

COL axioms

COL001-0.ax Type-respecting combinators

$\text{apply}(k(x), y) = x$ $\text{cnf}(\text{k_definition}, \text{axiom})$
 $\text{apply}(\text{projection}_1, \text{pair}(x, y)) = x$ $\text{cnf}(\text{projection}_1, \text{axiom})$
 $\text{apply}(\text{projection}_2, \text{pair}(x, y)) = y$ $\text{cnf}(\text{projection}_2, \text{axiom})$
 $\text{pair}(\text{apply}(\text{projection}_1, x), \text{apply}(\text{projection}_2, x)) = x$ $\text{cnf}(\text{pairing}, \text{axiom})$
 $\text{apply}(\text{pair}(x, y), z) = \text{pair}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{pairwise_application}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, x), y), z) = \text{apply}(\text{apply}(x, k(z)), \text{apply}(y, z))$ $\text{cnf}(\text{abstraction}, \text{axiom})$
 $\text{apply}(\text{eq}, \text{pair}(x, x)) = \text{projection}_1$ $\text{cnf}(\text{equality}, \text{axiom})$
 $x = y$ or $\text{apply}(\text{eq}, \text{pair}(x, y)) = \text{projection}_2$ $\text{cnf}(\text{extensionality}_1, \text{axiom})$
 $\text{apply}(x, n(x, y)) = \text{apply}(y, n(x, y)) \Rightarrow x = y$ $\text{cnf}(\text{extensionality}_2, \text{axiom})$
 $\text{projection}_1 \neq \text{projection}_2$ $\text{cnf}(\text{different_projections}, \text{axiom})$

COL problems

COL001-1.p Weak fixed point for S and K

The weak fixed point property holds for the set P consisting of the combinators S and K alone, where $((Sx)y)z = (xz)(yz)$ and $(Kx)y = x$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(k, x), y) = x$ $\text{cnf}(\text{k_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL001-2.p Weak fixed point for S and K

The weak fixed point property holds for the set P consisting of the combinators S and K alone, where $((Sx)y)z = (xz)(yz)$ and $(Kx)y = x$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(k, x), y) = x$ $\text{cnf}(\text{k_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(i, x) = x$ $\text{cnf}(\text{i_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(s, \text{apply}(b, x)), i), \text{apply}(\text{apply}(s, \text{apply}(b, x)), i)) = \text{apply}(x, \text{apply}(\text{apply}(\text{apply}(s, \text{apply}(b, x)), i), \text{apply}(\text{apply}(s, \text{apply}(b, x)), i)))$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL002-1.p Weak fixed point for S, B, C, and I

The weak fixed point property holds for the set P consisting of the combinators S, B, C, and I, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$, $((Cx)y)z = (xz)y$, and $Ix = x$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(c, x), y), z) = \text{apply}(\text{apply}(x, z), y)$ $\text{cnf}(\text{c_definition}, \text{axiom})$
 $\text{apply}(i, x) = x$ $\text{cnf}(\text{i_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{fixed_pt}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL002-2.p Weak fixed point for S, B, C, and I

The weak fixed point property holds for the set P consisting of the combinators S, B, C, and I, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$, $((Cx)y)z = (xz)y$, and $Ix = x$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(c, x), y), z) = \text{apply}(\text{apply}(x, z), y)$ $\text{cnf}(\text{c_definition}, \text{axiom})$
 $\text{apply}(i, x) = x$ $\text{cnf}(\text{i_definition}, \text{axiom})$
 $\text{weak_sage} = \text{apply}(\text{fixed_pt}, \text{weak_sage}) \Rightarrow \text{fixed_point}(\text{weak_sage})$ $\text{cnf}(\text{weak_fixed_point}, \text{axiom})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(\text{apply}(s, \text{apply}(b, x)), i), \text{apply}(\text{apply}(s, \text{apply}(b, x)), i)))$ $\text{cnf}(\text{prove_weak_fixed_point}, \text{negated_conjecture})$

COL002-3.p Weak fixed point for S, B, C, and I

The weak fixed point property holds for the set P consisting of the combinators S, B, C, and I, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$, $((Cx)y)z = (xz)y$, and $Ix = x$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(c, x), y), z) = \text{apply}(\text{apply}(x, z), y)$ $\text{cnf}(\text{c_definition}, \text{axiom})$
 $\text{apply}(i, x) = x$ $\text{cnf}(\text{i_definition}, \text{axiom})$
 $\text{weak_sage} = \text{apply}(\text{fixed_pt}, \text{weak_sage}) \Rightarrow \text{fixed_point}(\text{weak_sage})$ $\text{cnf}(\text{weak_fixed_point}, \text{axiom})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(\text{apply}(s, \text{apply}(c, \text{apply}(b, x))), \text{apply}(s, \text{apply}(c, \text{apply}(b, x)))), \text{apply}(s, \text{apply}(c, \text{apply}(b, x)))))$ $\text{cnf}(\text{prove_weak_fixed_point}, \text{negated_conjecture})$

$\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$
 $\text{apply}(\text{strong_fixed_point}, \text{fixed_pt}) = \text{apply}(\text{fixed_pt}, \text{apply}(\text{strong_fixed_point}, \text{fixed_pt})) \Rightarrow \text{fixed_point}(\text{strong_fixed_point})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(w, w))), \text{apply}(\text{apply}(b, w), b))), b)$ $\text{cnf}(\text{prove_strong_fixed_point}, \text{negated})$

COL003-5.p Strong fixed point for B and W

The strong fixed point property holds for the set P consisting of the combinators B and W alone, where $((Bx)y)z = x(yz)$ and $(Wx)y = (xy)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$
 $\text{apply}(\text{strong_fixed_point}, \text{fixed_pt}) = \text{apply}(\text{fixed_pt}, \text{apply}(\text{strong_fixed_point}, \text{fixed_pt})) \Rightarrow \text{fixed_point}(\text{strong_fixed_point})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(w, w))), w), \text{apply}(\text{apply}(b, b), b)))$ $\text{cnf}(\text{prove_strong_fixed_point}, \text{negated})$

COL003-6.p Strong fixed point for B and W

The strong fixed point property holds for the set P consisting of the combinators B and W alone, where $((Bx)y)z = x(yz)$ and $(Wx)y = (xy)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$
 $\text{apply}(\text{strong_fixed_point}, \text{fixed_pt}) = \text{apply}(\text{fixed_pt}, \text{apply}(\text{strong_fixed_point}, \text{fixed_pt})) \Rightarrow \text{fixed_point}(\text{strong_fixed_point})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(w, w))), w))), b))$ $\text{cnf}(\text{prove_strong_fixed_point}, \text{negated})$

COL003-7.p Strong fixed point for B and W

The strong fixed point property holds for the set P consisting of the combinators B and W alone, where $((Bx)y)z = x(yz)$ and $(Wx)y = (xy)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$
 $\text{apply}(\text{strong_fixed_point}, \text{fixed_pt}) = \text{apply}(\text{fixed_pt}, \text{apply}(\text{strong_fixed_point}, \text{fixed_pt})) \Rightarrow \text{fixed_point}(\text{strong_fixed_point})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(w, w))), \text{apply}(b, w))), b), b))$ $\text{cnf}(\text{prove_strong_fixed_point}, \text{negated})$

COL003-8.p Strong fixed point for B and W

The strong fixed point property holds for the set P consisting of the combinators B and W alone, where $((Bx)y)z = x(yz)$ and $(Wx)y = (xy)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$
 $\text{apply}(\text{strong_fixed_point}, \text{fixed_pt}) = \text{apply}(\text{fixed_pt}, \text{apply}(\text{strong_fixed_point}, \text{fixed_pt})) \Rightarrow \text{fixed_point}(\text{strong_fixed_point})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(w, w))), \text{apply}(b, w))), \text{apply}(\text{apply}(b, b), b)))$ $\text{cnf}(\text{prove_strong_fixed_point}, \text{negated})$

COL003-9.p Strong fixed point for B and W

The strong fixed point property holds for the set P consisting of the combinators B and W alone, where $((Bx)y)z = x(yz)$ and $(Wx)y = (xy)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$
 $\text{apply}(\text{strong_fixed_point}, \text{fixed_pt}) = \text{apply}(\text{fixed_pt}, \text{apply}(\text{strong_fixed_point}, \text{fixed_pt})) \Rightarrow \text{fixed_point}(\text{strong_fixed_point})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(w, w))), \text{apply}(\text{apply}(b, \text{apply}(b, w))), b)), b))$ $\text{cnf}(\text{prove_strong_fixed_point}, \text{negated})$

COL004-1.p Find combinator equivalent to U from S and K

Construct from S and K alone a combinator that behaves as the combinator U does, where $((Sx)y)z = (xz)(yz)$, $(Kx)y = x$, $(Ux)y = y((xx)y)$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(k, x), y) = x$ $\text{cnf}(\text{k_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(z, f(z)), g(z)) \neq \text{apply}(g(z), \text{apply}(\text{apply}(f(z), f(z)), g(z)))$ $\text{cnf}(\text{prove_u_combinator}, \text{negated_conjecture})$

COL004-3.p Find combinator equivalent to U from S and K.

Construct from S and K alone a combinator that behaves as the combinator U does, where $((Sx)y)z = (xz)(yz)$, $(Kx)y = x$, $(Ux)y = y((xx)y)$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(k, x), y) = x$ $\text{cnf}(\text{k_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(\text{apply}(s, \text{apply}(k, \text{apply}(s, \text{apply}(\text{apply}(s, k), k))))), \text{apply}(\text{apply}(s, \text{apply}(\text{apply}(s, k), k))), \text{apply}(\text{apply}(s, k), \text{apply}(y, \text{apply}(\text{apply}(x, x), y))))$ $\text{cnf}(\text{prove_u_combinator}, \text{negated_conjecture})$

COL005-1.p Find a model for S and W but not a weak fixed point

The model one is seeking must satisfy S and W and fail to satisfy the weak fixed point property, where $((Sx)y)z = (xz)(yz)$, $(Wx)y = (xy)y$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(m, x) = \text{apply}(x, x)$ $\text{cnf}(\text{m_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL009-1.p Weak fixed point for B and L2

The weak fixed point property holds for the set P consisting of the combinators B and L2, where $((Bx)y)z = x(yz)$, $((L2x)y)z = y(xx)$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(l_2, x), y) = \text{apply}(y, \text{apply}(x, x))$ $\text{cnf}(\text{l2_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL010-1.p Weak fixed point for B and S2

The weak fixed point property holds for the set P consisting of the combinators B and S2, where $((Bx)y)z = x(yz)$, $((S2x)y)z = (xz)(yy)$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(s_2, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, y))$ $\text{cnf}(\text{s2_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL011-1.p Weak fixed point for O and Q1

The weak fixed point property holds for the set P consisting of the combinators O and Q1, where $(Ox)y = y(xy)$, $((Q1x)y)z = x(zy)$.

$\text{apply}(\text{apply}(o, x), y) = \text{apply}(y, \text{apply}(x, y))$ $\text{cnf}(\text{o_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(q_1, x), y), z) = \text{apply}(x, \text{apply}(z, y))$ $\text{cnf}(\text{q1_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL012-1.p Weak fixed point for U

The weak fixed point property holds for the set P consisting of the combinator U, where $(Ux)y = y((xx)y)$.

$\text{apply}(\text{apply}(u, x), y) = \text{apply}(y, \text{apply}(\text{apply}(x, x), y))$ $\text{cnf}(\text{u_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL013-1.p Weak fixed point for S and L

The weak fixed point property holds for the set P consisting of the combinators S and L, where $((Sx)y)z = (xz)(yz)$, $(Lx)y = x(yy)$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y))$ $\text{cnf}(\text{l_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL014-1.p Weak fixed point for L and O

The weak fixed point property holds for the set P consisting of the combinators L and O, where $(Lx)y = x(yy)$, $(Ox)y = y(xy)$.

