SEU axioms
SEU problems

SEU019+1.p Functions and their basic properties, theorem 52
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and function(a)) $\Rightarrow$ (one_to_one(a) $\iff$ $\forall b, c$: ((in(b, relation_dom(a)) and in(c, relation_dom(a)) and appl
apply(a, c)) $\Rightarrow$ $b = c$))) fof(d8_funct$_1$, axiom)
$\forall a$: relation(identity_relation(a)) fof(dt_k6_relat$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a$: (relation(identity_relation(a)) and function(identity_relation(a))) fof(fc2_funct$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_dom(a)) and relation(relation_dom(a)))) fof(fc7_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element(b, powerset(a)) and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: ((relation(b) and function(b)) $\Rightarrow$ (b = identity_relation(a) $\iff$ (relation_dom(b) $= a$ and $\forall c$: (in(c, a) $\Rightarrow$
apply(b, c) $= c$)))) fof(t34_funct$_1$, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a$: one_to_one(identity_relation(a)) fof(t52_funct$_1$, conjecture)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU116+1.p Boolean domains, theorem 30
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a$: ($\neg$ empty(powerset(a)) and cup_closed(powerset(a)) and diff_closed(powerset(a)) and preboolean(powerset(a))) fof(fc
$\forall a$: (preboolean(a) $\Rightarrow$ (cup_closed(a) and diff_closed(a))) fof(cc1_finsub$_1$, axiom)
$\forall a$: ((cup_closed(a) and diff_closed(a)) $\Rightarrow$ preboolean(a)) fof(cc2_finsub$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ finite(a)) fof(cc1_finset$_1$, axiom)
$\forall a$: (finite(a) $\Rightarrow$ $\forall b$: (element(b, powerset(a)) $\Rightarrow$ finite(b))) fof(cc2_finset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)

∀a: preboolean(finite_subsets(a))   fof(dt_k5_finsub$_1$, axiom)
∀a: (¬ empty(finite_subsets(a)) and cup_closed(finite_subsets(a)) and diff_closed(finite_subsets(a)) and preboolean(finite_subs
∀a, b: (element(b, finite_subsets(a)) ⇒ finite(b))   fof(cc3_finsub$_1$, axiom)
∃a: (¬ empty(a) and cup_closed(a) and cap_closed(a) and diff_closed(a) and preboolean(a))   fof(rc1_finsub$_1$, axiom)
∃a: (¬ empty(a) and finite(a))   fof(rc1_finset$_1$, axiom)
∀a: ∃b: (element(b, powerset(a)) and empty(b) and relation(b) and function(b) and one_to_one(b) and epsilon_transitive(b) an
∀a: (¬ empty(a) ⇒ ∃b: (element(b, powerset(a)) and ¬ empty(b) and finite(b)))   fof(rc3_finset$_1$, axiom)
∀a: (¬ empty(a) ⇒ ∃b: (element(b, powerset(a)) and ¬ empty(b) and finite(b)))   fof(rc4_finset$_1$, axiom)
∀a: (¬ empty(a) ⇒ ∃b: (element(b, powerset(a)) and ¬ empty(b)))   fof(rc1_subset$_1$, axiom)
∀a: ∃b: (element(b, powerset(a)) and empty(b))   fof(rc2_subset$_1$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (element(b, finite_subsets(a)) ⇒ finite(b))   fof(t30_finsub$_1$, conjecture)

SEU119+1.p MPTP bushy problem t3_xboole_0
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)   fof(commutativity_k3_xboole$_0$, axiom)
∀a: (a = empty_set ⟺ ∀b: ¬ in(b, a))   fof(d1_xboole$_0$, axiom)
∀a, b, c: (c = set_intersection$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) and in(d, b))))   fof(d3_xboole$_0$, axiom)
∀a, b: (disjoint(a, b) ⟺ set_intersection$_2$(a, b) = empty_set)   fof(d7_xboole$_0$, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$true   fof(dt_k3_xboole$_0$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a, b: set_intersection$_2$(a, a) = a   fof(idempotence_k3_xboole$_0$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (disjoint(a, b) ⇒ disjoint(b, a))   fof(symmetry_r1_xboole$_0$, axiom)
∀a, b: (¬ ¬ disjoint(a, b) and ∀c: ¬ in(c, a) and in(c, b) and ¬ ∃c: (in(c, a) and in(c, b)) and disjoint(a, b))   fof(t3_xboole$_0$, co

SEU119+2.p MPTP chainy problem t3_xboole_0
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)   fof(commutativity_k3_xboole$_0$, axiom)
∀a: (a = empty_set ⟺ ∀b: ¬ in(b, a))   fof(d1_xboole$_0$, axiom)
∀a, b, c: (c = set_intersection$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) and in(d, b))))   fof(d3_xboole$_0$, axiom)
∀a, b: (disjoint(a, b) ⟺ set_intersection$_2$(a, b) = empty_set)   fof(d7_xboole$_0$, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$true   fof(dt_k3_xboole$_0$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a, b: set_intersection$_2$(a, a) = a   fof(idempotence_k3_xboole$_0$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (disjoint(a, b) ⇒ disjoint(b, a))   fof(symmetry_r1_xboole$_0$, axiom)
∀a, b: (¬ ¬ disjoint(a, b) and ∀c: ¬ in(c, a) and in(c, b) and ¬ ∃c: (in(c, a) and in(c, b)) and disjoint(a, b))   fof(t3_xboole$_0$, co

SEU120+1.p MPTP bushy problem t4_xboole_0
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)   fof(commutativity_k3_xboole$_0$, axiom)
∀a: (a = empty_set ⟺ ∀b: ¬ in(b, a))   fof(d1_xboole$_0$, axiom)
∀a, b: (disjoint(a, b) ⟺ set_intersection$_2$(a, b) = empty_set)   fof(d7_xboole$_0$, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$true   fof(dt_k3_xboole$_0$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a, b: set_intersection$_2$(a, a) = a   fof(idempotence_k3_xboole$_0$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (disjoint(a, b) ⇒ disjoint(b, a))   fof(symmetry_r1_xboole$_0$, axiom)
∀a, b: (¬ ¬ disjoint(a, b) and ∀c: ¬ in(c, set_intersection$_2$(a, b)) and ¬ ∃c: in(c, set_intersection$_2$(a, b)) and disjoint(a, b))   fof(

SEU120+2.p MPTP chainy problem t4_xboole_0
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)   fof(commutativity_k3_xboole$_0$, axiom)

$\forall a$: (a = empty_set $\iff$ $\forall b$: ¬ in(b, a)) fof(d1_xboole$_0$, axiom)
$\forall a, b, c$: (c = set_intersection$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: (disjoint(a, b) $\iff$ set_intersection$_2$(a, b) = empty_set) fof(d7_xboole$_0$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (disjoint(a, b) $\Rightarrow$ disjoint(b, a)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: (¬ ¬ disjoint(a, b) and $\forall c$: ¬ in(c, a) and in(c, b) and ¬ $\exists c$: (in(c, a) and in(c, b)) and disjoint(a, b)) fof(t3_xboole$_0$, len
$\forall a, b$: (¬ ¬ disjoint(a, b) and $\forall c$: ¬ in(c, set_intersection$_2$(a, b)) and ¬ $\exists c$: in(c, set_intersection$_2$(a, b)) and disjoint(a, b)) fof(

SEU121+1.p MPTP bushy problem t1_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow$ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$) fof(t1_xboole$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: ¬ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU121+2.p MPTP chainy problem t1_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow$ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a$: (a = empty_set $\iff$ $\forall b$: ¬ in(b, a)) fof(d1_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: (c = set_intersection$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: (disjoint(a, b) $\iff$ set_intersection$_2$(a, b) = empty_set) fof(d7_xboole$_0$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (disjoint(a, b) $\Rightarrow$ disjoint(b, a)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$) fof(t1_xboole$_1$, conjecture)
$\forall a, b$: (¬ ¬ disjoint(a, b) and $\forall c$: ¬ in(c, a) and in(c, b) and ¬ $\exists c$: (in(c, a) and in(c, b)) and disjoint(a, b)) fof(t3_xboole$_0$, len
$\forall a, b$: (¬ ¬ disjoint(a, b) and $\forall c$: ¬ in(c, set_intersection$_2$(a, b)) and ¬ $\exists c$: in(c, set_intersection$_2$(a, b)) and disjoint(a, b)) fof(
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: ¬ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU122+1.p MPTP bushy problem t2_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow$ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: empty_set $\subseteq a$ fof(t2_xboole$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: ¬ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU122+2.p MPTP chainy problem t2_xboole_1
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2(a, b) =$ set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a$: $(a =$ empty_set $\iff \forall b$: $\neg\,\mathrm{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c$: $(\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b, c$: $(c =$ set_intersection$_2(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{disjoint}(a, b) \iff$ set_intersection$_2(a, b) =$ empty_set$)$ fof(d7_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $(\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b, c$: $((a \subseteq b$ and $b \subseteq c) \Rightarrow a \subseteq c)$ fof(t1_xboole$_1$, lemma)
$\forall a$: empty_set $\subseteq a$ fof(t2_xboole$_1$, conjecture)
$\forall a, b$: $(\neg\neg\,\mathrm{disjoint}(a, b)$ and $\forall c$: $\neg\,\mathrm{in}(c, a)$ and $\mathrm{in}(c, b)$ and $\neg\,\exists c$: $(\mathrm{in}(c, a)$ and $\mathrm{in}(c, b))$ and $\mathrm{disjoint}(a, b))$ fof(t3_xboole$_0$, len
$\forall a, b$: $(\neg\neg\,\mathrm{disjoint}(a, b)$ and $\forall c$: $\neg\,\mathrm{in}(c,$ set_intersection$_2(a, b))$ and $\neg\,\exists c$: $\mathrm{in}(c,$ set_intersection$_2(a, b))$ and $\mathrm{disjoint}(a, b))$ fof(
$\forall a$: $(\mathrm{empty}(a) \Rightarrow a =$ empty_set$)$ fof(t6_boole, axiom)
$\forall a, b$: $\neg\,\mathrm{in}(a, b)$ and $\mathrm{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b$: $\neg\,\mathrm{empty}(a)$ and $a \neq b$ and $\mathrm{empty}(b)$ fof(t8_boole, axiom)

SEU123+1.p MPTP bushy problem t3_xboole_1
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $(a = b \iff (a \subseteq b$ and $b \subseteq a))$ fof(d10_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: empty_set $\subseteq a$ fof(t2_xboole$_1$, axiom)
$\forall a$: $(a \subseteq$ empty_set $\Rightarrow a =$ empty_set$)$ fof(t3_xboole$_1$, conjecture)
$\forall a$: $(\mathrm{empty}(a) \Rightarrow a =$ empty_set$)$ fof(t6_boole, axiom)
$\forall a, b$: $\neg\,\mathrm{in}(a, b)$ and $\mathrm{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b$: $\neg\,\mathrm{empty}(a)$ and $a \neq b$ and $\mathrm{empty}(b)$ fof(t8_boole, axiom)

SEU123+2.p MPTP chainy problem t3_xboole_1
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2(a, b) =$ set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: $(a = b \iff (a \subseteq b$ and $b \subseteq a))$ fof(d10_xboole$_0$, axiom)
$\forall a$: $(a =$ empty_set $\iff \forall b$: $\neg\,\mathrm{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c$: $(\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b, c$: $(c =$ set_intersection$_2(a, b) \iff \forall d$: $(\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{disjoint}(a, b) \iff$ set_intersection$_2(a, b) =$ empty_set$)$ fof(d7_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: $\mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $(\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b, c$: $((a \subseteq b$ and $b \subseteq c) \Rightarrow a \subseteq c)$ fof(t1_xboole$_1$, lemma)
$\forall a$: empty_set $\subseteq a$ fof(t2_xboole$_1$, lemma)
$\forall a, b$: $(\neg\neg\,\mathrm{disjoint}(a, b)$ and $\forall c$: $\neg\,\mathrm{in}(c, a)$ and $\mathrm{in}(c, b)$ and $\neg\,\exists c$: $(\mathrm{in}(c, a)$ and $\mathrm{in}(c, b))$ and $\mathrm{disjoint}(a, b))$ fof(t3_xboole$_0$, len
$\forall a$: $(a \subseteq$ empty_set $\Rightarrow a =$ empty_set$)$ fof(t3_xboole$_1$, conjecture)
$\forall a, b$: $(\neg\neg\,\mathrm{disjoint}(a, b)$ and $\forall c$: $\neg\,\mathrm{in}(c,$ set_intersection$_2(a, b))$ and $\neg\,\exists c$: $\mathrm{in}(c,$ set_intersection$_2(a, b))$ and $\mathrm{disjoint}(a, b))$ fof(
$\forall a$: $(\mathrm{empty}(a) \Rightarrow a =$ empty_set$)$ fof(t6_boole, axiom)
$\forall a, b$: $\neg\,\mathrm{in}(a, b)$ and $\mathrm{empty}(b)$ fof(t7_boole, axiom)

$\forall a, b\colon \neg\,\mathrm{empty}(a)$ and $a \neq b$ and $\mathrm{empty}(b)$ fof(t8_boole, axiom)

SEU124+1.p MPTP bushy problem t7_xboole_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \mathrm{set\_union}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ or $\mathrm{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
$\mathrm{empty}(\mathrm{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a\colon \mathrm{set\_union}_2(a, \mathrm{empty\_set}) = a$ fof(t1_boole, axiom)
$\forall a\colon (\mathrm{empty}(a) \Rightarrow a = \mathrm{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b\colon \neg\,\mathrm{in}(a, b)$ and $\mathrm{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b\colon a \subseteq \mathrm{set\_union}_2(a, b)$ fof(t7_xboole$_1$, conjecture)
$\forall a, b\colon \neg\,\mathrm{empty}(a)$ and $a \neq b$ and $\mathrm{empty}(b)$ fof(t8_boole, axiom)

SEU124+2.p MPTP chainy problem t7_xboole_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b\colon (a = b \iff (a \subseteq b$ and $b \subseteq a))$ fof(d10_xboole$_0$, axiom)
$\forall a\colon (a = \mathrm{empty\_set} \iff \forall b\colon \neg\,\mathrm{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \mathrm{set\_union}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ or $\mathrm{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b, c\colon (c = \mathrm{set\_intersection}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \iff \mathrm{set\_intersection}_2(a, b) = \mathrm{empty\_set})$ fof(d7_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
$\mathrm{empty}(\mathrm{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a\colon \mathrm{set\_union}_2(a, \mathrm{empty\_set}) = a$ fof(t1_boole, axiom)
$\forall a, b, c\colon ((a \subseteq b$ and $b \subseteq c) \Rightarrow a \subseteq c)$ fof(t1_xboole$_1$, lemma)
$\forall a\colon \mathrm{empty\_set} \subseteq a$ fof(t2_xboole$_1$, lemma)
$\forall a, b\colon (\neg\neg\,\mathrm{disjoint}(a, b)$ and $\forall c\colon \neg\,\mathrm{in}(c, a)$ and $\mathrm{in}(c, b)$ and $\neg\,\exists c\colon (\mathrm{in}(c, a)$ and $\mathrm{in}(c, b))$ and $\mathrm{disjoint}(a, b))$ fof(t3_xboole$_0$, len
$\forall a\colon (a \subseteq \mathrm{empty\_set} \Rightarrow a = \mathrm{empty\_set})$ fof(t3_xboole$_1$, lemma)
$\forall a, b\colon (\neg\neg\,\mathrm{disjoint}(a, b)$ and $\forall c\colon \neg\,\mathrm{in}(c, \mathrm{set\_intersection}_2(a, b))$ and $\neg\,\exists c\colon \mathrm{in}(c, \mathrm{set\_intersection}_2(a, b))$ and $\mathrm{disjoint}(a, b))$ fof(
$\forall a\colon (\mathrm{empty}(a) \Rightarrow a = \mathrm{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b\colon \neg\,\mathrm{in}(a, b)$ and $\mathrm{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b\colon a \subseteq \mathrm{set\_union}_2(a, b)$ fof(t7_xboole$_1$, conjecture)
$\forall a, b\colon \neg\,\mathrm{empty}(a)$ and $a \neq b$ and $\mathrm{empty}(b)$ fof(t8_boole, axiom)

SEU125+1.p MPTP bushy problem t8_xboole_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \mathrm{set\_union}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ or $\mathrm{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)

\$true fof(dt_k2_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: $\mathrm{set\_union}_2(a, \mathrm{empty\_set}) = a$ fof(t1_boole, axiom)
$\forall a$: $(\mathrm{empty}(a) \Rightarrow a = \mathrm{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b$: $\neg\,\mathrm{in}(a, b)$ and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg\,\mathrm{empty}(a)$ and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a, b, c$: $((a \subseteq b$ and $c \subseteq b) \Rightarrow \mathrm{set\_union}_2(a, c) \subseteq b)$ fof(t8_xboole$_1$, conjecture)

SEU125+2.p MPTP chainy problem t8_xboole_1
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: $(a = b \iff (a \subseteq b$ and $b \subseteq a))$ fof(d10_xboole$_0$, axiom)
$\forall a$: $(a = \mathrm{empty\_set} \iff \forall b\colon \neg\,\mathrm{in}(b, a))$ fof(d1_xboole$_0$, axiom)
$\forall a, b, c$: $(c = \mathrm{set\_union}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ or $\mathrm{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b, c$: $(c = \mathrm{set\_intersection}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ and $\mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b$: $(\mathrm{disjoint}(a, b) \iff \mathrm{set\_intersection}_2(a, b) = \mathrm{empty\_set})$ fof(d7_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $(\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a$: $\mathrm{set\_union}_2(a, \mathrm{empty\_set}) = a$ fof(t1_boole, axiom)
$\forall a, b, c$: $((a \subseteq b$ and $b \subseteq c) \Rightarrow a \subseteq c)$ fof(t1_xboole$_1$, lemma)
$\forall a$: $\mathrm{empty\_set} \subseteq a$ fof(t2_xboole$_1$, lemma)
$\forall a, b$: $(\neg\neg\,\mathrm{disjoint}(a, b)$ and $\forall c\colon \neg\,\mathrm{in}(c, a)$ and $\mathrm{in}(c, b)$ and $\neg\,\exists c\colon (\mathrm{in}(c, a)$ and $\mathrm{in}(c, b))$ and $\mathrm{disjoint}(a, b))$ fof(t3_xboole$_0$, len
$\forall a$: $(a \subseteq \mathrm{empty\_set} \Rightarrow a = \mathrm{empty\_set})$ fof(t3_xboole$_1$, lemma)
$\forall a, b$: $(\neg\neg\,\mathrm{disjoint}(a, b)$ and $\forall c\colon \neg\,\mathrm{in}(c, \mathrm{set\_intersection}_2(a, b))$ and $\neg\,\exists c\colon \mathrm{in}(c, \mathrm{set\_intersection}_2(a, b))$ and $\mathrm{disjoint}(a, b))$ fof(
$\forall a$: $(\mathrm{empty}(a) \Rightarrow a = \mathrm{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b$: $\neg\,\mathrm{in}(a, b)$ and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $a \subseteq \mathrm{set\_union}_2(a, b)$ fof(t7_xboole$_1$, lemma)
$\forall a, b$: $\neg\,\mathrm{empty}(a)$ and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a, b, c$: $((a \subseteq b$ and $c \subseteq b) \Rightarrow \mathrm{set\_union}_2(a, c) \subseteq b)$ fof(t8_xboole$_1$, conjecture)

SEU126+1.p MPTP bushy problem t12_xboole_1
$\forall a, b$: $(\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: $(a = b \iff (a \subseteq b$ and $b \subseteq a))$ fof(d10_xboole$_0$, axiom)
$\forall a, b, c$: $(c = \mathrm{set\_union}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a)$ or $\mathrm{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b$: $(\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b$: $\mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)

$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: ($a \subseteq b \Rightarrow$ set_union$_2$(a, b) $= b$) fof(t12_xboole$_1$, conjecture)
$\forall a$: set_union$_2$(a, empty_set) $= a$ fof(t1_boole, axiom)
$\forall a$: (empty(a) $\Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU126+2.p MPTP chainy problem t12_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2$(a, b) $=$ set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, b) $=$ set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($a = b \iff$ ($a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
$\forall a$: ($a =$ empty_set $\iff \forall b$: $\neg$ in(b, a)) fof(d1_xboole$_0$, axiom)
$\forall a, b, c$: ($c =$ set_union$_2$(a, b) $\iff \forall d$: (in(d, c) $\iff$ (in(d, a) or in(d, b)))) fof(d2_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in($c, a$) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c =$ set_intersection$_2$(a, b) $\iff \forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: (disjoint(a, b) $\iff$ set_intersection$_2$(a, b) $=$ empty_set) fof(d7_xboole$_0$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k2_xboole$_0$, axiom)
$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow \neg$ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow \neg$ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$(a, a) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) $= a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (disjoint(a, b) $\Rightarrow$ disjoint(b, a)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \Rightarrow$ set_union$_2$(a, b) $= b$) fof(t12_xboole$_1$, conjecture)
$\forall a$: set_union$_2$(a, empty_set) $= a$ fof(t1_boole, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow a \subseteq c$) fof(t1_xboole$_1$, lemma)
$\forall a$: empty_set $\subseteq a$ fof(t2_xboole$_1$, lemma)
$\forall a, b$: ($\neg\neg$ disjoint(a, b) and $\forall c$: $\neg$ in(c, a) and in(c, b) and $\neg \exists c$: (in(c, a) and in(c, b)) and disjoint(a, b)) fof(t3_xboole$_0$, len
$\forall a$: ($a \subseteq$ empty_set $\Rightarrow a =$ empty_set) fof(t3_xboole$_1$, lemma)
$\forall a, b$: ($\neg\neg$ disjoint(a, b) and $\forall c$: $\neg$ in(c, set_intersection$_2$(a, b)) and $\neg \exists c$: in(c, set_intersection$_2$(a, b)) and disjoint(a, b)) fof(
$\forall a$: (empty(a) $\Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $a \subseteq$ set_union$_2$(a, b) fof(t7_xboole$_1$, lemma)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a, b, c$: (($a \subseteq b$ and $c \subseteq b$) $\Rightarrow$ set_union$_2$(a, c) $\subseteq b$) fof(t8_xboole$_1$, lemma)

SEU127+1.p MPTP bushy problem t17_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2$(a, b) $=$ set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in($c, a$) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c =$ set_intersection$_2$(a, b) $\iff \forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) $= a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: set_intersection$_2$(a, b) $\subseteq a$ fof(t17_xboole$_1$, conjecture)
$\forall a$: set_intersection$_2$(a, empty_set) $=$ empty_set fof(t2_boole, axiom)
$\forall a$: (empty(a) $\Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)

∀a, b: ¬ empty(a) and a ≠ b and empty(b)   fof(t8_boole, axiom)

SEU127+2.p MPTP chainy problem t17_xboole_1
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_union$_2$(a, b) = set_union$_2$(b, a)   fof(commutativity_k2_xboole$_0$, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)   fof(commutativity_k3_xboole$_0$, axiom)
∀a, b: (a = b ⟺ (a ⊆ b and b ⊆ a))   fof(d10_xboole$_0$, axiom)
∀a: (a = empty_set ⟺ ∀b: ¬ in(b, a))   fof(d1_xboole$_0$, axiom)
∀a, b, c: (c = set_union$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) or in(d, b))))   fof(d2_xboole$_0$, axiom)
∀a, b: (a ⊆ b ⟺ ∀c: (in(c, a) ⇒ in(c, b)))   fof(d3_tarski, axiom)
∀a, b, c: (c = set_intersection$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) and in(d, b))))   fof(d3_xboole$_0$, axiom)
∀a, b: (disjoint(a, b) ⟺ set_intersection$_2$(a, b) = empty_set)   fof(d7_xboole$_0$, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$true   fof(dt_k2_xboole$_0$, axiom)
$true   fof(dt_k3_xboole$_0$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a, b: (¬ empty(a) ⇒ ¬ empty(set_union$_2$(a, b)))   fof(fc2_xboole$_0$, axiom)
∀a, b: (¬ empty(a) ⇒ ¬ empty(set_union$_2$(b, a)))   fof(fc3_xboole$_0$, axiom)
∀a, b: set_union$_2$(a, a) = a   fof(idempotence_k2_xboole$_0$, axiom)
∀a, b: set_intersection$_2$(a, a) = a   fof(idempotence_k3_xboole$_0$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: a ⊆ a   fof(reflexivity_r1_tarski, axiom)
∀a, b: (disjoint(a, b) ⇒ disjoint(b, a))   fof(symmetry_r1_xboole$_0$, axiom)
∀a, b: (a ⊆ b ⇒ set_union$_2$(a, b) = b)   fof(t12_xboole$_1$, lemma)
∀a, b: set_intersection$_2$(a, b) ⊆ a   fof(t17_xboole$_1$, conjecture)
∀a: set_union$_2$(a, empty_set) = a   fof(t1_boole, axiom)
∀a, b, c: ((a ⊆ b and b ⊆ c) ⇒ a ⊆ c)   fof(t1_xboole$_1$, lemma)
∀a: set_intersection$_2$(a, empty_set) = empty_set   fof(t2_boole, axiom)
∀a: empty_set ⊆ a   fof(t2_xboole$_1$, lemma)
∀a, b: (¬ ¬ disjoint(a, b) and ∀c: ¬ in(c, a) and in(c, b) and ¬ ∃c: (in(c, a) and in(c, b)) and disjoint(a, b))   fof(t3_xboole$_0$, len
∀a: (a ⊆ empty_set ⇒ a = empty_set)   fof(t3_xboole$_1$, lemma)
∀a, b: (¬ ¬ disjoint(a, b) and ∀c: ¬ in(c, set_intersection$_2$(a, b)) and ¬ ∃c: in(c, set_intersection$_2$(a, b)) and disjoint(a, b))   fof(
∀a: (empty(a) ⇒ a = empty_set)   fof(t6_boole, axiom)
∀a, b: ¬ in(a, b) and empty(b)   fof(t7_boole, axiom)
∀a, b: a ⊆ set_union$_2$(a, b)   fof(t7_xboole$_1$, lemma)
∀a, b: ¬ empty(a) and a ≠ b and empty(b)   fof(t8_boole, axiom)
∀a, b, c: ((a ⊆ b and c ⊆ b) ⇒ set_union$_2$(a, c) ⊆ b)   fof(t8_xboole$_1$, lemma)

SEU128+1.p MPTP bushy problem t19_xboole_1
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)   fof(commutativity_k3_xboole$_0$, axiom)
∀a, b: (a ⊆ b ⟺ ∀c: (in(c, a) ⇒ in(c, b)))   fof(d3_tarski, axiom)
∀a, b, c: (c = set_intersection$_2$(a, b) ⟺ ∀d: (in(d, c) ⟺ (in(d, a) and in(d, b))))   fof(d3_xboole$_0$, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$true   fof(dt_k3_xboole$_0$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a, b: set_intersection$_2$(a, a) = a   fof(idempotence_k3_xboole$_0$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: a ⊆ a   fof(reflexivity_r1_tarski, axiom)
∀a, b, c: ((a ⊆ b and a ⊆ c) ⇒ a ⊆ set_intersection$_2$(b, c))   fof(t19_xboole$_1$, conjecture)
∀a: set_intersection$_2$(a, empty_set) = empty_set   fof(t2_boole, axiom)
∀a: (empty(a) ⇒ a = empty_set)   fof(t6_boole, axiom)
∀a, b: ¬ in(a, b) and empty(b)   fof(t7_boole, axiom)
∀a, b: ¬ empty(a) and a ≠ b and empty(b)   fof(t8_boole, axiom)

SEU129+1.p MPTP bushy problem t26_xboole_1
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a)   fof(commutativity_k3_xboole$_0$, axiom)
∀a, b: (a ⊆ b ⟺ ∀c: (in(c, a) ⇒ in(c, b)))   fof(d3_tarski, axiom)

$\forall a, b, c$: (c = set_intersection$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: ($a \subseteq b \Rightarrow$ set_intersection$_2(a, c) \subseteq$ set_intersection$_2(b, c)$) fof(t26_xboole$_1$, conjecture)
$\forall a$: set_intersection$_2(a$, empty_set) = empty_set fof(t2_boole, axiom)
$\forall a$: (empty$(a) \Rightarrow a$ = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU130+1.p MPTP bushy problem t28_xboole_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2(a, b)$ = set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($a = b \iff$ ($a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: (c = set_intersection$_2(a, b) \iff \forall d$: (in$(d, c) \iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: set_intersection$_2(a, b) \subseteq a$ fof(t17_xboole$_1$, axiom)
$\forall a, b$: ($a \subseteq b \Rightarrow$ set_intersection$_2(a, b) = a$) fof(t28_xboole$_1$, conjecture)
$\forall a$: set_intersection$_2(a$, empty_set) = empty_set fof(t2_boole, axiom)
$\forall a$: (empty$(a) \Rightarrow a$ = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU131+1.p MPTP bushy problem l32_xboole_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: (c = set_difference$(a, b) \iff \forall d$: (in$(d, c) \iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (set_difference(a, b) = empty_set $\iff a \subseteq b$) fof(l32_xboole$_1$, conjecture)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: ($\forall c$: (in$(c, a) \iff$ in(c, b)) $\Rightarrow a = b$) fof(t2_tarski, axiom)
$\forall a$: set_difference$(a$, empty_set$) = a$ fof(t3_boole, axiom)
$\forall a$: set_difference(empty_set, a) = empty_set fof(t4_boole, axiom)
$\forall a$: (empty$(a) \Rightarrow a$ = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU132+1.p MPTP bushy problem t33_xboole_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: (c = set_difference$(a, b) \iff \forall d$: (in$(d, c) \iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)

$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c\colon (a \subseteq b \Rightarrow \text{set\_difference}(a, c) \subseteq \text{set\_difference}(b, c))$ fof(t33_xboole$_1$, conjecture)
$\forall a\colon \text{set\_difference}(a, \text{empty\_set}) = a$ fof(t3_boole, axiom)
$\forall a\colon \text{set\_difference}(\text{empty\_set}, a) = \text{empty\_set}$ fof(t4_boole, axiom)
$\forall a\colon (\text{empty}(a) \Rightarrow a = \text{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b\colon \neg\, \text{in}(a, b)$ and $\text{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b\colon \neg\, \text{empty}(a)$ and $a \neq b$ and $\text{empty}(b)$ fof(t8_boole, axiom)

SEU133+1.p MPTP bushy problem t36_xboole_1
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\, \text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\text{in}(c, a) \Rightarrow \text{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b, c\colon (c = \text{set\_difference}(a, b) \iff \forall d\colon (\text{in}(d, c) \iff (\text{in}(d, a)$ and $\neg\, \text{in}(d, b))))$ fof(d4_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\, \text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b\colon \text{set\_difference}(a, b) \subseteq a$ fof(t36_xboole$_1$, conjecture)
$\forall a\colon \text{set\_difference}(a, \text{empty\_set}) = a$ fof(t3_boole, axiom)
$\forall a\colon \text{set\_difference}(\text{empty\_set}, a) = \text{empty\_set}$ fof(t4_boole, axiom)
$\forall a\colon (\text{empty}(a) \Rightarrow a = \text{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b\colon \neg\, \text{in}(a, b)$ and $\text{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b\colon \neg\, \text{empty}(a)$ and $a \neq b$ and $\text{empty}(b)$ fof(t8_boole, axiom)

SEU134+1.p MPTP bushy problem t37_xboole_1
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\, \text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\, \text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon \neg\, \text{in}(a, b)$ and $\text{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b\colon \neg\, \text{empty}(a)$ and $a \neq b$ and $\text{empty}(b)$ fof(t8_boole, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a\colon \text{set\_difference}(a, \text{empty\_set}) = a$ fof(t3_boole, axiom)
$\forall a\colon \text{set\_difference}(\text{empty\_set}, a) = \text{empty\_set}$ fof(t4_boole, axiom)
$\forall a\colon (\text{empty}(a) \Rightarrow a = \text{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b\colon (\text{set\_difference}(a, b) = \text{empty\_set} \iff a \subseteq b)$ fof(t37_xboole$_1$, conjecture)
$\forall a, b\colon (\text{set\_difference}(a, b) = \text{empty\_set} \iff a \subseteq b)$ fof(l32_xboole$_1$, axiom)

SEU135+1.p MPTP bushy problem t39_xboole_1
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\, \text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \text{set\_union}_2(a, b) = \text{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b\colon (a = b \iff (a \subseteq b$ and $b \subseteq a))$ fof(d10_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \text{set\_union}_2(a, b) \iff \forall d\colon (\text{in}(d, c) \iff (\text{in}(d, a)$ or $\text{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\text{in}(c, a) \Rightarrow \text{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b, c\colon (c = \text{set\_difference}(a, b) \iff \forall d\colon (\text{in}(d, c) \iff (\text{in}(d, a)$ and $\neg\, \text{in}(d, b))))$ fof(d4_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon (\neg\, \text{empty}(a) \Rightarrow \neg\, \text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\, \text{empty}(a) \Rightarrow \neg\, \text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\, \text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a\colon \text{set\_union}_2(a, \text{empty\_set}) = a$ fof(t1_boole, axiom)
$\forall a, b\colon \text{set\_union}_2(a, \text{set\_difference}(b, a)) = \text{set\_union}_2(a, b)$ fof(t39_xboole$_1$, conjecture)
$\forall a\colon \text{set\_difference}(a, \text{empty\_set}) = a$ fof(t3_boole, axiom)

$\forall a$: set_difference(empty_set, a) $=$ empty_set fof(t4_boole, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU136+1.p MPTP bushy problem t40_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2$(a, b) $=$ set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b$: ($a = b$ $\iff$ ($a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
$\forall a, b, c$: ($c =$ set_union$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) or in(d, b)))) fof(d2_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c =$ set_difference(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$(a, a) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: set_union$_2$(a, empty_set) $= a$ fof(t1_boole, axiom)
$\forall a$: set_difference(a, empty_set) $= a$ fof(t3_boole, axiom)
$\forall a, b$: set_difference(set_union$_2$(a, b), b) $=$ set_difference(a, b) fof(t40_xboole$_1$, conjecture)
$\forall a$: set_difference(empty_set, a) $=$ empty_set fof(t4_boole, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU137+1.p MPTP bushy problem t45_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2$(a, b) $=$ set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b, c$: ($c =$ set_union$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) or in(d, b)))) fof(d2_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c =$ set_difference(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$(a, a) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: set_union$_2$(a, empty_set) $= a$ fof(t1_boole, axiom)
$\forall a, b$: ($\forall c$: (in(c, a) $\iff$ in(c, b)) $\Rightarrow$ $a = b$) fof(t2_tarski, axiom)
$\forall a$: set_difference(a, empty_set) $= a$ fof(t3_boole, axiom)
$\forall a, b$: ($a \subseteq b$ $\Rightarrow$ $b =$ set_union$_2$(a, set_difference(b, a))) fof(t45_xboole$_1$, conjecture)
$\forall a$: set_difference(empty_set, a) $=$ empty_set fof(t4_boole, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU138+1.p MPTP bushy problem t48_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2$(a, b) $=$ set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($a = b$ $\iff$ ($a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c =$ set_intersection$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)

$\forall a, b, c$: (c = set_difference(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: set_intersection$_2$(a, empty_set) = empty_set fof(t2_boole, axiom)
$\forall a$: set_difference(a, empty_set) = a fof(t3_boole, axiom)
$\forall a, b$: set_difference(a, set_difference(a, b)) = set_intersection$_2$(a, b) fof(t48_xboole$_1$, conjecture)
$\forall a$: set_difference(empty_set, a) = empty_set fof(t4_boole, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU139+1.p MPTP bushy problem t60_xboole_1
$\forall a, b$: (proper_subset(a, b) $\Rightarrow$ $\neg$ proper_subset(b, a)) fof(antisymmetry_r2_xboole$_0$, axiom)
$\forall a, b$: ($a = b$ $\iff$ ($a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
$\forall a, b$: (proper_subset(a, b) $\iff$ ($a \subseteq b$ and $a \neq b$)) fof(d8_xboole$_0$, axiom)
$\forall a, b$: $\neg$ proper_subset(a, a) fof(irreflexivity_r2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $\neg\, a \subseteq b$ and proper_subset(b, a) fof(t60_xboole$_1$, conjecture)

SEU140+1.p MPTP bushy problem t63_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: (disjoint(a, b) $\iff$ set_intersection$_2$(a, b) = empty_set) fof(d7_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (disjoint(a, b) $\Rightarrow$ disjoint(b, a)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b, c$: ($a \subseteq b$ $\Rightarrow$ set_intersection$_2$(a, c) $\subseteq$ set_intersection$_2$(b, c)) fof(t26_xboole$_1$, axiom)
$\forall a$: set_intersection$_2$(a, empty_set) = empty_set fof(t2_boole, axiom)
$\forall a$: ($a \subseteq$ empty_set $\Rightarrow$ a = empty_set) fof(t3_xboole$_1$, axiom)
$\forall a, b, c$: (($a \subseteq b$ and disjoint(b, c)) $\Rightarrow$ disjoint(a, c)) fof(t63_xboole$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU141+1.p MPTP bushy problem t83_xboole_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: ($a = b$ $\iff$ ($a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: (c = set_intersection$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$\forall a, b, c$: (c = set_difference(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
$\forall a, b$: (disjoint(a, b) $\iff$ set_intersection$_2$(a, b) = empty_set) fof(d7_xboole$_0$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)

$\forall a, b$: (disjoint$(a, b) \Rightarrow$ disjoint(b, a)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a$: set_intersection$_2(a,$ empty_set$)$ = empty_set fof(t2_boole, axiom)
$\forall a$: set_difference$(a,$ empty_set$) = a$ fof(t3_boole, axiom)
$\forall a$: set_difference(empty_set$, a)$ = empty_set fof(t4_boole, axiom)
$\forall a, b$: ($\neg\neg$ disjoint(a, b) and $\forall c$: $\neg$ in$(c,$ set_intersection$_2(a, b))$ and $\neg\exists c$: in$(c,$ set_intersection$_2(a, b))$ and disjoint(a, b)) fof(
$\forall a$: (empty$(a) \Rightarrow a$ = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: (disjoint$(a, b) \iff$ set_difference$(a, b) = a$) fof(t83_xboole$_1$, conjecture)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU142+1.p MPTP bushy problem t69_enumset1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
$\forall a, b, c$: (c = unordered_pair$(a, b) \iff \forall d$: (in$(d, c) \iff (d = a$ or $d = b$))) fof(d2_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k2_tarski, axiom)
$\forall a, b$: ($\forall c$: (in$(c, a) \iff$ in(c, b)) $\Rightarrow a = b$) fof(t2_tarski, axiom)
$\forall a$: unordered_pair(a, a) = singleton(a) fof(t69_enumset$_1$, conjecture)

