MSC axioms
MSC problems

MSC001-1.p A Blind Hand Problem
at(a, there, b) ⇒ ¬ at(a, here, b) cnf(clause_1, axiom)
¬ hold(thing(a), do(let_go, b)) cnf(clause_2, axiom)
¬ red(hand) cnf(clause_3, axiom)
at(hand, a, do(go(a), b)) cnf(clause_5, axiom)
at(thing(s), here, now) cnf(clause_6, axiom)
at(thing(a), b, do(go(b), c)) ⇒ (at(thing(a), b, c) or hold(thing(a), c)) cnf(clause_7, axiom)
(at(hand, a, b) and at(thing(c), a, b)) ⇒ hold(thing(taken(b)), do(pick_up, b)) cnf(clause_8, axiom)
(hold(thing(a), b) and at(hand, c, b)) ⇒ at(thing(a), c, b) cnf(clause_9, axiom)
(hold(thing(a), b) and at(thing(a), c, b)) ⇒ at(hand, c, b) cnf(clause_{10}, axiom)
(red(a) and at(a, there, b)) ⇒ answer(b) cnf(clause_{11}, axiom)
at(thing(a), b, c) ⇒ at(thing(a), b, do(go(b), c)) cnf(clause_{12}, axiom)
hold(thing(a), b) ⇒ hold(thing(a), do(go(c), b)) cnf(clause_{13}, axiom)
at(hand, a, b) ⇒ at(thing(taken(b)), a, b) cnf(clause_{14}, axiom)
at(a, b, c) ⇒ at(a, b, do(pick_up, c)) cnf(clause_{15}, axiom)
at(a, b, c) ⇒ at(a, b, do(let_go, c)) cnf(clause_{16}, axiom)
at(a, b, do(let_go, c)) ⇒ at(a, b, c) cnf(clause_{17}, axiom)
at(a, here, now) ⇒ red(a) cnf(things_here_now_are_red, hypothesis)
¬ answer(a) cnf(prove_there_is_a_red_thing, negated_conjecture)

MSC002-1.p A Blind Hand Problem
at(something, here, now) cnf(something_is_here_now, axiom)
hand_at(place, situation) ⇒ hand_at(place, let_go(situation)) cnf(hand_let_go, axiom)
hand_at(place, situation) ⇒ hand_at(another_place, go(another_place, situation)) cnf(hand_go, axiom)
¬ held(thing, let_go(situation)) cnf(cant_hold_and_let_go, axiom)
at(thing, here, situation) ⇒ red(thing) cnf(everything_is_red, axiom)
at(thing, place, situation) ⇒ at(thing, place, let_go(situation)) cnf(situation_let_go, axiom)
at(thing, place, situation) ⇒ at(thing, place, pick_up(situation)) cnf(situation_pick_up, axiom)
at(thing, place, situation) ⇒ grabbed(thing, pick_up(go(place, let_go(situation)))) cnf(can_grab_if_previously_let_go, axiom
(red(thing) and put(thing, there, situation)) ⇒ answer(situation) cnf(answer_if_red_and_put_there, axiom)
(at(thing, place, situation) and grabbed(thing, situation)) ⇒ put(thing, another_place, go(another_place, situation)) cnf(c
at(thing, place, situation) ⇒ (held(thing, situation) or at(thing, place, go(another_place, situation))) cnf(thing_either_held
(hand_at(one_place, situation) and held(thing, situation)) ⇒ at(thing, place, go(place, situation)) cnf(thing_goes_in_hand,
(hand_at(place, situation) and at(thing, place, situation)) ⇒ held(thing, pick_up(situation)) cnf(thing_picked_up_by_hand
¬ answer(situation) cnf(prove_there_is_an_answer_situation, negated_conjecture)

MSC002-2.p A Blind Hand Problem
at(something, here, now) cnf(something_is_here_now, axiom)
¬ held(thing, let_go(situation)) cnf(cant_hold_and_let_go, axiom)
at(thing, here, situation) ⇒ red(thing) cnf(everything_is_red, axiom)
at(thing, place, situation) ⇒ at(thing, place, let_go(situation)) cnf(situation_let_go, axiom)
at(thing, place, situation) ⇒ at(thing, place, pick_up(situation)) cnf(situation_pick_up, axiom)
at(thing, place, situation) ⇒ grabbed(thing, pick_up(go(place, let_go(situation)))) cnf(can_grab_if_previously_let_go, axiom
(red(thing) and put(thing, there, situation)) ⇒ answer(situation) cnf(answer_if_red_and_put_there, axiom)
(at(thing, place, situation) and grabbed(thing, situation)) ⇒ put(thing, another_place, go(another_place, situation)) cnf(c
at(thing, place, situation) ⇒ (held(thing, situation) or at(thing, place, go(another_place, situation))) cnf(thing_either_held
¬ answer(situation) cnf(prove_there_is_an_answer_situation, negated_conjecture)

