

LAT axioms

LAT001-0.ax Lattice theory (equality) axioms

$x \wedge x = x$ cnf(idempotence_of_meet, axiom)
 $x \vee x = x$ cnf(idempotence_of_join, axiom)
 $x \wedge (x \vee y) = x$ cnf(absorption₁, axiom)
 $x \vee (x \wedge y) = x$ cnf(absorption₂, axiom)
 $x \wedge y = y \wedge x$ cnf(commutativity_of_meet, axiom)
 $x \vee y = y \vee x$ cnf(commutativity_of_join, axiom)
 $(x \wedge y) \wedge z = x \wedge (y \wedge z)$ cnf(associativity_of_meet, axiom)
 $(x \vee y) \vee z = x \vee (y \vee z)$ cnf(associativity_of_join, axiom)

LAT001-1.ax Lattice theory modularity (equality) axioms

$x \wedge n_0 = n_0$ cnf(x_meet₀, axiom)
 $x \vee n_0 = x$ cnf(x_join₀, axiom)
 $x \wedge n_1 = x$ cnf(x_meet₁, axiom)
 $x \vee n_1 = n_1$ cnf(x_join₁, axiom)
 $x \wedge z = x \Rightarrow z \wedge (x \vee y) = x \vee (y \wedge z)$ cnf(modular, axiom)

LAT001-2.ax Lattice theory complement (equality) axioms

$x' = y \Rightarrow x \wedge y = n_0$ cnf(complement_meet, axiom)
 $x' = y \Rightarrow x \vee y = n_1$ cnf(complement_join, axiom)
 $(x \wedge y = n_0 \text{ and } x \vee y = n_1) \Rightarrow x' = y$ cnf(meet_join_complement, axiom)

LAT001-3.ax Lattice theory unique complement (equality) axioms

unique_complement(x, y) $\Rightarrow x' = y$ cnf(unique_complement₁, axiom)
(unique_complement(x, y) and $x' = z$) $\Rightarrow z = y$ cnf(unique_complement₂, axiom)
 $x' = y \Rightarrow (\text{unique_complement}(x, y) \text{ or } x' = f(x, y))$ cnf(unique_complement₃, axiom)
 $(x' = y \text{ and } f(x, y) = y) \Rightarrow \text{unique_complement}(x, y)$ cnf(unique_complement₄, axiom)

LAT001-4.ax Lattice theory unique complementation (equality) axioms

$x \vee x' = 1$ cnf(complement_join, axiom)
 $x \wedge x' = 0$ cnf(complement_meet, axiom)
 $(x \wedge y = 0 \text{ and } x \vee y = 1) \Rightarrow x' = y$ cnf(meet_join_complement, axiom)

LAT002-0.ax Lattice theory axioms

$n_1 \vee x = n_1$ cnf(join_1_and_x, axiom)
 $x \vee x = x$ cnf(join_x_and_x, axiom)
 $n_0 \vee x = x$ cnf(join_0_and_x, axiom)
 $n_0 \wedge x = n_0$ cnf(meet_0_and_x, axiom)
 $x \wedge x = x$ cnf(meet_x_and_x, axiom)
 $n_1 \wedge x = x$ cnf(meet_1_and_x, axiom)
 $x \wedge y = z \Rightarrow y \wedge x = z$ cnf(commutativity_of_meet, axiom)
 $x \vee y = z \Rightarrow y \vee x = z$ cnf(commutativity_of_join, axiom)
 $x \wedge y = z \Rightarrow x \vee z = x$ cnf(absorbtion₁, axiom)
 $x \vee y = z \Rightarrow x \wedge z = x$ cnf(absorbtion₂, axiom)
 $(x \wedge y = xy \text{ and } y \wedge z = yz \text{ and } x \wedge yz = xyz) \Rightarrow xy \wedge z = xyz$ cnf(associativity_of_meet₁, axiom)
 $(x \wedge y = xy \text{ and } y \wedge z = yz \text{ and } xy \wedge z = xyz) \Rightarrow x \wedge yz = xyz$ cnf(associativity_of_meet₂, axiom)
 $(x \vee y = xy \text{ and } y \vee z = yz \text{ and } x \vee yz = xyz) \Rightarrow xy \vee z = xyz$ cnf(associativity_of_join₁, axiom)
 $(x \vee y = xy \text{ and } y \vee z = yz \text{ and } xy \vee z = xyz) \Rightarrow x \vee yz = xyz$ cnf(associativity_of_join₂, axiom)
 $(x \wedge z = x \text{ and } x \vee y = x_1 \text{ and } y \wedge z = y_1 \text{ and } z \wedge x_1 = z_1) \Rightarrow x \vee y_1 = z_1$ cnf(modularity₁, axiom)
 $(x \wedge z = x \text{ and } x \vee y = x_1 \text{ and } y \wedge z = y_1 \text{ and } x \vee y_1 = z_1) \Rightarrow z \wedge x_1 = z_1$ cnf(modularity₂, axiom)
 $x \wedge y = \text{meet_of}(x, y)$ cnf(meet_total_function₁, axiom)
 $(x \wedge y = z_1 \text{ and } x \wedge y = z_2) \Rightarrow z_1 = z_2$ cnf(meet_total_function₂, axiom)
 $x \vee y = \text{join_of}(x, y)$ cnf(join_total_function₁, axiom)
 $(x \vee y = z_1 \text{ and } x \vee y = z_2) \Rightarrow z_1 = z_2$ cnf(join_total_function₂, axiom)

LAT003-0.ax Ortholattice theory (equality) axioms

$x' \vee x = n_1$ cnf(top, axiom)
 $x' \wedge x = n_0$ cnf(bottom, axiom)
 $x \vee (x \wedge y) = x$ cnf(absorbtion₂, axiom)
 $x \wedge y = y \wedge x$ cnf(commutativity_of_meet, axiom)

$x \vee y = y \vee x$ cnf(commutativity_of_join, axiom)
 $(x \wedge y) \wedge z = x \wedge (y \wedge z)$ cnf(associativity_of_meet, axiom)
 $(x \vee y) \vee z = x \vee (y \vee z)$ cnf(associativity_of_join, axiom)
 $(x')' = x$ cnf(complement_involution, axiom)
 $x \vee (y \vee y') = y \vee y'$ cnf(join_complement, axiom)
 $x \wedge y = (x' \vee y')'$ cnf(meet_complement, axiom)

LAT004-0.ax Quasilattice theory (equality) axioms

$x \wedge x = x$ cnf(idempotence_of_meet, axiom)
 $x \vee x = x$ cnf(idempotence_of_join, axiom)
 $x \wedge y = y \wedge x$ cnf(commutativity_of_meet, axiom)
 $x \vee y = y \vee x$ cnf(commutativity_of_join, axiom)
 $(x \wedge y) \wedge z = x \wedge (y \wedge z)$ cnf(associativity_of_meet, axiom)
 $(x \vee y) \vee z = x \vee (y \vee z)$ cnf(associativity_of_join, axiom)
 $(x \wedge (y \vee z)) \vee (x \wedge y) = x \wedge (y \vee z)$ cnf(quasi_lattice₁, axiom)
 $(x \vee (y \wedge z)) \wedge (x \vee y) = x \vee (y \wedge z)$ cnf(quasi_lattice₂, axiom)

LAT005-0.ax Weakly Associative Lattices theory (equality) axioms

$x \wedge x = x$ cnf(idempotence_of_meet, axiom)
 $x \vee x = x$ cnf(idempotence_of_join, axiom)
 $x \wedge y = y \wedge x$ cnf(commutativity_of_meet, axiom)
 $x \vee y = y \vee x$ cnf(commutativity_of_join, axiom)
 $((x \vee y) \wedge (z \vee y)) \wedge y = y$ cnf(wal₁, axiom)
 $((x \wedge y) \vee (z \wedge y)) \vee y = y$ cnf(wal₂, axiom)

LAT006-0.ax Tarski's fixed point theorem (equality) axioms

$c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}}) = c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset_H}}, v_{\text{order_H}}, v_{\text{more_H}}, t_{\text{a}}, t_{\text{z}})$
 $v_{\text{pset}} = v_{\text{pset_H}}$ cnf(cls_Tarski_Opotype_Oext_inject₀, axiom)
 $c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}}) = c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset_H}}, v_{\text{order_H}}, v_{\text{more_H}}, t_{\text{a}}, t_{\text{z}})$
 $v_{\text{order}} = v_{\text{order_H}}$ cnf(cls_Tarski_Opotype_Oext_inject₁, axiom)
 $c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}}) = c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset_H}}, v_{\text{order_H}}, v_{\text{more_H}}, t_{\text{a}}, t_{\text{z}})$
 $v_{\text{more}} = v_{\text{more_H}}$ cnf(cls_Tarski_Opotype_Oext_inject₂, axiom)
 $c_{\text{Tarski_Opotype_Opset}}(c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{y}}, v_{\text{order}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}}), t_{\text{a}}, t_{\text{z}}) = v_{\text{y}}$ cnf(cls_Tarski_Opotype_Opset, v_y, v_order, v_more, t_a, t_z, t_a, t_z)
 $c_{\text{Tarski_Opotype_Order}}(c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{y}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}}), t_{\text{a}}, t_{\text{z}}) = v_{\text{y}}$ cnf(cls_Tarski_Opotype_Order, v_pset, v_y, v_more, t_a, t_z, t_a, t_z)
 $c_{\text{Tarski_Opotype_Omore}}(c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order}}, v_{\text{y}}, t_{\text{a}}, t_{\text{a}}), t_{\text{a}}, t_{\text{a}}) = v_{\text{y}}$ cnf(cls_Tarski_Opotype_Omore, v_pset, v_order, v_y, t_a, t_a, t_a, t_a)
 $c_{\text{Tarski_Opotype_Opset_update}}(v_{\text{pset_H}}, c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}}), t_{\text{a}}, t_{\text{z}}) =$
 $c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset_H}}, v_{\text{order}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}})$ cnf(cls_Tarski_Opotype_Oupdate_Convs_1_0, axiom)
 $c_{\text{Tarski_Opotype_Order_update}}(v_{\text{order_H}}, c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}}), t_{\text{a}}, t_{\text{z}}) =$
 $c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order_H}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}})$ cnf(cls_Tarski_Opotype_Oupdate_Convs_2_0, axiom)
 $c_{\text{Tarski_Opotype_Omore_update}}(v_{\text{more_H}}, c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order}}, v_{\text{more}}, t_{\text{a}}, t_{\text{z}}), t_{\text{z}}, t_{\text{a}}) =$
 $c_{\text{Tarski_Opotype_Opotype_ext}}(v_{\text{pset}}, v_{\text{order}}, v_{\text{more_H}}, t_{\text{a}}, t_{\text{z}})$ cnf(cls_Tarski_Opotype_Oupdate_Convs_3_0, axiom)

LAT006-1.ax Tarski's fixed point theorem GLB (equality) axioms

$v_{\text{A}} = c_{\text{Tarski_Opotype_Opset}}(v_{\text{cl}}, t_{\text{a}}, tc_{\text{Product_Type_Ounit}})$ cnf(cls_Tarski_OA_A_61_61_Apset_Acl₀, axiom)
 $(c_{\text{in}}(v_{\text{x}}, v_{\text{S}}, t_{\text{a}}) \text{ and } c_{\text{in}}(v_{\text{cl}}, c_{\text{Tarski_OCompleteLattice}}, tc_{\text{Tarski_Opotype_Opotype_ext_type}}(t_{\text{a}}, tc_{\text{Product_Type_Ounit}})))$
 $c_{\text{in}}(c_{\text{Pair}}(v_{\text{x}}, c_{\text{Tarski_Olub}}(v_{\text{S}}, v_{\text{cl}}, t_{\text{a}}), t_{\text{a}}, t_{\text{a}}), c_{\text{Tarski_Opotype_Order}}(v_{\text{cl}}, t_{\text{a}}, tc_{\text{Product_Type_Ounit}}), tc_{\text{prod}}(t_{\text{a}}, t_{\text{a}}))$
 $c_{\text{in}}(c_{\text{Pair}}(v_{\text{x}}, v_{\text{y}}, t_{\text{a}}, t_{\text{a}}), c_{\text{Tarski_Opotype_Order}}(c_{\text{Tarski_Odual}}(v_{\text{cl}}, t_{\text{a}}), t_{\text{a}}, tc_{\text{Product_Type_Ounit}}), tc_{\text{prod}}(t_{\text{a}}, t_{\text{a}}))$
 $c_{\text{in}}(c_{\text{Pair}}(v_{\text{y}}, v_{\text{x}}, t_{\text{a}}, t_{\text{a}}), c_{\text{Tarski_Opotype_Order}}(v_{\text{cl}}, t_{\text{a}}, tc_{\text{Product_Type_Ounit}}), tc_{\text{prod}}(t_{\text{a}}, t_{\text{a}}))$ cnf(cls_Tarski_Opotype_Oorder, v_y, v_x, t_a, t_a, t_a, t_a)
 $c_{\text{in}}(c_{\text{Pair}}(v_{\text{y}}, v_{\text{x}}, t_{\text{a}}, t_{\text{a}}), c_{\text{Tarski_Opotype_Order}}(v_{\text{cl}}, t_{\text{a}}, tc_{\text{Product_Type_Ounit}}), tc_{\text{prod}}(t_{\text{a}}, t_{\text{a}})) \Rightarrow c_{\text{in}}(c_{\text{Pair}}(v_{\text{y}}, v_{\text{x}}, t_{\text{a}}, t_{\text{a}}), c_{\text{Tarski_Opotype_Order}}(v_{\text{cl}}, t_{\text{a}}, tc_{\text{Product_Type_Ounit}}), tc_{\text{prod}}(t_{\text{a}}, t_{\text{a}}))$
 $c_{\text{in}}(v_{\text{cl}}, c_{\text{Tarski_OCompleteLattice}}, tc_{\text{Tarski_Opotype_Opotype_ext_type}}(t_{\text{a}}, tc_{\text{Product_Type_Ounit}})) \Rightarrow c_{\text{Relation_O}}$
 $c_{\text{in}}(v_{\text{cl}}, c_{\text{Tarski_OCompleteLattice}}, tc_{\text{Tarski_Opotype_Opotype_ext_type}}(t_{\text{a}}, tc_{\text{Product_Type_Ounit}})) \Rightarrow c_{\text{Relation_O}}$
 $c_{\text{in}}(v_{\text{cl}}, c_{\text{Tarski_OCompleteLattice}}, tc_{\text{Tarski_Opotype_Opotype_ext_type}}(t_{\text{a}}, tc_{\text{Product_Type_Ounit}})) \Rightarrow c_{\text{Relation_O}}$
 $c_{\text{in}}(v_{\text{cl}}, c_{\text{Tarski_OCompleteLattice}}, tc_{\text{Tarski_Opotype_Opotype_ext_type}}(t_{\text{a}}, tc_{\text{Product_Type_Ounit}}))$ cnf(cls_Tarski_Opotype_Oext_type, v_cl, t_a, tc_Prod)
 $c_{\text{in}}(c_{\text{Tarski_Odual}}(v_{\text{cl}}, t_{\text{a}}), c_{\text{Tarski_OCompleteLattice}}, tc_{\text{Tarski_Opotype_Opotype_ext_type}}(t_{\text{a}}, tc_{\text{Product_Type_Ounit}}))$
 $c_{\text{in}}(c_{\text{Tarski_Odual}}(v_{\text{cl}}, t_{\text{a}}), c_{\text{Tarski_OPartialOrder}}, tc_{\text{Tarski_Opotype_Opotype_ext_type}}(t_{\text{a}}, tc_{\text{Product_Type_Ounit}}))$
 $c_{\text{Tarski_Oglb}}(v_{\text{S}}, v_{\text{cl}}, t_{\text{a}}) = c_{\text{Tarski_Olub}}(v_{\text{S}}, c_{\text{Tarski_Odual}}(v_{\text{cl}}, t_{\text{a}}), t_{\text{a}})$ cnf(cls_Tarski_Oglb_dual_lub₀, axiom)
 $c_{\text{Tarski_Opotype_Opset}}(c_{\text{Tarski_Odual}}(v_{\text{cl}}, t_{\text{a}}), t_{\text{a}}, tc_{\text{Product_Type_Ounit}}) = c_{\text{Tarski_Opotype_Opset}}(v_{\text{cl}}, t_{\text{a}}, tc_{\text{Prod}})$
 $v_{\text{r}} = c_{\text{Tarski_Opotype_Order}}(v_{\text{cl}}, t_{\text{a}}, tc_{\text{Product_Type_Ounit}})$ cnf(cls_Tarski_Or_A_61_61_Aorder_Acl₀, axiom)

LAT problems

LAT001-1.p If $X' = U \vee V$ and $Y' = U \wedge V$, then $U' = X \vee (Y \wedge V)$

The theorem states that there is a complement of "a" in a modular lattice with 0 and 1.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-1.ax')
include('Axioms/LAT001-2.ax')
r'_1=a ∨ b      cnf(complement_of_a_join_b, hypothesis)
r'_2=a ∧ b      cnf(complement_of_a_meet_b, hypothesis)
¬a'=r_1 ∨ (r_2 ∧ b)    cnf(prove_complement, negated_conjecture)
```

LAT002-1.p If $X' = U \vee V$ and $Y' = U \wedge V$, then U' exists

The theorem states that there is a complement of "a" in a modular lattice with 0 and 1.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-1.ax')
include('Axioms/LAT001-2.ax')
r'_1=a ∨ b      cnf(complement_of_a_join_b, hypothesis)
r'_2=a ∧ b      cnf(complement_of_a_meet_b, hypothesis)
¬a'=w      cnf(prove_complement_exists, negated_conjecture)
```

LAT003-1.p A fairly complex equation to establish

If $X' = U \vee V$ and $Y' = U \wedge V$ and $U'' = UC$ and $V'' = VC$ then $(U \vee V)' = UC \wedge VC$. " means unique complement.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-1.ax')
include('Axioms/LAT001-2.ax')
include('Axioms/LAT001-3.ax')
r'_1=a ∨ b      cnf(complement_of_a_join_b, hypothesis)
r'_2=a ∧ b      cnf(complement_of_a_meet_b, hypothesis)
unique_complement(a, a_2)      cnf(unique_complement_of_a, hypothesis)
unique_complement(b, b_2)      cnf(unique_complement_of_b, hypothesis)
¬a ∨ b'=a_2 ∧ b_2      cnf(prove_complement, negated_conjecture)
```

LAT004-1.p A fairly complex equation to establish

If $X' = U \vee V$ and $Y' = U \wedge V$ and $U'' = UC$ and $V'' = VC$ then $(U \vee V)'' = (UC \wedge VC)$. " means unique complement.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-1.ax')
include('Axioms/LAT001-2.ax')
include('Axioms/LAT001-3.ax')
r'_1=a ∨ b      cnf(complement_of_a_join_b, hypothesis)
r'_2=a ∧ b      cnf(complement_of_a_meet_b, hypothesis)
unique_complement(a, a_2)      cnf(unique_complement_of_a, hypothesis)
unique_complement(b, b_2)      cnf(unique_complement_of_b, hypothesis)
¬unique_complement(a ∨ b, a_2 ∧ b_2)      cnf(prove_unique_complement, negated_conjecture)
```

LAT005-3.p SAM's lemma

Let L be a modular lattice with 0 and 1. Suppose that A and B are elements of L such that $(A \vee B)$ and $(A \wedge B)$ both have not necessarily unique complements. Then, $(A \vee B)' = ((A \vee B)' \vee ((A \wedge B)' \wedge B)) \wedge ((A \vee B)' \vee ((A \wedge B)' \wedge A))$.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-1.ax')
include('Axioms/LAT001-2.ax')
r'_1=a ∨ b      cnf(complement_of_a_join_b, hypothesis)
r'_2=a ∧ b      cnf(complement_of_a_meet_b, hypothesis)
r_1 ≠ (r_1 ∨ (r_2 ∧ b)) ∧ (r_1 ∨ (r_2 ∧ a))      cnf(prove_lemma, negated_conjecture)
```

LAT005-4.p SAM's lemma

Let L be a modular lattice with 0 and 1. Suppose that A and B are elements of L such that $(A \vee B)$ and $(A \wedge B)$ both have not necessarily unique complements. Then, $(A \vee B)' = ((A \vee B)' \vee ((A \wedge B)' \wedge B)) \wedge ((A \vee B)' \vee ((A \wedge B)' \wedge A))$.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-1.ax')
r_2 ∨ (a ∧ b) = n_1      cnf(r2_complement_meet_a_b_1, negated_conjecture)
r_2 ∧ (a ∧ b) = n_0      cnf(r2_complement_meet_a_b_2, negated_conjecture)
r_1 ∨ (a ∨ b) = n_1      cnf(r1_complement_join_a_b_1, negated_conjecture)
```

$r_1 \wedge (a \vee b) = n_0$ cnf(r1_complement_join_a_b2, negated_conjecture)
 $r_1 \vee (a \wedge r_2) = b_2$ cnf(define_b2, negated_conjecture)
 $r_1 \vee (b \wedge r_2) = a_2$ cnf(define_a2, negated_conjecture)
 $a_2 \wedge b_2 \neq r_1$ cnf(prove_SAMs_lemma, negated_conjecture)

LAT005-5.p SAM's lemma

Let L be a modular lattice with 0 and 1. Suppose that A and B are elements of L such that $(A \vee B)$ and $(A \wedge B)$ both have not necessarily unique complements. Then, $(A \vee B)' = ((A \vee B)' \vee ((A \wedge B)' \wedge B)) \wedge ((A \vee B)' \vee ((A \wedge B)' \wedge A))$.

```

include('Axioms/LAT002-0.ax')

a ∧ b = c      cnf(meet_a_and_b, negated_conjecture)
c ∨ r2 = n1    cnf(join_c_and_r2, negated_conjecture)
c ∧ r2 = n0    cnf(meet_c_and_r2, negated_conjecture)
r2 ∧ b = e     cnf(meet_r2_and_b, negated_conjecture)
a ∨ b = c2     cnf(join_a_and_b, negated_conjecture)
c2 ∨ r1 = n1   cnf(join_c2_and_r1, negated_conjecture)
c2 ∧ r1 = n0   cnf(meet_c2_and_r1, negated_conjecture)
r2 ∧ a = d     cnf(meet_r2_and_a, negated_conjecture)
r1 ∨ e = a2    cnf(join_r1_and_e, negated_conjecture)
r1 ∨ d = b2    cnf(join_r1_and_d, negated_conjecture)
¬a2 ∧ b2 = r1  cnf(meet_a2_and_b2, negated_conjecture)
  
```

LAT005-6.p SAM's lemma

Let L be a modular lattice with 0 and 1. Suppose that A and B are elements of L such that $(A \vee B)$ and $(A \wedge B)$ both have not necessarily unique complements. Then, $(A \vee B)' = ((A \vee B)' \vee ((A \wedge B)' \wedge B)) \wedge ((A \vee B)' \vee ((A \wedge B)' \wedge A))$.

```

include('Axioms/LAT002-0.ax')

x ∨ n1 = n1    cnf(join_x_and1, axiom)
x ∨ n0 = x     cnf(join_x_and0, axiom)
x ∧ n0 = n0    cnf(meet_x_and0, axiom)
x ∧ n1 = x     cnf(meet_x_and1, axiom)

(x ∧ z = z ∧ y ∨ z = y1 ∧ x ∧ y = x1 ∧ x ∧ y1 = z1) ⇒ z ∨ x1 = z1      cnf(modularity3, axiom)
(x ∧ z = z ∧ y ∨ z = y1 ∧ x ∧ y = x1 ∧ z ∨ x1 = z1) ⇒ x ∧ y1 = z1      cnf(modularity4, axiom)

a ∧ b = c      cnf(meet_a_and_b, negated_conjecture)
c ∨ r2 = n1    cnf(join_c_and_r2, negated_conjecture)
c ∧ r2 = n0    cnf(meet_c_and_r2, negated_conjecture)
r2 ∧ b = e     cnf(meet_r2_and_b, negated_conjecture)
a ∨ b = c2     cnf(join_a_and_b, negated_conjecture)
c2 ∨ r1 = n1   cnf(join_c2_and_r1, negated_conjecture)
c2 ∧ r1 = n0   cnf(meet_c2_and_r1, negated_conjecture)
r2 ∧ a = d     cnf(meet_r2_and_a, negated_conjecture)
r1 ∨ e = a2    cnf(join_r1_and_e, negated_conjecture)
r1 ∨ d = b2    cnf(join_r1_and_d, negated_conjecture)
¬a2 ∧ b2 = r1  cnf(meet_a2_and_b2, negated_conjecture)
  
```

LAT006-1.p Sholander's basis for distributive lattices, part 2 (of 6).

