjecture_1, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_b)  cnf(tfree_tcs, negated_conjecture)

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ANA017-2.p Problem about Big-O notation

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c_lessequals(c_HOL_Oabs(v_b(v_U), t_b), c_times(v_ca, c_HOL_Oabs(v_f(v_U), t_b), t_b), t_b)  cnf(cls_conjecture_0, negated_conjecture)
¬ c_lessequals(c_HOL_Oabs(c_times(v_c, v_b(v_x(v_U))), t_b), t_b), c_times(v_U, c_HOL_Oabs(v_f(v_x(v_U))), t_b), t_b), t_b)  cnf(cls_conjecture_1, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_b)  cnf(tfree_tcs, negated_conjecture)
class_OrderedGroup_Olordered_ab_group_abs(t_a) ⇒ c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)  cnf(cls_OrderedGroup_Olordered_ab_group_abs)
class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)
class_Ring_and_Field_Ordered_idom(t_a) ⇒ c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals(c_times(v_a, v_b, t_a), c_times(v_b, v_a, t_a), t_a)
class_Ring_and_Field_Ordered_idom(t) ⇒ class_OrderedGroup_Osemigroup_mult(t)  cnf(clsrel_Ring_and_Field_Ordered_idom)
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Ring_and_Field_Oordered_semiring(t)  cnf(clsrel_Ring_and_Field_Ordered_idom)
class_Ring_and_Field_Ordered_idom(t) ⇒ class_OrderedGroup_Olordered_ab_group_abs(t)  cnf(clsrel_Ring_and_Field_Ordered_idom)

```

ANA018-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
class_OrderedGroup_Osemigroup_mult(t_a) ⇒ c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)
class_OrderedGroup_Oab_semigroup_mult(t_a) ⇒ c_times(v_a, v_b, t_a) = c_times(v_b, v_a, t_a)  cnf(cls_OrderedGroup_Oab_semigroup_mult)
class_OrderedGroup_Oab_semigroup_mult(t_a) ⇒ c_times(v_a, c_times(v_b, v_c, t_a), t_a) = c_times(v_b, c_times(v_a, v_c, t_a), t_a)
(class_Orderings_Oorder(t_a) and c_lessequals(v_y, v_z, t_a) and c_lessequals(v_x, v_y, t_a)) ⇒ c_lessequals(v_x, v_z, t_a)  cnf(cls_Orderings_Oorder_lessequals_imp)
(class_Orderings_Oorder(t_a) and c_less(v_x, v_y, t_a)) ⇒ c_lessequals(v_x, v_y, t_a)  cnf(cls_Orderings_Oorder_less_imp)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals(c_times(v_a, v_b, t_a), c_times(v_b, v_a, t_a), t_a)
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y  cnf(cls_SetsAndFunctions_Oelt_set_times)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y  cnf(cls_SetsAndFunctions_Oelt_set_plus)
c_less(c_0, v_c, t_b)  cnf(cls_conjecture_0, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_f(v_U), t_b), c_times(v_c, c_HOL_Oabs(v_g(v_U), t_b), t_b), t_b)  cnf(cls_conjecture_1, negated_conjecture)
c_less(c_0, v_ca, t_b)  cnf(cls_conjecture_2, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_x(v_U), t_b), c_times(v_ca, c_HOL_Oabs(v_f(v_U), t_b), t_b), t_b)  cnf(cls_conjecture_3, negated_conjecture)
¬ c_lessequals(c_HOL_Oabs(v_x(v_xa), t_b), c_times(c_times(v_ca, v_c, t_b), c_HOL_Oabs(v_g(v_xa), t_b), t_b), t_b), t_b)  cnf(cls_conjecture_4, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_b)  cnf(tfree_tcs, negated_conjecture)

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ANA018-2.p Problem about Big-O notation

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c_lessequals(c_HOL_Oabs(v_f(v_U), t_b), c_times(v_c, c_HOL_Oabs(v_g(v_U), t_b), t_b), t_b)  cnf(cls_conjecture_1, negated_conjecture)
c_less(c_0, v_ca, t_b)  cnf(cls_conjecture_2, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_x(v_U), t_b), c_times(v_ca, c_HOL_Oabs(v_f(v_U), t_b), t_b), t_b)  cnf(cls_conjecture_3, negated_conjecture)

```

$\neg c_lesseqs(c_HOL_Oabs(v_x(v_xa), t_b), c_times(c_times(v_ca, v_c, t_b), c_HOL_Oabs(v_g(v_xa), t_b), t_b), t_b)$ $cnf(cls_c_times, negated_conjecture)$
 $class_Ring_and_Field_Ordered_idom(t_b)$ $cnf(tfree_tcs, negated_conjecture)$
 $class_OrderedGroup_Osemigroup_mult(t_a) \Rightarrow c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)$
 $(class_Orderings_Order(t_a) \text{ and } c_lesseqs(v_y, v_z, t_a) \text{ and } c_lesseqs(v_x, v_y, t_a)) \Rightarrow c_lesseqs(v_x, v_z, t_a)$ $cnf($
 $(class_Orderings_Order(t_a) \text{ and } c_less(v_x, v_y, t_a)) \Rightarrow c_lesseqs(v_x, v_y, t_a)$ $cnf($
 $(class_Ring_and_Field_Ordered_cancel_semiring(t_a) \text{ and } c_lesseqs(v_a, v_b, t_a) \text{ and } c_lesseqs(c_0, v_c, t_a)) \Rightarrow c_lesseqs$
 $class_LOrder_Ojoin_semilorder(t) \Rightarrow class_Orderings_Order(t)$ $cnf($
 $class_Ring_and_Field_Ordered_idom(t) \Rightarrow class_OrderedGroup_Osemigroup_mult(t)$ $cnf($
 $class_Ring_and_Field_Ordered_idom(t) \Rightarrow class_LOrder_Ojoin_semilorder(t)$ $cnf($
 $class_Ring_and_Field_Ordered_idom(t) \Rightarrow class_Ring_and_Field_Ordered_cancel_semiring(t)$ $cnf($

ANA019-1.p Problem about Big-O notation

$include('Axioms/ANA003-0.ax')$
 $include('Axioms/MSC001-1.ax')$
 $include('Axioms/MSC001-0.ax')$
 $c_less(c_0, v_x, tc_nat) \Rightarrow v_x = c_Suc(c_minus(v_x, c_1, tc_nat))$ $cnf($
 $(class_Orderings_Order(t_a) \text{ and } c_less(v_x, v_y, t_a)) \Rightarrow c_lesseqs(v_x, v_y, t_a)$ $cnf($
 $(class_Ring_and_Field_Ordered_cancel_semiring(t_a) \text{ and } c_lesseqs(c_0, v_b, t_a) \text{ and } c_lesseqs(c_0, v_a, t_a)) \Rightarrow$
 $c_lesseqs(c_0, c_times(v_a, v_b, t_a), t_a)$ $cnf($
 $class_OrderedGroup_Ocomm_monoid_mult(t_a) \Rightarrow c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y$ $cnf($
 $class_OrderedGroup_Ocomm_monoid_add(t_a) \Rightarrow c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y$ $cnf($
 $v_f(c_0) = c_0$ $cnf($
 $c_less(c_0, v_x, t_a)$ $cnf($
 $c_lesseqs(c_HOL_Oabs(v_f(c_Suc(v_U))), t_a), c_times(v_x, c_HOL_Oabs(v_h(c_Suc(v_U))), t_a), t_a), t_a)$ $cnf($
 $c_lesseqs(c_HOL_Oabs(v_f(v_xa(v_U))), t_a), c_times(v_U, c_HOL_Oabs(v_h(v_xa(v_U))), t_a), t_a), t_a) \Rightarrow \neg c_less(c_0, v_U, t_a)$
 $class_Ring_and_Field_Ordered_idom(t_a)$ $cnf($