$\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y))$ $\text{cnf}(\text{l_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(o, x), y) = \text{apply}(y, \text{apply}(x, y))$ $\text{cnf}(\text{o_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL015-1.p Weak fixed point for Q and M

The weak fixed point property holds for the set P consisting of the combinators Q and M, where $Mx = xx$, $((Qx)y)z = y(xz)$.

$\text{apply}(m, x) = \text{apply}(x, x)$ $\text{cnf}(\text{m_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(q, x), y), z) = \text{apply}(y, \text{apply}(x, z))$ $\text{cnf}(\text{q_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL016-1.p Weak fixed point for B, M and L

The weak fixed point property holds for the set P consisting of the combinators B, M and L, where $((Bx)y)z = x(yz)$, $(Lx)y = x(yy)$, $Mx = xx$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y))$ $\text{cnf}(\text{l_definition}, \text{axiom})$
 $\text{apply}(m, x) = \text{apply}(x, x)$ $\text{cnf}(\text{m_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL017-1.p Weak fixed point for B, M, and T

The weak fixed point property holds for the set P consisting of the combinators B, M, and T, where $((Bx)y)z = x(yz)$, $Mx = xx$, $(Tx)y = yx$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(m, x) = \text{apply}(x, x)$ $\text{cnf}(\text{m_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(t, x), y) = \text{apply}(y, x)$ $\text{cnf}(\text{t_definition}, \text{axiom})$

$y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL018-1.p Weak fixed point for W, Q, and L

The weak fixed point property holds for the set P consisting of the combinators W, Q, and L, where $(Lx)y = x(yy)$, $(Wx)y = (xy)y$, $((Qx)y)z = y(xz)$.

$\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y)) \quad \text{cnf}(l_definition, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y) \quad \text{cnf}(w_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(q, x), y), z) = \text{apply}(y, \text{apply}(x, z)) \quad \text{cnf}(q_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL019-1.p Weak fixed point for B, S, and T

The weak fixed point property holds for the set P consisting of the combinators B, S, and T, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$, $(Tx)y = yx$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z)) \quad \text{cnf}(s_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z)) \quad \text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(\text{apply}(t, x), y) = \text{apply}(y, x) \quad \text{cnf}(t_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL020-1.p Weak fixed point for B, S, and C

The weak fixed point property holds for the set P consisting of the combinators B, S, and C, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$, $((Cx)y)z = (xz)y$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z)) \quad \text{cnf}(s_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z)) \quad \text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(c, x), y), z) = \text{apply}(\text{apply}(x, z), y) \quad \text{cnf}(c_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL021-1.p Weak fixed point for B, M, and V

The weak fixed point property holds for the set P consisting of the combinators B, M, and V, where $((Bx)y)z = x(yz)$, $Mx = xx$, $((Vx)y)z = (zx)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z)) \quad \text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(m, x) = \text{apply}(x, x) \quad \text{cnf}(m_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(v, x), y), z) = \text{apply}(\text{apply}(z, x), y) \quad \text{cnf}(v_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL022-1.p Weak fixed point for B, O, and M

The weak fixed point property holds for the set P consisting of the combinators B, O, and M, where $((Bx)y)z = x(yz)$, $Mx = xx$, $(Ox)y = y(xy)$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z)) \quad \text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(m, x) = \text{apply}(x, x) \quad \text{cnf}(m_definition, \text{axiom})$
 $\text{apply}(\text{apply}(o, x), y) = \text{apply}(y, \text{apply}(x, y)) \quad \text{cnf}(o_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL023-1.p Weak fixed point for B and N

The weak fixed point property holds for the set P consisting of the combinators B and N, where $((Bx)y)z = x(yz)$, $((Nx)y)z = ((xz)y)z$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z)) \quad \text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(n, x), y), z) = \text{apply}(\text{apply}(\text{apply}(x, z), y), z) \quad \text{cnf}(n_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL024-1.p Weak fixed point for B, M, and C

The weak fixed point property holds for the set P consisting of the combinators B, M, and C, where $((Bx)y)z = x(yz)$, $Mx = xx$, $((Cx)y)z = (xz)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z)) \quad \text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(m, x) = \text{apply}(x, x) \quad \text{cnf}(m_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(c, x), y), z) = \text{apply}(\text{apply}(x, z), y) \quad \text{cnf}(c_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL025-1.p Weak fixed point for B and W

The weak fixed point property holds for the set P consisting of the combinators B and W, where $((Bx)y)z = x(yz)$, $(Wx)y = (xy)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z)) \quad \text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y) \quad \text{cnf}(w_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y) \quad \text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL026-1.p Weak fixed point for B and W1

The weak fixed point property holds for the set P consisting of the combinators B and W1, where $((Bx)y)z = x(yz)$, $(W1x)y = (yx)x$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(w_1, x), y) = \text{apply}(\text{apply}(y, x), x)$ $\text{cnf}(\text{w1_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL027-1.p Weak fixed point for B and H

The weak fixed point property holds for the set P consisting of the combinators B and H, where $((Bx)y)z = x(yz)$, $((Hx)y)z = ((xy)z)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(h, x), y), z) = \text{apply}(\text{apply}(\text{apply}(x, y), z), y)$ $\text{cnf}(\text{h_definition}, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL029-1.p Strong fixed point for U

The strong fixed point property holds for the set P consisting of the combinator U, where $(Ux)y = y((xx)y)$.

$\text{apply}(\text{apply}(u, x), y) = \text{apply}(y, \text{apply}(\text{apply}(x, x), y))$ $\text{cnf}(\text{u_definition}, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL030-1.p Strong fixed point for S and L

The strong fixed point property holds for the set P consisting of the combinators S and L, where $((Sx)y)z = (xz)(yz)$, $(Lx)y = x(yy)$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y))$ $\text{cnf}(\text{l_definition}, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL031-1.p Strong fixed point for L and O

The strong fixed point property holds for the set P consisting of the combinators L and O, where $(Lx)y = x(yy)$, $(Ox)y = y(xy)$.

$\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y))$ $\text{cnf}(\text{l_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(o, x), y) = \text{apply}(y, \text{apply}(x, y))$ $\text{cnf}(\text{o_definition}, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL032-1.p Strong fixed point for Q and M

The strong fixed point property holds for the set P consisting of the combinators Q and M, where $Mx = xx$, $((Qx)y)z = y(xz)$.

$\text{apply}(m, x) = \text{apply}(x, x)$ $\text{cnf}(\text{m_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(q, x), y), z) = \text{apply}(y, \text{apply}(x, z))$ $\text{cnf}(\text{q_definition}, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL033-1.p Strong fixed point for B, M and L

The strong fixed point property holds for the set P consisting of the combinators B, M and L, where $((Bx)y)z = x(yz)$, $(Lx)y = x(yy)$, $Mx = xx$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y))$ $\text{cnf}(\text{l_definition}, \text{axiom})$
 $\text{apply}(m, x) = \text{apply}(x, x)$ $\text{cnf}(\text{m_definition}, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL034-1.p Strong fixed point for B, M, and T

The strong fixed point property holds for the set P consisting of the combinators B, M, and T, where $((Bx)y)z = x(yz)$, $Mx = xx$, $(Tx)y = yx$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$
 $\text{apply}(m, x) = \text{apply}(x, x)$ $\text{cnf}(\text{m_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(t, x), y) = \text{apply}(y, x)$ $\text{cnf}(\text{t_definition}, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL035-1.p Strong fixed point for W, Q, and L

The strong fixed point property holds for the set P consisting of the combinators W, Q, and L, where $(Lx)y = x(yy)$, $(Wx)y = (xy)y$, $((Qx)y)z = y(xz)$.

$\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y))$ $\text{cnf}(\text{l_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(q, x), y), z) = \text{apply}(y, \text{apply}(x, z))$ $\text{cnf}(\text{q_definition}, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL036-1.p Strong fixed point for B, S, and T

The strong fixed point property holds for the set P consisting of the combinators B, S, and T, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$, $(Tx)y = yx$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ cnf(s_definition, axiom)
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ cnf(b_definition, axiom)
 $\text{apply}(\text{apply}(t, x), y) = \text{apply}(y, x)$ cnf(t_definition, axiom)
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ cnf(prove_fixed_point, negated_conjecture)

COL037-1.p Strong fixed point for B, S, and C

The strong fixed point property holds for the set P consisting of the combinators B, S, and C, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$, $((Cx)y)z = (xz)y$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ cnf(s_definition, axiom)
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ cnf(b_definition, axiom)
 $\text{apply}(\text{apply}(\text{apply}(c, x), y), z) = \text{apply}(\text{apply}(x, z), y)$ cnf(c_definition, axiom)
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ cnf(prove_fixed_point, negated_conjecture)

COL038-1.p Strong fixed point for B, M, and V

The strong fixed point property holds for the set P consisting of the combinators B, M, and V, where $((Bx)y)z = x(yz)$, $Mx = xx$, $((Vx)y)z = (zx)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ cnf(b_definition, axiom)
 $\text{apply}(m, x) = \text{apply}(x, x)$ cnf(m_definition, axiom)
 $\text{apply}(\text{apply}(\text{apply}(v, x), y), z) = \text{apply}(\text{apply}(z, x), y)$ cnf(v_definition, axiom)
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ cnf(prove_fixed_point, negated_conjecture)

COL039-1.p Strong fixed point for B, O, and M

The strong fixed point property holds for the set P consisting of the combinators B, O, and M, where $((Bx)y)z = x(yz)$, $Mx = xx$, $(Ox)y = y(xy)$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ cnf(b_definition, axiom)
 $\text{apply}(m, x) = \text{apply}(x, x)$ cnf(m_definition, axiom)
 $\text{apply}(\text{apply}(o, x), y) = \text{apply}(y, \text{apply}(x, y))$ cnf(o_definition, axiom)
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ cnf(prove_fixed_point, negated_conjecture)

COL041-1.p Strong fixed point for B, M, and C

The strong fixed point property holds for the set P consisting of the combinators B, M, and C, where $((Bx)y)z = x(yz)$, $Mx = xx$, $((Cx)y)z = (xz)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ cnf(b_definition, axiom)
 $\text{apply}(m, x) = \text{apply}(x, x)$ cnf(m_definition, axiom)
 $\text{apply}(\text{apply}(\text{apply}(c, x), y), z) = \text{apply}(\text{apply}(x, z), y)$ cnf(c_definition, axiom)
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ cnf(prove_fixed_point, negated_conjecture)

COL042-1.p Strong fixed point for B and W1

The strong fixed point property holds for the set P consisting of the combinators B and W1, where $((Bx)y)z = x(yz)$, $(W1x)y = (yx)x$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ cnf(b_definition, axiom)
 $\text{apply}(\text{apply}(w_1, x), y) = \text{apply}(\text{apply}(y, x), x)$ cnf(w1_definition, axiom)
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ cnf(prove_fixed_point, negated_conjecture)

COL042-2.p Strong fixed point for B and W1

The strong fixed point property holds for the set P consisting of the combinators B and W1, where $((Bx)y)z = x(yz)$, $(W1x)y = (yx)x$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ cnf(b_definition, axiom)
 $\text{apply}(\text{apply}(w_1, x), y) = \text{apply}(\text{apply}(y, x), x)$ cnf(w1_definition, axiom)
 $\text{apply}(\text{strong_fixed_point}, \text{fixed_pt}) = \text{apply}(\text{fixed_pt}, \text{apply}(\text{strong_fixed_point}, \text{fixed_pt})) \Rightarrow \text{fixed_point}(\text{strong_fixed_point})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(w_1, w_1)), \text{apply}(b, w_1))), b)), b))$ cnf(prove_strong_fixed_point)

COL042-3.p Strong fixed point for B and W1

The strong fixed point property holds for the set P consisting of the combinators B and W1, where $((Bx)y)z = x(yz)$, $(W1x)y = (yx)x$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ cnf(b_definition, axiom)
 $\text{apply}(\text{apply}(w_1, x), y) = \text{apply}(\text{apply}(y, x), x)$ cnf(w1_definition, axiom)
 $\text{apply}(\text{strong_fixed_point}, \text{fixed_pt}) = \text{apply}(\text{fixed_pt}, \text{apply}(\text{strong_fixed_point}, \text{fixed_pt})) \Rightarrow \text{fixed_point}(\text{strong_fixed_point})$
 $\neg \text{fixed_point}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(w_1, w_1)), \text{apply}(b, w_1))), \text{apply}(\text{apply}(b, b), b)))$ cnf(prove_strong_fixed_point)

COL042-4.p Strong fixed point for B and W1

$\text{response}(\text{mocking_bird}, y) = \text{response}(y, y) \quad \text{cnf}(\text{mocking_bird_exists}, \text{axiom})$
 $\text{response}(x \circ y, w) = \text{response}(x, \text{response}(y, w)) \quad \text{cnf}(\text{composer_exists}, \text{hypothesis})$
 $\text{response}(x, x) \neq x \quad \text{cnf}(\text{prove_the_bird_exists}, \text{negated_conjecture})$

COL052-1.p A Question on Agreeable Birds

For all birds x and y , there exists a bird z that composes x with y for all birds w . Prove that if C is agreeable then A is agreeable.