MSC003-1.p Show that the boy, John, has 2 hands
has_parts(big_part, number_of_mid_parts, mid_part) ⇒ (in(object_in(big_part, mid_part, small_part, number_of_mid_parts, nur
(has_parts(big_part, number_of_mid_parts, mid_part) and has_parts(object_in(big_part, mid_part, small_part, number_of_mid_pa
has_parts(big_part, times(number_of_mid_parts, number_of_small_parts), small_part) cnf(part_inheritance, axiom)
in(john, boy) cnf(john_is_a_boy, hypothesis)
in(x, boy) ⇒ in(x, human) cnf(in_boy_in_human, hypothesis)
in(x, hand) ⇒ has_parts(x, n_5, fingers) cnf(hands_have_5_fingers, hypothesis)
in(x, human) ⇒ has_parts(x, n_2, arm) cnf(humans_have_two_arms, hypothesis)

in(x, arm) $\Rightarrow$ has_parts(x, n_1, hand)   cnf(arms_have_one_hand, hypothesis)
$\neg$ has_parts(john, times(n_2, n_1), hand)   cnf(prove_john_has_2_hands, negated_conjecture)

MSC004-1.p Show that the boy, John, has 10 fingers
has_parts(big_part, number_of_mid_parts, mid_part) $\Rightarrow$ (in(object_in(big_part, mid_part, small_part, number_of_mid_parts, nur
(has_parts(big_part, number_of_mid_parts, mid_part) and has_parts(object_in(big_part, mid_part, small_part, number_of_mid_pa
has_parts(big_part, times(number_of_mid_parts, number_of_small_parts), small_part)   cnf(part_inheritance, axiom)
in(john, boy)   cnf(john_is_a_boy, hypothesis)
in(x, boy) $\Rightarrow$ in(x, human)   cnf(in_boy_in_human, hypothesis)
in(x, hand) $\Rightarrow$ has_parts(x, n_5, fingers)   cnf(hands_have_5_fingers, hypothesis)
in(x, human) $\Rightarrow$ has_parts(x, n_2, arm)   cnf(humans_have_two_arms, hypothesis)
in(x, arm) $\Rightarrow$ has_parts(x, n_1, hand)   cnf(arms_have_one_hand, hypothesis)
$\neg$ has_parts(john, times(times(n_2, n_1), n_5), fingers)   cnf(prove_john_has_10_fingers, negated_conjecture)

MSC005-1.p The evaluation of XOR expressions
value(truth, truth)   cnf(true_is_true, axiom)
value(falsity, falsity)   cnf(false_is_false, axiom)
(value(x, truth) and value(y, truth)) $\Rightarrow$ value(xor(x, y), falsity)   cnf(true_xor_true, axiom)
(value(x, truth) and value(y, falsity)) $\Rightarrow$ value(xor(x, y), truth)   cnf(true_xor_false, axiom)
(value(x, falsity) and value(y, truth)) $\Rightarrow$ value(xor(x, y), truth)   cnf(false_xor_true, axiom)
(value(x, falsity) and value(y, falsity)) $\Rightarrow$ value(xor(x, y), falsity)   cnf(false_xor_false, axiom)
$\neg$ value(xor(xor(xor(xor(truth, falsity), falsity), truth), falsity), value)   cnf(evaluate_expression, negated_conjecture)

MSC006-1.p A "non-obvious" problem
Suppose there are two relations, P and Q. P is transitive, and Q is both transitive and symmetric. Suppose further the "squareness" of P and Q: any two things are related either in the P manner or the Q manner. Prove that either P is total or Q is total.
($p(x, y)$ and $p(y, z)$) $\Rightarrow$ $p(x, z)$ cnf(p_transitivity, hypothesis)
($q(x, y)$ and $q(y, z)$) $\Rightarrow$ $q(x, z)$ cnf(q_transitivity, hypothesis)
$q(x, y)$ $\Rightarrow$ $q(y, x)$ cnf(q_symmetry, hypothesis)
$p(x, y)$ or $q(x, y)$ cnf(all_related, hypothesis)
$\neg\, p(a, b)$ cnf(p_is_not_total, hypothesis)
$\neg\, q(c, d)$ cnf(prove_q_is_total, negated_conjecture)