ANA019-2.p Problem about Big-O notation

$v_f(c_0) = c_0$ $cnf($
 $c_less(c_0, v_x, t_a)$ $cnf($
 $c_lesseqs(c_HOL_Oabs(v_f(c_Suc(v_U))), t_a), c_times(v_x, c_HOL_Oabs(v_h(c_Suc(v_U))), t_a), t_a), t_a)$ $cnf($
 $c_lesseqs(c_HOL_Oabs(v_f(v_xa(v_U))), t_a), c_times(v_U, c_HOL_Oabs(v_h(v_xa(v_U))), t_a), t_a), t_a) \Rightarrow \neg c_less(c_0, v_U, t_a)$
 $class_Ring_and_Field_Ordered_idom(t_a)$ $cnf($
 $c_less(c_0, v_x, tc_nat) \Rightarrow v_x = c_Suc(c_minus(v_x, c_1, tc_nat))$ $cnf($
 $c_less(c_0, v_n, tc_nat) \text{ or } v_n = c_0$ $cnf($
 $class_OrderedGroup_Oordered_ab_group_abs(t_a) \Rightarrow c_HOL_Oabs(c_0, t_a) = c_0$ $cnf($
 $class_OrderedGroup_Oordered_ab_group_abs(t_a) \Rightarrow c_lesseqs(c_0, c_HOL_Oabs(v_a, t_a), t_a)$ $cnf($
 $(class_Orderings_Order(t_a) \text{ and } c_less(v_x, v_y, t_a)) \Rightarrow c_lesseqs(v_x, v_y, t_a)$ $cnf($
 $(class_Ring_and_Field_Ordered_cancel_semiring(t_a) \text{ and } c_lesseqs(c_0, v_b, t_a) \text{ and } c_lesseqs(c_0, v_a, t_a)) \Rightarrow$
 $c_lesseqs(c_0, c_times(v_a, v_b, t_a), t_a)$ $cnf($
 $class_LOrder_Ojoin_semilorder(t) \Rightarrow class_Orderings_Order(t)$ $cnf($
 $class_Ring_and_Field_Ordered_idom(t) \Rightarrow class_LOrder_Ojoin_semilorder(t)$ $cnf($
 $class_Ring_and_Field_Ordered_idom(t) \Rightarrow class_Ring_and_Field_Ordered_cancel_semiring(t)$ $cnf($
 $class_Ring_and_Field_Ordered_idom(t) \Rightarrow class_OrderedGroup_Oordered_ab_group_abs(t)$ $cnf($

ANA020-1.p Problem about Big-O notation

$include('Axioms/ANA003-0.ax')$
 $include('Axioms/MSC001-1.ax')$
 $include('Axioms/MSC001-0.ax')$
 $c_less(c_0, v_x, tc_nat) \Rightarrow v_x = c_Suc(c_minus(v_x, c_1, tc_nat))$ $cnf($
 $(class_Orderings_Order(t_a) \text{ and } c_less(v_x, v_y, t_a)) \Rightarrow c_lesseqs(v_x, v_y, t_a)$ $cnf($
 $(class_Ring_and_Field_Ordered_cancel_semiring(t_a) \text{ and } c_lesseqs(c_0, v_b, t_a) \text{ and } c_lesseqs(c_0, v_a, t_a)) \Rightarrow$
 $c_lesseqs(c_0, c_times(v_a, v_b, t_a), t_a)$ $cnf($
 $class_OrderedGroup_Ocomm_monoid_mult(t_a) \Rightarrow c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y$ $cnf($
 $class_OrderedGroup_Ocomm_monoid_add(t_a) \Rightarrow c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y$ $cnf($
 $v_f(c_0) = c_0$ $cnf($
 $c_less(c_0, v_c, t_a)$ $cnf($
 $c_lesseqs(c_HOL_Oabs(v_f(c_Suc(v_U))), t_a), c_times(v_c, c_HOL_Oabs(v_h(c_Suc(v_U))), t_a), t_a), t_a)$ $cnf($
 $v_x = c_0$ $cnf($
 $\neg c_lesseqs(c_HOL_Oabs(v_f(v_x), t_a), c_times(v_c, c_HOL_Oabs(v_h(v_x), t_a), t_a), t_a), t_a)$ $cnf($
 $class_Ring_and_Field_Ordered_idom(t_a)$ $cnf($

ANA020-2.p Problem about Big-O notation

$v_f(c_0) = c_0$ $\text{cnf}(\text{cls_conjecture}_0, \text{negated_conjecture})$
 $c_less(c_0, v_c, t_a)$ $\text{cnf}(\text{cls_conjecture}_1, \text{negated_conjecture})$
 $v_x = c_0$ $\text{cnf}(\text{cls_conjecture}_3, \text{negated_conjecture})$
 $\neg c_lessequals(c_HOL_Oabs(v_f(v_x), t_a), c_times(v_c, c_HOL_Oabs(v_h(v_x), t_a), t_a), t_a)$ $\text{cnf}(\text{cls_conjecture}_4, \text{negated_conjecture})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_a)$ $\text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$
 $\text{class_OrderedGroup_Oordered_ab_group_abs}(t_a) \Rightarrow c_HOL_Oabs(c_0, t_a) = c_0$ $\text{cnf}(\text{cls_OrderedGroup_Oabs_eq_0}_1, \text{axiom})$
 $\text{class_OrderedGroup_Oordered_ab_group_abs}(t_a) \Rightarrow c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)$ $\text{cnf}(\text{cls_OrderedGroup_Oabs_eq_0}_1, \text{axiom})$
 $(\text{class_Orderings_Oorder}(t_a) \text{ and } c_less(v_x, v_y, t_a)) \Rightarrow c_lessequals(v_x, v_y, t_a)$ $\text{cnf}(\text{cls_Orderings_Oorder_less_imp}, \text{axiom})$
 $(\text{class_Ring_and_Field_Oordered_cancel_semiring}(t_a) \text{ and } c_lessequals(c_0, v_b, t_a) \text{ and } c_lessequals(c_0, v_a, t_a)) \Rightarrow$
 $c_lessequals(c_0, c_times(v_a, v_b, t_a), t_a)$ $\text{cnf}(\text{cls_Ring_and_Field_Omult_nonneg_nonneg}_0, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_Ring_and_Field_Oordered_cancel_semiring}(t)$ $\text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{44}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_Orderings_Oorder}(t)$ $\text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{44}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_OrderedGroup_Oordered_ab_group_abs}(t)$ $\text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{44}, \text{axiom})$

ANA021-1.p Problem about Big-O notation

$\text{include}('Axioms/ANA003-0.ax')$
 $\text{include}('Axioms/MSC001-1.ax')$
 $\text{include}('Axioms/MSC001-0.ax')$
 $c_less(c_0, v_x, tc_nat) \Rightarrow v_x = c_Suc(c_minus(v_x, c_1, tc_nat))$ $\text{cnf}(\text{cls_NatBin_OSuc_pred_H}_0, \text{axiom})$
 $(\text{class_Orderings_Oorder}(t_a) \text{ and } c_less(v_x, v_y, t_a)) \Rightarrow c_lessequals(v_x, v_y, t_a)$ $\text{cnf}(\text{cls_Orderings_Oorder_less_imp}, \text{axiom})$
 $(\text{class_Ring_and_Field_Oordered_cancel_semiring}(t_a) \text{ and } c_lessequals(c_0, v_b, t_a) \text{ and } c_lessequals(c_0, v_a, t_a)) \Rightarrow$
 $c_lessequals(c_0, c_times(v_a, v_b, t_a), t_a)$ $\text{cnf}(\text{cls_Ring_and_Field_Omult_nonneg_nonneg}_0, \text{axiom})$
 $\text{class_OrderedGroup_Ocomm_monoid_mult}(t_a) \Rightarrow c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y$ $\text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_times}, \text{axiom})$
 $\text{class_OrderedGroup_Ocomm_monoid_add}(t_a) \Rightarrow c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y$ $\text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_plus}, \text{axiom})$
 $v_f(c_0) = c_0$ $\text{cnf}(\text{cls_conjecture}_0, \text{negated_conjecture})$
 $c_less(c_0, v_c, t_a)$ $\text{cnf}(\text{cls_conjecture}_1, \text{negated_conjecture})$
 $c_lessequals(c_HOL_Oabs(v_f(c_Suc(v_U))), t_a), c_times(v_c, c_HOL_Oabs(v_h(c_Suc(v_U))), t_a), t_a)$ $\text{cnf}(\text{cls_conjecture}_2, \text{negated_conjecture})$
 $v_x \neq c_0$ $\text{cnf}(\text{cls_conjecture}_3, \text{negated_conjecture})$
 $\neg c_lessequals(c_HOL_Oabs(v_f(v_x), t_a), c_times(v_c, c_HOL_Oabs(v_h(v_x), t_a), t_a), t_a)$ $\text{cnf}(\text{cls_conjecture}_4, \text{negated_conjecture})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_a)$ $\text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$

