

# HWC axioms

**HWC001-0.ax** Definitions of AND, OR and NOT

```
and( $n_0, n_0$ ) =  $n_0$       cnf(and_definition1, axiom)
and( $n_0, n_1$ ) =  $n_0$       cnf(and_definition2, axiom)
and( $n_1, n_0$ ) =  $n_0$       cnf(and_definition3, axiom)
and( $n_1, n_1$ ) =  $n_1$       cnf(and_definition4, axiom)
or( $n_0, n_0$ ) =  $n_0$       cnf(or_definition1, axiom)
or( $n_0, n_1$ ) =  $n_1$       cnf(or_definition2, axiom)
or( $n_1, n_0$ ) =  $n_1$       cnf(or_definition3, axiom)
or( $n_1, n_1$ ) =  $n_1$       cnf(or_definition4, axiom)
not( $n_0$ ) =  $n_1$       cnf(not_definition1, axiom)
not( $n_1$ ) =  $n_0$       cnf(not_definition2, axiom)
```

**HWC002-0.ax** Definitions of AND, OR and NOT

```
and( $x, n_0$ ) =  $n_0$       cnf(and_definition1, axiom)
and( $x, n_1$ ) =  $x$       cnf(and_definition2, axiom)
or( $x, n_0$ ) =  $x$       cnf(or_definition1, axiom)
or( $x, n_1$ ) =  $n_1$       cnf(or_definition2, axiom)
not( $n_0$ ) =  $n_1$       cnf(not_definition1, axiom)
not( $n_1$ ) =  $n_0$       cnf(not_definition2, axiom)
```

# HWC problems

**HWC001-1.p** Design an OR gate using NAND gates

```
 $x \uparrow y = \text{not}(\text{and}(x, y))$       cnf(nand_definition, axiom)
and( $x, n_0$ ) =  $n_0$       cnf(and_definition1, axiom)
and( $x, n_1$ ) =  $x$       cnf(and_definition2, axiom)
not( $n_0$ ) =  $n_1$       cnf(not_definition1, axiom)
not( $n_1$ ) =  $n_0$       cnf(not_definition2, axiom)
(output( $x_1, x_2, x_3, x_4$ ) and output( $y_1, y_2, y_3, y_4$ ))  $\Rightarrow$  output( $x_1 \uparrow y_1, x_2 \uparrow y_2, x_3 \uparrow y_3, x_4 \uparrow y_4$ )      cnf(nand_table_definition, ne)
output( $n_0, n_0, n_1, n_1$ )      cnf(input1, negated_conjecture)
output( $n_0, n_1, n_0, n_1$ )      cnf(input2, negated_conjecture)
 $\neg$ output( $n_0, n_1, n_1, n_1$ )      cnf(prove_cannot_make_or, negated_conjecture)
```

**HWC003-1.p** Invert 3 inputs with 2 not gates

```
include('Axioms/HWC002-0.ax')
add_inverter(list( $x, y$ ),  $z$ ) = list( $x, \text{add\_inverter}(y, z)$ )      cnf(add_inverter_definition1, axiom)
add_inverter( $x, y$ ) = list( $y, x$ )      cnf(add_inverter_definition2, axiom)
make_reverse_list(list(inverter_table( $x_{000}, x_{001}, x_{010}, x_{011}, x_{100}, x_{101}, x_{110}, x_{111}$ ),  $v$ )) = list_reversion(possible_reversion(r00m,  $x_{000}, x_{001}, x_{010}, x_{011}, x_{100}, x_{101}, x_{110}, x_{111}$ ),  $v$ )
make_reverse_list( $v$ ) = end      cnf(make_reverse_list_definition2, axiom)
possible_reversion(xname,  $n_1, n_0$ ) = xname      cnf(possible_reversion1, axiom)
possible_reversion(xname,  $n_0, n_1$ ) = not_reversion      cnf(possible_reversion2, axiom)
possible_reversion(xname,  $x, x$ ) = not_reversion      cnf(possible_reversion3, axiom)
list_reversion(not_reversion,  $x$ ) =  $x$       cnf(list_reversion_definition1, axiom)
list_reversion( $x, \text{list\_reversion}(y, z)$ ) = list_reversion( $y, \text{list\_reversion}(x, z)$ )      cnf(list_reversion_definition2, axiom)
list_reversion( $x, \text{list\_reversion}(x, y)$ ) = list_reversion( $x, y$ )      cnf(list_reversion_definition3, axiom)
(output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ ) and output( $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, v$ ))  $\Rightarrow$  output(and( $x_1, y_1$ ), and( $x_2, y_2$ ), and( $x_3, y_3$ ), and( $x_4, y_4$ ))
(output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ ) and output( $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, v$ ))  $\Rightarrow$  output(or( $x_1, y_1$ ), or( $x_2, y_2$ ), or( $x_3, y_3$ ), or( $x_4, y_4$ ))
output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ )  $\Rightarrow$  test(not( $x_1$ ), not( $x_2$ ), not( $x_3$ ), not( $x_4$ ), not( $x_5$ ), not( $x_6$ ), not( $x_7$ ), not( $x_8$ ), add_inverter( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ ))
test( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v, \text{xrevlist}$ )  $\Rightarrow$  output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ )      cnf(create_test, axiom)
output( $n_0, n_0, n_0, n_0, n_1, n_1, n_1, x$ )      cnf(input1, negated_conjecture)
output( $n_0, n_0, n_1, n_1, n_0, n_0, n_1, n_1, x$ )      cnf(input2, negated_conjecture)
output( $n_0, n_1, n_0, n_1, n_0, n_1, n_0, n_1, x$ )      cnf(input3, negated_conjecture)
(output( $n_1, n_1, n_1, n_1, n_0, n_0, n_0, v$ ) and output( $n_1, n_1, n_0, n_0, n_1, n_1, n_0, n_0, v$ ))  $\Rightarrow$   $\neg$ output( $n_1, n_0, n_1, n_0, n_1, n_0, n_1, n_0, v$ )
```