MSC007-2.005.p Cook pigeon-hole problem
Suppose there are N holes and N+1 pigeons to put in the holes. Every pigeon is in a hole and no hole contains more than one pigeon. Prove that this is impossible. The size is the number of pigeons.
$x = x$ cnf(reflexivity, axiom)
$x = y$ $\Rightarrow$ $y = x$ cnf(symmetry, axiom)
($x = y$ and $y = z$) $\Rightarrow$ $x = z$ cnf(transitivity, axiom)
pigeon(pigeon$_1$)   cnf(pigeon$_1$, axiom)
pigeon(pigeon$_2$)   cnf(pigeon$_2$, axiom)
pigeon(pigeon$_3$)   cnf(pigeon$_3$, axiom)
pigeon(pigeon$_4$)   cnf(pigeon$_4$, axiom)
pigeon(pigeon$_5$)   cnf(pigeon$_5$, axiom)
pigeon$_1$ $\neq$ pigeon$_2$ cnf(pigeon_1_is_not_pigeon$_2$, axiom)
pigeon$_1$ $\neq$ pigeon$_3$ cnf(pigeon_1_is_not_pigeon$_3$, axiom)
pigeon$_1$ $\neq$ pigeon$_4$ cnf(pigeon_1_is_not_pigeon$_4$, axiom)
pigeon$_1$ $\neq$ pigeon$_5$ cnf(pigeon_1_is_not_pigeon$_5$, axiom)
pigeon$_2$ $\neq$ pigeon$_3$ cnf(pigeon_2_is_not_pigeon$_3$, axiom)
pigeon$_2$ $\neq$ pigeon$_4$ cnf(pigeon_2_is_not_pigeon$_4$, axiom)
pigeon$_2$ $\neq$ pigeon$_5$ cnf(pigeon_2_is_not_pigeon$_5$, axiom)
pigeon$_3$ $\neq$ pigeon$_4$ cnf(pigeon_3_is_not_pigeon$_4$, axiom)
pigeon$_3$ $\neq$ pigeon$_5$ cnf(pigeon_3_is_not_pigeon$_5$, axiom)
pigeon$_4$ $\neq$ pigeon$_5$ cnf(pigeon_4_is_not_pigeon$_5$, axiom)
hole(hole$_1$)   cnf(hole$_1$, axiom)
hole(hole$_2$)   cnf(hole$_2$, axiom)
hole(hole$_3$)   cnf(hole$_3$, axiom)
hole(hole$_4$)   cnf(hole$_4$, axiom)
hole$_1$ $\neq$ hole$_2$ cnf(hole_1_is_not_hole$_2$, axiom)
hole$_1$ $\neq$ hole$_3$ cnf(hole_1_is_not_hole$_3$, axiom)

hole$_1$ $\neq$ hole$_4$ cnf(hole_1_is_not_hole$_4$, axiom)
hole$_2$ $\neq$ hole$_3$ cnf(hole_2_is_not_hole$_3$, axiom)
hole$_2$ $\neq$ hole$_4$ cnf(hole_2_is_not_hole$_4$, axiom)
hole$_3$ $\neq$ hole$_4$ cnf(hole_3_is_not_hole$_4$, axiom)
hole(hole) $\Rightarrow$ (hole = hole$_1$ or hole = hole$_2$ or hole = hole$_3$ or hole = hole$_4$) cnf(all_holes, axiom)
pigeon(x) $\Rightarrow$ hole(hole_of(x)) cnf(each_pigeons_hole$_1$, axiom)
pigeon(x) $\Rightarrow$ in(x, hole_of(x)) cnf(each_pigeons_hole$_2$, axiom)
(hole(hole) and pigeon(pigeon$_1$) and pigeon(pigeon$_2$) and in(pigeon$_1$, hole) and in(pigeon$_2$, hole)) $\Rightarrow$ pigeon$_1$ = pigeon$_2$ c

MSC007∧1.003.004.p Cook pigeon-hole problem, 4 pigeons and 3 holes
Suppose there are N holes and more pigeons to put in the holes. Every pigeon is in a hole and no hole contains more than one pigeon. Prove that some pigoen has no hole.
hole: \$tType thf(hole, type)
pigeon: \$tType thf(pigeon, type)
hole$_1$: hole thf(hole$_1$, type)
hole$_2$: hole thf(hole$_2$, type)
hole$_3$: hole thf(hole$_3$, type)
pigeon$_1$: pigeon thf(pigeon$_1$, type)
pigeon$_2$: pigeon thf(pigeon$_2$, type)
pigeon$_3$: pigeon thf(pigeon$_3$, type)
pigeon$_4$: pigeon thf(pigeon$_4$, type)
pigeon_hole: pigeon $\rightarrow$ hole thf(pigeon_hole_t, type)
$\forall p$: hole $\rightarrow$ \$o: ((p@hole$_1$ and p@hole$_2$ and p@hole$_3$) $\Rightarrow$ $\forall x$: hole: (p@x)) thf(holecover, axiom)
pigeon$_1$ $\neq$ pigeon$_2$ thf(pigeon1pigeon$_2$, axiom)
pigeon$_1$ $\neq$ pigeon$_3$ thf(pigeon1pigeon$_3$, axiom)
pigeon$_2$ $\neq$ pigeon$_3$ thf(pigeon2pigeon$_3$, axiom)
pigeon$_1$ $\neq$ pigeon$_4$ thf(pigeon1pigeon$_4$, axiom)
pigeon$_2$ $\neq$ pigeon$_4$ thf(pigeon2pigeon$_4$, axiom)
pigeon$_3$ $\neq$ pigeon$_4$ thf(pigeon3pigeon$_4$, axiom)
$\exists x$: pigeon, y: pigeon: ((pigeon_hole@x) = (pigeon_hole@y) and $x \neq y$) thf(sharing_a_hole, conjecture)