ANA021-2.p Problem about Big-O notation

$c_less(c_0, v_x, tc_nat) \Rightarrow v_x = c_Suc(c_minus(v_x, c_1, tc_nat))$ $\text{cnf}(\text{cls_NatBin_OSuc_pred_H}_0, \text{axiom})$
 $c_less(c_0, v_n, tc_nat) \text{ or } v_n = c_0$ $\text{cnf}(\text{cls_Nat_Onot_gr}_0, \text{axiom})$
 $c_lessequals(c_HOL_Oabs(v_f(c_Suc(v_U))), t_a), c_times(v_c, c_HOL_Oabs(v_h(c_Suc(v_U))), t_a), t_a)$ $\text{cnf}(\text{cls_conjecture}_2, \text{negated_conjecture})$
 $v_x \neq c_0$ $\text{cnf}(\text{cls_conjecture}_3, \text{negated_conjecture})$
 $\neg c_lessequals(c_HOL_Oabs(v_f(v_x), t_a), c_times(v_c, c_HOL_Oabs(v_h(v_x), t_a), t_a), t_a)$ $\text{cnf}(\text{cls_conjecture}_4, \text{negated_conjecture})$

ANA023-2.p Problem about Big-O notation

$\text{class_OrderedGroup_Ocomm_monoid_add}(t_a) \Rightarrow c_plus(c_0, v_y, t_a) = v_y$ $\text{cnf}(\text{cls_OrderedGroup_Ocomm_monoid_add}, \text{axiom})$
 $(\text{class_OrderedGroup_Oordered_ab_group_add}(t_a) \text{ and } c_lessequals(v_a, c_minus(v_c, v_b, t_a), t_a)) \Rightarrow c_lessequals(c_plus(v_a, v_b, t_a), v_c, t_a)$
 $(\text{class_OrderedGroup_Oordered_ab_group_add}(t_a) \text{ and } c_lessequals(c_plus(v_a, v_b, t_a), v_c, t_a)) \Rightarrow c_lessequals(v_a, c_minus(v_c, v_b, t_a), t_a)$
 $(\text{class_Orderings_Oorder}(t_a) \text{ and } c_lessequals(v_y, v_z, t_a) \text{ and } c_lessequals(v_x, v_y, t_a)) \Rightarrow c_lessequals(v_x, v_z, t_a)$ $\text{cnf}(\text{cls_Orderings_Oorder_less_imp}, \text{axiom})$
 $c_lessequals(c_0, c_minus(v_k(v_x), v_g(v_x), t_b), t_b)$ $\text{cnf}(\text{cls_conjecture}_1, \text{negated_conjecture})$
 $c_lessequals(v_k(v_x), v_f(v_x), t_b)$ $\text{cnf}(\text{cls_conjecture}_2, \text{negated_conjecture})$
 $\neg c_lessequals(c_0, c_minus(v_f(v_x), v_g(v_x), t_b), t_b)$ $\text{cnf}(\text{cls_conjecture}_3, \text{negated_conjecture})$
 $\text{class_Orderings_Olinorder}(t) \Rightarrow \text{class_Orderings_Oorder}(t)$ $\text{cnf}(\text{clsrel_Orderings_Olinorder}_4, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_OrderedGroup_Ocomm_monoid_add}(t)$ $\text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{33}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_Orderings_Olinorder}(t)$ $\text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{33}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_OrderedGroup_Oordered_ab_group_add}(t)$ $\text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{33}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_b)$ $\text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$

ANA024-2.p Problem about Big-O notation

$\text{class_OrderedGroup_Oab_group_add}(t_a) \Rightarrow v_a = c_plus(c_minus(v_a, v_b, t_a), v_b, t_a)$ $\text{cnf}(\text{cls_OrderedGroup_Ocomm_monoid_add}, \text{axiom})$
 $(\text{class_OrderedGroup_Oordered_ab_group_add}(t_a) \text{ and } c_lessequals(c_plus(v_a, v_b, t_a), v_c, t_a)) \Rightarrow c_lessequals(v_a, c_minus(v_c, v_b, t_a), t_a)$
 $c_lessequals(v_k(v_U), v_f(v_U), t_b)$ $\text{cnf}(\text{cls_conjecture}_1, \text{negated_conjecture})$
 $\neg c_lessequals(c_minus(v_k(v_x), v_g(v_x), t_b), c_minus(v_f(v_x), v_g(v_x), t_b), t_b)$ $\text{cnf}(\text{cls_conjecture}_3, \text{negated_conjecture})$
 $\text{class_OrderedGroup_Oordered_ab_group_add}(t) \Rightarrow \text{class_OrderedGroup_Oab_group_add}(t)$ $\text{cnf}(\text{clsrel_OrderedGroup_Oab_group_add}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t) \Rightarrow \text{class_OrderedGroup_Oordered_ab_group_add}(t)$ $\text{cnf}(\text{clsrel_Ring_and_Field_Ordered_idom}_{33}, \text{axiom})$
 $\text{class_Ring_and_Field_Ordered_idom}(t_b)$ $\text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$


```

c_lessequals(c_HOL_Oabs(c_Orderings_Omax(c_minus(v_f(v_U), v_g(v_U), t_b), c_0, t_b), t_b), c_times(v_c, c_HOL_Oabs(v_h(v_U)
- c_lessequals(v_f(v_x(v_U)), c_plus(v_g(v_x(v_U)), c_times(v_U, c_HOL_Oabs(v_h(v_x(v_U)), t_b), t_b), t_b), t_b)    cnf(cls_conj
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)
(class_OrderedGroup_Olordered_ab_group_abs(t_a) and c_lessequals(c_0, v_y, t_a)) => c_HOL_Oabs(v_y, t_a) = v_y    cnf(c
class_OrderedGroup_Oab_group_add(t_a) => c_plus(v_a, c_uminus(v_b, t_a), t_a) = c_minus(v_a, v_b, t_a)    cnf(cls_Ordered
(class_OrderedGroup_Oordered_ab_group_add(t_a) and c_lessequals(c_minus(v_a, v_b, t_a), v_c, t_a)) => c_lessequals(v_a, c
class_OrderedGroup_Oab_group_add(t_a) => c_minus(v_a, c_uminus(v_b, t_a), t_a) = c_plus(v_a, v_b, t_a)    cnf(cls_Ordered
class_OrderedGroup_Oab_group_add(t_a) => c_uminus(c_plus(v_a, v_b, t_a), t_a) = c_plus(c_uminus(v_a, t_a), c_uminus(v_b, t
class_OrderedGroup_Oab_group_add(t_a) => c_uminus(c_minus(v_a, v_b, t_a), t_a) = c_minus(v_b, v_a, t_a)    cnf(cls_Order
class_OrderedGroup_Oab_group_add(t_a) => c_uminus(c_uminus(v_y, t_a), t_a) = v_y    cnf(cls_OrderedGroup_Ominus_m
class_Orderings_Olinorder(t_b) => c_lessequals(v_y, c_Orderings_Omax(v_x, v_y, t_b), t_b)    cnf(cls_Orderings_Ole_maxI2_0,
(class_Orderings_Olinorder(t_b) and c_lessequals(c_Orderings_Omax(v_x, v_y, t_b), v_z, t_b)) => c_lessequals(v_x, v_z, t_b)
class_OrderedGroup_Olordered_ab_group_abs(t) => class_OrderedGroup_Oordered_ab_group_add(t)    cnf(clsrel_Order
class_Ring_and_Field_Ordered_idom(t) => class_Orderings_Olinorder(t)    cnf(clsrel_Ring_and_Field_Ordered_idom_33
class_Ring_and_Field_Ordered_idom(t) => class_OrderedGroup_Oab_group_add(t)    cnf(clsrel_Ring_and_Field_Oorde
class_Ring_and_Field_Ordered_idom(t) => class_OrderedGroup_Olordered_ab_group_abs(t)    cnf(clsrel_Ring_and_Fi