SEU143+1.p MPTP bushy problem l1_zfmisc_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
$\forall a$: (a = empty_set $\iff \forall b$: $\neg$ in(b, a)) fof(d1_xboole$_0$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a$: singleton$(a) \neq$ empty_set fof(l1_zfmisc$_1$, conjecture)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)

SEU144+1.p MPTP bushy problem l2_zfmisc_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
$\forall a, b$: (singleton$(a) \subseteq b \iff$ in(a, b)) fof(l2_zfmisc$_1$, conjecture)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)

SEU145+1.p MPTP bushy problem l3_zfmisc_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: (c = set_difference$(a, b) \iff \forall d$: (in$(d, c) \iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
$\forall a, b, c$: ($a \subseteq b \Rightarrow$ (in(c, a) or $a \subseteq$ set_difference$(b,$ singleton(c)))) fof(l3_zfmisc$_1$, conjecture)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)

SEU146+1.p MPTP bushy problem l4_zfmisc_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a = b \iff (a \subseteq b$ and $b \subseteq a$)) fof(d10_xboole$_0$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (singleton$(a) \subseteq b \iff$ in(a, b)) fof(l2_zfmisc$_1$, axiom)
$\forall a, b, c$: ($a \subseteq b \Rightarrow$ (in(c, a) or $a \subseteq$ set_difference$(b,$ singleton(c)))) fof(l3_zfmisc$_1$, axiom)
$\forall a, b$: ($a \subseteq$ singleton$(b) \iff$ (a = empty_set or a = singleton(b))) fof(l4_zfmisc$_1$, conjecture)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)

$\forall a$: empty_set $\subseteq a$ fof($t2\_xboole_1$, axiom)
$\forall a, b$: (set_difference(a, b) = empty_set $\iff a \subseteq b$) fof($t37\_xboole_1$, axiom)
$\forall a$: ($a \subseteq$ empty_set $\Rightarrow a$ = empty_set) fof($t3\_xboole_1$, axiom)

SEU147+1.p MPTP bushy problem t1_zfmisc_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: (b = powerset$(a) \iff \forall c$: (in$(c, b) \iff c \subseteq a$)) fof($d1\_zfmisc_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof($dt\_k1\_xboole_0$, axiom)
\$true fof($dt\_k1\_zfmisc_1$, axiom)
empty(empty_set) fof($fc1\_xboole_0$, axiom)
$\exists a$: empty(a) fof($rc1\_xboole_0$, axiom)
$\exists a$: $\neg$ empty(a) fof($rc2\_xboole_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
powerset(empty_set) = singleton(empty_set) fof($t1\_zfmisc_1$, conjecture)
$\forall a, b$: ($\forall c$: (in$(c, a) \iff$ in(c, b)) $\Rightarrow a = b$) fof(t2_tarski, axiom)
$\forall a$: ($a \subseteq$ empty_set $\Rightarrow a$ = empty_set) fof($t3\_xboole_1$, axiom)

SEU147+3.p Basic properties of sets, theorem 1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
$\forall a, b$: (b = powerset$(a) \iff \forall c$: (in$(c, b) \iff c \subseteq a$)) fof($d1\_zfmisc_1$, axiom)
empty(empty_set) fof($fc1\_xboole_0$, axiom)
$\exists a$: empty(a) fof($rc1\_xboole_0$, axiom)
$\exists a$: $\neg$ empty(a) fof($rc2\_xboole_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
powerset(empty_set) = singleton(empty_set) fof($t1\_zfmisc_1$, conjecture)
$\forall a, b$: ($\forall c$: (in$(c, a) \iff$ in(c, b)) $\Rightarrow a = b$) fof(t2_tarski, axiom)
$\forall a$: ($a \subseteq$ empty_set $\Rightarrow a$ = empty_set) fof($t3\_xboole_1$, axiom)

SEU148+1.p MPTP bushy problem t6_zfmisc_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof($dt\_k1\_xboole_0$, axiom)
empty(empty_set) fof($fc1\_xboole_0$, axiom)
$\forall a$: singleton$(a) \neq$ empty_set fof($l1\_zfmisc_1$, axiom)
$\forall a, b$: ($a \subseteq$ singleton$(b) \iff$ (a = empty_set or a = singleton(b))) fof($l4\_zfmisc_1$, axiom)
$\exists a$: empty(a) fof($rc1\_xboole_0$, axiom)
$\exists a$: $\neg$ empty(a) fof($rc2\_xboole_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (singleton$(a) \subseteq$ singleton$(b) \Rightarrow a = b$) fof($t6\_zfmisc_1$, conjecture)

SEU148+3.p Basic properties of sets, theorem 6
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
empty(empty_set) fof($fc1\_xboole_0$, axiom)
$\forall a$: singleton$(a) \neq$ empty_set fof($l1\_zfmisc_1$, axiom)
$\forall a, b$: ($a \subseteq$ singleton$(b) \iff$ (a = empty_set or a = singleton(b))) fof($l4\_zfmisc_1$, axiom)
$\exists a$: empty(a) fof($rc1\_xboole_0$, axiom)
$\exists a$: $\neg$ empty(a) fof($rc2\_xboole_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (singleton$(a) \subseteq$ singleton$(b) \Rightarrow a = b$) fof($t6\_zfmisc_1$, conjecture)

SEU149+1.p MPTP bushy problem t8_zfmisc_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: (b = singleton$(a) \iff \forall c$: (in$(c, b) \iff c = a$)) fof(d1_tarski, axiom)
$\forall a, b, c$: (c = unordered_pair$(a, b) \iff \forall d$: (in$(d, c) \iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k2_tarski, axiom)

$\forall a, b, c$: (singleton(a) = unordered_pair(b, c) $\Rightarrow$ $a = b$) fof(t8_zfmisc$_1$, conjecture)

SEU149+3.p Basic properties of sets, theorem 8
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: (b = singleton(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a, b, c$: (c = unordered_pair(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (singleton(a) = unordered_pair(b, c) $\Rightarrow$ $a = b$) fof(t8_zfmisc$_1$, conjecture)

SEU150+1.p MPTP bushy problem t9_zfmisc_1
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k2_tarski, axiom)
$\forall a, b, c$: (singleton(a) = unordered_pair(b, c) $\Rightarrow$ $a = b$) fof(t8_zfmisc$_1$, axiom)
$\forall a, b, c$: (singleton(a) = unordered_pair(b, c) $\Rightarrow$ $b = c$) fof(t9_zfmisc$_1$, conjecture)

SEU150+3.p Basic properties of sets, theorem
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (singleton(a) = unordered_pair(b, c) $\Rightarrow$ $a = b$) fof(t8_zfmisc$_1$, axiom)
$\forall a, b, c$: (singleton(a) = unordered_pair(b, c) $\Rightarrow$ $b = c$) fof(t9_zfmisc$_1$, conjecture)

SEU151+1.p MPTP bushy problem t10_zfmisc_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: (c = unordered_pair(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)
\$true fof(dt_k2_tarski, axiom)
$\forall a, b, c, d$: $\neg$ unordered_pair(a, b) = unordered_pair(c, d) and $a \neq c$ and $a \neq d$ fof(t10_zfmisc$_1$, conjecture)

SEU151+3.p Basic properties of sets, theorem 10
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: (c = unordered_pair(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c, d$: $\neg$ unordered_pair(a, b) = unordered_pair(c, d) and $a \neq c$ and $a \neq d$ fof(t10_zfmisc$_1$, conjecture)

SEU152+1.p MPTP bushy problem l23_zfmisc_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_union$_2$(a, b) = set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$(a, a) = a fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ set_union$_2$(singleton(a), b) = b) fof(l23_zfmisc$_1$, conjecture)
$\forall a, b$: (singleton(a) $\subseteq b$ $\iff$ in(a, b)) fof(l2_zfmisc$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: ($a \subseteq b$ $\Rightarrow$ set_union$_2$(a, b) = b) fof(t12_xboole$_1$, axiom)

SEU153+1.p MPTP bushy problem l25_zfmisc_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: (b = singleton(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a$: (a = empty_set $\iff$ $\forall b$: $\neg$ in(b, a)) fof(d1_xboole$_0$, axiom)
$\forall a, b, c$: (c = set_intersection$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: (disjoint(a, b) $\iff$ set_intersection$_2$(a, b) = empty_set) fof(d7_xboole$_0$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)

$\forall a, b\colon \neg\,\mathrm{disjoint}(\mathrm{singleton}(a), b) \text{ and } \mathrm{in}(a, b)$ fof(l25_zfmisc$_1$, conjecture)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)

SEU154+1.p MPTP bushy problem l28_zfmisc_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \mathrm{set\_intersection}_2(a, b) = \mathrm{set\_intersection}_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b\colon (a = b \iff (a \subseteq b \text{ and } b \subseteq a))$ fof(d10_xboole$_0$, axiom)
$\forall a, b\colon (b = \mathrm{singleton}(a) \iff \forall c\colon (\mathrm{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b, c\colon (c = \mathrm{set\_intersection}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff (\mathrm{in}(d, a) \text{ and } \mathrm{in}(d, b))))$ fof(d3_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \iff \mathrm{set\_intersection}_2(a, b) = \mathrm{empty\_set})$ fof(d7_xboole$_0$, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k3_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_intersection}_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\mathrm{in}(a, b) \Rightarrow \mathrm{disjoint}(\mathrm{singleton}(a), b))$ fof(l28_zfmisc$_1$, conjecture)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a\colon \mathrm{empty\_set} \subseteq a$ fof(t2_xboole$_1$, axiom)

SEU155+1.p MPTP bushy problem l50_zfmisc_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b\colon (b = \mathrm{union}(a) \iff \forall c\colon (\mathrm{in}(c, b) \iff \exists d\colon (\mathrm{in}(c, d) \text{ and } \mathrm{in}(d, a))))$ fof(d4_tarski, axiom)
$true fof(dt_k3_tarski, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow a \subseteq \mathrm{union}(b))$ fof(l50_zfmisc$_1$, conjecture)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)

SEU156+1.p MPTP bushy problem t33_zfmisc_1
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k2_tarski, axiom)
$true fof(dt_k4_tarski, axiom)
$\forall a, b\colon \neg\,\mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d\colon \neg\,\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(c, d) \text{ and } a \neq c \text{ and } a \neq d$ fof(t10_zfmisc$_1$, axiom)
$\forall a, b, c, d\colon (\mathrm{ordered\_pair}(a, b) = \mathrm{ordered\_pair}(c, d) \Rightarrow (a = c \text{ and } b = d))$ fof(t33_zfmisc$_1$, conjecture)
$\forall a\colon \mathrm{unordered\_pair}(a, a) = \mathrm{singleton}(a)$ fof(t69_enumset$_1$, axiom)
$\forall a, b\colon (\mathrm{singleton}(a) \subseteq \mathrm{singleton}(b) \Rightarrow a = b)$ fof(t6_zfmisc$_1$, axiom)
$\forall a, b, c\colon (\mathrm{singleton}(a) = \mathrm{unordered\_pair}(b, c) \Rightarrow a = b)$ fof(t8_zfmisc$_1$, axiom)
$\forall a, b, c\colon (\mathrm{singleton}(a) = \mathrm{unordered\_pair}(b, c) \Rightarrow b = c)$ fof(t9_zfmisc$_1$, axiom)

SEU156+3.p Basic properties of sets, theorem 33
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\,\mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d\colon \neg\,\mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(c, d) \text{ and } a \neq c \text{ and } a \neq d$ fof(t10_zfmisc$_1$, axiom)
$\forall a, b, c, d\colon (\mathrm{ordered\_pair}(a, b) = \mathrm{ordered\_pair}(c, d) \Rightarrow (a = c \text{ and } b = d))$ fof(t33_zfmisc$_1$, conjecture)
$\forall a\colon \mathrm{unordered\_pair}(a, a) = \mathrm{singleton}(a)$ fof(t69_enumset$_1$, axiom)
$\forall a, b\colon (\mathrm{singleton}(a) \subseteq \mathrm{singleton}(b) \Rightarrow a = b)$ fof(t6_zfmisc$_1$, axiom)

$\forall a, b, c$: (singleton(a) = unordered_pair(b, c) $\Rightarrow$ $a = b$) fof(t8_zfmisc$_1$, axiom)
$\forall a, b, c$: (singleton(a) = unordered_pair(b, c) $\Rightarrow$ $b = c$) fof(t9_zfmisc$_1$, axiom)

SEU157+1.p MPTP bushy problem l55_zfmisc_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: (c = cartesian_product$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ $\exists e, f$: (in(e, a) and in(f, b) and d = ordered_pair(e, f)))) fof
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k2_zfmisc$_1$, axiom)
\$true fof(dt_k4_tarski, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a, b, c, d$: (in(ordered_pair(a, b), cartesian_product$_2$(c, d)) $\iff$ (in(a, c) and in(b, d))) fof(l55_zfmisc$_1$, conjecture)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c, d$: (ordered_pair(a, b) = ordered_pair(c, d) $\Rightarrow$ ($a = c$ and $b = d$)) fof(t33_zfmisc$_1$, axiom)

SEU158+1.p MPTP bushy problem t37_zfmisc_1
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
\$true fof(dt_k1_tarski, axiom)
$\forall a, b$: (singleton(a) $\subseteq b$ $\iff$ in(a, b)) fof(t37_zfmisc$_1$, conjecture)
$\forall a, b$: (singleton(a) $\subseteq b$ $\iff$ in(a, b)) fof(l2_zfmisc$_1$, axiom)

SEU158+3.p Basic properties of sets, theorem 37
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (singleton(a) $\subseteq b$ $\iff$ in(a, b)) fof(t37_zfmisc$_1$, conjecture)
$\forall a, b$: (singleton(a) $\subseteq b$ $\iff$ in(a, b)) fof(l2_zfmisc$_1$, axiom)

SEU159+1.p MPTP bushy problem t38_zfmisc_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: (c = unordered_pair(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
\$true fof(dt_k2_tarski, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: (unordered_pair(a, b) $\subseteq c$ $\iff$ (in(a, c) and in(b, c))) fof(t38_zfmisc$_1$, conjecture)

SEU159+3.p Basic properties of sets, theorem 38
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: (c = unordered_pair(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ ($d = a$ or $d = b$))) fof(d2_tarski, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: (unordered_pair(a, b) $\subseteq c$ $\iff$ (in(a, c) and in(b, c))) fof(t38_zfmisc$_1$, conjecture)

SEU160+1.p MPTP bushy problem t39_zfmisc_1
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($a \subseteq$ singleton(b) $\iff$ (a = empty_set or a = singleton(b))) fof(t39_zfmisc$_1$, conjecture)
$\forall a, b$: ($a \subseteq$ singleton(b) $\iff$ (a = empty_set or a = singleton(b))) fof(l4_zfmisc$_1$, axiom)

SEU160+3.p Basic properties of sets, theorem 39

$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (a \subseteq \mathrm{singleton}(b) \iff (a = \mathrm{empty\_set} \text{ or } a = \mathrm{singleton}(b)))$ fof(t39_zfmisc$_1$, conjecture)
$\forall a, b\colon (a \subseteq \mathrm{singleton}(b) \iff (a = \mathrm{empty\_set} \text{ or } a = \mathrm{singleton}(b)))$ fof(l4_zfmisc$_1$, axiom)

SEU161+1.p MPTP bushy problem t46_zfmisc_1
$\forall a, b\colon \mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \mathrm{set\_union}_2(\mathrm{singleton}(a), b) = b)$ fof(t46_zfmisc$_1$, conjecture)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \mathrm{set\_union}_2(\mathrm{singleton}(a), b) = b)$ fof(l23_zfmisc$_1$, axiom)

SEU161+3.p Basic properties of sets, theorem 46
$\forall a, b\colon (\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon (\neg\,\mathrm{empty}(a) \Rightarrow \neg\,\mathrm{empty}(\mathrm{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, b) = \mathrm{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b\colon \mathrm{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \mathrm{set\_union}_2(\mathrm{singleton}(a), b) = b)$ fof(t46_zfmisc$_1$, conjecture)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \mathrm{set\_union}_2(\mathrm{singleton}(a), b) = b)$ fof(l23_zfmisc$_1$, axiom)

SEU162+1.p MPTP bushy problem t65_zfmisc_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
$\forall a, b\colon \neg\,\mathrm{disjoint}(\mathrm{singleton}(a), b) \text{ and } \mathrm{in}(a, b)$ fof(l25_zfmisc$_1$, axiom)
$\forall a, b\colon (\neg\,\mathrm{in}(a, b) \Rightarrow \mathrm{disjoint}(\mathrm{singleton}(a), b))$ fof(l28_zfmisc$_1$, axiom)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{set\_difference}(a, \mathrm{singleton}(b)) = a \iff \neg\,\mathrm{in}(b, a))$ fof(t65_zfmisc$_1$, conjecture)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \iff \mathrm{set\_difference}(a, b) = a)$ fof(t83_xboole$_1$, axiom)

SEU162+3.p Basic properties of sets, theorem 65
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \neg\,\mathrm{disjoint}(\mathrm{singleton}(a), b) \text{ and } \mathrm{in}(a, b)$ fof(l25_zfmisc$_1$, axiom)
$\forall a, b\colon (\neg\,\mathrm{in}(a, b) \Rightarrow \mathrm{disjoint}(\mathrm{singleton}(a), b))$ fof(l28_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \Rightarrow \mathrm{disjoint}(b, a))$ fof(symmetry_r1_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{set\_difference}(a, \mathrm{singleton}(b)) = a \iff \neg\,\mathrm{in}(b, a))$ fof(t65_zfmisc$_1$, conjecture)
$\forall a, b\colon (\mathrm{disjoint}(a, b) \iff \mathrm{set\_difference}(a, b) = a)$ fof(t83_xboole$_1$, axiom)

SEU163+1.p MPTP bushy problem t92_zfmisc_1
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
\$true fof(dt_k3_tarski, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow a \subseteq \mathrm{union}(b))$ fof(t92_zfmisc$_1$, conjecture)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow a \subseteq \mathrm{union}(b))$ fof(l50_zfmisc$_1$, axiom)

SEU163+3.p Basic properties of sets, theorem 92
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow a \subseteq \mathrm{union}(b))$ fof(t92_zfmisc$_1$, conjecture)
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow a \subseteq \mathrm{union}(b))$ fof(l50_zfmisc$_1$, axiom)

SEU164+1.p MPTP bushy problem t99_zfmisc_1

$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $(b = \text{singleton}(a) \iff \forall c$: $(\text{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b$: $(b = \text{powerset}(a) \iff \forall c$: $(\text{in}(c, b) \iff c \subseteq a))$ fof(d1_zfmisc$_1$, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c$: $(\text{in}(c, a) \Rightarrow \text{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b$: $(b = \text{union}(a) \iff \forall c$: $(\text{in}(c, b) \iff \exists d$: $(\text{in}(c, d)$ and $\text{in}(d, a))))$ fof(d4_tarski, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k1_zfmisc$_1$, axiom)
$true fof(dt_k3_tarski, axiom)
$\forall a, b$: $(\text{singleton}(a) \subseteq b \iff \text{in}(a, b))$ fof(l2_zfmisc$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $(\forall c$: $(\text{in}(c, a) \iff \text{in}(c, b)) \Rightarrow a = b)$ fof(t2_tarski, axiom)
$\forall a$: $\text{union}(\text{powerset}(a)) = a$ fof(t99_zfmisc$_1$, conjecture)

SEU164+3.p Basic properties of sets, theorem 99
$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $(b = \text{singleton}(a) \iff \forall c$: $(\text{in}(c, b) \iff c = a))$ fof(d1_tarski, axiom)
$\forall a, b$: $(b = \text{powerset}(a) \iff \forall c$: $(\text{in}(c, b) \iff c \subseteq a))$ fof(d1_zfmisc$_1$, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c$: $(\text{in}(c, a) \Rightarrow \text{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b$: $(b = \text{union}(a) \iff \forall c$: $(\text{in}(c, b) \iff \exists d$: $(\text{in}(c, d)$ and $\text{in}(d, a))))$ fof(d4_tarski, axiom)
$\forall a, b$: $(\text{singleton}(a) \subseteq b \iff \text{in}(a, b))$ fof(l2_zfmisc$_1$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: $(\forall c$: $(\text{in}(c, a) \iff \text{in}(c, b)) \Rightarrow a = b)$ fof(t2_tarski, axiom)
$\forall a$: $\text{union}(\text{powerset}(a)) = a$ fof(t99_zfmisc$_1$, conjecture)

SEU165+1.p MPTP bushy problem t106_zfmisc_1
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k2_tarski, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$true fof(dt_k2_zfmisc$_1$, axiom)
$true fof(dt_k4_tarski, axiom)
$\forall a, b$: $\neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b$: $\text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b, c, d$: $(\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, d)) \iff (\text{in}(a, c)$ and $\text{in}(b, d)))$ fof(t106_zfmisc$_1$, conjecture)
$\forall a, b, c, d$: $(\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, d)) \iff (\text{in}(a, c)$ and $\text{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)

SEU165+3.p Basic properties of sets, theorem 106
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a$: $\text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $\text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b, c, d$: $(\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, d)) \iff (\text{in}(a, c)$ and $\text{in}(b, d)))$ fof(t106_zfmisc$_1$, conjecture)
$\forall a, b, c, d$: $(\text{in}(\text{ordered\_pair}(a, b), \text{cartesian\_product}_2(c, d)) \iff (\text{in}(a, c)$ and $\text{in}(b, d)))$ fof(l55_zfmisc$_1$, axiom)

SEU166+1.p MPTP bushy problem t118_zfmisc_1
$\forall a, b$: $(\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: $\text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b, c$: $(c = \text{cartesian\_product}_2(a, b) \iff \forall d$: $(\text{in}(d, c) \iff \exists e, f$: $(\text{in}(e, a)$ and $\text{in}(f, b)$ and $d = \text{ordered\_pair}(e, f))))$ fof
$\forall a, b$: $(a \subseteq b \iff \forall c$: $(\text{in}(c, a) \Rightarrow \text{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b$: $\text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k2_tarski, axiom)
$true fof(dt_k2_zfmisc$_1$, axiom)
$true fof(dt_k4_tarski, axiom)
$\forall a, b$: $\neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)

$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c\colon (a \subseteq b \Rightarrow (\text{cartesian\_product}_2(a, c) \subseteq \text{cartesian\_product}_2(b, c)$ and $\text{cartesian\_product}_2(c, a) \subseteq \text{cartesian\_product}_2(c, b$

SEU166+3.p Basic properties of sets, theorem 118
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b, c\colon (c = \text{cartesian\_product}_2(a, b) \iff \forall d\colon (\mathrm{in}(d, c) \iff \exists e, f\colon (\mathrm{in}(e, a)$ and $\mathrm{in}(f, b)$ and $d = \text{ordered\_pair}(e, f))))$ fof
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\forall a, b\colon \text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a, b\colon \neg\,\mathrm{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c\colon (a \subseteq b \Rightarrow (\text{cartesian\_product}_2(a, c) \subseteq \text{cartesian\_product}_2(b, c)$ and $\text{cartesian\_product}_2(c, a) \subseteq \text{cartesian\_product}_2(c, b$

SEU167+1.p MPTP bushy problem t119_zfmisc_1
\$true fof(dt_k2_zfmisc$_1$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c\colon (a \subseteq b \Rightarrow (\text{cartesian\_product}_2(a, c) \subseteq \text{cartesian\_product}_2(b, c)$ and $\text{cartesian\_product}_2(c, a) \subseteq \text{cartesian\_product}_2(c, b$
$\forall a, b, c, d\colon ((a \subseteq b$ and $c \subseteq d) \Rightarrow \text{cartesian\_product}_2(a, c) \subseteq \text{cartesian\_product}_2(b, d))$ fof(t119_zfmisc$_1$, conjecture)
$\forall a, b, c\colon ((a \subseteq b$ and $b \subseteq c) \Rightarrow a \subseteq c)$ fof(t1_xboole$_1$, axiom)

SEU167+3.p Basic properties of sets, theorem 119
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c\colon (a \subseteq b \Rightarrow (\text{cartesian\_product}_2(a, c) \subseteq \text{cartesian\_product}_2(b, c)$ and $\text{cartesian\_product}_2(c, a) \subseteq \text{cartesian\_product}_2(c, b$
$\forall a, b, c, d\colon ((a \subseteq b$ and $c \subseteq d) \Rightarrow \text{cartesian\_product}_2(a, c) \subseteq \text{cartesian\_product}_2(b, d))$ fof(t119_zfmisc$_1$, conjecture)
$\forall a, b, c\colon ((a \subseteq b$ and $b \subseteq c) \Rightarrow a \subseteq c)$ fof(t1_xboole$_1$, axiom)

SEU168+1.p MPTP bushy problem t136_zfmisc_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon (b = \mathrm{powerset}(a) \iff \forall c\colon (\mathrm{in}(c, b) \iff c \subseteq a))$ fof(d1_zfmisc$_1$, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a\colon \exists b\colon (\mathrm{in}(a, b)$ and $\forall c, d\colon ((\mathrm{in}(c, b)$ and $d \subseteq c) \Rightarrow \mathrm{in}(d, b))$ and $\forall c\colon (\mathrm{in}(c, b) \Rightarrow \mathrm{in}(\mathrm{powerset}(c), b))$ and $\forall c\colon \neg\, c \subseteq b$ and $\neg\,\text{are\_equipotent}(c, b)$ and $\neg\,\mathrm{in}(c, b))$ fof(t136_zfmisc$_1$, conjecture)
$\forall a\colon \exists b\colon (\mathrm{in}(a, b)$ and $\forall c, d\colon ((\mathrm{in}(c, b)$ and $d \subseteq c) \Rightarrow \mathrm{in}(d, b))$ and $\forall c\colon \neg\,\mathrm{in}(c, b)$ and $\forall d\colon \neg\,\mathrm{in}(d, b)$ and $\forall e\colon (e \subseteq c \Rightarrow \mathrm{in}(e, d))$ and $\forall c\colon \neg\, c \subseteq b$ and $\neg\,\text{are\_equipotent}(c, b)$ and $\neg\,\mathrm{in}(c, b))$ fof(t9_tarski, axiom)

SEU168+3.p Basic properties of sets, theorem 136
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon (b = \mathrm{powerset}(a) \iff \forall c\colon (\mathrm{in}(c, b) \iff c \subseteq a))$ fof(d1_zfmisc$_1$, axiom)
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
$\exists a\colon \mathrm{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\mathrm{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a\colon \exists b\colon (\mathrm{in}(a, b)$ and $\forall c, d\colon ((\mathrm{in}(c, b)$ and $d \subseteq c) \Rightarrow \mathrm{in}(d, b))$ and $\forall c\colon (\mathrm{in}(c, b) \Rightarrow \mathrm{in}(\mathrm{powerset}(c), b))$ and $\forall c\colon \neg\, c \subseteq b$ and $\neg\,\text{are\_equipotent}(c, b)$ and $\neg\,\mathrm{in}(c, b))$ fof(t136_zfmisc$_1$, conjecture)
$\forall a\colon \exists b\colon (\mathrm{in}(a, b)$ and $\forall c, d\colon ((\mathrm{in}(c, b)$ and $d \subseteq c) \Rightarrow \mathrm{in}(d, b))$ and $\forall c\colon \neg\,\mathrm{in}(c, b)$ and $\forall d\colon \neg\,\mathrm{in}(d, b)$ and $\forall e\colon (e \subseteq c \Rightarrow \mathrm{in}(e, d))$ and $\forall c\colon \neg\, c \subseteq b$ and $\neg\,\text{are\_equipotent}(c, b)$ and $\neg\,\mathrm{in}(c, b))$ fof(t9_tarski, axiom)

SEU169+1.p MPTP bushy problem l3_subset_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg\,\mathrm{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon (b = \mathrm{powerset}(a) \iff \forall c\colon (\mathrm{in}(c, b) \iff c \subseteq a))$ fof(d1_zfmisc$_1$, axiom)
$\forall a, b\colon ((\neg\,\mathrm{empty}(a) \Rightarrow (\mathrm{element}(b, a) \iff \mathrm{in}(b, a)))$ and $(\mathrm{empty}(a) \Rightarrow (\mathrm{element}(b, a) \iff \mathrm{empty}(b))))$ fof(d2_subset$_1$, a
$\forall a, b\colon (a \subseteq b \iff \forall c\colon (\mathrm{in}(c, a) \Rightarrow \mathrm{in}(c, b)))$ fof(d3_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)

$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ $\forall c$: (in(c, b) $\Rightarrow$ in(c, a))) fof(l3_subset$_1$, conjecture)
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element$(b$, powerset$(a))$ and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU170+1.p MPTP bushy problem t43_subset_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b, c$: ($c =$ set_difference(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ subset_complement$(a, b) =$ set_difference(a, b)) fof(d5_subset$_1$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k1_zfmisc$_1$, axiom)
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ element(subset_complement(a, b), powerset(a))) fof(dt_k3_subset$_1$, axiom)
$true fof(dt_k4_xboole$_0$, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ subset_complement$(a$, subset_complement$(a, b)) = b$) fof(involutiveness_k3_subset$_1$, axio
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ $\forall c$: (in(c, b) $\Rightarrow$ in(c, a))) fof(l3_subset$_1$, axiom)
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element$(b$, powerset$(a))$ and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\forall a$: $\exists b$: (element$(b$, powerset$(a))$ and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (disjoint(a, b) $\Rightarrow$ disjoint(b, a)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a$: set_difference$(a$, empty_set$) = a$ fof(t3_boole, axiom)
$\forall a, b$: ($\neg\neg$ disjoint(a, b) and $\forall c$: $\neg$ in(c, a) and in(c, b) and $\neg\exists c$: (in(c, a) and in(c, b)) and disjoint(a, b)) fof(t3_xboole$_0$, axi
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ $\forall c$: (element$(c$, powerset$(a))$ $\Rightarrow$ (disjoint(b, c) $\iff$ $b \subseteq$ subset_complement(a, c)))) fof
$\forall a$: set_difference(empty_set$, a) =$ empty_set fof(t4_boole, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU171+1.p MPTP bushy problem t50_subset_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (($\neg$ empty(a) $\Rightarrow$ (element(b, a) $\iff$ in(b, a))) and (empty(a) $\Rightarrow$ (element(b, a) $\iff$ empty(b)))) fof(d2_subset$_1$, a
$\forall a, b, c$: ($c =$ set_difference(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ subset_complement$(a, b) =$ set_difference(a, b)) fof(d5_subset$_1$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k1_zfmisc$_1$, axiom)
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ element(subset_complement(a, b), powerset(a))) fof(dt_k3_subset$_1$, axiom)
$true fof(dt_k4_xboole$_0$, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (element$(b$, powerset$(a))$ $\Rightarrow$ subset_complement$(a$, subset_complement$(a, b)) = b$) fof(involutiveness_k3_subset$_1$, axio
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element$(b$, powerset$(a))$ and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\forall a$: $\exists b$: (element$(b$, powerset$(a))$ and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a$: set_difference$(a$, empty_set$) = a$ fof(t3_boole, axiom)

$\forall a$: set_difference(empty_set, a) = empty_set fof(t4_boole, axiom)
$\forall a$: ($a \neq$ empty_set $\Rightarrow \forall b$: (element(b, powerset(a)) $\Rightarrow \forall c$: (element(c, a) $\Rightarrow$ ($\neg$ in(c, b) $\Rightarrow$ in(c, subset_complement(a, b)))))
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU172+1.p MPTP bushy problem t54_subset_1
\$true fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a$: set_difference(a, empty_set) = a fof(t3_boole, axiom)
$\forall a$: set_difference(empty_set, a) = empty_set fof(t4_boole, axiom)
$\forall a$: ($\neg$ empty(a) $\Rightarrow \exists b$: (element(b, powerset(a)) and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ subset_complement(a, subset_complement(a, b)) = b) fof(involutiveness_k3_subset$_1$, axio
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ element(subset_complement(a, b), powerset(a))) fof(dt_k3_subset$_1$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ subset_complement(a, b) = set_difference(a, b)) fof(d5_subset$_1$, axiom)
$\forall a, b, c$: (element(c, powerset(a)) $\Rightarrow$ $\neg$ in(b, subset_complement(a, c)) and in(b, c)) fof(t54_subset$_1$, conjecture)
$\forall a, b, c$: (c = set_difference(a, b) $\iff \forall d$: (in(d, c) $\iff$ (in(d, a) and $\neg$ in(d, b)))) fof(d4_xboole$_0$, axiom)

SEU173+1.p MPTP bushy problem l71_subset_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (b = powerset(a) $\iff \forall c$: (in(c, b) $\iff$ $c \subseteq a$)) fof(d1_zfmisc$_1$, axiom)
$\forall a, b$: (($\neg$ empty(a) $\Rightarrow$ (element(b, a) $\iff$ in(b, a))) and (empty(a) $\Rightarrow$ (element(b, a) $\iff$ empty(b)))) fof(d2_subset$_1$, a
$\forall a, b$: ($a \subseteq b$ $\iff \forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\forall c$: (in(c, a) $\Rightarrow$ in(c, b)) $\Rightarrow$ element(a, powerset(b))) fof(l71_subset$_1$, conjecture)
$\forall a$: ($\neg$ empty(a) $\Rightarrow \exists b$: (element(b, powerset(a)) and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU174+1.p MPTP bushy problem t46_setfam_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff \forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b$: (element(b, powerset(powerset(a))) $\Rightarrow \forall c$: (element(c, powerset(powerset(a))) $\Rightarrow$ (c = complements_of_subsets(a, b) $\Leftarrow$
$\forall d$: (element(d, powerset(a)) $\Rightarrow$ (in(d, c) $\iff$ in(subset_complement(a, d), b)))))) fof(d8_setfam$_1$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ element(subset_complement(a, b), powerset(a))) fof(dt_k3_subset$_1$, axiom)
$\forall a, b$: (element(b, powerset(powerset(a))) $\Rightarrow$ element(complements_of_subsets(a, b), powerset(powerset(a)))) fof(dt_k7_set
\$true fof(dt_m1_subset$_1$, axiom)

$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (element$(b$, powerset$(a)) \Rightarrow$ subset_complement$(a$, subset_complement$(a, b)) = b)$ fof(involutiveness_k3_subset$_1$, axio
$\forall a, b$: (element$(b$, powerset(powerset$(a))) \Rightarrow$ complements_of_subsets$(a$, complements_of_subsets$(a, b)) = b)$ fof(involutive
$\forall a$: $(\neg$empty$(a) \Rightarrow \exists b$: (element$(b$, powerset$(a))$ and $\neg$empty$(b)))$ fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\forall a$: $\exists b$: (element$(b$, powerset$(a))$ and empty$(b))$ fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow$ element$(a, b))$ fof(t1_subset, axiom)
$\forall a, b$: (element$(a, b) \Rightarrow$ (empty(b) or in$(a, b)))$ fof(t2_subset, axiom)
$\forall a, b$: (element$(a$, powerset$(b)) \iff a \subseteq b)$ fof(t3_subset, axiom)
$\forall a, b$: (element$(b$, powerset(powerset$(a))) \Rightarrow \neg b \neq$ empty_set and complements_of_subsets$(a, b) =$ empty_set) fof(t46_setf
$\forall a, b, c$: ((in(a, b) and element$(b$, powerset$(c))) \Rightarrow$ element$(a, c))$ fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$in(a, b) and element$(b$, powerset$(c))$ and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty$(a) \Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU177+1.p MPTP bushy problem t20_relat_1
$true fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k2_tarski, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$\exists a$: (empty(a) and relation$(a))$ fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a$: $\neg$empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
$\forall a, b$: (element$(a, b) \Rightarrow$ (empty(b) or in$(a, b)))$ fof(t2_subset, axiom)
$\forall a$: (empty$(a) \Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$in$(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$true fof(dt_k1_relat$_1$, axiom)
$true fof(dt_k2_relat$_1$, axiom)
$true fof(dt_k4_tarski, axiom)
$\forall a, b$: $\neg$empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow$ element$(a, b))$ fof(t1_subset, axiom)
$\forall a, b$: $\neg$in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: ordered_pair$(a, b) =$ unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
$\forall a, b, c$: (relation$(c) \Rightarrow$ (in(ordered_pair$(a, b), c) \Rightarrow$ (in$(a$, relation_dom$(c))$ and in$(b$, relation_rng$(c)))))$ fof(t20_relat$_1$, con
$\forall a$: (relation$(a) \Rightarrow \forall b$: $(b =$ relation_dom$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: in(ordered_pair$(c, d), a))))$ fof(d4_relat$_1$, axiom)
$\forall a$: (relation$(a) \Rightarrow \forall b$: $(b =$ relation_rng$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: in(ordered_pair$(d, c), a))))$ fof(d5_relat$_1$, axiom)

SEU179+1.p MPTP bushy problem t25_relat_1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$in$(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: $(a \subseteq b \iff \forall c$: (in$(c, a) \Rightarrow$ in$(c, b)))$ fof(d3_tarski, axiom)
$\forall a$: (relation$(a) \Rightarrow \forall b$: $(b =$ relation_dom$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: in(ordered_pair$(c, d), a))))$ fof(d4_relat$_1$, axiom)
$\forall a$: (relation$(a) \Rightarrow \forall b$: $(b =$ relation_rng$(a) \iff \forall c$: (in$(c, b) \iff \exists d$: in(ordered_pair$(d, c), a))))$ fof(d5_relat$_1$, axiom)
$\forall a, b$: ordered_pair$(a, b) =$ unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
$true fof(dt_k1_relat$_1$, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k1_zfmisc$_1$, axiom)
$true fof(dt_k2_relat$_1$, axiom)

\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k4_tarski, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a\colon \exists b\colon \text{element}(b, a)$ fof(existence_m1_subset$_1$, axiom)
$\forall a\colon \neg\,\text{empty}(\text{powerset}(a))$ fof(fc1_subset$_1$, axiom)
$\text{empty}(\text{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon \neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a\colon \neg\,\text{empty}(\text{singleton}(a))$ fof(fc2_subset$_1$, axiom)
$\forall a, b\colon \neg\,\text{empty}(\text{unordered\_pair}(a, b))$ fof(fc3_subset$_1$, axiom)
$\exists a\colon (\text{empty}(a) \text{ and } \text{relation}(a))$ fof(rc1_relat$_1$, axiom)
$\forall a\colon (\neg\,\text{empty}(a) \Rightarrow \exists b\colon (\text{element}(b, \text{powerset}(a)) \text{ and } \neg\,\text{empty}(b)))$ fof(rc1_subset$_1$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\forall a\colon \exists b\colon (\text{element}(b, \text{powerset}(a)) \text{ and } \text{empty}(b))$ fof(rc2_subset$_1$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a, b\colon a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \text{element}(a, b))$ fof(t1_subset, axiom)
$\forall a\colon (\text{relation}(a) \Rightarrow \forall b\colon (\text{relation}(b) \Rightarrow (a \subseteq b \Rightarrow (\text{relation\_dom}(a) \subseteq \text{relation\_dom}(b) \text{ and } \text{relation\_rng}(a) \subseteq \text{relation\_rng}(b)))))$ fof(t25_relat$_1$, conjecture)
$\forall a, b\colon (\text{element}(a, b) \Rightarrow (\text{empty}(b) \text{ or } \text{in}(a, b)))$ fof(t2_subset, axiom)
$\forall a, b\colon (\text{element}(a, \text{powerset}(b)) \iff a \subseteq b)$ fof(t3_subset, axiom)
$\forall a, b, c\colon ((\text{in}(a, b) \text{ and } \text{element}(b, \text{powerset}(c))) \Rightarrow \text{element}(a, c))$ fof(t4_subset, axiom)
$\forall a, b, c\colon \neg\,\text{in}(a, b) \text{ and } \text{element}(b, \text{powerset}(c)) \text{ and } \text{empty}(c)$ fof(t5_subset, axiom)
$\forall a\colon (\text{empty}(a) \Rightarrow a = \text{empty\_set})$ fof(t6_boole, axiom)
$\forall a, b\colon \neg\,\text{in}(a, b) \text{ and } \text{empty}(b)$ fof(t7_boole, axiom)
$\forall a, b\colon \neg\,\text{empty}(a) \text{ and } a \neq b \text{ and } \text{empty}(b)$ fof(t8_boole, axiom)