MSC008-1.002.p The (in)constructability of Graeco-Latin Squares
The constructibility of Graeco-Latin squares of order 4t+2. This is impossible for t=0,1, but possible for all other cases. The size is the size of the squares.
$\neg$ eq(p_1, p_2) cnf(p_1_is_not_p$_2$, axiom)
eq(x, x) cnf(reflexivity, axiom)
eq(x, y) $\Rightarrow$ eq(y, x) cnf(symmetry, axiom)
(latin(row, column, label$_1$) and latin(row, column, label$_2$)) $\Rightarrow$ eq(label$_1$, label$_2$) cnf(latin_element_is_unique, axiom)
(latin(row, column$_1$, label) and latin(row, column$_2$, label)) $\Rightarrow$ eq(column$_1$, column$_2$) cnf(latin_column_is_unique, axiom)
(latin(row$_1$, column, label) and latin(row$_2$, column, label)) $\Rightarrow$ eq(row$_1$, row$_2$) cnf(latin_row_is_unique, axiom)
(greek(row, column, label$_1$) and greek(row, column, label$_2$)) $\Rightarrow$ eq(label$_1$, label$_2$) cnf(greek_element_is_unique, axiom)
(greek(row, column$_1$, label) and greek(row, column$_2$, label)) $\Rightarrow$ eq(column$_1$, column$_2$) cnf(greek_column_is_unique, axiom)
(greek(row$_1$, column, label) and greek(row$_2$, column, label)) $\Rightarrow$ eq(row$_1$, row$_2$) cnf(greek_row_is_unique, axiom)
latin(row, column, p_1) or latin(row, column, p_2) cnf(latin_cell_element, axiom)
latin(row, p_1, label) or latin(row, p_2, label) cnf(latin_column_required, axiom)
latin(p_1, column, label) or latin(p_2, column, label) cnf(latin_row_required, axiom)
greek(row, column, p_1) or greek(row, column, p_2) cnf(greek_cell_element, axiom)
greek(row, p_1, label) or greek(row, p_2, label) cnf(greek_column_required, axiom)
greek(p_1, column, label) or greek(p_2, column, label) cnf(greek_row_required, axiom)
(greek(row$_1$, column$_1$, label$_1$) and latin(row$_1$, column$_1$, label$_2$) and greek(row$_2$, column$_2$, label$_1$) and latin(row$_2$, column$_2$, label
eq(column$_1$, column$_2$) cnf(no_two_same$_1$, negated_conjecture)
(greek(row$_1$, column$_1$, label$_1$) and latin(row$_1$, column$_1$, label$_2$) and greek(row$_2$, column$_2$, label$_1$) and latin(row$_2$, column$_2$, label
eq(row$_1$, row$_2$) cnf(no_two_same$_2$, negated_conjecture)

MSC008-2.002.p The inconstructability of a Graeco-Latin Square
The constructibility of Graeco-Latin squares of order 4t+2. This is impossible for t=0,1, but possible for all other cases.
$\neg$ eq(p_1, p_2) cnf(p1_is_not_p$_2$, axiom)
eq(a, a) cnf(reflexivity, axiom)
eq(a, b) $\Rightarrow$ eq(b, a) cnf(symmetry, axiom)
(latin(a, b, c) and latin(a, b, d)) $\Rightarrow$ eq(d, c) cnf(latin_element_is_unique, axiom)

(latin(a, b, c) and latin(a, d, c)) $\Rightarrow$ eq(d, b) cnf(latin_column_is_unique, axiom)
(latin(a, b, c) and latin(d, b, c)) $\Rightarrow$ eq(d, a) cnf(latin_row_is_unique, axiom)
(greek(a, b, c) and greek(a, b, d)) $\Rightarrow$ eq(d, c) cnf(greek_element_is_unique, axiom)
(greek(a, b, c) and greek(a, d, c)) $\Rightarrow$ eq(d, b) cnf(greek_column_is_unique, axiom)
(greek(a, b, c) and greek(d, b, c)) $\Rightarrow$ eq(d, a) cnf(greek_row_is_unique, axiom)
latin(e, f, p_1) or latin(e, f, p_2) cnf(latin_cell_element, axiom)
latin(g, p_1, h) or latin(g, p_2, h) cnf(latin_column_required, axiom)
latin(p_1, i, j) or latin(p_2, i, j) cnf(latin_row_required, axiom)
greek(k, l, p_1) or greek(k, l, p_2) cnf(greek_cell_element, axiom)
greek(m, p_1, n) or greek(m, p_2, n) cnf(greek_column_required, axiom)
greek(p_1, o, p) or greek(p_2, o, p) cnf(greek_row_required, axiom)
(greek(a, b, c) and latin(a, b, d) and greek(e, f, c) and latin(e, f, d)) $\Rightarrow$ (eq(f, b) or eq(e, a)) cnf(no_two_same, negated_con

