HAL axioms
HAL problems

HAL001+1.p Short Five Lemma, Part 1
include('Axioms/HAL001+0.ax')
morphism(alpha, a, b)   fof(alpha_morphism, axiom)
morphism(beta, b, c)   fof(beta_morphism, axiom)
morphism(gamma, d, e)   fof(gamma_morphism, axiom)
morphism(delta, e, r)   fof(delta_morphism, axiom)
morphism(f, a, d)   fof(f_morphism, axiom)
morphism(g, b, e)   fof(g_morphism, axiom)
morphism(h, c, r)   fof(h_morphism, axiom)
injection(alpha)   fof(alpha_injection, axiom)
injection(gamma)   fof(gamma_injection, axiom)
surjection(beta)   fof(beta_surjection, axiom)
surjection(delta)   fof(delta_surjection, axiom)
exact(alpha, beta)   fof(alpha_beta_exact, axiom)
exact(gammma, delta)   fof(gamma_delta_exact, axiom)
commute(alpha, g, f, gamma)   fof(alpha_g_f_gamma_commute, axiom)
commute(beta, h, g, delta)   fof(beta_h_g_delta_commute, axiom)
injection(f)   fof(f_injection, hypothesis)
injection(h)   fof(h_injection, hypothesis)
injection(g)   fof(g_injection, conjecture)

HAL001+2.p Short Five Lemma, Part 1
include('Axioms/HAL001+0.ax')
$\forall$morphism, dom, cod: ((injection(morphism) and morphism(morphism, dom, cod)) $\Rightarrow$ $\forall$el: ((element(el, dom) and apply(mor
0) $\Rightarrow$ el = 0))   fof(injection_properties$_2$, axiom)
$\forall$morphism, dom, cod: ((morphism(morphism, dom, cod) and $\forall$el: ((element(el, dom) and apply(morphism, el) = 0) $\Rightarrow$
el = 0)) $\Rightarrow$ injection(morphism))   fof(properties_for_injection$_2$, axiom)
morphism(alpha, a, b)   fof(alpha_morphism, axiom)
morphism(beta, b, c)   fof(beta_morphism, axiom)
morphism(gamma, d, e)   fof(gamma_morphism, axiom)
morphism(delta, e, r)   fof(delta_morphism, axiom)
morphism(f, a, d)   fof(f_morphism, axiom)
morphism(g, b, e)   fof(g_morphism, axiom)
morphism(h, c, r)   fof(h_morphism, axiom)
injection(alpha)   fof(alpha_injection, axiom)
injection(gamma)   fof(gamma_injection, axiom)
surjection(beta)   fof(beta_surjection, axiom)
surjection(delta)   fof(delta_surjection, axiom)
exact(alpha, beta)   fof(alpha_beta_exact, axiom)
exact(gammma, delta)   fof(gamma_delta_exact, axiom)
commute(alpha, g, f, gamma)   fof(alpha_g_f_gamma_commute, axiom)
commute(beta, h, g, delta)   fof(beta_h_g_delta_commute, axiom)
injection(f)   fof(f_injection, hypothesis)
injection(h)   fof(h_injection, hypothesis)
injection(g)   fof(g_injection, conjecture)

HAL002+1.p Equivalence of injection axioms
include('Axioms/HAL001+0.ax')
$\forall$morphism, dom, cod: ((injection$_2$(morphism) and morphism(morphism, dom, cod)) $\Rightarrow$ $\forall$el: ((element(el, dom) and apply(mo
0) $\Rightarrow$ el = 0))   fof(injection_properties$_2$, axiom)
$\forall$morphism, dom, cod: ((morphism(morphism, dom, cod) and $\forall$el: ((element(el, dom) and apply(morphism, el) = 0) $\Rightarrow$
el = 0)) $\Rightarrow$ injection$_2$(morphism))   fof(properties_for_injection$_2$, axiom)
morphism(x, any$_1$, any$_2$)   fof(x_morphism, hypothesis)
injection(x) $\iff$ injection$_2$(x)   fof(my, conjecture)

HAL003+1.p Short Five Lemma, Part 2

include('Axioms/HAL001+0.ax')
morphism(alpha, a, b)   fof(alpha_morphism, axiom)
morphism(beta, b, c)   fof(beta_morphism, axiom)
morphism(gamma, d, e)   fof(gamma_morphism, axiom)
morphism(delta, e, r)   fof(delta_morphism, axiom)
morphism(f, a, d)   fof(f_morphism, axiom)
morphism(g, b, e)   fof(g_morphism, axiom)
morphism(h, c, r)   fof(h_morphism, axiom)
injection(alpha)   fof(alpha_injection, axiom)
injection(gamma)   fof(gamma_injection, axiom)
surjection(beta)   fof(beta_surjection, axiom)
surjection(delta)   fof(delta_surjection, axiom)
exact(alpha, beta)   fof(alpha_beta_exact, axiom)
exact(gammma, delta)   fof(gamma_delta_exact, axiom)
commute(alpha, g, f, gamma)   fof(alpha_g_f_gamma_commute, axiom)
commute(beta, h, g, delta)   fof(beta_h_g_delta_commute, axiom)
surjection(f)   fof(f_surjection, hypothesis)
surjection(h)   fof(h_surjection, hypothesis)
surjection(g)   fof(g_surjection, conjecture)