This is part of the proof that Sholander's 2-basis for distributive lattices is correct. Here we prove associativity of meet.

$x \wedge (x \vee y) = x$ cnf(absorption, axiom)
 $x \wedge (y \vee z) = (z \wedge x) \vee (y \wedge x)$ cnf(distribution, axiom)
 $(a \wedge b) \wedge c \neq a \wedge (b \wedge c)$ cnf(prove_associativity_of_meet, negated_conjecture)

LAT007-1.p Sholander's basis for distributive lattices, part 5 (of 6).

This is part of the proof that Sholander's 2-basis for distributive lattices is correct. Here we prove associativity of join.

$x \wedge (x \vee y) = x$ cnf(absorption, axiom)
 $x \wedge (y \vee z) = (z \wedge x) \vee (y \wedge x)$ cnf(distribution, axiom)
 $(a \vee b) \vee c \neq a \vee (b \vee c)$ cnf(prove_associativity_of_join, negated_conjecture)

LAT008-1.p Sholander's basis for distributive lattices, part 5 (of 6).

This is part of the proof that Sholander's 2-basis for distributive lattices is correct. Here we prove the absorption law $x \vee (x \wedge y) = x$.

$x \wedge (x \vee y) = x$ cnf(absorption, axiom)
 $x \wedge (y \vee z) = (z \wedge x) \vee (y \wedge x)$ cnf(distribution, axiom)
 $a \vee (a \wedge b) \neq a$ cnf(prove_absorbtion_dual, negated_conjecture)

LAT009-1.p A self-dual form of distributivity for lattice theory.

From lattice theory axioms and a self-dual form of distributivity, we prove ordinary distributivity.

include('Axioms/LAT001-0.ax')

$((x \wedge y) \vee z) \wedge y \vee (z \wedge x) = (((x \vee y) \wedge z) \vee y) \wedge (z \vee x)$ cnf(distributivity_dual, axiom)
 $a \vee (b \wedge c) \neq (a \vee b) \wedge (a \vee c)$ cnf(prove_distributivity, negated_conjecture)

LAT010-1.p McKenzie's basis for the variety generated by N5.

McKenzie's basis for the variety generated by N5.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \wedge u))) = (x \wedge (y \vee (x \wedge z))) \vee (x \wedge ((x \wedge y) \vee (z \wedge u)))$ cnf(n5₁, hypothesis)
 $x \vee (y \wedge (z \vee (x \wedge u))) = (x \vee (y \wedge (x \vee z))) \wedge (x \vee ((x \vee y) \wedge (z \vee u)))$ cnf(n5₂, hypothesis)
 $(x \vee (y \wedge z)) \wedge (z \vee (x \wedge y)) = (z \wedge (x \vee (y \wedge z))) \vee (x \wedge (y \vee z))$ cnf(nr₃, hypothesis)
 $a \wedge ((b \vee c) \wedge (b \vee d)) \neq (a \wedge ((b \vee c) \wedge (b \vee d))) \wedge ((a \wedge (b \vee (c \wedge d))) \vee ((a \wedge c) \vee (a \wedge d)))$ cnf(prove_this, negated_conjecture)

LAT011-1.p Uniqueness of meet (dually join) in lattice theory

Let's say we have a lattice with two meet operations, say meet₁ and meet₂. In other words, join,meet₁ is a lattice, and join,meet₂ is a lattice. Then, we can prove that the two meet operations are really the same.

include('Axioms/LAT001-0.ax')

$x \wedge_2 x = x$ cnf(idempotence_of_meet₂, axiom)
 $x \wedge_2 y = y \wedge_2 x$ cnf(commutativity_of_meet₂, axiom)
 $x \wedge_2 (x \vee y) = x$ cnf(absorption1₂, axiom)
 $x \vee (x \wedge_2 y) = x$ cnf(absorption2₂, axiom)
 $(x \wedge_2 y) \wedge_2 z = x \wedge_2 (y \wedge_2 z)$ cnf(associativity_of_meet₂, axiom)
 $a \wedge b \neq a \wedge_2 b$ cnf(prove_meets_are_same, negated_conjecture)

LAT012-1.p McKenzie's 4-basis for lattice theory, part 1 (of 3)

This is part of a proof that McKenzie's 4-basis axiomatizes lattice theory. We prove half of the standard basis. The other half follows by duality. In this part we prove commutativity of meet.

$x \vee (y \wedge (x \wedge z)) = x$ cnf(mckenzie₁, axiom)
 $x \wedge (y \vee (x \vee z)) = x$ cnf(mckenzie₂, axiom)
 $((x \wedge y) \vee (y \wedge z)) \vee y = y$ cnf(mckenzie₃, axiom)
 $((x \vee y) \wedge (y \vee z)) \wedge y = y$ cnf(mckenzie₄, axiom)
 $b \wedge a \neq a \wedge b$ cnf(prove_commutativity_of_meet, negated_conjecture)

LAT013-1.p McKenzie's 4-basis for lattice theory, part 2 (of 3)

This is part of a proof that McKenzie's 4-basis axiomatizes lattice theory. We prove half of the standard basis. The other half follows by duality. In this part we prove associativity of meet.

$x \vee (y \wedge (x \wedge z)) = x$ cnf(mckenzie₁, axiom)
 $x \wedge (y \vee (x \vee z)) = x$ cnf(mckenzie₂, axiom)
 $((x \wedge y) \vee (y \wedge z)) \vee y = y$ cnf(mckenzie₃, axiom)
 $((x \vee y) \wedge (y \vee z)) \wedge y = y$ cnf(mckenzie₄, axiom)
 $(a \wedge b) \wedge c \neq a \wedge (b \wedge c)$ cnf(prove_associativity_of_meet, negated_conjecture)

LAT014-1.p McKenzie's 4-basis for lattice theory, part 3 (of 3)

This is part of a proof that McKenzie's 4-basis axiomatizes lattice theory. We prove half of the standard basis. The other half follows by duality. In this part we prove absorbtion of meet.

$x \vee (y \wedge (x \wedge z)) = x$ cnf(mckenzie₁, axiom)
 $x \wedge (y \vee (x \vee z)) = x$ cnf(mckenzie₂, axiom)
 $((x \wedge y) \vee (y \wedge z)) \vee y = y$ cnf(mckenzie₃, axiom)
 $((x \vee y) \wedge (y \vee z)) \wedge y = y$ cnf(mckenzie₄, axiom)
 $a \wedge (a \vee b) \neq a$ cnf(prove_absorbtion, negated_conjecture)

LAT015-1.p Single axiom for lattice theory

This starts with a single axiom for lattice theory and derives a standard basis for lattice theory.

$((((x \wedge y) \vee (y \wedge (x \vee y))) \wedge z) \vee (((x \wedge (((x_1 \wedge y) \vee (y \wedge x_2)) \vee y)) \wedge (((y \wedge (((x_1 \vee (y \vee x_2)) \wedge (x_3 \vee y)) \wedge y)) \vee (u \wedge (y \vee (((x_1 \vee (y \vee x_2)) \wedge (x_3 \vee y)) \wedge y)))) \wedge (x \vee (((x_1 \wedge y) \vee (y \wedge x_2)) \vee y)))) \wedge (((x \wedge y) \vee (y \wedge (x \vee y))) \vee z)) = y$ cnf(single_axiom, axiom)
 $(a \wedge a = a \text{ and } a \wedge b = b \wedge a \text{ and } (a \wedge b) \wedge c = a \wedge (b \wedge c) \text{ and } a \vee a = a \text{ and } a \vee b = b \vee a \text{ and } (a \vee b) \vee c = a \vee (b \vee c) \text{ and } a \wedge (a \vee b) = a) \Rightarrow a \vee (a \wedge b) \neq a$ cnf(prove_normal_axioms, negated_conjecture)

LAT016-1.p E1 fails for Ortholattices.

Show that Ortholattices do not necessarily satisfy equation E1.

```
include('Axioms/LAT003-0.ax')
((a ∧ b') ∨ a')' ∨ ((a ∧ b') ∨ ((a' ∧ ((a ∨ b') ∧ (a ∨ b))) ∨ (a' ∧ ((a ∨ b') ∧ (a ∨ b))')))) ≠ n1      cnf(prove_e1, negated_conjecture)
```

LAT017-1.p E2 holds in Ortholattices.

Prove that from ortholattice axioms, one can derive equation E2.

```
include('Axioms/LAT003-0.ax')
a ∨ ((a' ∧ ((a ∨ b') ∧ (a ∨ b))) ∨ (a' ∧ ((a' ∧ b) ∨ (a' ∧ b')))) ≠ n1      cnf(prove_e2, negated_conjecture)
```

LAT018-1.p E3 holds in Ortholattices.

Prove that from ortholattice axioms, one can derive equation E3.

```
include('Axioms/LAT003-0.ax')
(((a' ∧ b) ∨ (a' ∧ b')) ∨ (a ∧ (a' ∨ b)))' ∨ (a' ∨ b) ≠ n1      cnf(prove_e3, negated_conjecture)
```

LAT019-1.p In quasilattices, a distributive law implies its dual.

```
include('Axioms/LAT004-0.ax')
x ∧ (y ∨ z) = (x ∧ y) ∨ (x ∧ z)      cnf(distributivity_law, hypothesis)
a ∨ (b ∧ c) ≠ (a ∨ b) ∧ (a ∨ c)      cnf(prove_distributivity_law_dual, negated_conjecture)
```

LAT020-1.p Self-dual distributivity for quasilattices.

```
include('Axioms/LAT004-0.ax')
(((x ∧ y) ∨ z) ∧ y) ∨ (z ∧ x) = (((x ∨ y) ∧ z) ∨ y) ∧ (z ∨ x)      cnf(self_dual_distributivity, hypothesis)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT021-1.p Bowden's inequality gives distributivity in lattice theory.

```
include('Axioms/LAT004-0.ax')
(x ∨ (y ∧ z)) ∨ ((x ∨ y) ∧ z) = x ∨ (y ∧ z)      cnf(bowden, hypothesis)
a ∧ (b ∨ (a ∧ c)) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT022-1.p Self-dual modularity for quasilattices.

```
include('Axioms/LAT004-0.ax')
(x ∧ y) ∨ (z ∧ (x ∨ y)) = (x ∨ y) ∧ (z ∨ (x ∧ y))      cnf(self_dual_modularity, hypothesis)
a ∧ (b ∨ (a ∧ c)) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_modularity, negated_conjecture)
```

LAT023-1.p Yet another modularity equation for quasilattices.

```
include('Axioms/LAT004-0.ax')
((x ∨ y) ∧ z) ∨ y = ((z ∨ y) ∧ x) ∨ y      cnf(modularity_axiom, hypothesis)
a ∧ (b ∨ (a ∧ c)) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_modularity, negated_conjecture)
```

LAT024-1.p Meet (dually join) is not necessarily unique for quasilattices.

Let's say we have a quasilattice with two meet operations, say meet₁ and meet₂. In other words, join,meet₁ is a lattice, and join,meet₂ is a lattice. Then, we can show that the two meet operations not necessarily the same.

```
include('Axioms/LAT004-0.ax')
x ∧2 x = x      cnf(idempotence_of_meet2, axiom)
x ∧2 y = y ∧2 x      cnf(commutativity_of_meet2, axiom)
(x ∧2 y) ∧2 z = x ∧2 (y ∧2 z)      cnf(associativity_of_meet2, axiom)
(x ∧2 (y ∨ z)) ∨ (x ∧2 y) = x ∧2 (y ∨ z)      cnf(quasi_lattice12, axiom)
(x ∨ (y ∧2 z)) ∧2 (x ∨ y) = x ∨ (y ∧2 z)      cnf(quasi_lattice22, axiom)
a ∧ b ≠ a ∧2 b      cnf(prove_meets_equal, negated_conjecture)
```

LAT025-1.p Non-uniqueness of meet (dually join) in TNL

Let's say we have a ternary near-lattice (TNL) with two meet operations, say meet₁ and meet₂. In other words, join,meet₁ and join,meet₂ are TNLs. Are the two meets necessarily the same? No, they aren't. Here is a counterexample.

```
x ∧ x = x      cnf(idempotence_of_meet, axiom)
x ∨ x = x      cnf(idempotence_of_join, axiom)
x ∧ (x ∨ y) = x      cnf(absorption1, axiom)
x ∨ (x ∧ y) = x      cnf(absorption2, axiom)
x ∧ y = y ∧ x      cnf(commutativity_of_meet, axiom)
x ∨ y = y ∨ x      cnf(commutativity_of_join, axiom)
x ∨ (y ∧ (x ∨ z)) = x      cnf(tnl1, axiom)
x ∧ (y ∨ (x ∨ z)) = x      cnf(tnl2, axiom)
x ∧2 x = x      cnf(idempotence_of_meet2, axiom)
x ∧2 (x ∨ y) = x      cnf(absorption12, axiom)
x ∨ (x ∧2 y) = x      cnf(absorption22, axiom)
```

$x \wedge_2 y = y \wedge_2 x$ cnf(commutativity_of_meet₂, axiom)
 $x \vee (y \wedge_2 (x \wedge_2 z)) = x$ cnf(tnl_1₂, axiom)
 $x \wedge_2 (y \vee (x \vee z)) = x$ cnf(tnl_2₂, axiom)
 $a \wedge b \neq a \wedge_2 b$ cnf(prove_meets_equal, negated_conjecture)

LAT026-1.p WAL + absorption gives LT, part 1.

A Weakly associative lattice (WAL) satisfying an absorption law is associative, and therefore a full lattice, part 1.

```
include('Axioms/LAT005-0.ax')
x ∧ (y ∨ (x ∨ z)) = x cnf(absorbtion, hypothesis)
(a ∧ b) ∧ c ≠ a ∧ (b ∧ c) cnf(prove_associativity_of_meet, negated_conjecture)
```

LAT027-1.p WAL + absorption gives LT, part 2.

A Weakly associative lattice (WAL) satisfying an absorption law is associative, and therefore a full lattice, part 2.

```
include('Axioms/LAT005-0.ax')
x ∧ (y ∨ (x ∨ z)) = x cnf(absorption, hypothesis)
(a ∨ b) ∨ c ≠ a ∨ (b ∨ c) cnf(prove_associativity_of_join, negated_conjecture)
```

LAT028-1.p Uniqueness of meet (dually join) in WAL

Let's say we have a weakly-associative lattice (WAL) with 2 meet operations, say meet₁ and meet₂. In other words, join,meet₁ is a WAL, and join,meet₂ is a WAL. Then, we can prove that the two meet operations are really the same.

```
include('Axioms/LAT005-0.ax')
x ∧2 x = x cnf(idempotence_of_meet2, axiom)
x ∧2 y = y ∧2 x cnf(commutativity_of_meet2, axiom)
((x ∨ y) ∧2 (z ∨ y)) ∧2 y = y cnf(wal_12, axiom)
((x ∧2 y) ∨ (z ∧2 y)) ∨ y = y cnf(wal_22, axiom)
a ∧ b ≠ a ∧2 b cnf(name, negated_conjecture)
```

LAT029-1.p Absorption basis for WAL

Prove that the 5 absorption equations below are a basis for weakly associative lattices. This can be done by deriving commutativity and idempotence of the two operations.

```
(x ∧ y) ∨ (x ∧ (x ∨ y)) = x cnf(wal_absorbtion1, axiom)
(x ∧ x) ∨ (y ∧ (x ∨ x)) = x cnf(wal_absorbtion2, axiom)
(x ∧ y) ∨ (y ∧ (x ∨ y)) = y cnf(wal_absorbtion3, axiom)
((x ∨ y) ∧ (z ∨ x)) ∧ x = x cnf(wal_absorbtion4, axiom)
((x ∧ y) ∨ (z ∧ x)) ∨ x = x cnf(wal_absorbtion5, axiom)
(a ∧ a = a and b ∧ a = a ∧ b and a ∨ a = a) ⇒ b ∨ a ≠ a ∨ b cnf(prove_normal_axioms, negated_conjecture)
```

LAT030-1.p Single axiom for weakly associative lattices (WAL)

This starts with a single axiom for WAL and derives a standard basis for WAL.

```
((((x ∧ y) ∨ (y ∧ (x ∨ y))) ∧ z) ∨ (((x ∧ (((y ∧ x1) ∨ (x2 ∧ y)) ∨ y)) ∨ (((y ∧ (((y ∨ x1) ∧ (x2 ∨ y)) ∧ y)) ∨ (u ∧ (y ∨ (((y ∨ x1) ∧ (x2 ∨ y)) ∧ y)))))) ∧ (x ∨ (((y ∧ x1) ∨ (x2 ∧ y)) ∨ y))) ∨ (((x ∧ y) ∨ (y ∧ (x ∨ y))) ∨ z)) = y cnf(single_axiom, axiom)
(a ∧ a = a and b ∧ a = a ∧ b and a ∨ a = a and b ∨ a = a ∨ b and ((a ∨ b) ∧ (c ∨ b)) ∧ b = b) ⇒ ((a ∧ b) ∨ (c ∧ b)) ∨ b ≠ b cnf(prove_wal_axioms, negated_conjecture)
```

LAT031-1.p Distributivity of meet implies distributivity of join

```
include('Axioms/LAT001-0.ax')
x ∧ (y ∨ z) = (x ∧ y) ∨ (x ∧ z) cnf(dist_meet, hypothesis)
xx ∨ (yy ∧ zz) ≠ (xx ∨ yy) ∧ (xx ∨ zz) cnf(dist_join, negated_conjecture)
```

LAT032-1.p Distributivity of join implies distributivity of meet

```
include('Axioms/LAT001-0.ax')
x ∨ (y ∧ z) = (x ∨ y) ∧ (x ∨ z) cnf(dist_join, hypothesis)
xx ∧ (yy ∨ zz) ≠ (xx ∧ yy) ∨ (xx ∧ zz) cnf(dist_meet, negated_conjecture)
```

LAT033-1.p Idempotency of join

```
x ∧ (x ∨ y) = x cnf(absorbtion1, axiom)
x ∨ (x ∧ y) = x cnf(absorbtion2, axiom)
x ∧ y = y ∧ x cnf(commutativity_of_meet, axiom)
x ∨ y = y ∨ x cnf(commutativity_of_join, axiom)
(x ∧ y) ∧ z = x ∧ (y ∧ z) cnf(associativity_of_meet, axiom)
(x ∨ y) ∨ z = x ∨ (y ∨ z) cnf(associativity_of_join, axiom)
xx ∨ xx ≠ xx cnf(idempotence_of_join, negated_conjecture)
```

LAT034-1.p Idempotency of meet

```

x ∧ (x ∨ y) = x      cnf(absorption1, axiom)
x ∨ (x ∧ y) = x      cnf(absorption2, axiom)
x ∧ y = y ∧ x      cnf(commutativity_of_meet, axiom)
x ∨ y = y ∨ x      cnf(commutativity_of_join, axiom)
(x ∧ y) ∧ z = x ∧ (y ∧ z)    cnf(associativity_of_meet, axiom)
(x ∨ y) ∨ z = x ∨ (y ∨ z)    cnf(associativity_of_join, axiom)
xx ∧ xx ≠ xx      cnf(idempotence_of_meet, negated_conjecture)

```

LAT035-1.p Composition to form a join hemimorphism

In a lattice with 0,1, the composition of a unary join antihemimorphism and a lattice antimorphism is a join hemimorphism.

```

include('Axioms/LAT001-0.ax')
x ∧ n0 = n0      cnf(x_meet0, axiom)
x ∨ n0 = x      cnf(x_join0, axiom)
x ∧ n1 = x      cnf(x_meet1, axiom)
x ∨ n1 = n1      cnf(x_join1, axiom)
x ∧ z = x ⇒ z ∧ (x ∨ y) = x ∨ (y ∧ z)    cnf(modular, axiom)
k(u ∨ v) = k(u) ∧ k(v)      cnf(k_on_join, axiom)
k(u ∧ v) = k(u) ∨ k(v)      cnf(k_on_meet, axiom)
k(n0) = n1      cnf(k_on_bottom, axiom)
k(n1) = n0      cnf(k_on_top, axiom)
f(u ∧ v) = f(u) ∨ f(v)      cnf(f_on_meet, axiom)
f(n1) = n0      cnf(f_on_top, axiom)
f(k(aa ∨ bb)) = f(k(aa)) ∨ f(k(bb)) ⇒ f(k(n0)) ≠ n0      cnf(comp_join_hemimorphism, negated_conjecture)

```

LAT036-1.p Property of a distributive lattice with an antimorphism

In every distributive lattice with 0,1 and an antimorphism k: if $k \wedge 2(a) \leq a \vee k(a)$ and $k \wedge 3(b) \leq a \vee k(a)$ and $k \wedge 2(a) \leq k(a) \vee k(b) \vee k(c)$ and $k \wedge 3(b) \leq k(a) \vee k(b) \vee k(c)$ then $k \wedge 2(a \vee k(b)) \leq (a \wedge k(b \wedge c)) \vee k(a)$

```

include('Axioms/LAT001-0.ax')
x ∨ (y ∧ z) = (x ∨ y) ∧ (x ∨ z)      cnf(dist_join, hypothesis)
x ∧ (y ∨ z) = (x ∧ y) ∨ (x ∧ z)      cnf(dist_meet, hypothesis)
x ∧ n0 = n0      cnf(x_meet0, axiom)
x ∨ n0 = x      cnf(x_join0, axiom)
x ∧ n1 = x      cnf(x_meet1, axiom)
x ∨ n1 = n1      cnf(x_join1, axiom)
x ∧ z = x ⇒ z ∧ (x ∨ y) = x ∨ (y ∧ z)    cnf(modular, axiom)
k(u ∨ v) = k(u) ∧ k(v)      cnf(k_on_join, axiom)
k(u ∧ v) = k(u) ∨ k(v)      cnf(k_on_meet, axiom)
k(n0) = n1      cnf(k_on_bottom, axiom)
k(n1) = n0      cnf(k_on_top, axiom)
k(k(aa)) ∨ (aa ∨ k(aa)) = aa ∨ k(aa)      cnf(lhs1, hypothesis)
k(k(bb)) ∨ (aa ∨ k(aa)) = aa ∨ k(aa)      cnf(lhs2, hypothesis)
k(k(aa)) ∨ (k(aa) ∨ (k(bb) ∨ k(cc))) = k(aa) ∨ (k(bb) ∨ k(cc))    cnf(lhs3, hypothesis)
k(k(bb)) ∨ (k(aa) ∨ (k(bb) ∨ k(cc))) = k(aa) ∨ (k(bb) ∨ k(cc))    cnf(lhs4, hypothesis)
k(k(aa ∨ k(bb))) ∨ ((aa ∧ k(bb ∧ cc)) ∨ k(aa)) ≠ (aa ∧ k(bb ∧ cc)) ∨ k(aa)      cnf(rhs, negated_conjecture)

