CAT axioms

CAT001-0.ax Category theory axioms

defined(x, y) $\Rightarrow$ $x \cdot y = x \circ y$ cnf(closure_of_composition, axiom)
$x \cdot y = z$ $\Rightarrow$ defined(x, y) cnf(associative_property$_1$, axiom)
($x \cdot y$=xy and defined(xy, z)) $\Rightarrow$ defined(y, z) cnf(associative_property$_2$, axiom)
($x \cdot y$=xy and $y \cdot z$=yz and defined(xy, z)) $\Rightarrow$ defined(x, yz) cnf(category_theory_axiom$_1$, axiom)
($x \cdot y$=xy and xy $\cdot z$=xyz and $y \cdot z$=yz) $\Rightarrow$ $x \cdot$ yz=xyz cnf(category_theory_axiom$_2$, axiom)
($y \cdot z$=yz and defined(x, yz)) $\Rightarrow$ defined(x, y) cnf(category_theory_axiom$_3$, axiom)
($y \cdot z$=yz and $x \cdot y$=xy and defined(x, yz)) $\Rightarrow$ defined(xy, z) cnf(category_theory_axiom$_4$, axiom)
($y \cdot z$=yz and $x \cdot$ yz=xyz and $x \cdot y$=xy) $\Rightarrow$ xy $\cdot z$=xyz cnf(category_theory_axiom$_5$, axiom)
(defined(x, y) and defined(y, z) and identity_map(y)) $\Rightarrow$ defined(x, z) cnf(category_theory_axiom$_6$, axiom)
identity_map(dom(x)) cnf(domain_is_an_identity_map, axiom)
identity_map(cod(x)) cnf(codomain_is_an_identity_map, axiom)
defined(x, dom(x)) cnf(mapping_from_x_to_its_domain, axiom)
defined(cod(x), x) cnf(mapping_from_codomain_of_x_to_x, axiom)
$x \cdot$ dom(x)=x cnf(product_on_domain, axiom)
cod(x) $\cdot x = x$ cnf(product_on_codomain, axiom)
(defined(x, y) and identity_map(x)) $\Rightarrow$ $x \cdot y = y$ cnf(identity$_1$, axiom)
(defined(x, y) and identity_map(y)) $\Rightarrow$ $x \cdot y = x$ cnf(identity$_2$, axiom)
($x \cdot y = z$ and $x \cdot y = w$) $\Rightarrow$ $z = w$ cnf(composition_is_well_defined, axiom)

CAT002-0.ax Category theory (equality) axioms

cod(dom(x)) = dom(x) cnf(codomain_of_domain_is_domain, axiom)
dom(cod(x)) = cod(x) cnf(domain_of_codomain_is_codomain, axiom)
dom(x) $\circ x = x$ cnf(domain_composition, axiom)
$x \circ$ cod(x) $= x$ cnf(codomain_composition, axiom)
cod(x) = dom(y) $\Rightarrow$ dom($x \circ y$) = dom(x) cnf(codomain_domain$_1$, axiom)
cod(x) = dom(y) $\Rightarrow$ cod($x \circ y$) = cod(y) cnf(codomain_domain$_2$, axiom)
(cod(x) = dom(y) and cod(y) = dom(z)) $\Rightarrow$ $x \circ (y \circ z) = (x \circ y) \circ z$ cnf(star_property, axiom)

CAT003-0.ax Category theory axioms

equivalent(x, y) $\Rightarrow$ there_exists(x) cnf(equivalence_implies_existence$_1$, axiom)
equivalent(x, y) $\Rightarrow$ $x = y$ cnf(equivalence_implies_existence$_2$, axiom)
(there_exists(x) and $x = y$) $\Rightarrow$ equivalent(x, y) cnf(existence_and_equality_implies_equivalence$_1$, axiom)
there_exists(dom(x)) $\Rightarrow$ there_exists(x) cnf(domain_has_elements, axiom)
there_exists(cod(x)) $\Rightarrow$ there_exists(x) cnf(codomain_has_elements, axiom)
there_exists($x \circ y$) $\Rightarrow$ there_exists(dom(x)) cnf(composition_implies_domain, axiom)
there_exists($x \circ y$) $\Rightarrow$ dom(x) = cod(y) cnf(domain_codomain_composition$_1$, axiom)
(there_exists(dom(x)) and dom(x) = cod(y)) $\Rightarrow$ there_exists($x \circ y$) cnf(domain_codomain_composition$_2$, axiom)
$x \circ (y \circ z) = (x \circ y) \circ z$ cnf(associativity_of_compose, axiom)
$x \circ$ dom(x) $= x$ cnf(compose_domain, axiom)
cod(x) $\circ x = x$ cnf(compose_codomain, axiom)
equivalent(x, y) $\Rightarrow$ there_exists(y) cnf(equivalence_implies_existence$_3$, axiom)
(there_exists(x) and there_exists(y) and $x = y$) $\Rightarrow$ equivalent(x, y) cnf(existence_and_equality_implies_equivalence$_2$, axio
there_exists($x \circ y$) $\Rightarrow$ there_exists(cod(x)) cnf(composition_implies_codomain, axiom)
there_exists($f_1(x, y)$) or $x = y$ cnf(indiscernibles$_1$, axiom)
$x = f_1(x, y)$ or $y = f_1(x, y)$ or $x = y$ cnf(indiscernibles$_2$, axiom)
($x = f_1(x, y)$ and $y = f_1(x, y)$) $\Rightarrow$ $x = y$ cnf(indiscernibles$_3$, axiom)