$\text{response}(x \circ y, w) = \text{response}(x, \text{response}(y, w)) \quad \text{cnf}(\text{composer_exists}, \text{axiom})$
 $\text{response}(c, \text{common_bird}(x)) = \text{response}(x, \text{common_bird}(x)) \quad \text{cnf}(\text{agreeable}_1, \text{hypothesis})$
 $\text{response}(a, v) \neq \text{response}(\text{odd_bird}, v) \quad \text{cnf}(\text{prove_a_is_agreeable}, \text{negated_conjecture})$
 $c = a \circ b \quad \text{cnf}(c_composes_a_with_b, \text{hypothesis})$

COL052-2.p A Question on Agreeable Birds

For all birds x and y , there exists a bird z that composes x with y for all birds w . Prove that if C is agreeable then A is agreeable.

$\text{response}(x \circ y, w) = \text{response}(x, \text{response}(y, w)) \quad \text{cnf}(\text{composer_exists}, \text{axiom})$
 $\text{agreeable}(x) \Rightarrow \text{response}(x, \text{common_bird}(y)) = \text{response}(y, \text{common_bird}(y)) \quad \text{cnf}(\text{agreeable}_1, \text{axiom})$
 $\text{response}(x, z) = \text{response}(\text{compatible}(x), z) \Rightarrow \text{agreeable}(x) \quad \text{cnf}(\text{agreeable}_2, \text{axiom})$
 $\text{agreeable}(c) \quad \text{cnf}(c_is_agreeable, \text{hypothesis})$
 $\neg \text{agreeable}(a) \quad \text{cnf}(\text{prove_a_is_agreeable}, \text{negated_conjecture})$
 $c = a \circ b \quad \text{cnf}(c_composes_a_with_b, \text{hypothesis})$

COL053-1.p An Exercise in Composition

For all birds x and y , there exists a bird z that composes x with y for all birds w . Prove that for all birds x, y , and z , there exists a bird u such that for all w , $uw = x(y(zw))$.

$\text{response}(x \circ y, w) = \text{response}(x, \text{response}(y, w)) \quad \text{cnf}(\text{composer_exists}, \text{axiom})$
 $\text{response}(u, f(u)) \neq \text{response}(a, \text{response}(b, \text{response}(c, f(u)))) \quad \text{cnf}(\text{prove_bird_exists}, \text{negated_conjecture})$

COL054-1.p Compatible Birds

There exists a mockingbird. For all birds x and y , there exists a bird z that composes x with y for all birds w . Prove that any two birds are compatible.

$\text{response}(\text{mocking_bird}, y) = \text{response}(y, y) \quad \text{cnf}(\text{mocking_bird_exists}, \text{axiom})$
 $\text{response}(x \circ y, w) = \text{response}(x, \text{response}(y, w)) \quad \text{cnf}(\text{composer_exists}, \text{hypothesis})$
 $\text{response}(a, x) = y \Rightarrow \text{response}(b, y) \neq x \quad \text{cnf}(\text{prove_birds_are_compatible}, \text{negated_conjecture})$

COL055-1.p Happy Birds

There exists a bird which is fond of some other bird. Prove that any bird that is fond of at least one bird must be happy.

$\text{response}(a, b) = b \quad \text{cnf}(\text{fond_bird_exists}, \text{hypothesis})$
 $\text{response}(a, z) = w \Rightarrow \text{response}(a, w) \neq z \quad \text{cnf}(\text{prove_happiness}, \text{negated_conjecture})$

COL056-1.p Normal Birds

For all birds x and y , there exists a bird z that composes x with y for all birds w . Prove that if there exists a happy bird then there exists a normal bird.

$\text{response}(x \circ y, w) = \text{response}(x, \text{response}(y, w)) \quad \text{cnf}(\text{composer_exists}, \text{axiom})$
 $\text{response}(a, b) = c \quad \text{cnf}(a_to_b_responds_c, \text{hypothesis})$
 $\text{response}(a, c) = b \quad \text{cnf}(a_to_c_responds_b, \text{hypothesis})$
 $\text{response}(w, v) \neq v \quad \text{cnf}(\text{prove_there_exists_a_happy_bird}, \text{negated_conjecture})$

COL057-1.p Strong fixed point for S, B, C, and I

The strong fixed point property holds for the set P consisting of the combinators $S, B, C,$ and I , where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$, $((Cx)y)z = (xz)y$, and $Ix = x$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z)) \quad \text{cnf}(s_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z)) \quad \text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(c, x), y), z) = \text{apply}(\text{apply}(x, z), y) \quad \text{cnf}(c_definition, \text{axiom})$
 $\text{apply}(i, x) = x \quad \text{cnf}(i_definition, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y))) \quad \text{cnf}(\text{prove_strong_fixed_point}, \text{negated_conjecture})$

COL058-1.p If there's a lark, then there's an egocentric bird.

Suppose we are given a forest that contains a lark, and we are not given any other information. Prove that at least one bird in the forest must be egocentric.

$\text{response}(\text{response}(\text{lark}, x_1), x_2) = \text{response}(x_1, \text{response}(x_2, x_2)) \quad \text{cnf}(\text{lark_exists}, \text{axiom})$
 $\text{response}(x, x) \neq x \quad \text{cnf}(\text{prove_the_bird_exists}, \text{negated_conjecture})$

COL058-2.p If there's a lark, then there's an egocentric bird.

Suppose we are given a forest that contains a lark, and we are not given any other information. Prove that at least one bird in the forest must be egocentric.

response(response(lark, x_1), x_2) = response(x_1 , response(x_2 , x_2)) cnf(lark_exists, axiom)
 response(response(response(lark, response(response(lark, response(lark, lark))), response(lark, response(lark, lark)))), response(response(lark, response(response(lark, response(lark, lark))), response(lark, response(lark, lark))), response(lark, response(lark, lark))), response(lark, response(lark, lark))), response(lark, response(lark, lark))) cnf(prove_col058_3_p)

COL058-3.p If there's a lark, then there's an egocentric bird.

Suppose we are given a forest that contains a lark, and we are not given any other information. Prove that at least one bird in the forest must be egocentric.

response(response(lark, x_1), x_2) = response(x_1 , response(x_2 , x_2)) cnf(lark_exists, axiom)
 response(response(response(response(lark, lark), response(lark, response(lark, lark))), response(lark, response(lark, lark))), response(lark, response(lark, lark))), response(lark, response(lark, lark)) cnf(prove_col059_1_p)

COL059-1.p L3 ((lark lark) lark) is not egocentric.

response(response(kestrel, x_1), x_2) = x_1 cnf(kestrel_exists, axiom)
 response(response(lark, x_1), x_2) = response(x_1 , response(x_2 , x_2)) cnf(lark_exists, axiom)
 response(response(response(lark, lark), x_1), x_2) = response(response(x_1 , x_1), response(x_2 , x_2)) cnf(lark_lemma1, axiom)
 response(response(response(response(lark, lark), lark), x_1), x_2) = response(response(response(x_1 , x_1), response(x_1 , x_1)), response(x_2 , x_2)) cnf(lark_lemma1, axiom)
 response(l_2 , l_2) \neq l_2 cnf(lark_not_egocentric, axiom)
 response(lark, lark) = l_2 cnf(l2_definition, axiom)
 response(l_2 , lark) = l_3 cnf(l3_definition, axiom)
 response(l_3 , l_3) = l_3 cnf(prove_l3_not_egocentric, negated_conjecture)

COL060-1.p Find combinator equivalent to Q from B and T

Construct from B and T alone a combinator that behaves as the combinator Q does, where ((Bx)y)z = x(yz), (Tx)y = yx, ((Qx)y)z = y(xz).

apply(apply(apply(b , x), y), z) = apply(x , apply(y , z)) cnf(b_definition, axiom)
 apply(apply(t , x), y) = apply(y , x) cnf(t_definition, axiom)
 apply(apply(apply(x , $f(x)$), $g(x)$), $h(x)$) \neq apply($g(x)$, apply($f(x)$, $h(x)$)) cnf(prove_q_combinator, negated_conjecture)

COL060-2.p Find combinator equivalent to Q from B and T

Construct from B and T alone a combinator that behaves as the combinator Q does, where ((Bx)y)z = x(yz), (Tx)y = yx, ((Qx)y)z = y(xz).

apply(apply(apply(b , x), y), z) = apply(x , apply(y , z)) cnf(b_definition, axiom)
 apply(apply(t , x), y) = apply(y , x) cnf(t_definition, axiom)
 apply(apply(apply(apply(apply(b , apply(t , b)), apply(apply(b , b), t)), x), y), z) \neq apply(y , apply(x , z)) cnf(prove_q_combinator, negated_conjecture)

COL060-3.p Find combinator equivalent to Q from B and T

Construct from B and T alone a combinator that behaves as the combinator Q does, where ((Bx)y)z = x(yz), (Tx)y = yx, ((Qx)y)z = y(xz).

apply(apply(apply(b , x), y), z) = apply(x , apply(y , z)) cnf(b_definition, axiom)
 apply(apply(t , x), y) = apply(y , x) cnf(t_definition, axiom)
 apply(apply(apply(apply(apply(b , apply(apply(b , apply(t , b))), b)), t), x), y), z) \neq apply(y , apply(x , z)) cnf(prove_q_combinator, negated_conjecture)

COL061-1.p Find combinator equivalent to Q1 from B and T

Construct from B and T alone a combinator that behaves as the combinator Q1 does, where ((Bx)y)z = x(yz), (Tx)y = yx, ((Q1x)y)z = x(z y).

apply(apply(apply(b , x), y), z) = apply(x , apply(y , z)) cnf(b_definition, axiom)
 apply(apply(t , x), y) = apply(y , x) cnf(t_definition, axiom)
 apply(apply(apply(x , $f(x)$), $g(x)$), $h(x)$) \neq apply($f(x)$, apply($h(x)$, $g(x)$)) cnf(prove_q1_combinator, negated_conjecture)

COL061-2.p Find combinator equivalent to Q1 from B and T

Construct from B and T alone a combinator that behaves as the combinator Q1 does, where ((Bx)y)z = x(yz), (Tx)y = yx, ((Q1x)y)z = x(z y).

apply(apply(apply(b , x), y), z) = apply(x , apply(y , z)) cnf(b_definition, axiom)
 apply(apply(t , x), y) = apply(y , x) cnf(t_definition, axiom)
 apply(apply(apply(apply(apply(b , apply(t , t)), apply(apply(b , b), b)), x), y), z) \neq apply(x , apply(z , y)) cnf(prove_q1_combinator, negated_conjecture)