```

LAT037-1.p Uniqueness of complement

Distributive lattice complements are unique whenever they exist.

```

include('Axioms/LAT001-0.ax')
x ∨ (y ∧ z) = (x ∨ y) ∧ (x ∨ z)      cnf(dist_join, hypothesis)
x ∧ (y ∨ z) = (x ∧ y) ∨ (x ∧ z)      cnf(dist_meet, hypothesis)
x ∧ n0 = n0      cnf(x_meet0, axiom)
x ∨ n0 = x      cnf(x_join0, axiom)
x ∧ n1 = x      cnf(x_meet1, axiom)
x ∨ n1 = n1      cnf(x_join1, axiom)
x ∧ z = x ⇒ z ∧ (x ∨ y) = x ∨ (y ∧ z)    cnf(modular, axiom)
xx ∨ yy = n1      cnf(lhs1, axiom)
xx ∨ zz = n1      cnf(lhs2, axiom)
xx ∧ yy = n0      cnf(lhs3, axiom)
xx ∧ zz = n0      cnf(lhs4, axiom)
yy ≠ zz      cnf(rhs, negated_conjecture)

```

LAT038-1.p Simplification rule in a distributive lattice

In a distributive lattice, the following simplification rule holds: forall a, b, c, d: if $f(a \vee b, d) = f(c \vee b, d)$ and $f(a, d) & f(b, d) = f(c, d)$ & $f(b, d)$ then $f(a, d) = f(c, d)$.

include('Axioms/LAT001-0.ax')

$$\begin{aligned} x \vee (y \wedge z) &= (x \vee y) \wedge (x \vee z) & \text{cnf(dist_join, hypothesis)} \\ x \wedge (y \vee z) &= (x \wedge y) \vee (x \wedge z) & \text{cnf(dist_meet, hypothesis)} \\ f(u \vee v, w) &= f(u, w) \vee f(v, w) & \text{cnf(f_on_left_join, axiom)} \\ f(n_0, w) &= n_0 & \text{cnf(f_on_left_bottom, axiom)} \\ f(w, u \vee v) &= f(w, u) \vee f(w, v) & \text{cnf(f_on_right_join, axiom)} \\ f(w, n_0) &= n_0 & \text{cnf(f_on_right_bottom, axiom)} \\ f(aa \vee bb, dd) &= f(cc \vee bb, dd) & \text{cnf(lhs}_1\text{, hypothesis)} \\ f(aa, dd) \wedge f(bb, dd) &= f(cc, dd) \wedge f(bb, dd) & \text{cnf(lhs}_2\text{, hypothesis)} \\ f(aa, dd) \neq f(cc, dd) & & \text{cnf(rhs, negated_conjecture)} \end{aligned}$$

LAT039-1.p Every distributive lattice is modular

include('Axioms/LAT001-0.ax')

$$\begin{aligned} x \vee (y \wedge z) &= (x \vee y) \wedge (x \vee z) & \text{cnf(dist_join, hypothesis)} \\ x \wedge (y \vee z) &= (x \wedge y) \vee (x \wedge z) & \text{cnf(dist_meet, hypothesis)} \\ xx \vee yy &= yy & \text{cnf(lhs, hypothesis)} \\ xx \vee (yy \wedge zz) &\neq yy \wedge (xx \vee zz) & \text{cnf(rhs, negated_conjecture)} \end{aligned}$$

LAT039-2.p Every distributive lattice is modular

Theorem formulation : Modularity is expressed by: $x \leq y \rightarrow x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

include('Axioms/LAT001-0.ax')

$$\begin{aligned} x \vee (y \wedge z) &= (x \vee y) \wedge (x \vee z) & \text{cnf(dist_join, hypothesis)} \\ x \wedge (y \vee z) &= (x \wedge y) \vee (x \wedge z) & \text{cnf(dist_meet, hypothesis)} \\ xx \vee yy &= yy & \text{cnf(lhs, hypothesis)} \\ xx \vee (yy \wedge zz) &\neq (xx \vee yy) \wedge (xx \vee zz) & \text{cnf(rhs, negated_conjecture)} \end{aligned}$$

LAT040-1.p Another simplification rule for distributive lattices

In every distributive lattice the simplification rule holds: forall x, y, z: $(x \vee y = x \vee z, x \wedge y = x \wedge z \rightarrow y = z)$.

include('Axioms/LAT001-0.ax')

$$\begin{aligned} x \vee (y \wedge z) &= (x \vee y) \wedge (x \vee z) & \text{cnf(dist_join, hypothesis)} \\ x \wedge (y \vee z) &= (x \wedge y) \vee (x \wedge z) & \text{cnf(dist_meet, hypothesis)} \\ xx \vee yy &= xx \vee zz & \text{cnf(lhs}_1\text{, hypothesis)} \\ xx \wedge yy &= xx \wedge zz & \text{cnf(lhs}_2\text{, hypothesis)} \\ yy \neq zz & & \text{cnf(rhs, negated_conjecture)} \end{aligned}$$

LAT041-1.p A hyperbase for type <2,2> lattice hyperidentities

$$\text{big_p(term}_1, x, y) = x & \text{cnf(big_p_term}_1\text{, axiom)}$$

$$\text{big_p(term}_2, x, y) = y & \text{cnf(big_p_term}_2\text{, axiom)}$$

$$\text{big_p(term}_3, x, y) = \text{times}(x, y) & \text{cnf(big_p_term}_3\text{, axiom)}$$

$$\text{big_p(term}_4, x, y) = \text{times}(y, x) & \text{cnf(big_p_term}_4\text{, axiom)}$$

$$\text{big_p(term}_5, x, y) = x + y & \text{cnf(big_p_term}_5\text{, axiom)}$$

$$\text{big_p(term}_6, x, y) = y + x & \text{cnf(big_p_term}_6\text{, axiom)}$$

$$\text{big_t}(w, x, y) = \text{big_p}(w, x, y) & \text{cnf(big_p_and_big_t, axiom)}$$

$$\text{term(term}_1) & \text{cnf(term}_1\text{, axiom)}$$

$$\text{term(term}_2) & \text{cnf(term}_2\text{, axiom)}$$

$$\text{term(term}_3) & \text{cnf(term}_3\text{, axiom)}$$

$$\text{term(term}_4) & \text{cnf(term}_4\text{, axiom)}$$

$$\text{term(term}_5) & \text{cnf(term}_5\text{, axiom)}$$

$$\text{term(term}_6) & \text{cnf(term}_6\text{, axiom)}$$

$$\text{term}(w) \Rightarrow \text{big_p}(w, \text{big_p}(w, x, y), z) = \text{big_p}(w, x, \text{big_p}(w, y, z)) & \text{cnf(q}_2\text{, hypothesis)}$$

$$\text{term}(w) \Rightarrow \text{big_p}(w, \text{big_p}(w, x, y), \text{big_p}(w, z, v)) = \text{big_p}(w, \text{big_p}(w, x, z), \text{big_p}(w, y, v)) & \text{cnf(q}_5\text{, hypothesis)}$$

$$(\text{term}(w_1) \text{ and } \text{term}(w_2)) \Rightarrow \text{big_t}(w_1, \text{big_p}(w_2, \text{big_t}(w_1, x, y), z), \text{big_p}(w_2, y, z)) = \text{big_p}(w_2, \text{big_t}(w_1, x, y), z) & \text{cnf(q}_3\text{, hypothesis)}$$

$$\text{times}(a, b + \text{times}(d, \text{times}(c, e))) \neq \text{times}(a, b + \text{times}(b + c, \text{times}(d, \text{times}(c, e)))) & \text{cnf(prove_q}_4\text{, negated_conjecture)}$$

LAT042-1.p Lattice modularity from Boolean algebra

include('Axioms/LAT001-0.ax')

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z) & \text{cnf(distributivity, axiom)}$$

$$x' \vee x = n_1 & \text{cnf(invertability}_1\text{, axiom)}$$

$$x' \wedge x = n_0 & \text{cnf(invertability}_2\text{, axiom)}$$

$(x')' = x$ cnf(invertability₃, axiom)
 $a \vee (b \wedge (a \vee c)) \neq (a \vee b) \wedge (a \vee c)$ cnf(prove_modular_law, negated_conjecture)

LAT043-1.p Lattice compatibility from Boolean algebra

include('Axioms/LAT001-0.ax')
 $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ cnf(distributivity, axiom)
 $x' \vee x = n_1$ cnf(invertability₁, axiom)
 $x' \wedge x = n_0$ cnf(invertability₂, axiom)
 $(x')' = x$ cnf(invertability₃, axiom)
 $(c \vee d)' \neq c' \wedge d'$ cnf(prove_compatibility_law, negated_conjecture)

LAT044-1.p Lattice weak orthomodular law from orthomodular lattice

include('Axioms/LAT001-0.ax')
 $(x \vee y)' = x' \wedge y'$ cnf(compatibility₁, axiom)
 $(x \wedge y)' = x' \vee y'$ cnf(compatibility₂, axiom)
 $x' \vee x = n_1$ cnf(invertability₁, axiom)
 $x' \wedge x = n_0$ cnf(invertability₂, axiom)
 $(x')' = x$ cnf(invertability₃, axiom)
 $x \vee (x' \wedge (x \vee y)) = x \vee y$ cnf(orthomodular_law, axiom)
 $(a' \wedge (a \vee b)) \vee (b' \vee (a \wedge b)) \neq n_1$ cnf(prove_weak_orthomodular_law, negated_conjecture)

LAT045-1.p Lattice orthomodular law from modular lattice

include('Axioms/LAT001-0.ax')
 $(x \vee y)' = x' \wedge y'$ cnf(compatibility₁, axiom)
 $(x \wedge y)' = x' \vee y'$ cnf(compatibility₂, axiom)
 $x' \vee x = n_1$ cnf(invertability₁, axiom)
 $x' \wedge x = n_0$ cnf(invertability₂, axiom)
 $(x')' = x$ cnf(invertability₃, axiom)
 $x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z)$ cnf(modular_law, axiom)
 $a \vee (a' \wedge (a \vee b)) \neq a \vee b$ cnf(prove_orthomodular_law, negated_conjecture)

LAT046-1.p Modular ortholattice is not Boolean algebra

include('Axioms/LAT001-0.ax')
 $(x \vee y)' = x' \wedge y'$ cnf(compatibility₁, axiom)
 $(x \wedge y)' = x' \vee y'$ cnf(compatibility₂, axiom)
 $x' \vee x = n_1$ cnf(invertability₁, axiom)
 $x' \wedge x = n_0$ cnf(invertability₂, axiom)
 $(x')' = x$ cnf(invertability₃, axiom)
 $x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z)$ cnf(modular_law, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT047-1.p Lattice is not modular lattice

include('Axioms/LAT001-0.ax')
 $a \vee (b \wedge (a \vee c)) \neq (a \vee b) \wedge (a \vee c)$ cnf(prove_modularity, negated_conjecture)

LAT048-1.p Weakly orthomodular lattice is not orthomodular lattice

include('Axioms/LAT001-0.ax')
 $(x \vee y)' = x' \wedge y'$ cnf(compatibility₁, axiom)
 $(x \wedge y)' = x' \vee y'$ cnf(compatibility₂, axiom)
 $x' \vee x = n_1$ cnf(invertability₁, axiom)
 $x' \wedge x = n_0$ cnf(invertability₂, axiom)
 $(x')' = x$ cnf(invertability₃, axiom)
 $(x' \wedge (x \vee y)) \vee (y' \vee (x \wedge y)) = n_1$ cnf(weak_orthomodular_law, hypothesis)
 $a \vee (a' \wedge (a \vee b)) \neq a \vee b$ cnf(prove_orthomodular_law, negated_conjecture)

LAT049-1.p Ortholattice is not weakly orthomodular lattice

include('Axioms/LAT001-0.ax')
 $(x \vee y)' = x' \wedge y'$ cnf(compatibility₁, axiom)
 $(x \wedge y)' = x' \vee y'$ cnf(compatibility₂, axiom)
 $x' \vee x = n_1$ cnf(invertability₁, axiom)
 $x' \wedge x = n_0$ cnf(invertability₂, axiom)
 $(x')' = x$ cnf(invertability₃, axiom)
 $(a' \wedge (a \vee b)) \vee (b' \vee (a \wedge b)) \neq n_1$ cnf(prove_weak_orthomodular_law, negated_conjecture)

LAT050-1.p Orthomodular lattice is not modular lattice

```
include('Axioms/LAT001-0.ax')
(x ∨ y)' = x' ∧ y'      cnf(compatibility1, axiom)
(x ∧ y)' = x' ∨ y'      cnf(compatibility2, axiom)
x' ∨ x = n1            cnf(invertability1, axiom)
x' ∧ x = n0            cnf(invertability2, axiom)
(x')' = x               cnf(invertability3, axiom)
x ∨ (x' ∧ (x ∨ y)) = x ∨ y      cnf(orthomodular_law, axiom)
a ∨ (b ∧ (a ∨ c)) ≠ (a ∨ b) ∧ (a ∨ c)      cnf(prove_modular_law, negated_conjecture)
```

LAT051-1.p Lattice is not ortholattice

```
include('Axioms/LAT001-0.ax')
x' ∨ x = n1            cnf(invertability1, axiom)
x' ∧ x = n0            cnf(invertability2, axiom)
(x')' = x               cnf(invertability3, axiom)
(a ∨ b)' ≠ a' ∧ b'      cnf(prove_compatibility_law, negated_conjecture)
```

LAT052-1.p Modular lattice is not modular ortholattice

```
include('Axioms/LAT001-0.ax')
x' ∨ x = n1            cnf(invertability1, axiom)
x' ∧ x = n0            cnf(invertability2, axiom)
(x')' = x               cnf(invertability3, axiom)
x ∨ (y ∧ (x ∨ z)) = (x ∨ y) ∧ (x ∨ z)      cnf(modular_law, axiom)
(a ∨ b)' ≠ a' ∧ b'      cnf(prove_compatibility_law, negated_conjecture)
```

LAT053-1.p Countermodel for Megill equation for weakly orthomodular lattices

```
include('Axioms/LAT001-0.ax')
(x ∨ y)' = x' ∧ y'      cnf(compatibility1, axiom)
(x ∧ y)' = x' ∨ y'      cnf(compatibility2, axiom)
x' ∨ x = n1            cnf(invertability1, axiom)
x' ∧ x = n0            cnf(invertability2, axiom)
(x')' = x               cnf(invertability3, axiom)
(x' ∧ (x ∨ y)) ∨ (y' ∨ (x ∧ y)) = n1      cnf(megill, axiom)
a ∧ (b ∨ (a ∧ (a' ∨ (a ∧ b)))) ≠ a ∧ (a' ∨ (a ∧ b))      cnf(prove_this, negated_conjecture)
```

LAT054-1.p Countermodel for Megill equation for ortholattices

```
include('Axioms/LAT001-0.ax')
(x ∨ y)' = x' ∧ y'      cnf(compatibility1, axiom)
(x ∧ y)' = x' ∨ y'      cnf(compatibility2, axiom)
x' ∨ x = n1            cnf(invertability1, axiom)
x' ∧ x = n0            cnf(invertability2, axiom)
(x')' = x               cnf(invertability3, axiom)
a ∨ (b' ∧ (a' ∨ (b' ∧ (a ∨ (b' ∧ a'))))) ≠ a ∨ (b' ∧ (a' ∨ (b' ∧ (a ∨ (b' ∧ (a' ∨ (b' ∧ a)))))))      cnf(prove_this, negated_conjecture)
```

LAT055-1.p Lattice theory axioms

```
include('Axioms/LAT002-0.ax')
```

LAT055-2.p Lattice theory (equality) axioms

```
include('Axioms/LAT001-0.ax')
```

LAT056-1.p Lattice theory modularity (equality) axioms

```
include('Axioms/LAT001-0.ax')
```

```
include('Axioms/LAT001-1.ax')
```

LAT057-1.p Lattice theory complement (equality) axioms

```
include('Axioms/LAT001-0.ax')
```

```
include('Axioms/LAT001-2.ax')
```

LAT058-1.p Lattice theory unique complement (equality) axioms

```
include('Axioms/LAT001-0.ax')
```

```
include('Axioms/LAT001-1.ax')
```

```
include('Axioms/LAT001-3.ax')
```

LAT059-1.p Ortholattice theory (equality) axioms

```
include('Axioms/LAT003-0.ax')
```

LAT060-1.p Quasilattice theory (equality) axioms
 include('Axioms/LAT004-0.ax')

LAT061-1.p Weakly Associative Lattices theory (equality) axioms
 include('Axioms/LAT005-0.ax')

LAT062-1.p E51 does not necessarily hold in ortholattices

include('Axioms/LAT001-0.ax')

$a' \vee a = n_1$ cnf(top, axiom)

$a' \wedge a = n_0$ cnf(bottom, axiom)

$a \wedge b = (a' \vee b')'$ cnf(compatibility, axiom)

$(a \vee b') \wedge (((a \wedge b) \vee (a' \wedge b)) \vee (a' \wedge b')) \neq (a \wedge b) \vee (a' \wedge b')$ cnf(prove_e₅₁, negated_conjecture)

LAT063-1.p E62 does not necessarily hold in ortholattices

include('Axioms/LAT001-0.ax')

$a' \vee a = n_1$ cnf(top, axiom)

$a' \wedge a = n_0$ cnf(bottom, axiom)

$a \wedge b = (a' \vee b')'$ cnf(compatibility, axiom)

$a \wedge (b \vee (a \wedge (a' \vee (a \wedge b)))) \neq a \wedge (a' \vee (a \wedge b))$ cnf(prove_e₆₂, negated_conjecture)

LAT064-1.p Weak property 94-6 to make a uniquely complemented lattice Boolean

include('Axioms/LAT001-0.ax')

$a \vee a' = n_1$ cnf(top, axiom)

$a \wedge a' = n_0$ cnf(bottom, axiom)

$(a \vee b = n_1 \text{ and } a \wedge b = n_0) \Rightarrow a' = b$ cnf(complements_are_unique, axiom)

$a \wedge (b \vee (c \wedge (a \vee (b \wedge c)))) = a \wedge (b \vee (a \wedge c))$ cnf(c94₆, axiom)

$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT065-1.p Weak property 94-37 to make uniquely complemented lattice Boolean

include('Axioms/LAT001-0.ax')

$a \vee a' = n_1$ cnf(top, axiom)

$a \wedge a' = n_0$ cnf(bottom, axiom)

$(a \vee b = n_1 \text{ and } a \wedge b = n_0) \Rightarrow a' = b$ cnf(complements_are_unique, axiom)

$a \wedge ((b \wedge (c \vee (a \wedge b))) \vee (c \wedge (a \vee b))) = (a \wedge b) \vee (a \wedge c)$ cnf(c94₃₇, axiom)

$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT066-1.p Weak property G61 to make a uniquely complemented lattice Boolean

include('Axioms/LAT001-0.ax')

$a \vee a' = n_1$ cnf(top, axiom)

$a \wedge a' = n_0$ cnf(bottom, axiom)

$(a \vee b = n_1 \text{ and } a \wedge b = n_0) \Rightarrow a' = b$ cnf(complements_are_unique, axiom)

$a \wedge (b \vee (c \vee (d \wedge (a \vee (b \wedge c))))) = a \wedge (b \vee (c \vee (a \wedge d)))$ cnf(g₆₁, axiom)

$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT067-1.p Weak property 94-3 to make a uniquely complemented lattice Boolean

include('Axioms/LAT001-0.ax')

$a \vee a' = n_1$ cnf(top, axiom)

$a \wedge a' = n_0$ cnf(bottom, axiom)

$(a \vee b = n_1 \text{ and } a \wedge b = n_0) \Rightarrow a' = b$ cnf(complements_are_unique, axiom)

$a \wedge (b \vee ((a \vee b) \wedge (c \vee (b \wedge (a \vee c))))) = a \wedge (b \vee c)$ cnf(c94₃, axiom)

$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT068-1.p Weak property F53 to make a uniquely complemented lattice Boolean

include('Axioms/LAT001-0.ax')

$a \vee a' = n_1$ cnf(top, axiom)

$a \wedge a' = n_0$ cnf(bottom, axiom)

$(a \vee b = n_1 \text{ and } a \wedge b = n_0) \Rightarrow a' = b$ cnf(complements_are_unique, axiom)

$a \wedge (b \vee (c \wedge (d \vee (a \wedge (b \vee c))))) = a \wedge (b \vee (c \wedge (a \vee d)))$ cnf(f₅₃, axiom)

$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT069-1.p Weak property G113 to make a uniquely complemented lattice Boolean

include('Axioms/LAT001-0.ax')

$a \vee a' = n_1$ cnf(top, axiom)

$a \wedge a' = n_0$ cnf(bottom, axiom)

$(a \vee b = n_1 \text{ and } a \wedge b = n_0) \Rightarrow a' = b$ cnf(complements_are_unique, axiom)

$$a \wedge (b \vee (c \wedge ((a \wedge c) \vee (d \wedge (b \vee c))))) = a \wedge (b \vee ((a \wedge c) \vee (c \wedge d))) \quad \text{cnf}(g_{113}, \text{axiom})$$

$$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c) \quad \text{cnf}(\text{prove_distributivity}, \text{negated_conjecture})$$

LAT070-1.p Given single axiom OL-23A, prove associativity

Given a single axiom OL-23A for ortholattices (OL) in terms of the Sheffer Stroke, prove a Sheffer stroke form of associativity.

$$f(f(f(f(b, a), f(a, c)), d), f(a, f(f(a, f(f(b, b), b)), c))) = a \quad \text{cnf(ol_23A, axiom)}$$

$$f(a, f(f(b, c), f(b, c))) \neq f(c, f(f(b, a), f(b, a))) \quad \text{cnf(associativity, negated_conjecture)}$$

LAT071-1.p Given single axiom OML-21C, prove associativity

Given a single axiom candidate OML-21C for orthomodular lattices (OML) in terms of the Sheffer Stroke, prove a Sheffer stroke form of associativity.

$$f(f(b, a), f(f(f(f(b, a), a), f(c, a)), f(f(a, a), d))) = a \quad \text{cnf(oml_21C, axiom)}$$

$$f(a, f(f(b, c), f(b, c))) \neq f(c, f(f(b, a), f(b, a))) \quad \text{cnf(associativity, negated_conjecture)}$$

LAT072-1.p Given single axiom OML-23A, prove associativity

Given a single axiom candidate OML-23A for orthomodular lattices (OML) in terms of the Sheffer Stroke, prove a Sheffer stroke form of associativity.

$$f(f(f(f(b, a), f(a, c)), d), f(a, f(f(c, f(f(a, a), c)), c))) = a \quad \text{cnf(oml_23A, axiom)}$$

$$f(a, f(f(b, c), f(b, c))) \neq f(c, f(f(b, a), f(b, a))) \quad \text{cnf(associativity, negated_conjecture)}$$