CAT004-0.ax Category theory axioms

equivalent(x, y) $\Rightarrow$ there_exists(x) cnf(equivalence_implies_existence$_1$, axiom)
equivalent(x, y) $\Rightarrow$ $x = y$ cnf(equivalence_implies_existence$_2$, axiom)
(there_exists(x) and $x = y$) $\Rightarrow$ equivalent(x, y) cnf(existence_and_equality_implies_equivalence$_1$, axiom)
there_exists(dom(x)) $\Rightarrow$ there_exists(x) cnf(domain_has_elements, axiom)
there_exists(cod(x)) $\Rightarrow$ there_exists(x) cnf(codomain_has_elements, axiom)
there_exists($x \circ y$) $\Rightarrow$ there_exists(dom(x)) cnf(composition_implies_domain, axiom)
there_exists($x \circ y$) $\Rightarrow$ dom(x) = cod(y) cnf(domain_codomain_composition$_1$, axiom)
(there_exists(dom(x)) and dom(x) = cod(y)) $\Rightarrow$ there_exists($x \circ y$) cnf(domain_codomain_composition$_2$, axiom)
$x \circ (y \circ z) = (x \circ y) \circ z$ cnf(associativity_of_compose, axiom)

$x \circ \mathrm{dom}(x) = x$ cnf(compose_domain, axiom)
$\mathrm{cod}(x) \circ x = x$ cnf(compose_codomain, axiom)