```

ANA031-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MS001-1.ax')
include('Axioms/MS001-0.ax')
class_OrderedGroup_Osemigroup_mult(t_a) => c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, v_b, t_a) = c_times(v_b, v_a, t_a)    cnf(cls_OrderedGroup_O
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, c_times(v_b, v_c, t_a), t_a) = c_times(v_b, c_times(v_a, v_c, t_a), t_a)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) => c_lessequals
class_OrderedGroup_Ocomm_monoid_mult(t_a) => c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAn
class_OrderedGroup_Ocomm_monoid_add(t_a) => c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAn
c_lessequals(c_HOL_Oabs(v_b(v_U), t_b), c_times(v_c, c_HOL_Oabs(v_g(v_U), t_b), t_b), t_b)    cnf(cls_conjecture_0, negated_co
- c_lessequals(c_times(c_HOL_Oabs(v_b(v_x(v_U))), t_b), c_HOL_Oabs(v_f(v_x(v_U))), t_b), t_b), c_times(v_U, c_times(c_HOL_Oa
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA031-2.p Problem about Big-O notation

```

c_lessequals(c_HOL_Oabs(v_b(v_U), t_b), c_times(v_c, c_HOL_Oabs(v_g(v_U), t_b), t_b), t_b)    cnf(cls_conjecture_0, negated_co
- c_lessequals(c_times(c_HOL_Oabs(v_b(v_x(v_U))), t_b), c_HOL_Oabs(v_f(v_x(v_U))), t_b), t_b), c_times(v_U, c_times(c_HOL_Oa
class_OrderedGroup_Olordered_ab_group_abs(t_a) => c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)    cnf(cls_OrderedGroup
class_OrderedGroup_Osemigroup_mult(t_a) => c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, v_b, t_a) = c_times(v_b, v_a, t_a)    cnf(cls_OrderedGroup_O
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) => c_lessequals
class_Ring_and_Field_Ordered_idom(t) => class_OrderedGroup_Oab_semigroup_mult(t)    cnf(clsrel_Ring_and_Field_O
class_Ring_and_Field_Ordered_idom(t) => class_OrderedGroup_Osemigroup_mult(t)    cnf(clsrel_Ring_and_Field_Oorde
class_Ring_and_Field_Ordered_idom(t) => class_Ring_and_Field_Oordered_semiring(t)    cnf(clsrel_Ring_and_Field
class_Ring_and_Field_Ordered_idom(t) => class_OrderedGroup_Olordered_ab_group_abs(t)    cnf(clsrel_Ring_and_Fi
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA032-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MS001-1.ax')
include('Axioms/MS001-0.ax')
class_OrderedGroup_Osemigroup_mult(t_a) => c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, v_b, t_a) = c_times(v_b, v_a, t_a)    cnf(cls_OrderedGroup_O
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, c_times(v_b, v_c, t_a), t_a) = c_times(v_b, c_times(v_a, v_c, t_a), t_a)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) => c_lessequals
class_OrderedGroup_Ocomm_monoid_mult(t_a) => c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAn
class_OrderedGroup_Ocomm_monoid_add(t_a) => c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAn
c_lessequals(c_HOL_Oabs(v_b(v_x), t_b), c_times(v_c, c_HOL_Oabs(v_g(v_x), t_b), t_b), t_b)    cnf(cls_conjecture_0, negated_co
- c_lessequals(c_times(c_HOL_Oabs(v_b(v_x), t_b), c_HOL_Oabs(v_f(v_x), t_b), t_b), c_times(v_c, c_times(c_HOL_Oabs(v_f(v_x),
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA032-2.p Problem about Big-O notation

```

class_OrderedGroup_Olordered_ab_group_abs(t_a) => c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)    cnf(cls_OrderedGroup
class_OrderedGroup_Osemigroup_mult(t_a) => c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)

```

```

class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, v_b, t_a) = c_times(v_b, v_a, t_a)    cnf(cls_OrderedGroup_Oab_semigroup_mult, axiom)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) => c_lessequals(v_c, v_a, v_b, t_a)
c_lessequals(c_HOL_Oabs(v_b(v_x), t_b), c_times(v_c, c_HOL_Oabs(v_g(v_x), t_b), t_b), t_b)    cnf(cls_conjecture_0, negated_conjecture)
~ c_lessequals(c_times(c_HOL_Oabs(v_b(v_x), t_b), c_HOL_Oabs(v_f(v_x), t_b), t_b), c_times(v_c, c_times(c_HOL_Oabs(v_f(v_x), t_b), t_b), t_b))
class_Ring_and_Field_Oordered_idom(t) => class_OrderedGroup_Oab_semigroup_mult(t)    cnf(clsrel_Ring_and_Field_Oordered_idom, axiom)
class_Ring_and_Field_Oordered_idom(t) => class_OrderedGroup_Osemigroup_mult(t)    cnf(clsrel_Ring_and_Field_Oordered_idom, axiom)
class_Ring_and_Field_Oordered_idom(t) => class_Ring_and_Field_Oordered_semiring(t)    cnf(clsrel_Ring_and_Field_Oordered_idom, axiom)
class_Ring_and_Field_Oordered_idom(t) => class_OrderedGroup_Oordered_ab_group_abs(t)    cnf(clsrel_Ring_and_Field_Oordered_idom, axiom)
class_Ring_and_Field_Oordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA033-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MS001-1.ax')
include('Axioms/MS001-0.ax')
class_OrderedGroup_Osemigroup_mult(t_a) => c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)    cnf(cls_OrderedGroup_Osemigroup_mult, axiom)
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, v_b, t_a) = c_times(v_b, v_a, t_a)    cnf(cls_OrderedGroup_Oab_semigroup_mult, axiom)
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, c_times(v_b, v_c, t_a), t_a) = c_times(v_b, c_times(v_a, v_c, t_a), t_a)    cnf(cls_OrderedGroup_Oab_semigroup_mult, axiom)
(class_Orderings_Oorder(t_a) and c_less(v_x, v_y, t_a)) => c_lessequals(v_x, v_y, t_a)    cnf(cls_Orderings_Oorder_less_imp, axiom)
class_Ring_and_Field_Oordered_idom(t_a) => c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)    cnf(cls_Ring_and_Field_Oordered_idom, axiom)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_c, v_d, t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(v_c, v_a, v_b, t_a)) =>
c_lessequals(c_times(v_a, v_c, t_a), c_times(v_b, v_d, t_a), t_a)    cnf(cls_Ring_and_Field_Omult_mono_0, axiom)
(class_Ring_and_Field_Oordered_cancel_semiring(t_a) and c_lessequals(c_0, v_b, t_a) and c_lessequals(c_0, v_a, t_a)) =>
c_lessequals(c_0, c_times(v_a, v_b, t_a), t_a)    cnf(cls_Ring_and_Field_Omult_nonneg_nonneg_0, axiom)
class_OrderedGroup_Ocomm_monoid_mult(t_a) => c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, axiom)
class_OrderedGroup_Ocomm_monoid_add(t_a) => c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, axiom)
c_less(c_0, v_c, t_b)    cnf(cls_conjecture_0, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_a(v_U), t_b), c_times(v_c, c_HOL_Oabs(v_f(v_U), t_b), t_b), t_b)    cnf(cls_conjecture_1, negated_conjecture)
c_less(c_0, v_ca, t_b)    cnf(cls_conjecture_2, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_b(v_U), t_b), c_times(v_ca, c_HOL_Oabs(v_g(v_U), t_b), t_b), t_b)    cnf(cls_conjecture_3, negated_conjecture)
~ c_lessequals(c_HOL_Oabs(c_times(v_a(v_x(v_U)), v_b(v_x(v_U))), t_b), t_b), c_times(v_U, c_HOL_Oabs(c_times(v_f(v_x(v_U))), t_b), t_b)
class_Ring_and_Field_Oordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA034-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MS001-1.ax')
include('Axioms/MS001-0.ax')
class_OrderedGroup_Osemigroup_mult(t_a) => c_times(c_times(v_a, v_b, t_a), v_c, t_a) = c_times(v_a, c_times(v_b, v_c, t_a), t_a)    cnf(cls_OrderedGroup_Osemigroup_mult, axiom)
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, v_b, t_a) = c_times(v_b, v_a, t_a)    cnf(cls_OrderedGroup_Oab_semigroup_mult, axiom)
class_OrderedGroup_Oab_semigroup_mult(t_a) => c_times(v_a, c_times(v_b, v_c, t_a), t_a) = c_times(v_b, c_times(v_a, v_c, t_a), t_a)    cnf(cls_OrderedGroup_Oab_semigroup_mult, axiom)
(class_Orderings_Oorder(t_a) and c_less(v_x, v_y, t_a)) => c_lessequals(v_x, v_y, t_a)    cnf(cls_Orderings_Oorder_less_imp, axiom)
class_Ring_and_Field_Oordered_idom(t_a) => c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)    cnf(cls_Ring_and_Field_Oordered_idom, axiom)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_c, v_d, t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(v_c, v_a, v_b, t_a)) =>
c_lessequals(c_times(v_a, v_c, t_a), c_times(v_b, v_d, t_a), t_a)    cnf(cls_Ring_and_Field_Omult_mono_0, axiom)
(class_Ring_and_Field_Oordered_cancel_semiring(t_a) and c_lessequals(c_0, v_b, t_a) and c_lessequals(c_0, v_a, t_a)) =>
c_lessequals(c_0, c_times(v_a, v_b, t_a), t_a)    cnf(cls_Ring_and_Field_Omult_nonneg_nonneg_0, axiom)
class_OrderedGroup_Ocomm_monoid_mult(t_a) => c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, axiom)
class_OrderedGroup_Ocomm_monoid_add(t_a) => c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, axiom)
c_less(c_0, v_c, t_b)    cnf(cls_conjecture_0, negated_conjecture)
c_less(c_0, v_ca, t_b)    cnf(cls_conjecture_1, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_a(v_x), t_b), c_times(v_c, c_HOL_Oabs(v_f(v_x), t_b), t_b), t_b)    cnf(cls_conjecture_2, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_b(v_x), t_b), c_times(v_ca, c_HOL_Oabs(v_g(v_x), t_b), t_b), t_b)    cnf(cls_conjecture_3, negated_conjecture)
c_times(c_times(v_c, v_ca, t_b), c_HOL_Oabs(c_times(v_f(v_x), v_g(v_x), t_b), t_b), t_b) = c_times(c_times(v_c, c_HOL_Oabs(v_f(v_x), t_b), t_b), t_b)
~ c_lessequals(c_HOL_Oabs(c_times(v_a(v_x), v_b(v_x), t_b), t_b), c_times(c_times(v_c, v_ca, t_b), c_HOL_Oabs(c_times(v_f(v_x), t_b), t_b), t_b))
class_Ring_and_Field_Oordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA034-2.p Problem about Big-O notation

```

class_Ring_and_Field_Oordered_idom(t_a) => c_HOL_Oabs(c_times(v_a, v_b, t_a), t_a) = c_times(c_HOL_Oabs(v_a, t_a), c_HOL_Oabs(v_b, t_a), t_a)    cnf(cls_Ring_and_Field_Oordered_idom, axiom)
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_c, v_d, t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(v_c, v_a, v_b, t_a)) =>
c_lessequals(c_times(v_a, v_c, t_a), c_times(v_b, v_d, t_a), t_a)    cnf(cls_Ring_and_Field_Omult_mono_0, axiom)
(class_Ring_and_Field_Oordered_cancel_semiring(t_a) and c_lessequals(c_0, v_b, t_a) and c_lessequals(c_0, v_a, t_a)) =>
c_lessequals(c_0, c_times(v_a, v_b, t_a), t_a)    cnf(cls_Ring_and_Field_Omult_nonneg_nonneg_0, axiom)