LAT073-1.p Given single axiom MOL-23C, prove modularity

Given a single axiom candidate MOL-23C for modular ortholattices (MOL) in terms of the Sheffer Stroke, prove a Sheffer stroke form of modularity.

$$f(f(f(b, f(a, b)), b), f(a, f(c, f(f(a, b), f(f(c, c), d))))) = a \quad \text{cnf(mol_23C, axiom)}$$

$$f(a, f(b, f(a, f(c, c)))) \neq f(a, f(c, f(a, f(b, b)))) \quad \text{cnf(modularity, negated_conjecture)}$$

LAT074-1.p Given single axiom MOL-25A, prove associativity

Given a single axiom candidate MOL-25A for modular ortholattices (MOL) in terms of the Sheffer Stroke, prove a Sheffer stroke form of associativity.

$$f(f(b, a), f(f(f(f(a, a), c), f(f(f(f(f(a, b), c), c), a), f(a, d)))) = a \quad \text{cnf(mol_25A, axiom)}$$

$$f(a, f(f(b, c), f(b, c))) \neq f(c, f(f(b, a), f(b, a))) \quad \text{cnf(associativity, negated_conjecture)}$$

LAT075-1.p Given single axiom MOL-25A, prove modularity

Given a single axiom candidate MOL-25A for modular ortholattices (MOL) in terms of the Sheffer Stroke, prove a Sheffer stroke form of modularity.

$$f(f(b, a), f(f(f(f(a, a), c), f(f(f(f(f(a, b), c), c), a), f(a, d)))) = a \quad \text{cnf(mol_25A, axiom)}$$

$$f(a, f(b, f(a, f(c, c)))) \neq f(a, f(c, f(a, f(b, b)))) \quad \text{cnf(modularity, negated_conjecture)}$$

LAT076-1.p Given single axiom MOL-27B1, prove associativity

Given a single axiom candidate MOL-27B1 for modular ortholattices (MOL) in terms of the Sheffer Stroke, prove a Sheffer stroke form of associativity.

$$f(f(f(f(b, a), f(c, a)), d), f(a, f(f(f(f(f(b, b), a), c), a), b))) = a \quad \text{cnf(mol_27B1, axiom)}$$

$$f(a, f(f(b, c), f(b, c))) \neq f(c, f(f(b, a), f(b, a))) \quad \text{cnf(associativity, negated_conjecture)}$$

LAT077-1.p Given single axiom MOL-27B1, prove modularity

Given a single axiom candidate MOL-27B1 for modular ortholattices (MOL) in terms of the Sheffer Stroke, prove a Sheffer stroke form of modularity.

$$f(f(f(f(b, a), f(c, a)), d), f(a, f(f(f(f(f(b, b), a), c), c), a), b))) = a \quad \text{cnf(mol_27B1, axiom)}$$

$$f(a, f(b, f(a, f(c, c)))) \neq f(a, f(c, f(a, f(b, b)))) \quad \text{cnf(modularity, negated_conjecture)}$$

LAT078-1.p Given single axiom MOL-27B2, prove associativity

Given a single axiom candidate MOL-27B2 for modular ortholattices (MOL) in terms of the Sheffer Stroke, prove a Sheffer stroke form of associativity.

$$f(f(f(f(b, a), f(a, c)), d), f(a, f(f(f(b, f(b, f(f(c, c), a))), a), c))) = a \quad \text{cnf(mol_27B2, axiom)}$$

$$f(a, f(f(b, c), f(b, c))) \neq f(c, f(f(b, a), f(b, a))) \quad \text{cnf(associativity, negated_conjecture)}$$

LAT079-1.p Given single axiom MOL-27B2, prove modularity

Given a single axiom candidate MOL-27B2 for modular ortholattices (MOL) in terms of the Sheffer Stroke, prove a Sheffer stroke form of modularity.

$$f(f(f(f(b, a), f(a, c)), d), f(a, f(f(f(b, f(b, f(f(c, c), a))), a), c))) = a \quad \text{cnf(mol_27B2, axiom)}$$

$$f(a, f(b, f(a, f(c, c)))) \neq f(a, f(c, f(a, f(b, b)))) \quad \text{cnf(modularity, negated_conjecture)}$$

LAT080-1.p Axiom for lattice theory, part 1

$$(((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((d \wedge b) \vee (b \wedge e)) \vee b)) \vee (((b \wedge (((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)) \vee (g \wedge (b \vee (((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)))) \wedge (a \vee (((d \wedge b) \vee (b \wedge e)) \vee b)))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$$

$a \wedge a \neq a$ cnf(prove_normal_axioms₁, negated_conjecture)

LAT081-1.p Axiom for lattice theory, part 2

$((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((d \wedge b) \vee (b \wedge e)) \vee b)) \vee (((b \wedge (((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)) \vee (g \wedge (b \vee ((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)))) \wedge (a \vee (((d \wedge b) \vee (b \wedge e)) \vee b))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b$ cnf(single_axiom, axiom)
 $a \wedge b \neq b \wedge a$ cnf(prove_normal_axioms₂, negated_conjecture)