CAT problems

CAT001-1.p XY monomorphism => Y monomorphism
If xy is a monomorphism, then y is a monomorphism.
include('Axioms/CAT001-0.ax')
$a \cdot b{=}c$ cnf(ab_equals_c, hypothesis)
$(c \cdot x_1{=}x_2 \text{ and } c \cdot x_3{=}x_2) \Rightarrow x_1 = x_3$ cnf(cancellation_for_product, hypothesis)
$b \cdot h{=}d$ cnf(bh_equals_d, hypothesis)
$b \cdot g{=}d$ cnf(bg_equals_d, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT001-2.p XY monomorphism => Y monomorphism
If xy is a monomorphism, then y is a monomorphism.
include('Axioms/CAT002-0.ax')
$(\mathrm{cod}(x) = \mathrm{dom}(a \circ b) \text{ and } x \circ (a \circ b) = y \text{ and } \mathrm{cod}(z) = \mathrm{dom}(a \circ b) \text{ and } z \circ (a \circ b) = y) \Rightarrow x = z$ cnf(c_1, hypothesis)
$\mathrm{cod}(a) = \mathrm{dom}(b)$ cnf(codomain_of_a_equals_domain_of_b, hypothesis)
$\mathrm{cod}(a) = \mathrm{dom}(h)$ cnf(codomain_of_a_equals_domain_of_h, hypothesis)
$\mathrm{cod}(a) = \mathrm{dom}(g)$ cnf(codomain_of_a_equals_domain_of_g, hypothesis)
$a \circ h = a \circ g$ cnf(ah_equals_ag, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT001-3.p XY monomorphism => Y monomorphism
If xy is a monomorphism, then y is a monomorphism.
include('Axioms/CAT003-0.ax')
there_exists($a \circ b$) cnf(assume_ab_exists, hypothesis)
$((a \circ b) \circ x = y \text{ and } (a \circ b) \circ z = y) \Rightarrow x = z$ cnf(monomorphism, hypothesis)
there_exists($b \circ h$) cnf(assume_bh_exists, hypothesis)
$b \circ h = b \circ g$ cnf(bh_equals_bg, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT001-4.p XY monomorphism => Y monomorphism
If xy is a monomorphism, then y is a monomorphism.
include('Axioms/CAT004-0.ax')
there_exists($a \circ b$) cnf(assume_ab_exists, hypothesis)
$((a \circ b) \circ x = y \text{ and } (a \circ b) \circ z = y) \Rightarrow x = z$ cnf(monomorphism, hypothesis)
there_exists($b \circ h$) cnf(assume_bh_exists, hypothesis)
$b \circ h = b \circ g$ cnf(bh_equals_bg, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT002-1.p X and Y monomorphisms, XY well-defined => XY monomorphism
If x and y are monomorphisms and xy is well-defined then xy is a monomorphism.
include('Axioms/CAT001-0.ax')
$(a \cdot x{=}w \text{ and } a \cdot y{=}w) \Rightarrow x = y$ cnf(cancellation_for_product$_1$, hypothesis)
$(b \cdot x{=}w \text{ and } b \cdot y{=}w) \Rightarrow x = y$ cnf(cancellation_for_product$_2$, hypothesis)
$a \cdot b{=}c$ cnf(ab_equals_c, hypothesis)
$c \cdot h{=}d$ cnf(ch_equals_d, hypothesis)
$c \cdot g{=}d$ cnf(cg_equals_d, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT002-2.p X and Y monomorphisms, XY well-defined => XY monomorphism
If x and y are monomorphisms and xy is well-defined then xy is a monomorphism.
include('Axioms/CAT002-0.ax')
$\mathrm{cod}(a) = \mathrm{dom}(b)$ cnf(codomain_of_a_equals_domain_of_b, hypothesis)
$(\mathrm{cod}(x) = \mathrm{dom}(a) \text{ and } x \circ a = y \text{ and } \mathrm{cod}(z) = \mathrm{dom}(b) \text{ and } z \circ a = y) \Rightarrow x = z$ cnf(monomorphism$_1$, hypothesis)
$(\mathrm{cod}(x) = \mathrm{dom}(a) \text{ and } x \circ b = y \text{ and } \mathrm{cod}(z) = \mathrm{dom}(b) \text{ and } z \circ b = y) \Rightarrow x = z$ cnf(monomorphism$_2$, hypothesis)
$\mathrm{cod}(h) = \mathrm{dom}(a \circ b)$ cnf(codomain_of_h_equals_domain_of_ab, hypothesis)
$\mathrm{cod}(g) = \mathrm{dom}(a \circ b)$ cnf(codomain_of_g_equals_domain_of_ab, hypothesis)
$h \circ (a \circ b) = g \circ (a \circ b)$ cnf(h_ab_equals_g_ab, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT002-3.p X and Y monomorphisms, XY well-defined => XY monomorphism
If x and y are monomorphisms and xy is well-defined then xy is a monomorphism.
include('Axioms/CAT003-0.ax')
there_exists($a \circ b$) cnf(assume_ab_exists, hypothesis)
$(a \circ x = y \text{ and } a \circ z = y) \Rightarrow x = z$ cnf(cancellation_for_compose$_1$, hypothesis)
$(b \circ x = y \text{ and } b \circ z = y) \Rightarrow x = z$ cnf(cancellation_for_compose$_2$, hypothesis)
there_exists(h) cnf(assume_h_exists, hypothesis)
$(a \circ b) \circ h = (a \circ b) \circ g$ cnf(ab_h_equals_ab_g, hypothesis)
$g \neq h$ cnf(prove_g_equals_h, negated_conjecture)

CAT002-4.p X and Y monomorphisms, XY well-defined => XY monomorphism
If x and y are monomorphisms and xy is well-defined then xy is a monomorphism.
include('Axioms/CAT004-0.ax')
there_exists($a \circ b$) cnf(assume_ab_exists, hypothesis)
$(a \circ x = y \text{ and } a \circ z = y) \Rightarrow x = z$ cnf(cancellation_for_compose$_1$, hypothesis)
$(b \circ x = y \text{ and } b \circ z = y) \Rightarrow x = z$ cnf(cancellation_for_compose$_2$, hypothesis)
there_exists(h) cnf(assume_h_exists, hypothesis)
$(a \circ b) \circ h = (a \circ b) \circ g$ cnf(ab_h_equals_ab_g, hypothesis)
$g \neq h$ cnf(prove_g_equals_h, negated_conjecture)