SEU180+1.p MPTP bushy problem t30_relat_1
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \neg\,\text{in}(b, a))$ fof(antisymmetry_r2_hidden, axiom)
$\forall a, b\colon \text{unordered\_pair}(a, b) = \text{unordered\_pair}(b, a)$ fof(commutativity_k2_tarski, axiom)
$\forall a, b\colon \text{set\_union}_2(a, b) = \text{set\_union}_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a, b, c\colon (c = \text{set\_union}_2(a, b) \iff \forall d\colon (\text{in}(d, c) \iff (\text{in}(d, a) \text{ or } \text{in}(d, b))))$ fof(d2_xboole$_0$, axiom)
$\forall a\colon (\text{relation}(a) \Rightarrow \forall b\colon (b = \text{relation\_dom}(a) \iff \forall c\colon (\text{in}(c, b) \iff \exists d\colon \text{in}(\text{ordered\_pair}(c, d), a))))$ fof(d4_relat$_1$, axiom)
$\forall a\colon (\text{relation}(a) \Rightarrow \forall b\colon (b = \text{relation\_rng}(a) \iff \forall c\colon (\text{in}(c, b) \iff \exists d\colon \text{in}(\text{ordered\_pair}(d, c), a))))$ fof(d5_relat$_1$, axiom)
$\forall a, b\colon \text{ordered\_pair}(a, b) = \text{unordered\_pair}(\text{unordered\_pair}(a, b), \text{singleton}(a))$ fof(d5_tarski, axiom)
$\forall a\colon (\text{relation}(a) \Rightarrow \text{relation\_field}(a) = \text{set\_union}_2(\text{relation\_dom}(a), \text{relation\_rng}(a)))$ fof(d6_relat$_1$, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k3_relat$_1$, axiom)
\$true fof(dt_k4_tarski, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a\colon \exists b\colon \text{element}(b, a)$ fof(existence_m1_subset$_1$, axiom)
$\text{empty}(\text{empty\_set})$ fof(fc1_xboole$_0$, axiom)
$\forall a, b\colon \neg\,\text{empty}(\text{ordered\_pair}(a, b))$ fof(fc1_zfmisc$_1$, axiom)
$\forall a, b\colon ((\text{relation}(a) \text{ and } \text{relation}(b)) \Rightarrow \text{relation}(\text{set\_union}_2(a, b)))$ fof(fc2_relat$_1$, axiom)
$\forall a\colon \neg\,\text{empty}(\text{singleton}(a))$ fof(fc2_subset$_1$, axiom)
$\forall a, b\colon (\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(a, b)))$ fof(fc2_xboole$_0$, axiom)
$\forall a, b\colon \neg\,\text{empty}(\text{unordered\_pair}(a, b))$ fof(fc3_subset$_1$, axiom)
$\forall a, b\colon (\neg\,\text{empty}(a) \Rightarrow \neg\,\text{empty}(\text{set\_union}_2(b, a)))$ fof(fc3_xboole$_0$, axiom)
$\forall a, b\colon \text{set\_union}_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a\colon (\text{empty}(a) \text{ and } \text{relation}(a))$ fof(rc1_relat$_1$, axiom)
$\exists a\colon \text{empty}(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a\colon \neg\,\text{empty}(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a\colon \text{set\_union}_2(a, \text{empty\_set}) = a$ fof(t1_boole, axiom)
$\forall a, b\colon (\text{in}(a, b) \Rightarrow \text{element}(a, b))$ fof(t1_subset, axiom)
$\forall a, b\colon (\text{element}(a, b) \Rightarrow (\text{empty}(b) \text{ or } \text{in}(a, b)))$ fof(t2_subset, axiom)
$\forall a, b, c\colon (\text{relation}(c) \Rightarrow (\text{in}(\text{ordered\_pair}(a, b), c) \Rightarrow (\text{in}(a, \text{relation\_field}(c)) \text{ and } \text{in}(b, \text{relation\_field}(c)))))$ fof(t30_relat$_1$, co

$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set)   fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b)   fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b)   fof(t8_boole, axiom)

SEU182+1.p MPTP bushy problem t44_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a)   fof(commutativity_k2_tarski, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in($c, a$) $\Rightarrow$ in(c, b)))   fof(d3_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: ($b =$ relation_dom(a) $\iff \forall c$: (in(c, b) $\iff \exists d$: in(ordered_pair(c, d), a))))   fof(d4_relat$_1$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ $\forall c$: (relation(c) $\Rightarrow$ ($c =$ relation_composition(a, b) $\iff \forall d, e$: (in(ordered_pair(d, e), c
$\exists f$: (in(ordered_pair(d, f), a) and in(ordered_pair(f, e), b)))))))   fof(d8_relat$_1$, axiom)
\$true   fof(dt_k1_relat$_1$, axiom)
\$true   fof(dt_k1_tarski, axiom)
\$true   fof(dt_k1_xboole$_0$, axiom)
\$true   fof(dt_k1_zfmisc$_1$, axiom)
\$true   fof(dt_k2_tarski, axiom)
\$true   fof(dt_k4_tarski, axiom)
$\forall a, b$: ((relation(a) and relation(b)) $\Rightarrow$ relation(relation_composition(a, b)))   fof(dt_k5_relat$_1$, axiom)
\$true   fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a)   fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a))   fof(fc1_subset$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a))   fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b))   fof(fc3_subset$_1$, axiom)
$\exists a$: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element(b, powerset(a)) and $\neg$ empty(b)))   fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a)   fof(rc1_xboole$_0$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b))   fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a)   fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b))   fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff a \subseteq b$)   fof(t3_subset, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ relation_dom(relation_composition(a, b)) $\subseteq$ relation_dom(a)))   fof(t44_relat$_1$, conjec
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c))   fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c)   fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set)   fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b)   fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b)   fof(t8_boole, axiom)

SEU183+1.p MPTP bushy problem t45_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a)   fof(commutativity_k2_tarski, axiom)
$\forall a, b$: ($a \subseteq b \iff \forall c$: (in($c, a$) $\Rightarrow$ in(c, b)))   fof(d3_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: ($b =$ relation_rng(a) $\iff \forall c$: (in(c, b) $\iff \exists d$: in(ordered_pair(d, c), a))))   fof(d5_relat$_1$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ $\forall c$: (relation(c) $\Rightarrow$ ($c =$ relation_composition(a, b) $\iff \forall d, e$: (in(ordered_pair(d, e), c
$\exists f$: (in(ordered_pair(d, f), a) and in(ordered_pair(f, e), b)))))))   fof(d8_relat$_1$, axiom)
\$true   fof(dt_k1_tarski, axiom)
\$true   fof(dt_k1_xboole$_0$, axiom)
\$true   fof(dt_k1_zfmisc$_1$, axiom)
\$true   fof(dt_k2_relat$_1$, axiom)
\$true   fof(dt_k2_tarski, axiom)
\$true   fof(dt_k4_tarski, axiom)
$\forall a, b$: ((relation(a) and relation(b)) $\Rightarrow$ relation(relation_composition(a, b)))   fof(dt_k5_relat$_1$, axiom)
\$true   fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a)   fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a))   fof(fc1_subset$_1$, axiom)

empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a))   fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b))   fof(fc3_subset$_1$, axiom)
$\exists a$: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element(b, powerset(a)) and $\neg$ empty(b)))   fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a)   fof(rc1_xboole$_0$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b))   fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a)   fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b))   fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$)   fof(t3_subset, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ relation_rng(relation_composition(a, b)) $\subseteq$ relation_rng(b)))   fof(t45_relat$_1$, conjectu
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c))   fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c)   fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set)   fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b)   fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b)   fof(t8_boole, axiom)

SEU186+1.p MPTP bushy problem t56_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a))   fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a)   fof(commutativity_k2_tarski, axiom)
$\forall a$: (relation(a) $\iff$ $\forall b$: $\neg$ in(b, a) and $\forall c, d$: $b \neq$ ordered_pair(c, d))   fof(d1_relat$_1$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
\$true   fof(dt_k1_tarski, axiom)
\$true   fof(dt_k1_xboole$_0$, axiom)
\$true   fof(dt_k2_tarski, axiom)
\$true   fof(dt_k4_tarski, axiom)
\$true   fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a)   fof(existence_m1_subset$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a))   fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b))   fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set)   fof(fc4_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a)   fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a))   fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a)   fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b))   fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
$\forall a$: (relation(a) $\Rightarrow$ ($\forall b, c$: $\neg$ in(ordered_pair(b, c), a) $\Rightarrow$ a = empty_set))   fof(t56_relat$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set)   fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b)   fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b)   fof(t8_boole, axiom)

SEU187+1.p MPTP bushy problem t60_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a))   fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a)   fof(commutativity_k2_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (b = relation_dom(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair(c, d), a))))   fof(d4_relat$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (b = relation_rng(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair(d, c), a))))   fof(d5_relat$_1$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
\$true   fof(dt_k1_relat$_1$, axiom)
\$true   fof(dt_k1_tarski, axiom)
\$true   fof(dt_k1_xboole$_0$, axiom)
\$true   fof(dt_k2_relat$_1$, axiom)
\$true   fof(dt_k2_tarski, axiom)

$true fof(dt_k4_tarski, axiom)

$true fof(dt_m1_subset$_1$, axiom)

∀a: ∃b: element(b, a) fof(existence_m1_subset$_1$, axiom)

empty(empty_set) fof(fc1_xboole$_0$, axiom)

∀a, b: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)

∀a: ¬ empty(singleton(a)) fof(fc2_subset$_1$, axiom)

∀a, b: ¬ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)

empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)

∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)

∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_rng(a))) fof(fc6_relat$_1$, axiom)

∃a: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)

∃a: empty(a) fof(rc1_xboole$_0$, axiom)

∃a: (¬ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)

∃a: ¬ empty(a) fof(rc2_xboole$_0$, axiom)

∀a, b: (in(a, b) ⇒ element(a, b)) fof(t1_subset, axiom)

∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b))) fof(t2_subset, axiom)

∀a, b: (∀c: (in(c, a) ⟺ in(c, b)) ⇒ a = b) fof(t2_tarski, axiom)

relation_dom(empty_set) = empty_set and relation_rng(empty_set) = empty_set fof(t60_relat$_1$, conjecture)

∀a: (empty(a) ⇒ a = empty_set) fof(t6_boole, axiom)

∀a, b: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)

∀a, b: ¬ empty(a) and a ≠ b and empty(b) fof(t8_boole, axiom)

SEU188+1.p MPTP bushy problem t64_relat_1

∀a, b: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)

∀a: (empty(a) ⇒ relation(a)) fof(cc1_relat$_1$, axiom)

∀a, b: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)

∀a: (relation(a) ⇒ ∀b: (b = relation_dom(a) ⟺ ∀c: (in(c, b) ⟺ ∃d: in(ordered_pair(c, d), a)))) fof(d4_relat$_1$, axiom)

∀a: (relation(a) ⇒ ∀b: (b = relation_rng(a) ⟺ ∀c: (in(c, b) ⟺ ∃d: in(ordered_pair(d, c), a)))) fof(d5_relat$_1$, axiom)

∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)

$true fof(dt_k1_relat$_1$, axiom)

$true fof(dt_k1_tarski, axiom)

$true fof(dt_k1_xboole$_0$, axiom)

$true fof(dt_k2_relat$_1$, axiom)

$true fof(dt_k2_tarski, axiom)

$true fof(dt_k4_tarski, axiom)

$true fof(dt_m1_subset$_1$, axiom)

∀a: ∃b: element(b, a) fof(existence_m1_subset$_1$, axiom)

empty(empty_set) fof(fc1_xboole$_0$, axiom)

∀a, b: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)

∀a: ¬ empty(singleton(a)) fof(fc2_subset$_1$, axiom)

∀a, b: ¬ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)

empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)

∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)

∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_rng(a))) fof(fc6_relat$_1$, axiom)

∀a: (empty(a) ⇒ (empty(relation_dom(a)) and relation(relation_dom(a)))) fof(fc7_relat$_1$, axiom)

∀a: (empty(a) ⇒ (empty(relation_rng(a)) and relation(relation_rng(a)))) fof(fc8_relat$_1$, axiom)

∃a: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)

∃a: empty(a) fof(rc1_xboole$_0$, axiom)

∃a: (¬ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)

∃a: ¬ empty(a) fof(rc2_xboole$_0$, axiom)

∀a, b: (in(a, b) ⇒ element(a, b)) fof(t1_subset, axiom)

∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b))) fof(t2_subset, axiom)

∀a: (relation(a) ⇒ (∀b, c: ¬ in(ordered_pair(b, c), a) ⇒ a = empty_set)) fof(t56_relat$_1$, axiom)

∀a: (relation(a) ⇒ ((relation_dom(a) = empty_set or relation_rng(a) = empty_set) ⇒ a = empty_set)) fof(t64_relat$_1$, co

∀a: (empty(a) ⇒ a = empty_set) fof(t6_boole, axiom)

∀a, b: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)

∀a, b: ¬ empty(a) and a ≠ b and empty(b) fof(t8_boole, axiom)

SEU189+1.p MPTP bushy problem t65_relat_1

∀a: ∃b: element(b, a) fof(existence_m1_subset$_1$, axiom)

\$true   fof(dt_m1_subset$_1$, axiom)
∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a, b: (in(a, b) ⇒ element(a, b))   fof(t1_subset, axiom)
∃a: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
∀a: (empty(a) ⇒ relation(a))   fof(cc1_relat$_1$, axiom)
∃a: (¬ empty(a) and relation(a))   fof(rc2_relat$_1$, axiom)
∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_dom(a)))   fof(fc5_relat$_1$, axiom)
∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_rng(a)))   fof(fc6_relat$_1$, axiom)
∀a: (empty(a) ⇒ (empty(relation_dom(a)) and relation(relation_dom(a))))   fof(fc7_relat$_1$, axiom)
∀a: (empty(a) ⇒ (empty(relation_rng(a)) and relation(relation_rng(a))))   fof(fc8_relat$_1$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: ¬ in(a, b) and empty(b)   fof(t7_boole, axiom)
∀a, b: ¬ empty(a) and $a \neq b$ and empty(b)   fof(t8_boole, axiom)
\$true   fof(dt_k1_relat$_1$, axiom)
\$true   fof(dt_k1_xboole$_0$, axiom)
\$true   fof(dt_k2_relat$_1$, axiom)
empty(empty_set) and relation(empty_set)   fof(fc4_relat$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a: (empty(a) ⇒ a = empty_set)   fof(t6_boole, axiom)
∀a: (relation(a) ⇒ (relation_dom(a) = empty_set ⟺ relation_rng(a) = empty_set))   fof(t65_relat$_1$, conjecture)
relation_dom(empty_set) = empty_set and relation_rng(empty_set) = empty_set   fof(t60_relat$_1$, axiom)
∀a: (relation(a) ⇒ ((relation_dom(a) = empty_set or relation_rng(a) = empty_set) ⇒ a = empty_set))   fof(t64_relat$_1$, axi

SEU190+1.p MPTP bushy problem t71_relat_1
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a: (empty(a) ⇒ relation(a))   fof(cc1_relat$_1$, axiom)
∀a, b: unordered_pair(a, b) = unordered_pair(b, a)   fof(commutativity_k2_tarski, axiom)
∀a, b: (relation(b) ⇒ (b = identity_relation(a) ⟺ ∀c, d: (in(ordered_pair(c, d), b) ⟺ (in(c, a) and c = d))))   fof(d10_relat$_1$, axiom)
∀a: (relation(a) ⇒ ∀b: (b = relation_dom(a) ⟺ ∀c: (in(c, b) ⟺ ∃d: in(ordered_pair(c, d), a))))   fof(d4_relat$_1$, axiom)
∀a: (relation(a) ⇒ ∀b: (b = relation_rng(a) ⟺ ∀c: (in(c, b) ⟺ ∃d: in(ordered_pair(d, c), a))))   fof(d5_relat$_1$, axiom)
∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
\$true   fof(dt_k1_relat$_1$, axiom)
\$true   fof(dt_k1_tarski, axiom)
\$true   fof(dt_k1_xboole$_0$, axiom)
\$true   fof(dt_k2_relat$_1$, axiom)
\$true   fof(dt_k2_tarski, axiom)
\$true   fof(dt_k4_tarski, axiom)
∀a: relation(identity_relation(a))   fof(dt_k6_relat$_1$, axiom)
\$true   fof(dt_m1_subset$_1$, axiom)
∀a: ∃b: element(b, a)   fof(existence_m1_subset$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a, b: ¬ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
∀a: ¬ empty(singleton(a))   fof(fc2_subset$_1$, axiom)
∀a, b: ¬ empty(unordered_pair(a, b))   fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set)   fof(fc4_relat$_1$, axiom)
∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_dom(a)))   fof(fc5_relat$_1$, axiom)
∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_rng(a)))   fof(fc6_relat$_1$, axiom)
∀a: (empty(a) ⇒ (empty(relation_dom(a)) and relation(relation_dom(a))))   fof(fc7_relat$_1$, axiom)
∀a: (empty(a) ⇒ (empty(relation_rng(a)) and relation(relation_rng(a))))   fof(fc8_relat$_1$, axiom)
∃a: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: (¬ empty(a) and relation(a))   fof(rc2_relat$_1$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (in(a, b) ⇒ element(a, b))   fof(t1_subset, axiom)
∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
∀a, b: (∀c: (in(c, a) ⟺ in(c, b)) ⇒ a = b)   fof(t2_tarski, axiom)
∀a: (empty(a) ⇒ a = empty_set)   fof(t6_boole, axiom)

∀a: (relation_dom(identity_relation(a)) = a and relation_rng(identity_relation(a)) = a)   fof(t71_relat$_1$, conjecture)
∀a, b: ¬ in(a, b) and empty(b)   fof(t7_boole, axiom)
∀a, b: ¬ empty(a) and a ≠ b and empty(b)   fof(t8_boole, axiom)

SEU191+1.p MPTP bushy problem t74_relat_1
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a: (empty(a) ⇒ relation(a))   fof(cc1_relat$_1$, axiom)
∀a, b: unordered_pair(a, b) = unordered_pair(b, a)   fof(commutativity_k2_tarski, axiom)
∀a, b: (relation(b) ⇒ (b = identity_relation(a) ⟺ ∀c, d: (in(ordered_pair(c, d), b) ⟺ (in(c, a) and c = d))))   fof(d10_relat$_1$, axiom)
∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
∀a: (relation(a) ⇒ ∀b: (relation(b) ⇒ ∀c: (relation(c) ⇒ (c = relation_composition(a, b) ⟺ ∀d, e: (in(ordered_pair(d, e), c
∃f: (in(ordered_pair(d, f), a) and in(ordered_pair(f, e), b)))))))   fof(d8_relat$_1$, axiom)
$true   fof(dt_k1_tarski, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$true   fof(dt_k2_tarski, axiom)
$true   fof(dt_k4_tarski, axiom)
∀a, b: ((relation(a) and relation(b)) ⇒ relation(relation_composition(a, b)))   fof(dt_k5_relat$_1$, axiom)
∀a: relation(identity_relation(a))   fof(dt_k6_relat$_1$, axiom)
$true   fof(dt_m1_subset$_1$, axiom)
∀a: ∃b: element(b, a)   fof(existence_m1_subset$_1$, axiom)
∀a, b: ((empty(a) and relation(b)) ⇒ (empty(relation_composition(b, a)) and relation(relation_composition(b, a))))   fof(fc
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a, b: ¬ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
∀a: ¬ empty(singleton(a))   fof(fc2_subset$_1$, axiom)
∀a, b: ¬ empty(unordered_pair(a, b))   fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set)   fof(fc4_relat$_1$, axiom)
∀a, b: ((empty(a) and relation(b)) ⇒ (empty(relation_composition(a, b)) and relation(relation_composition(a, b))))   fof(fc
∃a: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: (¬ empty(a) and relation(a))   fof(rc2_relat$_1$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (in(a, b) ⇒ element(a, b))   fof(t1_subset, axiom)
∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
∀a: (empty(a) ⇒ a = empty_set)   fof(t6_boole, axiom)
∀a, b, c, d: (relation(d) ⇒ (in(ordered_pair(a, b), relation_composition(identity_relation(c), d)) ⟺ (in(a, c) and in(ordered_p
∀a, b: ¬ in(a, b) and empty(b)   fof(t7_boole, axiom)
∀a, b: ¬ empty(a) and a ≠ b and empty(b)   fof(t8_boole, axiom)

SEU192+1.p MPTP bushy problem t86_relat_1
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a: (empty(a) ⇒ relation(a))   fof(cc1_relat$_1$, axiom)
∀a, b: unordered_pair(a, b) = unordered_pair(b, a)   fof(commutativity_k2_tarski, axiom)
∀a: (relation(a) ⇒ ∀b, c: (relation(c) ⇒ (c = relation_dom_restriction(a, b) ⟺ ∀d, e: (in(ordered_pair(d, e), c) ⟺ (in(d, b) and in(ordered_pair(d, e), a))))))   fof(d11_relat$_1$, axiom)
∀a: (relation(a) ⇒ ∀b: (b = relation_dom(a) ⟺ ∀c: (in(c, b) ⟺ ∃d: in(ordered_pair(c, d), a))))   fof(d4_relat$_1$, axiom)
∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
$true   fof(dt_k1_relat$_1$, axiom)
$true   fof(dt_k1_tarski, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$true   fof(dt_k2_tarski, axiom)
$true   fof(dt_k4_tarski, axiom)
∀a, b: (relation(a) ⇒ relation(relation_dom_restriction(a, b)))   fof(dt_k7_relat$_1$, axiom)
$true   fof(dt_m1_subset$_1$, axiom)
∀a: ∃b: element(b, a)   fof(existence_m1_subset$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a, b: ¬ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
∀a: ¬ empty(singleton(a))   fof(fc2_subset$_1$, axiom)
∀a, b: ¬ empty(unordered_pair(a, b))   fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set)   fof(fc4_relat$_1$, axiom)

$\forall a$: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)
$\forall a$: (empty(a) ⇒ (empty(relation_dom(a)) and relation(relation_dom(a)))) fof(fc7_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (¬ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (in(a, b) ⇒ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) ⇒ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) ⇒ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b, c$: (relation(c) ⇒ (in(a, relation_dom(relation_dom_restriction(c, b))) ⟺ (in(a, b) and in(a, relation_dom(c))))) fo
$\forall a, b$: ¬ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU193+1.p MPTP bushy problem t88_relat_1
$\forall a, b$: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) ⇒ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a$: (relation(a) ⇒ $\forall b, c$: (relation(c) ⇒ (c = relation_dom_restriction(a, b) ⟺ $\forall d, e$: (in(ordered_pair(d, e), c) ⟺ (in(d, b) and in(ordered_pair(d, e), a)))))) fof(d11_relat$_1$, axiom)
$\forall a$: (relation(a) ⇒ $\forall b$: (relation(b) ⇒ ($a \subseteq b$ ⟺ $\forall c, d$: (in(ordered_pair(c, d), a) ⇒ in(ordered_pair(c, d), b))))) fof(d3_
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k1_zfmisc$_1$, axiom)
$true fof(dt_k2_tarski, axiom)
$true fof(dt_k4_tarski, axiom)
$\forall a, b$: (relation(a) ⇒ relation(relation_dom_restriction(a, b))) fof(dt_k7_relat$_1$, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: ¬ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a$: ¬ empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: ¬ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\forall a$: (¬ empty(a) ⇒ $\exists b$: (element(b, powerset(a)) and ¬ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (¬ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: ¬ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) ⇒ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) ⇒ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) ⟺ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) ⇒ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: ¬ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) ⇒ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: (relation(b) ⇒ relation_dom_restriction(b, a) $\subseteq b$) fof(t88_relat$_1$, conjecture)
$\forall a, b$: ¬ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU194+1.p MPTP bushy problem t90_relat_1
$\forall a, b$: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) ⇒ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b, c$: (c = set_intersection$_2$(a, b) ⟺ $\forall d$: (in(d, c) ⟺ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$true fof(dt_k1_relat$_1$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k3_xboole$_0$, axiom)

$\forall a, b\colon (\mathrm{relation}(a) \Rightarrow \mathrm{relation}(\mathrm{relation\_dom\_restriction}(a, b)))$ $\mathrm{fof}(\mathrm{dt\_k7\_relat}_1, \mathrm{axiom})$
$true $\mathrm{fof}(\mathrm{dt\_m1\_subset}_1, \mathrm{axiom})$
$\forall a\colon \exists b\colon \mathrm{element}(b, a)$ $\mathrm{fof}(\mathrm{existence\_m1\_subset}_1, \mathrm{axiom})$
$\forall a, b\colon ((\mathrm{relation}(a) \text{ and } \mathrm{relation}(b)) \Rightarrow \mathrm{relation}(\mathrm{set\_intersection}_2(a, b)))$ $\mathrm{fof}(\mathrm{fc1\_relat}_1, \mathrm{axiom})$
$\mathrm{empty}(\mathrm{empty\_set})$ $\mathrm{fof}(\mathrm{fc1\_xboole}_0, \mathrm{axiom})$
$\mathrm{empty}(\mathrm{empty\_set}) \text{ and } \mathrm{relation}(\mathrm{empty\_set})$ $\mathrm{fof}(\mathrm{fc4\_relat}_1, \mathrm{axiom})$
$\forall a\colon ((\neg \mathrm{empty}(a) \text{ and } \mathrm{relation}(a)) \Rightarrow \neg \mathrm{empty}(\mathrm{relation\_dom}(a)))$ $\mathrm{fof}(\mathrm{fc5\_relat}_1, \mathrm{axiom})$
$\forall a\colon (\mathrm{empty}(a) \Rightarrow (\mathrm{empty}(\mathrm{relation\_dom}(a)) \text{ and } \mathrm{relation}(\mathrm{relation\_dom}(a))))$ $\mathrm{fof}(\mathrm{fc7\_relat}_1, \mathrm{axiom})$
$\forall a, b\colon \mathrm{set\_intersection}_2(a, a) = a$ $\mathrm{fof}(\mathrm{idempotence\_k3\_xboole}_0, \mathrm{axiom})$
$\exists a\colon (\mathrm{empty}(a) \text{ and } \mathrm{relation}(a))$ $\mathrm{fof}(\mathrm{rc1\_relat}_1, \mathrm{axiom})$
$\exists a\colon \mathrm{empty}(a)$ $\mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a\colon (\neg \mathrm{empty}(a) \text{ and } \mathrm{relation}(a))$ $\mathrm{fof}(\mathrm{rc2\_relat}_1, \mathrm{axiom})$
$\exists a\colon \neg \mathrm{empty}(a)$ $\mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \mathrm{element}(a, b))$ $\mathrm{fof}(\mathrm{t1\_subset}, \mathrm{axiom})$
$\forall a\colon \mathrm{set\_intersection}_2(a, \mathrm{empty\_set}) = \mathrm{empty\_set}$ $\mathrm{fof}(\mathrm{t2\_boole}, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{element}(a, b) \Rightarrow (\mathrm{empty}(b) \text{ or } \mathrm{in}(a, b)))$ $\mathrm{fof}(\mathrm{t2\_subset}, \mathrm{axiom})$
$\forall a, b\colon (\forall c\colon (\mathrm{in}(c, a) \iff \mathrm{in}(c, b)) \Rightarrow a = b)$ $\mathrm{fof}(\mathrm{t2\_tarski}, \mathrm{axiom})$
$\forall a\colon (\mathrm{empty}(a) \Rightarrow a = \mathrm{empty\_set})$ $\mathrm{fof}(\mathrm{t6\_boole}, \mathrm{axiom})$
$\forall a, b\colon \neg \mathrm{in}(a, b) \text{ and } \mathrm{empty}(b)$ $\mathrm{fof}(\mathrm{t7\_boole}, \mathrm{axiom})$
$\forall a, b, c\colon (\mathrm{relation}(c) \Rightarrow (\mathrm{in}(a, \mathrm{relation\_dom}(\mathrm{relation\_dom\_restriction}(c, b))) \iff (\mathrm{in}(a, b) \text{ and } \mathrm{in}(a, \mathrm{relation\_dom}(c)))))$ fof
$\forall a, b\colon \neg \mathrm{empty}(a) \text{ and } a \neq b \text{ and } \mathrm{empty}(b)$ $\mathrm{fof}(\mathrm{t8\_boole}, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{relation}(b) \Rightarrow \mathrm{relation\_dom}(\mathrm{relation\_dom\_restriction}(b, a)) = \mathrm{set\_intersection}_2(\mathrm{relation\_dom}(b), a))$ $\mathrm{fof}(\mathrm{t90\_relat}_1, \mathrm{c}$

SEU195+1.p MPTP bushy problem t94_relat_1
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \neg \mathrm{in}(b, a))$ $\mathrm{fof}(\mathrm{antisymmetry\_r2\_hidden}, \mathrm{axiom})$
$\forall a\colon (\mathrm{empty}(a) \Rightarrow \mathrm{relation}(a))$ $\mathrm{fof}(\mathrm{cc1\_relat}_1, \mathrm{axiom})$
$\forall a, b\colon \mathrm{unordered\_pair}(a, b) = \mathrm{unordered\_pair}(b, a)$ $\mathrm{fof}(\mathrm{commutativity\_k2\_tarski}, \mathrm{axiom})$
$\forall a\colon (\mathrm{relation}(a) \Rightarrow \forall b, c\colon (\mathrm{relation}(c) \Rightarrow (c = \mathrm{relation\_dom\_restriction}(a, b) \iff \forall d, e\colon (\mathrm{in}(\mathrm{ordered\_pair}(d, e), c) \iff$
$(\mathrm{in}(d, b) \text{ and } \mathrm{in}(\mathrm{ordered\_pair}(d, e), a))))))$ $\mathrm{fof}(\mathrm{d11\_relat}_1, \mathrm{axiom})$
$\forall a\colon (\mathrm{relation}(a) \Rightarrow \forall b\colon (\mathrm{relation}(b) \Rightarrow (a = b \iff \forall c, d\colon (\mathrm{in}(\mathrm{ordered\_pair}(c, d), a) \iff \mathrm{in}(\mathrm{ordered\_pair}(c, d), b)))))$ fof(d
$\forall a, b\colon \mathrm{ordered\_pair}(a, b) = \mathrm{unordered\_pair}(\mathrm{unordered\_pair}(a, b), \mathrm{singleton}(a))$ $\mathrm{fof}(\mathrm{d5\_tarski}, \mathrm{axiom})$
$true $\mathrm{fof}(\mathrm{dt\_k1\_tarski}, \mathrm{axiom})$
$true $\mathrm{fof}(\mathrm{dt\_k1\_xboole}_0, \mathrm{axiom})$
$true $\mathrm{fof}(\mathrm{dt\_k2\_tarski}, \mathrm{axiom})$
$true $\mathrm{fof}(\mathrm{dt\_k4\_tarski}, \mathrm{axiom})$
$\forall a, b\colon ((\mathrm{relation}(a) \text{ and } \mathrm{relation}(b)) \Rightarrow \mathrm{relation}(\mathrm{relation\_composition}(a, b)))$ $\mathrm{fof}(\mathrm{dt\_k5\_relat}_1, \mathrm{axiom})$
$\forall a\colon \mathrm{relation}(\mathrm{identity\_relation}(a))$ $\mathrm{fof}(\mathrm{dt\_k6\_relat}_1, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{relation}(a) \Rightarrow \mathrm{relation}(\mathrm{relation\_dom\_restriction}(a, b)))$ $\mathrm{fof}(\mathrm{dt\_k7\_relat}_1, \mathrm{axiom})$
$true $\mathrm{fof}(\mathrm{dt\_m1\_subset}_1, \mathrm{axiom})$
$\forall a\colon \exists b\colon \mathrm{element}(b, a)$ $\mathrm{fof}(\mathrm{existence\_m1\_subset}_1, \mathrm{axiom})$
$\forall a, b\colon ((\mathrm{empty}(a) \text{ and } \mathrm{relation}(b)) \Rightarrow (\mathrm{empty}(\mathrm{relation\_composition}(b, a)) \text{ and } \mathrm{relation}(\mathrm{relation\_composition}(b, a))))$ fof(fc
$\mathrm{empty}(\mathrm{empty\_set})$ $\mathrm{fof}(\mathrm{fc1\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon \neg \mathrm{empty}(\mathrm{ordered\_pair}(a, b))$ $\mathrm{fof}(\mathrm{fc1\_zfmisc}_1, \mathrm{axiom})$
$\forall a\colon \neg \mathrm{empty}(\mathrm{singleton}(a))$ $\mathrm{fof}(\mathrm{fc2\_subset}_1, \mathrm{axiom})$
$\forall a, b\colon \neg \mathrm{empty}(\mathrm{unordered\_pair}(a, b))$ $\mathrm{fof}(\mathrm{fc3\_subset}_1, \mathrm{axiom})$
$\mathrm{empty}(\mathrm{empty\_set}) \text{ and } \mathrm{relation}(\mathrm{empty\_set})$ $\mathrm{fof}(\mathrm{fc4\_relat}_1, \mathrm{axiom})$
$\forall a, b\colon ((\mathrm{empty}(a) \text{ and } \mathrm{relation}(b)) \Rightarrow (\mathrm{empty}(\mathrm{relation\_composition}(a, b)) \text{ and } \mathrm{relation}(\mathrm{relation\_composition}(a, b))))$ fof(fc
$\exists a\colon (\mathrm{empty}(a) \text{ and } \mathrm{relation}(a))$ $\mathrm{fof}(\mathrm{rc1\_relat}_1, \mathrm{axiom})$
$\exists a\colon \mathrm{empty}(a)$ $\mathrm{fof}(\mathrm{rc1\_xboole}_0, \mathrm{axiom})$
$\exists a\colon (\neg \mathrm{empty}(a) \text{ and } \mathrm{relation}(a))$ $\mathrm{fof}(\mathrm{rc2\_relat}_1, \mathrm{axiom})$
$\exists a\colon \neg \mathrm{empty}(a)$ $\mathrm{fof}(\mathrm{rc2\_xboole}_0, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{in}(a, b) \Rightarrow \mathrm{element}(a, b))$ $\mathrm{fof}(\mathrm{t1\_subset}, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{element}(a, b) \Rightarrow (\mathrm{empty}(b) \text{ or } \mathrm{in}(a, b)))$ $\mathrm{fof}(\mathrm{t2\_subset}, \mathrm{axiom})$
$\forall a\colon (\mathrm{empty}(a) \Rightarrow a = \mathrm{empty\_set})$ $\mathrm{fof}(\mathrm{t6\_boole}, \mathrm{axiom})$
$\forall a, b, c, d\colon (\mathrm{relation}(d) \Rightarrow (\mathrm{in}(\mathrm{ordered\_pair}(a, b), \mathrm{relation\_composition}(\mathrm{identity\_relation}(c), d)) \iff (\mathrm{in}(a, c) \text{ and } \mathrm{in}(\mathrm{ordered\_p}$
$\forall a, b\colon \neg \mathrm{in}(a, b) \text{ and } \mathrm{empty}(b)$ $\mathrm{fof}(\mathrm{t7\_boole}, \mathrm{axiom})$
$\forall a, b\colon \neg \mathrm{empty}(a) \text{ and } a \neq b \text{ and } \mathrm{empty}(b)$ $\mathrm{fof}(\mathrm{t8\_boole}, \mathrm{axiom})$
$\forall a, b\colon (\mathrm{relation}(b) \Rightarrow \mathrm{relation\_dom\_restriction}(b, a) = \mathrm{relation\_composition}(\mathrm{identity\_relation}(a), b))$ $\mathrm{fof}(\mathrm{t94\_relat}_1, \mathrm{conjectu}$

SEU196+1.p MPTP bushy problem t99_relat_1

$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
$\forall a, b$: (relation(a) $\Rightarrow$ relation(relation_dom_restriction(a, b))) fof(dt_k7_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$empty(a) and relation(a)) $\Rightarrow$ $\neg$empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)
$\forall a$: (($\neg$empty(a) and relation(a)) $\Rightarrow$ $\neg$empty(relation_rng(a))) fof(fc6_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_dom(a)) and relation(relation_dom(a)))) fof(fc7_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_rng(a)) and relation(relation_rng(a)))) fof(fc8_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\forall a$: ($\neg$empty(a) $\Rightarrow$ $\exists b$: (element$(b$, powerset$(a))$ and $\neg$empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\forall a$: $\exists b$: (element$(b$, powerset$(a))$ and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ $(a \subseteq b$ $\Rightarrow$ (relation_dom$(a) \subseteq$ relation_dom(b) and relation_rng$(a) \subseteq$ relation_rng(b))))) fof(t25_relat$_1$, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element$(a$, powerset$(b))$ $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element$(b$, powerset$(c))$) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$in(a, b) and element$(b$, powerset$(c))$ and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation_dom_restriction$(b, a) \subseteq b$) fof(t88_relat$_1$, axiom)
$\forall a, b$: $\neg$empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation_rng(relation_dom_restriction(b, a)) $\subseteq$ relation_rng(b)) fof(t99_relat$_1$, conjecture)

SEU197+1.p MPTP bushy problem t115_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ $\forall c$: (relation(c) $\Rightarrow$ $(c =$ relation_rng_restriction(a, b) $\iff$ $\forall d, e$: (in(ordered_pair$(d, e), c)$ $\iff$ (in(e, a) and in(ordered_pair$(d, e), b)$)))))) fof(d12_relat$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: $(b =$ relation_rng(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair$(d, c), a)$))) fof(d5_relat$_1$, axiom)
$\forall a, b$: ordered_pair$(a, b) =$ unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k4_tarski, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation(relation_rng_restriction(a, b))) fof(dt_k8_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$empty(a) and relation(a)) $\Rightarrow$ $\neg$empty(relation_rng(a))) fof(fc6_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_rng(a)) and relation(relation_rng(a)))) fof(fc8_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)