LAT082-1.p Axiom for lattice theory, part 3

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((((a ∧ b) ∨ (b ∧ (a ∨ b))) ∧ c) ∨ (((a ∧ (((d ∧ b) ∨ (b ∧ e)) ∨ b)) ∨ (((b ∧ (((d ∨ (b ∨ e)) ∧ (f ∨ b)) ∧ b)) ∨ (g ∧ (b ∨ (((d ∨ (b ∨ e)) ∧ (f ∨ b)) ∧ b)))))) ∧ (a ∨ (((d ∧ b) ∨ (b ∧ e)) ∨ b))) ∨ (((a ∧ b) ∨ (b ∧ (a ∨ b))) ∨ c)) = b      cnf(single_axiom, axiom)
(a ∧ b) ∧ c ≠ a ∧ (b ∧ c)      cnf(prove_normal_axioms3, negated_conjecture)
```

LAT083-1.p Axiom for lattice theory, part 4

$((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((d \wedge b) \vee (b \wedge e)) \vee b)) \vee (((b \wedge (((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)) \vee (g \wedge (b \vee ((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)))) \wedge (a \vee (((d \wedge b) \vee (b \wedge e)) \vee b))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b$ cnf(single_axiom, axiom)
 $a \vee a \neq a$ cnf(prove_normal_axioms₄, negated_conjecture)

LAT084-1.p Axiom for lattice theory, part 5

$((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge ((d \wedge b) \vee (b \wedge e))) \vee b)) \vee (((b \wedge ((d \vee (b \vee e)) \wedge (f \vee b))) \wedge b)) \vee (g \wedge (b \vee ((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)))) \wedge (a \vee (((d \wedge b) \vee (b \wedge e)) \vee b))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $a \vee b \neq b \vee a \quad \text{cnf(prove_normal_axioms}_5\text{, negated_conjecture)}$

LAT085-1.p Axiom for lattice theory, part 6

$((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge ((d \wedge b) \vee (b \wedge e)) \vee b)) \vee (((b \wedge ((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)) \vee (g \wedge (b \vee ((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)))) \wedge (a \vee (((d \wedge b) \vee (b \wedge e)) \vee b))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $(a \vee b) \vee c \neq a \vee (b \vee c) \quad \text{cnf(prove_normal_axioms}_6, \text{negated_conjecture})$

LAT086-1.p Axiom for lattice theory, part 7

$((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((d \wedge b) \vee (b \wedge e)) \vee b)) \vee (((b \wedge (((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)) \vee (g \wedge (b \vee (((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)))) \wedge (a \vee (((d \wedge b) \vee (b \wedge e)) \vee b)))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b$ cnf(single_axiom, axiom)
 $a \wedge (a \vee b) \neq a$ cnf(prove_normal_axioms7, negated_conjecture)

LAT087-1.p Axiom for lattice theory, part 8

$((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge ((d \wedge b) \vee (b \wedge e)) \vee b)) \vee (((b \wedge ((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)) \vee (g \wedge (b \vee ((d \vee (b \vee e)) \wedge (f \vee b)) \wedge b)))) \wedge (a \vee (((d \wedge b) \vee (b \wedge e)) \vee b))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $a \vee (a \wedge b) \neq a \quad \text{cnf(prove_normal_axioms}_8\text{, negated_conjecture)}$

LAT088-1.p Absorption basis for WAL, part 1

$(a \wedge b) \vee (a \wedge (a \vee b)) = a$	cnf(wal_absorbtion ₁ , axiom)
$(a \wedge a) \vee (b \wedge (a \vee a)) = a$	cnf(wal_absorbtion ₂ , axiom)
$(a \wedge b) \vee (b \wedge (a \vee b)) = b$	cnf(wal_absorbtion ₃ , axiom)
$((a \vee b) \wedge (c \vee a)) \wedge a = a$	cnf(wal_absorbtion ₄ , axiom)
$((a \wedge b) \vee (c \wedge a)) \vee a = a$	cnf(wal_absorbtion ₅ , axiom)
$a \wedge a \neq a$	cnf(prove_normal_axioms ₁ , negated_conjecture)

LAT089-1.p Absorption basis for WAL, part 2

$(a \wedge b) \vee (a \wedge (a \vee b)) = a$	cnf(wal_absorbtion ₁ , axiom)
$(a \wedge a) \vee (b \wedge (a \vee a)) = a$	cnf(wal_absorbtion ₂ , axiom)
$(a \wedge b) \vee (b \wedge (a \vee b)) = b$	cnf(wal_absorbtion ₃ , axiom)
$((a \vee b) \wedge (c \vee a)) \wedge a = a$	cnf(wal_absorbtion ₄ , axiom)
$((a \wedge b) \vee (c \wedge a)) \vee a = a$	cnf(wal_absorbtion ₅ , axiom)
$b \wedge a \neq a \wedge b$	cnf(prove_normal_axioms ₅ , negated_conjecture)

LAT090-1.p Absorption basis for WAI; part 3

LTL3C Top Absorption laws for WTA, part 3	
$(a \wedge b) \vee (a \wedge (a \vee b)) = a$	cnf(wal_absorbtion1, axiom)
$(a \wedge a) \vee (b \wedge (a \vee a)) = a$	cnf(wal_absorbtion2, axiom)
$(a \wedge b) \vee (b \wedge (a \vee b)) = b$	cnf(wal_absorbtion3, axiom)
$((a \vee b) \wedge (c \vee a)) \wedge a = a$	cnf(wal_absorbtion4, axiom)
$((a \wedge b) \vee (c \wedge a)) \vee a = a$	cnf(wal_absorbtion5, axiom)
$a \vee a \neq a$	cnf(prove_normal_axioms, negated_conjecture)

LAT091-1.p Absorption basis for WAI, part 4

EAT091-1.p Absorption basis for WAL, part 4	
$(a \wedge b) \vee (a \wedge (a \vee b)) = a$	cnf(wal_absorbtion1, axiom)
$(a \wedge a) \vee (b \wedge (a \vee a)) = a$	cnf(wal_absorbtion2, axiom)
$(a \wedge b) \vee (b \wedge (a \vee b)) = b$	cnf(wal_absorbtion3, axiom)
$((a \vee b) \wedge (c \vee a)) \wedge a = a$	cnf(wal_absorbtion4, axiom)

$((a \wedge b) \vee (c \wedge a)) \vee a = a \quad \text{cnf(wal_absorbtion}_5\text{, axiom)}$
 $b \vee a \neq a \vee b \quad \text{cnf(prove_normal_axioms}_4\text{, negated_conjecture)}$

LAT092-1.p Axiom for weakly associative lattices (WAL), part 1
 $((((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((b \wedge d) \vee (e \wedge b)) \vee b)) \vee (((b \wedge (((b \vee d) \wedge (e \vee b)) \wedge b)))) \wedge (a \vee (((b \wedge d) \vee (e \wedge b)) \vee b)))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $a \wedge a \neq a \quad \text{cnf(prove_wal_axioms}_1\text{, negated_conjecture)}$

LAT093-1.p Axiom for weakly associative lattices (WAL), part 2
 $((((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((b \wedge d) \vee (e \wedge b)) \vee b)) \vee (((b \wedge (((b \vee d) \wedge (e \vee b)) \wedge b)))) \wedge (a \vee (((b \wedge d) \vee (e \wedge b)) \vee b)))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $b \wedge a \neq a \wedge b \quad \text{cnf(prove_wal_axioms}_2\text{, negated_conjecture)}$

LAT094-1.p Axiom for weakly associative lattices (WAL), part 3
 $((((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((b \wedge d) \vee (e \wedge b)) \vee b)) \vee (((b \wedge (((b \vee d) \wedge (e \vee b)) \wedge b)))) \wedge (a \vee (((b \wedge d) \vee (e \wedge b)) \vee b)))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $a \vee a \neq a \quad \text{cnf(prove_wal_axioms}_3\text{, negated_conjecture)}$

LAT095-1.p Axiom for weakly associative lattices (WAL), part 4
 $((((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((b \wedge d) \vee (e \wedge b)) \vee b)) \vee (((b \wedge (((b \vee d) \wedge (e \vee b)) \wedge b)))) \wedge (a \vee (((b \wedge d) \vee (e \wedge b)) \vee b)))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $b \vee a \neq a \vee b \quad \text{cnf(prove_wal_axioms}_4\text{, negated_conjecture)}$

LAT096-1.p Axiom for weakly associative lattices (WAL), part 5
 $((((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((b \wedge d) \vee (e \wedge b)) \vee b)) \vee (((b \wedge (((b \vee d) \wedge (e \vee b)) \wedge b)))) \wedge (a \vee (((b \wedge d) \vee (e \wedge b)) \vee b)))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $((a \vee b) \wedge (c \vee b)) \wedge b \neq b \quad \text{cnf(prove_wal_axioms}_5\text{, negated_conjecture)}$

LAT097-1.p Single axiom for weakly associative lattices (WAL), part 6
 $((((a \wedge b) \vee (b \wedge (a \vee b))) \wedge c) \vee (((a \wedge (((b \wedge d) \vee (e \wedge b)) \vee b)) \vee (((b \wedge (((b \vee d) \wedge (e \vee b)) \wedge b)))) \wedge (a \vee (((b \wedge d) \vee (e \wedge b)) \vee b)))) \wedge (((a \wedge b) \vee (b \wedge (a \vee b))) \vee c)) = b \quad \text{cnf(single_axiom, axiom)}$
 $((a \wedge b) \vee (c \wedge b)) \vee b \neq b \quad \text{cnf(prove_wal_axioms}_6\text{, negated_conjecture)}$

LAT098-1.p Huntington equation H3 is independent of H2

Show that Huntington equation H2 does not imply Huntington equation H3 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (z \wedge ((x \wedge (y \vee z)) \vee (y \wedge z)))) \quad \text{cnf(equation_H}_2\text{, axiom)}$
 $a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge (b \vee (a \wedge (c \vee (a \wedge b)))))) \quad \text{cnf(prove_H}_3\text{, negated_conjecture)}$

LAT099-1.p Huntington equation H2 is independent of H3

Show that Huntington equation H3 does not imply Huntington equation H2 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (z \wedge (y \vee (x \wedge (z \vee (x \wedge y)))))) \quad \text{cnf(equation_H}_3\text{, axiom)}$
 $a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge ((a \wedge (b \vee c)) \vee (b \wedge c)))) \quad \text{cnf(prove_H}_2\text{, negated_conjecture)}$

LAT100-1.p Huntington equation H4 is independent of H6

Show that Huntington equation H6 does not imply Huntington equation H4 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge ((x \wedge (y \vee (x \wedge z))) \vee (z \wedge (x \vee y))) \quad \text{cnf(equation_H}_6\text{, axiom)}$
 $a \wedge (b \vee (a \wedge (c \vee d))) \neq a \wedge (b \vee ((a \vee (b \wedge d)) \wedge (c \vee d))) \quad \text{cnf(prove_H}_4\text{, negated_conjecture)}$

LAT101-1.p Huntington equation H10 is independent of H6

Show that Huntington equation H6 does not imply Huntington equation H10 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge ((x \wedge (y \vee (x \wedge z))) \vee (z \wedge (x \vee y))) \quad \text{cnf(equation_H}_6\text{, axiom)}$
 $a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge (a \vee (b \wedge c)))) \quad \text{cnf(prove_H}_10\text{, negated_conjecture)}$

LAT102-1.p Huntington equation H4 is independent of H7

Show that Huntington equation H7 does not imply Huntington equation H4 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (x \wedge ((x \wedge y) \vee (z \wedge (x \vee y))))) \quad \text{cnf(equation_H}_7\text{, axiom)}$
 $a \wedge (b \vee (a \wedge (c \vee d))) \neq a \wedge (b \vee ((a \vee (b \wedge d)) \wedge (c \vee d))) \quad \text{cnf(prove_H}_4\text{, negated_conjecture)}$

LAT103-1.p Huntington equation H6 is independent of H10

Show that Huntington equation H10 does not imply Huntington equation H6 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (z \wedge (x \vee (y \wedge z)))) \quad \text{cnf(equation_H}_10\text{, axiom)}$

$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H₆, negated_conjecture)

LAT104-1.p Huntington equation H3 is independent of H21

Show that Huntington equation H21 does not imply Huntington equation H3 in lattice theory.
include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (x \vee (y \wedge z))) \vee (z \wedge (x \vee y)))$ cnf(equation_H₂₁, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge (b \vee (a \wedge (c \vee (a \wedge b))))))$ cnf(prove_H₃, negated_conjecture)

LAT105-1.p Huntington equation H10 is independent of H21

Show that Huntington equation H21 does not imply Huntington equation H10 in lattice theory.
include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (x \vee (y \wedge z))) \vee (z \wedge (x \vee y)))$ cnf(equation_H₂₁, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge (a \vee (b \wedge c))))$ cnf(prove_H₁₀, negated_conjecture)

LAT106-1.p Huntington equation H3 is independent of H22

Show that Huntington equation H22 does not imply Huntington equation H3 in lattice theory.
include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (z \vee (x \wedge y))) \vee (z \wedge (x \vee y)))$ cnf(equation_H₂₂, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge (b \vee (a \wedge (c \vee (a \wedge b))))))$ cnf(prove_H₃, negated_conjecture)

LAT107-1.p Huntington equation H17 is independent of H22

Show that Huntington equation H22 does not imply Huntington equation H17 in lattice theory.

include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (z \vee (x \wedge y))) \vee (z \wedge (x \vee y)))$ cnf(equation_H₂₂, axiom)

$a \wedge ((a \wedge b) \vee (a \wedge c)) \neq a \wedge ((b \wedge (a \vee (b \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H₁₇, negated_conjecture)

LAT108-1.p Huntington equation H42 is independent of H31

Show that Huntington equation H31 does not imply Huntington equation H42 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge (z \wedge u))) = x \wedge (y \vee (z \wedge (u \wedge (y \vee (x \wedge z)))))$ cnf(equation_H₃₁, axiom)

$a \wedge (b \vee (c \wedge (a \wedge d))) \neq a \wedge (b \vee (c \wedge (b \vee (d \vee (a \wedge c)))))$ cnf(prove_H₄₂, negated_conjecture)

LAT109-1.p Huntington equation H40 is independent of H37

Show that Huntington equation H37 does not imply Huntington equation H40 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \vee (x \wedge u))) = x \wedge (y \vee (z \vee (u \wedge (x \vee (y \wedge z)))))$ cnf(equation_H₃₇, axiom)

$a \wedge (b \vee (c \wedge (a \wedge d))) \neq a \wedge (b \vee (c \wedge (b \vee (d \vee (c \wedge (a \wedge b))))))$ cnf(prove_H₄₀, negated_conjecture)

LAT110-1.p Huntington equation H42 is independent of H37

Show that Huntington equation H37 does not imply Huntington equation H42 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \vee (x \wedge u))) = x \wedge (y \vee (z \vee (u \wedge (x \vee (y \wedge z)))))$ cnf(equation_H₃₇, axiom)

$a \wedge (b \vee (c \wedge (a \wedge d))) \neq a \wedge (b \vee (c \wedge (b \vee (d \vee (a \wedge c)))))$ cnf(prove_H₄₂, negated_conjecture)

LAT111-1.p Huntington equation H40 is independent of H45

Show that Huntington equation H45 does not imply Huntington equation H40 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \wedge (z \vee (x \wedge u))) = x \wedge (y \wedge (z \vee (u \wedge (x \vee (y \wedge z)))))$ cnf(equation_H₄₅, axiom)

$a \wedge (b \vee (c \wedge (a \wedge d))) \neq a \wedge (b \vee (c \wedge (d \vee (c \wedge (a \wedge b)))))$ cnf(prove_H₄₀, negated_conjecture)

LAT112-1.p Huntington equation H42 is independent of H47

Show that Huntington equation H47 does not imply Huntington equation H42 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \wedge (z \vee (y \wedge u))) = x \wedge (y \wedge (z \vee (u \wedge (y \vee (x \wedge z)))))$ cnf(equation_H₄₇, axiom)

$a \wedge (b \vee (c \wedge (a \wedge d))) \neq a \wedge (b \vee (c \wedge (b \vee (d \vee (a \wedge c)))))$ cnf(prove_H₄₂, negated_conjecture)

LAT113-1.p Huntington equation H40 is independent of H50

Show that Huntington equation H50 does not imply Huntington equation H40 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (x \vee (z \wedge (y \vee u)))))$ cnf(equation_H₅₀, axiom)

$a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (c \wedge (a \vee b)))))$ cnf(prove_H₄₀, negated_conjecture)

LAT114-1.p Huntington equation H56 is independent of H55

Show that Huntington equation H55 does not imply Huntington equation H56 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (x \vee z)) = x \vee (y \wedge (z \vee (x \wedge (z \vee y))))$ cnf(equation_H₅₅, axiom)

$(a \wedge b) \vee (a \wedge (b \vee c)) \neq a \wedge (b \vee ((a \vee b) \wedge (c \vee (a \wedge b))))$ cnf(prove_H₅₆, negated_conjecture)

LAT115-1.p Huntington equation H59 is independent of H55

Show that Huntington equation H55 does not imply Huntington equation H59 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (x \vee z)) = x \vee (y \wedge (z \vee (x \wedge (z \vee y))))$ cnf(equation_H₅₅, axiom)

$a \wedge ((b \vee c) \wedge (b \vee d)) \neq a \wedge (b \vee ((b \vee d) \wedge (c \vee (a \wedge b))))$ cnf(prove_H₅₉, negated_conjecture)

LAT116-1.p Huntington equation H60 is independent of H55

Show that Huntington equation H55 does not imply Huntington equation H60 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (x \vee z)) = x \vee (y \wedge (z \vee (x \wedge (z \vee y))))$ cnf(equation_H₅₅, axiom)

$a \wedge ((b \vee c) \wedge (b \vee d)) \neq a \wedge (b \vee ((b \vee c) \wedge (d \vee (a \wedge b))))$ cnf(prove_H₆₀, negated_conjecture)

LAT117-1.p Huntington equation H69 is independent of H65

Show that Huntington equation H65 does not imply Huntington equation H69 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge u)) = x \wedge (y \vee (x \wedge ((x \wedge y) \vee (z \wedge u))))$ cnf(equation_H₆₅, axiom)

$a \wedge (b \vee c) \neq (a \wedge (c \vee (a \wedge b))) \vee (a \wedge (b \vee (a \wedge c)))$ cnf(prove_H₆₉, negated_conjecture)

LAT118-1.p Huntington equation H69 is independent of H79

Show that Huntington equation H79 does not imply Huntington equation H69 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge ((x \wedge (y \vee (x \wedge z))) \vee (z \wedge u))$ cnf(equation_H₇₉, axiom)

$a \wedge (b \vee c) \neq (a \wedge (c \vee (a \wedge b))) \vee (a \wedge (b \vee (a \wedge c)))$ cnf(prove_H₆₉, negated_conjecture)

LAT119-1.p Huntington equation H3 is independent of H82

Show that Huntington equation H82 does not imply Huntington equation H3 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \wedge ((y \wedge (x \vee z)) \vee (z \wedge (x \vee y))) = (x \wedge y) \vee (x \wedge z)$ cnf(equation_H₈₂, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee ((b \vee (c \wedge (b \wedge (a \wedge (c \vee b)))))))$ cnf(prove_H₃, negated_conjecture)

LAT120-1.p Huntington equation H58 is independent of H10_dual

Show that Huntington equation H10_dual does not imply Huntington equation H58 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (x \vee z)) = x \vee (y \wedge (z \vee (x \wedge (y \vee z))))$ cnf(equation_H10_dual, axiom)

$a \wedge (b \vee c) \neq a \wedge (b \vee ((a \vee b) \wedge (c \vee (a \wedge b))))$ cnf(prove_H₅₈, negated_conjecture)

LAT121-1.p Huntington equation H55 is independent of H18_dual

Show that Huntington equation H18_dual does not imply Huntington equation H55 in lattice theory.

include('Axioms/LAT001-0.ax')

$(x \vee y) \wedge (x \vee z) = x \vee ((x \vee y) \wedge ((x \vee z) \wedge (y \vee (x \wedge z))))$ cnf(equation_H18_dual, axiom)

$a \vee (b \wedge (a \vee c)) \neq a \vee (b \wedge (c \vee (a \wedge (c \vee b))))$ cnf(prove_H₅₅, negated_conjecture)

LAT122-1.p Huntington equation H55 is independent of H21_dual

Show that Huntington equation H21_dual does not imply Huntington equation H55 in lattice theory.

include('Axioms/LAT001-0.ax')

$(x \vee y) \wedge (x \vee z) = x \vee ((y \vee (x \wedge (y \vee z))) \wedge (z \vee (x \wedge y)))$ cnf(equation_H21_dual, axiom)

$a \vee (b \wedge (a \vee c)) \neq a \vee (b \wedge (c \vee (a \wedge (c \vee b))))$ cnf(prove_H₅₅, negated_conjecture)

LAT123-1.p Huntington equation H55 is independent of H22_dual

Show that Huntington equation H22_dual does not imply Huntington equation H55 in lattice theory.

include('Axioms/LAT001-0.ax')

$(x \vee y) \wedge (x \vee z) = x \vee ((y \vee (z \wedge (x \vee y))) \wedge (z \vee (x \wedge y)))$ cnf(equation_H22_dual, axiom)

$a \vee (b \wedge (a \vee c)) \neq a \vee (b \wedge (c \vee (a \wedge (c \vee b))))$ cnf(prove_H₅₅, negated_conjecture)

LAT124-1.p Huntington equation H69 is independent of H32_dual

Show that Huntington equation H32_dual does not imply Huntington equation H69 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (x \vee (z \vee u))) = x \vee (y \wedge (z \vee ((x \vee u) \wedge (y \vee u))))$ cnf(equation_H32_dual, axiom)

$a \wedge (b \vee c) \neq (a \wedge (c \vee (a \wedge b))) \vee (a \wedge (b \vee (a \wedge c)))$ cnf(prove_H₆₉, negated_conjecture)

LAT125-1.p Huntington equation H69 is independent of H34_dual

Show that Huntington equation H34_dual does not imply Huntington equation H69 in lattice theory.

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (z \vee u)) = x \vee (y \wedge (z \vee (y \wedge (u \vee (y \wedge z)))))$ cnf(equation_H34_dual, axiom)

$a \wedge (b \vee c) \neq (a \wedge (c \vee (a \wedge b))) \vee (a \wedge (b \vee (a \wedge c)))$ cnf(prove_H₆₉, negated_conjecture)

LAT126-1.p Huntington equation H69 is independent of H39_dual

Show that Huntington equation H39_dual does not imply Huntington equation H69 in lattice theory.
include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (z \vee (x \wedge u))) = x \vee (y \wedge (z \vee (u \wedge (x \vee z))))$ cnf(equation_H39_dual, axiom)

$a \wedge (b \vee c) \neq (a \wedge (c \vee (a \wedge b))) \vee (a \wedge (b \vee (a \wedge c)))$ cnf(prove_H₆₉, negated_conjecture)

LAT127-1.p Huntington equation H6 is independent of H55_dual

Show that Huntington equation H55_dual does not imply Huntington equation H6 in lattice theory.
include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (z \wedge (x \vee y)))$ cnf(equation_H55_dual, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H₆, negated_conjecture)

LAT128-1.p Huntington equation H3 is independent of H58_dual

Show that Huntington equation H58_dual does not imply Huntington equation H3 in lattice theory.
include('Axioms/LAT001-0.ax')

$x \vee (y \wedge z) = x \vee (y \wedge ((x \wedge y) \vee (z \wedge (x \vee y))))$ cnf(equation_H58_dual, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge (b \vee (a \wedge (c \vee (a \wedge b))))))$ cnf(prove_H₃, negated_conjecture)

LAT129-1.p Huntington equation H10 is independent of H58_dual

Show that Huntington equation H58_dual does not imply Huntington equation H10 in lattice theory.
include('Axioms/LAT001-0.ax')

$x \vee (y \wedge z) = x \vee (y \wedge ((x \wedge y) \vee (z \wedge (x \vee y))))$ cnf(equation_H58_dual, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge (a \vee (b \wedge c))))$ cnf(prove_H₁₀, negated_conjecture)

LAT130-1.p Huntington equation H39 is independent of H68_dual

Show that Huntington equation H68_dual does not imply Huntington equation H39 in lattice theory.
include('Axioms/LAT001-0.ax')

$x \vee (y \wedge z) = x \vee (y \wedge (x \vee (z \wedge (x \vee y))))$ cnf(equation_H68_dual, axiom)

$a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (a \wedge c))))$ cnf(prove_H₃₉, negated_conjecture)

LAT131-1.p Huntington equation H42 is independent of H68_dual

Show that Huntington equation H68_dual does not imply Huntington equation H42 in lattice theory.
include('Axioms/LAT001-0.ax')

$x \vee (y \wedge z) = x \vee (y \wedge (x \vee (z \wedge (x \vee y))))$ cnf(equation_H68_dual, axiom)

$a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee (c \wedge (b \vee (d \vee (a \wedge c))))))$ cnf(prove_H₄₂, negated_conjecture)

LAT132-1.p Huntington equation H42 is independent of H69_dual

Show that Huntington equation H69_dual does not imply Huntington equation H42 in lattice theory.
include('Axioms/LAT001-0.ax')

$x \vee (y \wedge z) = (x \vee (z \wedge (x \vee y))) \wedge (x \vee (y \wedge (x \vee z)))$ cnf(equation_H69_dual, axiom)

$a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee (c \wedge (b \vee (d \vee (a \wedge c))))))$ cnf(prove_H₄₂, negated_conjecture)

LAT133-1.p Huntington equation H6_dual is independent of H55

Show that Huntington equation H55 does not imply Huntington equation H6_dual in lattice theory.
include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (x \vee z)) = x \vee (y \wedge (z \vee (x \wedge (z \vee y))))$ cnf(equation_H₅₅, axiom)

$a \vee (b \wedge (a \vee c)) \neq a \vee ((a \vee (b \wedge (a \vee c))) \wedge (c \vee (a \wedge b)))$ cnf(prove_H₆_dual, negated_conjecture)

LAT134-1.p Huntington equation H22_dual is independent of H61

Show that Huntington equation H61 does not imply Huntington equation H22_dual in lattice theory.
include('Axioms/LAT001-0.ax')

$(x \vee y) \wedge (x \vee z) = x \vee ((x \vee y) \wedge ((x \wedge y) \vee z))$ cnf(equation_H₆₁, axiom)

$(a \vee b) \wedge (a \vee c) \neq a \vee ((b \vee (c \wedge (a \vee b))) \wedge (c \vee (a \wedge b)))$ cnf(prove_H₂₂_dual, negated_conjecture)

LAT135-1.p Huntington equation H39_dual is independent of H68

Show that Huntington equation H68 does not imply Huntington equation H39_dual in lattice theory.
include('Axioms/LAT001-0.ax')

$x \wedge (y \vee z) = x \wedge (y \vee (x \wedge (z \vee (x \wedge y))))$ cnf(equation_H₆₈, axiom)

$a \vee (b \wedge (c \wedge (a \wedge d))) \neq a \vee (b \wedge (c \wedge (d \wedge (a \wedge c))))$ cnf(prove_H₃₉_dual, negated_conjecture)

LAT136-1.p Huntington equation H39_dual is independent of H69

Show that Huntington equation H69 does not imply Huntington equation H39_dual in lattice theory.
include('Axioms/LAT001-0.ax')

$x \wedge (y \vee z) = (x \wedge (z \vee (x \wedge y))) \vee (x \wedge (y \vee (x \wedge z)))$ cnf(equation_H₆₉, axiom)

$a \vee (b \wedge (c \vee (a \wedge d))) \neq a \vee (b \wedge (c \vee (d \wedge (a \vee c))))$ cnf(prove_H39_dual, negated_conjecture)

LAT137-1.p Huntington equation H40_dual is independent of H69

Show that Huntington equation H69 does not imply Huntington equation H40_dual in lattice theory.
include('Axioms/LAT001-0.ax')

$x \wedge (y \vee z) = (x \wedge (z \vee (x \wedge y))) \vee (x \wedge (y \vee (x \wedge z)))$ cnf(equation_H69, axiom)

$a \wedge (b \wedge (c \vee (a \wedge d))) \neq a \wedge (b \wedge (c \vee (d \wedge (c \vee (a \wedge b)))))$ cnf(prove_H40_dual, negated_conjecture)

LAT138-1.p Huntington equation H7 implies H6

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (x \wedge ((x \wedge y) \vee (z \wedge (x \vee y)))))$ cnf(equation_H7, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H6, negated_conjecture)

LAT139-1.p Huntington equation H11 implies H10

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (z \wedge (x \vee (y \wedge (z \vee (x \wedge y))))))$ cnf(equation_H11, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge (a \vee (b \wedge c))))$ cnf(prove_H10, negated_conjecture)

LAT140-1.p Huntington equation H21 implies H2

include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (x \vee (y \wedge z))) \vee (z \wedge (x \vee y)))$ cnf(equation_H21, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (b \wedge c))$ cnf(prove_H2, negated_conjecture)

LAT141-1.p Huntington equation H21 implies H6

include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (x \vee (y \wedge z))) \vee (z \wedge (x \vee y)))$ cnf(equation_H21, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H6, negated_conjecture)

LAT142-1.p Huntington equation H22 implies H6

include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (z \vee (x \wedge y))) \vee (z \wedge (x \vee y)))$ cnf(equation_H22, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H6, negated_conjecture)

LAT143-1.p Huntington equation H24 implies H15

include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (y \wedge z) = (x \wedge y) \vee (y \wedge ((x \wedge y) \vee (z \wedge (x \vee y))))$ cnf(equation_H24, axiom)

$a \wedge ((a \wedge b) \vee (a \wedge c)) \neq a \wedge ((a \wedge b) \vee ((a \wedge c) \vee (c \wedge (a \vee b))))$ cnf(prove_H15, negated_conjecture)

LAT144-1.p Huntington equation H32 implies H2

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge (z \wedge u))) = x \wedge (y \vee (z \wedge ((x \wedge u) \vee (y \wedge u))))$ cnf(equation_H32, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge ((a \wedge (b \vee c)) \vee (b \wedge c))))$ cnf(prove_H2, negated_conjecture)

LAT145-1.p Huntington equation H32 implies H6

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (x \wedge (z \wedge u))) = x \wedge (y \vee (z \wedge ((x \wedge u) \vee (y \wedge u))))$ cnf(equation_H32, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H6, negated_conjecture)

LAT146-1.p Huntington equation H34 implies H28

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge u)) = x \wedge (y \vee (z \wedge (y \vee (u \wedge (y \vee z)))))$ cnf(equation_H34, axiom)

$a \wedge (b \vee (a \wedge c \wedge d)) \neq a \wedge (b \vee (c \wedge (d \wedge (a \vee (b \wedge d)))))$ cnf(prove_H28, negated_conjecture)

LAT147-1.p Huntington equation H34 implies H45

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge u)) = x \wedge (y \vee (z \wedge (y \vee (u \wedge (y \vee z)))))$ cnf(equation_H34, axiom)

$a \wedge (b \vee (c \vee (a \wedge d))) \neq a \wedge (b \vee (c \vee (d \wedge (a \vee (b \wedge d)))))$ cnf(prove_H45, negated_conjecture)

LAT148-1.p Huntington equation H34 implies H7

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge u)) = x \wedge (y \vee (z \wedge (y \vee (u \wedge (y \vee z)))))$ cnf(equation_H34, axiom)

$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (a \wedge ((a \wedge b) \vee (c \wedge (a \vee b)))))$ cnf(prove_H7, negated_conjecture)

LAT149-1.p Huntington equation H37 implies H43

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \vee (x \wedge u))) = x \wedge (y \vee (z \vee (u \wedge (x \vee (y \wedge z)))))$ cnf(equation_H37, axiom)

$a \wedge (b \vee (c \wedge (b \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (a \wedge (b \vee d)))))$ cnf(prove_H43, negated_conjecture)

LAT150-1.p Huntington equation H39 implies H40

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (u \vee (x \wedge z))))$	cnf(equation_H ₃₉ , axiom)
$a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (c \wedge (a \vee b)))))$	cnf(prove_H ₄₀ , negated_conjecture)

LAT151-1.p Huntington equation H39 implies H42

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (u \vee (x \wedge z))))$	cnf(equation_H ₃₉ , axiom)
$a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee (c \wedge (b \vee (d \vee (a \wedge c)))))$	cnf(prove_H ₄₂ , negated_conjecture)

LAT152-1.p Huntington equation H40 implies H6

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (u \vee (z \wedge (x \vee y)))))$	cnf(equation_H ₄₀ , axiom)
$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$	cnf(prove_H ₆ , negated_conjecture)

LAT153-1.p Huntington equation H40 implies H7

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (u \vee (z \wedge (x \vee y)))))$	cnf(equation_H ₄₀ , axiom)
$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$	cnf(prove_H ₇ , negated_conjecture)

LAT154-1.p Huntington equation H42 implies H6

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (y \vee (u \vee (x \wedge z)))))$	cnf(equation_H ₄₂ , axiom)
$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$	cnf(prove_H ₆ , negated_conjecture)

LAT155-1.p Huntington equation H49 implies H2

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee ((x \wedge z) \vee (z \wedge (y \vee u))))$	cnf(equation_H ₄₉ , axiom)
$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$	cnf(prove_H ₂ , negated_conjecture)

LAT156-1.p Huntington equation H49 implies H6

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee ((x \wedge z) \vee (z \wedge (y \vee u))))$	cnf(equation_H ₄₉ , axiom)
$a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$	cnf(prove_H ₆ , negated_conjecture)

LAT157-1.p Huntington equation H50 implies H2

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (x \vee (z \wedge (y \vee u)))))$	cnf(equation_H ₅₀ , axiom)
$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (c \wedge ((a \wedge (b \vee c)) \vee (b \wedge c))))$	cnf(prove_H ₂ , negated_conjecture)

LAT158-1.p Huntington equation H50 implies H49

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (x \vee (z \wedge (y \vee u)))))$	cnf(equation_H ₅₀ , axiom)
$a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee ((a \wedge c) \vee (c \wedge (b \vee d))))$	cnf(prove_H ₄₉ , negated_conjecture)

LAT159-1.p Huntington equation H50 implies H7

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (x \vee (z \wedge (y \vee u)))))$	cnf(equation_H ₅₀ , axiom)
$a \wedge (b \vee (a \wedge c)) \neq a \wedge (b \vee (a \wedge ((a \wedge b) \vee (c \wedge (a \vee b)))))$	cnf(prove_H ₇ , negated_conjecture)

LAT160-1.p Huntington equation H52 implies H51

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee ((z \wedge u) \vee (z \wedge (x \vee y))))$	cnf(equation_H ₅₂ , axiom)
$a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee ((a \wedge c) \vee (c \wedge (a \vee d))))$	cnf(prove_H ₅₁ , negated_conjecture)

LAT161-1.p Huntington equation H58 implies H59

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee z) = x \wedge (y \vee ((x \wedge y) \wedge (z \vee (x \wedge y))))$	cnf(equation_H ₅₈ , axiom)
$a \wedge ((b \vee c) \wedge (b \vee d)) \neq a \wedge (b \vee ((b \vee d) \wedge (c \vee (a \wedge b))))$	cnf(prove_H ₅₉ , negated_conjecture)

LAT162-1.p Huntington equation H68 implies H73

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee z) = x \wedge (y \vee (x \wedge (z \vee (x \wedge y))))$	cnf(equation_H ₆₈ , axiom)
$a \wedge (b \wedge (c \vee d)) \neq a \wedge (b \wedge (c \vee (a \wedge (d \vee (b \wedge c)))))$	cnf(prove_H ₇₃ , negated_conjecture)

LAT163-1.p Huntington equation H76 implies H32

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (y \vee u))) = x \wedge (y \vee (z \wedge (u \vee (x \wedge y))))$ cnf(equation_H₇₆, axiom)
 $a \wedge (b \vee (a \wedge (c \wedge d))) \neq a \wedge (b \vee (c \wedge ((a \wedge d) \vee (b \wedge d))))$ cnf(prove_H₃₂, negated_conjecture)

LAT164-1.p Huntington equation H76 implies H6

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (y \vee u))) = x \wedge (y \vee (z \wedge (u \vee (x \wedge y))))$ cnf(equation_H₇₆, axiom)
 $a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H₆, negated_conjecture)

LAT165-1.p Huntington equation H76 implies H77

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (y \vee u))) = x \wedge (y \vee (z \wedge (u \vee (x \wedge y))))$ cnf(equation_H₇₆, axiom)
 $a \wedge (b \vee (c \wedge (b \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (a \wedge (b \wedge c))))))$ cnf(prove_H₇₇, negated_conjecture)

LAT166-1.p Huntington equation H77 implies H78

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (y \vee u))) = x \wedge (y \vee (z \wedge (u \vee (x \wedge (y \wedge z))))))$ cnf(equation_H₇₇, axiom)
 $a \wedge (b \vee (c \wedge (b \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (b \wedge (a \vee d))))))$ cnf(prove_H₇₈, negated_conjecture)

LAT167-1.p Huntington equation H78 implies H77

include('Axioms/LAT001-0.ax')

$x \wedge (y \vee (z \wedge (y \vee u))) = x \wedge (y \vee (z \wedge (u \vee (y \wedge (x \vee u))))))$ cnf(equation_H₇₈, axiom)
 $a \wedge (b \vee (c \wedge (b \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (a \wedge (b \wedge c))))))$ cnf(prove_H₇₇, negated_conjecture)

LAT168-1.p Huntington equation H18_dual implies H58

include('Axioms/LAT001-0.ax')

$(x \vee y) \wedge (x \vee z) = x \vee ((x \vee y) \wedge ((x \vee z) \wedge (y \vee (x \wedge z))))$ cnf(equation_H18_dual, axiom)
 $a \wedge (b \vee c) \neq a \wedge (b \vee ((a \vee b) \wedge (c \vee (a \wedge b))))$ cnf(prove_H₅₈, negated_conjecture)

LAT169-1.p Huntington equation H21_dual implies H58

include('Axioms/LAT001-0.ax')

$(x \vee y) \wedge (x \vee z) = x \vee ((y \vee (x \wedge (y \vee z))) \wedge (z \vee (x \wedge y)))$ cnf(equation_H21_dual, axiom)
 $a \wedge (b \vee c) \neq a \wedge (b \vee ((a \vee b) \wedge (c \vee (a \wedge b))))$ cnf(prove_H₅₈, negated_conjecture)

LAT170-1.p Huntington equation H49_dual implies H58

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (z \vee (x \wedge u))) = x \vee (y \wedge ((x \vee z) \wedge (z \vee (y \wedge u))))$ cnf(equation_H49_dual, axiom)
 $a \wedge (b \vee c) \neq a \wedge (b \vee ((a \vee b) \wedge (c \vee (a \wedge b))))$ cnf(prove_H₅₈, negated_conjecture)

LAT171-1.p Huntington equation H61_dual implies H6

include('Axioms/LAT001-0.ax')

$(x \wedge y) \vee (x \wedge z) = x \wedge ((x \wedge y) \vee ((x \vee y) \wedge z))$ cnf(equation_H61_dual, axiom)
 $a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H₆, negated_conjecture)

LAT172-1.p Huntington equation H76_dual implies H32

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (z \vee (y \wedge u))) = x \vee (y \wedge (z \vee (u \wedge (x \vee y))))$ cnf(equation_H76_dual, axiom)
 $a \wedge (b \vee (a \wedge (c \wedge d))) \neq a \wedge (b \vee (c \wedge ((a \wedge d) \vee (b \wedge d))))$ cnf(prove_H₃₂, negated_conjecture)

LAT173-1.p Huntington equation H76_dual implies H40

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (z \vee (y \wedge u))) = x \vee (y \wedge (z \vee (u \wedge (x \vee y))))$ cnf(equation_H76_dual, axiom)
 $a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (c \wedge (a \vee b))))))$ cnf(prove_H₄₀, negated_conjecture)

LAT174-1.p Huntington equation H76_dual implies H6

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (z \vee (y \wedge u))) = x \vee (y \wedge (z \vee (u \wedge (x \vee y))))$ cnf(equation_H76_dual, axiom)
 $a \wedge (b \vee (a \wedge c)) \neq a \wedge ((a \wedge (b \vee (a \wedge c))) \vee (c \wedge (a \vee b)))$ cnf(prove_H₆, negated_conjecture)

LAT175-1.p Huntington equation H79_dual implies H32

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (z \vee (x \wedge u))) = x \vee ((x \vee (y \wedge (x \vee z))) \wedge (z \vee u))$ cnf(equation_H79_dual, axiom)
 $a \wedge (b \vee (a \wedge (c \wedge d))) \neq a \wedge (b \vee (c \wedge ((a \wedge d) \vee (b \wedge d))))$ cnf(prove_H₃₂, negated_conjecture)

LAT176-1.p Huntington equation H79_dual implies H42

include('Axioms/LAT001-0.ax')

$x \vee (y \wedge (z \vee (x \wedge u))) = x \vee ((x \vee (y \wedge (x \vee z))) \wedge (z \vee u))$ cnf(equation_H79_dual, axiom)
 $a \wedge (b \vee (c \wedge (a \vee d))) \neq a \wedge (b \vee (c \wedge (d \vee (a \wedge c))))$ cnf(prove_H₄₂, negated_conjecture)

LAT177-1.p Huntington equation H79_dual implies H6