CAT003-1.p XY epimorphism => X epimorphism
If xy is an epimorphism, then x is an epimorphism.
include('Axioms/CAT001-0.ax')
$a \cdot b{=}c$ cnf(ab_equals_c, hypothesis)
$(x \cdot c{=}w \text{ and } y \cdot c{=}w) \Rightarrow x = y$ cnf(cancellation_for_product, hypothesis)
$h \cdot a{=}d$ cnf(ha_equals_d, hypothesis)
$g \cdot a{=}d$ cnf(ga_equals_d, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT003-2.p XY epimorphism => X epimorphism
If xy is an epimorphism, then x is an epimorphism.
include('Axioms/CAT002-0.ax')
$(\mathrm{cod}(a{\circ}b) = \mathrm{dom}(x) \text{ and } (a{\circ}b){\circ}x = y \text{ and } \mathrm{cod}(a{\circ}b) = \mathrm{dom}(z) \text{ and } (a{\circ}b){\circ}z = y) \Rightarrow x = z$ cnf(endomorphism, hypothesis)
$\mathrm{cod}(a) = \mathrm{dom}(b)$ cnf(codomain_of_a_equals_domain_of_b, hypothesis)
$\mathrm{cod}(b) = \mathrm{dom}(h)$ cnf(codomain_of_b_equals_domain_of_h, hypothesis)
$\mathrm{cod}(b) = \mathrm{dom}(g)$ cnf(codomain_of_b_equals_domain_of_g, hypothesis)
$b \circ h = b \circ g$ cnf(bh_equals_bg, hypothesis)
$g \neq h$ cnf(prove_g_equals_h, negated_conjecture)

CAT003-3.p XY epimorphism => X epimorphism
If xy is an epimorphism, then x is an epimorphism.
include('Axioms/CAT003-0.ax')
there_exists($a \circ b$) cnf(assume_ab_exists, hypothesis)
$(x \circ (a \circ b) = y \text{ and } z \circ (a \circ b) = y) \Rightarrow x = z$ cnf(epimorphism, hypothesis)
there_exists(h) cnf(assume_h_exists, hypothesis)
$h \circ a = g \circ a$ cnf(ha_equals_ga, hypothesis)
$g \neq h$ cnf(prove_g_equals_h, negated_conjecture)

CAT003-4.p XY epimorphism => X epimorphism
If xy is an epimorphism, then x is an epimorphism.
include('Axioms/CAT004-0.ax')
there_exists($a \circ b$) cnf(assume_ab_exists, hypothesis)
$(x \circ (a \circ b) = y \text{ and } z \circ (a \circ b) = y) \Rightarrow x = z$ cnf(epimorphism, hypothesis)
there_exists(h) cnf(assume_h_exists, hypothesis)
$h \circ a = g \circ a$ cnf(ha_equals_ga, hypothesis)
$g \neq h$ cnf(prove_g_equals_h, negated_conjecture)

CAT004-1.p X and Y epimorphisms, XY well-defined => XY epimorphism
If x and y are epimorphisms and xy is well-defined, then xy is an epimorphism.
include('Axioms/CAT001-0.ax')
$(x \cdot a{=}w \text{ and } y \cdot a{=}w) \Rightarrow x = y$ cnf(cancellation_for_product$_1$, hypothesis)
$(x \cdot b{=}w \text{ and } y \cdot b{=}w) \Rightarrow x = y$ cnf(cancellation_for_product$_2$, hypothesis)
$a \cdot b{=}c$ cnf(ab_equals_c, hypothesis)

$h \cdot c = d$ cnf(hc_equals_d, hypothesis)
$g \cdot c = d$ cnf(gc_equals_d, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT004-2.p X and Y epimorphisms, XY well-defined => XY epimorphism
If x and y are epimorphisms and xy is well-defined, then xy is an epimorphism.
include('Axioms/CAT002-0.ax')
$(\mathrm{cod}(a) = \mathrm{dom}(x)$ and $a \circ x = y$ and $\mathrm{cod}(a) = \mathrm{dom}(z)$ and $a \circ z = y) \Rightarrow x = z$ cnf(epimorphism$_1$, hypothesis)
$(\mathrm{cod}(b) = \mathrm{dom}(x)$ and $b \circ x = y$ and $\mathrm{cod}(b) = \mathrm{dom}(z)$ and $b \circ z = y) \Rightarrow x = z$ cnf(epimorphism$_2$, hypothesis)
$\mathrm{cod}(a) = \mathrm{dom}(b)$ cnf(codomain_of_a_equals_domain_of_b, hypothesis)
$\mathrm{cod}(a \circ b) = \mathrm{dom}(h)$ cnf(codomain_of_ab_equals_domain_of_h, hypothesis)
$\mathrm{cod}(a \circ b) = \mathrm{dom}(g)$ cnf(codomain_of_ab_equals_domain_of_g, hypothesis)
$(a \circ b) \circ h = (a \circ b) \circ g$ cnf(ab_h_equals_ab_g, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT004-3.p X and Y epimorphisms, XY well-defined => XY epimorphism
If x and y are epimorphisms and xy is well-defined, then xy is an epimorphism.
include('Axioms/CAT003-0.ax')
there_exists$(a \circ b)$ cnf(assume_ab_exists, hypothesis)
$(x \circ a = y$ and $z \circ a = y) \Rightarrow x = z$ cnf(cancellation_for_product$_1$, hypothesis)
$(x \circ b = y$ and $z \circ b = y) \Rightarrow x = z$ cnf(cancellation_for_product$_2$, hypothesis)
there_exists(h) cnf(assume_h_exists, hypothesis)
$h \circ (a \circ b) = g \circ (a \circ b)$ cnf(h_ab_equals_g_ab, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT004-4.p X and Y epimorphisms, XY well-defined => XY epimorphism
If x and y are epimorphisms and xy is well-defined, then xy is an epimorphism.
include('Axioms/CAT004-0.ax')
there_exists$(a \circ b)$ cnf(assume_ab_exists, hypothesis)
$(x \circ a = y$ and $z \circ a = y) \Rightarrow x = z$ cnf(cancellation_for_product$_1$, hypothesis)
$(x \circ b = y$ and $z \circ b = y) \Rightarrow x = z$ cnf(cancellation_for_product$_2$, hypothesis)
there_exists(h) cnf(assume_h_exists, hypothesis)
$h \circ (a \circ b) = g \circ (a \circ b)$ cnf(h_ab_equals_g_ab, hypothesis)
$h \neq g$ cnf(prove_h_equals_g, negated_conjecture)