COL061-3.p Find combinator equivalent to Q1 from B and T

Construct from B and T alone a combinator that behaves as the combinator Q1 does, where ((Bx)y)z = x(yz), (Tx)y = yx, ((Q1x)y)z = x(z y).

apply(apply(apply(b , x), y), z) = apply(x , apply(y , z)) cnf(b_definition, axiom)
 apply(apply(t , x), y) = apply(y , x) cnf(t_definition, axiom)
 apply(apply(apply(apply(apply(b , apply(apply(b , apply(t , t))), b)), b), x), y), z) \neq apply(x , apply(z , y)) cnf(prove_q1_combinator, negated_conjecture)

Construct from B and T alone a combinator that behaves as the combinator V does, where $((Bx)y)z = x(yz)$, $(Tx)y = yx$, $((Vx)y)z = (zx)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$

$\text{apply}(\text{apply}(t, x), y) = \text{apply}(y, x)$ $\text{cnf}(\text{t_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{apply}(b, \text{apply}(t, \text{apply}(\text{apply}(b, b), t))), \text{apply}(\text{apply}(b, b), \text{apply}(\text{apply}(b, t), t))), x), y), z) \neq \text{apply}(\text{apply}(z, x), y)$ $\text{cnf}(\text{prove_v_combinator}, \text{negated_conjecture})$

COL064-8.p Find combinator equivalent to V from B and T

Construct from B and T alone a combinator that behaves as the combinator V does, where $((Bx)y)z = x(yz)$, $(Tx)y = yx$, $((Vx)y)z = (zx)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$

$\text{apply}(\text{apply}(t, x), y) = \text{apply}(y, x)$ $\text{cnf}(\text{t_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, \text{apply}(t, \text{apply}(\text{apply}(b, b), t))), b)), \text{apply}(\text{apply}(b, t), t)), x), y), z) \neq \text{apply}(\text{apply}(z, x), y)$ $\text{cnf}(\text{prove_v_combinator}, \text{negated_conjecture})$

COL064-9.p Find combinator equivalent to V from B and T

Construct from B and T alone a combinator that behaves as the combinator V does, where $((Bx)y)z = x(yz)$, $(Tx)y = yx$, $((Vx)y)z = (zx)y$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$

$\text{apply}(\text{apply}(t, x), y) = \text{apply}(y, x)$ $\text{cnf}(\text{t_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{apply}(b, \text{apply}(t, \text{apply}(\text{apply}(b, b), t))), \text{apply}(\text{apply}(b, \text{apply}(\text{apply}(b, b), t))), t)), x), y), z) \neq \text{apply}(\text{apply}(z, x), y)$ $\text{cnf}(\text{prove_v_combinator}, \text{negated_conjecture})$

COL065-1.p Find combinator equivalent to G from B and T

Construct from B and T alone a combinator that behaves as the combinator G does, where $((Bx)y)z = x(yz)$, $(Tx)y = yx$, $((Gx)y)z = (xw)(yz)$

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$

$\text{apply}(\text{apply}(t, x), y) = \text{apply}(y, x)$ $\text{cnf}(\text{t_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{apply}(x, f(x)), g(x)), h(x)), i(x)) \neq \text{apply}(\text{apply}(f(x), i(x)), \text{apply}(g(x), h(x)))$ $\text{cnf}(\text{prove_g_combinator}, \text{negated_conjecture})$

COL066-1.p Find combinator equivalent to P from B, Q and W

Construct from B, Q and W alone a combinator that behaves as the combinator P does, where $((Bx)y)z = x(yz)$, $((Qx)y)z = y(xz)$, $(Wx)y = (xy)y$, $((Px)y)y)z = (xy)((xy)z)$

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(q, x), y), z) = \text{apply}(y, \text{apply}(x, z))$ $\text{cnf}(\text{q_definition}, \text{axiom})$

$\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{apply}(x, f(x)), g(x)), g(x)), h(x)) \neq \text{apply}(\text{apply}(f(x), g(x)), \text{apply}(\text{apply}(f(x), g(x)), h(x)))$ $\text{cnf}(\text{prove_p_combinator}, \text{negated_conjecture})$

COL066-2.p Find combinator equivalent to P from B, Q and W

Construct from B, Q and W alone a combinator that behaves as the combinator P does, where $((Bx)y)z = x(yz)$, $((Qx)y)z = y(xz)$, $(Wx)y = (xy)y$, $((Px)y)y)z = (xy)((xy)z)$

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(q, x), y), z) = \text{apply}(y, \text{apply}(x, z))$ $\text{cnf}(\text{q_definition}, \text{axiom})$

$\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{apply}(q, q), \text{apply}(w, \text{apply}(q, \text{apply}(q, q))))), x), y), y), z) \neq \text{apply}(\text{apply}(x, y), \text{apply}(\text{apply}(x, y), z))$ $\text{cnf}(\text{prove_p_combinator}, \text{negated_conjecture})$

COL066-3.p Find combinator equivalent to P from B, Q and W

Construct from B, Q and W alone a combinator that behaves as the combinator P does, where $((Bx)y)z = x(yz)$, $((Qx)y)z = y(xz)$, $(Wx)y = (xy)y$, $((Px)y)y)z = (xy)((xy)z)$

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(q, x), y), z) = \text{apply}(y, \text{apply}(x, z))$ $\text{cnf}(\text{q_definition}, \text{axiom})$

$\text{apply}(\text{apply}(w, x), y) = \text{apply}(\text{apply}(x, y), y)$ $\text{cnf}(\text{w_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{apply}(b, \text{apply}(w, \text{apply}(q, \text{apply}(q, q))))), q), x), y), y), z) \neq \text{apply}(\text{apply}(x, y), \text{apply}(\text{apply}(x, y), z))$ $\text{cnf}(\text{prove_p_combinator}, \text{negated_conjecture})$

COL067-1.p Strong fixed point for B and S

The strong fixed point property holds for the set P consisting of the combinators B and S, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{s_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(\text{b_definition}, \text{axiom})$

$\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL068-1.p Weak fixed point for B and S

The weak fixed point property holds for the set P consisting of the combinators B and S, where $((Sx)y)z = (xz)(yz)$, $((Bx)y)z = x(yz)$.

$\text{apply}(\text{apply}(\text{apply}(s, x), y), z) = \text{apply}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(s_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(b_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL069-1.p Strong fixed point for B and L

The strong fixed point property holds for the set P consisting of the combinators B and L, where $((Bx)y)z = x(yz)$, $(Lx)y = x(yy)$.

$\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(\text{apply}(l, x), y) = \text{apply}(x, \text{apply}(y, y))$ $\text{cnf}(l_definition, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL070-1.p Weak fixed point for B and N1

The weak fixed point property holds for the set P consisting of the combinators B and N1, where $N1xyz = xyyz$, $((Bx)y)z = x(yz)$.

$\text{apply}(\text{apply}(\text{apply}(n_1, x), y), z) = \text{apply}(\text{apply}(\text{apply}(x, y), y), z)$ $\text{cnf}(n1_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(b_definition, \text{axiom})$
 $y \neq \text{apply}(\text{combinator}, y)$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL071-1.p Strong fixed point for N and Q

The strong fixed point property holds for the set P consisting of the combinators N and Q, where $((Nx)y)z = ((xz)y)z$, $((Qx)y)z = y(xz)$.

$\text{apply}(\text{apply}(\text{apply}(n, x), y), z) = \text{apply}(\text{apply}(\text{apply}(x, z), y), z)$ $\text{cnf}(n_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(q, x), y), z) = \text{apply}(y, \text{apply}(x, z))$ $\text{cnf}(q_definition, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_fixed_point}, \text{negated_conjecture})$

COL073-1.p Strong fixed point for B and N1

The strong fixed point property holds for the set P consisting of the combinators B and N1, where $N1xyz = xyyz$, $((Bx)y)z = x(yz)$.

$\text{apply}(\text{apply}(\text{apply}(n_1, x), y), z) = \text{apply}(\text{apply}(\text{apply}(x, y), y), z)$ $\text{cnf}(n1_definition, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(b, x), y), z) = \text{apply}(x, \text{apply}(y, z))$ $\text{cnf}(b_definition, \text{axiom})$
 $\text{apply}(y, f(y)) \neq \text{apply}(f(y), \text{apply}(y, f(y)))$ $\text{cnf}(\text{prove_strong_fixed_point}, \text{negated_conjecture})$

COL074-1.p Unsatisfiable variant of TRC

If the function symbol K is replaced by the K combinator then the resultant system is inconsistent.

$\text{apply}(\text{apply}(k, x), y) = x$ $\text{cnf}(k_definition, \text{negated_conjecture})$
 $\text{apply}(\text{projection}_1, \text{pair}(x, y)) = x$ $\text{cnf}(\text{projection}_1, \text{axiom})$
 $\text{apply}(\text{projection}_2, \text{pair}(x, y)) = y$ $\text{cnf}(\text{projection}_2, \text{axiom})$
 $\text{pair}(\text{apply}(\text{projection}_1, x), \text{apply}(\text{projection}_2, x)) = x$ $\text{cnf}(\text{pairing}, \text{axiom})$
 $\text{apply}(\text{pair}(x, y), z) = \text{pair}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{pairwise_application}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, x), y), z) = \text{apply}(\text{apply}(x, \text{apply}(k, z)), \text{apply}(y, z))$ $\text{cnf}(\text{abstraction}, \text{negated_conjecture})$
 $\text{apply}(\text{eq}, \text{pair}(x, x)) = \text{projection}_1$ $\text{cnf}(\text{equality}, \text{axiom})$
 $x = y$ or $\text{apply}(\text{eq}, \text{pair}(x, y)) = \text{projection}_2$ $\text{cnf}(\text{extensionality}_1, \text{axiom})$
 $\text{apply}(x, n(x, y)) = \text{apply}(y, n(x, y)) \Rightarrow x = y$ $\text{cnf}(\text{extensionality}_2, \text{axiom})$
 $\text{projection}_1 \neq \text{projection}_2$ $\text{cnf}(\text{different_projections}, \text{axiom})$

COL074-2.p Unsatisfiable variant of TRC

If the function symbol K is replaced by the K combinator then the resultant system is inconsistent.

$\text{apply}(\text{apply}(k, x), y) = x$ $\text{cnf}(k_definition, \text{negated_conjecture})$
 $\text{apply}(\text{projection}_1, \text{pair}(x, y)) = x$ $\text{cnf}(\text{projection}_1, \text{axiom})$
 $\text{apply}(\text{projection}_2, \text{pair}(x, y)) = y$ $\text{cnf}(\text{projection}_2, \text{axiom})$
 $\text{pair}(\text{apply}(\text{projection}_1, x), \text{apply}(\text{projection}_2, x)) = x$ $\text{cnf}(\text{pairing}, \text{axiom})$
 $\text{apply}(\text{pair}(x, y), z) = \text{pair}(\text{apply}(x, z), \text{apply}(y, z))$ $\text{cnf}(\text{pairwise_application}, \text{axiom})$
 $\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, x), y), z) = \text{apply}(\text{apply}(x, \text{apply}(k, z)), \text{apply}(y, z))$ $\text{cnf}(\text{abstraction}, \text{negated_conjecture})$
 $\text{apply}(\text{eq}, \text{pair}(x, x)) = \text{projection}_1$ $\text{cnf}(\text{equality}, \text{axiom})$
 $x = y$ or $\text{apply}(\text{eq}, \text{pair}(x, y)) = \text{projection}_2$ $\text{cnf}(\text{extensionality}_1, \text{axiom})$
 $\text{apply}(x, n(x, y)) = \text{apply}(y, n(x, y)) \Rightarrow x = y$ $\text{cnf}(\text{extensionality}_2, \text{axiom})$
 $\text{projection}_1 \neq \text{projection}_2$ $\text{cnf}(\text{different_projections}, \text{axiom})$
 $\text{apply}(\text{apply}(f, x), y) = \text{apply}(x, x)$ $\text{cnf}(\text{diagonal_combinator}, \text{axiom})$

COL074-3.p Unsatisfiable variant of TRC

If the function symbol K is replaced by the K combinator then the resultant system is inconsistent.