$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ (in(a, relation_rng(relation_rng_restriction(b, c))) $\iff$ (in(a, b) and in(a, relation_rng(c))))) fof(t1
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU198+1.p MPTP bushy problem t116_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation(relation_rng_restriction(a, b))) fof(dt_k8_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$empty(a) and relation(a)) $\Rightarrow$ $\neg$empty(relation_rng(a))) fof(fc6_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_rng(a)) and relation(relation_rng(a)))) fof(fc8_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\forall a$: ($\neg$empty(a) $\Rightarrow$ $\exists b$: (element(b, powerset(a)) and $\neg$empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ (in(a, relation_rng(relation_rng_restriction(b, c))) $\iff$ (in(a, b) and in(a, relation_rng(c))))) fof(t1
$\forall a, b$: (relation(b) $\Rightarrow$ relation_rng(relation_rng_restriction(a, b)) $\subseteq a$) fof(t116_relat$_1$, conjecture)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU199+1.p MPTP bushy problem t117_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ $\forall c$: (relation(c) $\Rightarrow$ ($c =$ relation_rng_restriction(a, b) $\iff$ $\forall d, e$: (in(ordered_pair(d, e), c) $\iff$ (in(e, a) and in(ordered_pair(d, e), b)))))) fof(d12_relat$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ ($a \subseteq b$ $\iff$ $\forall c, d$: (in(ordered_pair(c, d), a) $\Rightarrow$ in(ordered_pair(c, d), b))))) fof(d3_
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k4_tarski, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation(relation_rng_restriction(a, b))) fof(dt_k8_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$empty(powerset(a)) fof(fc1_subset$_1$, axiom)

empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element(b, powerset(a)) and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation_rng_restriction(a, b) $\subseteq b$) fof(t117_relat$_1$, conjecture)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU200+1.p MPTP bushy problem t118_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$true fof(dt_k1_relat$_1$, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k1_zfmisc$_1$, axiom)
$true fof(dt_k2_relat$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation(relation_rng_restriction(a, b))) fof(dt_k8_relat$_1$, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_rng(a))) fof(fc6_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_dom(a)) and relation(relation_dom(a)))) fof(fc7_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_rng(a)) and relation(relation_rng(a)))) fof(fc8_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element(b, powerset(a)) and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation_rng_restriction(a, b) $\subseteq b$) fof(t117_relat$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation_rng(relation_rng_restriction(a, b)) $\subseteq$ relation_rng(b)) fof(t118_relat$_1$, conjecture)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ ($a \subseteq b$ $\Rightarrow$ (relation_dom(a) $\subseteq$ relation_dom(b) and relation_rng(a) $\subseteq$ relation_rng(b))))) fof(t25_relat$_1$, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU202+1.p MPTP bushy problem t140_relat_1

$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b, c$: (relation(c) $\Rightarrow$ (c = relation_dom_restriction(a, b) $\iff$ $\forall d, e$: (in(ordered_pair$(d, e), c)$ $\iff$ (in(d, b) and in(ordered_pair$(d, e), a)$)))))) fof(d11_relat$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ $\forall c$: (relation(c) $\Rightarrow$ (c = relation_rng_restriction(a, b) $\iff$ $\forall d, e$: (in(ordered_pair$(d, e), c)$ $\iff$ (in(e, a) and in(ordered_pair$(d, e), b)$)))))) fof(d12_relat$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ ($a = b$ $\iff$ $\forall c, d$: (in(ordered_pair$(c, d), a)$ $\iff$ in(ordered_pair$(c, d), b)$))))) fof(d
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k4_tarski, axiom)
$\forall a, b$: (relation(a) $\Rightarrow$ relation(relation_dom_restriction(a, b))) fof(dt_k7_relat$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation(relation_rng_restriction(a, b))) fof(dt_k8_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ relation_dom_restriction(relation_rng_restriction$(a, c), b)$ = relation_rng_restriction(a, relation_dom_res
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU203+1.p MPTP bushy problem t143_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b, c$: (c = relation_image(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ $\exists e$: (in(ordered_pair$(e, d), a)$ and in(e, b)))))) fo
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (b = relation_dom(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair$(c, d), a)$))) fof(d4_relat$_1$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k4_tarski, axiom)
\$true fof(dt_k9_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_dom(a)) and relation(relation_dom(a)))) fof(fc7_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ (in(a, relation_image(c, b)) $\iff$ $\exists d$: (in(d, relation_dom(c)) and in(ordered_pair$(d, a), c)$ and in(d, b))))

$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU205+1.p MPTP bushy problem t145_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b, c$: (c = relation_image(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ $\exists e$: (in(ordered_pair(e, d), a) and in(e, b))))) fo
$\forall a, b, c$: (c = set_intersection$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
\$true fof(dt_k4_tarski, axiom)
\$true fof(dt_k9_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a, b$: ((relation(a) and relation(b)) $\Rightarrow$ relation(set_intersection$_2$(a, b))) fof(fc1_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_dom(a)) and relation(relation_dom(a)))) fof(fc7_relat$_1$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ (in(a, relation_image(c, b)) $\iff$ $\exists d$: (in(d, relation_dom(c)) and in(ordered_pair(d, a), c) and in(d, b)))
$\forall a, b$: (relation(b) $\Rightarrow$ relation_image(b, a) = relation_image(b, set_intersection$_2$(relation_dom(b), a))) fof(t145_relat$_1$, conjec
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a$: set_intersection$_2$(a, empty_set) = empty_set fof(t2_boole, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: ($\forall c$: (in(c, a) $\iff$ in(c, b)) $\Rightarrow$ $a = b$) fof(t2_tarski, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU208+1.p MPTP bushy problem t166_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b, c$: (c = relation_inverse_image(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ $\exists e$: (in(ordered_pair(d, e), a) and in(e, b))))
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (b = relation_rng(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair(d, c), a)))) fof(d5_relat$_1$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k10_relat$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k4_tarski, axiom)
\$true fof(dt_m1_subset$_1$, axiom)

$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_rng(a))) fof(fc6_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_rng(a)) and relation(relation_rng(a)))) fof(fc8_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ (in(a, relation_inverse_image(c, b)) $\iff$ $\exists d$: (in(d, relation_rng(c)) and in(ordered_pair(a, d), c) and in
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU211+1.p MPTP bushy problem t178_relat_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b, c$: (c = relation_inverse_image(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ $\exists e$: (in(ordered_pair(d, e), a) and in(e, b))))
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k10_relat$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k4_tarski, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a$: $\neg$ empty(powerset(a)) fof(fc1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a)) fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b)) fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\forall a$: ($\neg$ empty(a) $\Rightarrow$ $\exists b$: (element(b, powerset(a)) and $\neg$ empty(b))) fof(rc1_subset$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\forall a$: $\exists b$: (element(b, powerset(a)) and empty(b)) fof(rc2_subset$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ ($a \subseteq b$ $\Rightarrow$ relation_inverse_image(c, a) $\subseteq$ relation_inverse_image(c, b))) fof(t178_relat$_1$, conjecture)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU212+1.p MPTP bushy problem t8_funct_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)

$\forall a$: (empty(a) $\Rightarrow$ relation(a))   fof(cc1_relat$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a)   fof(commutativity_k2_tarski, axiom)
$\forall a$: ((relation(a) and function(a)) $\Rightarrow$ $\forall b, c$: ((in(b, relation_dom(a)) $\Rightarrow$ (c = apply(a, b) $\iff$ in(ordered_pair(b, c), a))) and (
(c = apply(a, b) $\iff$ c = empty_set))))   fof(d4_funct$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (b = relation_dom(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair(c, d), a))))   fof(d4_relat$_1$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
\$true   fof(dt_k1_funct$_1$, axiom)
\$true   fof(dt_k1_relat$_1$, axiom)
\$true   fof(dt_k1_tarski, axiom)
\$true   fof(dt_k1_xboole$_0$, axiom)
\$true   fof(dt_k2_tarski, axiom)
\$true   fof(dt_k4_tarski, axiom)
\$true   fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a)   fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set)   fof(fc12_relat$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
$\forall a$: $\neg$ empty(singleton(a))   fof(fc2_subset$_1$, axiom)
$\forall a, b$: $\neg$ empty(unordered_pair(a, b))   fof(fc3_subset$_1$, axiom)
empty(empty_set) and relation(empty_set)   fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_dom(a)))   fof(fc5_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_dom(a)) and relation(relation_dom(a))))   fof(fc7_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a))   fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a)   fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a))   fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a)   fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a))   fof(rc3_relat$_1$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b))   fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set)   fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b)   fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b)   fof(t8_boole, axiom)
$\forall a, b, c$: ((relation(c) and function(c)) $\Rightarrow$ (in(ordered_pair(a, b), c) $\iff$ (in(a, relation_dom(c)) and b = apply(c, a))))   fof

SEU217+1.p MPTP bushy problem t35_funct_1
$\exists a$: (relation(a) and relation_empty_yielding(a))   fof(rc3_relat$_1$, axiom)
\$true   fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set)   fof(fc4_relat$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set)   fof(fc12_relat$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\forall a$: $\exists b$: element(b, a)   fof(existence_m1_subset$_1$, axiom)
\$true   fof(dt_m1_subset$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a))   fof(cc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a))   fof(cc1_relat$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a))   fof(rc2_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_dom(a)))   fof(fc5_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_dom(a)) and relation(relation_dom(a))))   fof(fc7_relat$_1$, axiom)
$\exists a$: empty(a)   fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a)   fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set)   fof(t6_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b)   fof(t8_boole, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
\$true   fof(dt_k1_funct$_1$, axiom)
\$true   fof(dt_k1_relat$_1$, axiom)
$\forall a$: relation(identity_relation(a))   fof(dt_k6_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a))   fof(rc1_funct$_1$, axiom)
$\forall a$: (relation(identity_relation(a)) and function(identity_relation(a)))   fof(fc2_funct$_1$, axiom)

∀a, b: (in(a, b) ⇒ element(a, b))   fof(t1_subset, axiom)
∀a, b: ¬ in(a, b) and empty(b)   fof(t7_boole, axiom)
∀a, b: (in(b, a) ⇒ apply(identity_relation(a), b) = b)   fof(t35_funct$_1$, conjecture)
∀a, b: ((relation(b) and function(b)) ⇒ (b = identity_relation(a) ⟺ (relation_dom(b) = a and ∀c: (in(c, a) ⇒ apply(b, c) = c))))   fof(t34_funct$_1$, axiom)

SEU217+3.p Functions and their basic properties, theorem 35
∀a, b: a ⊆ a   fof(reflexivity_r1_tarski, axiom)
empty(empty_set) and relation(empty_set)   fof(fc4_relat$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set)   fof(fc12_relat$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
∀a: ∃b: element(b, a)   fof(existence_m1_subset$_1$, axiom)
∀a: (empty(a) ⇒ function(a))   fof(cc1_funct$_1$, axiom)
∀a: ¬ empty(powerset(a))   fof(fc1_subset$_1$, axiom)
∀a: ((¬ empty(a) and relation(a)) ⇒ ¬ empty(relation_dom(a)))   fof(fc5_relat$_1$, axiom)
∀a: (empty(a) ⇒ (empty(relation_dom(a)) and relation(relation_dom(a))))   fof(fc7_relat$_1$, axiom)
∀a: (empty(a) ⇒ relation(a))   fof(cc1_relat$_1$, axiom)
∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
∀a, b: (element(a, powerset(b)) ⟺ a ⊆ b)   fof(t3_subset, axiom)
∀a, b, c: ((in(a, b) and element(b, powerset(c))) ⇒ element(a, c))   fof(t4_subset, axiom)
∀a, b, c: ¬ in(a, b) and element(b, powerset(c)) and empty(c)   fof(t5_subset, axiom)
∀a: (empty(a) ⇒ a = empty_set)   fof(t6_boole, axiom)
∀a, b: ¬ empty(a) and a ≠ b and empty(b)   fof(t8_boole, axiom)
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a: relation(identity_relation(a))   fof(dt_k6_relat$_1$, axiom)
∀a: (relation(identity_relation(a)) and function(identity_relation(a)))   fof(fc2_funct$_1$, axiom)
∃a: (relation(a) and function(a))   fof(rc1_funct$_1$, axiom)
∀a: (¬ empty(a) ⇒ ∃b: (element(b, powerset(a)) and ¬ empty(b)))   fof(rc1_subset$_1$, axiom)
∀a: ∃b: (element(b, powerset(a)) and empty(b))   fof(rc2_subset$_1$, axiom)
∃a: (empty(a) and relation(a))   fof(rc1_relat$_1$, axiom)
∃a: (¬ empty(a) and relation(a))   fof(rc2_relat$_1$, axiom)
∃a: (relation(a) and relation_empty_yielding(a))   fof(rc3_relat$_1$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (in(a, b) ⇒ element(a, b))   fof(t1_subset, axiom)
∀a, b: ¬ in(a, b) and empty(b)   fof(t7_boole, axiom)
∀a, b: (in(b, a) ⇒ apply(identity_relation(a), b) = b)   fof(t35_funct$_1$, conjecture)
∀a, b: ((relation(b) and function(b)) ⇒ (b = identity_relation(a) ⟺ (relation_dom(b) = a and ∀c: (in(c, a) ⇒ apply(b, c) = c))))   fof(t34_funct$_1$, axiom)

SEU219+1.p MPTP bushy problem t55_funct_1
∀a: ((relation(a) and function(a)) ⇒ (one_to_one(a) ⇒ function_inverse(a) = relation_inverse(a)))   fof(d9_funct$_1$, axiom)
$true   fof(dt_k1_relat$_1$, axiom)
∀a: ((relation(a) and function(a)) ⇒ (relation(function_inverse(a)) and function(function_inverse(a))))   fof(dt_k2_funct$_1$,
$true   fof(dt_k2_relat$_1$, axiom)
∀a: (relation(a) ⇒ relation(relation_inverse(a)))   fof(dt_k4_relat$_1$, axiom)
∀a: ((relation(a) and function(a) and one_to_one(a)) ⇒ (relation(relation_inverse(a)) and function(relation_inverse(a))))
∀a: (relation(a) ⇒ relation_inverse(relation_inverse(a)) = a)   fof(involutiveness_k4_relat$_1$, axiom)
∃a: (relation(a) and function(a))   fof(rc1_funct$_1$, axiom)
∃a: (relation(a) and function(a) and one_to_one(a))   fof(rc3_funct$_1$, axiom)
∀a: (relation(a) ⇒ (relation_rng(a) = relation_dom(relation_inverse(a)) and relation_dom(a) = relation_rng(relation_inverse(
∀a: ((relation(a) and function(a)) ⇒ (one_to_one(a) ⇒ (relation_rng(a) = relation_dom(function_inverse(a)) and relation_d
relation_rng(function_inverse(a)))))   fof(t55_funct$_1$, conjecture)

SEU229+1.p MPTP bushy problem t3_ordinal1
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a: (empty(a) ⇒ function(a))   fof(cc1_funct$_1$, axiom)
∀a: (empty(a) ⇒ relation(a))   fof(cc1_relat$_1$, axiom)
∀a: ((relation(a) and empty(a) and function(a)) ⇒ (relation(a) and function(a) and one_to_one(a)))   fof(cc2_funct$_1$, axio
∀a, b, c, d: (d = unordered_triple(a, b, c) ⟺ ∀e: (in(e, d) ⟺ ¬ e ≠ a and e ≠ b and e ≠ c))   fof(d1_enumset$_1$, axiom)
$true   fof(dt_k1_enumset$_1$, axiom)

\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and in(b, c) and in(c, a) fof(t3_ordinal$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and $\forall c$: $\neg$ in(c, b) and $\forall d$: $\neg$ in(d, b) and in(d, c) fof(t7_tarski, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU229+3.p Ordinal numbers, theorem 3
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b, c, d$: ($d =$ unordered_triple(a, b, c) $\iff$ $\forall e$: (in(e, d) $\iff$ $\neg$ $e \neq a$ and $e \neq b$ and $e \neq c$)) fof(d1_enumset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and in(b, c) and in(c, a) fof(t3_ordinal$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and $\forall c$: $\neg$ in(c, b) and $\forall d$: $\neg$ in(d, b) and in(d, c) fof(t7_tarski, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU230+1.p MPTP bushy problem t10_ordinal1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: set_union$_2(a, b) =$ set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: succ$(a) =$ set_union$_2(a,$ singleton$(a))$ fof(d1_ordinal$_1$, axiom)
$\forall a, b$: ($b =$ singleton(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a, b, c$: ($c =$ set_union$_2(a, b)$ $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) or in(d, b)))) fof(d2_xboole$_0$, axiom)

$true fof(dt_k1_ordinal$_1$, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k2_xboole$_0$, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
$\forall a$: $\neg$ empty(succ(a)) fof(fc1_ordinal$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ((relation(a) and relation(b)) $\Rightarrow$ relation(set_union$_2$(a, b))) fof(fc2_relat$_1$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a, b$: set_union$_2$(a, a) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\forall a$: in(a, succ(a)) fof(t10_ordinal$_1$, conjecture)
$\forall a$: set_union$_2$(a, empty_set) $= a$ fof(t1_boole, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU230+3.p Ordinal numbers, theorem Ordinal numbers, theorem 10
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: set_union$_2$(a, b) $=$ set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: succ(a) $=$ set_union$_2$(a, singleton(a)) fof(d1_ordinal$_1$, axiom)
$\forall a, b$: ($b =$ singleton(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $c = a$)) fof(d1_tarski, axiom)
$\forall a, b, c$: ($c =$ set_union$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) or in(d, b)))) fof(d2_xboole$_0$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
$\forall a$: $\neg$ empty(succ(a)) fof(fc1_ordinal$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ((relation(a) and relation(b)) $\Rightarrow$ relation(set_union$_2$(a, b))) fof(fc2_relat$_1$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a, b$: set_union$_2$(a, a) $= a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a) and function(a)) fof(rc4_funct$_1$, axiom)
$\exists a$: (relation(a) and relation_non_empty(a) and function(a)) fof(rc5_funct$_1$, axiom)
$\forall a$: in(a, succ(a)) fof(t10_ordinal$_1$, conjecture)

$\forall a$: set_union$_2(a, \text{empty_set}) = a$ fof(t1_boole, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element$(a, b) \Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty$(a) \Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU239+1.p MPTP bushy problem l1_wellord1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty$(a) \Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2(a, b) =$ set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: (relation$(a) \Rightarrow \forall b$: (is_reflexive_in$(a, b) \iff \forall c$: (in$(c, b) \Rightarrow$ in(ordered_pair$(c, c), a$)))) fof(d1_relat$_2$, axiom)
$\forall a, b$: ordered_pair$(a, b) =$ unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
$\forall a$: (relation$(a) \Rightarrow$ relation_field$(a) =$ set_union$_2$(relation_dom(a), relation_rng(a))) fof(d6_relat$_1$, axiom)
$\forall a$: (relation$(a) \Rightarrow$ (reflexive$(a) \iff$ is_reflexive_in$(a$, relation_field(a)))) fof(d9_relat$_2$, axiom)
$true fof(dt_k1_relat$_1$, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k2_relat$_1$, axiom)
$true fof(dt_k2_tarski, axiom)
$true fof(dt_k2_xboole$_0$, axiom)
$true fof(dt_k3_relat$_1$, axiom)
$true fof(dt_k4_tarski, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(a, b)$)) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty$(a) \Rightarrow \neg$ empty(set_union$_2(b, a)$)) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a$: (relation$(a) \Rightarrow$ (reflexive$(a) \iff \forall b$: (in$(b$, relation_field$(a)) \Rightarrow$ in(ordered_pair$(b, b), a$)))) fof(l1_wellord$_1$, conjecture
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\forall a$: set_union$_2(a, \text{empty_set}) = a$ fof(t1_boole, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element$(a, b) \Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty$(a) \Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU241+1.p MPTP bushy problem l3_wellord1
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty$(a) \Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2(a, b) =$ set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: (relation$(a) \Rightarrow$ (antisymmetric$(a) \iff$ is_antisymmetric_in$(a$, relation_field(a)))) fof(d12_relat$_2$, axiom)
$\forall a$: (relation$(a) \Rightarrow \forall b$: (is_antisymmetric_in$(a, b) \iff \forall c, d$: ((in$(c, b)$ and in(d, b) and in(ordered_pair$(c, d), a)$ and in(ordere
$c = d$))) fof(d4_relat$_2$, axiom)
$\forall a, b$: ordered_pair$(a, b) =$ unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
$\forall a$: (relation$(a) \Rightarrow$ relation_field$(a) =$ set_union$_2$(relation_dom(a), relation_rng(a))) fof(d6_relat$_1$, axiom)
$true fof(dt_k1_relat$_1$, axiom)
$true fof(dt_k1_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k2_relat$_1$, axiom)

$true   fof(dt_k2_tarski, axiom)
$true   fof(dt_k2_xboole$_0$, axiom)
$true   fof(dt_k3_relat$_1$, axiom)
$true   fof(dt_k4_tarski, axiom)
$true   fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2(a, b)$))   fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2(b, a)$))   fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (antisymmetric(a) $\iff$ $\forall b, c$: ((in(ordered_pair$(b, c), a)$ and in(ordered_pair$(c, b), a)$) $\Rightarrow$ $b = c$)))   fof(l3_wellord$_1$, conjecture)
$\exists a$: (relation(a) and function(a))   fof(rc1_funct$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a))   fof(rc2_funct$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a))   fof(rc3_funct$_1$, axiom)
$\forall a$: set_union$_2(a$, empty_set$) = a$ fof(t1_boole, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b))   fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b)))   fof(t2_subset, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ (in(ordered_pair$(a, b), c)$ $\Rightarrow$ (in$(a$, relation_field$(c))$ and in$(b$, relation_field(c)))))   fof(t30_relat$_1$, ax
$\forall a$: (empty(a) $\Rightarrow$ $a =$ empty_set)   fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU242+1.p MPTP bushy problem l4_wellord1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a))   fof(cc1_funct$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a)))   fof(cc2_funct$_1$, axio
$\forall a, b$: unordered_pair$(a, b) =$ unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: set_union$_2(a, b) =$ set_union$_2(b, a)$ fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (connected(a) $\iff$ is_connected_in$(a$, relation_field(a))))   fof(d14_relat$_2$, axiom)
$\forall a, b$: ordered_pair$(a, b) =$ unordered_pair(unordered_pair(a, b), singleton(a))   fof(d5_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ relation_field$(a) =$ set_union$_2$(relation_dom(a), relation_rng(a)))   fof(d6_relat$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (is_connected_in(a, b) $\iff$ $\forall c, d$: $\neg$ in(c, b) and in(d, b) and $c \neq d$ and $\neg$ in(ordered_pair$(c, d), a)$ and $\neg$
$true   fof(dt_k1_relat$_1$, axiom)
$true   fof(dt_k1_tarski, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$true   fof(dt_k2_relat$_1$, axiom)
$true   fof(dt_k2_tarski, axiom)
$true   fof(dt_k2_xboole$_0$, axiom)
$true   fof(dt_k3_relat$_1$, axiom)
$true   fof(dt_k4_tarski, axiom)
$true   fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2(a, b)$))   fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2(b, a)$))   fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (connected(a) $\iff$ $\forall b, c$: $\neg$ in$(b$, relation_field(a)) and in$(c$, relation_field(a)) and $b \neq c$ and $\neg$ in(ordered_
$\exists a$: (relation(a) and function(a))   fof(rc1_funct$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a))   fof(rc2_funct$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a))   fof(rc3_funct$_1$, axiom)
$\forall a$: set_union$_2(a$, empty_set$) = a$ fof(t1_boole, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b))   fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b)))   fof(t2_subset, axiom)

$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU243+1.p MPTP bushy problem t5_wellord1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: set_union$_2$(a, b) = set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (well_founded_relation(a) $\iff$ $\forall b$: $\neg$ $b \subseteq$ relation_field(a) and $b \neq$ empty_set and $\forall c$: $\neg$ in(c, b) and disjoi
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (is_well_founded_in(a, b) $\iff$ $\forall c$: $\neg$ $c \subseteq b$ and $c \neq$ empty_set and $\forall d$: $\neg$ in(d, c) and disjoint(fiber(a, d)
$\forall a$: (relation(a) $\Rightarrow$ relation_field(a) = set_union$_2$(relation_dom(a), relation_rng(a))) fof(d6_relat$_1$, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_wellord$_1$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k3_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(a, b))) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ empty(a) $\Rightarrow$ $\neg$ empty(set_union$_2$(b, a))) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: set_union$_2$(a, a) = a fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b$: (disjoint(a, b) $\Rightarrow$ disjoint(b, a)) fof(symmetry_r1_xboole$_0$, axiom)
$\forall a$: set_union$_2$(a, empty_set) = a fof(t1_boole, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (well_founded_relation(a) $\iff$ is_well_founded_in(a, relation_field(a)))) fof(t5_wellord$_1$, conjecture)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU244+1.p MPTP bushy problem t8_wellord1
$\forall a, b$: set_union$_2$(a, b) = set_union$_2$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (antisymmetric(a) $\iff$ is_antisymmetric_in(a, relation_field(a)))) fof(d12_relat$_2$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (connected(a) $\iff$ is_connected_in(a, relation_field(a)))) fof(d14_relat$_2$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (transitive(a) $\iff$ is_transitive_in(a, relation_field(a)))) fof(d16_relat$_2$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (well_ordering(a) $\iff$ (reflexive(a) and transitive(a) and antisymmetric(a) and connected(a) and well_f
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (well_orders(a, b) $\iff$ (is_reflexive_in(a, b) and is_transitive_in(a, b) and is_antisymmetric_in(a, b) and
$\forall a$: (relation(a) $\Rightarrow$ relation_field(a) = set_union$_2$(relation_dom(a), relation_rng(a))) fof(d6_relat$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (reflexive(a) $\iff$ is_reflexive_in(a, relation_field(a)))) fof(d9_relat$_2$, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k3_relat$_1$, axiom)
$\forall a, b$: set_union$_2$(a, a) = a fof(idempotence_k2_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (well_founded_relation(a) $\iff$ is_well_founded_in(a, relation_field(a)))) fof(t5_wellord$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (well_orders(a, relation_field(a)) $\iff$ well_ordering(a))) fof(t8_wellord$_1$, conjecture)

SEU245+1.p MPTP bushy problem t16_wellord1

$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a$: set_intersection$_2$(a, empty_set) = empty_set fof(t2_boole, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)
$\forall a, b$: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b$: (relation(a) $\Rightarrow$ relation(relation_restriction(a, b))) fof(dt_k2_wellord$_1$, axiom)
\$true fof(dt_k2_zfmisc$_1$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: relation_restriction(a, b) = set_intersection$_2$(a, cartesian_product$_2$(b, b))) fof(d6_wellord$_1$, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ (in(a, relation_restriction(c, b)) $\iff$ (in(a, c) and in(a, cartesian_product$_2$(b, b))))) fof(t16_wellord
$\forall a, b, c$: (c = set_intersection$_2$(a, b) $\iff$ $\forall d$: (in(d, c) $\iff$ (in(d, a) and in(d, b)))) fof(d3_xboole$_0$, axiom)

SEU247+1.p MPTP bushy problem t18_wellord1
$\forall a, b$: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: relation_restriction(a, b) = set_intersection$_2$(a, cartesian_product$_2$(b, b))) fof(d6_wellord$_1$, axiom)
$\forall a, b$: (relation(a) $\Rightarrow$ relation(relation_restriction(a, b))) fof(dt_k2_wellord$_1$, axiom)
\$true fof(dt_k2_zfmisc$_1$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
$\forall a, b$: (relation(a) $\Rightarrow$ relation(relation_dom_restriction(a, b))) fof(dt_k7_relat$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation(relation_rng_restriction(a, b))) fof(dt_k8_relat$_1$, axiom)
$\forall a, b$: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ relation_dom_restriction(relation_rng_restriction(a, c), b) = relation_rng_restriction(a, relation_dom_res
$\forall a, b$: (relation(b) $\Rightarrow$ relation_restriction(b, a) = relation_dom_restriction(relation_rng_restriction(a, b), a)) fof(t17_wellord
$\forall a, b$: (relation(b) $\Rightarrow$ relation_restriction(b, a) = relation_rng_restriction(a, relation_dom_restriction(b, a))) fof(t18_wellord

SEU248+1.p MPTP bushy problem l29_wellord1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: ((relation(a) and empty(a) and function(a)) $\Rightarrow$ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ $\forall c$: (relation(c) $\Rightarrow$ (c = relation_rng_restriction(a, b) $\iff$ $\forall d, e$: (in(ordered_pair(d, e), c) $\iff$ (in(e, a) and in(ordered_pair(d, e), b)))))) fof(d12_relat$_1$, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (b = relation_dom(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair(c, d), a)))) fof(d4_relat$_1$, axiom)
$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k4_tarski, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ relation(relation_rng_restriction(a, b))) fof(dt_k8_relat$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)

∀a, b: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)

∀a, b: ((relation(b) and function(b)) ⇒ (relation(relation_rng_restriction(a, b)) and function(relation_rng_restriction(a, b))))

∀a, b: (relation(b) ⇒ relation_dom(relation_rng_restriction(a, b)) ⊆ relation_dom(b)) fof(l29_wellord$_1$, conjecture)

∃a: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)

∃a: empty(a) fof(rc1_xboole$_0$, axiom)

∃a: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)

∃a: ¬ empty(a) fof(rc2_xboole$_0$, axiom)

∃a: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)

∀a, b: a ⊆ a fof(reflexivity_r1_tarski, axiom)

∀a, b: (in(a, b) ⇒ element(a, b)) fof(t1_subset, axiom)

∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b))) fof(t2_subset, axiom)

∀a, b: (element(a, powerset(b)) ⟺ a ⊆ b) fof(t3_subset, axiom)

∀a, b, c: ((in(a, b) and element(b, powerset(c))) ⇒ element(a, c)) fof(t4_subset, axiom)

∀a, b, c: ¬ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)

∀a: (empty(a) ⇒ a = empty_set) fof(t6_boole, axiom)

∀a, b: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)

∀a, b: ¬ empty(a) and a ≠ b and empty(b) fof(t8_boole, axiom)

SEU254+1.p MPTP bushy problem t24_wellord1

∀a, b: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)

∀a: (empty(a) ⇒ function(a)) fof(cc1_funct$_1$, axiom)

∀a: ((relation(a) and empty(a) and function(a)) ⇒ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio

∀a, b: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)

∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)

∀a, b: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)

∀a: (relation(a) ⇒ ∀b: relation_restriction(a, b) = set_intersection$_2$(a, cartesian_product$_2$(b, b))) fof(d6_wellord$_1$, axiom)

$true fof(dt_k1_tarski, axiom)

$true fof(dt_k1_xboole$_0$, axiom)

$true fof(dt_k2_tarski, axiom)

∀a, b: (relation(a) ⇒ relation(relation_restriction(a, b))) fof(dt_k2_wellord$_1$, axiom)

$true fof(dt_k2_zfmisc$_1$, axiom)

$true fof(dt_k3_xboole$_0$, axiom)

$true fof(dt_k4_tarski, axiom)

$true fof(dt_m1_subset$_1$, axiom)

∀a: ∃b: element(b, a) fof(existence_m1_subset$_1$, axiom)

empty(empty_set) fof(fc1_xboole$_0$, axiom)

∀a, b: ¬ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)

∀a, b: set_intersection$_2$(a, a) = a fof(idempotence_k3_xboole$_0$, axiom)

∀a: (relation(a) ⇒ (transitive(a) ⟺ ∀b, c, d: ((in(ordered_pair(b, c), a) and in(ordered_pair(c, d), a)) ⇒ in(ordered_pair(b,

∃a: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)

∃a: empty(a) fof(rc1_xboole$_0$, axiom)

∃a: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)

∃a: ¬ empty(a) fof(rc2_xboole$_0$, axiom)

∃a: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)

∀a, b, c, d: (in(ordered_pair(a, b), cartesian_product$_2$(c, d)) ⟺ (in(a, c) and in(b, d))) fof(t106_zfmisc$_1$, axiom)

∀a, b, c: (relation(c) ⇒ (in(a, relation_restriction(c, b)) ⟺ (in(a, c) and in(a, cartesian_product$_2$(b, b))))) fof(t16_wellor

∀a, b: (in(a, b) ⇒ element(a, b)) fof(t1_subset, axiom)

∀a, b: (relation(b) ⇒ (transitive(b) ⇒ transitive(relation_restriction(b, a)))) fof(t24_wellord$_1$, conjecture)

∀a: set_intersection$_2$(a, empty_set) = empty_set fof(t2_boole, axiom)

∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b))) fof(t2_subset, axiom)

∀a: (empty(a) ⇒ a = empty_set) fof(t6_boole, axiom)

∀a, b: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)

∀a, b: ¬ empty(a) and a ≠ b and empty(b) fof(t8_boole, axiom)

SEU255+1.p MPTP bushy problem t25_wellord1

∀a, b: (in(a, b) ⇒ ¬ in(b, a)) fof(antisymmetry_r2_hidden, axiom)

∀a: (empty(a) ⇒ function(a)) fof(cc1_funct$_1$, axiom)

∀a: ((relation(a) and empty(a) and function(a)) ⇒ (relation(a) and function(a) and one_to_one(a))) fof(cc2_funct$_1$, axio

∀a, b: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)

∀a, b: set_intersection$_2$(a, b) = set_intersection$_2$(b, a) fof(commutativity_k3_xboole$_0$, axiom)

$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: relation_restriction(a, b) = set_intersection$_2(a$, cartesian_product$_2(b, b)$)) fof(d6_wellord$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_tarski, axiom)
$\forall a, b$: (relation(a) $\Rightarrow$ relation(relation_restriction(a, b))) fof(dt_k2_wellord$_1$, axiom)
\$true fof(dt_k2_zfmisc$_1$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
\$true fof(dt_k4_tarski, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (antisymmetric(a) $\iff$ $\forall b, c$: ((in(ordered_pair$(b, c), a)$ and in(ordered_pair$(c, b), a)$) $\Rightarrow$ $b = c$))) fof(l3_wellord$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: (relation(a) and empty(a) and function(a)) fof(rc2_funct$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and function(a) and one_to_one(a)) fof(rc3_funct$_1$, axiom)
$\forall a, b, c$: (relation(c) $\Rightarrow$ (in$(a$, relation_restriction$(c, b))$ $\iff$ (in(a, c) and in$(a$, cartesian_product$_2(b, b)$)))) fof(t16_wellord
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ (antisymmetric(b) $\Rightarrow$ antisymmetric(relation_restriction(b, a)))) fof(t25_wellord$_1$, conjecture)
$\forall a$: set_intersection$_2(a$, empty_set) = empty_set fof(t2_boole, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU257+1.p MPTP bushy problem t32_wellord1
$\forall a, b$: set_intersection$_2(a, b)$ = set_intersection$_2(b, a)$ fof(commutativity_k3_xboole$_0$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ (well_ordering(a) $\iff$ (reflexive(a) and transitive(a) and antisymmetric(a) and connected(a) and well_f
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: relation_restriction(a, b) = set_intersection$_2(a$, cartesian_product$_2(b, b)$)) fof(d6_wellord$_1$, axiom)
$\forall a, b$: (relation(a) $\Rightarrow$ relation(relation_restriction(a, b))) fof(dt_k2_wellord$_1$, axiom)
\$true fof(dt_k2_zfmisc$_1$, axiom)
\$true fof(dt_k3_xboole$_0$, axiom)
$\forall a, b$: set_intersection$_2(a, a) = a$ fof(idempotence_k3_xboole$_0$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ (reflexive(b) $\Rightarrow$ reflexive(relation_restriction(b, a)))) fof(t22_wellord$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ (connected(b) $\Rightarrow$ connected(relation_restriction(b, a)))) fof(t23_wellord$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ (transitive(b) $\Rightarrow$ transitive(relation_restriction(b, a)))) fof(t24_wellord$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ (antisymmetric(b) $\Rightarrow$ antisymmetric(relation_restriction(b, a)))) fof(t25_wellord$_1$, axiom)
$\forall a, b$: (relation(b) $\Rightarrow$ (well_founded_relation(b) $\Rightarrow$ well_founded_relation(relation_restriction(b, a)))) fof(t31_wellord$_1$, axic
$\forall a, b$: (relation(b) $\Rightarrow$ (well_ordering(b) $\Rightarrow$ well_ordering(relation_restriction(b, a)))) fof(t32_wellord$_1$, conjecture)

SEU261+1.p MPTP bushy problem t54_wellord1
$\forall a$: (relation(a) $\Rightarrow$ (well_ordering(a) $\iff$ (reflexive(a) and transitive(a) and antisymmetric(a) and connected(a) and well_f
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ $\forall c$: ((relation(c) and function(c)) $\Rightarrow$ (relation_isomorphism(a, b, c) $\Rightarrow$ ((reflexive(a) $\Rightarrow$ reflexive(b)) and (transitive(a) $\Rightarrow$ transitive(b)) and (connected(a) $\Rightarrow$ connected(b)) and (antisymmetric(antisymmetric(b)) and (well_founded_relation(a) $\Rightarrow$ well_founded_relation(b))))))) fof(t53_wellord$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (relation(b) $\Rightarrow$ $\forall c$: ((relation(c) and function(c)) $\Rightarrow$ ((well_ordering(a) and relation_isomorphism$(a,$ well_ordering(b)))))) fof(t54_wellord$_1$, conjecture)

SEU262+1.p MPTP bushy problem t12_relset_1
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
$\forall a, b, c$: (element$(c$, powerset(cartesian_product$_2(a, b)$)) $\Rightarrow$ relation(c)) fof(cc1_relset$_1$, axiom)
$\forall a, b$: unordered_pair(a, b) = unordered_pair(b, a) fof(commutativity_k2_tarski, axiom)
$\forall a, b$: ($a \subseteq b$ $\iff$ $\forall c$: (in(c, a) $\Rightarrow$ in(c, b))) fof(d3_tarski, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (b = relation_dom(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair$(c, d), a)$))) fof(d4_relat$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $\forall b$: (b = relation_rng(a) $\iff$ $\forall c$: (in(c, b) $\iff$ $\exists d$: in(ordered_pair$(d, c), a)$))) fof(d5_relat$_1$, axiom)