```

$(\text{class_Orderings_Oorder}(t_a) \text{ and } c.\text{less}(v_x, v_y, t_a)) \Rightarrow c.\text{lessequals}(v_x, v_y, t_a) \quad \text{cnf}(\text{cls_Orderings_Oorder_less_imp})$
 $\text{class_OrderedGroup_Oordered_ab_group_abs}(t_a) \Rightarrow c.\text{lessequals}(c_0, c.\text{HOL_Oabs}(v_a, t_a), t_a) \quad \text{cnf}(\text{cls_OrderedGroup_Oordered_ab_group_abs})$
 $c.\text{less}(c_0, v_c, t_b) \quad \text{cnf}(\text{cls_conjecture}_0, \text{negated_conjecture})$
 $c.\text{lessequals}(c.\text{HOL_Oabs}(v_a(v_x), t_b), c.\text{times}(v_c, c.\text{HOL_Oabs}(v_f(v_x), t_b), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_2, \text{negated_conjecture})$
 $c.\text{lessequals}(c.\text{HOL_Oabs}(v_b(v_x), t_b), c.\text{times}(v_ca, c.\text{HOL_Oabs}(v_g(v_x), t_b), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_3, \text{negated_conjecture})$
 $c.\text{times}(c.\text{times}(v_c, v_ca, t_b), c.\text{HOL_Oabs}(c.\text{times}(v_f(v_x), v_g(v_x), t_b), t_b), t_b) = c.\text{times}(c.\text{times}(v_c, c.\text{HOL_Oabs}(v_f(v_x), v_g(v_x), t_b), t_b), c.\text{times}(v_c, v_ca, t_b), c.\text{HOL_Oabs}(c.\text{times}(v_f(v_x), v_g(v_x), t_b), t_b))$
 $\text{class_Ring_and_Field_Oordered_idom}(t_b) \quad \text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$
 $\text{class_OrderedGroup_Oordered_ab_group_abs}(t) \Rightarrow \text{class_Orderings_Oorder}(t) \quad \text{cnf}(\text{clsrel_OrderedGroup_Oordered_ab_group_abs})$
 $\text{class_Ring_and_Field_Oordered_idom}(t) \Rightarrow \text{class_Ring_and_Field_Oordered_cancel_semiring}(t) \quad \text{cnf}(\text{clsrel_Ring_and_Field_Oordered_cancel_semiring})$
 $\text{class_Ring_and_Field_Oordered_idom}(t) \Rightarrow \text{class_Ring_and_Field_Oordered_semiring}(t) \quad \text{cnf}(\text{clsrel_Ring_and_Field_Oordered_semiring})$
 $\text{class_Ring_and_Field_Oordered_idom}(t) \Rightarrow \text{class_OrderedGroup_Oordered_ab_group_abs}(t) \quad \text{cnf}(\text{clsrel_Ring_and_Field_Oordered_idom})$

ANA035-1.p Problem about Big-O notation

$\text{include}(' \text{Axioms/ANA003-0.ax}')$
 $\text{include}(' \text{Axioms/MS001-1.ax}')$
 $\text{include}(' \text{Axioms/MS001-0.ax}')$
 $\text{class_OrderedGroup_Osemigroup_mult}(t_a) \Rightarrow c.\text{times}(c.\text{times}(v_a, v_b, t_a), v_c, t_a) = c.\text{times}(v_a, c.\text{times}(v_b, v_c, t_a), t_a)$
 $\text{class_OrderedGroup_Oab_semigroup_mult}(t_a) \Rightarrow c.\text{times}(v_a, v_b, t_a) = c.\text{times}(v_b, v_a, t_a) \quad \text{cnf}(\text{cls_OrderedGroup_Oab_semigroup_mult})$
 $\text{class_OrderedGroup_Oab_semigroup_mult}(t_a) \Rightarrow c.\text{times}(v_a, c.\text{times}(v_b, v_c, t_a), t_a) = c.\text{times}(v_b, c.\text{times}(v_a, v_c, t_a), t_a)$
 $(\text{class_Orderings_Oorder}(t_a) \text{ and } c.\text{less}(v_x, v_y, t_a)) \Rightarrow c.\text{lessequals}(v_x, v_y, t_a) \quad \text{cnf}(\text{cls_Orderings_Oorder_less_imp})$
 $\text{class_Ring_and_Field_Oordered_idom}(t_a) \Rightarrow c.\text{HOL_Oabs}(c.\text{times}(v_a, v_b, t_a), t_a) = c.\text{times}(c.\text{HOL_Oabs}(v_a, t_a), c.\text{HOL_Oabs}(v_b, t_a))$
 $(\text{class_Ring_and_Field_Oordered_cancel_semiring}(t_a) \text{ and } c.\text{lessequals}(v_c, v_d, t_a) \text{ and } c.\text{lessequals}(v_a, v_b, t_a) \text{ and } c.\text{lessequals}(v_c, v_d, t_a)) \Rightarrow c.\text{lessequals}(c.\text{times}(v_a, v_c, t_a), c.\text{times}(v_b, v_d, t_a), t_a) \quad \text{cnf}(\text{cls_Ring_and_Field_Omult_mono}_0, \text{axiom})$
 $(\text{class_Ring_and_Field_Oordered_cancel_semiring}(t_a) \text{ and } c.\text{lessequals}(c_0, v_b, t_a) \text{ and } c.\text{lessequals}(c_0, v_a, t_a)) \Rightarrow c.\text{lessequals}(c_0, c.\text{times}(v_a, v_b, t_a), t_a) \quad \text{cnf}(\text{cls_Ring_and_Field_Omult_nonneg_nonneg}_0, \text{axiom})$
 $\text{class_OrderedGroup_Ocomm_monoid_mult}(t_a) \Rightarrow c.\text{SetsAndFunctions_Oelt_set_times}(c_1, v_y, t_a) = v_y \quad \text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_times})$
 $\text{class_OrderedGroup_Ocomm_monoid_add}(t_a) \Rightarrow c.\text{SetsAndFunctions_Oelt_set_plus}(c_0, v_y, t_a) = v_y \quad \text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_plus})$
 $c.\text{less}(c_0, v_c, t_b) \quad \text{cnf}(\text{cls_conjecture}_0, \text{negated_conjecture})$
 $c.\text{less}(c_0, v_ca, t_b) \quad \text{cnf}(\text{cls_conjecture}_1, \text{negated_conjecture})$
 $c.\text{lessequals}(c.\text{HOL_Oabs}(v_a(v_x), t_b), c.\text{times}(v_c, c.\text{HOL_Oabs}(v_f(v_x), t_b), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_2, \text{negated_conjecture})$
 $c.\text{lessequals}(c.\text{HOL_Oabs}(v_b(v_x), t_b), c.\text{times}(v_ca, c.\text{HOL_Oabs}(v_g(v_x), t_b), t_b), t_b) \quad \text{cnf}(\text{cls_conjecture}_3, \text{negated_conjecture})$
 $c.\text{times}(c.\text{times}(v_c, v_ca, t_b), c.\text{HOL_Oabs}(c.\text{times}(v_f(v_x), v_g(v_x), t_b), t_b), t_b) \neq c.\text{times}(c.\text{times}(v_c, c.\text{HOL_Oabs}(v_f(v_x), v_g(v_x), t_b), t_b), c.\text{times}(v_c, v_ca, t_b), c.\text{HOL_Oabs}(c.\text{times}(v_f(v_x), v_g(v_x), t_b), t_b))$
 $\text{class_Ring_and_Field_Oordered_idom}(t_b) \quad \text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$