CAT005-1.p Domain is the unique right identity
domain(x) is the unique identity i such that xi is defined.
include('Axioms/CAT001-0.ax')
defined(a, d) cnf(ad_defined, hypothesis)
identity_map(d) cnf(d_is_identity_map, hypothesis)
$\mathrm{dom}(a) \neq d$ cnf(prove_domain_of_a_is_d, negated_conjecture)

CAT005-3.p Domain is the unique right identity
domain(x) is the unique identity i such that xi is defined.
include('Axioms/CAT003-0.ax')
there_exists$(a \circ d)$ cnf(ad_exists, hypothesis)
there_exists$(x \circ d) \Rightarrow x \circ d = x$ cnf(xd_equals_x, hypothesis)
there_exists$(d \circ y) \Rightarrow d \circ y = y$ cnf(dy_equals_y, hypothesis)
$\mathrm{dom}(a) \neq d$ cnf(prove_domain_of_a_is_d, negated_conjecture)

CAT005-4.p Domain is the unique right identity
domain(x) is the unique identity i such that xi is defined.
include('Axioms/CAT004-0.ax')
there_exists$(a \circ d)$ cnf(ad_exists, hypothesis)
there_exists$(x \circ d) \Rightarrow x \circ d = x$ cnf(xd_equals_x, hypothesis)
there_exists$(d \circ y) \Rightarrow d \circ y = y$ cnf(dy_equals_y, hypothesis)
$\mathrm{dom}(a) \neq d$ cnf(prove_domain_of_a_is_d, negated_conjecture)

CAT006-1.p Codomain is the unique left identity
codomain(x) is the unique identity j such that jx is defined.
include('Axioms/CAT001-0.ax')
defined(h, a) cnf(ha_defined, hypothesis)
identity_map(h) cnf(h_is_the_identity_map, hypothesis)

$\mathrm{cod}(a) \neq h$ cnf(prove_codomain_of_a_is_h, negated_conjecture)

CAT006-3.p Codomain is the unique left identity
codomain(x) is the unique identity j such that jx is defined.
include('Axioms/CAT003-0.ax')
there_exists($d \circ a$) cnf(da_exists, hypothesis)
there_exists($x \circ d$) $\Rightarrow$ $x \circ d = x$ cnf(xd_equals_x, hypothesis)
there_exists($d \circ x$) $\Rightarrow$ $d \circ x = x$ cnf(dx_equals_x, hypothesis)
$\mathrm{cod}(a) \neq d$ cnf(prove_codomain_of_a_is_d, negated_conjecture)

CAT006-4.p Codomain is the unique left identity
codomain(x) is the unique identity j such that jx is defined.
include('Axioms/CAT004-0.ax')
there_exists($d \circ a$) cnf(da_exists, hypothesis)
there_exists($x \circ d$) $\Rightarrow$ $x \circ d = x$ cnf(xd_equals_x, hypothesis)
there_exists($d \circ x$) $\Rightarrow$ $d \circ x = x$ cnf(dx_equals_x, hypothesis)
$\mathrm{cod}(a) \neq d$ cnf(prove_codomain_of_a_is_d, negated_conjecture)

CAT007-1.p If domain(x) = codomain(y) then xy is defined
include('Axioms/CAT001-0.ax')
$\mathrm{dom}(a) = \mathrm{cod}(b)$ cnf(domain_of_a_equals_codomain_of_b, hypothesis)
$\neg$ defined(a, b) cnf(prove_ab_is_defined, negated_conjecture)