$\forall a, b$: ordered_pair(a, b) = unordered_pair(unordered_pair(a, b), singleton(a)) fof(d5_tarski, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k2_tarski, axiom)
\$true fof(dt_k2_zfmisc$_1$, axiom)
\$true fof(dt_k4_tarski, axiom)
\$true fof(dt_m1_relset$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a, b, c$: (relation_of2_as_subset(c, a, b) $\Rightarrow$ element(c, powerset(cartesian_product$_2(a, b)$))) fof(dt_m2_relset$_1$, axiom)
$\forall a, b$: $\exists c$: relation_of$_2(c, a, b)$ fof(existence_m1_relset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a, b$: $\exists c$: relation_of2_as_subset(c, a, b) fof(existence_m2_relset$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b, c$: (relation_of2_as_subset(c, a, b) $\iff$ relation_of$_2(c, a, b)$) fof(redefinition_m2_relset$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d$: (in(ordered_pair(a, b), cartesian_product$_2(c, d)$) $\iff$ (in(a, c) and in(b, d))) fof(t106_zfmisc$_1$, axiom)
$\forall a, b, c$: (relation_of2_as_subset(c, a, b) $\Rightarrow$ (relation_dom$(c) \subseteq a$ and relation_rng$(c) \subseteq b$)) fof(t12_relset$_1$, conjecture)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)
$\forall a, b, c$: ((in(a, b) and element(b, powerset(c))) $\Rightarrow$ element(a, c)) fof(t4_subset, axiom)
$\forall a, b, c$: $\neg$ in(a, b) and element(b, powerset(c)) and empty(c) fof(t5_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU263+1.p MPTP bushy problem t14_relset_1
$\forall a, b, c$: (element(c, powerset(cartesian_product$_2(a, b)$)) $\Rightarrow$ relation(c)) fof(cc1_relset$_1$, axiom)
$\forall a, b, c$: (relation_of$_2(c, a, b)$ $\iff$ $c \subseteq$ cartesian_product$_2(a, b)$) fof(d1_relset$_1$, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k2_zfmisc$_1$, axiom)
\$true fof(dt_m1_relset$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a, b, c$: (relation_of2_as_subset(c, a, b) $\Rightarrow$ element(c, powerset(cartesian_product$_2(a, b)$))) fof(dt_m2_relset$_1$, axiom)
$\forall a, b$: $\exists c$: relation_of$_2(c, a, b)$ fof(existence_m1_relset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a, b$: $\exists c$: relation_of2_as_subset(c, a, b) fof(existence_m2_relset$_1$, axiom)
$\forall a, b, c$: (relation_of2_as_subset(c, a, b) $\iff$ relation_of$_2(c, a, b)$) fof(redefinition_m2_relset$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c, d$: (($a \subseteq b$ and $c \subseteq d$) $\Rightarrow$ cartesian_product$_2(a, c) \subseteq$ cartesian_product$_2(b, d)$) fof(t119_zfmisc$_1$, axiom)
$\forall a, b, c$: (relation_of2_as_subset(c, a, b) $\Rightarrow$ (relation_dom$(c) \subseteq a$ and relation_rng$(c) \subseteq b$)) fof(t12_relset$_1$, axiom)
$\forall a, b, c, d$: (relation_of2_as_subset(d, c, a) $\Rightarrow$ (relation_rng$(d) \subseteq b$ $\Rightarrow$ relation_of2_as_subset(d, c, b))) fof(t14_relset$_1$, conjec
$\forall a, b, c$: (($a \subseteq b$ and $b \subseteq c$) $\Rightarrow$ $a \subseteq c$) fof(t1_xboole$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ $a \subseteq$ cartesian_product$_2$(relation_dom(a), relation_rng(a))) fof(t21_relat$_1$, axiom)
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$) fof(t3_subset, axiom)

SEU264+1.p MPTP bushy problem t16_relset_1
$\forall a, b, c$: (element(c, powerset(cartesian_product$_2(a, b)$)) $\Rightarrow$ relation(c)) fof(cc1_relset$_1$, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_zfmisc$_1$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k2_zfmisc$_1$, axiom)

\$true fof(dt_m1_relset$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a, b, c$: (relation_of2_as_subset$(c, a, b) \Rightarrow$ element$(c$, powerset(cartesian_product$_2(a, b)))$) fof(dt_m2_relset$_1$, axiom)
$\forall a, b$: $\exists c$: relation_of$_2(c, a, b)$ fof(existence_m1_relset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a, b$: $\exists c$: relation_of2_as_subset(c, a, b) fof(existence_m2_relset$_1$, axiom)
$\forall a, b, c$: (relation_of2_as_subset$(c, a, b) \iff$ relation_of$_2(c, a, b)$) fof(redefinition_m2_relset$_1$, axiom)
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a, b, c$: (relation_of2_as_subset$(c, a, b) \Rightarrow$ (relation_dom$(c) \subseteq a$ and relation_rng$(c) \subseteq b$)) fof(t12_relset$_1$, axiom)
$\forall a, b, c, d$: (relation_of2_as_subset$(d, c, a) \Rightarrow$ (relation_rng$(d) \subseteq b \Rightarrow$ relation_of2_as_subset(d, c, b))) fof(t14_relset$_1$, axiom
$\forall a, b, c, d$: (relation_of2_as_subset$(d, c, a) \Rightarrow (a \subseteq b \Rightarrow$ relation_of2_as_subset(d, c, b))) fof(t16_relset$_1$, conjecture)
$\forall a, b, c$: $((a \subseteq b$ and $b \subseteq c) \Rightarrow a \subseteq c)$ fof(t1_xboole$_1$, axiom)
$\forall a, b$: (element$(a$, powerset$(b)) \iff a \subseteq b$) fof(t3_subset, axiom)

SEU267+1.p MPTP bushy problem t7_mcart_1
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a, b$: (element$(a, b) \Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a$: (empty$(a) \Rightarrow a =$ empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)
\$true fof(dt_k1_mcart$_1$, axiom)
\$true fof(dt_k2_mcart$_1$, axiom)
\$true fof(dt_k4_tarski, axiom)
$\forall a, b$: $\neg$ empty(ordered_pair(a, b)) fof(fc1_zfmisc$_1$, axiom)
$\forall a, b$: (pair_first(ordered_pair$(a, b)) = a$ and pair_second(ordered_pair$(a, b)) = b$) fof(t7_mcart$_1$, conjecture)
$\forall a$: ($\exists b, c$: $a =$ ordered_pair$(b, c) \Rightarrow \forall b$: ($b =$ pair_first$(a) \iff \forall c, d$: ($a =$ ordered_pair$(c, d) \Rightarrow b = c$))) fof(d1_mcart$_1$, axiom)
$\forall a$: ($\exists b, c$: $a =$ ordered_pair$(b, c) \Rightarrow \forall b$: ($b =$ pair_second$(a) \iff \forall c, d$: ($a =$ ordered_pair$(c, d) \Rightarrow b = d$))) fof(d2_mcart$_1$, axiom)

SEU272+1.p MPTP bushy problem s1_xboole_0__e3_38_1__ordinal1
$\forall a, b$: (ordinal$(b) \Rightarrow \exists c$: $\forall d$: (in$(d, c) \iff$ (in$(d$, succ$(b))$ and $\exists e$: (ordinal(e) and $d = e$ and in(e, a))))) fof(s1_xboole_0__e
$\forall a$: (empty$(a) \Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: ((epsilon_transitive(a) and epsilon_connected$(a)) \Rightarrow$ ordinal(a)) fof(cc2_ordinal$_1$, axiom)
$\exists a$: (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a)) fof(rc1_ordinal$_1$, axiom)
$\forall a$: (empty$(a) \Rightarrow$ (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a))) fof(cc3_ordinal$_1$, axiom)
$\exists a$: ($\neg$ empty(a) and epsilon_transitive(a) and epsilon_connected(a) and ordinal(a)) fof(rc3_ordinal$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\forall a, b$: (in$(a, b) \Rightarrow \neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
\$true fof(dt_k1_ordinal$_1$, axiom)
$\forall a$: $\neg$ empty(succ(a)) fof(fc1_ordinal$_1$, axiom)
$\forall a$: (ordinal$(a) \Rightarrow$ (epsilon_transitive(a) and epsilon_connected(a))) fof(cc1_ordinal$_1$, axiom)
$\forall a$: (ordinal$(a) \Rightarrow$ ($\neg$ empty(succ(a)) and epsilon_transitive(succ(a)) and epsilon_connected(succ(a)) and ordinal(succ(a))))
$\forall a, b$: (ordinal$(b) \Rightarrow$ ($\forall c, d, e$: (($c = d$ and $\exists f$: (ordinal(f) and $d = f$ and in(f, a)) and $c = e$ and $\exists g$: (ordinal(g) and $e = g$ and in(g, a))) $\Rightarrow d = e$) $\Rightarrow \exists c$: $\forall d$: (in$(d, c) \iff \exists e$: (in$(e$, succ$(b))$ and $e = d$ and $\exists h$: (ordinal(h) and $d = h$ and in(h, a)))))) fof(s1_tarski__e8_6__wellord2$_1$, axiom)

SEU275+1.p MPTP bushy problem t7_wellord2
$\forall a$: (ordinal$(a) \Rightarrow$ (epsilon_transitive(a) and epsilon_connected(a))) fof(cc1_ordinal$_1$, axiom)
$\forall a$: ((epsilon_transitive(a) and epsilon_connected$(a)) \Rightarrow$ ordinal(a)) fof(cc2_ordinal$_1$, axiom)
$\forall a$: (relation$(a) \Rightarrow$ (well_ordering$(a) \iff$ (reflexive(a) and transitive(a) and antisymmetric(a) and connected(a) and well_f
$\forall a$: relation(inclusion_relation(a)) fof(dt_k1_wellord$_2$, axiom)
$\exists a$: (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a)) fof(rc1_ordinal$_1$, axiom)

∀a: reflexive(inclusion_relation(a))   fof(t2_wellord$_2$, axiom)
∀a: transitive(inclusion_relation(a))   fof(t3_wellord$_2$, axiom)
∀a: (ordinal(a) ⇒ connected(inclusion_relation(a)))   fof(t4_wellord$_2$, axiom)
∀a: antisymmetric(inclusion_relation(a))   fof(t5_wellord$_2$, axiom)
∀a: (ordinal(a) ⇒ well_founded_relation(inclusion_relation(a)))   fof(t6_wellord$_2$, axiom)
∀a: (ordinal(a) ⇒ well_ordering(inclusion_relation(a)))   fof(t7_wellord$_2$, conjecture)

SEU277+1.p MPTP bushy problem s1_xboole_0__e1_8_1_1__relat_1
∀a, b, c: ((relation(b) and relation(c) and function(c)) ⇒ ∃d: ∀e: (in(e, d) ⟺ (in(e, cartesian_product$_2$(a, a)) and ∃f, g: (e = ordered_pair(f, g) and in(ordered_pair(apply(c, f), apply(c, g)), b)))))   fof(s1_xboole_0__e6_21__wellord2$_{-1}$, conjecture)
∃a: (relation(a) and function(a) and one_to_one(a))   fof(rc3_funct$_1$, axiom)
∀a: (ordinal(a) ⇒ (epsilon_transitive(a) and epsilon_connected(a)))   fof(cc1_ordinal$_1$, axiom)
∀a: ((epsilon_transitive(a) and epsilon_connected(a)) ⇒ ordinal(a))   fof(cc2_ordinal$_1$, axiom)
∃a: (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a))   fof(rc1_ordinal$_1$, axiom)
∃a: (relation(a) and function(a) and one_to_one(a) and empty(a) and epsilon_transitive(a) and epsilon_connected(a) and ord
∃a: (¬ empty(a) and epsilon_transitive(a) and epsilon_connected(a) and ordinal(a))   fof(rc3_ordinal$_1$, axiom)
∀a: (empty(a) ⇒ function(a))   fof(cc1_funct$_1$, axiom)
∃a: (relation(a) and empty(a) and function(a))   fof(rc2_funct$_1$, axiom)
∀a: ((relation(a) and empty(a) and function(a)) ⇒ (relation(a) and function(a) and one_to_one(a)))   fof(cc2_funct$_1$, axio
∀a: (empty(a) ⇒ (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a)))   fof(cc3_ordinal$_1$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
$true   fof(dt_k1_funct$_1$, axiom)
$true   fof(dt_k2_zfmisc$_1$, axiom)
$true   fof(dt_k4_tarski, axiom)
∃a: (relation(a) and function(a))   fof(rc1_funct$_1$, axiom)
∀a, b: ¬ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
∀a, b, c: ((relation(b) and relation(c) and function(c)) ⇒ (∀d, e, f: ((d = e and ∃g, h: (e = ordered_pair(g, h) and in(ordered_p
f and ∃i, j: (f = ordered_pair(i, j) and in(ordered_pair(apply(c, i), apply(c, j)), b))) ⇒ e = f) ⇒ ∃d: ∀e: (in(e, d) ⟺
∃f: (in(f, cartesian_product$_2$(a, a)) and f = e and ∃k, l: (e = ordered_pair(k, l) and in(ordered_pair(apply(c, k), apply(c, l)), b

SEU280+1.p MPTP bushy problem s1_xboole_0__e6_22__wellord2
∀a: ∃b: ∀c: (in(c, b) ⟺ (in(c, a) and ordinal(c)))   fof(s1_xboole_0__e6_22__wellord$_2$, conjecture)
∀a: ((epsilon_transitive(a) and epsilon_connected(a)) ⇒ ordinal(a))   fof(cc2_ordinal$_1$, axiom)
∃a: (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a))   fof(rc1_ordinal$_1$, axiom)
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
∀a: (ordinal(a) ⇒ (epsilon_transitive(a) and epsilon_connected(a)))   fof(cc1_ordinal$_1$, axiom)
∀a: (∀b, c, d: ((b = c and ordinal(c) and b = d and ordinal(d)) ⇒ c = d) ⇒ ∃b: ∀c: (in(c, b) ⟺ ∃d: (in(d, a) and d = c and ordinal(c))))   fof(s1_tarski__e6_22__wellord2$_{-1}$, axiom)

SEU281+1.p MPTP bushy problem s1_xboole_0__e4_5_1__funct_1
∀a, b: ∃c: ∀d: (in(d, c) ⟺ (in(d, cartesian_product$_2$(a, b)) and ∃e, f: (ordered_pair(e, f) = d and in(e, a) and f = singleton(e))))   fof(s1_xboole_0__e16_22__wellord2$_{-1}$, conjecture)
∀a: (ordinal(a) ⇒ (epsilon_transitive(a) and epsilon_connected(a)))   fof(cc1_ordinal$_1$, axiom)
∀a: ((epsilon_transitive(a) and epsilon_connected(a)) ⇒ ordinal(a))   fof(cc2_ordinal$_1$, axiom)
∃a: (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a))   fof(rc1_ordinal$_1$, axiom)
∃a: (¬ empty(a) and epsilon_transitive(a) and epsilon_connected(a) and ordinal(a))   fof(rc3_ordinal$_1$, axiom)
∀a: (empty(a) ⇒ function(a))   fof(cc1_funct$_1$, axiom)
∀a: (empty(a) ⇒ (epsilon_transitive(a) and epsilon_connected(a) and ordinal(a)))   fof(cc3_ordinal$_1$, axiom)
∃a: empty(a)   fof(rc1_xboole$_0$, axiom)
∃a: ¬ empty(a)   fof(rc2_xboole$_0$, axiom)
∀a, b: (in(a, b) ⇒ ¬ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
$true   fof(dt_k1_tarski, axiom)
$true   fof(dt_k2_zfmisc$_1$, axiom)
$true   fof(dt_k4_tarski, axiom)
∀a, b: ¬ empty(ordered_pair(a, b))   fof(fc1_zfmisc$_1$, axiom)
∀a, b: (∀c, d, e: ((c = d and ∃f, g: (ordered_pair(f, g) = d and in(f, a) and g = singleton(f)) and c = e and ∃h, i: (ordered_pair
e and in(h, a) and i = singleton(h))) ⇒ d = e) ⇒ ∃c: ∀d: (in(d, c) ⟺ ∃e: (in(e, cartesian_product$_2$(a, b)) and e = d and ∃j, k: (ordered_pair(j, k) = d and in(j, a) and k = singleton(j)))))   fof(s1_tarski__e16_22__wellord2$_{-2}$, axiom)

SEU284+1.p MPTP bushy problem s3_funct_1__e16_22__wellord2
$\forall a$: $\exists b$: (relation(b) and function(b) and relation_dom(b) $= a$ and $\forall c$: (in(c, a) $\Rightarrow$ apply(b, c) = singleton(c))) fof(s3_funct_
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a)) fof(antisymmetry_r2_hidden, axiom)
\$true fof(dt_k1_funct$_1$, axiom)
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k1_tarski, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\forall a$: (($\forall b, c, d$: ((in(b, a) and c = singleton(b) and d = singleton(b)) $\Rightarrow$ $c = d$) and $\forall b$: $\neg$ in(b, a) and $\forall c$: $c \neq$ singleton(b)) $\Rightarrow$ $\exists b$: (relation(b) and function(b) and relation_dom(b) $= a$ and $\forall c$: (in(c, a) $\Rightarrow$ apply(b, c) = singleton(c)))) fof(s2_funct_1__e16_22__wellord2$_{-1}$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ function(a)) fof(cc1_funct$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ relation(a)) fof(cc1_relat$_1$, axiom)
\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
empty(empty_set) and relation(empty_set) and relation_empty_yielding(empty_set) fof(fc12_relat$_1$, axiom)
empty(empty_set) fof(fc1_xboole$_0$, axiom)
empty(empty_set) and relation(empty_set) fof(fc4_relat$_1$, axiom)
$\forall a$: (($\neg$ empty(a) and relation(a)) $\Rightarrow$ $\neg$ empty(relation_dom(a))) fof(fc5_relat$_1$, axiom)
$\forall a$: (empty(a) $\Rightarrow$ (empty(relation_dom(a)) and relation(relation_dom(a)))) fof(fc7_relat$_1$, axiom)
$\exists a$: (empty(a) and relation(a)) fof(rc1_relat$_1$, axiom)
$\exists a$: empty(a) fof(rc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and relation(a)) fof(rc2_relat$_1$, axiom)
$\exists a$: $\neg$ empty(a) fof(rc2_xboole$_0$, axiom)
$\exists a$: (relation(a) and relation_empty_yielding(a)) fof(rc3_relat$_1$, axiom)
$\forall a, b$: (in(a, b) $\Rightarrow$ element(a, b)) fof(t1_subset, axiom)
$\forall a, b$: (element(a, b) $\Rightarrow$ (empty(b) or in(a, b))) fof(t2_subset, axiom)
$\forall a$: (empty(a) $\Rightarrow$ a = empty_set) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ in(a, b) and empty(b) fof(t7_boole, axiom)
$\forall a, b$: $\neg$ empty(a) and $a \neq b$ and empty(b) fof(t8_boole, axiom)

SEU303+1.p MPTP bushy problem t26_finset_1
\$true fof(dt_k1_relat$_1$, axiom)
\$true fof(dt_k2_relat$_1$, axiom)
\$true fof(dt_k9_relat$_1$, axiom)
$\forall a, b$: ((relation(a) and function(a) and finite(b)) $\Rightarrow$ finite(relation_image(a, b))) fof(fc13_finset$_1$, axiom)
$\exists a$: (relation(a) and function(a)) fof(rc1_funct$_1$, axiom)
$\forall a$: (relation(a) $\Rightarrow$ relation_image(a, relation_dom(a)) = relation_rng(a)) fof(t146_relat$_1$, axiom)
$\forall a, b$: ((relation(b) and function(b)) $\Rightarrow$ (finite(a) $\Rightarrow$ finite(relation_image(b, a)))) fof(t17_finset$_1$, axiom)
$\forall a$: ((relation(a) and function(a)) $\Rightarrow$ (finite(relation_dom(a)) $\Rightarrow$ finite(relation_rng(a)))) fof(t26_finset$_1$, conjecture)

SEU319+1.p MPTP bushy problem t29_tops_1
$\forall a$: (top_str(a) $\Rightarrow$ $\forall b$: (element(b, powerset(the_carrier(a))) $\Rightarrow$ (closed_subset(b, a) $\iff$ open_subset(subset_difference(the_c
\$true fof(dt_k1_zfmisc$_1$, axiom)
$\forall a$: (one_sorted_str(a) $\Rightarrow$ element(cast_as_carrier_subset(a), powerset(the_carrier(a)))) fof(dt_k2_pre_topc, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ element(subset_complement(a, b), powerset(a))) fof(dt_k3_subset$_1$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
$\forall a, b, c$: ((element(b, powerset(a)) and element(c, powerset(a))) $\Rightarrow$ element(subset_difference(a, b, c), powerset(a))) fof(dt_
$\forall a$: (top_str(a) $\Rightarrow$ one_sorted_str(a)) fof(dt_l1_pre_topc, axiom)
\$true fof(dt_l1_struct$_0$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
\$true fof(dt_u1_struct$_0$, axiom)
$\exists a$: top_str(a) fof(existence_l1_pre_topc, axiom)
$\exists a$: one_sorted_str(a) fof(existence_l1_struct$_0$, axiom)
$\forall a$: $\exists b$: element(b, a) fof(existence_m1_subset$_1$, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ subset_complement(a, subset_complement(a, b)) $= b$) fof(involutiveness_k3_subset$_1$, axio
$\forall a, b, c$: ((element(b, powerset(a)) and element(c, powerset(a))) $\Rightarrow$ subset_difference(a, b, c) = set_difference(b, c)) fof(rede
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: (one_sorted_str(a) $\Rightarrow$ $\forall b$: (element(b, powerset(the_carrier(a))) $\Rightarrow$ subset_complement(the_carrier(a), b) = subset_difference(the_carrier(a), cast_as_carrier_subset(a), b))) fof(t17_pre_topc, axiom)

$\forall a$: (top_str(a) $\Rightarrow$ $\forall b$: (element(b, powerset(the_carrier(a))) $\Rightarrow$ (closed_subset(b, a) $\iff$ open_subset(subset_complement(th
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$)   fof(t3_subset, axiom)

SEU320+1.p MPTP bushy problem t30_tops_1
$true   fof(dt_k1_zfmisc$_1$, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ element(subset_complement(a, b), powerset(a)))   fof(dt_k3_subset$_1$, axiom)
$\forall a$: (top_str(a) $\Rightarrow$ one_sorted_str(a))   fof(dt_l1_pre_topc, axiom)
$true   fof(dt_l1_struct$_0$, axiom)
$true   fof(dt_m1_subset$_1$, axiom)
$true   fof(dt_u1_struct$_0$, axiom)
$\exists a$: top_str(a)   fof(existence_l1_pre_topc, axiom)
$\exists a$: one_sorted_str(a)   fof(existence_l1_struct$_0$, axiom)
$\forall a$: $\exists b$: element(b, a)   fof(existence_m1_subset$_1$, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ subset_complement(a, subset_complement(a, b)) $= b$)   fof(involutiveness_k3_subset$_1$, axio
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\forall a$: (top_str(a) $\Rightarrow$ $\forall b$: (element(b, powerset(the_carrier(a))) $\Rightarrow$ (closed_subset(b, a) $\iff$ open_subset(subset_complement(th
$\forall a$: (top_str(a) $\Rightarrow$ $\forall b$: (element(b, powerset(the_carrier(a))) $\Rightarrow$ (open_subset(b, a) $\iff$ closed_subset(subset_complement(th
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$)   fof(t3_subset, axiom)

SEU323+1.p MPTP bushy problem t51_tops_1
$\forall a, b$: ((topological_space(a) and top_str(a) and closed_subset(b, a) and element(b, powerset(the_carrier(a)))) $\Rightarrow$ open_subset(
$\forall a$: ((topological_space(a) and top_str(a)) $\Rightarrow$ $\exists b$: (element(b, powerset(the_carrier(a))) and closed_subset(b, a)))   fof(rc6_p
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ subset_complement(a, subset_complement(a, b)) $= b$)   fof(involutiveness_k3_subset$_1$, axio
$\forall a, b$: $a \subseteq a$ fof(reflexivity_r1_tarski, axiom)
$\exists a$: one_sorted_str(a)   fof(existence_l1_struct$_0$, axiom)
$\forall a, b$: (element(b, powerset(a)) $\Rightarrow$ element(subset_complement(a, b), powerset(a)))   fof(dt_k3_subset$_1$, axiom)
$\forall a, b$: ((top_str(a) and element(b, powerset(the_carrier(a)))) $\Rightarrow$ element(topstr_closure(a, b), powerset(the_carrier(a))))   fo
$true   fof(dt_l1_struct$_0$, axiom)
$\forall a, b$: ((topological_space(a) and top_str(a) and element(b, powerset(the_carrier(a)))) $\Rightarrow$ closed_subset(topstr_closure(a, b), a)
$\forall a, b$: ((topological_space(a) and top_str(a) and open_subset(b, a) and element(b, powerset(the_carrier(a)))) $\Rightarrow$ closed_subset(
$\exists a$: top_str(a)   fof(existence_l1_pre_topc, axiom)
$\forall a$: $\exists b$: element(b, a)   fof(existence_m1_subset$_1$, axiom)
$\forall a, b$: ((top_str(a) and element(b, powerset(the_carrier(a)))) $\Rightarrow$ element(interior(a, b), powerset(the_carrier(a))))   fof(dt_k1
$true   fof(dt_k1_zfmisc$_1$, axiom)
$\forall a$: (top_str(a) $\Rightarrow$ one_sorted_str(a))   fof(dt_l1_pre_topc, axiom)
$true   fof(dt_m1_subset$_1$, axiom)
$true   fof(dt_u1_struct$_0$, axiom)
$\forall a$: ((topological_space(a) and top_str(a)) $\Rightarrow$ $\exists b$: (element(b, powerset(the_carrier(a))) and open_subset(b, a)))   fof(rc1_top
$\forall a, b$: (element(a, powerset(b)) $\iff$ $a \subseteq b$)   fof(t3_subset, axiom)
$\forall a$: (top_str(a) $\Rightarrow$ $\forall b$: (element(b, powerset(the_carrier(a))) $\Rightarrow$ interior(a, b) = subset_complement(the_carrier(a), topstr_clos
$\forall a$: ((topological_space(a) and top_str(a)) $\Rightarrow$ $\forall b$: (element(b, powerset(the_carrier(a))) $\Rightarrow$ open_subset(interior(a, b), a)))

SEU355+1.p MPTP bushy problem t6_yellow_0
$\forall a, b$: (in(a, b) $\Rightarrow$ $\neg$ in(b, a))   fof(antisymmetry_r2_hidden, axiom)
$\forall a$: (empty(a) $\Rightarrow$ finite(a))   fof(cc1_finset$_1$, axiom)
$\forall a$: (rel_str(a) $\Rightarrow$ $\forall b, c$: (element(c, the_carrier(a)) $\Rightarrow$ (relstr_element_smaller(a, b, c) $\iff$ $\forall d$: (element(d, the_carrier(a)) $\Rightarrow$
(in(d, b) $\Rightarrow$ related(a, c, d))))))   fof(d8_lattice$_3$, axiom)
$\forall a$: (rel_str(a) $\Rightarrow$ $\forall b, c$: (element(c, the_carrier(a)) $\Rightarrow$ (relstr_set_smaller(a, b, c) $\iff$ $\forall d$: (element(d, the_carrier(a)) $\Rightarrow$
(in(d, b) $\Rightarrow$ related(a, d, c))))))   fof(d9_lattice$_3$, axiom)
$true   fof(dt_k1_xboole$_0$, axiom)
$\forall a$: (rel_str(a) $\Rightarrow$ one_sorted_str(a))   fof(dt_l1_orders$_2$, axiom)
$true   fof(dt_l1_struct$_0$, axiom)
$true   fof(dt_m1_subset$_1$, axiom)
$true   fof(dt_u1_struct$_0$, axiom)
$\exists a$: rel_str(a)   fof(existence_l1_orders$_2$, axiom)
$\exists a$: one_sorted_str(a)   fof(existence_l1_struct$_0$, axiom)
$\forall a$: $\exists b$: element(b, a)   fof(existence_m1_subset$_1$, axiom)
empty(empty_set)   fof(fc1_xboole$_0$, axiom)
$\exists a$: ($\neg$ empty(a) and finite(a))   fof(rc1_finset$_1$, axiom)
$\exists a$: empty(a)   fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ empty(a)   fof(rc2_xboole$_0$, axiom)

∀a, b: (in(a, b) ⇒ element(a, b)) fof(t1_subset, axiom)
∀a, b: (element(a, b) ⇒ (empty(b) or in(a, b))) fof(t2_subset, axiom)
∀a: (empty(a) ⇒ a = empty_set) fof(t6_boole, axiom)
∀a: (rel_str(a) ⇒ ∀b: (element(b, the_carrier(a)) ⇒ (relstr_set_smaller(a, empty_set, b) and relstr_element_smaller(a, empty_s
∀a, b: ¬ in(a, b) and empty(b) fof(t7_boole, axiom)
∀a, b: ¬ empty(a) and a ≠ b and empty(b) fof(t8_boole, axiom)

SEU356+1.p MPTP bushy problem t15_yellow_0
∀a: (rel_str(a) ⇒ ∀b: (ex_sup_of_relstr_set(a, b) ⟺ ∃c: (element(c, the_carrier(a)) and relstr_set_smaller(a, b, c) and ∀d: (ele
(relstr_set_smaller(a, b, d) ⇒ related(a, c, d))) and ∀d: (element(d, the_carrier(a)) ⇒ ((relstr_set_smaller(a, b, d) and ∀e: (elem
(relstr_set_smaller(a, b, e) ⇒ related(a, d, e)))) ⇒ d = c))))) fof(d7_yellow$_0$, axiom)
∀a: (rel_str(a) ⇒ one_sorted_str(a)) fof(dt_l1_orders$_2$, axiom)
$true fof(dt_l1_struct$_0$, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$true fof(dt_u1_struct$_0$, axiom)
∃a: rel_str(a) fof(existence_l1_orders$_2$, axiom)
∃a: one_sorted_str(a) fof(existence_l1_struct$_0$, axiom)
∀a: ∃b: element(b, a) fof(existence_m1_subset$_1$, axiom)
∀a: ((antisymmetric_relstr(a) and rel_str(a)) ⇒ ∀b: (ex_sup_of_relstr_set(a, b) ⟺ ∃c: (element(c, the_carrier(a)) and relstr_
(relstr_set_smaller(a, b, d) ⇒ related(a, c, d)))))) fof(t15_yellow$_0$, conjecture)
∀a: ((antisymmetric_relstr(a) and rel_str(a)) ⇒ ∀b: (element(b, the_carrier(a)) ⇒ ∀c: (element(c, the_carrier(a)) ⇒
((related(a, b, c) and related(a, c, b)) ⇒ b = c)))) fof(t25_orders$_2$, axiom)

SEU357+1.p MPTP bushy problem t16_yellow_0
∀a: (rel_str(a) ⇒ ∀b: (ex_inf_of_relstr_set(a, b) ⟺ ∃c: (element(c, the_carrier(a)) and relstr_element_smaller(a, b, c) and ∀d:
(relstr_element_smaller(a, b, d) ⇒ related(a, d, c))) and ∀d: (element(d, the_carrier(a)) ⇒ ((relstr_element_smaller(a, b, d) and
(relstr_element_smaller(a, b, e) ⇒ related(a, e, d)))) ⇒ d = c))))) fof(d8_yellow$_0$, axiom)
∀a: (rel_str(a) ⇒ one_sorted_str(a)) fof(dt_l1_orders$_2$, axiom)
$true fof(dt_l1_struct$_0$, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$true fof(dt_u1_struct$_0$, axiom)
∃a: rel_str(a) fof(existence_l1_orders$_2$, axiom)
∃a: one_sorted_str(a) fof(existence_l1_struct$_0$, axiom)
∀a: ∃b: element(b, a) fof(existence_m1_subset$_1$, axiom)
∀a: ((antisymmetric_relstr(a) and rel_str(a)) ⇒ ∀b: (ex_inf_of_relstr_set(a, b) ⟺ ∃c: (element(c, the_carrier(a)) and relstr_e
(relstr_element_smaller(a, b, d) ⇒ related(a, d, c)))))) fof(t16_yellow$_0$, conjecture)
∀a: ((antisymmetric_relstr(a) and rel_str(a)) ⇒ ∀b: (element(b, the_carrier(a)) ⇒ ∀c: (element(c, the_carrier(a)) ⇒
((related(a, b, c) and related(a, c, b)) ⇒ b = c)))) fof(t25_orders$_2$, axiom)

SEU359+1.p MPTP bushy problem t30_yellow_0
∀a: (rel_str(a) ⇒ ∀b, c: (element(c, the_carrier(a)) ⇒ (ex_sup_of_relstr_set(a, b) ⇒ (c = join_on_relstr(a, b) ⟺
(relstr_set_smaller(a, b, c) and ∀d: (element(d, the_carrier(a)) ⇒ (relstr_set_smaller(a, b, d) ⇒ related(a, c, d)))))))) fof(d9
∀a, b: (rel_str(a) ⇒ element(join_on_relstr(a, b), the_carrier(a))) fof(dt_k1_yellow$_0$, axiom)
∀a: (rel_str(a) ⇒ one_sorted_str(a)) fof(dt_l1_orders$_2$, axiom)
$true fof(dt_l1_struct$_0$, axiom)
$true fof(dt_m1_subset$_1$, axiom)
$true fof(dt_u1_struct$_0$, axiom)
∃a: rel_str(a) fof(existence_l1_orders$_2$, axiom)
∃a: one_sorted_str(a) fof(existence_l1_struct$_0$, axiom)
∀a: ∃b: element(b, a) fof(existence_m1_subset$_1$, axiom)
∀a: ((antisymmetric_relstr(a) and rel_str(a)) ⇒ ∀b: (ex_sup_of_relstr_set(a, b) ⟺ ∃c: (element(c, the_carrier(a)) and relstr_
(relstr_set_smaller(a, b, d) ⇒ related(a, c, d)))))) fof(t15_yellow$_0$, axiom)
∀a: ((antisymmetric_relstr(a) and rel_str(a)) ⇒ ∀b: (element(b, the_carrier(a)) ⇒ ∀c: (((b = join_on_relstr(a, c) and ex_sup_o
(relstr_set_smaller(a, c, b) and ∀d: (element(d, the_carrier(a)) ⇒ (relstr_set_smaller(a, c, d) ⇒ related(a, b, d))))) and ((relstr_
(relstr_set_smaller(a, c, d) ⇒ related(a, b, d)))) ⇒ (b = join_on_relstr(a, c) and ex_sup_of_relstr_set(a, c)))))) fof(t30_yellow

SEU406+1.p The Operation of Addition of Relational Structures T01
∀a, b, c, d: ¬ r2_hidden(a, k2_xboole$_0$(c, d)) and r2_hidden(b, k2_xboole$_0$(c, d)) and ¬ r2_hidden(a, k4_xboole$_0$(c, d)) and r2_hidd
∀a, b: (r2_hidden(a, b) ⇒ ¬ r2_hidden(b, a)) fof(antisymmetry_r2_hidden, axiom)
∀a, b: k2_xboole$_0$(a, b) = k2_xboole$_0$(b, a) fof(commutativity_k2_xboole$_0$, axiom)
∀a, b, c: (c = k2_xboole$_0$(a, b) ⟺ ∀d: (r2_hidden(d, c) ⟺ (r2_hidden(d, a) or r2_hidden(d, b)))) fof(d2_xboole$_0$, axiom)

\$true fof(dt_k1_xboole$_0$, axiom)
\$true fof(dt_k2_xboole$_0$, axiom)
\$true fof(dt_k4_xboole$_0$, axiom)
\$true fof(dt_m1_subset$_1$, axiom)
$\forall a$: $\exists b$: m1_subset$_1(b, a)$ fof(existence_m1_subset$_1$, axiom)
v1_xboole$_0$(k1_xboole$_0$) fof(fc1_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ v1_xboole$_0(a)$ $\Rightarrow$ $\neg$ v1_xboole$_0$(k2_xboole$_0(a, b)$)) fof(fc2_xboole$_0$, axiom)
$\forall a, b$: ($\neg$ v1_xboole$_0(a)$ $\Rightarrow$ $\neg$ v1_xboole$_0$(k2_xboole$_0(b, a)$)) fof(fc3_xboole$_0$, axiom)
$\forall a, b$: k2_xboole$_0(a, a) = a$ fof(idempotence_k2_xboole$_0$, axiom)
$\exists a$: v1_xboole$_0(a)$ fof(rc1_xboole$_0$, axiom)
$\exists a$: $\neg$ v1_xboole$_0(a)$ fof(rc2_xboole$_0$, axiom)
$\forall a$: k2_xboole$_0(a$, k1_xboole$_0) = a$ fof(t1_boole, axiom)
$\forall a, b$: (r2_hidden(a, b) $\Rightarrow$ m1_subset$_1(a, b)$) fof(t1_subset, axiom)
$\forall a, b$: (m1_subset$_1(a, b)$ $\Rightarrow$ (v1_xboole$_0(b)$ or r2_hidden(a, b))) fof(t2_subset, axiom)
$\forall a, b$: k2_xboole$_0(a$, k4_xboole$_0(b, a)) =$ k2_xboole$_0(a, b)$ fof(t39_xboole$_1$, axiom)
$\forall a$: k4_xboole$_0(a$, k1_xboole$_0) = a$ fof(t3_boole, axiom)
$\forall a$: k4_xboole$_0$(k1_xboole$_0, a) =$ k1_xboole$_0$ fof(t4_boole, axiom)
$\forall a$: (v1_xboole$_0(a)$ $\Rightarrow$ $a =$ k1_xboole$_0$) fof(t6_boole, axiom)
$\forall a, b$: $\neg$ r2_hidden(a, b) and v1_xboole$_0(b)$ fof(t7_boole, axiom)
$\forall a, b$: $\neg$ v1_xboole$_0(a)$ and $a \neq b$ and v1_xboole$_0(b)$ fof(t8_boole, axiom)

SEU406+2.p The Operation of Addition of Relational Structures T01
include('Axioms/SET007/SET007+0.ax')
include('Axioms/SET007/SET007+1.ax')
include('Axioms/SET007/SET007+2.ax')
include('Axioms/SET007/SET007+3.ax')
include('Axioms/SET007/SET007+4.ax')
include('Axioms/SET007/SET007+6.ax')
include('Axioms/SET007/SET007+7.ax')
include('Axioms/SET007/SET007+9.ax')
include('Axioms/SET007/SET007+10.ax')
include('Axioms/SET007/SET007+11.ax')
include('Axioms/SET007/SET007+13.ax')
include('Axioms/SET007/SET007+14.ax')
include('Axioms/SET007/SET007+16.ax')
include('Axioms/SET007/SET007+17.ax')
include('Axioms/SET007/SET007+19.ax')
include('Axioms/SET007/SET007+20.ax')
include('Axioms/SET007/SET007+24.ax')
include('Axioms/SET007/SET007+25.ax')
include('Axioms/SET007/SET007+26.ax')
include('Axioms/SET007/SET007+31.ax')
include('Axioms/SET007/SET007+35.ax')
include('Axioms/SET007/SET007+54.ax')
include('Axioms/SET007/SET007+55.ax')
include('Axioms/SET007/SET007+59.ax')
include('Axioms/SET007/SET007+60.ax')
include('Axioms/SET007/SET007+64.ax')
include('Axioms/SET007/SET007+80.ax')
include('Axioms/SET007/SET007+200.ax')
include('Axioms/SET007/SET007+205.ax')
include('Axioms/SET007/SET007+218.ax')
include('Axioms/SET007/SET007+242.ax')
include('Axioms/SET007/SET007+295.ax')
include('Axioms/SET007/SET007+335.ax')
include('Axioms/SET007/SET007+480.ax')
include('Axioms/SET007/SET007+481.ax')
include('Axioms/SET007/SET007+483.ax')
include('Axioms/SET007/SET007+484.ax')
include('Axioms/SET007/SET007+485.ax')

include('Axioms/SET007/SET007+492.ax')
∀a, b: ((l1_orders$_2$(a) and l1_orders$_2$(b)) ⇒ (v1_orders$_2$(k1_latsum$_1$(a, b)) and l1_orders$_2$(k1_latsum$_1$(a, b)))) fof(dt_k1_lats
∀a, b, c, d: ¬ r2_hidden(a, k2_xboole$_0$(c, d)) and r2_hidden(b, k2_xboole$_0$(c, d)) and ¬ r2_hidden(a, k4_xboole$_0$(c, d)) and r2_hidd