ANA035-2.p Problem about Big-O notation

$c.\text{times}(c.\text{times}(v_c, v_ca, t_b), c.\text{HOL_Oabs}(c.\text{times}(v_f(v_x), v_g(v_x), t_b), t_b), t_b) \neq c.\text{times}(c.\text{times}(v_c, c.\text{HOL_Oabs}(v_f(v_x), v_g(v_x), t_b), t_b), c.\text{times}(v_c, v_ca, t_b), c.\text{HOL_Oabs}(c.\text{times}(v_f(v_x), v_g(v_x), t_b), t_b))$
 $\text{class_Ring_and_Field_Oordered_idom}(t_b) \quad \text{cnf}(\text{tfree_tcs}, \text{negated_conjecture})$
 $\text{class_OrderedGroup_Oab_semigroup_mult}(t_a) \Rightarrow c.\text{times}(v_a, v_b, t_a) = c.\text{times}(v_b, v_a, t_a) \quad \text{cnf}(\text{cls_OrderedGroup_Oab_semigroup_mult})$
 $\text{class_OrderedGroup_Oab_semigroup_mult}(t_a) \Rightarrow c.\text{times}(v_a, c.\text{times}(v_b, v_c, t_a), t_a) = c.\text{times}(v_b, c.\text{times}(v_a, v_c, t_a), t_a)$
 $\text{class_Ring_and_Field_Oordered_idom}(t_a) \Rightarrow c.\text{HOL_Oabs}(c.\text{times}(v_a, v_b, t_a), t_a) = c.\text{times}(c.\text{HOL_Oabs}(v_a, t_a), c.\text{HOL_Oabs}(v_b, t_a))$
 $\text{class_Ring_and_Field_Oordered_idom}(t) \Rightarrow \text{class_OrderedGroup_Oab_semigroup_mult}(t) \quad \text{cnf}(\text{clsrel_Ring_and_Field_Oordered_idom})$

ANA036-1.p Problem about Big-O notation

$\text{include}(' \text{Axioms/ANA003-0.ax}')$
 $\text{include}(' \text{Axioms/MS001-1.ax}')$
 $\text{include}(' \text{Axioms/MS001-0.ax}')$
 $\text{class_OrderedGroup_Oordered_ab_group_abs}(t_a) \Rightarrow c.\text{lessequals}(c.\text{HOL_Oabs}(c.\text{plus}(v_a, v_b, t_a), t_a), c.\text{plus}(c.\text{HOL_Oabs}(v_a, t_a), c.\text{HOL_Oabs}(v_b, t_a)))$
 $(\text{class_OrderedGroup_Oordered_ab_semigroup_add}(t_a) \text{ and } c.\text{lessequals}(v_c, v_d, t_a) \text{ and } c.\text{lessequals}(v_a, v_b, t_a)) \Rightarrow c.\text{lessequals}(c.\text{plus}(v_a, v_c, t_a), c.\text{plus}(v_b, v_d, t_a), t_a) \quad \text{cnf}(\text{cls_OrderedGroup_Oadd_mono}_0, \text{axiom})$
 $(\text{class_OrderedGroup_Ocomm_monoid_add}(t_a) \text{ and } \text{class_OrderedGroup_Oordered_cancel_ab_semigroup_add}(t_a) \text{ and } c.\text{lessequals}(c_0, c.\text{plus}(v_x, v_y, t_a), t_a)) \Rightarrow c.\text{lessequals}(c_0, c.\text{plus}(v_x, v_y, t_a), t_a) \quad \text{cnf}(\text{cls_OrderedGroup_Oadd_nonneg_nonneg}_0, \text{axiom})$
 $\text{class_Orderings_Olinorder}(t_b) \Rightarrow c.\text{lessequals}(v_x, c.\text{Orderings_Omax}(v_x, v_y, t_b), t_b) \quad \text{cnf}(\text{cls_Orderings_Ole_maxI1}_0)$
 $\text{class_Orderings_Olinorder}(t_b) \Rightarrow c.\text{lessequals}(v_y, c.\text{Orderings_Omax}(v_x, v_y, t_b), t_b) \quad \text{cnf}(\text{cls_Orderings_Ole_maxI2}_0)$
 $(\text{class_Orderings_Oorder}(t_a) \text{ and } c.\text{lessequals}(v_y, v_z, t_a) \text{ and } c.\text{lessequals}(v_x, v_y, t_a)) \Rightarrow c.\text{lessequals}(v_x, v_z, t_a) \quad \text{cnf}(\text{cls_Orderings_Ole_maxI1}_0)$
 $(\text{class_Orderings_Oorder}(t_a) \text{ and } c.\text{less}(v_x, v_y, t_a) \text{ and } c.\text{lessequals}(v_y, v_z, t_a)) \Rightarrow c.\text{less}(v_x, v_z, t_a) \quad \text{cnf}(\text{cls_Orderings_Ole_maxI2}_0)$
 $(\text{class_Ring_and_Field_Oordered_cancel_semiring}(t_a) \text{ and } c.\text{lessequals}(v_a, v_b, t_a) \text{ and } c.\text{lessequals}(c_0, v_c, t_a)) \Rightarrow c.\text{lessequals}(c.\text{times}(v_a, v_c, t_a), c.\text{times}(v_b, v_c, t_a), t_a)$
 $\text{class_Ring_and_Field_Osemiring}(t_a) \Rightarrow c.\text{times}(v_a, c.\text{plus}(v_b, v_c, t_a), t_a) = c.\text{plus}(c.\text{times}(v_a, v_b, t_a), c.\text{times}(v_a, v_c, t_a))$
 $\text{class_Ring_and_Field_Osemiring}(t_a) \Rightarrow c.\text{times}(c.\text{plus}(v_a, v_b, t_a), v_c, t_a) = c.\text{plus}(c.\text{times}(v_a, v_c, t_a), c.\text{times}(v_b, v_c, t_a))$
 $\text{class_OrderedGroup_Ocomm_monoid_mult}(t_a) \Rightarrow c.\text{SetsAndFunctions_Oelt_set_times}(c_1, v_y, t_a) = v_y \quad \text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_times})$
 $\text{class_OrderedGroup_Ocomm_monoid_add}(t_a) \Rightarrow c.\text{SetsAndFunctions_Oelt_set_plus}(c_0, v_y, t_a) = v_y \quad \text{cnf}(\text{cls_SetsAndFunctions_Oelt_set_plus})$


```

c_lessequals(c_0, v_f(v_U), t_b)    cnf(cls_conjecture_0, negated_conjecture)
c_lessequals(c_0, v_g(v_U), t_b)    cnf(cls_conjecture_1, negated_conjecture)
c_less(c_0, v_c, t_b)               cnf(cls_conjecture_2, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_a(v_U), t_b), c_times(v_c, v_f(v_U), t_b), t_b)    cnf(cls_conjecture_3, negated_conjecture)
c_less(c_0, v_ca, t_b)              cnf(cls_conjecture_4, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_b(v_U), t_b), c_times(v_ca, v_g(v_U), t_b), t_b)    cnf(cls_conjecture_5, negated_conjecture)
c_lessequals(c_HOL_Oabs(c_plus(v_a(v_xa(v_U)), v_b(v_xa(v_U))), t_b), t_b), c_times(v_U, c_HOL_Oabs(c_plus(v_f(v_xa(v_U))), v
¬ c_less(c_0, v_U, t_b)              cnf(cls_conjecture_6, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA037-2.p Problem about Big-O notation