CAT007-3.p If domain(x) = codomain(y) then xy is defined
$x{=}x$ cnf(reflexivity, axiom)
$x{=}y$ $\Rightarrow$ $y{=}x$ cnf(symmetry, axiom)
($x{=}y$ and $y{=}z$) $\Rightarrow$ $x{=}z$ cnf(transitivity, axiom)
there_exists(dom(x)) $\Rightarrow$ there_exists(x) cnf(domain_has_elements, axiom)
(there_exists(dom(x)) and dom(x)=cod(y)) $\Rightarrow$ there_exists($x \circ y$) cnf(domain_codomain_composition$_2$, axiom)
there_exists($f_1(x, y)$) or $x{=}y$ cnf(indiscernibles$_1$, axiom)
$x{=}f_1(x, y)$ or $y{=}f_1(x, y)$ or $x{=}y$ cnf(indiscernibles$_2$, axiom)
($x{=}f_1(x, y)$ and $y{=}f_1(x, y)$) $\Rightarrow$ $x{=}y$ cnf(indiscernibles$_3$, axiom)
there_exists(dom(c_2)) cnf(domain_of_c2_exists, hypothesis)
there_exists(dom(c_1)) cnf(domain_of_c1_exists, hypothesis)
dom(c_2)=cod(c_1) cnf(domain_of_c2_equals_codomain_of_c$_1$, hypothesis)
$\neg$ there_exists($c_2 \circ c_1$) cnf(prove_c1_c2_is_defined, negated_conjecture)

CAT008-1.p If xy is defined then domain(x) = codomain(y)
include('Axioms/CAT001-0.ax')
defined(a, b) cnf(ab_defined, hypothesis)
$\mathrm{dom}(a) \neq \mathrm{cod}(b)$ cnf(prove_domain_of_a_equals_codomain_of_b, negated_conjecture)

CAT009-1.p If xy is defined, then domain(xy) = domain(y)
include('Axioms/CAT001-0.ax')
defined(b, a) cnf(ba_defined, hypothesis)
$\mathrm{dom}(b \circ a) \neq \mathrm{dom}(a)$ cnf(prove_domain_of_ba_equals_domain_of_a, negated_conjecture)

CAT009-3.p If xy is defined, then domain(xy) = domain(y)
include('Axioms/CAT003-0.ax')
there_exists($a \circ b$) cnf(ab_exists, hypothesis)
$\mathrm{dom}(a \circ b) \neq \mathrm{dom}(b)$ cnf(prove_domain_of_ab_equals_domain_of_b, negated_conjecture)

CAT009-4.p If xy is defined, then domain(xy) = domain(y)
include('Axioms/CAT004-0.ax')
there_exists($a \circ b$) cnf(ab_exists, hypothesis)
$\mathrm{dom}(a \circ b) \neq \mathrm{dom}(b)$ cnf(prove_domain_of_ab_equals_domain_of_b, negated_conjecture)

CAT010-1.p If xy is defined, then codomain(xy) = codomain(x)
include('Axioms/CAT001-0.ax')
defined(b, a) cnf(ba_defined, hypothesis)
$\mathrm{cod}(b \circ a) \neq \mathrm{cod}(b)$ cnf(prove_codomain_of_ba_equals_codomain_of_b, negated_conjecture)

CAT010-4.p If xy is defined, then codomain(xy) = codomain(x)
include('Axioms/CAT004-0.ax')
there_exists($a \circ b$) cnf(ab_exists, hypothesis)

$\mathrm{cod}(a \circ b) \neq \mathrm{cod}(a)$ cnf(prove_codomain_of_ab_equals_codomain_of_a, negated_conjecture)

CAT011-1.p domain(domain(x)) = domain(x)
include('Axioms/CAT001-0.ax')
$\mathrm{dom}(\mathrm{dom}(a)) \neq \mathrm{dom}(a)$ cnf(prove_domain_is_idempotent, negated_conjecture)

CAT011-2.p domain(domain(x)) = domain(x)
include('Axioms/CAT002-0.ax')
$\mathrm{dom}(\mathrm{dom}(a)) \neq \mathrm{dom}(a)$ cnf(prove_domain_is_idempotent, negated_conjecture)

CAT011-3.p domain(domain(x)) = domain(x)
include('Axioms/CAT003-0.ax')
there_exists($\mathrm{dom}(a)$) cnf(assume_domain_exists, hypothesis)
$\mathrm{dom}(\mathrm{dom}(a)) \neq \mathrm{dom}(a)$ cnf(prove_domain_is_idempotent, negated_conjecture)

CAT011-4.p domain(domain(x)) = domain(x)
include('Axioms/CAT004-0.ax')
there_exists($\mathrm{dom}(a)$) cnf(assume_domain_exists, hypothesis)
$\mathrm{dom}(\mathrm{dom}(a)) \neq \mathrm{dom}(a)$ cnf(prove_domain_is_idempotent, negated_conjecture)