**HWC003-2.p** Invert 3 inputs with 2 not gates

```
include('Axioms/HWC002-0.ax')
add_inverter(list( $x, y$ ),  $z$ ) = list( $x, \text{add\_inverter}(y, z)$ )      cnf(add_inverter_definition1, axiom)
add_inverter( $x, y$ ) = list( $y, x$ )      cnf(add_inverter_definition2, axiom)
make_reverse_list(list(inverter_table( $x_{000}, x_{001}, x_{010}, x_{011}, x_{100}, x_{101}, x_{110}, x_{111}$ ),  $v$ )) = list_reversion(possible_reversion(r00m,  $x_{000}, x_{001}, x_{010}, x_{011}, x_{100}, x_{101}, x_{110}, x_{111}$ ),  $v$ )
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make_reverse_list( $v$ ) = end      cnf(make_reverse_list_definition2, axiom)
possible_reversion(xname,  $n_1, n_0$ ) = xname      cnf(possible_reversion1, axiom)
possible_reversion(xname,  $n_0, n_1$ ) = not_reversion      cnf(possible_reversion2, axiom)
possible_reversion(xname,  $x, x$ ) = not_reversion      cnf(possible_reversion3, axiom)
list_reversion(not_reversion,  $x$ ) =  $x$       cnf(list_reversion_definition1, axiom)
list_reversion( $x, \text{list\_reversion}(y, z)$ ) = list_reversion( $y, \text{list\_reversion}(x, z)$ )      cnf(list_reversion_definition2, axiom)
list_reversion( $x, \text{list\_reversion}(x, y)$ ) = list_reversion( $x, y$ )      cnf(list_reversion_definition3, axiom)
(basic_output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ ) and basic_output( $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, v$ ))  $\Rightarrow$  basic_output(and( $x_1, y_1$ ), and( $y_1, x_2$ ))
(basic_output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ ) and output( $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, v$ ))  $\Rightarrow$  output(or( $x_1, y_1$ ), or( $x_2, y_2$ ), or( $x_3, y_3$ ))
output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ )  $\Rightarrow$  test(not( $x_1$ ), not( $x_2$ ), not( $x_3$ ), not( $x_4$ ), not( $x_5$ ), not( $x_6$ ), not( $x_7$ ), not( $x_8$ ), add_inverter( $v$ ))
basic_output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ )  $\Rightarrow$  output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ )      cnf(basic_output_definition, axiom)
test( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v, \text{xrevlist}$ )  $\Rightarrow$  basic_output( $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, v$ )      cnf(create_test, axiom)
output( $n_0, n_0, n_0, n_0, n_1, n_1, n_1, x$ )      cnf(input1, negated_conjecture)
output( $n_0, n_0, n_1, n_1, n_0, n_0, n_1, n_1, x$ )      cnf(input2, negated_conjecture)
output( $n_0, n_1, n_0, n_1, n_0, n_1, n_0, n_1, x$ )      cnf(input3, negated_conjecture)
(output( $n_1, n_1, n_1, n_1, n_0, n_0, n_0, v$ ) and output( $n_1, n_1, n_0, n_0, n_1, n_1, n_0, n_0, v$ ))  $\Rightarrow$   $\neg$ output( $n_1, n_0, n_1, n_0, n_1, n_0, n_1, n_0, v$ )

```

**HWC004-1.p** Definitions of AND, OR and NOT

```
include('Axioms/HWC001-0.ax')
```

**HWC004-2.p** Definitions of AND, OR and NOT

```
include('Axioms/HWC002-0.ax')
```