```

c_lessequals(c_0, v_f(v_xa), t_b)    cnf(cls_conjecture_2, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_a(v_xa), t_b), c_times(v_c, v_f(v_xa), t_b), t_b)    cnf(cls_conjecture_4, negated_conjecture)
¬ c_lessequals(c_HOL_Oabs(v_a(v_xa), t_b), c_times(c_Orderings_Omax(v_c, v_ca, t_b), v_f(v_xa), t_b), t_b)    cnf(cls_conjectur
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)
class_Orderings_Olinorder(t_b) ⇒ c_lessequals(v_x, c_Orderings_Omax(v_x, v_y, t_b), t_b)    cnf(cls_Orderings_Ole_maxI1_0,
(class_Orderings_Oorder(t_a) and c_lessequals(v_y, v_z, t_a) and c_lessequals(v_x, v_y, t_a)) ⇒ c_lessequals(v_x, v_z, t_a)    cr
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Orderings_Olinorder(t)    cnf(clsrel_Ring_and_Field_Ordered_idom_33
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Ring_and_Field_Oordered_semiring(t)    cnf(clsrel_Ring_and_Field
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Orderings_Oorder(t)    cnf(clsrel_Ring_and_Field_Ordered_idom_44, a

```

ANA038-2.p Problem about Big-O notation

```

c_lessequals(c_0, v_g(v_xa), t_b)    cnf(cls_conjecture_3, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_b(v_xa), t_b), c_times(v_ca, v_g(v_xa), t_b), t_b)    cnf(cls_conjecture_5, negated_conjecture)
¬ c_lessequals(c_HOL_Oabs(v_b(v_xa), t_b), c_times(c_Orderings_Omax(v_c, v_ca, t_b), v_g(v_xa), t_b), t_b)    cnf(cls_conjectur
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)
class_Orderings_Olinorder(t_b) ⇒ c_lessequals(v_y, c_Orderings_Omax(v_x, v_y, t_b), t_b)    cnf(cls_Orderings_Ole_maxI2_0,
(class_Orderings_Oorder(t_a) and c_lessequals(v_y, v_z, t_a) and c_lessequals(v_x, v_y, t_a)) ⇒ c_lessequals(v_x, v_z, t_a)    cr
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Orderings_Olinorder(t)    cnf(clsrel_Ring_and_Field_Ordered_idom_33
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Ring_and_Field_Oordered_semiring(t)    cnf(clsrel_Ring_and_Field
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Orderings_Oorder(t)    cnf(clsrel_Ring_and_Field_Ordered_idom_44, a

```

ANA039-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MS001-2.ax')
include('Axioms/MS001-0.ax')
class_OrderedGroup_Olordered_ab_group_abs(t_a) ⇒ c_lessequals(v_a, c_HOL_Oabs(v_a, t_a), t_a)    cnf(cls_OrderedGroup
class_OrderedGroup_Olordered_ab_group_abs(t_a) ⇒ c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)    cnf(cls_OrderedGroup
(class_Orderings_Oorder(t_a) and c_lessequals(v_y, v_z, t_a) and c_lessequals(v_x, v_y, t_a)) ⇒ c_lessequals(v_x, v_z, t_a)    cr
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals
c_lessequals(c_HOL_Oabs(v_h(v_U), t_a), c_times(v_c, c_HOL_Oabs(v_f(v_U), t_a), t_a), t_a)    cnf(cls_conjecture_0, negated_conj
v_c ≠ c_0    cnf(cls_conjecture_1, negated_conjecture)
¬ c_lessequals(c_HOL_Oabs(v_h(v_x), t_a), c_times(c_HOL_Oabs(v_c, t_a), c_HOL_Oabs(v_f(v_x), t_a), t_a), t_a)    cnf(cls_conj
class_Ring_and_Field_Ordered_idom(t_a)    cnf(tfree_tcs, negated_conjecture)

```

ANA039-2.p Problem about Big-O notation

```

c_lessequals(c_HOL_Oabs(v_h(v_U), t_a), c_times(v_c, c_HOL_Oabs(v_f(v_U), t_a), t_a), t_a)    cnf(cls_conjecture_0, negated_conj
¬ c_lessequals(c_HOL_Oabs(v_h(v_x), t_a), c_times(c_HOL_Oabs(v_c, t_a), c_HOL_Oabs(v_f(v_x), t_a), t_a), t_a)    cnf(cls_conj
class_Ring_and_Field_Ordered_idom(t_a)    cnf(tfree_tcs, negated_conjecture)
class_OrderedGroup_Olordered_ab_group_abs(t_a) ⇒ c_lessequals(v_a, c_HOL_Oabs(v_a, t_a), t_a)    cnf(cls_OrderedGroup
class_OrderedGroup_Olordered_ab_group_abs(t_a) ⇒ c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)    cnf(cls_OrderedGroup
(class_Orderings_Oorder(t_a) and c_lessequals(v_y, v_z, t_a) and c_lessequals(v_x, v_y, t_a)) ⇒ c_lessequals(v_x, v_z, t_a)    cr
(class_Ring_and_Field_Oordered_semiring(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Ring_and_Field_Oordered_semiring(t)    cnf(clsrel_Ring_and_Field
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Orderings_Oorder(t)    cnf(clsrel_Ring_and_Field_Ordered_idom_44, a
class_Ring_and_Field_Ordered_idom(t) ⇒ class_OrderedGroup_Olordered_ab_group_abs(t)    cnf(clsrel_Ring_and_Field

```

ANA041-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MS001-1.ax')

```

```

include('Axioms/MSC001-0.ax')
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)
c_lessequals(c_0, v_h(v_U, v_V), t_c)    cnf(cls_conjecture_0, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_f(v_U, v_V), t_c), c_times(v_x, v_h(v_U, v_V), t_c), t_c)    cnf(cls_conjecture_1, negated_conjecture)
c_in(v_xb, v_A(v_xa), t_b)    cnf(cls_conjecture_2, negated_conjecture)
¬ c_lessequals(c_0, v_h(v_xa, v_xb), t_c)    cnf(cls_conjecture_3, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_c)    cnf(tfree_tcs, negated_conjecture)

```

ANA041-2.p Problem about Big-O notation

```

c_lessequals(c_0, v_h(v_U, v_V), t_c)    cnf(cls_conjecture_0, negated_conjecture)
¬ c_lessequals(c_0, v_h(v_xa, v_xb), t_c)    cnf(cls_conjecture_3, negated_conjecture)

```

ANA042-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)
c_lessequals(c_0, v_h(v_U, v_V), t_c)    cnf(cls_conjecture_0, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_f(v_U, v_V), t_c), c_times(v_x, v_h(v_U, v_V), t_c), t_c)    cnf(cls_conjecture_1, negated_conjecture)
c_in(v_xb(v_U), v_A(v_xa(v_U)), t_b)    cnf(cls_conjecture_2, negated_conjecture)
¬ c_lessequals(c_HOL_Oabs(v_f(v_xa(v_U), v_xb(v_U)), t_c), c_times(v_U, v_h(v_xa(v_U), v_xb(v_U)), t_c), t_c)    cnf(cls_conjecture_3, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t_c)    cnf(tfree_tcs, negated_conjecture)

```

ANA042-2.p Problem about Big-O notation

```

c_lessequals(c_HOL_Oabs(v_f(v_U, v_V), t_c), c_times(v_x, v_h(v_U, v_V), t_c), t_c)    cnf(cls_conjecture_1, negated_conjecture)
¬ c_lessequals(c_HOL_Oabs(v_f(v_xa(v_U), v_xb(v_U)), t_c), c_times(v_U, v_h(v_xa(v_U), v_xb(v_U)), t_c), t_c)    cnf(cls_conjecture_3, negated_conjecture)