HAL003+2.p Short Five Lemma, Part 2
include('Axioms/HAL001+0.ax')
morphism(alpha, a, b)   fof(alpha_morphism, axiom)
morphism(beta, b, c)   fof(beta_morphism, axiom)
morphism(gamma, d, e)   fof(gamma_morphism, axiom)
morphism(delta, e, r)   fof(delta_morphism, axiom)
morphism(f, a, d)   fof(f_morphism, axiom)
morphism(g, b, e)   fof(g_morphism, axiom)
morphism(h, c, r)   fof(h_morphism, axiom)
surjection(beta)   fof(beta_surjection, axiom)
exact(gammma, delta)   fof(gamma_delta_exact, axiom)
commute(alpha, g, f, gamma)   fof(alpha_g_f_gamma_commute, axiom)
commute(beta, h, g, delta)   fof(beta_h_g_delta_commute, axiom)
surjection(f)   fof(f_surjection, hypothesis)
surjection(h)   fof(h_surjection, hypothesis)
surjection(g)   fof(g_surjection, conjecture)

HAL003+3.p Short Five Lemma, Part 2
include('Axioms/HAL001+0.ax')
morphism(alpha, a, b)   fof(alpha_morphism, axiom)
morphism(beta, b, c)   fof(beta_morphism, axiom)
morphism(gamma, d, e)   fof(gamma_morphism, axiom)
morphism(delta, e, r)   fof(delta_morphism, axiom)
morphism(f, a, d)   fof(f_morphism, axiom)
morphism(g, b, e)   fof(g_morphism, axiom)
morphism(h, c, r)   fof(h_morphism, axiom)
injection(alpha)   fof(alpha_injection, axiom)
injection(gamma)   fof(gamma_injection, axiom)
surjection(beta)   fof(beta_surjection, axiom)
surjection(delta)   fof(delta_surjection, axiom)
exact(alpha, beta)   fof(alpha_beta_exact, axiom)
exact(gammma, delta)   fof(gamma_delta_exact, axiom)
commute(alpha, g, f, gamma)   fof(alpha_g_f_gamma_commute, axiom)
commute(beta, h, g, delta)   fof(beta_h_g_delta_commute, axiom)
surjection(f)   fof(f_surjection, hypothesis)
surjection(h)   fof(h_surjection, hypothesis)
$\forall e$: (element(e, e) $\Rightarrow$ $\exists r, b_1$: (element(r, r) and apply(delta, e) $= r$ and element(b_1, b) and apply(h, apply(beta, b_1)) $=$ r and apply(delta, apply(g, b_1)) $= r$))   fof(lemma$_3$, axiom)
$\forall e$: (element(e, e) $\Rightarrow$ $\exists b_1, e_1, a$: (element(b_1, b) and element(e_1, e) and subtract(e, apply(g, b_1), e) $= e_1$ and element(a, a) and e_1 and apply(g, apply(alpha, a)) $= e_1$))   fof(lemma$_8$, axiom)

$\forall e$: (element$(e, e) \Rightarrow \exists b_1, b_2$: (element$(b_1, b)$ and element(b_2, b) and apply$(g, \text{subtract}(b, b_1, b_2)) = e$)) fof(lemma$_{12}$, axiom)
surjection(g) fof(g_surjection, conjecture)

HAL004+1.p Lemma for the short Five Lemma, Part 2
include('Axioms/HAL001+0.ax')
morphism(alpha, a, b) fof(alpha_morphism, axiom)
morphism(beta, b, c) fof(beta_morphism, axiom)
morphism(gamma, d, e) fof(gamma_morphism, axiom)
morphism(delta, e, r) fof(delta_morphism, axiom)
morphism(f, a, d) fof(f_morphism, axiom)
morphism(g, b, e) fof(g_morphism, axiom)
morphism(h, c, r) fof(h_morphism, axiom)
injection(alpha) fof(alpha_injection, axiom)
injection(gamma) fof(gamma_injection, axiom)
surjection(beta) fof(beta_surjection, axiom)
surjection(delta) fof(delta_surjection, axiom)
exact(alpha, beta) fof(alpha_beta_exact, axiom)
exact(gammma, delta) fof(gamma_delta_exact, axiom)
commute(alpha, g, f, gamma) fof(alpha_g_f_gamma_commute, axiom)
commute(beta, h, g, delta) fof(beta_h_g_delta_commute, axiom)
surjection(f) fof(f_surjection, hypothesis)
surjection(h) fof(h_surjection, hypothesis)
$\forall e$: (element$(e, e) \Rightarrow \exists r, b_1$: (element$(r, r)$ and apply(delta, e) $= r$ and element(b_1, b) and apply$(h$, apply(beta, $b_1)) = r$ and apply(delta, apply$(g, b_1)) = r$)) fof(lemma$_3$, conjecture)