$\text{apply}(\text{apply}(k, x), y) = x$ $\text{cnf}(k_definition, \text{negated_conjecture})$
 $\text{apply}(\text{projection}_1, \text{pair}(x, y)) = x$ $\text{cnf}(\text{projection}_1, \text{axiom})$

```

apply(projection2, pair(x, y)) = y      cnf(projection2, axiom)
pair(apply(projection1, x), apply(projection2, x)) = x      cnf(pairing, axiom)
apply(pair(x, y), z) = pair(apply(x, z), apply(y, z))      cnf(pairwise_application, axiom)
apply(apply(apply(abstraction, x), y), z) = apply(apply(x, apply(k, z)), apply(y, z))      cnf(abstraction, negated_conjecture)
apply(eq, pair(x, x)) = projection1      cnf(equality, axiom)
x = y or apply(eq, pair(x, y)) = projection2      cnf(extensionality1, axiom)
apply(x, n(x, y)) = apply(y, n(x, y)) ⇒ x = y      cnf(extensionality2, axiom)
projection1 ≠ projection2      cnf(different_projections, axiom)
apply(k, projection1) = k_projection1      cnf(k_projection1, axiom)
apply(k, projection2) = k_projection2      cnf(k_projection2, axiom)
s = apply(eq, pair(apply(k, s), apply(k, projection2)))      cnf(self_referential, axiom)

```

COL075-1.p Lemma 1 for showing the unsatisfiable variant of TRC

Searching for a diagonal combinator F with the property $f X Y = X X$.

```

apply(apply(k, x), y) = x      cnf(k_definition, axiom)
apply(projection1, pair(x, y)) = x      cnf(projection1, axiom)
apply(projection2, pair(x, y)) = y      cnf(projection2, axiom)
pair(apply(projection1, x), apply(projection2, x)) = x      cnf(pairing, axiom)
apply(pair(x, y), z) = pair(apply(x, z), apply(y, z))      cnf(pairwise_application, axiom)
apply(apply(apply(abstraction, x), y), z) = apply(apply(x, apply(k, z)), apply(y, z))      cnf(abstraction, axiom)
apply(eq, pair(x, x)) = projection1      cnf(equality, axiom)
x = y or apply(eq, pair(x, y)) = projection2      cnf(extensionality1, axiom)
apply(x, n(x, y)) = apply(y, n(x, y)) ⇒ x = y      cnf(extensionality2, axiom)
projection1 ≠ projection2      cnf(different_projections, axiom)
apply(apply(y, b(y)), c(y)) ≠ apply(b(y), b(y))      cnf(prove_diagonal_combinator, negated_conjecture)

```

COL075-2.p Lemma 1 for showing the unsatisfiable variant of TRC

Searching for a diagonal combinator F with the property $f X Y = X X$.

```

apply(apply(k, x), y) = x      cnf(k_definition, axiom)
apply(apply(apply(abstraction, x), y), z) = apply(apply(x, apply(k, z)), apply(y, z))      cnf(abstraction, axiom)
apply(apply(y, b(y)), c(y)) ≠ apply(b(y), b(y))      cnf(prove_diagonal_combinator, negated_conjecture)

```

COL076-1.p Lemma 2 for showing the unsatisfiable variant of TRC

Searching for the self-referential combinator with the property $s = eq <k s, k p2>$.

```

apply(apply(k, x), y) = x      cnf(k_definition, axiom)
apply(projection1, pair(x, y)) = x      cnf(projection1, axiom)
apply(projection2, pair(x, y)) = y      cnf(projection2, axiom)
pair(apply(projection1, x), apply(projection2, x)) = x      cnf(pairing, axiom)
apply(pair(x, y), z) = pair(apply(x, z), apply(y, z))      cnf(pairwise_application, axiom)
apply(apply(apply(abstraction, x), y), z) = apply(apply(x, apply(k, z)), apply(y, z))      cnf(abstraction, axiom)
apply(eq, pair(x, x)) = projection1      cnf(equality, axiom)
x = y or apply(eq, pair(x, y)) = projection2      cnf(extensionality1, axiom)
apply(x, n(x, y)) = apply(y, n(x, y)) ⇒ x = y      cnf(extensionality2, axiom)
projection1 ≠ projection2      cnf(different_projections, axiom)
apply(apply(f, x), y) = apply(x, x)      cnf(diagonal_combinator, axiom)
y ≠ apply(eq, pair(apply(k, y), apply(k, projection2)))      cnf(prove_self_referential, negated_conjecture)

```

COL076-2.p Lemma 2 for showing the unsatisfiable variant of TRC

Searching for the self-referential combinator with the property $s = eq <k s, k p2>$.

```

apply(apply(k, x), y) = x      cnf(k_definition, axiom)
apply(projection1, pair(x, y)) = x      cnf(projection1, axiom)
apply(projection2, pair(x, y)) = y      cnf(projection2, axiom)
pair(apply(projection1, x), apply(projection2, x)) = x      cnf(pairing, axiom)
apply(pair(x, y), z) = pair(apply(x, z), apply(y, z))      cnf(pairwise_application, axiom)
apply(apply(apply(abstraction, x), y), z) = apply(apply(x, apply(k, z)), apply(y, z))      cnf(abstraction, axiom)
apply(x, n(x, y)) = apply(y, n(x, y)) ⇒ x = y      cnf(extensionality2, axiom)
apply(apply(f, x), y) = apply(x, x)      cnf(diagonal_combinator, axiom)
y ≠ apply(eq, pair(apply(k, y), apply(k, projection2)))      cnf(prove_self_referential, negated_conjecture)

```

COL077-1.p Abst Abst Abst Abst Abst Abst = Id

include('Axioms/COL001-0.ax')

```

apply(identity, x) = x      cnf(identity_definition, axiom)

```

$\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, \text{abstraction}), \text{abstraction}), \text{abstraction}), \text{abstraction}), \text{abstraction}) \neq \text{identity}$

COL078-1.p Abst Abst Abst Abst = k(k(id))

include('Axioms/COL001-0.ax')

$\text{apply}(\text{identity}, x) = x$ $\text{cnf}(\text{identity_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, \text{abstraction}), \text{abstraction}), \text{abstraction}) \neq k(k(\text{identity}))$ $\text{cnf}(\text{prove_TRC1b}, \text{negated_conjecture})$

COL078-2.p Abst Abst Abst Abst = k(k(id))

$\text{apply}(k(x), y) = x$ $\text{cnf}(k_definition, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, x), y), z) = \text{apply}(\text{apply}(x, k(z)), \text{apply}(y, z))$ $\text{cnf}(\text{abstraction}, \text{axiom})$

$\text{apply}(x, n(x, y)) = \text{apply}(y, n(x, y)) \Rightarrow x = y$ $\text{cnf}(\text{extensionality}_2, \text{axiom})$

$\text{apply}(\text{identity}, x) = x$ $\text{cnf}(\text{identity_definition}, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, \text{abstraction}), \text{abstraction}), \text{abstraction}) \neq k(k(\text{identity}))$ $\text{cnf}(\text{prove_TRC1b}, \text{negated_conjecture})$

COL079-1.p Abst(Abst(Abst X)) = Abst X

include('Axioms/COL001-0.ax')

$\text{apply}(\text{abstraction}, \text{apply}(\text{abstraction}, \text{apply}(\text{abstraction}, b))) \neq \text{apply}(\text{abstraction}, b)$ $\text{cnf}(\text{prove_TRC2a}, \text{negated_conjecture})$

COL079-2.p Abst(Abst(Abst X)) = Abst X

$\text{apply}(k(x), y) = x$ $\text{cnf}(k_definition, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, x), y), z) = \text{apply}(\text{apply}(x, k(z)), \text{apply}(y, z))$ $\text{cnf}(\text{abstraction}, \text{axiom})$

$\text{apply}(x, n(x, y)) = \text{apply}(y, n(x, y)) \Rightarrow x = y$ $\text{cnf}(\text{extensionality}_2, \text{axiom})$

$\text{apply}(\text{abstraction}, \text{apply}(\text{abstraction}, \text{apply}(\text{abstraction}, b))) \neq \text{apply}(\text{abstraction}, b)$ $\text{cnf}(\text{prove_TRC2a}, \text{negated_conjecture})$

COL080-1.p Abst(Abst k(X)) = k(X)

include('Axioms/COL001-0.ax')

$\text{apply}(\text{identity}, x) = x$ $\text{cnf}(\text{identity_definition}, \text{axiom})$

$k(b) \neq \text{apply}(\text{abstraction}, \text{apply}(\text{abstraction}, k(b)))$ $\text{cnf}(\text{prove_TRC2b}, \text{negated_conjecture})$

COL080-2.p Abst(Abst k(X)) = k(X)

$\text{apply}(k(x), y) = x$ $\text{cnf}(k_definition, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, x), y), z) = \text{apply}(\text{apply}(x, k(z)), \text{apply}(y, z))$ $\text{cnf}(\text{abstraction}, \text{axiom})$

$\text{apply}(x, n(x, y)) = \text{apply}(y, n(x, y)) \Rightarrow x = y$ $\text{cnf}(\text{extensionality}_2, \text{axiom})$

$\text{apply}(\text{identity}, x) = x$ $\text{cnf}(\text{identity_definition}, \text{axiom})$

$k(b) \neq \text{apply}(\text{abstraction}, \text{apply}(\text{abstraction}, k(b)))$ $\text{cnf}(\text{prove_TRC2b}, \text{negated_conjecture})$

COL081-1.p Abst k(k(X)) = k(k(X))

include('Axioms/COL001-0.ax')

$\text{apply}(\text{identity}, x) = x$ $\text{cnf}(\text{identity_definition}, \text{axiom})$

$k(k(b)) \neq \text{apply}(\text{abstraction}, k(k(b)))$ $\text{cnf}(\text{prove_TRC2c}, \text{negated_conjecture})$

COL081-2.p Abst k(k(X)) = k(k(X))

$\text{apply}(k(x), y) = x$ $\text{cnf}(k_definition, \text{axiom})$

$\text{apply}(\text{apply}(\text{apply}(\text{abstraction}, x), y), z) = \text{apply}(\text{apply}(x, k(z)), \text{apply}(y, z))$ $\text{cnf}(\text{abstraction}, \text{axiom})$

$\text{apply}(x, n(x, y)) = \text{apply}(y, n(x, y)) \Rightarrow x = y$ $\text{cnf}(\text{extensionality}_2, \text{axiom})$

$\text{apply}(\text{identity}, x) = x$ $\text{cnf}(\text{identity_definition}, \text{axiom})$

$k(k(b)) \neq \text{apply}(\text{abstraction}, k(k(b)))$ $\text{cnf}(\text{prove_TRC2c}, \text{negated_conjecture})$

COL082-1.p Type-respecting combinators

include('Axioms/COL001-0.ax')

COL083-1.p Compatible Birds, part 1

$\text{response}(\text{mocking_bird}, a) = \text{response}(a, a)$ $\text{cnf}(\text{mocking_bird_exists}, \text{axiom})$

$\text{response}(a \circ b, c) = \text{response}(a, \text{response}(b, c))$ $\text{cnf}(\text{composer_exists}, \text{hypothesis})$

$\text{response}(a, a) \neq b$ $\text{cnf}(\text{prove_birds_are_compatible}_1, \text{negated_conjecture})$

COL084-1.p Compatible Birds, part 2

$\text{response}(\text{mocking_bird}, a) = \text{response}(a, a)$ $\text{cnf}(\text{mocking_bird_exists}, \text{axiom})$

$\text{response}(a \circ b, c) = \text{response}(a, \text{response}(b, c))$ $\text{cnf}(\text{composer_exists}, \text{hypothesis})$

$\text{response}(b, b) \neq a$ $\text{cnf}(\text{prove_birds_are_compatible}_2, \text{negated_conjecture})$