MSC009+1.p Definitions of a family structure
$\forall x$: (female(x) $\iff$ $\neg$ male(x)) fof(female, axiom)
$\forall x$: (person(x) $\iff$ $\exists y$: (sex(x, y) and (male(y) or female(y)))) fof(person, axiom)
$\forall x$: (parent(x) $\iff$ (person(x) and $\exists y$: (child(x, y) and person(y)))) fof(parent, axiom)
$\forall x$: (mother(x) $\iff$ (parent(x) and $\exists y$: (sex(x, y) and female(y)))) fof(mother, axiom)
$\forall x$: (father(x) $\iff$ (parent(x) and $\neg$ mother(x))) fof(father, axiom)
$\forall x$: (grandparent(x) $\iff$ (parent(x) and $\exists y$: (child(x, y) and parent(y)))) fof(grandparent, axiom)
$\forall x$: (parent_with_sons_only(x) $\iff$ (parent(x) and $\forall y$: (child(x, y) $\Rightarrow$ $\exists z$: (sex(y, z) and male(z))))) fof(parent_with_son

MSC009-1.p Definitions of a family structure
female(x) $\Rightarrow$ $\neg$ male(x) cnf(female$_1$, negated_conjecture)
male(x) or female(x) cnf(female$_2$, negated_conjecture)
person(x) $\Rightarrow$ sex(x, sex_of$_1$(x)) cnf(person$_1$, negated_conjecture)
person(x) $\Rightarrow$ (male(sex_of$_1$(x)) or female(sex_of$_1$(x))) cnf(person$_2$, negated_conjecture)
(sex(x, y) and male(y)) $\Rightarrow$ person(x) cnf(person$_3$, negated_conjecture)
(sex(x, y) and female(y)) $\Rightarrow$ person(x) cnf(person$_4$, negated_conjecture)
parent(x) $\Rightarrow$ person(x) cnf(parent$_1$, negated_conjecture)
parent(x) $\Rightarrow$ child(x, child_of$_1$(x)) cnf(parent$_2$, negated_conjecture)
parent(x) $\Rightarrow$ person(child_of$_1$(x)) cnf(parent$_3$, negated_conjecture)
(person(x) and child(x, y) and person(y)) $\Rightarrow$ parent(x) cnf(parent$_4$, negated_conjecture)
mother(x) $\Rightarrow$ parent(x) cnf(mother$_1$, negated_conjecture)
mother(x) $\Rightarrow$ sex(x, sex_of$_2$(x)) cnf(mother$_2$, negated_conjecture)
mother(x) $\Rightarrow$ female(sex_of$_2$(x)) cnf(mother$_3$, negated_conjecture)
(parent(x) and sex(x, y) and female(y)) $\Rightarrow$ mother(x) cnf(mother$_4$, negated_conjecture)
father(x) $\Rightarrow$ parent(x) cnf(father$_1$, negated_conjecture)
father(x) $\Rightarrow$ $\neg$ mother(x) cnf(father$_2$, negated_conjecture)
parent(x) $\Rightarrow$ (mother(x) or father(x)) cnf(father$_3$, negated_conjecture)
grandparent(x) $\Rightarrow$ parent(x) cnf(grandparent$_1$, negated_conjecture)
grandparent(x) $\Rightarrow$ child(x, child_of$_2$(x)) cnf(grandparent$_2$, negated_conjecture)
grandparent(x) $\Rightarrow$ parent(child_of$_2$(x)) cnf(grandparent$_3$, negated_conjecture)
(parent(x) and child(x, y) and parent(y)) $\Rightarrow$ grandparent(x) cnf(grandparent$_4$, negated_conjecture)
parent_with_sons_only(x) $\Rightarrow$ parent(x) cnf(parent_with_sons_only$_1$, negated_conjecture)
(parent_with_sons_only(x) and child(x, y)) $\Rightarrow$ child_with_parent(y) cnf(parent_with_sons_only$_2$, negated_conjecture)
parent(x) $\Rightarrow$ (child(x, female_child_of(x)) or parent_with_sons_only(x)) cnf(parent_with_sons_only$_3$, negated_conjecture)
(parent(x) and child_with_parent(female_child_of(x))) $\Rightarrow$ parent_with_sons_only(x) cnf(parent_with_sons_only$_4$, negated_c
child_with_parent(y) $\Rightarrow$ sex(y, sex_of$_3$(y)) cnf(parent_with_sons_only$_5$, negated_conjecture)
child_with_parent(y) $\Rightarrow$ male(sex_of$_3$(y)) cnf(parent_with_sons_only$_6$, negated_conjecture)
(sex(y, z) and male(z)) $\Rightarrow$ child_with_parent(y) cnf(parent_with_sons_only$_7$, negated_conjecture)