CAT012-1.p codomain(domain(x)) = domain(x)
include('Axioms/CAT001-0.ax')
$\mathrm{cod}(\mathrm{dom}(a)) \neq \mathrm{dom}(a)$ cnf(prove_codomain_of_domain_is_domain, negated_conjecture)

CAT012-3.p codomain(domain(x)) = domain(x)
include('Axioms/CAT003-0.ax')
there_exists($\mathrm{dom}(a)$) cnf(assume_domain_exists, hypothesis)
$\mathrm{cod}(\mathrm{dom}(a)) \neq \mathrm{dom}(a)$ cnf(prove_codomain_of_domain_is_domain, negated_conjecture)

CAT012-4.p codomain(domain(x)) = domain(x)
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$\mathrm{cod}(\mathrm{dom}(a)) \neq \mathrm{dom}(a)$ cnf(prove_codomain_of_domain_is_domain, negated_conjecture)

CAT013-1.p domain(codomain(x)) = codomain(x)
include('Axioms/CAT001-0.ax')
$\mathrm{dom}(\mathrm{cod}(a)) \neq \mathrm{cod}(a)$ cnf(prove_domain_of_codomain_is_codomain, negated_conjecture)

CAT013-3.p domain(codomain(x)) = codomain(x)
include('Axioms/CAT003-0.ax')
there_exists($\mathrm{cod}(a)$) cnf(assume_codomain_exists, hypothesis)
$\mathrm{dom}(\mathrm{cod}(a)) \neq \mathrm{cod}(a)$ cnf(prove_domain_of_codomain_is_codomain, negated_conjecture)

CAT013-4.p domain(codomain(x)) = codomain(x)
include('Axioms/CAT004-0.ax')
there_exists($\mathrm{cod}(a)$) cnf(assume_codomain_exists, hypothesis)
$\mathrm{dom}(\mathrm{cod}(a)) \neq \mathrm{cod}(a)$ cnf(prove_domain_of_codomain_is_codomain, negated_conjecture)

CAT014-1.p codomain(codomain(x)) = codomain(x)
include('Axioms/CAT001-0.ax')
$\mathrm{cod}(\mathrm{cod}(a)) \neq \mathrm{cod}(a)$ cnf(prove_codomain_is_idempotent, negated_conjecture)

CAT014-2.p codomain(codomain(x)) = codomain(x)
include('Axioms/CAT002-0.ax')
$\mathrm{cod}(\mathrm{cod}(a)) \neq \mathrm{cod}(a)$ cnf(prove_codomain_is_idempotent, negated_conjecture)

CAT014-3.p codomain(codomain(x)) = codomain(x)
include('Axioms/CAT003-0.ax')
there_exists($\mathrm{cod}(a)$) cnf(assume_codomain_exists, hypothesis)
$\mathrm{cod}(\mathrm{cod}(a)) \neq \mathrm{cod}(a)$ cnf(prove_codomain_is_idempotent, negated_conjecture)

CAT014-4.p codomain(codomain(x)) = codomain(x)
include('Axioms/CAT004-0.ax')
there_exists($\mathrm{cod}(a)$) cnf(assume_codomain_exists, hypothesis)
$\mathrm{cod}(\mathrm{cod}(a)) \neq \mathrm{cod}(a)$ cnf(prove_codomain_is_idempotent, negated_conjecture)

CAT015-3.p Prove something exists
Can anything be proven to exist, directly from the axioms? Using all the kinds of resolution steps possible, no.
include('Axioms/CAT003-0.ax')

$x = y$ or there_exists(x) or there_exists(y) cnf(equal_things_exist, axiom)
$\neg$ there_exists(x) cnf(prove_something_exists, negated_conjecture)

CAT015-4.p Prove something exists
Can anything be proven to exist, directly from the axioms? Using all the kinds of resolution steps possible, no.
include('Axioms/CAT004-0.ax')
$x = y$ or there_exists(x) or there_exists(y) cnf(equal_things_exist, axiom)
$\neg$ there_exists(x) cnf(prove_something_exists, negated_conjecture)

CAT016-3.p If x exists, then domain(x) exists
include('Axioms/CAT003-0.ax')
there_exists(a) cnf(assume_a_exists, hypothesis)
$\neg$ there_exists(dom(a)) cnf(prove_domain_of_a_exists, negated_conjecture)

CAT016-4.p If x exists, then domain(x) exists
include('Axioms/CAT004-0.ax')
there_exists(a) cnf(assume_a_exists, hypothesis)
$\neg$ there_exists(dom(a)) cnf(prove_domain_of_a_exists, negated_conjecture)

CAT017-3.p If x exists, then codomain(x) exists
include('Axioms/CAT003-0.ax')
there_exists(a) cnf(assume_a_exists, hypothesis)
$\neg$ there_exists(cod(a)) cnf(prove_codomain_of_a_exists, negated_conjecture)