COL085-1.p Happy Birds, part 1

$\text{response}(a, b) = b$ $\text{cnf}(\text{fond_bird_exists}, \text{hypothesis})$

$\text{response}(a, a) \neq b$ $\text{cnf}(\text{prove_happiness}_1, \text{negated_conjecture})$

COL086-1.p Happy Birds, part 2

$\text{response}(a, b) = b$ $\text{cnf}(\text{fond_bird_exists}, \text{hypothesis})$

response(a, b) $\neq a$ cnf(prove_happiness₂, negated_conjecture)

COL087-1.p Strong fixed point for B and M

The strong fixed point property holds for the set with the combinators B and M as a basis, where $Bxyz = x(yz)$ and $Mx = xx$.

apply(apply(apply(b, x), y), z) = apply(x , apply(y, z)) cnf(definition_B, axiom)

apply(m, x) = apply(x, x) cnf(definition_M, axiom)

apply($y, f(y)$) \neq apply($f(y)$, apply($y, f(y)$)) cnf(strong_fixpoint, negated_conjecture)

COL090-1.p i_contract_E

combK \neq combS cnf(k_s, axiom)

combK \neq comb_app(p, q) cnf(k_app, axiom)

combS \neq comb_app(p, q) cnf(s_app, axiom)

comb_app(p_1, q_1) = comb_app(p_2, q_2) $\Rightarrow p_1 = p_2$ cnf(app_app₁, axiom)

comb_app(p_1, q_1) = comb_app(p_2, q_2) $\Rightarrow q_1 = q_2$ cnf(app_app₂, axiom)

($p_1 = p_2$ and $q_1 = q_2$) \Rightarrow comb_app(p_1, q_1) = comb_app(p_2, q_2) cnf(app_app₃, axiom)

comb_app(p, q) \in comb $\Rightarrow p \in$ comb cnf(ap_E₁, axiom)

comb_app(p, q) \in comb $\Rightarrow q \in$ comb cnf(ap_E₂, axiom)

combK \in comb cnf(comb_intros₁, axiom)

combS \in comb cnf(comb_intros₂, axiom)

($p \in$ comb and $q \in$ comb) \Rightarrow comb_app(p, q) \in comb cnf(comb_intros₃, axiom)

$a \in$ comb \Rightarrow pair(a, a) \in rtranc1(contract) cnf(reduction_refl, axiom)

($p \in$ comb and $q \in$ comb) \Rightarrow pair(comb_app(comb_app(combK, p), q), p) \in contract cnf(contract_K, axiom)

($p \in$ comb and $q \in$ comb and $r \in$ comb) \Rightarrow pair(comb_app(comb_app(comb_app(combS, p), q), r), comb_app(comb_app(p, r), contract) cnf(contract_S, axiom)

pair(comb_app(p, q), r) \in contract \Rightarrow (ap_contractE_c₁(p, q, r) or ap_contractE_c₂(p, q, r) or ap_contractE_c₃(p, q, r) or ap-co

ap_contractE_c₁(p, q, r) $\Rightarrow r \in$ comb cnf(ap_contractE₂, axiom)

ap_contractE_c₁(p, q, r) $\Rightarrow q \in$ comb cnf(ap_contractE₃, axiom)

ap_contractE_c₁(p, q, r) $\Rightarrow p =$ comb_app(combK, r) cnf(ap_contractE₄, axiom)

ap_contractE_c₂(p, q, r) \Rightarrow ap_contractE_sk1p(p, q, r) \in comb cnf(ap_contractE₅, axiom)

ap_contractE_c₂(p, q, r) \Rightarrow ap_contractE_sk1q(p, q, r) \in comb cnf(ap_contractE₆, axiom)

ap_contractE_c₂(p, q, r) $\Rightarrow q \in$ comb cnf(ap_contractE₇, axiom)

ap_contractE_c₂(p, q, r) $\Rightarrow r =$ comb_app(comb_app(ap_contractE_sk1p(p, q, r), q), comb_app(ap_contractE_sk1q(p, q, r), q))

ap_contractE_c₂(p, q, r) $\Rightarrow p =$ comb_app(comb_app(combS, ap_contractE_sk1p(p, q, r)), ap_contractE_sk1q(p, q, r)) cnf(a

ap_contractE_c₃(p, q, r) \Rightarrow pair(p , ap_contractE_sk2q(p, q, r)) \in contract cnf(ap_contractE₁₀, axiom)

ap_contractE_c₃(p, q, r) $\Rightarrow q \in$ comb cnf(ap_contractE₁₁, axiom)

ap_contractE_c₃(p, q, r) $\Rightarrow r =$ comb_app(ap_contractE_sk2q(p, q, r), q) cnf(ap_contractE₁₂, axiom)

ap_contractE_c₄(p, q, r) \Rightarrow pair(q , ap_contractE_sk3q(p, q, r)) \in contract cnf(ap_contractE₁₃, axiom)

ap_contractE_c₄(p, q, r) $\Rightarrow p \in$ comb cnf(ap_contractE₁₄, axiom)

ap_contractE_c₄(p, q, r) $\Rightarrow r =$ comb_app(p , ap_contractE_sk3q(p, q, r)) cnf(ap_contractE₁₅, axiom)

\neg pair(combK, r) \in contract cnf(k_contractE, axiom)

\neg pair(combS, r) \in contract cnf(s_contractE, axiom)

pair(comb_app(comb_app(combS, combK), combK), r) \in contract cnf(i_contract_E, negated_conjecture)

COL090-3.p i_contract_E

combK \neq combS cnf(k_s, axiom)

combK \neq comb_app(p, q) cnf(k_app, axiom)

combS \neq comb_app(p, q) cnf(s_app, axiom)

comb_app(p_1, q_1) = comb_app(p_2, q_2) $\Rightarrow p_1 = p_2$ cnf(app_app₁, axiom)

comb_app(p_1, q_1) = comb_app(p_2, q_2) $\Rightarrow q_1 = q_2$ cnf(app_app₂, axiom)

($p_1 = p_2$ and $q_1 = q_2$) \Rightarrow comb_app(p_1, q_1) = comb_app(p_2, q_2) cnf(app_app₃, axiom)

pair(comb_app(p, q), r) \in contract \Rightarrow (ap_contractE_c₁(p, q, r) or ap_contractE_c₂(p, q, r) or ap_contractE_c₃(p, q, r) or ap-co

ap_contractE_c₁(p, q, r) $\Rightarrow r \in$ comb cnf(ap_contractE₂, axiom)

ap_contractE_c₁(p, q, r) $\Rightarrow q \in$ comb cnf(ap_contractE₃, axiom)

ap_contractE_c₁(p, q, r) $\Rightarrow p =$ comb_app(combK, r) cnf(ap_contractE₄, axiom)

ap_contractE_c₂(p, q, r) \Rightarrow ap_contractE_sk1p(p, q, r) \in comb cnf(ap_contractE₅, axiom)

ap_contractE_c₂(p, q, r) \Rightarrow ap_contractE_sk1q(p, q, r) \in comb cnf(ap_contractE₆, axiom)

ap_contractE_c₂(p, q, r) $\Rightarrow q \in$ comb cnf(ap_contractE₇, axiom)

ap_contractE_c₂(p, q, r) $\Rightarrow r =$ comb_app(comb_app(ap_contractE_sk1p(p, q, r), q), comb_app(ap_contractE_sk1q(p, q, r), q))

ap_contractE_c₂(p, q, r) $\Rightarrow p =$ comb_app(comb_app(combS, ap_contractE_sk1p(p, q, r)), ap_contractE_sk1q(p, q, r)) cnf(a

ap_contractE_c₃(p, q, r) \Rightarrow pair(p , ap_contractE_sk2q(p, q, r)) \in contract cnf(ap_contractE₁₀, axiom)

$\text{ap_contractE_c}_3(p, q, r) \Rightarrow q \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_{11}, \text{axiom})$
 $\text{ap_contractE_c}_3(p, q, r) \Rightarrow r = \text{comb_app}(\text{ap_contractE_sk2q}(p, q, r), q) \quad \text{cnf}(\text{ap_contractE}_{12}, \text{axiom})$
 $\text{ap_contractE_c}_4(p, q, r) \Rightarrow \text{pair}(q, \text{ap_contractE_sk3q}(p, q, r)) \in \text{contract} \quad \text{cnf}(\text{ap_contractE}_{13}, \text{axiom})$
 $\text{ap_contractE_c}_4(p, q, r) \Rightarrow p \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_{14}, \text{axiom})$
 $\text{ap_contractE_c}_4(p, q, r) \Rightarrow r = \text{comb_app}(p, \text{ap_contractE_sk3q}(p, q, r)) \quad \text{cnf}(\text{ap_contractE}_{15}, \text{axiom})$
 $\neg \text{pair}(\text{combK}, r) \in \text{contract} \quad \text{cnf}(\text{k_contractE}, \text{axiom})$
 $\neg \text{pair}(\text{combS}, r) \in \text{contract} \quad \text{cnf}(\text{s_contractE}, \text{axiom})$
 $\text{pair}(\text{comb_app}(\text{comb_app}(\text{combS}, \text{combK}), \text{combK}), r) \in \text{contract} \quad \text{cnf}(\text{i_contract_E}, \text{negated_conjecture})$