SEU430+1.p First and Second Order Cutting of Binary Relations T30
∀a, b: (m1_subset$_1$(b, k1_zfmisc$_1$(k1_zfmisc$_1$(a))) ⇒ (k5_setfam$_1$(a, b) = k1_xboole$_0$ ⟺ ∀c: (r2_hidden(c, b) ⇒ c = k1_xboole$_0$))) fof(t30_relset$_2$, conjecture)
∀a, b: (r2_hidden(a, b) ⇒ ¬ r2_hidden(b, a)) fof(antisymmetry_r2_hidden, axiom)
∀a: (v1_xboole$_0$(a) ⇒ v1_relat$_1$(a)) fof(cc1_relat$_1$, axiom)
∀a: (a = k1_xboole$_0$ ⟺ ∀b: ¬ r2_hidden(b, a)) fof(d1_xboole$_0$, axiom)
∀a, b: (b = k3_tarski(a) ⟺ ∀c: (r2_hidden(c, b) ⟺ ∃d: (r2_hidden(c, d) and r2_hidden(d, a)))) fof(d4_tarski, axiom)
$true fof(dt_k1_xboole$_0$, axiom)
$true fof(dt_k1_zfmisc$_1$, axiom)
$true fof(dt_k3_tarski, axiom)
∀a, b: (m1_subset$_1$(b, k1_zfmisc$_1$(k1_zfmisc$_1$(a))) ⇒ m1_subset$_1$(k5_setfam$_1$(a, b), k1_zfmisc$_1$(a))) fof(dt_k5_setfam$_1$, axiom
$true fof(dt_m1_subset$_1$, axiom)
∀a: ∃b: m1_subset$_1$(b, a) fof(existence_m1_subset$_1$, axiom)
v1_xboole$_0$(k1_xboole$_0$) and v1_relat$_1$(k1_xboole$_0$) and v3_relat$_1$(k1_xboole$_0$) fof(fc12_relat$_1$, axiom)
∀a: ¬ v1_xboole$_0$(k1_zfmisc$_1$(a)) fof(fc1_subset$_1$, axiom)
v1_xboole$_0$(k1_xboole$_0$) and v1_relat$_1$(k1_xboole$_0$) fof(fc4_relat$_1$, axiom)
∃a: (v1_xboole$_0$(a) and v1_relat$_1$(a)) fof(rc1_relat$_1$, axiom)
∀a: (¬ v1_xboole$_0$(a) ⇒ ∃b: (m1_subset$_1$(b, k1_zfmisc$_1$(a)) and ¬ v1_xboole$_0$(b))) fof(rc1_subset$_1$, axiom)
∃a: (¬ v1_xboole$_0$(a) and v1_relat$_1$(a)) fof(rc2_relat$_1$, axiom)
∀a: ∃b: (m1_subset$_1$(b, k1_zfmisc$_1$(a)) and v1_xboole$_0$(b)) fof(rc2_subset$_1$, axiom)
∃a: (v1_relat$_1$(a) and v3_relat$_1$(a)) fof(rc3_relat$_1$, axiom)
∀a, b: (m1_subset$_1$(b, k1_zfmisc$_1$(k1_zfmisc$_1$(a))) ⇒ k5_setfam$_1$(a, b) = k3_tarski(b)) fof(redefinition_k5_setfam$_1$, axiom)
∀a, b: r1_tarski(a, a) fof(reflexivity_r1_tarski, axiom)
∀a, b: (r2_hidden(a, b) ⇒ m1_subset$_1$(a, b)) fof(t1_subset, axiom)
∀a, b: (m1_subset$_1$(a, b) ⇒ (v1_xboole$_0$(b) or r2_hidden(a, b))) fof(t2_subset, axiom)
∀a, b: (m1_subset$_1$(a, k1_zfmisc$_1$(b)) ⟺ r1_tarski(a, b)) fof(t3_subset, axiom)
∀a, b, c: ((r2_hidden(a, b) and m1_subset$_1$(b, k1_zfmisc$_1$(c))) ⇒ m1_subset$_1$(a, c)) fof(t4_subset, axiom)
∀a, b, c: ¬ r2_hidden(a, b) and m1_subset$_1$(b, k1_zfmisc$_1$(c)) and v1_xboole$_0$(c) fof(t5_subset, axiom)
∀a: (v1_xboole$_0$(a) ⇒ a = k1_xboole$_0$) fof(t6_boole, axiom)
∀a, b: ¬ r2_hidden(a, b) and v1_xboole$_0$(b) fof(t7_boole, axiom)
∀a, b: ¬ v1_xboole$_0$(a) and a ≠ b and v1_xboole$_0$(b) fof(t8_boole, axiom)

SEU452^1.p Hofman's Marktoberdorf exercise
The equivalence of two characterizations of the smallest "quasi-PER" containing a given binary relation R, one the obvious inductive characterization.
r: $i → $i → $o thf(r, type)
∀a: $i, b: $i: (∀s: $i → $i → $o: ((∀x: $i, y: $i: ((r@x@y) ⇒ (s@x@y)) and ∀w: $i, x: $i, y: $i, z: $i: ((s@x@y and s@z@y and s (s@x@w))) ⇒ (s@a@b)) ⟺ ∀p: $i → $o, q: $i → $o: (∀x: $i, y: $i: ((r@x@y) ⇒ ((p@x) ⟺ (q@y))) ⇒ ((p@a) ⟺ (q@b)))) thf(thm, conjecture)

SEU453^1.p The reflexive closure of a binary relation is reflexive
include('Axioms/SET009^0.ax')
∀r: $i → $i → $o: (refl@(rc@r)) thf(reflexive_closure_is_reflexive, conjecture)

SEU454^1.p The reflexive closure operator is idempotent
include('Axioms/SET009^0.ax')
idem@rc thf(reflexive_closure_op_is_idempotent, conjecture)

SEU455^1.p The reflexive closure operator is inflationary
include('Axioms/SET009^0.ax')
infl@rc thf(reflexive_closure_op_is_inflationary, conjecture)

SEU456^1.p The reflexive closure operator is monotonic
include('Axioms/SET009^0.ax')
mono@rc thf(reflexive_closure_op_is_monotonic, conjecture)

SEU457^1.p The symmetric closure of a binary relation is symmetric
include('Axioms/SET009^0.ax')
∀r: $i → $i → $o: (symm@(sc@r)) thf(symmetric_closure_is_symmetric, conjecture)

SEU458^1.p The symmetric closure operator is idempotent
include('Axioms/SET009^0.ax')
idem@sc thf(symmetric_closure_op_is_idempotent, conjecture)

SEU459^1.p The symmetric closure operator is inflationary
include('Axioms/SET009^0.ax')
infl@sc thf(symmetric_closure_op_is_inflationary, conjecture)

SEU460^1.p The symmetric closure operator is monotonic
include('Axioms/SET009^0.ax')
mono@sc thf(symmetric_closure_op_is_monotonic, conjecture)

SEU461^1.p The transitive closure of a binary relation is transitive, part 1
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon (\mathrm{trans}@(\mathrm{tc}@r))$ thf(transitive_closure_is_transitive, conjecture)

SEU462^1.p The transitive closure of a binary relation is transitive, part 2
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o, x\colon \$i, y\colon \$i, s\colon \$i \to \$i \to \$o\colon ((\mathrm{trans}@s \text{ and } \mathrm{subrel}@r@s \text{ and } \mathrm{tc}@r@x@y) \Rightarrow (s@x@y))$ thf(transitive_clo

SEU463^1.p The transitive closure of a binary relation is transitive, part 3
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o, x\colon \$i, y\colon \$i, z\colon \$i, s\colon \$i \to \$i \to \$o\colon ((\mathrm{trans}@s \text{ and } \mathrm{subrel}@r@s \text{ and } \mathrm{tc}@r@x@y \text{ and } \mathrm{tc}@r@y@z) \Rightarrow (s@x@z))$ thf(transitive_closure_is_transitive$_3$, conjecture)

SEU464^1.p The transitive closure of a binary relation is transitive, part 4
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o, x\colon \$i, y\colon \$i, s\colon \$i \to \$i \to \$o\colon ((\mathrm{trans}@s \text{ and } \mathrm{subrel}@r@s \text{ and } \forall s\colon \$i \to \$i \to \$o\colon ((\mathrm{trans}@s \text{ and } \mathrm{subrel}@r@s) =$
$(s@x@y))) \Rightarrow (s@x@y))$ thf(transitive_closure_is_transitive$_4$, conjecture)

SEU465^1.p The transitive closure of a binary relation is transitive, part 5
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o, x\colon \$i, y\colon \$i, s\colon \$i \to \$i \to \$o\colon ((\mathrm{trans}@s \text{ and } \mathrm{subrel}@r@s \text{ and } ((\mathrm{trans}@s \text{ and } \mathrm{subrel}@r@s) \Rightarrow (s@x@y))) \Rightarrow (s@x@y))$ thf(transitive_closure_is_transitive$_5$, conjecture)

SEU466^1.p The transitive closure operator is idempotent
include('Axioms/SET009^0.ax')
idem@tc thf(transitive_closure_op_is_idempotent, conjecture)

SEU467^1.p The transitive closure operator is inflationary
include('Axioms/SET009^0.ax')
infl@tc thf(transitive_closure_op_is_inflationary, conjecture)

SEU468^1.p The transitive closure operator is monotonic
include('Axioms/SET009^0.ax')
mono@tc thf(transitive_closure_op_is_monotonic, conjecture)

SEU469^1.p Transitive reflexive closure is transitive and reflexive
The transitive reflexive closure of a binary relation is transitive and reflexive.
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon (\mathrm{trans}@(\mathrm{trc}@r) \text{ and } \mathrm{refl}@(\mathrm{trc}@r))$ thf(transitive_reflexive_closure_is_transitive_reflexive, conjecture)

SEU470^1.p Transitive reflexive symmetric closure properties
The transitive reflexive symmetric closure of a binary relation is transitive, reflexive, and symmetric.
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon (\mathrm{trans}@(\mathrm{trsc}@r) \text{ and } \mathrm{refl}@(\mathrm{trsc}@r) \text{ and } \mathrm{symm}@(\mathrm{trsc}@r))$ thf(transitive_reflexive_symmetric_closure_is_t

SEU471^1.p The transitive reflexive symmetric closure operator is idempotent
include('Axioms/SET009^0.ax')
idem@trsc thf(transitive_reflexive_symmetric_closure_op_is_idempotent, conjecture)

SEU472^1.p The transitive reflexive symmetric closure operator is inflationary
include('Axioms/SET009^0.ax')
infl@trsc thf(transitive_reflexive_symmetric_closure_op_is_inflationary, conjecture)

SEU473^1.p The transitive reflexive symmetric closure operator is monotonic
include('Axioms/SET009^0.ax')
mono@trsc thf(transitive_reflexive_symmetric_closure_op_is_monotonic, conjecture)

SEU474∧1.p Swapping symmetric closure and reflexive closure
Taking the symmetric closure of the reflexive closure is the same as taking the reflexive closure of the symmetric closure
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ (sc@(rc@r)) = (rc@(sc@r))$ thf(composing_symmetric_closure_and_reflexive_closure, conjecture)

SEU475∧1.p Not swapping symmetric closure and transitive closure
Taking the symmetric closure of the transitive closure is NOT the same as taking the transitive closure of the symmetric closure.
include('Axioms/SET009^0.ax')
$\exists x{:}\ \$i, y{:}\ \$i, z{:}\ \$i{:}\ (x \neq y \text{ and } x \neq z \text{ and } y \neq z)$ thf(three_individuals, hypothesis)
$\neg \forall r{:}\ \$i \to \$i \to \$o{:}\ (sc@(tc@r)) = (tc@(sc@r))$ thf(composing_symmetric_closure_and_transitive_closure, conjecture)

SEU476∧1.p Swaping transitive closure and reflexive closure
Taking the transitive closure of the reflexive closure is the same as taking the reflexive closure of the transitive closure.
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ (tc@(rc@r)) = (rc@(tc@r))$ thf(composing_transitive_closure_and_reflexive_closure, conjecture)

SEU477∧1.p Another definition of terminating
The definition of terminating is the same as saying there is no non-empty subset A in which every element has an R successor (in A).
include('Axioms/SET009^0.ax')
$\text{term} = (\lambda r{:}\ \$i \to \$i \to \$o{:}\ \neg \exists a{:}\ \$i \to \$o{:}\ (\exists x{:}\ \$i{:}\ (a@x) \text{ and } \forall x{:}\ \$i{:}\ ((a@x) \Rightarrow \exists y{:}\ \$i{:}\ (a@y \text{ and } r@x@y))))$ thf(alternativ

SEU478∧1.p A terminating relation is normalizing
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{term}@r) \Rightarrow (\text{norm}@r))$ thf(terminating_implies_normalizing, conjecture)

SEU479∧1.p If a relation is terminating, then so is its transitive closure
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{term}@r) \Rightarrow (\text{term}@(tc@r)))$ thf(termination_implies_termination_of_tc, conjecture)

SEU480∧1.p Termination implies the induction principle
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{term}@r) \Rightarrow (\text{ind}@r))$ thf(termination_implies_induction, conjecture)

SEU481∧1.p Satisfying the induction principle implies termination
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{ind}@r) \Rightarrow (\text{term}@r))$ thf(induction_implies_termination, conjecture)

SEU482∧1.p A normalizing relation is not necessarily terminating
include('Axioms/SET009^0.ax')
$\exists y{:}\ \$i, z{:}\ \$i{:}\ y \neq z$ thf(two_individuals, hypothesis)
$\neg \forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{norm}@r) \Rightarrow (\text{term}@r))$ thf(normalizing_does_not_imply_terminating, conjecture)

SEU483∧1.p A symmetric relation is non-terminating
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ (\exists x{:}\ \$i, y{:}\ \$i{:}\ (r@x@y) \Rightarrow ((\text{symm}@r) \Rightarrow \neg\, \text{term}@r))$ thf(symmetric_implies_non_terminating, conjectu

SEU484∧1.p A reflexive relation is non-terminating
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{refl}@r) \Rightarrow \neg\, \text{term}@r)$ thf(reflexive_implies_non_terminating, conjecture)

SEU485∧1.p In a confluent relation every element has at most one normal form
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{confl}@r) \Rightarrow \forall x{:}\ \$i, y{:}\ \$i, z{:}\ \$i{:}\ ((\text{nfof}@r@y@x \text{ and } \text{nfof}@r@z@x) \Rightarrow y = z))$ thf(confluent_implies_at_

SEU486∧1.p Confluence implies local confluence
include('Axioms/SET009^0.ax')
$\forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{confl}@r) \Rightarrow (\text{lconfl}@r))$ thf(confluence_implies_local_confluence, conjecture)

SEU487∧1.p Local confluence does NOT imply confluence
include('Axioms/SET009^0.ax')
$\exists w{:}\ \$i, x{:}\ \$i, y{:}\ \$i, z{:}\ \$i{:}\ (w \neq x \text{ and } w \neq y \text{ and } w \neq z \text{ and } x \neq y \text{ and } x \neq z \text{ and } y \neq z)$ thf(four_individuals, hypothesis)
$\neg \forall r{:}\ \$i \to \$i \to \$o{:}\ ((\text{lconfl}@r) \Rightarrow (\text{confl}@r))$ thf(local_confluence_does_not_imply_confluence, conjecture)

SEU488∧1.p Confluence implies semi confluence

include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{confl}@r) \Rightarrow (\mathrm{sconfl}@r))$ thf(confluence_implies_semi_confluence, conjecture)

SEU489∧1.p Church-Rosser property implies confluence
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{cr}@r) \Rightarrow (\mathrm{confl}@r))$ thf(church_rosser_implies_confluence, conjecture)

SEU490∧1.p Semi confluence implies Church-Rosser property
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{sconfl}@r) \Rightarrow (\mathrm{cr}@r))$ thf(semi_confluence_implies_church_rosser, conjecture)

SEU491∧1.p Terminating relations and confluence and local confluence
For a terminating relation confluence and local confluence are the same.
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{term}@r) \Rightarrow ((\mathrm{confl}@r) \iff (\mathrm{lconfl}@r)))$ thf(termination_makes_confluence_equal_to_local_confluenc

SEU492∧1.p Alternative definition of a strict (partial) order: requiring
Alternative definition of a strict (partial) order: requiring irreflexibility instead of asymmetry.
include('Axioms/SET009^0.ax')
$\mathrm{so} = (\lambda r\colon \$i \to \$i \to \$o\colon (\mathrm{irrefl}@r \text{ and } \mathrm{trans}@r))$ thf(alternative_definition_of_strict_order, conjecture)

SEU493∧1.p The inverse of a partial order is again a partial order
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{po}@r) \Rightarrow (\mathrm{po}@(\mathrm{inv}@r)))$ thf(inverse_of_partial_order_is_partial_order, conjecture)

SEU494∧1.p Inverse of a strict (partial) order is a strict (partial) order
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{so}@r) \Rightarrow (\mathrm{so}@(\mathrm{inv}@r)))$ thf(inverse_of_strict_order_is_strict_order, conjecture)

SEU495∧1.p The inverse of a total relation is again a total relation
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{total}@r) \Rightarrow (\mathrm{total}@(\mathrm{inv}@r)))$ thf(inverse_of_total_relation_is_total_relation, conjecture)

SEU496∧1.p Transitive closure and strict (partial) orders
The transitive closure of a strict (partial) order is a strict (partial) order.
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{so}@r) \Rightarrow (\mathrm{so}@(\mathrm{tc}@r)))$ thf(transitive_closure_of_strict_order_is_strict_order, conjecture)

SEU497∧1.p Every strict (partial) order induces a partial order
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{so}@r) \Rightarrow (\mathrm{po}@(\mathrm{rc}@r)))$ thf(strict_order_induces_partial_order, conjecture)

SEU498∧1.p Every partial order induces a strict (partial) order
include('Axioms/SET009^0.ax')
$\forall r\colon \$i \to \$i \to \$o\colon ((\mathrm{po}@r) \Rightarrow (\mathrm{so}@\lambda x\colon \$i, y\colon \$i\colon (r@x@y \text{ and } x \neq y)))$ thf(partial_order_induces_strict_order, conjecture)

SEU499∧2.p Foundation - Axioms - Logical Axioms
(!phi:i>o.exu(∧x:i.phi x)→(?x:i.phi x & (!y:i.phi y → x = y)))
$\mathrm{exu}\colon (\$i \to \$o) \to \$o$ thf(exu_type, type)
$\mathrm{exu} = (\lambda \mathrm{xphi}\colon \$i \to \$o\colon \exists \mathrm{xx}\colon \$i\colon (\mathrm{xphi}@\mathrm{xx} \text{ and } \forall \mathrm{xy}\colon \$i\colon ((\mathrm{xphi}@\mathrm{xy}) \Rightarrow \mathrm{xx} = \mathrm{xy})))$ thf(exu, definition)
$\forall \mathrm{xphi}\colon \$i \to \$o\colon ((\mathrm{exu}@\lambda \mathrm{xx}\colon \$i\colon (\mathrm{xphi}@\mathrm{xx})) \Rightarrow \exists \mathrm{xx}\colon \$i\colon (\mathrm{xphi}@\mathrm{xx} \text{ and } \forall \mathrm{xy}\colon \$i\colon ((\mathrm{xphi}@\mathrm{xy}) \Rightarrow \mathrm{xx} = \mathrm{xy})))$ thf(exuE_1, conje

SEU500∧2.p Preliminary Notions - Propositions as Sets
(! phi:o.! x:i.in x (prop2set phi) → phi)
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{emptyset}\colon \$i$ thf(emptyset_type, type)
$\mathrm{powerset}\colon \$i \to \$i$ thf(powerset_type, type)
$\mathrm{dsetconstr}\colon \$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$\mathrm{dsetconstrER}\colon \$o$ thf(dsetconstrER_type, type)
$\mathrm{dsetconstrER} = (\forall a\colon \$i, \mathrm{xphi}\colon \$i \to \$o, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{dsetconstr}@a@\lambda \mathrm{xy}\colon \$i\colon (\mathrm{xphi}@\mathrm{xy}))) \Rightarrow (\mathrm{xphi}@\mathrm{xx})))$ thf(dsetcons
$\mathrm{prop2set}\colon \$o \to \$i$ thf(prop2set_type, type)
$\mathrm{prop2set} = (\lambda \mathrm{xphi}\colon \$o\colon (\mathrm{dsetconstr}@(\mathrm{powerset}@\mathrm{emptyset})@\lambda \mathrm{xx}\colon \$i\colon \mathrm{xphi}))$ thf(prop2set, definition)
$\mathrm{dsetconstrER} \Rightarrow \forall \mathrm{xphi}\colon \$o, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{prop2set}@\mathrm{xphi})) \Rightarrow \mathrm{xphi})$ thf(prop2setE, conjecture)

SEU501∧2.p Preliminary Notions - Basic Laws of Logic
(! x:i.in x emptyset → (! phi:o.phi))
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)

emptyset: \$i   thf(emptyset_type, type)
emptysetAx: \$o   thf(emptysetAx_type, type)
emptysetAx = ($\forall$xx: \$i: $\neg$ in@xx@emptyset)   thf(emptysetAx, definition)
emptysetAx $\Rightarrow$ $\forall$xx: \$i: ((in@xx@emptyset) $\Rightarrow$ $\forall$xphi: \$o: xphi)   thf(emptysetE, conjecture)

SEU502^2.p Preliminary Notions - Basic Laws of Logic
(! x:i.in x emptyset → false)
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
emptysetE: \$o   thf(emptysetE_type, type)
emptysetE = ($\forall$xx: \$i: ((in@xx@emptyset) $\Rightarrow$ $\forall$xphi: \$o: xphi))   thf(emptysetE, definition)
emptysetE $\Rightarrow$ $\forall$xx: \$i: ((in@xx@emptyset) $\Rightarrow$ \$false)   thf(emptysetimpfalse, conjecture)

SEU503^2.p Preliminary Notions - Basic Laws of Logic
(! x:i. (in x emptyset))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
emptysetE: \$o   thf(emptysetE_type, type)
emptysetE = ($\forall$xx: \$i: ((in@xx@emptyset) $\Rightarrow$ $\forall$xphi: \$o: xphi))   thf(emptysetE, definition)
emptysetE $\Rightarrow$ $\forall$xx: \$i: $\neg$ in@xx@emptyset   thf(notinemptyset, conjecture)

SEU504^2.p Preliminary Notions - Basic Laws of Logic
(! phi:i>o.exu (∧ x:i.phi x) → (? x:i.phi x))
exu: (\$i → \$o) → \$o   thf(exu_type, type)
exu = (λxphi: \$i → \$o: $\exists$xx: \$i: (xphi@xx and $\forall$xy: \$i: ((xphi@xy) $\Rightarrow$ xx = xy)))   thf(exu, definition)
exuE_1: \$o   thf(exuE1_type, type)
exuE_1 = ($\forall$xphi: \$i → \$o: ((exu@λxx: \$i: (xphi@xx)) $\Rightarrow$ $\exists$xx: \$i: (xphi@xx and $\forall$xy: \$i: ((xphi@xy) $\Rightarrow$ xx = xy))))   thf(exuE_1, definition)
exuE_1 $\Rightarrow$ $\forall$xphi: \$i → \$o: ((exu@λxx: \$i: (xphi@xx)) $\Rightarrow$ $\exists$xx: \$i: (xphi@xx))   thf(exuE3e, conjecture)

SEU505^2.p Preliminary Notions - Basic Laws of Logic
(! A:i.! B:i.(! x:i.in x A → in x B) → (! x:i.in x B → in x A) → A = B)
in: \$i → \$i → \$o   thf(in_type, type)
setextAx: \$o   thf(setextAx_type, type)
setextAx = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Longleftrightarrow$ (in@xx@b)) $\Rightarrow$ $a = b$))   thf(setextAx, definition)
setextAx $\Rightarrow$ $\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@b) $\Rightarrow$ (in@xx@a)) $\Rightarrow$ $a = b$))   thf(setext, conjecture)

SEU506^2.p Preliminary Notions - Basic Laws of Logic
(! A:i.(! x:i. (in x A)) → A = emptyset)
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
emptysetE: \$o   thf(emptysetE_type, type)
emptysetE = ($\forall$xx: \$i: ((in@xx@emptyset) $\Rightarrow$ $\forall$xphi: \$o: xphi))   thf(emptysetE, definition)
setext: \$o   thf(setext_type, type)
setext = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@b) $\Rightarrow$ (in@xx@a)) $\Rightarrow$ $a = b$)))   thf(setext, definition)
emptysetE $\Rightarrow$ (setext $\Rightarrow$ $\forall a$: \$i: ($\forall$xx: \$i: $\neg$ in@xx@a $\Rightarrow$ a = emptyset))   thf(emptyI, conjecture)

SEU508^2.p Preliminary Notions - Basic Laws of Logic
(! A:i.! phi:i>o.! x:i.in x A → (in x (dsetconstr A (∧ y:i.phi y)) <→ phi x))
in: \$i → \$i → \$o   thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrI: \$o   thf(dsetconstrI_type, type)
dsetconstrI = ($\forall a$: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) $\Rightarrow$ ((xphi@xx) $\Rightarrow$ (in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))))))
dsetconstrER: \$o   thf(dsetconstrER_type, type)
dsetconstrER = ($\forall a$: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) $\Rightarrow$ (xphi@xx)))   thf(dsetcons
dsetconstrI $\Rightarrow$ (dsetconstrER $\Rightarrow$ $\forall a$: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy)))
(xphi@xx))))   thf(setbeta, conjecture)

SEU509^2.p Preliminary Notions - Basic Laws of Logic
(! A:i.nonempty A → (? x:i.in x A))
in: \$i → \$i → \$o   thf(in_type, type)

emptyset: \$i thf(emptyset_type, type)
emptysetE: \$o thf(emptysetE_type, type)
emptysetE = ($\forall$xx: \$i: ((in@xx@emptyset) $\Rightarrow$ $\forall$xphi: \$o: xphi)) thf(emptysetE, definition)
setext: \$o thf(setext_type, type)
setext = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@b) $\Rightarrow$ (in@xx@a)) $\Rightarrow$ a = b))) thf(setext, definition)
nonempty: \$i $\rightarrow$ \$o thf(nonempty_type, type)
nonempty = (λxx: \$i: xx $\neq$ emptyset) thf(nonempty, definition)
emptysetE $\Rightarrow$ (setext $\Rightarrow$ $\forall a$: \$i: ((nonempty@a) $\Rightarrow$ $\exists$xx: \$i: (in@xx@a))) thf(nonemptyE$_1$, conjecture)

SEU510^2.p Preliminary Notions - Basic Laws of Logic
(! A:i.! phi:i>o.! x:i.in x A $\rightarrow$ phi x $\rightarrow$ nonempty (dsetconstr A ($\wedge$ y:i.phi y)))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
dsetconstr: \$i $\rightarrow$ (\$i $\rightarrow$ \$o) $\rightarrow$ \$i thf(dsetconstr_type, type)
dsetconstrI: \$o thf(dsetconstrI_type, type)
dsetconstrI = ($\forall a$: \$i, xphi: \$i $\rightarrow$ \$o, xx: \$i: ((in@xx@a) $\Rightarrow$ ((xphi@xx) $\Rightarrow$ (in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))))))
emptysetE: \$o thf(emptysetE_type, type)
emptysetE = ($\forall$xx: \$i: ((in@xx@emptyset) $\Rightarrow$ $\forall$xphi: \$o: xphi)) thf(emptysetE, definition)
nonempty: \$i $\rightarrow$ \$o thf(nonempty_type, type)
nonempty = (λxx: \$i: xx $\neq$ emptyset) thf(nonempty, definition)
dsetconstrI $\Rightarrow$ (emptysetE $\Rightarrow$ $\forall a$: \$i, xphi: \$i $\rightarrow$ \$o, xx: \$i: ((in@xx@a) $\Rightarrow$ ((xphi@xx) $\Rightarrow$ (nonempty@(dsetconstr@a@λxy:

SEU511^2.p Preliminary Notions - Basic Laws of Logic
(! A:i.(? x:i.in x A) $\rightarrow$ nonempty A)
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
emptysetE: \$o thf(emptysetE_type, type)
emptysetE = ($\forall$xx: \$i: ((in@xx@emptyset) $\Rightarrow$ $\forall$xphi: \$o: xphi)) thf(emptysetE, definition)
nonempty: \$i $\rightarrow$ \$o thf(nonempty_type, type)
nonempty = (λxx: \$i: xx $\neq$ emptyset) thf(nonempty, definition)
emptysetE $\Rightarrow$ $\forall a$: \$i: ($\exists$xx: \$i: (in@xx@a) $\Rightarrow$ (nonempty@a)) thf(nonemptyI$_1$, conjecture)

SEU512^2.p Preliminary Notions - Adjoining Elements to Sets
(! x:i.! y:i.in x (setadjoin x y))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
setadjoin: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(setadjoin_type, type)
setadjoinAx: \$o thf(setadjoinAx_type, type)
setadjoinAx = ($\forall$xx: \$i, a: \$i, xy: \$i: ((in@xy@(setadjoin@xx@a)) $\iff$ (xy = xx or in@xy@a))) thf(setadjoinAx, definitio
setadjoinAx $\Rightarrow$ $\forall$xx: \$i, xy: \$i: (in@xx@(setadjoin@xx@xy)) thf(setadjoinIL, conjecture)

SEU513^2.p Preliminary Notions - Adjoining Elements to Sets
in emptyset (setadjoin emptyset emptyset)
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(setadjoin_type, type)
setadjoinIL: \$o thf(setadjoinIL_type, type)
setadjoinIL = ($\forall$xx: \$i, xy: \$i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
setadjoinIL $\Rightarrow$ (in@emptyset@(setadjoin@emptyset@emptyset)) thf(emptyinunitempty, conjecture)

SEU514^2.p Preliminary Notions - Adjoining Elements to Sets
(! x:i.! A:i.! y:i.in y A $\rightarrow$ in y (setadjoin x A))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
setadjoin: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(setadjoin_type, type)
setadjoinAx: \$o thf(setadjoinAx_type, type)
setadjoinAx = ($\forall$xx: \$i, a: \$i, xy: \$i: ((in@xy@(setadjoin@xx@a)) $\iff$ (xy = xx or in@xy@a))) thf(setadjoinAx, definitio
setadjoinAx $\Rightarrow$ $\forall$xx: \$i, a: \$i, xy: \$i: ((in@xy@a) $\Rightarrow$ (in@xy@(setadjoin@xx@a))) thf(setadjoinIR, conjecture)

SEU515^2.p Preliminary Notions - Adjoining Elements to Sets
(! x:i.! A:i.! y:i.in y (setadjoin x A) $\rightarrow$ (! phi:o.(y = x $\rightarrow$ phi) $\rightarrow$ (in y A $\rightarrow$ phi) $\rightarrow$ phi))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
setadjoin: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(setadjoin_type, type)
setadjoinAx: \$o thf(setadjoinAx_type, type)

setadjoinAx = (∀xx: $i, a: $i, xy: $i: ((in@xy@(setadjoin@xx@a)) ⟺ (xy = xx or in@xy@a))) thf(setadjoinAx, definitio
setadjoinAx ⇒ ∀xx: $i, a: $i, xy: $i: ((in@xy@(setadjoin@xx@a)) ⇒ ∀xphi: $o: ((xy = xx ⇒ xphi) ⇒ (((in@xy@a) ⇒ xphi) ⇒ xphi))) thf(setadjoinE, conjecture)

SEU516^2.p Preliminary Notions - Adjoining Elements to Sets
(! x:i.! A:i.! y:i.in y (setadjoin x A) → y = x — in y A)
in: $i → $i → $o thf(in_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setadjoinAx: $o thf(setadjoinAx_type, type)
setadjoinAx = (∀xx: $i, a: $i, xy: $i: ((in@xy@(setadjoin@xx@a)) ⟺ (xy = xx or in@xy@a))) thf(setadjoinAx, definitio
setadjoinAx ⇒ ∀xx: $i, a: $i, xy: $i: ((in@xy@(setadjoin@xx@a)) ⇒ (xy = xx or in@xy@a)) thf(setadjoinOr, conjecture)

SEU517^2.p Preliminary Notions - Power Sets and Unions
(! A:i.dsetconstr A (∧ x:i.true) = A)
in: $i → $i → $o thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrI: $o thf(dsetconstrI_type, type)
dsetconstrI = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))))))
dsetconstrEL: $o thf(dsetconstrEL_type, type)
dsetconstrEL = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (in@xx@a))) thf(dsetcons
setext: $o thf(setext_type, type)
setext = (∀a: $i, b: $i: (∀xx: $i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (∀xx: $i: ((in@xx@b) ⇒ (in@xx@a)) ⇒ a = b))) thf(setext, definition)
dsetconstrI ⇒ (dsetconstrEL ⇒ (setext ⇒ ∀a: $i: (dsetconstr@a@λxx: $i: $true) = a)) thf(setoftrueEq, conjecture)

SEU518^2.p Preliminary Notions - Power Sets and Unions
(! A:i.! B:i.(! x:i.in x B → in x A) → in B (powerset A))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
powersetAx: $o thf(powersetAx_type, type)
powersetAx = (∀a: $i, b: $i: ((in@b@(powerset@a)) ⟺ ∀xx: $i: ((in@xx@b) ⇒ (in@xx@a)))) thf(powersetAx, definitio
powersetAx ⇒ ∀a: $i, b: $i: (∀xx: $i: ((in@xx@b) ⇒ (in@xx@a)) ⇒ (in@b@(powerset@a))) thf(powersetI, conjecture)

SEU519^2.p Preliminary Notions - Power Sets and Unions
(! A:i.in emptyset (powerset A))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
powerset: $i → $i thf(powerset_type, type)
emptysetE: $o thf(emptysetE_type, type)
emptysetE = (∀xx: $i: ((in@xx@emptyset) ⇒ ∀xphi: $o: xphi)) thf(emptysetE, definition)
powersetI: $o thf(powersetI_type, type)
powersetI = (∀a: $i, b: $i: (∀xx: $i: ((in@xx@b) ⇒ (in@xx@a)) ⇒ (in@b@(powerset@a)))) thf(powersetI, definition)
emptysetE ⇒ (powersetI ⇒ ∀a: $i: (in@emptyset@(powerset@a))) thf(emptyinPowerset, conjecture)

SEU521^2.p Preliminary Notions - Power Sets and Unions
(! A:i.! B:i.! x:i.in B (powerset A) → in x B → in x A)
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
powersetAx: $o thf(powersetAx_type, type)
powersetAx = (∀a: $i, b: $i: ((in@b@(powerset@a)) ⟺ ∀xx: $i: ((in@xx@b) ⇒ (in@xx@a)))) thf(powersetAx, definitio
powersetAx ⇒ ∀a: $i, b: $i, xx: $i: ((in@b@(powerset@a)) ⇒ ((in@xx@b) ⇒ (in@xx@a))) thf(powersetE, conjecture)

SEU522^2.p Preliminary Notions - Power Sets and Unions
(! A:i.! x:i.! B:i.in x B → in B A → in x (setunion A))
in: $i → $i → $o thf(in_type, type)
setunion: $i → $i thf(setunion_type, type)
setunionAx: $o thf(setunionAx_type, type)
setunionAx = (∀a: $i, xx: $i: ((in@xx@(setunion@a)) ⟺ ∃b: $i: (in@xx@b and in@b@a))) thf(setunionAx, definition)
setunionAx ⇒ ∀a: $i, xx: $i, b: $i: ((in@xx@b) ⇒ ((in@b@a) ⇒ (in@xx@(setunion@a)))) thf(setunionI, conjecture)

SEU523^2.p Preliminary Notions - Power Sets and Unions
(! A:i.! x:i.in x (setunion A) → (! phi:o.(! B:i.in x B → in B A → phi) → phi))
in: $i → $i → $o thf(in_type, type)

setunion: \$i → \$i thf(setunion_type, type)
setunionAx: \$o thf(setunionAx_type, type)
setunionAx = (∀*a*: \$i, xx: \$i: ((in@xx@(setunion@*a*)) ⟺ ∃*b*: \$i: (in@xx@*b* and in@*b*@*a*))) thf(setunionAx, definition)
setunionAx ⇒ ∀*a*: \$i, xx: \$i: ((in@xx@(setunion@*a*)) ⇒ ∀xphi: \$o: (∀*b*: \$i: ((in@xx@*b*) ⇒ ((in@*b*@*a*) ⇒ xphi)) ⇒ xphi)) thf(setunionE, conjecture)

SEU524∧2.p Preliminary Notions - Power Sets and Unions
(! A:i.! x:i.in x A → in x (powerset (setunion A)))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
setunion: \$i → \$i thf(setunion_type, type)
powersetI: \$o thf(powersetI_type, type)
powersetI = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*b*) ⇒ (in@xx@*a*)) ⇒ (in@*b*@(powerset@*a*)))) thf(powersetI, definition)
setunionI: \$o thf(setunionI_type, type)
setunionI = (∀*a*: \$i, xx: \$i, *b*: \$i: ((in@xx@*b*) ⇒ ((in@*b*@*a*) ⇒ (in@xx@(setunion@*a*))))) thf(setunionI, definition)
powersetI ⇒ (setunionI ⇒ ∀*a*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (in@xx@(powerset@(setunion@*a*))))) thf(subPowSU, conjectu

SEU525∧2.p Preliminary Notions - Equality Laws
(! phi:i>o.exu (∧ x:i.phi x) → (? x:i.! y:i.phi y <→ y = x))
exu: (\$i → \$o) → \$o thf(exu_type, type)
exu = (λxphi: \$i → \$o: ∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy))) thf(exu, definition)
exuE$_1$: \$o thf(exuE1_type, type)
exuE$_1$ = (∀xphi: \$i → \$o: ((exu@λxx: \$i: (xphi@xx)) ⇒ ∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy)))) thf(exuE$_1$, definition)
exuE$_1$ ⇒ ∀xphi: \$i → \$o: ((exu@λxx: \$i: (xphi@xx)) ⇒ ∃xx: \$i: ∀xy: \$i: ((xphi@xy) ⟺ xy = xx)) thf(exuE$_2$, conjectu