CAT017-4.p If x exists, then codomain(x) exists
include('Axioms/CAT004-0.ax')
there_exists(a) cnf(assume_a_exists, hypothesis)
$\neg$ there_exists(cod(a)) cnf(prove_codomain_of_a_exists, negated_conjecture)

CAT018-1.p If xy and yz exist, then so does x(yz)
include('Axioms/CAT001-0.ax')
defined(a, b) cnf(assume_ab_exists, hypothesis)
defined(b, c) cnf(assume_bc_exists, hypothesis)
$\neg$ defined($a, b \circ c$) cnf(prove_a_bc_exists, negated_conjecture)

CAT018-3.p If xy and yz exist, then so does x(yz)
include('Axioms/CAT003-0.ax')
there_exists($a \circ b$) cnf(assume_ab_exists, hypothesis)
there_exists($b \circ c$) cnf(assume_bc_exists, hypothesis)
$\neg$ there_exists($a \circ (b \circ c)$) cnf(prove_a_bc_exists, negated_conjecture)

CAT018-4.p If xy and yz exist, then so does x(yz)
include('Axioms/CAT004-0.ax')
there_exists($a \circ b$) cnf(assume_ab_exists, hypothesis)
there_exists($b \circ c$) cnf(assume_bc_exists, hypothesis)
$\neg$ there_exists($a \circ (b \circ c)$) cnf(prove_a_bc_exists, negated_conjecture)

CAT019-1.p Axiom of Indiscernibles
[all z (x=z <→ y=z)] → x=y.
include('Axioms/CAT001-0.ax')
$c_2 = z \Rightarrow c_1 = z$ cnf(equality_to_c1_and_c2$_1$, hypothesis)
$c_1 = z \Rightarrow c_2 = z$ cnf(equality_to_c1_and_c2$_2$, hypothesis)
$c_2 \neq c_1$ cnf(prove_c1_equals_c$_2$, negated_conjecture)

CAT019-2.p Axiom of Indiscernibles
[all z (x=z <→ y=z)] → x=y.
include('Axioms/CAT002-0.ax')
$c_2 = z \Rightarrow c_1 = z$ cnf(equality_to_c1_and_c2$_1$, hypothesis)
$c_1 = z \Rightarrow c_2 = z$ cnf(equality_to_c1_and_c2$_2$, hypothesis)
$c_2 \neq c_1$ cnf(prove_c1_equals_c$_2$, negated_conjecture)

CAT019-3.p Axiom of Indiscernibles
[all z (x=z <→ y=z)] → x=y.
include('Axioms/CAT003-0.ax')
(there_exists(z) and $a = z$) $\Rightarrow$ $b = z$ cnf(equality_of_a_and_b$_1$, hypothesis)
(there_exists(z) and $b = z$) $\Rightarrow$ $a = z$ cnf(equality_of_a_and_b$_2$, hypothesis)

$a \neq b$ cnf(prove_a_equals_b, negated_conjecture)

CAT019-4.p Axiom of Indiscernibles
[all z (x=z <→ y=z)] → x=y.
include('Axioms/CAT004-0.ax')
(there_exists(z) and $a = z$) $\Rightarrow$ $b = z$ cnf(equality_of_a_and_b$_1$, hypothesis)
(there_exists(z) and $b = z$) $\Rightarrow$ $a = z$ cnf(equality_of_a_and_b$_2$, hypothesis)
$a \neq b$ cnf(prove_a_equals_b, negated_conjecture)

CAT019-5.p Axiom of Indiscernibles
[all z (x=z <→ y=z)] → x=y.
include('Axioms/CAT004-0.ax')
there_exists(c) cnf(assume_c_exists, hypothesis)
(there_exists(z) and $a = z$) $\Rightarrow$ $b = z$ cnf(equality_of_a_and_b$_1$, hypothesis)
(there_exists(z) and $b = z$) $\Rightarrow$ $a = z$ cnf(equality_of_a_and_b$_2$, hypothesis)
$a \neq b$ cnf(prove_a_equals_b, negated_conjecture)

CAT020-1.p Category theory axioms
include('Axioms/CAT001-0.ax')

CAT020-2.p Category theory (equality) axioms
include('Axioms/CAT002-0.ax')

CAT020-3.p Category theory axioms
include('Axioms/CAT003-0.ax')

CAT020-4.p Category theory axioms
include('Axioms/CAT004-0.ax')

CAT038∧1.p Swapping function
The proposition can be interpreted in concrete categories and asserts the existence of a certain arrow.
a: \$i thf(a, type)
b: \$i thf(b, type)
$\exists f$: \$i → \$i: ((f@a) = b and (f@b) = a) thf(swap, conjecture)