```

ANA043-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
class_OrderedGroup_Oab_semigroup_mult(t_a) ⇒ c_times(v_a, c_times(v_b, v_c, t_a), t_a) = c_times(v_b, c_times(v_a, v_c, t_a), t_a)
(class_Ring_and_Field_Ordered_idom(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals(c_times(v_a, v_b, t_a), c_times(v_a, v_c, t_a), t_a)
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)
c_lessequals(c_HOL_Oabs(v_f(v_U), t_b), c_times(v_c, c_HOL_Oabs(v_h(v_U), t_b), t_b), t_b)    cnf(cls_conjecture_0, negated_conjecture)
¬ c_lessequals(c_times(c_HOL_Oabs(v_l(v_x(v_U), v_xa(v_U)), t_b), c_HOL_Oabs(v_f(v_k(v_x(v_U), v_xa(v_U))), t_b), t_b), c_times(v_c, c_HOL_Oabs(v_h(v_U), t_b), t_b), t_b)
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)

```

ANA043-2.p Problem about Big-O notation

```

c_lessequals(c_HOL_Oabs(v_f(v_U), t_b), c_times(v_c, c_HOL_Oabs(v_h(v_U), t_b), t_b), t_b)    cnf(cls_conjecture_0, negated_conjecture)
¬ c_lessequals(c_times(c_HOL_Oabs(v_l(v_x(v_U), v_xa(v_U)), t_b), c_HOL_Oabs(v_f(v_k(v_x(v_U), v_xa(v_U))), t_b), t_b), c_times(v_c, c_HOL_Oabs(v_h(v_U), t_b), t_b), t_b)
class_Ring_and_Field_Ordered_idom(t_b)    cnf(tfree_tcs, negated_conjecture)
class_OrderedGroup_Olordered_ab_group_abs(t_a) ⇒ c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)    cnf(cls_OrderedGroup_Olordered_ab_group_abs, negated_conjecture)
class_OrderedGroup_Oab_semigroup_mult(t_a) ⇒ c_times(v_a, c_times(v_b, v_c, t_a), t_a) = c_times(v_b, c_times(v_a, v_c, t_a), t_a)
(class_Ring_and_Field_Ordered_idom(t_a) and c_lessequals(v_a, v_b, t_a) and c_lessequals(c_0, v_c, t_a)) ⇒ c_lessequals(c_times(v_a, v_b, t_a), c_times(v_a, v_c, t_a), t_a)
class_Ring_and_Field_Ordered_idom(t) ⇒ class_OrderedGroup_Oab_semigroup_mult(t)    cnf(clsrel_Ring_and_Field_Ordered_idom, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t) ⇒ class_Ring_and_Field_Ordered_idom(t)    cnf(clsrel_Ring_and_Field_Ordered_idom, negated_conjecture)
class_Ring_and_Field_Ordered_idom(t) ⇒ class_OrderedGroup_Olordered_ab_group_abs(t)    cnf(clsrel_Ring_and_Field_Ordered_idom, negated_conjecture)

```

ANA044-1.p Problem about Big-O notation

```

include('Axioms/ANA003-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
(class_Ring_and_Field_Ordered_idom(t_a) and c_lessequals(c_0, v_b, t_a) and c_lessequals(c_0, v_a, t_a)) ⇒ c_lessequals(c_0, c_times(v_a, v_b, t_a), t_a)    cnf(cls_Ring_and_Field_Ordered_idom, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_mult(t_a) ⇒ c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_times, negated_conjecture)
class_OrderedGroup_Ocomm_monoid_add(t_a) ⇒ c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y    cnf(cls_SetsAndFunctions_Oelt_set_plus, negated_conjecture)
c_lessequals(c_0, v_l(v_U, v_V), t_b)    cnf(cls_conjecture_0, negated_conjecture)
c_lessequals(c_0, v_h(v_U), t_b)    cnf(cls_conjecture_1, negated_conjecture)
c_in(v_xa, v_A(v_x), t_d)    cnf(cls_conjecture_2, negated_conjecture)

```

$c_times(v_l(v_x, v_xa), v_h(v_k(v_x, v_xa)), t_b) \neq c_HOL_Oabs(c_times(v_l(v_x, v_xa), v_h(v_k(v_x, v_xa)), t_b), t_b)$ $cnf(cls_Ring_and_Field_Ordered_idom(t_b))$ $cnf(tfree_tcs, negated_conjecture)$

ANA044-2.p Problem about Big-O notation

$c_lessequals(c_0, v_l(v_U, v_V), t_b)$ $cnf(cls_conjecture_0, negated_conjecture)$
 $c_lessequals(c_0, v_h(v_U), t_b)$ $cnf(cls_conjecture_1, negated_conjecture)$
 $c_times(v_l(v_x, v_xa), v_h(v_k(v_x, v_xa)), t_b) \neq c_HOL_Oabs(c_times(v_l(v_x, v_xa), v_h(v_k(v_x, v_xa)), t_b), t_b)$ $cnf(cls_Ring_and_Field_Ordered_idom(t_b))$ $cnf(tfree_tcs, negated_conjecture)$
 $(class_OrderedGroup_Oordered_ab_group_abs(t_a) \text{ and } c_lessequals(c_0, v_y, t_a)) \Rightarrow c_HOL_Oabs(v_y, t_a) = v_y$ $cnf(c_lessequals(c_0, v_b, t_a) \text{ and } c_lessequals(c_0, v_a, t_a)) \Rightarrow$
 $c_lessequals(c_0, c_times(v_a, v_b, t_a), t_a)$ $cnf(cls_Ring_and_Field_Omult_nonneg_nonneg_0, axiom)$
 $class_Ring_and_Field_Ordered_idom(t) \Rightarrow class_Ring_and_Field_Oordered_cancel_semiring(t)$ $cnf(clsrel_Ring_and_Field_Ordered_idom(t) \Rightarrow class_OrderedGroup_Oordered_ab_group_abs(t))$ $cnf(clsrel_Ring_and_Field_Ordered_idom(t) \Rightarrow class_OrderedGroup_Oordered_ab_group_abs(t))$

ANA045-1.p Problem about Big-O notation

$include('Axioms/ANA003-0.ax')$
 $include('Axioms/MS001-1.ax')$
 $include('Axioms/MS001-0.ax')$
 $class_OrderedGroup_Ocomm_monoid_mult(t_a) \Rightarrow c_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y$ $cnf(cls_SetsAndFunctions_Oelt_set_times(c_1, v_y, t_a) = v_y)$
 $class_OrderedGroup_Ocomm_monoid_add(t_a) \Rightarrow c_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y$ $cnf(cls_SetsAndFunctions_Oelt_set_plus(c_0, v_y, t_a) = v_y)$
 $\neg c_lessequals(c_0, c_times(v_U, c_HOL_Oabs(v_g(v_x(v_U))), t_b), t_b)$ $cnf(cls_conjecture_0, negated_conjecture)$
 $class_Ring_and_Field_Ordered_idom(t_b)$ $cnf(tfree_tcs, negated_conjecture)$

ANA045-2.p Problem about Big-O notation

$\neg c_lessequals(c_0, c_times(v_U, c_HOL_Oabs(v_g(v_x(v_U))), t_b), t_b)$ $cnf(cls_conjecture_0, negated_conjecture)$
 $class_Ring_and_Field_Ordered_idom(t_b)$ $cnf(tfree_tcs, negated_conjecture)$
 $class_OrderedGroup_Oordered_ab_group_abs(t_a) \Rightarrow c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a)$ $cnf(cls_OrderedGroup_Oordered_ab_group_abs(t_a) \Rightarrow c_lessequals(c_0, c_HOL_Oabs(v_a, t_a), t_a))$
 $class_Ring_and_Field_Ordered_idom(t_a) \Rightarrow v_c = c_times(c_1, v_c, t_a)$ $cnf(cls_Ring_and_Field_Omult_cancel_right1)$
 $class_Ring_and_Field_Ordered_idom(t) \Rightarrow class_OrderedGroup_Oordered_ab_group_abs(t)$ $cnf(clsrel_Ring_and_Field_Ordered_idom(t) \Rightarrow class_OrderedGroup_Oordered_ab_group_abs(t))$