SEU526∧2.p Preliminary Notions - Equality Laws
(! A:i.nonempty A → (? x:i.in x A & true))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
emptysetE: \$o thf(emptysetE_type, type)
emptysetE = (∀xx: \$i: ((in@xx@emptyset) ⇒ ∀xphi: \$o: xphi)) thf(emptysetE, definition)
setext: \$o thf(setext_type, type)
setext = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (∀xx: \$i: ((in@xx@*b*) ⇒ (in@xx@*a*)) ⇒ *a* = *b*))) thf(setext, definition)
nonempty: \$i → \$o thf(nonempty_type, type)
nonempty = (λxx: \$i: xx ≠ emptyset) thf(nonempty, definition)
emptysetE ⇒ (setext ⇒ ∀*a*: \$i: ((nonempty@*a*) ⇒ ∃xx: \$i: (in@xx@*a* and \$true))) thf(nonemptyImpWitness, conjectu

SEU527∧2.p Preliminary Notions - Equality Laws
(! x:i.! y:i.in x (setadjoin y emptyset) → x = y)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
emptysetE: \$o thf(emptysetE_type, type)
emptysetE = (∀xx: \$i: ((in@xx@emptyset) ⇒ ∀xphi: \$o: xphi)) thf(emptysetE, definition)
setadjoinE: \$o thf(setadjoinE_type, type)
setadjoinE = (∀xx: \$i, *a*: \$i, xy: \$i: ((in@xy@(setadjoin@xx@*a*)) ⇒ ∀xphi: \$o: ((xy = xx ⇒ xphi) ⇒ (((in@xy@*a*) ⇒ xphi) ⇒ xphi)))) thf(setadjoinE, definition)
emptysetE ⇒ (setadjoinE ⇒ ∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, conjectu

SEU528∧2.p Preliminary Notions - Equality Laws
(! x:i.! y:i. (x = y) → (in y (setadjoin x emptyset)))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
uniqinunit: \$o thf(uniqinunit_type, type)
uniqinunit = (∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
uniqinunit ⇒ ∀xx: \$i, xy: \$i: (xx ≠ xy ⇒ ¬ in@xy@(setadjoin@xx@emptyset)) thf(notinsingleton, conjecture)

SEU529∧2.p Preliminary Notions - Equality Laws
(! x:i.! y:i.x = y → in x (setadjoin y emptyset))
in: \$i → \$i → \$o thf(in_type, type)

emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setadjoinIL: $o   thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy)))   thf(setadjoinIL, definition)
setadjoinIL ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ (in@xx@(setadjoin@xy@emptyset)))   thf(eqinunit, conjecture)

SEU530∧2.p Preliminary Notions - Equality Laws
(! x:i.! y:i.in x (setadjoin y emptyset) → in y (setadjoin x emptyset))
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
uniqinunit: $o   thf(uniqinunit_type, type)
uniqinunit = (∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy))   thf(uniqinunit, definition)
eqinunit: $o   thf(eqinunit_type, type)
eqinunit = (∀xx: $i, xy: $i: (xx = xy ⇒ (in@xx@(setadjoin@xy@emptyset))))   thf(eqinunit, definition)
uniqinunit ⇒ (eqinunit ⇒ ∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ (in@xy@(setadjoin@xx@emptyset))))

SEU531∧2.p Preliminary Notions - Equality Laws
(! x:i.! y:i.! z:i.in z (setadjoin x (setadjoin y emptyset)) → z = x — z = y)
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setadjoinE: $o   thf(setadjoinE_type, type)
setadjoinE = (∀xx: $i, a: $i, xy: $i: ((in@xy@(setadjoin@xx@a)) ⇒ ∀xphi: $o: ((xy = xx ⇒ xphi) ⇒ (((in@xy@a) ⇒ xphi) ⇒ xphi))))   thf(setadjoinE, definition)
uniqinunit: $o   thf(uniqinunit_type, type)
uniqinunit = (∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy))   thf(uniqinunit, definition)
setadjoinE ⇒ (uniqinunit ⇒ ∀xx: $i, xy: $i, xz: $i: ((in@xz@(setadjoin@xx@(setadjoin@xy@emptyset))) ⇒ (xz = xx or xz = xy)))   thf(upairsetE, conjecture)

SEU532∧2.p Preliminary Notions - Equality Laws
(! x:i.! y:i.in x (setadjoin x (setadjoin y emptyset)))
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setadjoinIL: $o   thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy)))   thf(setadjoinIL, definition)
setadjoinIL ⇒ ∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@(setadjoin@xy@emptyset)))   thf(upairsetIL, conjecture)

SEU533∧2.p Preliminary Notions - Equality Laws - Kuratowski Pairs
(! x:i.! y:i.in y (setadjoin x (setadjoin y emptyset)))
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setadjoinIL: $o   thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy)))   thf(setadjoinIL, definition)
setadjoinIR: $o   thf(setadjoinIR_type, type)
setadjoinIR = (∀xx: $i, a: $i, xy: $i: ((in@xy@a) ⇒ (in@xy@(setadjoin@xx@a))))   thf(setadjoinIR, definition)
setadjoinIL ⇒ (setadjoinIR ⇒ ∀xx: $i, xy: $i: (in@xy@(setadjoin@xx@(setadjoin@xy@emptyset))))   thf(upairsetIR, conj

SEU534∧2.p Preliminary Notions - Bounded Quantifier Laws
(! A:i.! phi:i>o.(? x:i.in x A & phi x) → dsetconstr A (∧ x:i.phi x) = emptyset → false)
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
dsetconstr: $i → ($i → $o) → $i   thf(dsetconstr_type, type)
dsetconstrI: $o   thf(dsetconstrI_type, type)
dsetconstrI = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))))))
emptysetE: $o   thf(emptysetE_type, type)
emptysetE = (∀xx: $i: ((in@xx@emptyset) ⇒ ∀xphi: $o: xphi))   thf(emptysetE, definition)
dsetconstrI ⇒ (emptysetE ⇒ ∀a: $i, xphi: $i → $o: (∃xx: $i: (in@xx@a and xphi@xx) ⇒ ((dsetconstr@a@λxx: $i: (xphi@x
emptyset ⇒ $false)))   thf(emptyE$_1$, conjecture)

SEU535∧2.p Preliminary Notions - Bounded Quantifier Laws

(! phi:i>o.! x:i.in x emptyset → phi x)
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
emptysetE: \$o   thf(emptysetE_type, type)
emptysetE = (∀xx: \$i: ((in@xx@emptyset) ⇒ ∀xphi: \$o: xphi))   thf(emptysetE, definition)
emptysetE ⇒ ∀xphi: \$i → \$o, xx: \$i: ((in@xx@emptyset) ⇒ (xphi@xx))   thf(vacuousDall, conjecture)

SEU536^2.p Preliminary Notions - Bounded Quantifier Laws
(! A:i.! phi:i>o. (! x:i.in x A → phi x) → (? x:i.in x A & (phi x)))
in: \$i → \$i → \$o   thf(in, type)
∀*a*: \$i, xphi: \$i → \$o: (¬∀xx: \$i: ((in@xx@*a*) ⇒ (xphi@xx)) ⇒ ∃xx: \$i: (in@xx@*a* and ¬xphi@xx))   thf(quantDeMorgan

SEU537^2.p Preliminary Notions - Bounded Quantifier Laws
(! A:i.! phi:i>o.(! x:i.in x A → (phi x)) → (? x:i.in x A & phi x))
in: \$i → \$i → \$o   thf(in, type)
∀*a*: \$i, xphi: \$i → \$o: (∀xx: \$i: ((in@xx@*a*) ⇒ ¬xphi@xx) ⇒ ¬∃xx: \$i: (in@xx@*a* and xphi@xx))   thf(quantDeMorgan_2,

SEU538^2.p Preliminary Notions - Bounded Quantifier Laws
(! A:i.! phi:i>o. (? x:i.in x A & phi x) → (! x:i.in x A → (phi x)))
in: \$i → \$i → \$o   thf(in, type)
∀*a*: \$i, xphi: \$i → \$o: (¬∃xx: \$i: (in@xx@*a* and xphi@xx) ⇒ ∀xx: \$i: ((in@xx@*a*) ⇒ ¬xphi@xx))   thf(quantDeMorgan_3,

SEU539^2.p Preliminary Notions - Bounded Quantifier Laws
(! A:i.! phi:i>o.(? x:i.in x A & (phi x)) → (! x:i.in x A → phi x))
in: \$i → \$i → \$o   thf(in, type)
∀*a*: \$i, xphi: \$i → \$o: (∃xx: \$i: (in@xx@*a* and ¬xphi@xx) ⇒ ¬∀xx: \$i: ((in@xx@*a*) ⇒ (xphi@xx)))   thf(quantDeMorgan

SEU540^2.p Preliminary Notions - Dependent Connective Laws
(! phi:o.phi → in emptyset (prop2set phi))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
powerset: \$i → \$i   thf(powerset_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrI: \$o   thf(dsetconstrI_type, type)
dsetconstrI = (∀*a*: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@*a*) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@*a*@λxy: \$i: (xphi@xy))))))
prop2set: \$o → \$i   thf(prop2set_type, type)
prop2set = (λxphi: \$o: (dsetconstr@(powerset@emptyset)@λxx: \$i: xphi))   thf(prop2set, definition)
powersetI: \$o   thf(powersetI_type, type)
powersetI = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*b*) ⇒ (in@xx@*a*)) ⇒ (in@*b*@(powerset@*a*))))   thf(powersetI, definition)
dsetconstrI ⇒ (powersetI ⇒ ∀xphi: \$o: (xphi ⇒ (in@emptyset@(prop2set@xphi))))   thf(prop2setI, conjecture)

SEU541^2.p Preliminary Notions - Dependent Connective Laws
(! phi:o.phi → set2prop (prop2set phi))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
prop2set: \$o → \$i   thf(prop2set_type, type)
prop2setI: \$o   thf(prop2setI_type, type)
prop2setI = (∀xphi: \$o: (xphi ⇒ (in@emptyset@(prop2set@xphi))))   thf(prop2setI, definition)
set2prop: \$i → \$o   thf(set2prop_type, type)
set2prop = (λ*a*: \$i: (in@emptyset@*a*))   thf(set2prop, definition)
prop2setI ⇒ ∀xphi: \$o: (xphi ⇒ (set2prop@(prop2set@xphi)))   thf(prop2set2propI, conjecture)

SEU544^2.p Preliminary Notions - Equivalence Laws
(! phi:i>o.(? x:i.phi x & (! y:i.phi y → x = y)) → exu (^ x:i.phi x))
exu: (\$i → \$o) → \$o   thf(exu_type, type)
exu = (λxphi: \$i → \$o: ∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy)))   thf(exu, definition)
∀xphi: \$i → \$o: (∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy)) ⇒ (exu@λxx: \$i: (xphi@xx)))   thf(exuI_1, conjec

SEU545^2.p Preliminary Notions - Equivalence Laws
(! phi:i>o.(? x:i.phi x) → (! x:i.! y:i.phi x → phi y → x = y) → exu (^ x:i.phi x))
exu: (\$i → \$o) → \$o   thf(exu_type, type)
exu = (λxphi: \$i → \$o: ∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy)))   thf(exu, definition)
exuI_1: \$o   thf(exuI1_type, type)
exuI_1 = (∀xphi: \$i → \$o: (∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy)) ⇒ (exu@λxx: \$i: (xphi@xx))))   thf(ex

exuI$_1$ ⇒ ∀xphi: $i → $o: (∃xx: $i: (xphi@xx) ⇒ (∀xx: $i, xy: $i: ((xphi@xx) ⇒ ((xphi@xy) ⇒ xx = xy)) ⇒ (exu@λxx: $i: (xphi@xx)))) thf(exuI$_3$, conjecture)

SEU546∧2.p Preliminary Notions - Equivalence Laws
(! phi:i>o.(? x:i.! y:i.phi y <→ y = x) → exu (∧ x:i.phi x))
exu: ($i → $o) → $o thf(exu_type, type)
exu = (λxphi: $i → $o: ∃xx: $i: (xphi@xx and ∀xy: $i: ((xphi@xy) ⇒ xx = xy))) thf(exu, definition)
exuI$_1$: $o thf(exuI1_type, type)
exuI$_1$ = (∀xphi: $i → $o: (∃xx: $i: (xphi@xx and ∀xy: $i: ((xphi@xy) ⇒ xx = xy)) ⇒ (exu@λxx: $i: (xphi@xx)))) thf(ex
exuI$_1$ ⇒ ∀xphi: $i → $o: (∃xx: $i: ∀xy: $i: ((xphi@xy) ⟺ xy = xx) ⇒ (exu@λxx: $i: (xphi@xx))) thf(exuI$_2$, conjectur

SEU547∧2.p Preliminary Notions - Equality Laws
(! A:i.! B:i.A = B → (! x:i.! y:i.x = y → in x A → in y B))
in: $i → $i → $o thf(in, type)
∀a: $i, b: $i: ($a$ = b ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@$a$) ⇒ (in@xy@$b$)))) thf(inCongP, conjecture)

SEU548∧2.p A simple congruence property of in
(forall A:i.forall B:i.A = B → (forall x:i.forall y:i.x = y → (in x A <→ in y B)))
in: $i → $i → $o thf(in_type, type)
inCongP: $o thf(inCongP_type, type)
inCongP = (∀a: $i, b: $i: ($a$ = b ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@$a$) ⇒ (in@xy@$b$))))) thf(inCongP, definition)
inCongP ⇒ ∀a: $i, b: $i: ($a$ = b ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@$a$) ⟺ (in@xy@$b$)))) thf(in__Cong, conjecture)

SEU549∧2.p Preliminary Notions - Equality Laws
(! phi:i>o.exu (∧ x:i.phi x) → (! x:i.! y:i.phi x → phi y → x = y))
exu: ($i → $o) → $o thf(exu_type, type)
exu = (λxphi: $i → $o: ∃xx: $i: (xphi@xx and ∀xy: $i: ((xphi@xy) ⇒ xx = xy))) thf(exu, definition)
∀xphi: $i → $o: ((exu@λxx: $i: (xphi@xx)) ⇒ ∀xx: $i, xy: $i: ((xphi@xx) ⇒ ((xphi@xy) ⇒ xx = xy))) thf(exuE3u, conj

SEU550∧2.p A simple congruence property of exu
(forall phi:i>o.forall psi:i>o.(forall x:i.forall y:i.x = y → (phi x <→ psi y)) → (exu (lambda x:i.phi x) <→ exu (lambda x:i.psi x)))
exu: ($i → $o) → $o thf(exu_type, type)
exu = (λxphi: $i → $o: ∃xx: $i: (xphi@xx and ∀xy: $i: ((xphi@xy) ⇒ xx = xy))) thf(exu_def, definition)
∀xphi: $i → $o, xpsi: $i → $o: (∀xx: $i, xy: $i: (xx = xy ⇒ ((xphi@xx) ⟺ (xpsi@xy))) ⇒ ((exu@λxx: $i: (xphi@xx)) ⟺ (exu@λxx: $i: (xpsi@xx)))) thf(exu__Cong, conjecture)

SEU551∧2.p A simple congruence property of emptyset
emptyset: $i thf(emptyset, type)
emptyset = emptyset thf(emptyset__Cong, conjecture)

SEU551∧3.p A simple congruence property of emptyset
Reflexivity.
a: $tType thf(a_type, type)
∀a: a: a = a thf(cER_eq_, conjecture)

SEU552∧2.p A simple congruence property of setadjoin
(forall x:i.forall y:i.x = y → (forall z:i.forall u:i.z = u → setadjoin x z = setadjoin y u))
setadjoin: $i → $i → $i thf(setadjoin, type)
∀xx: $i, xy: $i: (xx = xy ⇒ ∀xz: $i, xu: $i: (xz = xu ⇒ (setadjoin@xx@xz) = (setadjoin@xy@xu))) thf(setadjoin__Cong, c

SEU553∧2.p A simple congruence property of powerset
(forall A:i.forall B:i.A = B → powerset A = powerset B)
powerset: $i → $i thf(powerset, type)
∀a: $i, b: $i: ($a$ = b ⇒ (powerset@a) = (powerset@b)) thf(powerset__Cong, conjecture)

SEU557∧2.p A simple congruence property of descr
(forall phi:i>o.forall psi:i>o.(forall x:i.forall y:i.x = y → (phi x <→ psi y)) → exu (lambda x:i.phi x) → exu (lambda x:i.psi x) → descr (lambda x:i.phi x) = descr (lambda x:i.psi x))
exu: ($i → $o) → $o thf(exu_type, type)
exuEu: $o thf(exuEu_type, type)
descr: ($i → $o) → $i thf(descr_type, type)
descrp: $o thf(descrp_type, type)
exu = (λxphi: $i → $o: ∃xx: $i: (xphi@xx and ∀xy: $i: ((xphi@xy) ⇒ xx = xy))) thf(exu_def, definition)

exuEu = (∀xphi: $i → $o: ((exu@λxx: $i: (xphi@xx)) ⇒ ∀xx: $i, xy: $i: ((xphi@xx) ⇒ ((xphi@xy) ⇒ xx = xy)))) thf(exuEu_def, definition)
descrp = (∀xphi: $i → $o: ((exu@λxx: $i: (xphi@xx)) ⇒ (xphi@(descr@λxx: $i: (xphi@xx))))) thf(descrp_def, definition)
descrp ⇒ (exuEu ⇒ ∀xphi: $i → $o, xpsi: $i → $o: (∀xx: $i, xy: $i: (xx = xy ⇒ ((xphi@xx) ⟺ (xpsi@xy))) ⇒ ((exu@λxx: $i: (xphi@xx)) ⇒ ((exu@λxx: $i: (xpsi@xx)) ⇒ (descr@λxx: $i: (xphi@xx)) = (descr@λxx: $i: (xpsi@xx)))))) thf(descr__Cong, conjecture)

SEU558^2.p A simple congruence property of dsetconstr
(forall A:i.forall B:i.A = B → (forall phi:i>o.forall psi:i>o.(forall x:i.in x A → (forall y:i.in y B → x = y → (phi x <→ psi y))) → dsetconstr A (lambda x:i.phi x) = dsetconstr B (lambda x:i.psi x)))
in: $i → $i → $o thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrI: $o thf(dsetconstrI_type, type)
dsetconstrI = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@*a*) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@*a*@λxy: $i: (xphi@xy))))))
dsetconstrEL: $o thf(dsetconstrEL_type, type)
dsetconstrEL = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@*a*@λxy: $i: (xphi@xy))) ⇒ (in@xx@*a*))) thf(dsetcons
dsetconstrER: $o thf(dsetconstrER_type, type)
dsetconstrER = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@*a*@λxy: $i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
setext: $o thf(setext_type, type)
setext = (∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (∀xx: $i: ((in@xx@*b*) ⇒ (in@xx@*a*)) ⇒ *a* = *b*))) thf(setext, definition)
dsetconstrI ⇒ (dsetconstrEL ⇒ (dsetconstrER ⇒ (setext ⇒ ∀*a*: $i, *b*: $i: (*a* = *b* ⇒ ∀xphi: $i → $o, xpsi: $i → $o: (∀xx: $i: ((in@xx@*a*) ⇒ ∀xy: $i: ((in@xy@*b*) ⇒ (xx = xy ⇒ ((xphi@xx) ⟺ (xpsi@xy))))) ⇒ (dsetconstr@*a*@λxx: $i: (xphi@xx)) = (dsetconstr@*b*@λxx: $i: (xpsi@xx))))))) thf(dsetconstr__Cong, conjecture)

SEU559^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.(! x:i.in x A → in x B) → subset A B)
in: $i → $i → $o thf(in_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
⊆= (λ*a*: $i, *b*: $i: ∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*))) thf(subset, definition)
∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*)) thf(subsetI$_1$, conjecture)

SEU560^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.A = B → subset B A)
in: $i → $i → $o thf(in_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
⊆= (λ*a*: $i, *b*: $i: ∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*))) thf(subset, definition)
subsetI$_1$: $o thf(subsetI1_type, type)
subsetI$_1$ = (∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*))) thf(subsetI$_1$, definition)
subsetI$_1$ ⇒ ∀*a*: $i, *b*: $i: (*a* = *b* ⇒ (⊆ @*b*@*a*)) thf(eqimpsubset$_2$, conjecture)

SEU561^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.A = B → subset A B)
in: $i → $i → $o thf(in_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
⊆= (λ*a*: $i, *b*: $i: ∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*))) thf(subset, definition)
subsetI$_1$: $o thf(subsetI1_type, type)
subsetI$_1$ = (∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*))) thf(subsetI$_1$, definition)
subsetI$_1$ ⇒ ∀*a*: $i, *b*: $i: (*a* = *b* ⇒ (⊆ @*a*@*b*)) thf(eqimpsubset$_1$, conjecture)

SEU562^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.(! x:i.in x A → in x B) → subset A B)
in: $i → $i → $o thf(in_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
⊆= (λ*a*: $i, *b*: $i: ∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*))) thf(subset, definition)
subsetI$_1$: $o thf(subsetI1_type, type)
subsetI$_1$ = (∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*))) thf(subsetI$_1$, definition)
subsetI$_1$ ⇒ ∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*)) thf(subsetI$_2$, conjecture)

SEU563^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.subset emptyset A)
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)

emptysetimpfalse: \$o   thf(emptysetimpfalse_type, type)
emptysetimpfalse = (∀xx: \$i: ((in@xx@emptyset) ⇒ \$false))   thf(emptysetimpfalse, definition)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
⊆= (λ*a*: \$i, *b*: \$i: ∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)))   thf(subset, definition)
subsetI_2: \$o   thf(subsetI2_type, type)
subsetI_2 = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*)))   thf(subsetI_2, definition)
emptysetimpfalse ⇒ (subsetI_2 ⇒ ∀*a*: \$i: (⊆ @emptyset@*a*))   thf(emptysetsubset, conjecture)

SEU564^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.! x:i.subset A B → in x A → in x B)
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
⊆= (λ*a*: \$i, *b*: \$i: ∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)))   thf(subset, definition)
∀*a*: \$i, *b*: \$i, xx: \$i: ((⊆ @*a*@*b*) ⇒ ((in@xx@*a*) ⇒ (in@xx@*b*)))   thf(subsetE, conjecture)

SEU565^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.! x:i.subset A B → (in x B) → (in x A))
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
subsetE: \$o   thf(subsetE_type, type)
subsetE = (∀*a*: \$i, *b*: \$i, xx: \$i: ((⊆ @*a*@*b*) ⇒ ((in@xx@*a*) ⇒ (in@xx@*b*))))   thf(subsetE, definition)
subsetE ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i: ((⊆ @*a*@*b*) ⇒ (¬ in@xx@*b* ⇒ ¬ in@xx@*a*))   thf(subsetE_2, conjecture)

SEU566^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.! x:i.in x A → (in x B) → (subset A B))
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
subsetE: \$o   thf(subsetE_type, type)
subsetE = (∀*a*: \$i, *b*: \$i, xx: \$i: ((⊆ @*a*@*b*) ⇒ ((in@xx@*a*) ⇒ (in@xx@*b*))))   thf(subsetE, definition)
subsetE ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (¬ in@xx@*b* ⇒ ¬ ⊆ @*a*@*b*))   thf(notsubsetI, conjecture)

SEU567^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i. (subset A B) → (A = B))
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
subsetI_1: \$o   thf(subsetI1_type, type)
subsetI_1 = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*)))   thf(subsetI_1, definition)
subsetI_1 ⇒ ∀*a*: \$i, *b*: \$i: (¬ ⊆ @*a*@*b* ⇒ *a* ≠ *b*)   thf(notequalI_1, conjecture)

SEU568^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.! x:i.in x A → (in x B) → (A = B))
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
notsubsetI: \$o   thf(notsubsetI_type, type)
notsubsetI = (∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (¬ in@xx@*b* ⇒ ¬ ⊆ @*a*@*b*)))   thf(notsubsetI, definition)
notequalI_1: \$o   thf(notequalI1_type, type)
notequalI_1 = (∀*a*: \$i, *b*: \$i: (¬ ⊆ @*a*@*b* ⇒ *a* ≠ *b*))   thf(notequalI_1, definition)
notsubsetI ⇒ (notequalI_1 ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (¬ in@xx@*b* ⇒ *a* ≠ *b*)))   thf(notequalI_2, conjecture)

SEU569^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.subset A A)
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
subsetI_2: \$o   thf(subsetI2_type, type)
subsetI_2 = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*)))   thf(subsetI_2, definition)
subsetI_2 ⇒ ∀*a*: \$i: (⊆ @*a*@*a*)   thf(subsetRefl, conjecture)

SEU570^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.! C:i.subset A B → subset B C → subset A C)
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
subsetI_2: \$o   thf(subsetI2_type, type)
subsetI_2 = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*)))   thf(subsetI_2, definition)

subsetE: \$o thf(subsetE_type, type)
$\text{subsetE} = (\forall a{:}\ \$i, b{:}\ \$i, \text{xx}{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@\text{xx}@b))))$ thf(subsetE, definition)
$\text{subsetI}_2 \Rightarrow (\text{subsetE} \Rightarrow \forall a{:}\ \$i, b{:}\ \$i, c{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow ((\subseteq @b@c) \Rightarrow (\subseteq @a@c))))$ thf(subsetTrans, conjecture)

SEU571∧2.p Preliminary Notions - Relations on Sets - Subsets
(! x:i.! A:i.subset A (setadjoin x A))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
setadjoin: $\$i \to \$i \to \$i$ thf(setadjoin_type, type)
setadjoinIR: \$o thf(setadjoinIR_type, type)
$\text{setadjoinIR} = (\forall \text{xx}{:}\ \$i, a{:}\ \$i, \text{xy}{:}\ \$i{:}\ ((\text{in}@\text{xy}@a) \Rightarrow (\text{in}@\text{xy}@(\text{setadjoin}@\text{xx}@a))))$ thf(setadjoinIR, definition)
$\subseteq{:}\ \$i \to \$i \to \$o$ thf(subset_type, type)
subsetI_1: \$o thf(subsetI1_type, type)
$\text{subsetI}_1 = (\forall a{:}\ \$i, b{:}\ \$i{:}\ (\forall \text{xx}{:}\ \$i{:}\ ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@\text{xx}@b)) \Rightarrow (\subseteq @a@b)))$ thf(subsetI_1, definition)
$\text{setadjoinIR} \Rightarrow (\text{subsetI}_1 \Rightarrow \forall \text{xx}{:}\ \$i, a{:}\ \$i{:}\ (\subseteq @a@(\text{setadjoin}@\text{xx}@a)))$ thf(setadjoinSub, conjecture)

SEU572∧2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! x:i.! B:i.subset A B → subset A (setadjoin x B))
setadjoin: $\$i \to \$i \to \$i$ thf(setadjoin_type, type)
$\subseteq{:}\ \$i \to \$i \to \$o$ thf(subset_type, type)
subsetTrans: \$o thf(subsetTrans_type, type)
$\text{subsetTrans} = (\forall a{:}\ \$i, b{:}\ \$i, c{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow ((\subseteq @b@c) \Rightarrow (\subseteq @a@c))))$ thf(subsetTrans, definition)
setadjoinSub: \$o thf(setadjoinSub_type, type)
$\text{setadjoinSub} = (\forall \text{xx}{:}\ \$i, a{:}\ \$i{:}\ (\subseteq @a@(\text{setadjoin}@\text{xx}@a)))$ thf(setadjoinSub, definition)
$\text{subsetTrans} \Rightarrow (\text{setadjoinSub} \Rightarrow \forall a{:}\ \$i, \text{xx}{:}\ \$i, b{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow (\subseteq @a@(\text{setadjoin}@\text{xx}@b))))$ thf(setadjoinSub_2, conj

SEU573∧2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.subset A B → in A (powerset B))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
powerset: $\$i \to \$i$ thf(powerset_type, type)
powersetI: \$o thf(powersetI_type, type)
$\text{powersetI} = (\forall a{:}\ \$i, b{:}\ \$i{:}\ (\forall \text{xx}{:}\ \$i{:}\ ((\text{in}@\text{xx}@b) \Rightarrow (\text{in}@\text{xx}@a)) \Rightarrow (\text{in}@b@(\text{powerset}@a))))$ thf(powersetI, definition)
$\subseteq{:}\ \$i \to \$i \to \$o$ thf(subset_type, type)
subsetE: \$o thf(subsetE_type, type)
$\text{subsetE} = (\forall a{:}\ \$i, b{:}\ \$i, \text{xx}{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@\text{xx}@b))))$ thf(subsetE, definition)
$\text{powersetI} \Rightarrow (\text{subsetE} \Rightarrow \forall a{:}\ \$i, b{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow (\text{in}@a@(\text{powerset}@b))))$ thf(subset2powerset, conjecture)

SEU574∧2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.subset A B → subset B A → A = B)
in: $\$i \to \$i \to \$o$ thf(in_type, type)
setext: \$o thf(setext_type, type)
$\text{setext} = (\forall a{:}\ \$i, b{:}\ \$i{:}\ (\forall \text{xx}{:}\ \$i{:}\ ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@\text{xx}@b)) \Rightarrow (\forall \text{xx}{:}\ \$i{:}\ ((\text{in}@\text{xx}@b) \Rightarrow (\text{in}@\text{xx}@a)) \Rightarrow a = b)))$ thf(setext, definition)
$\subseteq{:}\ \$i \to \$i \to \$o$ thf(subset_type, type)
subsetE: \$o thf(subsetE_type, type)
$\text{subsetE} = (\forall a{:}\ \$i, b{:}\ \$i, \text{xx}{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@\text{xx}@b))))$ thf(subsetE, definition)
$\text{setext} \Rightarrow (\text{subsetE} \Rightarrow \forall a{:}\ \$i, b{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow ((\subseteq @b@a) \Rightarrow a = b)))$ thf(setextsub, conjecture)

SEU575∧2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.subset A emptyset → A = emptyset)
emptyset: \$i thf(emptyset_type, type)
$\subseteq{:}\ \$i \to \$i \to \$o$ thf(subset_type, type)
emptysetsubset: \$o thf(emptysetsubset_type, type)
$\text{emptysetsubset} = (\forall a{:}\ \$i{:}\ (\subseteq @\text{emptyset}@a))$ thf(emptysetsubset, definition)
setextsub: \$o thf(setextsub_type, type)
$\text{setextsub} = (\forall a{:}\ \$i, b{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow ((\subseteq @b@a) \Rightarrow a = b)))$ thf(setextsub, definition)
$\text{emptysetsubset} \Rightarrow (\text{setextsub} \Rightarrow \forall a{:}\ \$i{:}\ ((\subseteq @a@\text{emptyset}) \Rightarrow a = \text{emptyset}))$ thf(subsetemptysetimpeq, conjecture)

SEU576∧2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.subset B A → in B (powerset A))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
powerset: $\$i \to \$i$ thf(powerset_type, type)
powersetI: \$o thf(powersetI_type, type)
$\text{powersetI} = (\forall a{:}\ \$i, b{:}\ \$i{:}\ (\forall \text{xx}{:}\ \$i{:}\ ((\text{in}@\text{xx}@b) \Rightarrow (\text{in}@\text{xx}@a)) \Rightarrow (\text{in}@b@(\text{powerset}@a))))$ thf(powersetI, definition)

$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\mathrm{subsetE}\colon \$o$ thf(subsetE_type, type)
$\mathrm{subsetE} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b))))$ thf(subsetE, definition)
$\mathrm{powersetI} \Rightarrow (\mathrm{subsetE} \Rightarrow \forall a\colon \$i, b\colon \$i\colon ((\subseteq @b@a) \Rightarrow (\mathrm{in}@b@(\mathrm{powerset}@a))))$ thf($\mathrm{powersetI}_1$, conjecture)

SEU577^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.in B (powerset A) → subset B A)
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \to \$i$ thf(powerset_type, type)
$\mathrm{powersetE}\colon \$o$ thf(powersetE_type, type)
$\mathrm{powersetE} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@b@(\mathrm{powerset}@a)) \Rightarrow ((\mathrm{in}@\mathrm{xx}@b) \Rightarrow (\mathrm{in}@\mathrm{xx}@a))))$ thf(powersetE, definition)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\mathrm{subsetI}_1\colon \$o$ thf(subsetI1_type, type)
$\mathrm{subsetI}_1 = (\forall a\colon \$i, b\colon \$i\colon (\forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b)) \Rightarrow (\subseteq @a@b)))$ thf($\mathrm{subsetI}_1$, definition)
$\mathrm{powersetE} \Rightarrow (\mathrm{subsetI}_1 \Rightarrow \forall a\colon \$i, b\colon \$i\colon ((\mathrm{in}@b@(\mathrm{powerset}@a)) \Rightarrow (\subseteq @b@a)))$ thf($\mathrm{powersetE}_1$, conjecture)

SEU578^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.in A (powerset A))
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \to \$i$ thf(powerset_type, type)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\mathrm{subsetRefl}\colon \$o$ thf(subsetRefl_type, type)
$\mathrm{subsetRefl} = (\forall a\colon \$i\colon (\subseteq @a@a))$ thf(subsetRefl, definition)
$\mathrm{powersetI}_1\colon \$o$ thf(powersetI1_type, type)
$\mathrm{powersetI}_1 = (\forall a\colon \$i, b\colon \$i\colon ((\subseteq @b@a) \Rightarrow (\mathrm{in}@b@(\mathrm{powerset}@a))))$ thf($\mathrm{powersetI}_1$, definition)
$\mathrm{subsetRefl} \Rightarrow (\mathrm{powersetI}_1 \Rightarrow \forall a\colon \$i\colon (\mathrm{in}@a@(\mathrm{powerset}@a)))$ thf(inPowerset, conjecture)

SEU579^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! B:i.subset A B → subset (powerset A) (powerset B))
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \to \$i$ thf(powerset_type, type)
$\mathrm{powersetI}\colon \$o$ thf(powersetI_type, type)
$\mathrm{powersetI} = (\forall a\colon \$i, b\colon \$i\colon (\forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@b) \Rightarrow (\mathrm{in}@\mathrm{xx}@a)) \Rightarrow (\mathrm{in}@b@(\mathrm{powerset}@a))))$ thf(powersetI, definition)
$\mathrm{powersetE}\colon \$o$ thf(powersetE_type, type)
$\mathrm{powersetE} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@b@(\mathrm{powerset}@a)) \Rightarrow ((\mathrm{in}@\mathrm{xx}@b) \Rightarrow (\mathrm{in}@\mathrm{xx}@a))))$ thf(powersetE, definition)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\mathrm{subsetI}_2\colon \$o$ thf(subsetI2_type, type)
$\mathrm{subsetI}_2 = (\forall a\colon \$i, b\colon \$i\colon (\forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b)) \Rightarrow (\subseteq @a@b)))$ thf($\mathrm{subsetI}_2$, definition)
$\mathrm{subsetE}\colon \$o$ thf(subsetE_type, type)
$\mathrm{subsetE} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b))))$ thf(subsetE, definition)
$\mathrm{powersetI} \Rightarrow (\mathrm{powersetE} \Rightarrow (\mathrm{subsetI}_2 \Rightarrow (\mathrm{subsetE} \Rightarrow \forall a\colon \$i, b\colon \$i\colon ((\subseteq @a@b) \Rightarrow (\subseteq @(\mathrm{powerset}@a)@(\mathrm{powerset}@b))))))$

SEU580^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! phi:i>o.in (dsetconstr A (^ x:i.phi x)) (powerset A))
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \to \$i$ thf(powerset_type, type)
$\mathrm{dsetconstr}\colon \$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$\mathrm{dsetconstrEL}\colon \$o$ thf(dsetconstrEL_type, type)
$\mathrm{dsetconstrEL} = (\forall a\colon \$i, \mathrm{xphi}\colon \$i \to \$o, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{dsetconstr}@a@\lambda \mathrm{xy}\colon \$i\colon (\mathrm{xphi}@\mathrm{xy}))) \Rightarrow (\mathrm{in}@\mathrm{xx}@a)))$ thf(dsetcons
$\mathrm{powersetI}\colon \$o$ thf(powersetI_type, type)
$\mathrm{powersetI} = (\forall a\colon \$i, b\colon \$i\colon (\forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@b) \Rightarrow (\mathrm{in}@\mathrm{xx}@a)) \Rightarrow (\mathrm{in}@b@(\mathrm{powerset}@a))))$ thf(powersetI, definition)
$\mathrm{dsetconstrEL} \Rightarrow (\mathrm{powersetI} \Rightarrow \forall a\colon \$i, \mathrm{xphi}\colon \$i \to \$o\colon (\mathrm{in}@(\mathrm{dsetconstr}@a@\lambda \mathrm{xx}\colon \$i\colon (\mathrm{xphi}@\mathrm{xx}))@(\mathrm{powerset}@a)))$ thf(sepIn

SEU581^2.p Preliminary Notions - Relations on Sets - Subsets
(! A:i.! phi:i>o.subset (dsetconstr A (^ x:i.phi x)) A)
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \to \$i$ thf(powerset_type, type)
$\mathrm{dsetconstr}\colon \$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\mathrm{powersetE}_1\colon \$o$ thf(powersetE1_type, type)
$\mathrm{powersetE}_1 = (\forall a\colon \$i, b\colon \$i\colon ((\mathrm{in}@b@(\mathrm{powerset}@a)) \Rightarrow (\subseteq @b@a)))$ thf($\mathrm{powersetE}_1$, definition)
$\mathrm{sepInPowerset}\colon \$o$ thf(sepInPowerset_type, type)

sepInPowerset = (∀a: $i, xphi: $i → $o: (in@(dsetconstr@a@λxx: $i: (xphi@xx))@(powerset@a)))   thf(sepInPowerset, defin
powersetE$_1$ ⇒ (sepInPowerset ⇒ ∀a: $i, xphi: $i → $o: (⊆ @(dsetconstr@a@λxx: $i: (xphi@xx))@a))   thf(sepSubset, con

SEU582^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! x:i.in x A → in x (binunion A B))
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setunion: $i → $i   thf(setunion_type, type)
setadjoinIL: $o   thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy)))   thf(setadjoinIL, definition)
setunionI: $o   thf(setunionI_type, type)
setunionI = (∀a: $i, xx: $i, b: $i: ((in@xx@b) ⇒ ((in@b@a) ⇒ (in@xx@(setunion@a)))))   thf(setunionI, definition)
binunion: $i → $i → $i   thf(binunion_type, type)
binunion = (λxx: $i, xy: $i: (setunion@(setadjoin@xx@(setadjoin@xy@emptyset))))   thf(binunion, definition)
setadjoinIL ⇒ (setunionI ⇒ ∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ (in@xx@(binunion@a@b))))   thf(binunionIL, conjecture)