MSC011+1.p Drinker paradox
$\forall a$: ((drunk(a) and not_drunk(a)) $\Rightarrow$ goal) fof(d_cons, axiom)
$\forall a$: (drunk(a) and neg_psi) fof(neg_phi, axiom)
neg_psi $\Rightarrow$ $\exists a$: not_drunk(a) fof(neg_psi_ax, axiom)
goal fof(goal_to_be_proved, conjecture)

MSC012+1.p A serial and transitive relation inconsistent for non-empty domain
$\forall a, b$: (($p(a)$ and less(a, b) and $p(b)$) $\Rightarrow$ goal) fof(left_to_right, axiom)
$\forall a$: ($p(a)$ or $\exists b$: (less(a, b) and $p(b)$)) fof(right_to_left, axiom)

$\forall a, b, c$: ((less(a, b) and less(b, c)) $\Rightarrow$ less(a, c)) fof(transitive_less, axiom)
$\forall a$: $\exists b$: less(a, b) fof(serial_less, axiom)
goal fof(goal_to_be_proved, conjecture)

MSC013+1.p Single-valued relation between 5-tuple and domain element
The existence of a single-valued relation between a 5-tuple of Booleans and a domain element
$n_0{=}n_0$ and $n_1{=}n_1$ fof(n0_and_n1_reflexive, axiom)
$n_0{=}n_1 \Rightarrow$ goal fof(n0_equal_n$_1$, axiom)
$n_1{=}n_0 \Rightarrow$ goal fof(n1_equal_n$_0$, axiom)
$\forall a, b, c, d, e$: (($a{=}a$ and $b{=}b$ and $c{=}c$ and $d{=}d$ and $e{=}e$) $\Rightarrow$ $\exists f$: $f(a, b, c, d, e, f)$) fof(relation_exists, axiom)
$\forall a, b, c, d, e, f, g, h, i, j, k$: (($f(a, b, c, d, e, k)$ and $f(f, g, h, i, j, k)$) $\Rightarrow$ ($a{=}f$ and $b{=}g$ and $c{=}h$ and $d{=}i$ and $e{=}j$)) fof(relati
goal fof(goal_to_be_proved, conjecture)

MSC014+1.p Find a model with a functional relation which is injective, n=4
$n_0{=}n_0$ and $n_1{=}n_1$ fof(n0_and_n1_reflexive, axiom)
$\neg\, n_0{=}n_1$ and $\neg\, n_1{=}n_0$ fof(n0_not_n$_1$, axiom)
$\forall x_1, x_2, x_3, x_4$: (($x_1{=}x_1$ and $x_2{=}x_2$ and $x_3{=}x_3$ and $x_4{=}x_4$) $\Rightarrow$ $\exists z$: $f(x_1, x_2, x_3, x_4, z)$) fof(exists_f, axiom)
$\forall x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4, z$: (($f(x_1, x_2, x_3, x_4, z)$ and $f(y_1, y_2, y_3, y_4, z)$) $\Rightarrow$ ($x_1{=}y_1$ and $x_2{=}y_2$ and $x_3{=}y_3$ and $x_4{=}y_4$))

MSC015-1.005.p Binary counter k=05
Each instance of the problem asserts p(0*). // p[0] p(x*01*) → p(x*10*). // p[i] → p[i + 1] not p(1*). // not p[2∧n - 1] These problems are unsatisfiable and have exponentially large propositional resolution refutations, while there is a short (quadratic) first order refutation.
$p(s_0, s_0, s_0, s_0, s_0)$ cnf(init, axiom)
$p(x_0, x_1, x_2, x_3, s_0) \Rightarrow p(x_0, x_1, x_2, x_3, s_1)$ cnf(rule$_1$, axiom)
$p(x_0, x_1, x_2, s_0, s_1) \Rightarrow p(x_0, x_1, x_2, s_1, s_0)$ cnf(rule$_2$, axiom)
$p(x_0, x_1, s_0, s_1, s_1) \Rightarrow p(x_0, x_1, s_1, s_0, s_0)$ cnf(rule$_3$, axiom)
$p(x_0, s_0, s_1, s_1, s_1) \Rightarrow p(x_0, s_1, s_0, s_0, s_0)$ cnf(rule$_4$, axiom)
$p(s_0, s_1, s_1, s_1, s_1) \Rightarrow p(s_1, s_0, s_0, s_0, s_0)$ cnf(rule$_5$, axiom)
$\neg\, p(s_1, s_1, s_1, s_1, s_1)$ cnf(goal, negated_conjecture)