HAL005+1.p Lemma for the short Five Lemma, Part 2
include('Axioms/HAL001+0.ax')
morphism(alpha, a, b) fof(alpha_morphism, axiom)
morphism(beta, b, c) fof(beta_morphism, axiom)
morphism(gamma, d, e) fof(gamma_morphism, axiom)
morphism(delta, e, r) fof(delta_morphism, axiom)
morphism(f, a, d) fof(f_morphism, axiom)
morphism(g, b, e) fof(g_morphism, axiom)
morphism(h, c, r) fof(h_morphism, axiom)
injection(alpha) fof(alpha_injection, axiom)
injection(gamma) fof(gamma_injection, axiom)
surjection(beta) fof(beta_surjection, axiom)
surjection(delta) fof(delta_surjection, axiom)
exact(alpha, beta) fof(alpha_beta_exact, axiom)
exact(gammma, delta) fof(gamma_delta_exact, axiom)
commute(alpha, g, f, gamma) fof(alpha_g_f_gamma_commute, axiom)
commute(beta, h, g, delta) fof(beta_h_g_delta_commute, axiom)
surjection(f) fof(f_surjection, hypothesis)
surjection(h) fof(h_surjection, hypothesis)
$\forall e$: (element$(e, e) \Rightarrow \exists r, b_1$: (element$(r, r)$ and apply(delta, e) $= r$ and element(b_1, b) and apply$(h$, apply(beta, $b_1)) = r$ and apply(delta, apply$(g, b_1)) = r$)) fof(lemma$_3$, axiom)
$\forall e$: (element$(e, e) \Rightarrow \exists b_1, e_1, a$: (element$(b_1, b)$ and element(e_1, e) and subtract$(e$, apply$(g, b_1), e) = e_1$ and element(a, a) and e_1 and apply$(g$, apply(alpha, $a)) = e_1$)) fof(lemma$_8$, conjecture)

HAL006+1.p Lemma for the short Five Lemma, Part 2
include('Axioms/HAL001+0.ax')
morphism(alpha, a, b) fof(alpha_morphism, axiom)
morphism(beta, b, c) fof(beta_morphism, axiom)
morphism(gamma, d, e) fof(gamma_morphism, axiom)
morphism(delta, e, r) fof(delta_morphism, axiom)
morphism(f, a, d) fof(f_morphism, axiom)
morphism(g, b, e) fof(g_morphism, axiom)
morphism(h, c, r) fof(h_morphism, axiom)
injection(alpha) fof(alpha_injection, axiom)
injection(gamma) fof(gamma_injection, axiom)
surjection(beta) fof(beta_surjection, axiom)

surjection(delta)   fof(delta_surjection, axiom)
exact(alpha, beta)   fof(alpha_beta_exact, axiom)
exact(gammma, delta)   fof(gamma_delta_exact, axiom)
commute(alpha, g, f, gamma)   fof(alpha_g_f_gamma_commute, axiom)
commute(beta, h, g, delta)   fof(beta_h_g_delta_commute, axiom)
surjection(f)   fof(f_surjection, hypothesis)
surjection(h)   fof(h_surjection, hypothesis)
$\forall e$: (element(e, e) $\Rightarrow$ $\exists r, b_1$: (element(r, r) and apply(delta, e) $= r$ and element(b_1, b) and apply(h, apply(beta, b_1)) $=$ r and apply(delta, apply(g, b_1)) $= r$))   fof(lemma$_3$, axiom)
$\forall e$: (element(e, e) $\Rightarrow$ $\exists b_1, e_1, a$: (element(b_1, b) and element(e_1, e) and subtract(e, apply(g, b_1), e) $= e_1$ and element(a, a) and e_1 and apply(g, apply(alpha, a)) $= e_1$))   fof(lemma$_8$, axiom)
$\forall e$: (element(e, e) $\Rightarrow$ $\exists b_1, b_2$: (element(b_1, b) and element(b_2, b) and apply(g, subtract(b, b_1, b_2)) $= e$))   fof(lemma$_{12}$, conject

HAL007+1.p Standard homological algebra axioms
include('Axioms/HAL001+0.ax')