

NUN axioms

NUN problems

NUN019+1.p Peano greater and unequal

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 $\forall x: \text{greater}(s(x), x) \quad \text{fof}(\text{greater}_0, \text{axiom})$ 
 $\forall x, y: (\text{greater}(x, y) \Rightarrow \text{greater}(s(x), y)) \quad \text{fof}(\text{greater}_1, \text{axiom})$ 
 $\forall x, y: (\text{greater}(x, y) \Rightarrow x \neq y) \quad \text{fof}(\text{not_equal}_0, \text{axiom})$ 

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NUN020+1.p Axioms for RDN arithmetic

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include('Axioms/NUM005+0.ax')
include('Axioms/NUM005+1.ax')
include('Axioms/NUM005+2.ax')

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NUN021^1.p Axioms for Church Numerals in Simple Type Theory

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include('Axioms/NUM006^0.ax')
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NUN022^1.p Find this function

Does there exist a function f from reals to reals such that for all x and y, $f(x + y * y) - f(x) \geq y$?

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 $\exists f: \$\text{real} \rightarrow \$\text{real}: \forall x: \$\text{real}, y: \$\text{real}: (\$lesseq@(\$difference@((f@(\$sum@x@(\$product@y@y)))@((f@x))@y)) \quad \text{thf(jasmin, conjecture)})$ 
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NUN023^1.p Function h s.t. $h(0) = 1$, $h(1) = 0$, no witness

Using an axiomatiztion of if-then-else, find the if-then-else term that expresses the function H.

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0: $i      thf(n5, type)
s: $i → $i      thf(n6, type)
ite: $o → $i → $i → $i      thf(n7, type)
( $\forall x: \$o, u: \$i, v: \$i: (x \Rightarrow (\text{ite}@x@u@v) = u)$  and  $\forall x: \$o, u: \$i, v: \$i: (\neg x \Rightarrow (\text{ite}@x@u@v) = v)) \Rightarrow \exists h: \$i \rightarrow \$i: ((h@0) = (s@0) \text{ and } (h@(s@0)) = 0)$ )      thf(n8, conjecture)

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NUN023^2.p Function h s.t. $h(0) = 1$, $h(1) = 0$, with witness

Using an axiomatiztion of if-then-else, find the if-then-else term that expresses the function H.

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0: $i      thf(n6, type)
s: $i → $i      thf(n7, type)
ite: $o → $i → $i → $i      thf(n8, type)
h: $i → $i      thf(n9, type)
( $\forall x_{100}: \$o, u: \$i, v: \$i: (x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = u)$  and  $\forall x_{100}: \$o, u: \$i, v: \$i: (\neg x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = v)$  and  $\forall x: \$i: (h@x) = (\text{ite}@x = 0@(s@0)@0)) \Rightarrow \exists h: \$i \rightarrow \$i: ((h@0) = (s@0) \text{ and } (h@(s@0)) = 0)$ )      thf(n10, conjecture)

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NUN024^1.p Function h s.t. $h(0) = 1$, $h(1) = 0$, $h(2) = 0$, no witness

Using an axiomatiztion of if-then-else, find the if-then-else term that expresses the function H.

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0: $i      thf(n6, type)
s: $i → $i      thf(n7, type)
ite: $o → $i → $i → $i      thf(n8, type)
h: $i → $i      thf(n9, type)
( $\forall x_{100}: \$o, u: \$i, v: \$i: (x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = u)$  and  $\forall x_{100}: \$o, u: \$i, v: \$i: (\neg x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = v)$  and  $\forall x: \$i: (s@x) \neq 0$  and  $\forall x: \$i: (s@x) \neq x \Rightarrow \exists h: \$i \rightarrow \$i: ((h@0) = (s@0) \text{ and } (h@(s@0)) = 0 \text{ and } (h@(s@(s@0))) = 0)$ )      thf(n9, conjecture)

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NUN024^2.p Function h s.t. $h(0) = 1$, $h(1) = 0$, $h(2) = 0$, with witness