MSC015-1.010.p Binary counter k=10
Each instance of the problem asserts p(0*). // p[0] p(x*01*) → p(x*10*). // p[i] → p[i + 1] not p(1*). // not p[2∧n - 1] These problems are unsatisfiable and have exponentially large propositional resolution refutations, while there is a short (quadratic) first order refutation.
$p(s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(init, axiom)
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, s_0) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, s_1)$ cnf(rule$_1$, axiom)
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, s_0, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, s_1, s_0)$ cnf(rule$_2$, axiom)
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, s_0, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, s_1, s_0, s_0)$ cnf(rule$_3$, axiom)
$p(x_0, x_1, x_2, x_3, x_4, x_5, s_0, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, s_1, s_0, s_0, s_0)$ cnf(rule$_4$, axiom)
$p(x_0, x_1, x_2, x_3, x_4, s_0, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, s_1, s_0, s_0, s_0, s_0)$ cnf(rule$_5$, axiom)
$p(x_0, x_1, x_2, x_3, s_0, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, s_1, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_6$, axiom)
$p(x_0, x_1, x_2, s_0, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, s_1, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_7$, axiom)
$p(x_0, x_1, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_8$, axiom)
$p(x_0, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_9$, axiom)
$p(s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_{10}$, axiom)
$\neg\, p(s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1)$ cnf(goal, negated_conjecture)

MSC015-1.015.p Binary counter k=15
Each instance of the problem asserts p(0*). // p[0] p(x*01*) → p(x*10*). // p[i] → p[i + 1] not p(1*). // not p[2∧n - 1] These problems are unsatisfiable and have exponentially large propositional resolution refutations, while there is a short (quadratic) first order refutation.
$p(s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(init, axiom)
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, s_0) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, s_1)$ cnf(
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, s_0, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, s_1, s_0)$ cnf(ru
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, s_0, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, s_1, s_0, s_0)$ cnf(rule$_3$
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, s_0, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, s_1, s_0, s_0, s_0)$ cnf(rule$_4$, a
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, s_0, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, s_1, s_0, s_0, s_0, s_0)$ cnf(rule$_5$, axi
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, s_0, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, s_1, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_6$, axio
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, s_0, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, s_1, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_7$, axio
$p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, x_6, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_8$, axion
$p(x_0, x_1, x_2, x_3, x_4, x_5, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, x_5, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_9$, axiom

$p(x_0, x_1, x_2, x_3, x_4, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, x_4, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_{10}$, axior
$p(x_0, x_1, x_2, x_3, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, x_3, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_{11}$, axion
$p(x_0, x_1, x_2, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, x_2, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_{12}$, axiom
$p(x_0, x_1, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, x_1, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_{13}$, axiom
$p(x_0, s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(x_0, s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_{14}$, axiom)
$p(s_0, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1) \Rightarrow p(s_1, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0, s_0)$ cnf(rule$_{15}$, axiom)
$\neg p(s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1, s_1)$ cnf(goal, negated_conjecture)

MSC017+1.p Diving has no bad outcomes
$\forall d$: (dive(d) $\Rightarrow$ nitrogen(d) = padi(depth(d), time(d))) fof(dive_nitrogen, axiom)
$\forall d$: (dive(d) $\Rightarrow$ ((greater(depth(d), depth_limit) or greater(time(d), time_limit)) $\iff$ greater(nitrogen(d), nitrogen_limit)))
$\forall d$: (dive(d) $\Rightarrow$ (greater(nitrogen(d), nitrogen_limit) $\iff$ outcome(d, dci))) fof(too_much_nitrogen, axiom)
bad(dci) fof(bad_dci, axiom)
$\forall d$: (dive(d) $\Rightarrow$ $\forall o$: ((outcome(d, o) and bad(o)) $\Rightarrow$ o = dci)) fof(dci_is_the_only_bad_outcome, axiom)
$\forall d$: (dive(d) $\Rightarrow$ ($\neg$ greater(depth(d), depth_limit) and $\neg$ greater(time(d), time_limit))) fof(no_deep_long, axiom)
$\forall d$: (dive(d) $\Rightarrow$ $\forall o$: (outcome(d, o) $\Rightarrow$ $\neg$ bad(o))) fof(no_bad, conjecture)

MSC020^5.p TPS problem THM301
Relation between HALF and DOUBLE functions.
cHALF: \$i → \$i → \$o thf(cHALF, type)
cDOUBLE: \$i → \$i → \$o thf(cDOUBLE, type)
cS: \$i → \$i thf(cS, type)
c_0: \$i thf(c_0, type)
($\forall q$: \$i → \$i → \$o, xu: \$i, xv: \$i: ((cDOUBLE@xu@xv and q@c_0@c_0 and ∀xx: \$i, xy: \$i: ((q@xx@xy) $\Rightarrow$ (q@(cS@xx)@(cS@(c
(q@xu@xv)) and cHALF@c_0@c_0 and cHALF@(cS@c_0)@c_0 and ∀xx: \$i, xy: \$i: ((cHALF@xx@xy) $\Rightarrow$ (cHALF@(cS@(cS@xx)
∀xu: \$i, xv: \$i: ((cDOUBLE@xu@xv) $\Rightarrow$ (cHALF@xv@xu)) thf(cTHM$_{301}$, conjecture)