```
include('Axioms/LAT001-0.ax')
x ∨ (y ∨ (z ∨ (x ∨ u))) = x ∨ ((x ∨ (y ∨ (x ∨ z))) ∨ (z ∨ u))      cnf(equation_H79_dual, axiom)
a ∧ (b ∨ (a ∧ c)) ≠ a ∧ ((a ∧ (b ∨ (a ∧ c))) ∨ (c ∧ (a ∨ b)))      cnf(prove_H6, negated_conjecture)
```

LAT178-1.p Equation H1 is Huntington by distributivity

Show that H1 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∨ (x ∨ u))) = x ∧ (y ∨ (z ∨ (x ∨ (z ∨ u))))      cnf(equation_H1, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT179-1.p Equation H2 is Huntington by distributivity

Show that H2 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∨ z)) = x ∧ (y ∨ (z ∨ ((x ∧ (y ∨ z)) ∨ (y ∧ z))))      cnf(equation_H2, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT180-1.p Equation H3 is Huntington by distributivity

Show that H3 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∨ z)) = x ∧ (y ∨ (z ∨ (y ∨ (x ∧ (z ∨ (x ∧ y))))))      cnf(equation_H3, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT181-1.p Equation H4 is Huntington by distributivity

Show that H4 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∨ (z ∨ u))) = x ∧ (y ∨ ((x ∨ (y ∧ u)) ∧ (z ∨ u)))      cnf(equation_H4, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT182-1.p Equation H6 is Huntington by distributivity

Show that H6 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∨ z)) = x ∧ ((x ∧ (y ∨ (x ∨ z))) ∨ (z ∨ (x ∨ y)))      cnf(equation_H6, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT183-1.p Equation H7 is Huntington by distributivity

Show that H7 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∨ z)) = x ∧ (y ∨ (x ∧ ((x ∧ y) ∨ (z ∧ (x ∨ y)))))      cnf(equation_H7, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT184-1.p Equation H8 is Huntington by distributivity

Show that H8 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∨ z)) = (x ∧ y) ∨ (x ∧ (y ∨ (z ∧ (x ∨ (y ∧ z)))))      cnf(equation_H8, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT185-1.p Equation H10 is Huntington by distributivity

Show that H10 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∨ z)) = x ∧ (y ∨ (z ∧ (x ∨ (y ∧ z))))      cnf(equation_H10, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT186-1.p Equation H11 is Huntington by distributivity

Show that H11 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
```

$x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (z \wedge (x \vee (y \wedge (z \vee (x \wedge y))))))$ cnf(equation_H₁₁, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT187-1.p Equation H15 is Huntington by distributivity

Show that H15 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $x \wedge ((x \wedge y) \vee (x \wedge z)) = x \wedge ((x \wedge y) \vee ((x \wedge z) \vee (z \wedge (x \vee y))))$ cnf(equation_H₁₅, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT188-1.p Equation H16 is Huntington by distributivity

Show that H16 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $x \wedge ((x \wedge y) \vee (x \wedge z)) = x \wedge ((x \wedge y) \vee (z \wedge (y \vee (z \wedge (x \vee y))))))$ cnf(equation_H₁₆, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT189-1.p Equation H17 is Huntington by distributivity

Show that H17 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $x \wedge ((x \wedge y) \vee (x \wedge z)) = x \wedge ((y \wedge (x \vee (y \wedge z))) \vee (z \wedge (x \vee y)))$ cnf(equation_H₁₇, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT190-1.p Equation H18 is Huntington by distributivity

Show that H18 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $(x \wedge y) \vee (x \wedge z) = x \wedge ((x \wedge y) \vee ((x \wedge z) \vee (y \wedge (x \vee z))))$ cnf(equation_H₁₈, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT191-1.p Equation H21 is Huntington by distributivity

Show that H21 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (x \vee (y \wedge z))) \vee (z \wedge (x \vee y)))$ cnf(equation_H₂₁, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT192-1.p Equation H22 is Huntington by distributivity

Show that H22 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $(x \wedge y) \vee (x \wedge z) = x \wedge ((y \wedge (z \vee (x \wedge y))) \vee (z \wedge (x \vee y)))$ cnf(equation_H₂₂, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT193-1.p Equation H24 is Huntington by distributivity

Show that H24 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $(x \wedge y) \vee (y \wedge z) = (x \wedge y) \vee (y \wedge ((x \wedge y) \vee (z \wedge (x \vee y))))$ cnf(equation_H₂₄, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT194-1.p Equation H32 is Huntington by distributivity

Show that H32 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $x \wedge (y \vee (x \wedge (z \wedge u))) = x \wedge (y \vee (z \wedge ((x \wedge u) \vee (y \wedge u))))$ cnf(equation_H₃₂, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT195-1.p Equation H34 is Huntington by distributivity

Show that H34 is Huntington by deriving distributivity in uniquely complemented lattices.
 include('Axioms/LAT001-0.ax')
 include('Axioms/LAT001-4.ax')
 $x \wedge (y \vee (z \wedge u)) = x \wedge (y \vee (z \wedge (y \vee (u \wedge (y \vee z))))))$ cnf(equation_H₃₄, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT196-1.p Equation H39 is Huntington by distributivity

Show that H39 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ (z ∧ (u ∨ (x ∧ z))))      cnf(equation_H39, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT197-1.p Equation H40 is Huntington by distributivity

Show that H40 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ (z ∧ (u ∨ (z ∧ (x ∨ y)))))      cnf(equation_H40, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT198-1.p Equation H42 is Huntington by distributivity

Show that H42 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ (z ∧ (y ∨ (u ∨ (x ∧ z)))))      cnf(equation_H42, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT199-1.p Equation H49 is Huntington by distributivity

Show that H49 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ ((x ∧ z) ∨ (z ∧ (y ∨ u))))      cnf(equation_H49, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT200-1.p Equation H50 is Huntington by distributivity

Show that H50 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ (z ∧ (x ∨ (z ∧ (y ∨ u)))))      cnf(equation_H50, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT201-1.p Equation H51 is Huntington by distributivity

Show that H51 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ ((x ∧ z) ∨ (z ∧ u)))      cnf(equation_H51, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT202-1.p Equation H55 is Huntington by distributivity

Show that H55 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∨ (y ∧ (x ∨ z)) = x ∨ (y ∧ (z ∨ (x ∧ (z ∨ y))))      cnf(equation_H55, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT203-1.p Equation H57 is Huntington by distributivity

Show that H57 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∧ (y ∨ z))) = x ∧ (y ∨ ((x ∨ y) ∧ (z ∨ (x ∧ y))))      cnf(equation_H57, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT204-1.p Equation H58 is Huntington by distributivity

Show that H58 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ z) = x ∧ (y ∨ ((x ∨ y) ∧ (z ∨ (x ∧ y))))      cnf(equation_H58, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c)      cnf(prove_distributivity, negated_conjecture)
```

LAT205-1.p Equation H59 is Huntington by distributivity

Show that H59 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((y ∨ z) ∧ (y ∨ u)) = x ∧ (y ∨ ((y ∨ u) ∧ (z ∨ (x ∧ y)))) cnf(equation_H59, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT206-1.p Equation H60 is Huntington by distributivity

Show that H60 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((y ∨ z) ∧ (y ∨ u)) = x ∧ (y ∨ ((y ∨ z) ∧ (u ∨ (x ∧ y)))) cnf(equation_H60, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT207-1.p Equation H61 is Huntington by distributivity

Show that H61 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
(x ∨ y) ∧ (x ∨ z) = x ∨ ((x ∨ y) ∧ ((x ∧ y) ∨ z)) cnf(equation_H61, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT208-1.p Equation H63 is Huntington by distributivity

Show that H63 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((y ∨ z) ∧ (y ∨ u)) = x ∧ (y ∨ ((y ∨ z) ∧ (u ∨ (y ∧ z)))) cnf(equation_H63, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT209-1.p Equation H64 is Huntington by distributivity

Show that H64 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ z) = x ∧ (y ∨ (x ∧ (z ∨ (x ∧ (y ∨ (x ∧ z)))))) cnf(equation_H64, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT210-1.p Equation H68 is Huntington by distributivity

Show that H68 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ z) = x ∧ (y ∨ (x ∧ (z ∨ (x ∧ y)))) cnf(equation_H68, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT211-1.p Equation H69 is Huntington by distributivity

Show that H69 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ z) = (x ∧ (z ∨ (x ∧ y))) ∨ (x ∧ (y ∨ (x ∧ z))) cnf(equation_H69, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT212-1.p Equation H70 is Huntington by distributivity

Show that H70 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (y ∨ u))) = x ∧ (y ∨ (z ∧ (u ∨ (y ∧ (x ∨ z))))) cnf(equation_H70, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT213-1.p Equation H76 is Huntington by distributivity

Show that H76 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (y ∨ u))) = x ∧ (y ∨ (z ∧ (u ∨ (x ∧ y)))) cnf(equation_H76, axiom)
a ∧ (b ∨ c) ≠ (a ∧ b) ∨ (a ∧ c) cnf(prove_distributivity, negated_conjecture)

```

LAT214-1.p Equation H79 is Huntington by distributivity

Show that H79 is Huntington by deriving distributivity in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')

```

$x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge ((x \wedge (y \vee (x \wedge z))) \vee (z \wedge u))$ cnf(equation_H₇₉, axiom)
 $a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$ cnf(prove_distributivity, negated_conjecture)

LAT215-1.p Equation H80 is Huntington by distributivity

Show that H80 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x \wedge ((x \wedge y) \vee (z \wedge (x \vee (y \wedge (x \vee z))))) = (x \wedge y) \vee (x \wedge z) cnf(equation_H80, axiom)
a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c) cnf(prove_distributivity, negated_conjecture)
```

LAT216-1.p Equation H81 is Huntington by distributivity

Show that H81 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x \wedge ((x \wedge y) \vee (z \wedge (x \vee (y \wedge (x \vee z))))) = x \wedge ((x \wedge y) \vee (x \wedge z)) cnf(equation_H81, axiom)
a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c) cnf(prove_distributivity, negated_conjecture)
```

LAT217-1.p Equation H82 is Huntington by distributivity

Show that H82 is Huntington by deriving distributivity in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x \wedge ((y \wedge (x \vee z)) \vee (z \wedge (x \vee y))) = (x \wedge y) \vee (x \wedge z) cnf(equation_H82, axiom)
a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c) cnf(prove_distributivity, negated_conjecture)
```

LAT218-1.p Equation H1 is Huntington by implication

Show that H1 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x \wedge (y \vee (z \wedge (x \vee u))) = x \wedge (y \vee (z \wedge (x \vee (z \wedge u)))) cnf(equation_H1, axiom)
b \wedge a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' \vee a' \neq a' cnf(prove_distributivity, negated_conjecture)
```

LAT219-1.p Equation H2 is Huntington by implication

Show that H2 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (z \wedge ((x \wedge (y \vee z)) \vee (y \wedge z)))) cnf(equation_H2, axiom)
b \wedge a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' \vee a' \neq a' cnf(prove_distributivity, negated_conjecture)
```

LAT220-1.p Equation H3 is Huntington by implication

Show that H3 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x \wedge (y \vee (x \wedge z)) = x \wedge (y \vee (z \wedge (y \vee (x \wedge (z \vee (x \wedge y)))))) cnf(equation_H3, axiom)
b \wedge a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' \vee a' \neq a' cnf(prove_distributivity, negated_conjecture)
```

LAT221-1.p Equation H4 is Huntington by implication

Show that H4 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x \wedge (y \vee (x \wedge (z \vee u))) = x \wedge (y \vee ((x \vee (y \wedge u)) \wedge (z \vee u))) cnf(equation_H4, axiom)
b \wedge a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' \vee a' \neq a' cnf(prove_distributivity, negated_conjecture)
```

LAT222-1.p Equation H6 is Huntington by implication

Show that H6 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
```

```

include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∧ z)) = x ∧ ((x ∧ (y ∨ (x ∧ z))) ∨ (z ∧ (x ∨ y)))      cnf(equation_H6, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT223-1.p Equation H7 is Huntington by implication

Show that H7 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∧ z)) = x ∧ (y ∨ (x ∧ ((x ∧ y) ∨ (z ∧ (x ∨ y)))))      cnf(equation_H7, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT224-1.p Equation H8 is Huntington by implication

Show that H8 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∧ z)) = (x ∧ y) ∨ (x ∧ (y ∨ (z ∧ (x ∨ (y ∧ z)))))      cnf(equation_H8, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT225-1.p Equation H10 is Huntington by implication

Show that H10 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∧ z)) = x ∧ (y ∨ (z ∧ (x ∨ (y ∧ z))))      cnf(equation_H10, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT226-1.p Equation H11 is Huntington by implication

Show that H11 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∧ z)) = x ∧ (y ∨ (z ∧ (x ∨ (y ∧ (z ∨ (x ∧ y))))))      cnf(equation_H11, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT227-1.p Equation H15 is Huntington by implication

Show that H15 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((x ∧ y) ∨ (x ∧ z)) = x ∧ ((x ∧ y) ∨ ((x ∧ z) ∨ (z ∧ (x ∨ y))))      cnf(equation_H15, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT228-1.p Equation H16 is Huntington by implication

Show that H16 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((x ∧ y) ∨ (x ∧ z)) = x ∧ ((x ∧ y) ∨ (z ∧ (y ∨ (z ∧ (x ∨ y)))))      cnf(equation_H16, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT229-1.p Equation H17 is Huntington by implication

Show that H17 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
```

```

include('Axioms/LAT001-4.ax')
x ∧ ((x ∧ y) ∨ (x ∧ z)) = x ∧ ((y ∧ (x ∨ (y ∧ z))) ∨ (z ∧ (x ∨ y)))      cnf(equation_H17, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT230-1.p Equation H18 is Huntington by implication

Show that H18 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
(x ∧ y) ∨ (x ∧ z) = x ∧ ((x ∧ y) ∨ ((x ∧ z) ∨ (y ∧ (x ∨ z))))      cnf(equation_H18, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT231-1.p Equation H21 is Huntington by implication

Show that H21 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
(x ∧ y) ∨ (x ∧ z) = x ∧ ((y ∧ (x ∨ (y ∧ z))) ∨ (z ∧ (x ∨ y)))      cnf(equation_H21, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT232-1.p Equation H22 is Huntington by implication

Show that H22 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
(x ∧ y) ∨ (x ∧ z) = x ∧ ((y ∧ (z ∨ (x ∧ y))) ∨ (z ∧ (x ∨ y)))      cnf(equation_H22, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT233-1.p Equation H24 is Huntington by implication

Show that H24 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
(x ∧ y) ∨ (y ∧ z) = (x ∧ y) ∨ (y ∧ ((x ∧ y) ∨ (z ∧ (x ∨ y))))      cnf(equation_H24, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT234-1.p Equation H32 is Huntington by implication

Show that H32 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∧ (z ∧ u))) = x ∧ (y ∨ (z ∧ ((x ∧ u) ∨ (y ∧ u))))      cnf(equation_H32, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT235-1.p Equation H34 is Huntington by implication

Show that H34 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ u)) = x ∧ (y ∨ (z ∧ (y ∨ (u ∧ (y ∨ z)))))      cnf(equation_H34, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'    cnf(prove_distributivity, negated_conjecture)

```

LAT236-1.p Equation H39 is Huntington by implication

Show that H39 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
```

```

include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ (z ∧ (u ∨ (x ∧ z)))) cnf(equation_H39, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT237-1.p Equation H40 is Huntington by implication

Show that H40 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ (z ∧ (u ∨ (x ∧ z)))) cnf(equation_H40, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT238-1.p Equation H42 is Huntington by implication

Show that H42 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ (z ∧ (y ∨ (u ∨ (x ∧ z))))) cnf(equation_H42, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT239-1.p Equation H49 is Huntington by implication

Show that H49 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ ((x ∧ z) ∨ (z ∧ (y ∨ u)))) cnf(equation_H49, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT240-1.p Equation H50 is Huntington by implication

Show that H50 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ (z ∧ (x ∨ (z ∧ (y ∨ u))))) cnf(equation_H50, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT241-1.p Equation H51 is Huntington by implication

Show that H51 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ (y ∨ ((x ∧ z) ∨ (z ∧ u))) cnf(equation_H51, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT242-1.p Equation H55 is Huntington by implication

Show that H55 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∨ (y ∧ (x ∨ z)) = x ∨ (y ∧ (z ∨ (x ∧ (z ∨ y)))) cnf(equation_H55, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT243-1.p Equation H57 is Huntington by implication

Show that H57 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
```

```

include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (x ∧ (y ∨ z))) = x ∧ (y ∨ ((x ∨ y) ∧ (z ∨ (x ∧ y)))) cnf(equation_H57, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT244-1.p Equation H58 is Huntington by implication

Show that H58 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ z) = x ∧ (y ∨ ((x ∨ y) ∧ (z ∨ (x ∧ y)))) cnf(equation_H58, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT245-1.p Equation H59 is Huntington by implication

Show that H59 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((y ∨ z) ∧ (y ∨ u)) = x ∧ (y ∨ ((y ∨ u) ∧ (z ∨ (x ∧ y)))) cnf(equation_H59, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT246-1.p Equation H60 is Huntington by implication

Show that H60 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((y ∨ z) ∧ (y ∨ u)) = x ∧ (y ∨ ((y ∨ z) ∧ (u ∨ (x ∧ y)))) cnf(equation_H60, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT247-1.p Equation H61 is Huntington by implication

Show that H61 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
(x ∨ y) ∧ (x ∨ z) = x ∨ ((x ∨ y) ∧ ((x ∧ y) ∨ z)) cnf(equation_H61, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT248-1.p Equation H63 is Huntington by implication

Show that H63 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((y ∨ z) ∧ (y ∨ u)) = x ∧ (y ∨ ((y ∨ z) ∧ (u ∨ (y ∧ z)))) cnf(equation_H63, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT249-1.p Equation H64 is Huntington by implication

Show that H64 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ z) = x ∧ (y ∨ (x ∧ (z ∨ (x ∧ (y ∨ (x ∧ z)))))) cnf(equation_H64, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT250-1.p Equation H68 is Huntington by implication

Show that H68 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
```

```

include('Axioms/LAT001-4.ax')
x ∧ (y ∨ z) = x ∧ (y ∨ (x ∧ (z ∨ (x ∧ y)))) cnf(equation_H68, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT251-1.p Equation H69 is Huntington by implication

Show that H69 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ z) = (x ∧ (z ∨ (x ∧ y))) ∨ (x ∧ (y ∨ (x ∧ z))) cnf(equation_H69, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT252-1.p Equation H70 is Huntington by implication

Show that H70 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (y ∨ u))) = x ∧ (y ∨ (z ∧ (u ∨ (y ∧ (x ∨ z))))) cnf(equation_H70, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT253-1.p Equation H76 is Huntington by implication

Show that H76 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (y ∨ u))) = x ∧ (y ∨ (z ∧ (u ∨ (x ∧ y)))) cnf(equation_H76, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT254-1.p Equation H79 is Huntington by implication

Show that H79 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ (y ∨ (z ∧ (x ∨ u))) = x ∧ ((x ∧ (y ∨ (x ∧ z))) ∨ (z ∧ u)) cnf(equation_H79, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT255-1.p Equation H80 is Huntington by implication

Show that H80 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((x ∧ y) ∨ (z ∧ (x ∨ (y ∧ (x ∨ z))))) = (x ∧ y) ∨ (x ∧ z) cnf(equation_H80, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT256-1.p Equation H81 is Huntington by implication

Show that H81 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```

include('Axioms/LAT001-0.ax')
include('Axioms/LAT001-4.ax')
x ∧ ((x ∧ y) ∨ (z ∧ (x ∨ (y ∧ (x ∨ z))))) = x ∧ ((x ∧ y) ∨ (x ∧ z)) cnf(equation_H81, axiom)
b ∧ a = a cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a' cnf(prove_distributivity, negated_conjecture)

```

LAT257-1.p Equation H82 is Huntington by implication

Show that H82 is Huntington by deriving the Huntington implication $X \wedge Y = Y \rightarrow X' \vee Y' = Y'$ in uniquely complemented lattices.

```
include('Axioms/LAT001-0.ax')
```

```

include('Axioms/LAT001-4.ax')
x ∧ ((y ∧ (x ∨ z)) ∨ (z ∧ (x ∨ y))) = (x ∧ y) ∨ (x ∧ z)      cnf(equation_H82, axiom)
b ∧ a = a      cnf(prove_distributivity_hypothesis, hypothesis)
b' ∨ a' ≠ a'      cnf(prove_distributivity, negated_conjecture)

```

LAT258+1.p A duality result on distributivity in lattices

```

x ∨ y=t and x ∨ z=u      fof(join_assumption, axiom)
t ∧ u=v      fof(meet_assumption, axiom)
y ∧ z=w and x ∨ w=p      fof(meet_join_assumption, axiom)
v < p ⇒ goal      fof(goal_ax, axiom)
∀a: a < a      fof(less_than_reflexive, axiom)
∀a, b, c: ((a < b and b < c) ⇒ a < c)      fof(less_than_transitive, axiom)
∀a, b, c: (a ∧ b=c ⇒ (c < a and c < b))      fof(lower_bound_meet, axiom)
∀a, b, c, d: ((a ∧ b=c and d < a and d < b) ⇒ d < c)      fof(greatest_lower_bound_meet, axiom)
∀a, b, c: (a ∨ b=c ⇒ (a < c and b < c))      fof(upper_bound_join, axiom)
∀a, b, c, d: ((a ∨ b=c and a < d and b < d) ⇒ c < d)      fof(least_upper_bound_join, axiom)
∀a, b: (a < b ⇒ (a ∧ b=a and a ∨ b=b))      fof(less_than_meet_join, axiom)
∀a, b, c: (a ∧ b=c ⇒ b ∧ a=c)      fof(commutititivity_meet, axiom)
∀a, b, c: (a ∨ b=c ⇒ b ∨ a=c)      fof(commutititivity_join, axiom)
∀a, b, c, d, e, f: ((a ∧ b=d and d ∧ c=e and b ∧ c=f) ⇒ a ∧ f=e)      fof(associativity_meet, axiom)
∀a, b, c, d, e, f: ((a ∨ b=d and d ∨ c=e and b ∨ c=f) ⇒ a ∨ f=e)      fof(associativity_join, axiom)
∀a, b, c, d, e, f, g, h: ((b ∨ c=h and a ∧ h=d and a ∧ b=e and a ∧ c=f and e ∨ f=g) ⇒ d < g)      fof(lo_le_distr, axiom)
∀a, b: ∃c: a ∧ b=c      fof(do_lattice, axiom)
goal      fof(goal_to_be_proved, conjecture)

```

LAT259-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
(c_Relation_Oantisym(v_r, t_a) and c_in(c_Pair(v_V, v_U, t_a, t_a), v_r, tc_prod(t_a, t_a)) and c_in(c_Pair(v_U, v_V, t_a, t_a), v_r,
v_U = v_V      cnf(cls_Relation_Oantisym_def0, axiom)
c_Relation_Oantisym(v_r, t_a) or c_in(c_Pair(c_Main_Oantisym_def_1(v_r, t_a), c_Main_Oantisym_def_2(v_r, t_a), t_a, t_a), v_r,
c_Relation_Oantisym(v_r, t_a) or c_in(c_Pair(c_Main_Oantisym_def_2(v_r, t_a), c_Main_Oantisym_def_1(v_r, t_a), t_a, t_a), v_r,
c_Main_Oantisym_def_1(v_r, t_a) = c_Main_Oantisym_def_2(v_r, t_a) ⇒ c_Relation_Oantisym(v_r, t_a)      cnf(cls_Relation_Oantisym_def1, axiom)
v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl0, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Orefl(c_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Oantisym(c_Tarski_Opotype_Oorder(v_P, t_a, tc_Product_Type_Ounit), t_a) and c_Relation_Orefl(c_Tarski_Opotype_Opotype_Oorder(v_P, t_a, tc_Product_Type_Ounit))
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
v_r = c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_Or_A_61_61_Aorder_Acl0, axiom)
c_in(c_Pair(v_a, v_b, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture0, negated_conjecture)
c_in(c_Pair(v_b, v_a, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture1, negated_conjecture)
v_a ≠ v_b      cnf(cls_conjecture2, negated_conjecture)

```

LAT259-2.p Problem about Tarski's fixed point theorem