Using an axiomatiztion of if-then-else, find the if-then-else term that expresses the function H.

```

0: $i      thf(n6, type)
s: $i → $i      thf(n7, type)
ite: $o → $i → $i → $i      thf(n8, type)
h: $i → $i      thf(n9, type)
( $\forall x_{100}: \$o, u: \$i, v: \$i: (x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = u)$  and  $\forall x_{100}: \$o, u: \$i, v: \$i: (\neg x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = v)$  and  $\forall x: \$i: (s@x) \neq 0$  and  $\forall x: \$i: (s@x) \neq x$  and  $\forall x: \$i: (h@x) = (\text{ite}@x = 0@(s@0)@0)) \Rightarrow \exists h: \$i \rightarrow \$i: ((h@0) = (s@0) \text{ and } (h@(s@0)) = 0 \text{ and } (h@(s@(s@0))) = 0)$ )      thf(n10, conjecture)

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NUN025^1.p Function h s.t. $h(0) = 1$, $h(1) = 0$, $h(2) = 1$, no witness

Using an axiomatiztion of if-then-else, find the if-then-else term that expresses the function H.

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0: $i      thf(n6, type)
s: $i → $i      thf(n7, type)
ite: $o → $i → $i → $i      thf(n8, type)

```

$(\forall x_{100}: \$o, u: \$i, v: \$i: (x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = u) \text{ and } \forall x_{100}: \$o, u: \$i, v: \$i: (\neg x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = v) \text{ and } \forall x: \$i: (s@x) \neq x \Rightarrow \exists h: \$i \rightarrow \$i: ((h@0) = (s@0) \text{ and } (h@(s@0)) = 0 \text{ and } (h@(s@(s@0))) = (s@0)) \quad \text{thf}(n_9, \text{conj})$

NUN025^2.p Function h s.t. $h(0) = 1$, $h(1) = 0$, $h(2) = 1$, with witness

Using an axiomatiztion of if-then-else, find the if-then-else term that expresses the function H.

$0: \$i \quad \text{thf}(n_6, \text{type})$

$s: \$i \rightarrow \$i \quad \text{thf}(n_7, \text{type})$

$\text{ite}: \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(n_8, \text{type})$

$h: \$i \rightarrow \$i \quad \text{thf}(n_9, \text{type})$

$(\forall x_{100}: \$o, u: \$i, v: \$i: (x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = u) \text{ and } \forall x_{100}: \$o, u: \$i, v: \$i: (\neg x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = v) \text{ and } \forall x: \$i: (s@x) \neq x \text{ and } \forall x: \$i: (h@x) = (\text{ite}@x = (s@0)@0@(s@0))) \Rightarrow \exists h: \$i \rightarrow \$i: ((h@0) = (s@0) \text{ and } (h@(s@0)) = 0 \text{ and } (h@(s@(s@0))) = (s@0)) \quad \text{thf}(n_{10}, \text{conjecture})$

NUN025^3.p Function h s.t. $h(0) = 1$, $h(1) = 0$, $h(2) = 1$, with witness

Using an axiomatiztion of if-then-else, find the if-then-else term that expresses the function H.

$0: \$i \quad \text{thf}(n_6, \text{type})$

$s: \$i \rightarrow \$i \quad \text{thf}(n_7, \text{type})$

$\text{ite}: \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(n_8, \text{type})$

$h: \$i \rightarrow \$i \quad \text{thf}(n_9, \text{type})$

$(\forall x_{100}: \$o, u: \$i, v: \$i: (x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = u) \text{ and } \forall x_{100}: \$o, u: \$i, v: \$i: (\neg x_{100} \Rightarrow (\text{ite}@x_{100}@u@v) = v) \text{ and } \forall x: \$i: (s@x) \neq 0 \text{ and } \forall x: \$i: (s@x) \neq x \text{ and } \forall x: \$i: (h@x) = (\text{ite}@x = 0@(s@0)@(\text{ite}@x = (s@0)@0@(s@0)))) \Rightarrow \exists h: \$i \rightarrow \$i: ((h@0) = (s@0) \text{ and } (h@(s@0)) = 0 \text{ and } (h@(s@(s@0))) = (s@0)) \quad \text{thf}(n_{10}, \text{conjecture})$