MSC021^5.p TPS problem THM300
Relation between HALF and DOUBLE functions.
cS: \$i → \$i thf(cS, type)
cDOUBLE: \$i → \$i → \$o thf(cDOUBLE, type)
cHALF: \$i → \$i → \$o thf(cHALF, type)
c_0: \$i thf(c_0, type)
(cDOUBLE@c_0@c_0 and ∀xx: \$i, xy: \$i: ((cDOUBLE@xx@xy) $\Rightarrow$ (cDOUBLE@(cS@xx)@(cS@(cS@xy)))) and $\forall q$: \$i →
\$i → \$o, xu: \$i, xv: \$i: ((cHALF@xu@xv and q@c_0@c_0 and q@(cS@c_0)@c_0 and ∀xx: \$i, xy: \$i: ((q@xx@xy) $\Rightarrow$
(q@(cS@(cS@xx))@(cS@xy)))) $\Rightarrow$ (q@xu@xv))) $\Rightarrow$ ∀xu: \$i, xv: \$i: ((cHALF@xu@xv) $\Rightarrow$ (cDOUBLE@xv@xu or cDOUBLI

MSC022=2.p Celcius 37 to Fahrenheit conversion
celsius_fahrenheit_temperature_conversion: (\$real×\$real) → \$o tff(celsius_fahrenheit_temperature_conversion_type, type)
$\forall c$: \$real, f: \$real: (\$sum(\$product($1._8$, c), $32._0$) = f $\Rightarrow$ celsius_fahrenheit_temperature_conversion(c, f)) tff(celsius_fahren
$\exists f$: \$real: celsius_fahrenheit_temperature_conversion($37._0$, f) tff(celsius_37_to_fahrenheit, conjecture)

MSC023=2.p Fahrenheit 451 to Celcius conversion
celsius_fahrenheit_temperature_conversion: (\$real×\$real) → \$o tff(celsius_fahrenheit_temperature_conversion_type, type)
$\forall c$: \$real, f: \$real: (\$sum(\$product($1._8$, c), $32._0$) = f $\Rightarrow$ celsius_fahrenheit_temperature_conversion(c, f)) tff(celsius_fahren
$\exists c$: \$real: celsius_fahrenheit_temperature_conversion(c, $451._0$) tff(fahrenheit_451_to_celsius, conjecture)

MSC025^1.p Nik's Challenge
Force the \$i type to only have two elements, then conjecture the existence of a bijection between \$o and \$i.
1: \$i thf(one, type)
two: \$i thf(two, type)
$\forall x$: \$i: ($x = 1$ or x = two) thf(binary_exhaust, axiom)
$1 \neq$ two thf(binary_distinc, axiom)
$\exists f$: \$o → \$i: $\forall x$: \$o: (f@x) $\neq$ (f@$\neg x$) thf(goal, conjecture)

MSC025^2.p Nik's Challenge
Force the \$i type to only have two elements, then conjecture the existence of a bijection between \$o and \$i.
1: \$i thf(one, type)
two: \$i thf(two, type)
$\forall x$: \$i: ($x = 1$ or x = two) thf(binarity_exhaust, axiom)
$1 \neq$ two thf(binarity_distinc, axiom)
b_1: \$o → \$i thf(b1_ty, type)
$\forall x$: \$o: ((x $\Rightarrow$ (b_1@x) = 1) and ($\neg x$ $\Rightarrow$ (b_1@x) = two)) thf(b_1, axiom)

b_2: \$o $\rightarrow$ \$i thf(b2_ty, type)
$\forall x$: \$o: $((x \Rightarrow (b_2@x) = \text{two})$ and $(\neg x \Rightarrow (b_2@x) = 1))$ thf(b_2, axiom)
$\forall f$: \$o $\rightarrow$ \$i: ($\forall x$: \$o: $(f@x) \neq (f@\neg x) \Rightarrow (f = b_1$ or $f = b_2))$ thf(goal, conjecture)

MSC026-1.p Sets, numbers, lists, etc, that make up the Isabelle/HOL library
include('Axioms/MSC001-0.ax')
include('Axioms/MSC001-1.ax')

MSC027-1.p Sets, numbers, lists, etc, that make up the Isabelle/HOL library
include('Axioms/MSC001-0.ax')
include('Axioms/MSC001-2.ax')

MSC028=1.p 2 and 3 cents stamps
Suppose we have stamps of two different denominations, 3 cents and 5 cents. We want to show that it is possible to make up exactly any postage of 8 cents or more using stamps of these two denominations. The formula below asserts this, however "8" replaced by "some lower bound".
$\exists l$: \$int: $\forall k$: \$int: (\$greater$(k, l) \Rightarrow \exists s_1$: \$int, s_2: \$int: (\$greatereq$(s_1, 0)$ and \$greatereq$(s_2, 0)$ and $k =$ \$sum(\$product$(s_1, 3)$,