COL091-1.p k1_contractD_c1

$\text{comb_app}(K, p) \neg \rightarrow r \implies \text{EX } q. r = \text{comb_app}(K, q) \ \& \ p \neg \rightarrow q$
 $\text{combK} \neq \text{combS} \quad \text{cnf}(\text{k_s}, \text{axiom})$
 $\text{combK} \neq \text{comb_app}(p, q) \quad \text{cnf}(\text{k_app}, \text{axiom})$
 $\text{combS} \neq \text{comb_app}(p, q) \quad \text{cnf}(\text{s_app}, \text{axiom})$
 $\text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \Rightarrow p_1 = p_2 \quad \text{cnf}(\text{app_app}_1, \text{axiom})$
 $\text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \Rightarrow q_1 = q_2 \quad \text{cnf}(\text{app_app}_2, \text{axiom})$
 $(p_1 = p_2 \ \& \ q_1 = q_2) \Rightarrow \text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \quad \text{cnf}(\text{app_app}_3, \text{axiom})$
 $\text{comb_app}(p, q) \in \text{comb} \Rightarrow p \in \text{comb} \quad \text{cnf}(\text{ap_E}_1, \text{axiom})$
 $\text{comb_app}(p, q) \in \text{comb} \Rightarrow q \in \text{comb} \quad \text{cnf}(\text{ap_E}_2, \text{axiom})$
 $\text{combK} \in \text{comb} \quad \text{cnf}(\text{comb_intros}_1, \text{axiom})$
 $\text{combS} \in \text{comb} \quad \text{cnf}(\text{comb_intros}_2, \text{axiom})$
 $(p \in \text{comb} \ \& \ q \in \text{comb}) \Rightarrow \text{comb_app}(p, q) \in \text{comb} \quad \text{cnf}(\text{comb_intros}_3, \text{axiom})$
 $a \in \text{comb} \Rightarrow \text{pair}(a, a) \in \text{rtrancl}(\text{contract}) \quad \text{cnf}(\text{reduction_refl}, \text{axiom})$
 $(p \in \text{comb} \ \& \ q \in \text{comb}) \Rightarrow \text{pair}(\text{comb_app}(\text{comb_app}(\text{combK}, p), q), p) \in \text{contract} \quad \text{cnf}(\text{contract_K}, \text{axiom})$
 $(p \in \text{comb} \ \& \ q \in \text{comb} \ \& \ r \in \text{comb}) \Rightarrow \text{pair}(\text{comb_app}(\text{comb_app}(\text{comb_app}(\text{combS}, p), q), r), \text{comb_app}(\text{comb_app}(p, r), \text{contract})) \in \text{contract} \quad \text{cnf}(\text{contract_S}, \text{axiom})$
 $\text{pair}(\text{comb_app}(p, q), r) \in \text{contract} \Rightarrow (\text{ap_contractE_c}_1(p, q, r) \ \text{or} \ \text{ap_contractE_c}_2(p, q, r) \ \text{or} \ \text{ap_contractE_c}_3(p, q, r) \ \text{or} \ \text{ap_contractE_c}_4(p, q, r))$
 $\text{ap_contractE_c}_1(p, q, r) \Rightarrow r \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_2, \text{axiom})$
 $\text{ap_contractE_c}_1(p, q, r) \Rightarrow q \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_3, \text{axiom})$
 $\text{ap_contractE_c}_1(p, q, r) \Rightarrow p = \text{comb_app}(\text{combK}, r) \quad \text{cnf}(\text{ap_contractE}_4, \text{axiom})$
 $\text{ap_contractE_c}_2(p, q, r) \Rightarrow \text{ap_contractE_sk1p}(p, q, r) \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_5, \text{axiom})$
 $\text{ap_contractE_c}_2(p, q, r) \Rightarrow \text{ap_contractE_sk1q}(p, q, r) \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_6, \text{axiom})$
 $\text{ap_contractE_c}_2(p, q, r) \Rightarrow q \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_7, \text{axiom})$
 $\text{ap_contractE_c}_2(p, q, r) \Rightarrow r = \text{comb_app}(\text{comb_app}(\text{ap_contractE_sk1p}(p, q, r), q), \text{comb_app}(\text{ap_contractE_sk1q}(p, q, r), q))$
 $\text{ap_contractE_c}_2(p, q, r) \Rightarrow p = \text{comb_app}(\text{comb_app}(\text{combS}, \text{ap_contractE_sk1p}(p, q, r)), \text{ap_contractE_sk1q}(p, q, r)) \quad \text{cnf}(\text{ap_contractE}_8, \text{axiom})$
 $\text{ap_contractE_c}_3(p, q, r) \Rightarrow \text{pair}(p, \text{ap_contractE_sk2q}(p, q, r)) \in \text{contract} \quad \text{cnf}(\text{ap_contractE}_{10}, \text{axiom})$
 $\text{ap_contractE_c}_3(p, q, r) \Rightarrow q \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_{11}, \text{axiom})$
 $\text{ap_contractE_c}_3(p, q, r) \Rightarrow r = \text{comb_app}(\text{ap_contractE_sk2q}(p, q, r), q) \quad \text{cnf}(\text{ap_contractE}_{12}, \text{axiom})$
 $\text{ap_contractE_c}_4(p, q, r) \Rightarrow \text{pair}(q, \text{ap_contractE_sk3q}(p, q, r)) \in \text{contract} \quad \text{cnf}(\text{ap_contractE}_{13}, \text{axiom})$
 $\text{ap_contractE_c}_4(p, q, r) \Rightarrow p \in \text{comb} \quad \text{cnf}(\text{ap_contractE}_{14}, \text{axiom})$
 $\text{ap_contractE_c}_4(p, q, r) \Rightarrow r = \text{comb_app}(p, \text{ap_contractE_sk3q}(p, q, r)) \quad \text{cnf}(\text{ap_contractE}_{15}, \text{axiom})$
 $\neg \text{pair}(\text{combK}, r) \in \text{contract} \quad \text{cnf}(\text{k_contractE}, \text{axiom})$
 $\neg \text{pair}(\text{combS}, r) \in \text{contract} \quad \text{cnf}(\text{s_contractE}, \text{axiom})$
 $\text{pair}(\text{comb_app}(\text{combK}, p), r) \in \text{contract} \quad \text{cnf}(\text{k1_contractD_h}_1, \text{hypothesis})$
 $r = \text{comb_app}(\text{combK}, q) \Rightarrow \neg \text{pair}(p, q) \in \text{contract} \quad \text{cnf}(\text{k1_contractD_c}_1, \text{negated_conjecture})$

COL098-1.p diamond_strip_lemmaD_2c1

[rule_format]:[\neg diamond(r); $\langle x, y \rangle : r \wedge + \neg$]

$\text{combK} \neq \text{combS} \quad \text{cnf}(\text{k_s}, \text{axiom})$
 $\text{combK} \neq \text{comb_app}(p, q) \quad \text{cnf}(\text{k_app}, \text{axiom})$
 $\text{combS} \neq \text{comb_app}(p, q) \quad \text{cnf}(\text{s_app}, \text{axiom})$
 $\text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \Rightarrow p_1 = p_2 \quad \text{cnf}(\text{app_app}_1, \text{axiom})$
 $\text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \Rightarrow q_1 = q_2 \quad \text{cnf}(\text{app_app}_2, \text{axiom})$
 $(p_1 = p_2 \ \& \ q_1 = q_2) \Rightarrow \text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \quad \text{cnf}(\text{app_app}_3, \text{axiom})$
 $\text{pair}(a, b) \in r \Rightarrow \text{pair}(a, b) \in \text{tranc1}(r) \quad \text{cnf}(\text{r_into_tranc1}, \text{axiom})$
 $\text{trans}(\text{tranc1}(r)) \quad \text{cnf}(\text{trans_tranc1}, \text{axiom})$
 $(\text{trans}(r) \ \& \ \text{pair}(a, b) \in r \ \& \ \text{pair}(b, c) \in r) \Rightarrow \text{pair}(a, c) \in r \quad \text{cnf}(\text{transD}, \text{axiom})$
 $(\text{pair}(x, y) \in r \ \& \ \text{pair}(x, yP) \in r) \Rightarrow \text{pair}(y, \text{sk}_1(x, y, yP)) \in r \quad \text{cnf}(\text{diamond_strip_lemmaD_2h}_1, \text{hypothesis})$
 $(\text{pair}(x, y) \in r \ \& \ \text{pair}(x, yP) \in r) \Rightarrow \text{pair}(yP, \text{sk}_1(x, y, yP)) \in r \quad \text{cnf}(\text{diamond_strip_lemmaD_2h}_2, \text{hypothesis})$
 $\text{pair}(x, y) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_strip_lemmaD_2h}_3, \text{hypothesis})$

$\text{pair}(y, z) \in r \quad \text{cnf}(\text{diamond_strip_lemmaD_2h}_4, \text{hypothesis})$
 $\text{pair}(x, yP) \in r \Rightarrow \text{pair}(yP, \text{sk}_2(yP)) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_strip_lemmaD_2h}_5, \text{hypothesis})$
 $\text{pair}(x, yP) \in r \Rightarrow \text{pair}(y, \text{sk}_2(yP)) \in r \quad \text{cnf}(\text{diamond_strip_lemmaD_2h}_6, \text{hypothesis})$
 $\text{pair}(x, \text{sk}_3) \in r \quad \text{cnf}(\text{diamond_strip_lemmaD_2c}_1, \text{negated_conjecture})$
 $\text{pair}(\text{sk}_3, zA) \in \text{tranc1}(r) \Rightarrow \neg \text{pair}(z, zA) \in r \quad \text{cnf}(\text{diamond_strip_lemmaD_2c}_2, \text{negated_conjecture})$

COL099-1.p diamond_tranc1_1c1

$\text{diamond}(r) ==> \text{diamond}(r \wedge +)$
 $\text{combK} \neq \text{combS} \quad \text{cnf}(\text{k_s}, \text{axiom})$
 $\text{combK} \neq \text{comb_app}(p, q) \quad \text{cnf}(\text{k_app}, \text{axiom})$
 $\text{combS} \neq \text{comb_app}(p, q) \quad \text{cnf}(\text{s_app}, \text{axiom})$
 $\text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \Rightarrow p_1 = p_2 \quad \text{cnf}(\text{app_app}_1, \text{axiom})$
 $\text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \Rightarrow q_1 = q_2 \quad \text{cnf}(\text{app_app}_2, \text{axiom})$
 $(p_1 = p_2 \text{ and } q_1 = q_2) \Rightarrow \text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \quad \text{cnf}(\text{app_app}_3, \text{axiom})$
 $\text{pair}(a, b) \in r \Rightarrow \text{pair}(a, b) \in \text{tranc1}(r) \quad \text{cnf}(\text{r_into_tranc1}, \text{axiom})$
 $\text{trans}(\text{tranc1}(r)) \quad \text{cnf}(\text{trans_tranc1}, \text{axiom})$
 $(\text{trans}(r) \text{ and } \text{pair}(a, b) \in r \text{ and } \text{pair}(b, c) \in r) \Rightarrow \text{pair}(a, c) \in r \quad \text{cnf}(\text{transD}, \text{axiom})$
 $(\text{diamond}(r) \text{ and } \text{pair}(x, y) \in \text{tranc1}(r) \text{ and } \text{pair}(x, yP) \in r) \Rightarrow \text{pair}(yP, \text{diamond_strip_lemmaD_sk}_1(x, y, yP, r)) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_strip_lemmaD}_1, \text{axiom})$
 $(\text{diamond}(r) \text{ and } \text{pair}(x, y) \in \text{tranc1}(r) \text{ and } \text{pair}(x, yP) \in r) \Rightarrow \text{pair}(y, \text{diamond_strip_lemmaD_sk}_1(x, y, yP, r)) \in r \quad \text{cnf}(\text{diamond_strip_lemmaD}_2, \text{axiom})$
 $\text{diamond}(r) \quad \text{cnf}(\text{diamond_tranc1_1h}_1, \text{hypothesis})$
 $\text{pair}(y, ya) \in r \quad \text{cnf}(\text{diamond_tranc1_1h}_2, \text{hypothesis})$
 $\text{pair}(y, yp) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_tranc1_1c}_1, \text{negated_conjecture})$
 $\text{pair}(ya, z) \in \text{tranc1}(r) \Rightarrow \neg \text{pair}(yp, z) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_tranc1_1c}_2, \text{negated_conjecture})$

COL100-1.p diamond_tranc1_2c1

$\text{diamond}(r) ==> \text{diamond}(r \wedge +)$
 $\text{combK} \neq \text{combS} \quad \text{cnf}(\text{k_s}, \text{axiom})$
 $\text{combK} \neq \text{comb_app}(p, q) \quad \text{cnf}(\text{k_app}, \text{axiom})$
 $\text{combS} \neq \text{comb_app}(p, q) \quad \text{cnf}(\text{s_app}, \text{axiom})$
 $\text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \Rightarrow p_1 = p_2 \quad \text{cnf}(\text{app_app}_1, \text{axiom})$
 $\text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \Rightarrow q_1 = q_2 \quad \text{cnf}(\text{app_app}_2, \text{axiom})$
 $(p_1 = p_2 \text{ and } q_1 = q_2) \Rightarrow \text{comb_app}(p_1, q_1) = \text{comb_app}(p_2, q_2) \quad \text{cnf}(\text{app_app}_3, \text{axiom})$
 $\text{pair}(a, b) \in r \Rightarrow \text{pair}(a, b) \in \text{tranc1}(r) \quad \text{cnf}(\text{r_into_tranc1}, \text{axiom})$
 $\text{trans}(\text{tranc1}(r)) \quad \text{cnf}(\text{trans_tranc1}, \text{axiom})$
 $(\text{trans}(r) \text{ and } \text{pair}(a, b) \in r \text{ and } \text{pair}(b, c) \in r) \Rightarrow \text{pair}(a, c) \in r \quad \text{cnf}(\text{transD}, \text{axiom})$
 $(\text{diamond}(r) \text{ and } \text{pair}(x, y) \in \text{tranc1}(r) \text{ and } \text{pair}(x, yP) \in r) \Rightarrow \text{pair}(yP, \text{diamond_strip_lemmaD_sk}_1(x, y, yP, r)) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_strip_lemmaD}_1, \text{axiom})$
 $(\text{diamond}(r) \text{ and } \text{pair}(x, y) \in \text{tranc1}(r) \text{ and } \text{pair}(x, yP) \in r) \Rightarrow \text{pair}(y, \text{diamond_strip_lemmaD_sk}_1(x, y, yP, r)) \in r \quad \text{cnf}(\text{diamond_strip_lemmaD}_2, \text{axiom})$
 $\text{diamond}(r) \quad \text{cnf}(\text{diamond_tranc1_2h}_1, \text{hypothesis})$
 $\text{pair}(y, ya) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_tranc1_2h}_2, \text{hypothesis})$
 $\text{pair}(ya, z) \in r \quad \text{cnf}(\text{diamond_tranc1_2h}_3, \text{hypothesis})$
 $\text{pair}(y, yP) \in \text{tranc1}(r) \Rightarrow \text{pair}(ya, \text{sk}_1(yP)) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_tranc1_2h}_4, \text{hypothesis})$
 $\text{pair}(y, yP) \in \text{tranc1}(r) \Rightarrow \text{pair}(yP, \text{sk}_1(yP)) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_tranc1_2h}_5, \text{hypothesis})$
 $\text{pair}(y, yp) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_tranc1_2c}_1, \text{negated_conjecture})$
 $\text{pair}(z, zA) \in \text{tranc1}(r) \Rightarrow \neg \text{pair}(yp, zA) \in \text{tranc1}(r) \quad \text{cnf}(\text{diamond_tranc1_2c}_2, \text{negated_conjecture})$