```

c_in(c_Pair(v_a, v_b, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture0, negated_conjecture)
c_in(c_Pair(v_b, v_a, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture1, negated_conjecture)
v_a ≠ v_b      cnf(cls_conjecture2, negated_conjecture)
(c_Relation_Oantisym(v_r, t_a) and c_in(c_Pair(v_V, v_U, t_a, t_a), v_r, tc_prod(t_a, t_a)) and c_in(c_Pair(v_U, v_V, t_a, t_a), v_r,
v_U = v_V      cnf(cls_Relation_Oantisym_def0, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Oantisym(c_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
v_r = c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_Or_A_61_61_Aorder_Acl0, axiom)

```

LAT260-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl0, axiom)

```

LAT260-2.p Problem about Tarski's fixed point theorem

$\neg c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))$
 $c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))$

LAT261-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

v_A = c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl0, axiom)
(c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) and c_in(v_cl, c_
c_in(c_Tarski_Olub(v_S, v_cl, t_a), c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), t_a)      cnf(cls_Tarski_OCL_Ol
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarsk
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Oun
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_Tarski_Oglb(v_S, v_cl, t_a) = c_Tarski_Olub(v_S, c_Tarski_Odual(v_cl, t_a), t_a)      cnf(cls_Tarski_Oglb_dual_lub0, axiom)
c_Tarski_Optype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit) = c_Tarski_Optype_Opset(v_cl, t_a, tc_Prod
c_lessequals(v_S, c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_conjecture0, negated_conj
¬ c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ou

```

LAT261-2.p Problem about Tarski's fixed point theorem

$\neg c_{in}(c_{Tarski_Odual}(v_{cl}, t_a), c_{Tarski_OCompleteLattice}, tc_{Tarski_Opotype_Opotype_ext_type}(t_a, tc_{Product_Type_Out}))$
 $c_{in}(c_{Tarski_Odual}(v_{cl}, t_a), c_{Tarski_OCompleteLattice}, tc_{Tarski_Opotype_Opotype_ext_type}(t_a, tc_{Product_Type_Out}))$

LAT262-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
v_A = c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl0, axiom)
(c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) and c_in(v_cl, c_
c_in(c_Tarski_Olub(v_S, v_cl, t_a), c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), t_a)      cnf(cls_Tarski_OCL_Ol
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarsk
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ou
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_Tarski_Oglb(v_S, v_cl, t_a) = c_Tarski_Olub(v_S, c_Tarski_Odual(v_cl, t_a), t_a)      cnf(cls_Tarski_Oglb_dual_lub0, axiom)
c_Tarski_Optype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit) = c_Tarski_Optype_Opset(v_cl, t_a, tc_Prod
c_lessequals(v_S, c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_conjecture0, negated_conj
¬c_lessequals(v_S, c_Tarski_Optype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_co

```

LAT262-2.p Problem about Tarski's fixed point theorem

$c_{\text{lessequals}}(v_S, c_{\text{Tarski_Opotype_Opset}}(v_{\text{cl}}, t_a, tc_{\text{Product_Type_Ounit}}), tc_{\text{set}}(t_a)) \quad \text{cnf}(\text{cls_conjecture}_0, \text{negated_conj})$
 $\neg c_{\text{lessequals}}(v_S, c_{\text{Tarski_Opotype_Opset}}(c_{\text{Tarski_Odual}}(v_{\text{cl}}, t_a), t_a, tc_{\text{Product_Type_Ounit}}), tc_{\text{set}}(t_a)) \quad \text{cnf}(\text{cls_co})$

$$v_A = c\text{-}\text{Tarski}\text{-}\text{Opotype}\text{-}\text{Opset}(v\text{-}cl, t\text{-}a, tc\text{-}Product\text{-}Type\text{-}Ounit) \quad cnf(cls\text{-}\text{Tarski}\text{-}\text{OA}\text{-}A\text{-}61\text{-}61\text{-}\text{Apset}\text{-}\text{Acl}_0, axiom)$$

c_Tarski_Opotype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit) = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Prod

LAT263-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

v_A = c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl0, axiom)
(c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) and c_in(v_cl, c
c_in(c_Tarski_Olub(v_S, v_cl, t_a), c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), t_a)      cnf(cls_Tarski_OCL_Ol
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_O
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_O
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) => c_Relation_O
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_O
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_Tarski_Oglb(v_S, v_cl, t_a) = c_Tarski_Olub(v_S, c_Tarski_Odual(v_cl, t_a), t_a)      cnf(cls_Tarski_Oglb_dual_lub0, axiom)
c_Tarski_Optype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit) = c_Tarski_Optype_Opset(v_cl, t_a, tc_Pro
c_lessequals(v_S, v_A, tc_set(t_a))      cnf(cls_conjecture0, negated_conjecture)
¬ c_in(c_Tarski_Oglb(v_S, v_cl, t_a), v_A, t_a)      cnf(cls_conjecture1, negated_conjecture)

```

LAT263-2.p Problem about Tarski's fixed point theorem

```

c_lessequals(v_S, v_A, tc_set(t_a))      cnf(cls_conjecture0, negated_conjecture)
¬ c_in(c_Tarski_Oglb(v_S, v_cl, t_a), v_A, t_a)      cnf(cls_conjecture1, negated_conjecture)

v_A = c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl0, axiom)
(c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) and c_in(v_cl, c
c_in(c_Tarski_Olub(v_S, v_cl, t_a), c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), t_a)      cnf(cls_Tarski_OCL_Ol
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_Tarski_Oglb(v_S, v_cl, t_a) = c_Tarski_Olub(v_S, c_Tarski_Odual(v_cl, t_a), t_a)      cnf(cls_Tarski_Oglb_dual_lub0, axiom)
c_Tarski_Optype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit) = c_Tarski_Optype_Opset(v_cl, t_a, tc_Pro

```

LAT264-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-1.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

c_lessequals(v_S, c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_conjecture0, negated_conj
c_in(v_x, v_S, t_a)      cnf(cls_conjecture1, negated_conjecture)
¬ c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))

```

LAT264-2.p Problem about Tarski's fixed point theorem

```

¬ c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))

```

LAT265-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-1.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

c_lessequals(v_S, c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_conjecture0, negated_conj
c_in(v_x, v_S, t_a)      cnf(cls_conjecture1, negated_conjecture)
¬ c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))

```

LAT265-2.p Problem about Tarski's fixed point theorem

```

¬ c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Opotype_ext_type(t_a, tc_Product_Type_Ounit))

```

LAT266-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-1.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

c_lessequals(v_S, c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_conjecture0, negated_conj
c_in(v_x, v_S, t_a)      cnf(cls_conjecture1, negated_conjecture)
¬ c_lessequals(v_S, c_Tarski_Optype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_co

```

LAT266-2.p Problem about Tarski's fixed point theorem

```

c_lessequals(v_S, c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_conjecture_0, negated_conj)
¬c_lessequals(v_S, c_Tarski_Opotype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit), tc_set(t_a))    cnf(cls_conj_0, negated_conj)
v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl_0, axiom)
c_Tarski_Opotype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit) = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Prod_

```

LAT267-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-1.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSL001-1.ax')
include('Axioms/MSL001-0.ax')
c_lessequals(v_S, c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit), tc_set(t_a))      cnf(cls_conjecture_0, negated_conjecture)
c_in(v_x, v_S, t_a)      cnf(cls_conjecture_1, negated_conjecture)
¬c_in(v_x, v_S, t_a)     cnf(cls_conjecture_2, negated_conjecture)

```

LAT267-2.p Problem about Tarski's fixed point theorem

$c_in(v_x, v_S, t_a)$ cnf(cls_conjecture₁, negated_conjecture)
 $\neg c_in(v_x, v_S, t_a)$ cnf(cls_conjecture₂, negated_conjecture)

LAT268-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-1.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSL001-1.ax')
include('Axioms/MSL001-0.ax')
c_lessequals(v_S, v_A, tc_set(t_a))      cnf(cls_conjecture0, negated_conjecture)
c_in(v_x, v_S, t_a)          cnf(cls_conjecture1, negated_conjecture)
¬ c_in(c_Pair(c_Tarski_Oglb(v_S, v_cl, t_a), v_x, t_a, t_a), v_r, tc_prod(t_a, t_a))  cnf(cls_conjecture2, negated_conjecture)

```

LAT268-2.p Problem about Tarski's fixed point theorem

```

c_lessequals(v_S, v_A, tc_set(t_a))      cnf(cls_conjecture_0, negated_conjecture)
c_in(v_x, v_S, t_a)      cnf(cls_conjecture_1, negated_conjecture)
¬ c_in(c_Pair(c_Tarski_Oglb(v_S, v_cl, t_a), v_x, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_2, negated_conjecture)
v_A = c_Tarski_Optype_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl_0, axiom)
(c_in(v_x, v_S, t_a) and c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_Optype_Optype_ext_type(t_a, tc_Product_Type_Ounit)))
c_in(c_Pair(v_x, c_Tarski_Olub(v_S, v_cl, t_a), t_a, t_a), c_Tarski_Optype_Oorder(v_cl, t_a, tc_Product_Type_Ounit), tc_prod(t_a, t_a))
c_in(c_Pair(v_x, v_y, t_a, t_a), c_Tarski_Optype_Oorder(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit), tc_prod(t_a, t_a))
c_in(c_Pair(v_y, v_x, t_a, t_a), c_Tarski_Optype_Oorder(v_cl, t_a, tc_Product_Type_Ounit), tc_prod(t_a, t_a))      cnf(cls_Tarski_OA_A_61_61_Apset_Acl_0, axiom)
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OCompleteLattice, tc_Tarski_Optype_Optype_ext_type(t_a, tc_Product_Type_Ounit))
c_in(c_Tarski_Odual(v_cl, t_a), c_Tarski_OPartialOrder, tc_Tarski_Optype_Optype_ext_type(t_a, tc_Product_Type_Ounit))
c_Tarski_Oglb(v_S, v_cl, t_a) = c_Tarski_Olub(v_S, c_Tarski_Odual(v_cl, t_a), t_a)      cnf(cls_Tarski_Oglb_dual_lub_0, axiom)
c_Tarski_Optype_Opset(c_Tarski_Odual(v_cl, t_a), t_a, tc_Product_Type_Ounit) = c_Tarski_Optype_Opset(v_cl, t_a, tc_Prod)
v_r = c_Tarski_Optype_Oorder(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_Or_A_61_61_Aorder_Acl_0, axiom)

```

LAT269-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
c_in(c_Tarski_OTop(v_cl, t_a), v_A, t_a)      cnf(cls_Tarski_OTop_Acl_A_58_AA_A_61_61_ATrue0, axiom)
(c_in(v_b, v_A, t_a) and c_in(v_a, v_A, t_a)) ⇒ (c_in(c_Tarski_OTOTYPE_OTOTYPE_ext(c_Tarski_Ointerval(v_r, v_a, v_b, t_a), c_T
c_emptyset)      cnf(cls_Tarski_O_91_124_Aa2_A_58_AA_59_Ab2_A_58_AA_59_Ainterval_Ar_Aa2_Ab2_A_126_61_A_123_125_A_1
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_OTOTYPE_OTOTYPE_ext_type(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_OTOTYPE_OTOTYPE_ext_type(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_OTOTYPE_OTOTYPE_ext_type(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_C
c_in(v_cl, c_Tarski_OCompleteLattice, tc_Tarski_OTOTYPE_OTOTYPE_ext_type(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarsk
v_intY_1 = c_Tarski_Ointerval(v_r, c_Tarski_Olub(v_Y, v_cl, t_a), c_Tarski_OTop(v_cl, t_a), t_a)      cnf(cls_Tarski_OintY1_A_61_
c_in(c_Tarski_Olub(v_Y, v_cl, t_a), v_A, t_a)      cnf(cls_Tarski_Olub_AY_Acl_A_58_AA_A_61_61_ATrue0, axiom)
c_in(v_x, v_A, t_a) ⇒ c_Tarski_Ointerval(v_r, v_x, c_Tarski_OTop(v_cl, t_a), t_a) ≠ c_emptyset      cnf(cls_Tarski_Ox1_A_58_AA
¬c_in(c_Tarski_OTOTYPE_OTOTYPE_ext(v_intY_1, c_Tarski_Oinduced(v_intY_1, v_r, t_a), c_Product_Type_OUnity, t_a, tc_Produc

```

LAT269-2.p Problem about Tarski's fixed point theorem

$\neg c_in(c_Tarski_Opotype_Opotype_ext(v_intY_1, c_Tarski_Oinduced(v_intY_1, v_r, t_a), c_Product_Type_OUnity, t_a, tc_Product, c_in(c_Tarski_OTop(v_cl, t_a), v_A, t_a)) \quad cnf(cls_Tarski_OTop_Acl_A_58_AA_A_61_61_ATrue_0, axiom)$

LAT270-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-2.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
c_in(v_a, v_A, t_a)      cnf(cls_conjecture0, negated_conjecture)
c_in(v_b, v_A, t_a)      cnf(cls_conjecture1, negated_conjecture)
c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a))  cnf(cls_conjecture2, negated_conjecture)
v_S ≠ c_emptyset        cnf(cls_conjecture3, negated_conjecture)
c_Tarski_OisLub(v_S, v_cl, v_L, t_a)    cnf(cls_conjecture4, negated_conjecture)
c_Tarski_Ointerval(v_r, v_a, v_b, t_a) ≠ c_emptyset  cnf(cls_conjecture5, negated_conjecture)
¬ c_lessequals(v_S, v_A, tc_set(t_a))   cnf(cls_conjecture6, negated_conjecture)

```

LAT270-2.p Problem about Tarski's fixed point theorem

```

c_in(v_a, v_A, t_a)      cnf(cls_conjecture0, negated_conjecture)
c_in(v_b, v_A, t_a)      cnf(cls_conjecture1, negated_conjecture)
c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a))  cnf(cls_conjecture2, negated_conjecture)
¬c_lessequals(v_S, v_A, tc_set(t_a))  cnf(cls_conjecture6, negated_conjecture)
(c_in(v_b, v_A, t_a) and c_in(v_a, v_A, t_a) and c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a)))  ⇒
c_lessequals(v_S, v_A, tc_set(t_a))  cnf(cls_Tarski_O_91_124_Aa1_A_58_AA_59_Ab1_A_58_AA_59_AS1_A_60_61_Ainterval_Ar_

```

LAT271-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-2.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
c_in(v_a, v_A, t_a)      cnf(cls_conjecture_0, negated_conjecture)
c_in(v_b, v_A, t_a)      cnf(cls_conjecture_1, negated_conjecture)
c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a))      cnf(cls_conjecture_2, negated_conjecture)
v_S ≠ c_emptyset      cnf(cls_conjecture_3, negated_conjecture)
c_Tarski_OisLub(v_S, v_cl, v_L, t_a)      cnf(cls_conjecture_4, negated_conjecture)
c_Tarski_Ointerval(v_r, v_a, v_b, t_a) ≠ c_emptyset      cnf(cls_conjecture_5, negated_conjecture)
c_in(v_x, v_S, t_a)      cnf(cls_conjecture_6, negated_conjecture)
¬c_in(c_Pair(v_a, v_x, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_7, negated_conjecture)

```

LAT271-2.p Problem about Tarski's fixed point theorem

```

c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a))      cnf(cls_conjecture_2, negated_conjecture)
c_in(v_x, v_S, t_a)      cnf(cls_conjecture_6, negated_conjecture)
¬ c_in(c_Pair(v_a, v_x, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_7, negated_conjecture)
(c_in(v_x, v_S, t_a) and c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a))) ⇒ c_in(c_Pair(v_a, v_x, t_a, t_a), v_r)

```

LAT272-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-2.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
c_in(v_a, v_A, t_a)      cnf(cls_conjecture_0, negated_conjecture)
c_in(v_b, v_A, t_a)      cnf(cls_conjecture_1, negated_conjecture)
c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a))      cnf(cls_conjecture_2, negated_conjecture)
v_S ≠ c_emptyset      cnf(cls_conjecture_3, negated_conjecture)
c_Tarski_OisLub(v_S, v_cl, v_L, t_a)      cnf(cls_conjecture_4, negated_conjecture)
c_Tarski_Ointerval(v_r, v_a, v_b, t_a) ≠ c_emptyset      cnf(cls_conjecture_5, negated_conjecture)
c_in(v_x, v_S, t_a)      cnf(cls_conjecture_6, negated.conjecture)
¬ c_in(c_Pair(v_x, v_L, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_7, negated_conjecture)

```

LAT272-2.p Problem about Tarski's fixed point theorem

c_Tarski_OisLub(v_S, v_cl, v_L, t_a) cnf(cls_conjecture₄, negated_conjecture)

c_in(v_x, v_S, t_a) cnf(cls_conjecture_6, negated_conjecture)
 $\neg c_in(c_Pair(v_x, v_L, t_a, t_a), v_r, tc_prod(t_a, t_a))$ cnf(cls_conjecture_7, negated_conjecture)
 $(c_Tarski_OisLub(v_S, v_cl, v_L, t_a) \text{ and } c_in(v_y, v_S, t_a)) \Rightarrow c_in(c_Pair(v_y, v_L, t_a, t_a), v_r, tc_prod(t_a, t_a))$ cnf(cls_...)

LAT273-1.p Problem about Tarski's fixed point theorem

```
include('Axioms/LAT006-2.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

c_in(v_a, v_A, t_a) cnf(cls_conjecture_0, negated_conjecture)
c_in(v_b, v_A, t_a) cnf(cls_conjecture_1, negated_conjecture)
c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a)) cnf(cls_conjecture_2, negated_conjecture)
v_S ≠ c_emptyset cnf(cls_conjecture_3, negated_conjecture)
c_Tarski_OisLub(v_S, v_cl, v_L, t_a) cnf(cls_conjecture_4, negated_conjecture)
c_Tarski_Ointerval(v_r, v_a, v_b, t_a) ≠ c_emptyset cnf(cls_conjecture_5, negated_conjecture)
¬ c_in(c_Pair(v_L, v_b, t_a, t_a), v_r, tc_prod(t_a, t_a)) cnf(cls_conjecture_6, negated_conjecture)
```

LAT273-2.p Problem about Tarski's fixed point theorem

```
(c_in(v_x, v_S, t_a) and c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a))) ⇒ c_in(c_Pair(v_x, v_b, t_a, t_a), v_r, tc_prod(t_a, t_a))
(c_Tarski_OisLub(v_S, v_cl, v_L, t_a) and c_in(v_z, v_A, t_a)) ⇒ (c_in(c_Pair(v_L, v_z, t_a, t_a), v_r, tc_prod(t_a, t_a)) or c_in(v_sko_4mi(v_S, v_r, v_z), v_z, t_a, t_a), v_r, tc_prod(t_a, t_a))
(c_Tarski_OisLub(v_S, v_cl, v_L, t_a) and c_in(v_z, v_A, t_a) and c_in(c_Pair(v_sko_4mi(v_S, v_r, v_z), v_z, t_a, t_a), v_r, tc_prod(t_a, t_a)))
c_in(c_Pair(v_L, v_z, t_a, t_a), v_r, tc_prod(t_a, t_a)) cnf(cls_Tarski_O_91_124_AisLub_AS1_Acl_AL1_59_Az1_A_58_AA_59_AA)
c_in(v_b, v_A, t_a) cnf(cls_conjecture_1, negated_conjecture)
c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a)) cnf(cls_conjecture_2, negated_conjecture)
c_Tarski_OisLub(v_S, v_cl, v_L, t_a) cnf(cls_conjecture_4, negated_conjecture)
¬ c_in(c_Pair(v_L, v_b, t_a, t_a), v_r, tc_prod(t_a, t_a)) cnf(cls_conjecture_6, negated_conjecture)
```

LAT274-1.p Problem about Tarski's fixed point theorem

```
include('Axioms/LAT006-2.ax')
include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')

c_in(v_a, v_A, t_a) cnf(cls_conjecture_0, negated_conjecture)
c_in(v_b, v_A, t_a) cnf(cls_conjecture_1, negated_conjecture)
c_lessequals(v_S, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), tc_set(t_a)) cnf(cls_conjecture_2, negated_conjecture)
v_S ≠ c_emptyset cnf(cls_conjecture_3, negated_conjecture)
c_Tarski_OisLub(v_S, v_cl, v_L, t_a) cnf(cls_conjecture_4, negated_conjecture)
c_Tarski_Ointerval(v_r, v_a, v_b, t_a) ≠ c_emptyset cnf(cls_conjecture_5, negated_conjecture)
¬ c_in(v_L, c_Tarski_Ointerval(v_r, v_a, v_b, t_a), t_a) cnf(cls_conjecture_6, negated_conjecture)
```

LAT275-2.p Problem about Tarski's fixed point theorem

```
c_lessequals(v_S, v_A, tc_set(t_a)) cnf(cls_conjecture_0, negated_conjecture)
¬ c_in(c_Tarski_Olub(v_S, v_cl, t_a), c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit), t_a) cnf(cls_conjecture_2, negated_conjecture)
v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_OA_A_61_61_Apset_Acl_0, axiom)
c_lessequals(v_S, v_A, tc_set(t_a)) ⇒ c_in(c_Tarski_Olub(v_S, v_cl, t_a), v_A, t_a) cnf(cls_Tarski_OS_A_60_61_AA_A_61_61_Apset_Acl_0, axiom)
```

LAT276-2.p Problem about Tarski's fixed point theorem

```
c_lessequals(v_S, v_A, tc_set(t_a)) cnf(cls.conjecture_0, negated_conjecture)
(c_in(v_V, v_S, t_a) and c_in(c_Pair(v_xa, c_Tarski_Olub(v_S, v_cl, t_a), t_a, t_a), c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit), t_a)) cnf(cls.conjecture_1, negated_conjecture)
c_in(c_Pair(v_V, v_xa, t_a, t_a), c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit), tc_prod(t_a, t_a)) cnf(cls.conjecture_2, negated_conjecture)
c_in(v_xa, c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit), t_a) or c_in(v_xa, v_S, t_a) cnf(cls.conjecture_3, negated_conjecture)
c_in(c_Pair(c_Tarski_Olub(v_S, v_cl, t_a), v_xa, t_a, t_a), c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit), tc_prod(t_a, t_a)) cnf(cls.conjecture_4, negated_conjecture)
c_in(c_Pair(c_Tarski_Olub(v_S, v_cl, t_a), v_xa, t_a, t_a), c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit), tc_prod(t_a, t_a)) cnf(cls.conjecture_5, negated_conjecture)
¬ c_in(c_Pair(v_xa, c_Tarski_Olub(v_S, v_cl, t_a), t_a, t_a), c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit), tc_prod(t_a, t_a)) cnf(cls.conjecture_6, negated_conjecture)
c_in(v_U, v_S, t_a) ⇒ (c_in(c_Pair(v_U, v_xa, t_a, t_a), c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit), tc_prod(t_a, t_a)) and c_in(v_c, v_A, t_a)) cnf(cls_Set_OsubsetD_0, axiom)
(c_in(v_c, v_A, t_a) and c_lessequals(v_A, v_B, tc_set(t_a))) ⇒ c_in(v_c, v_B, t_a) cnf(cls_Set_OsubsetD_0, axiom)
v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_OA_A_61_61_Apset_Acl_0, axiom)
(c_in(v_L, v_A, t_a) and c_lessequals(v_S, v_A, tc_set(t_a))) ⇒ (c_in(c_Pair(c_Tarski_Olub(v_S, v_cl, t_a), v_L, t_a, t_a), v_r, tc_prod(t_a, t_a)) and c_in(c_Pair(v_sko_4mp(v_L, v_S, v_r), v_L, t_a, t_a), v_r, tc_prod(t_a, t_a))) and c_lessequals(v_S, v_A, tc_set(t_a))
c_in(c_Pair(c_Tarski_Olub(v_S, v_cl, t_a), v_L, t_a, t_a), v_r, tc_prod(t_a, t_a)) cnf(cls_Tarski_O_91_124_AS1_A_60_61_AA_59_AA)
(c_in(v_x, v_S, t_a) and c_lessequals(v_S, v_A, tc_set(t_a))) ⇒ c_in(c_Pair(v_x, c_Tarski_Olub(v_S, v_cl, t_a), t_a, t_a), v_r, tc_prod(t_a, t_a)) cnf(cls.Tarski_O_91_124_AS1_A_60_61_AA_59_AA)
```