COL101-1.p Problem about combinators

$\text{include}(\text{'Axioms/COL002-0.ax'})$
 $\text{include}(\text{'Axioms/MS C001-2.ax'})$
 $\text{include}(\text{'Axioms/MS C001-0.ax'})$
 $(\text{c_in}(\text{c_Pair}(\text{v_b}, \text{v_c}, \text{t_a}, \text{t_a}), \text{c_Transitive_Closure_Ortranc1}(\text{v_r}, \text{t_a}), \text{tc_prod}(\text{t_a}, \text{t_a})) \text{ and } \text{c_in}(\text{c_Pair}(\text{v_a}, \text{v_b}, \text{t_a}, \text{t_a}), \text{c_Tr}(\text{v_r}, \text{t_a}), \text{tc_prod}(\text{t_a}, \text{t_a}))) \text{ and } \text{c_in}(\text{c_Pair}(\text{v_a}, \text{v_c}, \text{t_a}, \text{t_a}), \text{c_Transitive_Closure_Ortranc1}(\text{v_r}, \text{t_a}), \text{tc_prod}(\text{t_a}, \text{t_a})) \quad \text{cnf}(\text{cls_Transitive_Closure_Ortranc1}, \text{axiom})$
 $\neg \text{c_in}(\text{c_Pair}(\text{c_Comb_Ocomb_Oop_A_D_D}(\text{v_x}, \text{v_z}), \text{c_Comb_Ocomb_Oop_A_D_D}(\text{v_x}, \text{v_z}), \text{tc_Comb_Ocomb}, \text{tc_Comb_Ocomb}(\text{v_r}, \text{t_a}), \text{tc_prod}(\text{t_a}, \text{t_a}))) \quad \text{cnf}(\text{cls_Transitive_Closure_Ortranc1}, \text{axiom})$

COL101-2.p Problem about combinators

$\neg \text{c_in}(\text{c_Pair}(\text{c_Comb_Ocomb_Oop_A_D_D}(\text{v_x}, \text{v_z}), \text{c_Comb_Ocomb_Oop_A_D_D}(\text{v_x}, \text{v_z}), \text{tc_Comb_Ocomb}, \text{tc_Comb_Ocomb}(\text{v_r}, \text{t_a}), \text{tc_prod}(\text{t_a}, \text{t_a}))) \text{ and } \text{c_in}(\text{c_Pair}(\text{v_a}, \text{v_a}, \text{t_a}, \text{t_a}), \text{c_Transitive_Closure_Ortranc1}(\text{v_r}, \text{t_a}), \text{tc_prod}(\text{t_a}, \text{t_a})) \quad \text{cnf}(\text{cls_Transitive_Closure_Ortranc1}, \text{axiom})$

COL102-1.p Problem about combinators

$(c_in(c_Pair(v_b, v_c, t_a, t_a), c_Transitive_Closure_Ortranc1(v_r, t_a), tc_prod(t_a, t_a))$ and $c_in(c_Pair(v_a, v_b, t_a, t_a), c_Tr$
 $c_in(c_Pair(v_a, v_c, t_a, t_a), c_Transitive_Closure_Ortranc1(v_r, t_a), tc_prod(t_a, t_a))$ $cnf(cls_Transitive_Closure_Ortranc1$
 $c_in(c_Pair(v_x, v_y, t_a, t_a), c_Transitive_Closure_Ortranc1(v_r, t_a), tc_prod(t_a, t_a))$ $cnf(cls_conjecture_0, negated_conjecture$
 $c_in(c_Pair(v_y, v_z, t_a, t_a), v_r, tc_prod(t_a, t_a))$ $cnf(cls_conjecture_1, negated_conjecture)$
 $c_in(c_Pair(v_x, v_xb, t_a, t_a), v_r, tc_prod(t_a, t_a))$ $cnf(cls_conjecture_2, negated_conjecture)$
 $c_in(c_Pair(v_x, v_U, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(c_Pair(v_U, v_xaa(v_U), t_a, t_a), c_Transitive_Closure_Ortranc1(v$
 $c_in(c_Pair(v_x, v_U, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(c_Pair(v_y, v_xaa(v_U), t_a, t_a), v_r, tc_prod(t_a, t_a))$ $cnf(cls_c$
 $c_in(c_Pair(v_z, v_U, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow \neg c_in(c_Pair(v_xb, v_U, t_a, t_a), c_Transitive_Closure_Ortranc1(v_r, t_a$
 $(c_in(c_Pair(v_U, v_W, t_a, t_a), v_r, tc_prod(t_a, t_a))$ and $c_in(c_Pair(v_U, v_V, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(c_Pair$
 $(c_in(c_Pair(v_U, v_W, t_a, t_a), v_r, tc_prod(t_a, t_a))$ and $c_in(c_Pair(v_U, v_V, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(c_Pair$

COL123-2.p Problem about combinators

$c_in(v_p, v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(v_p, c_Transitive_Closure_Ortranc1(v_r, t_a), tc_prod(t_a, t_a))$ $cnf(cls_Transitive_C$
 $(c_in(c_Pair(v_b, v_c, t_a, t_a), c_Transitive_Closure_Ortranc1(v_r, t_a), tc_prod(t_a, t_a))$ and $c_in(c_Pair(v_a, v_b, t_a, t_a), c_Tr$
 $c_in(c_Pair(v_a, v_c, t_a, t_a), c_Transitive_Closure_Ortranc1(v_r, t_a), tc_prod(t_a, t_a))$ $cnf(cls_Transitive_Closure_Ortranc1$
 $c_in(c_Pair(v_y, v_z, t_a, t_a), v_r, tc_prod(t_a, t_a))$ $cnf(cls_conjecture_1, negated_conjecture)$
 $c_in(c_Pair(v_x, v_xb, t_a, t_a), v_r, tc_prod(t_a, t_a))$ $cnf(cls_conjecture_2, negated_conjecture)$
 $c_in(c_Pair(v_x, v_U, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(c_Pair(v_U, v_xaa(v_U), t_a, t_a), c_Transitive_Closure_Ortranc1(v$
 $c_in(c_Pair(v_x, v_U, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(c_Pair(v_y, v_xaa(v_U), t_a, t_a), v_r, tc_prod(t_a, t_a))$ $cnf(cls_c$
 $c_in(c_Pair(v_z, v_U, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow \neg c_in(c_Pair(v_xb, v_U, t_a, t_a), c_Transitive_Closure_Ortranc1(v_r, t_a$
 $(c_in(c_Pair(v_U, v_W, t_a, t_a), v_r, tc_prod(t_a, t_a))$ and $c_in(c_Pair(v_U, v_V, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(c_Pair$
 $(c_in(c_Pair(v_U, v_W, t_a, t_a), v_r, tc_prod(t_a, t_a))$ and $c_in(c_Pair(v_U, v_V, t_a, t_a), v_r, tc_prod(t_a, t_a)) \Rightarrow c_in(c_Pair$

COL124-1.p Problem about combinators

include('Axioms/COL002-0.ax')
include('Axioms/MS001-2.ax')
include('Axioms/MS001-0.ax')

$c_in(c_Pair(v_x, v_y, tc_Comb_Ocomb, tc_Comb_Ocomb), c_Transitive_Closure_Ortranc1(c_Comb_Ocontract, tc_Comb_Ocomb)$
 $c_in(c_Pair(c_Comb_Ocomb_Oop_A_D_D(v_x, v_z), c_Comb_Ocomb_Oop_A_D_D(v_y, v_z), tc_Comb_Ocomb, tc_Comb_Ocomb),$
 $c_in(c_Pair(v_x, v_y, tc_Comb_Ocomb, tc_Comb_Ocomb), c_Transitive_Closure_Ortranc1(c_Comb_Ocontract, tc_Comb_Ocomb)$
 $c_in(c_Pair(c_Comb_Ocomb_Oop_A_D_D(v_z, v_x), c_Comb_Ocomb_Oop_A_D_D(v_z, v_y), tc_Comb_Ocomb, tc_Comb_Ocomb),$
 $c_in(c_Pair(c_Comb_Ocomb_Oop_A_D_D(c_Comb_Ocomb_Oop_A_D_D(c_Comb_Ocomb_OK, c_Comb_OI), c_Comb_Ocomb_Oop$
 $c_in(c_Pair(c_Comb_Ocomb_Oop_A_D_D(c_Comb_Ocomb_Oop_A_D_D(c_Comb_Ocomb_OK, c_Comb_OI), c_Comb_Ocomb_Oop$
 $c_in(c_Pair(c_Comb_Ocomb_Oop_A_D_D(c_Comb_Ocomb_Oop_A_D_D(c_Comb_Ocomb_OK, c_Comb_OI), c_Comb_Ocomb_Oop$
 $(c_in(c_Pair(v_U, v_W, tc_Comb_Ocomb, tc_Comb_Ocomb), c_Comb_Ocontract, tc_prod(tc_Comb_Ocomb, tc_Comb_Ocomb))$ a
 $c_in(c_Pair(v_V, v_x(v_U, v_V, v_W), tc_Comb_Ocomb, tc_Comb_Ocomb), c_Comb_Ocontract, tc_prod(tc_Comb_Ocomb, tc_Comb$
 $(c_in(c_Pair(v_U, v_W, tc_Comb_Ocomb, tc_Comb_Ocomb), c_Comb_Ocontract, tc_prod(tc_Comb_Ocomb, tc_Comb_Ocomb))$ a
 $c_in(c_Pair(v_W, v_x(v_U, v_V, v_W), tc_Comb_Ocomb, tc_Comb_Ocomb), c_Comb_Ocontract, tc_prod(tc_Comb_Ocomb, tc_Comb$

COL124-2.p Problem about combinators

$(c_in(c_Pair(v_U, v_W, tc_Comb_Ocomb, tc_Comb_Ocomb), c_Comb_Ocontract, tc_prod(tc_Comb_Ocomb, tc_Comb_Ocomb))$ a
 $c_in(c_Pair(v_V, v_x(v_U, v_V, v_W), tc_Comb_Ocomb, tc_Comb_Ocomb), c_Comb_Ocontract, tc_prod(tc_Comb_Ocomb, tc_Comb$
 $\neg c_in(c_Pair(c_Comb_OI, v_z, tc_Comb_Ocomb, tc_Comb_Ocomb), c_Comb_Ocontract, tc_prod(tc_Comb_Ocomb, tc_Comb_Oco$
 $c_in(c_Pair(c_Comb_Ocomb_Oop_A_D_D(c_Comb_Ocomb_Oop_A_D_D(c_Comb_Ocomb_OK, v_x), v_y), v_x, tc_Comb_Ocomb, t$