$v_r = c_{\text{Tarski_Opotype_Oorder}}(v_{\text{cl}}, t_a, tc_{\text{Product_Type_Ounit}})$ cnf(cls_Tarski_Or_A_61_61_Aorder_Acl₀, axiom)

LAT277-1.p Problem about Tarski's fixed point theorem

include('Axioms/LAT006-0.ax')

include('Axioms/MSC001-1.ax')

include('Axioms/MSC001-0.ax')

(c_Relation_Orefl(v_A, v_r, t_a) and c_in(v_a, v_A, t_a)) \Rightarrow c_in(c_Pair(v_a, v_a, t_a, t_a), v_r, tc_prod(t_a, t_a)) cnf(cls_Relat
v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_OA_A_61_61_Apset_Acl₀, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Orefl
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Oanti
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Otrans
(c_Relation_Oantisym(c_Tarski_Opotype_Oorder(v_P, t_a, tc_Product_Type_Ounit), t_a) and c_Relation_Orefl(c_Tarski_Opote
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) cnf(cls_Tarski_O
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) cnf(cls_Tarski_O
v_r = c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_Or_A_61_61_Aorder_Acl₀, axiom)
c_in(v_x, v_A, t_a) cnf(cls_conjecture₀, negated_conjecture)
 $\neg c_{\text{in}}(c_{\text{Pair}}(v_x, v_x, t_a, t_a), v_r, tc_{\text{prod}}(t_a, t_a))$ cnf(cls_conjecture₁, negated_conjecture)

LAT277-2.p Problem about Tarski's fixed point theorem

c_in(v_x, v_A, t_a) cnf(cls_conjecture₀, negated_conjecture)

$\neg c_{\text{in}}(c_{\text{Pair}}(v_x, v_x, t_a, t_a), v_r, tc_{\text{prod}}(t_a, t_a))$ cnf(cls_conjecture₁, negated_conjecture)

(c_Relation_Orefl(v_A, v_r, t_a) and c_in(v_a, v_A, t_a)) \Rightarrow c_in(c_Pair(v_a, v_a, t_a, t_a), v_r, tc_prod(t_a, t_a)) cnf(cls_Relat
v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_OA_A_61_61_Apset_Acl₀, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Orefl
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Otrans
v_r = c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_Or_A_61_61_Aorder_Acl₀, axiom)

LAT278-1.p Problem about Tarski's fixed point theorem

include('Axioms/LAT006-0.ax')

include('Axioms/MSC001-1.ax')

include('Axioms/MSC001-0.ax')

v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_OA_A_61_61_Apset_Acl₀, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Orefl
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Oanti
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Otrans
(c_Relation_Oantisym(c_Tarski_Opotype_Oorder(v_P, t_a, tc_Product_Type_Ounit), t_a) and c_Relation_Orefl(c_Tarski_Opote
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) cnf(cls_Tarski_O
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) cnf(cls_Tarski_O
v_r = c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_Or_A_61_61_Aorder_Acl₀, axiom)
 $\neg c_{\text{Relation_Orefl}}(v_A, v_r, t_a)$ cnf(cls_conjecture₀, negated_conjecture)

LAT278-2.p Problem about Tarski's fixed point theorem

$\neg c_{\text{Relation_Orefl}}(v_A, v_r, t_a)$ cnf(cls_conjecture₀, negated_conjecture)

v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_OA_A_61_61_Apset_Acl₀, axiom)

c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Orefl
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Otrans
v_r = c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_Or_A_61_61_Aorder_Acl₀, axiom)

LAT279-1.p Problem about Tarski's fixed point theorem

include('Axioms/LAT006-0.ax')

include('Axioms/MSC001-1.ax')

include('Axioms/MSC001-0.ax')

v_A = c_Tarski_Opotype_Opset(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_OA_A_61_61_Apset_Acl₀, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Orefl
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Oanti
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) \Rightarrow c_Relation_Otrans
(c_Relation_Oantisym(c_Tarski_Opotype_Oorder(v_P, t_a, tc_Product_Type_Ounit), t_a) and c_Relation_Orefl(c_Tarski_Opote
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) cnf(cls_Tarski_O
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_Opotype_Opotype_ext_type(t_a, tc_Product_Type_Ounit)) cnf(cls_Tarski_O
v_r = c_Tarski_Opotype_Oorder(v_cl, t_a, tc_Product_Type_Ounit) cnf(cls_Tarski_Or_A_61_61_Aorder_Acl₀, axiom)
 $\neg c_{\text{Relation_Oantisym}}(v_r, t_a)$ cnf(cls_conjecture₀, negated_conjecture)

LAT279-2.p Problem about Tarski's fixed point theorem

```

¬c_Relation_Oantisym(v_r, t_a)      cnf(cls_conjecture_0, negated_conjecture)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Oanti
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_O
v_r = c_Tarski_OTOTYPE_Order(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_Or_A_61_61_Aorder_Acl0, axiom)

```

LAT280-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
v_A = c_Tarski_OTOTYPE_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl0, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Orefl
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Oanti
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Otrans
(c_Relation_Oantisym(c_Tarski_OTOTYPE_Order(v_P, t_a, tc_Product_Type_Ounit), t_a) and c_Relation_Orefl(c_Tarski_Opote
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_O
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_O
v_r = c_Tarski_OTOTYPE_Order(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_Or_A_61_61_Aorder_Acl0, axiom)
¬c_Relation_Otrans(v_r, t_a)      cnf(cls_conjecture_0, negated_conjecture)

```

LAT280-2.p Problem about Tarski's fixed point theorem

```

¬c_Relation_Otrans(v_r, t_a)      cnf(cls_conjecture_0, negated_conjecture)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Otrans
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_O
v_r = c_Tarski_OTOTYPE_Order(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_Or_A_61_61_Aorder_Acl0, axiom)

```

LAT281-1.p Problem about Tarski's fixed point theorem

```

include('Axioms/LAT006-0.ax')
include('Axioms/MSC001-1.ax')
include('Axioms/MSC001-0.ax')
(c_Relation_Otrans(v_r, t_a) and c_in(c_Pair(v_V, v_W, t_a, t_a), v_r, tc_prod(t_a, t_a)) and c_in(c_Pair(v_U, v_V, t_a, t_a), v_r, tc
c_in(c_Pair(v_U, v_W, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_Relation_Otrans_def0, axiom)
c_Relation_Otrans(v_r, t_a) or c_in(c_Pair(c_Main_Otrans_def_1(v_r, t_a), c_Main_Otrans_def_2(v_r, t_a), t_a, t_a), v_r, tc_prod(
c_Relation_Otrans(v_r, t_a) or c_in(c_Pair(c_Main_Otrans_def_2(v_r, t_a), c_Main_Otrans_def_3(v_r, t_a), t_a, t_a), v_r, tc_prod(
c_in(c_Pair(c_Main_Otrans_def_1(v_r, t_a), c_Main_Otrans_def_3(v_r, t_a), t_a, t_a), v_r, tc_prod(t_a, t_a)) ⇒ c_Relation_Otrans
v_A = c_Tarski_OTOTYPE_Opset(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_OA_A_61_61_Apset_Acl0, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Orefl
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Oanti
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Otrans
(c_Relation_Oantisym(c_Tarski_OTOTYPE_Order(v_P, t_a, tc_Product_Type_Ounit), t_a) and c_Relation_Orefl(c_Tarski_Opote
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_O
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_O
v_r = c_Tarski_OTOTYPE_Order(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_Or_A_61_61_Aorder_Acl0, axiom)
c_in(c_Pair(v_a, v_b, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_0, negated_conjecture)
c_in(c_Pair(v_b, v_c, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_1, negated_conjecture)
¬c_in(c_Pair(v_a, v_c, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_2, negated_conjecture)

```

LAT281-2.p Problem about Tarski's fixed point theorem

```

c_in(c_Pair(v_a, v_b, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_0, negated_conjecture)
c_in(c_Pair(v_b, v_c, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_1, negated_conjecture)
¬c_in(c_Pair(v_a, v_c, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_conjecture_2, negated_conjecture)
(c_Relation_Otrans(v_r, t_a) and c_in(c_Pair(v_V, v_W, t_a, t_a), v_r, tc_prod(t_a, t_a)) and c_in(c_Pair(v_U, v_V, t_a, t_a), v_r, tc
c_in(c_Pair(v_U, v_W, t_a, t_a), v_r, tc_prod(t_a, t_a))      cnf(cls_Relation_Otrans_def0, axiom)
c_in(v_P, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit)) ⇒ c_Relation_Otrans
c_in(v_cl, c_Tarski_OPartialOrder, tc_Tarski_OTOTYPE_OTOTYPE_EXT_TYPE(t_a, tc_Product_Type_Ounit))      cnf(cls_Tarski_O
v_r = c_Tarski_OTOTYPE_Order(v_cl, t_a, tc_Product_Type_Ounit)      cnf(cls_Tarski_Or_A_61_61_Aorder_Acl0, axiom)

```

LAT381+1.p Tarski-Knaster fixed point theorem 01, 00 expansion

```

∀w0: (aSet0(w0) ⇒ $true)      fof(mSetSort, axiom)
∀w0: (aElement0(w0) ⇒ $true)      fof(mElmSort, axiom)
∀w0: (aSet0(w0) ⇒ ∀w1: (aElementOf0(w1, w0) ⇒ aElement0(w1)))      fof(mEOfElem, axiom)
∀w0: (aSet0(w0) ⇒ (isEmpty0(w0) ⇔ ¬∃w1: aElementOf0(w1, w0)))      fof(mDefEmpty, definition)
∀w0: (aSet0(w0) ⇒ ∀w1: (aSubsetOf0(w1, w0) ⇔ (aSet0(w1) and ∀w2: (aElementOf0(w2, w1) ⇒ aElementOf0(w2, w0)))))      fof(mDefSubset, definition)

```

$\forall w_0, w_1: ((\text{aElement}_0(w_0) \text{ and } \text{aElement}_0(w_1)) \Rightarrow (\text{sdtlseqdt}_0(w_0, w_1) \Rightarrow \$\text{true})) \quad \text{fof(mLessRel, axiom)}$
 $\forall w_0: (\text{aElement}_0(w_0) \Rightarrow \text{sdtlseqdt}_0(w_0, w_0)) \quad \text{fof(mARefl, axiom)}$
 $\forall w_0, w_1: ((\text{aElement}_0(w_0) \text{ and } \text{aElement}_0(w_1)) \Rightarrow ((\text{sdtlseqdt}_0(w_0, w_1) \text{ and } \text{sdtlseqdt}_0(w_1, w_0)) \Rightarrow w_0 = w_1)) \quad \text{fof(mASy}$
 $\forall w_0, w_1, w_2: ((\text{aElement}_0(w_0) \text{ and } \text{aElement}_0(w_1) \text{ and } \text{aElement}_0(w_2)) \Rightarrow ((\text{sdtlseqdt}_0(w_0, w_1) \text{ and } \text{sdtlseqdt}_0(w_1, w_2)) \Rightarrow \text{sdtlseqdt}_0(w_0, w_2))) \quad \text{fof(mTrans, axiom)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2: (\text{aLowerBoundOfIn}_0(w_2, w_1, w_0) \iff (\text{aElementOf}_0(w_2, w_0) \text{ and } \forall w_3: \text{sdtlseqdt}_0(w_2, w_3)))))) \quad \text{fof(mDefLB, definition)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2: (\text{aUpperBoundOfIn}_0(w_2, w_1, w_0) \iff (\text{aElementOf}_0(w_2, w_0) \text{ and } \forall w_3: \text{sdtlseqdt}_0(w_3, w_2)))))) \quad \text{fof(mDefUB, definition)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2: (\text{aInfimumOfIn}_0(w_2, w_1, w_0) \iff (\text{aElementOf}_0(w_2, w_0) \text{ and } \text{aLowerBoundOfIn}_0(w_3, w_2)))))) \quad \text{fof(mDefInf, definition)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2: (\text{aSupremumOfIn}_0(w_2, w_1, w_0) \iff (\text{aElementOf}_0(w_2, w_0) \text{ and } \text{aUpperBoundOfIn}_0(w_3, w_2)))))) \quad \text{fof(mDefSup, definition)}$
 $\text{aSet}_0(\text{xT}) \quad \text{fof(m_725, hypothesis)}$
 $\text{aSubsetOf}_0(\text{xS}, \text{xT}) \quad \text{fof(m_72501, hypothesis)}$
 $\text{aSupremumOfIn}_0(\text{xu}, \text{xS}, \text{xT}) \text{ and } \text{aSupremumOfIn}_0(\text{xv}, \text{xS}, \text{xT}) \quad \text{fof(m_744, hypothesis)}$
 $\text{xu} = \text{xv} \quad \text{fof(m__, conjecture)}$

LAT382+1.p Tarski-Knaster fixed point theorem 02, 00 expansion

$\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \$\text{true}) \quad \text{fof(mSetSort, axiom)}$
 $\forall w_0: (\text{aElement}_0(w_0) \Rightarrow \$\text{true}) \quad \text{fof(mElmSort, axiom)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aElementOf}_0(w_1, w_0) \Rightarrow \text{aElement}_0(w_1))) \quad \text{fof(mEOfElem, axiom)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow (\text{isEmpty}_0(w_0) \iff \neg \exists w_1: \text{aElementOf}_0(w_1, w_0))) \quad \text{fof(mDefEmpty, definition)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \iff (\text{aSet}_0(w_1) \text{ and } \forall w_2: (\text{aElementOf}_0(w_2, w_1) \Rightarrow \text{aElementOf}_0(w_2, w_0)))))) \quad \text{fof(mDefSubset, definition)}$
 $\forall w_0, w_1: ((\text{aElement}_0(w_0) \text{ and } \text{aElement}_0(w_1)) \Rightarrow (\text{sdtlseqdt}_0(w_0, w_1) \Rightarrow \$\text{true})) \quad \text{fof(mLessRel, axiom)}$
 $\forall w_0: (\text{aElement}_0(w_0) \Rightarrow \text{sdtlseqdt}_0(w_0, w_0)) \quad \text{fof(mARefl, axiom)}$
 $\forall w_0, w_1: ((\text{aElement}_0(w_0) \text{ and } \text{aElement}_0(w_1)) \Rightarrow ((\text{sdtlseqdt}_0(w_0, w_1) \text{ and } \text{sdtlseqdt}_0(w_1, w_0)) \Rightarrow w_0 = w_1)) \quad \text{fof(mASy}$
 $\forall w_0, w_1, w_2: ((\text{aElement}_0(w_0) \text{ and } \text{aElement}_0(w_1) \text{ and } \text{aElement}_0(w_2)) \Rightarrow ((\text{sdtlseqdt}_0(w_0, w_1) \text{ and } \text{sdtlseqdt}_0(w_1, w_2)) \Rightarrow \text{sdtlseqdt}_0(w_0, w_2))) \quad \text{fof(mTrans, axiom)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2: (\text{aLowerBoundOfIn}_0(w_2, w_1, w_0) \iff (\text{aElementOf}_0(w_2, w_0) \text{ and } \forall w_3: \text{sdtlseqdt}_0(w_2, w_3)))))) \quad \text{fof(mDefLB, definition)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2: (\text{aUpperBoundOfIn}_0(w_2, w_1, w_0) \iff (\text{aElementOf}_0(w_2, w_0) \text{ and } \forall w_3: \text{sdtlseqdt}_0(w_3, w_2)))))) \quad \text{fof(mDefUB, definition)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2: (\text{aInfimumOfIn}_0(w_2, w_1, w_0) \iff (\text{aElementOf}_0(w_2, w_0) \text{ and } \text{aLowerBoundOfIn}_0(w_3, w_2)))))) \quad \text{fof(mDefInf, definition)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2: (\text{aSupremumOfIn}_0(w_2, w_1, w_0) \iff (\text{aElementOf}_0(w_2, w_0) \text{ and } \text{aUpperBoundOfIn}_0(w_3, w_2)))))) \quad \text{fof(mDefSup, definition)}$
 $\forall w_0: (\text{aSet}_0(w_0) \Rightarrow \forall w_1: (\text{aSubsetOf}_0(w_1, w_0) \Rightarrow \forall w_2, w_3: ((\text{aSupremumOfIn}_0(w_2, w_1, w_0) \text{ and } \text{aSupremumOfIn}_0(w_3, w_1, w_0)) \text{ and } w_2 = w_3))) \quad \text{fof(mSupUn, axiom)}$
 $\text{aSet}_0(\text{xT}) \quad \text{fof(m_773, hypothesis)}$
 $\text{aSubsetOf}_0(\text{xS}, \text{xT}) \quad \text{fof(m_77301, hypothesis)}$
 $\text{aInfimumOfIn}_0(\text{xu}, \text{xS}, \text{xT}) \text{ and } \text{aInfimumOfIn}_0(\text{xv}, \text{xS}, \text{xT}) \quad \text{fof(m_792, hypothesis)}$
 $\text{xu} = \text{xv} \quad \text{fof(m__, conjecture)}$

LAT389-1.p Short axiom for lattice theory, part 1

$((a + b) \cdot (b + a \cdot b) + c) \cdot ((a + ((d + b) \cdot (b + e)) \cdot b) \cdot ((b + ((d \cdot (b \cdot e) + f \cdot b) + b)) \cdot (v_6 + b \cdot ((b \cdot v_7 + f \cdot b) + b)) + a \cdot (((d + b) \cdot (b + e)) \cdot b)) + ((a + b) \cdot (b + a \cdot b)) \cdot c) = b \quad \text{cnf(sos, axiom)}$
 $a \cdot (b + (a + c)) \neq a \quad \text{cnf(goals, negated_conjecture)}$

LAT390-1.p Short axiom for lattice theory, part 2

$((a + b) \cdot (b + a \cdot b) + c) \cdot ((a + ((d + b) \cdot (b + e)) \cdot b) \cdot ((b + ((d \cdot (b \cdot e) + f \cdot b) + b)) \cdot (v_6 + b \cdot ((b \cdot v_7 + f \cdot b) + b)) + a \cdot (((d + b) \cdot (b + e)) \cdot b)) + ((a + b) \cdot (b + a \cdot b)) \cdot c) = b \quad \text{cnf(sos, axiom)}$
 $a + b \cdot (a \cdot c) \neq a \quad \text{cnf(goals, negated_conjecture)}$

LAT391-1.p Short axiom for lattice theory, part 3

$((a + b) \cdot (b + a \cdot b) + c) \cdot ((a + ((d + b) \cdot (b + e)) \cdot b) \cdot ((b + ((d \cdot (b \cdot e) + f \cdot b) + b)) \cdot (v_6 + b \cdot ((b \cdot v_7 + f \cdot b) + b)) + a \cdot (((d + b) \cdot (b + e)) \cdot b)) + ((a + b) \cdot (b + a \cdot b)) \cdot c) = b \quad \text{cnf(sos, axiom)}$
 $((a + b) \cdot (b + c)) \cdot b \neq b \quad \text{cnf(goals, negated_conjecture)}$

LAT392-1.p Short axiom for lattice theory, part 4

$((a + b) \cdot (b + a \cdot b) + c) \cdot ((a + ((d + b) \cdot (b + e)) \cdot b) \cdot ((b + ((d \cdot (b \cdot e) + f \cdot b) + b)) \cdot (v_6 + b \cdot ((b \cdot v_7 + f \cdot b) + b)) + a \cdot (((d + b) \cdot (b + e)) \cdot b)) + ((a + b) \cdot (b + a \cdot b)) \cdot c) = b \quad \text{cnf(sos, axiom)}$

$(a \cdot b + b \cdot c) + b \neq b$ cnf(goals, negated_conjecture)

LAT393-1.p Ortholattices in terms of Sheffer stroke + ops: associativity

$f(f(f(f(a, b), f(b, c)), d), f(b, f(f(b, f(f(a, a), a)), c))) = b$ cnf(sos, axiom)

$\text{or}(a, b) = f(f(a, a), f(b, b))$ cnf(sos₀₀₁, axiom)

$\text{and}(a, b) = f(f(a, b), f(a, b))$ cnf(sos₀₀₂, axiom)

$\text{neg}(a) = f(a, a)$ cnf(sos₀₀₃, axiom)

$f(x_0, f(f(x_1, x_2), f(x_1, x_2))) \neq f(x_1, f(f(x_0, x_2), f(x_0, x_2)))$ cnf(goals, negated_conjecture)

LAT393-2.p Ortholattices in terms of Sheffer stroke: associativity

$f(f(f(f(a, b), f(b, c)), d), f(b, f(f(b, f(f(a, a), a)), c))) = b$ cnf(sos, axiom)

$f(x_0, f(f(x_1, x_2), f(x_1, x_2))) \neq f(x_1, f(f(x_0, x_2), f(x_0, x_2)))$ cnf(goals, negated_conjecture)

LAT394-1.p Ortholattices in terms of Sheffer stroke + usual operations: unit

$f(f(f(f(a, b), f(b, c)), d), f(b, f(f(b, f(f(a, a), a)), c))) = b$ cnf(sos, axiom)

$\text{or}(a, b) = f(f(a, a), f(b, b))$ cnf(sos₀₀₁, axiom)

$\text{and}(a, b) = f(f(a, b), f(a, b))$ cnf(sos₀₀₂, axiom)

$\text{neg}(a) = f(a, a)$ cnf(sos₀₀₃, axiom)

$f(x_0, f(x_0, x_0)) \neq f(x_1, f(x_1, x_1))$ cnf(goals, negated_conjecture)

LAT394-2.p Ortholattices in terms of Sheffer stroke: unit

$f(f(f(f(a, b), f(b, c)), d), f(b, f(f(b, f(f(a, a), a)), c))) = b$ cnf(sos, axiom)

$f(x_0, f(x_0, x_0)) \neq f(x_1, f(x_1, x_1))$ cnf(goals, negated_conjecture)

LAT395-1.p Lattice theory (equality) axioms

include('Axioms/LAT001-0.ax')

include('Axioms/LAT001-1.ax')

include('Axioms/LAT001-2.ax')

include('Axioms/LAT001-3.ax')

LAT396-1.p Lattice theory (equality) axioms

include('Axioms/LAT001-0.ax')

include('Axioms/LAT001-4.ax')