SEU584^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! x:i.in x B → in x (binunion A B))
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setunion: $i → $i   thf(setunion_type, type)
setunionI: $o   thf(setunionI_type, type)
setunionI = (∀a: $i, xx: $i, b: $i: ((in@xx@b) ⇒ ((in@b@a) ⇒ (in@xx@(setunion@a)))))   thf(setunionI, definition)
binunion: $i → $i → $i   thf(binunion_type, type)
binunion = (λxx: $i, xy: $i: (setunion@(setadjoin@xx@(setadjoin@xy@emptyset))))   thf(binunion, definition)
upairset2IR: $o   thf(upairset2IR_type, type)
upairset2IR = (∀xx: $i, xy: $i: (in@xy@(setadjoin@xx@(setadjoin@xy@emptyset))))   thf(upairset2IR, definition)
setunionI ⇒ (upairset2IR ⇒ ∀a: $i, b: $i, xx: $i: ((in@xx@b) ⇒ (in@xx@(binunion@a@b))))   thf(binunionIR, conjecture)

SEU585^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! x:i.! phi:o.in x (binunion A B) → (in x A → phi) → (in x B → phi) → phi)
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setunion: $i → $i   thf(setunion_type, type)
setadjoinE: $o   thf(setadjoinE_type, type)
setadjoinE = (∀xx: $i, a: $i, xy: $i: ((in@xy@(setadjoin@xx@a)) ⇒ ∀xphi: $o: ((xy = xx ⇒ xphi) ⇒ (((in@xy@a) ⇒ xphi) ⇒ xphi))))   thf(setadjoinE, definition)
setunionE: $o   thf(setunionE_type, type)
setunionE = (∀a: $i, xx: $i: ((in@xx@(setunion@a)) ⇒ ∀xphi: $o: (∀b: $i: ((in@xx@b) ⇒ ((in@b@a) ⇒ xphi)) ⇒ xphi)))   thf(setunionE, definition)
uniqinunit: $o   thf(uniqinunit_type, type)
uniqinunit = (∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy))   thf(uniqinunit, definition)
binunion: $i → $i → $i   thf(binunion_type, type)
binunion = (λxx: $i, xy: $i: (setunion@(setadjoin@xx@(setadjoin@xy@emptyset))))   thf(binunion, definition)
setadjoinE ⇒ (setunionE ⇒ (uniqinunit ⇒ ∀a: $i, b: $i, xx: $i, xphi: $o: ((in@xx@(binunion@a@b)) ⇒ (((in@xx@a) ⇒ xphi) ⇒ (((in@xx@b) ⇒ xphi) ⇒ xphi)))))   thf(binunionEcases, conjecture)

SEU586^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! x:i.in x (binunion A B) → in x A — in x B)
in: $i → $i → $o   thf(in_type, type)
binunion: $i → $i → $i   thf(binunion_type, type)
binunionEcases: $o   thf(binunionEcases_type, type)
binunionEcases = (∀a: $i, b: $i, xx: $i, xphi: $o: ((in@xx@(binunion@a@b)) ⇒ (((in@xx@a) ⇒ xphi) ⇒ (((in@xx@b) ⇒ xphi) ⇒ xphi))))   thf(binunionEcases, definition)
binunionEcases ⇒ ∀a: $i, b: $i, xx: $i: ((in@xx@(binunion@a@b)) ⇒ (in@xx@a or in@xx@b))   thf(binunionE, conjecture)

SEU587^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.subset A (binunion A B))
in: $i → $i → $o   thf(in_type, type)

$\subseteq$: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(subset_type, type)
subsetI_2: \$o thf(subsetI2_type, type)
$\text{subsetI}_2 = (\forall a\colon \$i, b\colon \$i\colon (\forall xx\colon \$i\colon ((in@xx@a) \Rightarrow (in@xx@b)) \Rightarrow (\subseteq @a@b)))$ thf(subsetI_2, definition)
binunion: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(binunion_type, type)
binunionIL: \$o thf(binunionIL_type, type)
$\text{binunionIL} = (\forall a\colon \$i, b\colon \$i, xx\colon \$i\colon ((in@xx@a) \Rightarrow (in@xx@(binunion@a@b))))$ thf(binunionIL, definition)
$\text{subsetI}_2 \Rightarrow (\text{binunionIL} \Rightarrow \forall a\colon \$i, b\colon \$i\colon (\subseteq @a@(binunion@a@b)))$ thf(binunionLsub, conjecture)

SEU588^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.subset B (binunion A B))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
$\subseteq$: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(subset_type, type)
subsetI_2: \$o thf(subsetI2_type, type)
$\text{subsetI}_2 = (\forall a\colon \$i, b\colon \$i\colon (\forall xx\colon \$i\colon ((in@xx@a) \Rightarrow (in@xx@b)) \Rightarrow (\subseteq @a@b)))$ thf(subsetI_2, definition)
binunion: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(binunion_type, type)
binunionIR: \$o thf(binunionIR_type, type)
$\text{binunionIR} = (\forall a\colon \$i, b\colon \$i, xx\colon \$i\colon ((in@xx@b) \Rightarrow (in@xx@(binunion@a@b))))$ thf(binunionIR, definition)
$\text{subsetI}_2 \Rightarrow (\text{binunionIR} \Rightarrow \forall a\colon \$i, b\colon \$i\colon (\subseteq @b@(binunion@a@b)))$ thf(binunionRsub, conjecture)

SEU589^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! x:i.in x A → in x B → in x (binintersect A B))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
dsetconstr: \$i $\rightarrow$ (\$i $\rightarrow$ \$o) $\rightarrow$ \$i thf(dsetconstr_type, type)
dsetconstrI: \$o thf(dsetconstrI_type, type)
$\text{dsetconstrI} = (\forall a\colon \$i, xphi\colon \$i \rightarrow \$o, xx\colon \$i\colon ((in@xx@a) \Rightarrow ((xphi@xx) \Rightarrow (in@xx@(dsetconstr@a@\lambda xy\colon \$i\colon (xphi@xy))))))$
binintersect: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(binintersect_type, type)
$\text{binintersect} = (\lambda a\colon \$i, b\colon \$i\colon (dsetconstr@a@\lambda xx\colon \$i\colon (in@xx@b)))$ thf(binintersect, definition)
$\text{dsetconstrI} \Rightarrow \forall a\colon \$i, b\colon \$i, xx\colon \$i\colon ((in@xx@a) \Rightarrow ((in@xx@b) \Rightarrow (in@xx@(binintersect@a@b))))$ thf(binintersectI, conje

SEU590^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! C:i.subset C A → subset C B → subset C (binintersect A B))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
dsetconstr: \$i $\rightarrow$ (\$i $\rightarrow$ \$o) $\rightarrow$ \$i thf(dsetconstr_type, type)
$\subseteq$: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(subset_type, type)
subsetI_1: \$o thf(subsetI1_type, type)
$\text{subsetI}_1 = (\forall a\colon \$i, b\colon \$i\colon (\forall xx\colon \$i\colon ((in@xx@a) \Rightarrow (in@xx@b)) \Rightarrow (\subseteq @a@b)))$ thf(subsetI_1, definition)
subsetE: \$o thf(subsetE_type, type)
$\text{subsetE} = (\forall a\colon \$i, b\colon \$i, xx\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((in@xx@a) \Rightarrow (in@xx@b))))$ thf(subsetE, definition)
binintersect: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(binintersect_type, type)
$\text{binintersect} = (\lambda a\colon \$i, b\colon \$i\colon (dsetconstr@a@\lambda xx\colon \$i\colon (in@xx@b)))$ thf(binintersect, definition)
binintersectI: \$o thf(binintersectI_type, type)
$\text{binintersectI} = (\forall a\colon \$i, b\colon \$i, xx\colon \$i\colon ((in@xx@a) \Rightarrow ((in@xx@b) \Rightarrow (in@xx@(binintersect@a@b)))))$ thf(binintersectI, defi
$\text{subsetI}_1 \Rightarrow (\text{subsetE} \Rightarrow (\text{binintersectI} \Rightarrow \forall a\colon \$i, b\colon \$i, c\colon \$i\colon ((\subseteq @c@a) \Rightarrow ((\subseteq @c@b) \Rightarrow (\subseteq @c@(binintersect@a@b))))))$

SEU591^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! x:i.in x (binintersect A B) → in x A)
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
dsetconstr: \$i $\rightarrow$ (\$i $\rightarrow$ \$o) $\rightarrow$ \$i thf(dsetconstr_type, type)
dsetconstrEL: \$o thf(dsetconstrEL_type, type)
$\text{dsetconstrEL} = (\forall a\colon \$i, xphi\colon \$i \rightarrow \$o, xx\colon \$i\colon ((in@xx@(dsetconstr@a@\lambda xy\colon \$i\colon (xphi@xy))) \Rightarrow (in@xx@a)))$ thf(dsetcons
binintersect: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(binintersect_type, type)
$\text{binintersect} = (\lambda a\colon \$i, b\colon \$i\colon (dsetconstr@a@\lambda xx\colon \$i\colon (in@xx@b)))$ thf(binintersect, definition)
$\text{dsetconstrEL} \Rightarrow \forall a\colon \$i, b\colon \$i, xx\colon \$i\colon ((in@xx@(binintersect@a@b)) \Rightarrow (in@xx@a))$ thf(binintersectEL, conjecture)

SEU592^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.subset (binintersect A B) A)
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
dsetconstr: \$i $\rightarrow$ (\$i $\rightarrow$ \$o) $\rightarrow$ \$i thf(dsetconstr_type, type)
$\subseteq$: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(subset_type, type)
subsetI_2: \$o thf(subsetI2_type, type)
$\text{subsetI}_2 = (\forall a\colon \$i, b\colon \$i\colon (\forall xx\colon \$i\colon ((in@xx@a) \Rightarrow (in@xx@b)) \Rightarrow (\subseteq @a@b)))$ thf(subsetI_2, definition)
binintersect: \$i $\rightarrow$ \$i $\rightarrow$ \$i thf(binintersect_type, type)

$\mathrm{binintersect} = (\lambda a\colon \$i, b\colon \$i\colon (\mathrm{dsetconstr}@a@\lambda \mathrm{xx}\colon \$i\colon (\mathrm{in}@\mathrm{xx}@b)))$ thf(binintersect, definition)
$\mathrm{binintersectEL}\colon \$o$ thf(binintersectEL_type, type)
$\mathrm{binintersectEL} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@a@b)) \Rightarrow (\mathrm{in}@\mathrm{xx}@a)))$ thf(binintersectEL, definition)
$\mathrm{subsetI}_2 \Rightarrow (\mathrm{binintersectEL} \Rightarrow \forall a\colon \$i, b\colon \$i\colon (\subseteq @(\mathrm{binintersect}@a@b)@a))$ thf(binintersectLsub, conjecture)

SEU593^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.subset A B → binintersect A B = A)
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{dsetconstr}\colon \$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\mathrm{subsetI}_1\colon \$o$ thf(subsetI1_type, type)
$\mathrm{subsetI}_1 = (\forall a\colon \$i, b\colon \$i\colon (\forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b)) \Rightarrow (\subseteq @a@b)))$ thf($\mathrm{subsetI}_1$, definition)
$\mathrm{subsetE}\colon \$o$ thf(subsetE_type, type)
$\mathrm{subsetE} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b))))$ thf(subsetE, definition)
$\mathrm{setextsub}\colon \$o$ thf(setextsub_type, type)
$\mathrm{setextsub} = (\forall a\colon \$i, b\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\subseteq @b@a) \Rightarrow a = b)))$ thf(setextsub, definition)
$\mathrm{binintersect}\colon \$i \to \$i \to \$i$ thf(binintersect_type, type)
$\mathrm{binintersect} = (\lambda a\colon \$i, b\colon \$i\colon (\mathrm{dsetconstr}@a@\lambda \mathrm{xx}\colon \$i\colon (\mathrm{in}@\mathrm{xx}@b)))$ thf(binintersect, definition)
$\mathrm{binintersectI}\colon \$o$ thf(binintersectI_type, type)
$\mathrm{binintersectI} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow ((\mathrm{in}@\mathrm{xx}@b) \Rightarrow (\mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@a@b)))))$ thf(binintersectI, defi
$\mathrm{binintersectLsub}\colon \$o$ thf(binintersectLsub_type, type)
$\mathrm{binintersectLsub} = (\forall a\colon \$i, b\colon \$i\colon (\subseteq @(\mathrm{binintersect}@a@b)@a))$ thf(binintersectLsub, definition)
$\mathrm{subsetI}_1 \Rightarrow (\mathrm{subsetE} \Rightarrow (\mathrm{setextsub} \Rightarrow (\mathrm{binintersectI} \Rightarrow (\mathrm{binintersectLsub} \Rightarrow \forall a\colon \$i, b\colon \$i\colon ((\subseteq @a@b) \Rightarrow (\mathrm{binintersect}@a@b) = a)))))$ thf($\mathrm{binintersectSubset}_2$, conjecture)

SEU594^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.binintersect A B = B → subset B A)
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{dsetconstr}\colon \$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$\mathrm{in\_\_Cong}\colon \$o$ thf(in__Cong_type, type)
$\mathrm{in\_\_Cong} = (\forall a\colon \$i, b\colon \$i\colon (a = b \Rightarrow \forall \mathrm{xx}\colon \$i, \mathrm{xy}\colon \$i\colon (\mathrm{xx} = \mathrm{xy} \Rightarrow ((\mathrm{in}@\mathrm{xx}@a) \iff (\mathrm{in}@\mathrm{xy}@b)))))$ thf(in__Cong, definition)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\mathrm{subsetI}_1\colon \$o$ thf(subsetI1_type, type)
$\mathrm{subsetI}_1 = (\forall a\colon \$i, b\colon \$i\colon (\forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b)) \Rightarrow (\subseteq @a@b)))$ thf($\mathrm{subsetI}_1$, definition)
$\mathrm{binintersect}\colon \$i \to \$i \to \$i$ thf(binintersect_type, type)
$\mathrm{binintersect} = (\lambda a\colon \$i, b\colon \$i\colon (\mathrm{dsetconstr}@a@\lambda \mathrm{xx}\colon \$i\colon (\mathrm{in}@\mathrm{xx}@b)))$ thf(binintersect, definition)
$\mathrm{binintersectEL}\colon \$o$ thf(binintersectEL_type, type)
$\mathrm{binintersectEL} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@a@b)) \Rightarrow (\mathrm{in}@\mathrm{xx}@a)))$ thf(binintersectEL, definition)
$\mathrm{in\_\_Cong} \Rightarrow (\mathrm{subsetI}_1 \Rightarrow (\mathrm{binintersectEL} \Rightarrow \forall a\colon \$i, b\colon \$i\colon ((\mathrm{binintersect}@a@b) = b \Rightarrow (\subseteq @b@a))))$ thf(binintersectSubs

SEU595^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! x:i.in x (binintersect A B) → in x B)
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{dsetconstr}\colon \$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$\mathrm{dsetconstrER}\colon \$o$ thf(dsetconstrER_type, type)
$\mathrm{dsetconstrER} = (\forall a\colon \$i, \mathrm{xphi}\colon \$i \to \$o, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{dsetconstr}@a@\lambda \mathrm{xy}\colon \$i\colon (\mathrm{xphi}@\mathrm{xy}))) \Rightarrow (\mathrm{xphi}@\mathrm{xx})))$ thf(dsetcons
$\mathrm{binintersect}\colon \$i \to \$i \to \$i$ thf(binintersect_type, type)
$\mathrm{binintersect} = (\lambda a\colon \$i, b\colon \$i\colon (\mathrm{dsetconstr}@a@\lambda \mathrm{xx}\colon \$i\colon (\mathrm{in}@\mathrm{xx}@b)))$ thf(binintersect, definition)
$\mathrm{dsetconstrER} \Rightarrow \forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@a@b)) \Rightarrow (\mathrm{in}@\mathrm{xx}@b))$ thf(binintersectER, conjecture)

SEU596^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i. (? x:i.in x A & in x B) → binintersect A B = emptyset)
$\mathrm{in}\colon \$i \to \$i \to \$o$ thf(in_type, type)
$\mathrm{emptyset}\colon \$i$ thf(emptyset_type, type)
$\mathrm{emptyI}\colon \$o$ thf(emptyI_type, type)
$\mathrm{emptyI} = (\forall a\colon \$i\colon (\forall \mathrm{xx}\colon \$i\colon \neg\, \mathrm{in}@\mathrm{xx}@a \Rightarrow a = \mathrm{emptyset}))$ thf(emptyI, definition)
$\mathrm{binintersect}\colon \$i \to \$i \to \$i$ thf(binintersect_type, type)
$\mathrm{binintersectEL}\colon \$o$ thf(binintersectEL_type, type)
$\mathrm{binintersectEL} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@a@b)) \Rightarrow (\mathrm{in}@\mathrm{xx}@a)))$ thf(binintersectEL, definition)
$\mathrm{binintersectER}\colon \$o$ thf(binintersectER_type, type)
$\mathrm{binintersectER} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@a@b)) \Rightarrow (\mathrm{in}@\mathrm{xx}@b)))$ thf(binintersectER, definition)

emptyI $\Rightarrow$ (binintersectEL $\Rightarrow$ (binintersectER $\Rightarrow$ $\forall a$: \$i, b: \$i: ($\neg\exists$xx: \$i: (in@xx@a and in@xx@b) $\Rightarrow$ (binintersect@a@b) = emptyset))) thf(disjointsetsI$_1$, conjecture)

SEU597^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.subset (binintersect A B) B)
in: \$i $\to$ \$i $\to$ \$o thf(in_type, type)
$\subseteq$: \$i $\to$ \$i $\to$ \$o thf(subset_type, type)
subsetI$_2$: \$o thf(subsetI2_type, type)
subsetI$_2$ = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\subseteq$ @a@b))) thf(subsetI$_2$, definition)
binintersect: \$i $\to$ \$i $\to$ \$i thf(binintersect_type, type)
binintersectER: \$o thf(binintersectER_type, type)
binintersectER = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@(binintersect@a@b)) $\Rightarrow$ (in@xx@b))) thf(binintersectER, definition)
subsetI$_2$ $\Rightarrow$ (binintersectER $\Rightarrow$ $\forall a$: \$i, b: \$i: ($\subseteq$ @(binintersect@a@b)@b)) thf(binintersectRsub, conjecture)

SEU598^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.subset B A $\to$ binintersect A B = B)
in: \$i $\to$ \$i $\to$ \$o thf(in_type, type)
$\subseteq$: \$i $\to$ \$i $\to$ \$o thf(subset_type, type)
subsetI$_1$: \$o thf(subsetI1_type, type)
subsetI$_1$ = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\subseteq$ @a@b))) thf(subsetI$_1$, definition)
subsetE: \$o thf(subsetE_type, type)
subsetE = ($\forall a$: \$i, b: \$i, xx: \$i: (($\subseteq$ @a@b) $\Rightarrow$ ((in@xx@a) $\Rightarrow$ (in@xx@b)))) thf(subsetE, definition)
setextsub: \$o thf(setextsub_type, type)
setextsub = ($\forall a$: \$i, b: \$i: (($\subseteq$ @a@b) $\Rightarrow$ (($\subseteq$ @b@a) $\Rightarrow$ $a = b$))) thf(setextsub, definition)
binintersect: \$i $\to$ \$i $\to$ \$i thf(binintersect_type, type)
binintersectI: \$o thf(binintersectI_type, type)
binintersectI = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@b) $\Rightarrow$ (in@xx@(binintersect@a@b))))) thf(binintersectI, defi
binintersectRsub: \$o thf(binintersectRsub_type, type)
binintersectRsub = ($\forall a$: \$i, b: \$i: ($\subseteq$ @(binintersect@a@b)@b)) thf(binintersectRsub, definition)
subsetI$_1$ $\Rightarrow$ (subsetE $\Rightarrow$ (setextsub $\Rightarrow$ (binintersectI $\Rightarrow$ (binintersectRsub $\Rightarrow$ $\forall a$: \$i, b: \$i: (($\subseteq$ @b@a) $\Rightarrow$ (binintersect@a@b) = b))))) thf(binintersectSubset$_4$, conjecture)

SEU599^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.binintersect A B = A $\to$ subset A B)
in: \$i $\to$ \$i $\to$ \$o thf(in_type, type)
in__Cong: \$o thf(in__Cong_type, type)
in__Cong = ($\forall a$: \$i, b: \$i: ($a = b$ $\Rightarrow$ $\forall$xx: \$i, xy: \$i: (xx = xy $\Rightarrow$ ((in@xx@a) $\iff$ (in@xy@b))))) thf(in__Cong, definition)
$\subseteq$: \$i $\to$ \$i $\to$ \$o thf(subset_type, type)
subsetI$_1$: \$o thf(subsetI1_type, type)
subsetI$_1$ = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\subseteq$ @a@b))) thf(subsetI$_1$, definition)
binintersect: \$i $\to$ \$i $\to$ \$i thf(binintersect_type, type)
binintersectER: \$o thf(binintersectER_type, type)
binintersectER = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@(binintersect@a@b)) $\Rightarrow$ (in@xx@b))) thf(binintersectER, definition)
in__Cong $\Rightarrow$ (subsetI$_1$ $\Rightarrow$ (binintersectER $\Rightarrow$ $\forall a$: \$i, b: \$i: ((binintersect@a@b) = a $\Rightarrow$ ($\subseteq$ @a@b)))) thf(binintersectSub

SEU600^2.p Preliminary Notions - Ops on Sets - Unions and Intersections
(! A:i.! B:i.! C:i.binintersect A (binunion B C) = binunion (binintersect A B) (binintersect A C))
in: \$i $\to$ \$i $\to$ \$o thf(in_type, type)
$\subseteq$: \$i $\to$ \$i $\to$ \$o thf(subset_type, type)
subsetI$_1$: \$o thf(subsetI1_type, type)
subsetI$_1$ = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\subseteq$ @a@b))) thf(subsetI$_1$, definition)
setextsub: \$o thf(setextsub_type, type)
setextsub = ($\forall a$: \$i, b: \$i: (($\subseteq$ @a@b) $\Rightarrow$ (($\subseteq$ @b@a) $\Rightarrow$ $a = b$))) thf(setextsub, definition)
binunion: \$i $\to$ \$i $\to$ \$i thf(binunion_type, type)
binunionIL: \$o thf(binunionIL_type, type)
binunionIL = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@(binunion@a@b)))) thf(binunionIL, definition)
binunionIR: \$o thf(binunionIR_type, type)
binunionIR = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@b) $\Rightarrow$ (in@xx@(binunion@a@b)))) thf(binunionIR, definition)
binunionEcases: \$o thf(binunionEcases_type, type)
binunionEcases = ($\forall a$: \$i, b: \$i, xx: \$i, xphi: \$o: ((in@xx@(binunion@a@b)) $\Rightarrow$ (((in@xx@a) $\Rightarrow$ xphi) $\Rightarrow$ (((in@xx@b) $\Rightarrow$ xphi) $\Rightarrow$ xphi)))) thf(binunionEcases, definition)

binintersect: \$i → \$i → \$i thf(binintersect_type, type)
binintersectI: \$o thf(binintersectI_type, type)
binintersectI = (∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ ((in@xx@*b*) ⇒ (in@xx@(binintersect@*a*@*b*))))) thf(binintersectI, defi
binintersectEL: \$o thf(binintersectEL_type, type)
binintersectEL = (∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@(binintersect@*a*@*b*)) ⇒ (in@xx@*a*))) thf(binintersectEL, definition)
binintersectER: \$o thf(binintersectER_type, type)
binintersectER = (∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@(binintersect@*a*@*b*)) ⇒ (in@xx@*b*))) thf(binintersectER, definition)
subsetI$_1$ ⇒ (setextsub ⇒ (binunionIL ⇒ (binunionIR ⇒ (binunionEcases ⇒ (binintersectI ⇒ (binintersectEL ⇒ (binintersectER ⇒ ∀*a*: \$i, *b*: \$i, *c*: \$i: (binintersect@*a*@(binunion@*b*@*c*)) = (binunion@(binintersect@*a*@*b*)@(binintersect@*a*@

SEU601∧2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.! x:i.in x A → (in x B) → in x (setminus A B))
in: \$i → \$i → \$o thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrI: \$o thf(dsetconstrI_type, type)
dsetconstrI = (∀*a*: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@*a*) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@*a*@λxy: \$i: (xphi@xy))))))
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminus = (λ*a*: \$i, *b*: \$i: (dsetconstr@*a*@λxx: \$i: ¬ in@xx@*b*)) thf(setminus, definition)
dsetconstrI ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (¬ in@xx@*b* ⇒ (in@xx@(setminus@*a*@*b*)))) thf(setminusI, conjecture)

SEU602∧2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.! x:i.in x (setminus A B) → in x A)
in: \$i → \$i → \$o thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrEL: \$o thf(dsetconstrEL_type, type)
dsetconstrEL = (∀*a*: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@*a*@λxy: \$i: (xphi@xy))) ⇒ (in@xx@*a*))) thf(dsetcons
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminus = (λ*a*: \$i, *b*: \$i: (dsetconstr@*a*@λxx: \$i: ¬ in@xx@*b*)) thf(setminus, definition)
dsetconstrEL ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@(setminus@*a*@*b*)) ⇒ (in@xx@*a*)) thf(setminusEL, conjecture)

SEU603∧2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.! x:i.in x (setminus A B) → (in x B))
in: \$i → \$i → \$o thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrER: \$o thf(dsetconstrER_type, type)
dsetconstrER = (∀*a*: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@*a*@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminus = (λ*a*: \$i, *b*: \$i: (dsetconstr@*a*@λxx: \$i: ¬ in@xx@*b*)) thf(setminus, definition)
dsetconstrER ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@(setminus@*a*@*b*)) ⇒ ¬ in@xx@*b*) thf(setminusER, conjecture)

SEU604∧2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.subset A B → setminus A B = emptyset)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
subsetI$_2$: \$o thf(subsetI2_type, type)
subsetI$_2$ = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*))) thf(subsetI$_2$, definition)
subsetE: \$o thf(subsetE_type, type)
subsetE = (∀*a*: \$i, *b*: \$i, xx: \$i: ((⊆ @*a*@*b*) ⇒ ((in@xx@*a*) ⇒ (in@xx@*b*)))) thf(subsetE, definition)
subsetemptysetimpeq: \$o thf(subsetemptysetimpeq_type, type)
subsetemptysetimpeq = (∀*a*: \$i: ((⊆ @*a*@emptyset) ⇒ *a* = emptyset)) thf(subsetemptysetimpeq, definition)
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusEL: \$o thf(setminusEL_type, type)
setminusEL = (∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@(setminus@*a*@*b*)) ⇒ (in@xx@*a*))) thf(setminusEL, definition)
setminusER: \$o thf(setminusER_type, type)
setminusER = (∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@(setminus@*a*@*b*)) ⇒ ¬ in@xx@*b*)) thf(setminusER, definition)
subsetI$_2$ ⇒ (subsetE ⇒ (subsetemptysetimpeq ⇒ (setminusEL ⇒ (setminusER ⇒ ∀*a*: \$i, *b*: \$i: ((⊆ @*a*@*b*) ⇒ (setminus@*a*@*b*) = emptyset))))) thf(setminusSubset$_2$, conjecture)

SEU605∧2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.! x:i. (in x (setminus A B)) → in x A → in x B)
in: \$i → \$i → \$o thf(in_type, type)

setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusI: $o thf(setminusI_type, type)
$\text{setminusI} = (\forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon ((\text{in@xx@}a) \Rightarrow (\neg\,\text{in@xx@}b \Rightarrow (\text{in@xx@(setminus@}a\text{@}b)))))$ thf(setminusI, definition)
$\text{setminusI} \Rightarrow \forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon (\neg\,\text{in@xx@(setminus@}a\text{@}b) \Rightarrow ((\text{in@xx@}a) \Rightarrow (\text{in@xx@}b)))$ thf(setminusERneg, conject

SEU606^2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.! x:i. (in x (setminus A B)) → (in x B) → (in x A))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusERneg: $o thf(setminusERneg_type, type)
$\text{setminusERneg} = (\forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon (\neg\,\text{in@xx@(setminus@}a\text{@}b) \Rightarrow ((\text{in@xx@}a) \Rightarrow (\text{in@xx@}b))))$ thf(setminusERneg, d
$\text{setminusERneg} \Rightarrow \forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon (\neg\,\text{in@xx@(setminus@}a\text{@}b) \Rightarrow (\neg\,\text{in@xx@}b \Rightarrow \neg\,\text{in@xx@}a))$ thf(setminusELneg, c

SEU607^2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.! x:i. (in x A) → (in x (setminus A B)))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusEL: $o thf(setminusEL_type, type)
$\text{setminusEL} = (\forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon ((\text{in@xx@(setminus@}a\text{@}b)) \Rightarrow (\text{in@xx@}a)))$ thf(setminusEL, definition)
$\text{setminusEL} \Rightarrow \forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon (\neg\,\text{in@xx@}a \Rightarrow \neg\,\text{in@xx@(setminus@}a\text{@}b))$ thf(setminusILneg, conjecture)

SEU608^2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.! x:i.in x B → (in x (setminus A B)))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusER: $o thf(setminusER_type, type)
$\text{setminusER} = (\forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon ((\text{in@xx@(setminus@}a\text{@}b)) \Rightarrow \neg\,\text{in@xx@}b))$ thf(setminusER, definition)
$\text{setminusER} \Rightarrow \forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon ((\text{in@xx@}b) \Rightarrow \neg\,\text{in@xx@(setminus@}a\text{@}b))$ thf(setminusIRneg, conjecture)

SEU609^2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.subset (setminus A B) A)
in: $\$i \to \$i \to \$o$ thf(in_type, type)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
subsetI_2: $o thf(subsetI2_type, type)
$\text{subsetI}_2 = (\forall a\colon \$i, b\colon \$i\colon (\forall \text{xx}\colon \$i\colon ((\text{in@xx@}a) \Rightarrow (\text{in@xx@}b)) \Rightarrow (\subseteq \text{@}a\text{@}b)))$ thf(subsetI_2, definition)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusEL: $o thf(setminusEL_type, type)
$\text{setminusEL} = (\forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon ((\text{in@xx@(setminus@}a\text{@}b)) \Rightarrow (\text{in@xx@}a)))$ thf(setminusEL, definition)
$\text{subsetI}_2 \Rightarrow (\text{setminusEL} \Rightarrow \forall a\colon \$i, b\colon \$i\colon (\subseteq \text{@(setminus@}a\text{@}b\text{)@}a))$ thf(setminusLsub, conjecture)

SEU610^2.p Preliminary Notions - Operations on Sets - Set Difference
(! A:i.! B:i.setminus A B = emptyset → subset A B)
in: $\$i \to \$i \to \$o$ thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
emptysetE: $o thf(emptysetE_type, type)
$\text{emptysetE} = (\forall \text{xx}\colon \$i\colon ((\text{in@xx@emptyset}) \Rightarrow \forall \text{xphi}\colon \$o\colon \text{xphi}))$ thf(emptysetE, definition)
in__Cong: $o thf(in__Cong_type, type)
$\text{in\_\_Cong} = (\forall a\colon \$i, b\colon \$i\colon (a = b \Rightarrow \forall \text{xx}\colon \$i, \text{xy}\colon \$i\colon (\text{xx} = \text{xy} \Rightarrow ((\text{in@xx@}a) \iff (\text{in@xy@}b)))))$ thf(in__Cong, definition)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
subsetI_2: $o thf(subsetI2_type, type)
$\text{subsetI}_2 = (\forall a\colon \$i, b\colon \$i\colon (\forall \text{xx}\colon \$i\colon ((\text{in@xx@}a) \Rightarrow (\text{in@xx@}b)) \Rightarrow (\subseteq \text{@}a\text{@}b)))$ thf(subsetI_2, definition)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusI: $o thf(setminusI_type, type)
$\text{setminusI} = (\forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon ((\text{in@xx@}a) \Rightarrow (\neg\,\text{in@xx@}b \Rightarrow (\text{in@xx@(setminus@}a\text{@}b)))))$ thf(setminusI, definition)
$\text{emptysetE} \Rightarrow (\text{in\_\_Cong} \Rightarrow (\text{subsetI}_2 \Rightarrow (\text{setminusI} \Rightarrow \forall a\colon \$i, b\colon \$i\colon ((\text{setminus@}a\text{@}b) = \text{emptyset} \Rightarrow (\subseteq \text{@}a\text{@}b)))))$ thf(setminusSubset_1, conjecture)

SEU611^2.p Preliminary Notions - Operations on Sets - Symmetric Difference
(! A:i.! B:i.! x:i.in x (symdiff A B) → (! phi:o.(in x A → (in x B) → phi) → ((in x A) → in x B → phi) → phi))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
dsetconstr: $\$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
dsetconstrEL: $o thf(dsetconstrEL_type, type)
$\text{dsetconstrEL} = (\forall a\colon \$i, \text{xphi}\colon \$i \to \$o, \text{xx}\colon \$i\colon ((\text{in@xx@(dsetconstr@}a\text{@}\lambda \text{xy}\colon \$i\colon (\text{xphi@xy}))) \Rightarrow (\text{in@xx@}a)))$ thf(dsetcons

dsetconstrER: $o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
binunion: $i → $i → $i thf(binunion_type, type)
binunionE: $o thf(binunionE_type, type)
binunionE = (∀a: $i, b: $i, xx: $i: ((in@xx@(binunion@a@b)) ⇒ (in@xx@a or in@xx@b))) thf(binunionE, definition)
symdiff: $i → $i → $i thf(symdiff_type, type)
symdiff = (λa: $i, b: $i: (dsetconstr@(binunion@a@b)@λxx: $i: (¬ in@xx@a or ¬ in@xx@b))) thf(symdiff, definition)
dsetconstrEL ⇒ (dsetconstrER ⇒ (binunionE ⇒ ∀a: $i, b: $i, xx: $i: ((in@xx@(symdiff@a@b)) ⇒ ∀xphi: $o: (((in@xx@a)
(¬ in@xx@b ⇒ xphi)) ⇒ ((¬ in@xx@a ⇒ ((in@xx@b) ⇒ xphi)) ⇒ xphi))))) thf(symdiffE, conjecture)

SEU612∧2.p Preliminary Notions - Operations on Sets - Symmetric Difference
(! A:i.! B:i.! x:i.in x A → (in x B) → in x (symdiff A B))
in: $i → $i → $o thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrI: $o thf(dsetconstrI_type, type)
dsetconstrI = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))))))
binunion: $i → $i → $i thf(binunion_type, type)
binunionIL: $o thf(binunionIL_type, type)
binunionIL = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ (in@xx@(binunion@a@b)))) thf(binunionIL, definition)
symdiff: $i → $i → $i thf(symdiff_type, type)
symdiff = (λa: $i, b: $i: (dsetconstr@(binunion@a@b)@λxx: $i: (¬ in@xx@a or ¬ in@xx@b))) thf(symdiff, definition)
dsetconstrI ⇒ (binunionIL ⇒ ∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ (¬ in@xx@b ⇒ (in@xx@(symdiff@a@b))))) thf(symdi

SEU613∧2.p Preliminary Notions - Operations on Sets - Symmetric Difference
(! A:i.! B:i.! x:i. (in x A) → in x B → in x (symdiff A B))
in: $i → $i → $o thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrI: $o thf(dsetconstrI_type, type)
dsetconstrI = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))))))
binunion: $i → $i → $i thf(binunion_type, type)
binunionIR: $o thf(binunionIR_type, type)
binunionIR = (∀a: $i, b: $i, xx: $i: ((in@xx@b) ⇒ (in@xx@(binunion@a@b)))) thf(binunionIR, definition)
symdiff: $i → $i → $i thf(symdiff_type, type)
symdiff = (λa: $i, b: $i: (dsetconstr@(binunion@a@b)@λxx: $i: (¬ in@xx@a or ¬ in@xx@b))) thf(symdiff, definition)
dsetconstrI ⇒ (binunionIR ⇒ ∀a: $i, b: $i, xx: $i: (¬ in@xx@a ⇒ ((in@xx@b) ⇒ (in@xx@(symdiff@a@b))))) thf(symdi

SEU614∧2.p Preliminary Notions - Operations on Sets - Symmetric Difference
(! A:i.! B:i.! x:i.in x A → in x B → (in x (symdiff A B)))
in: $i → $i → $o thf(in_type, type)
symdiff: $i → $i → $i thf(symdiff_type, type)
symdiffE: $o thf(symdiffE_type, type)
symdiffE = (∀a: $i, b: $i, xx: $i: ((in@xx@(symdiff@a@b)) ⇒ ∀xphi: $o: (((in@xx@a) ⇒ (¬ in@xx@b ⇒ xphi)) ⇒
((¬ in@xx@a ⇒ ((in@xx@b) ⇒ xphi)) ⇒ xphi)))) thf(symdiffE, definition)
symdiffE ⇒ ∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ((in@xx@b) ⇒ ¬ in@xx@(symdiff@a@b))) thf(symdiffIneg_1, conjecture)

SEU615∧2.p Preliminary Notions - Operations on Sets - Symmetric Difference
(! A:i.! B:i.! x:i. (in x A) → (in x B) → (in x (symdiff A B)))
in: $i → $i → $o thf(in_type, type)
symdiff: $i → $i → $i thf(symdiff_type, type)
symdiffE: $o thf(symdiffE_type, type)
symdiffE = (∀a: $i, b: $i, xx: $i: ((in@xx@(symdiff@a@b)) ⇒ ∀xphi: $o: (((in@xx@a) ⇒ (¬ in@xx@b ⇒ xphi)) ⇒
((¬ in@xx@a ⇒ ((in@xx@b) ⇒ xphi)) ⇒ xphi)))) thf(symdiffE, definition)
symdiffE ⇒ ∀a: $i, b: $i, xx: $i: (¬ in@xx@a ⇒ (¬ in@xx@b ⇒ ¬ in@xx@(symdiff@a@b))) thf(symdiffIneg_2, conjecture)

SEU617∧2.p Ordered Pairs - Kuratowski Pairs
(! x:i.! y:i.in x (setunion (setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) empty-
set))))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
setadjoinIL: $o thf(setadjoinIL_type, type)

setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
setunionI: $o thf(setunionI_type, type)
setunionI = (∀a: $i, xx: $i, b: $i: ((in@xx@$b$) ⇒ ((in@$b$@$a$) ⇒ (in@xx@(setunion@$a$))))) thf(setunionI, definition)
setadjoinIL ⇒ (setunionI ⇒ ∀xx: $i, xy: $i: (in@xx@(setunion@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoi

SEU618∧2.p Ordered Pairs - Kuratowski Pairs
(! x:i.! y:i.in y (setunion (setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset))))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
setunionI: $o thf(setunionI_type, type)
setunionI = (∀a: $i, xx: $i, b: $i: ((in@xx@$b$) ⇒ ((in@$b$@$a$) ⇒ (in@xx@(setunion@$a$))))) thf(setunionI, definition)
secondinupair: $o thf(secondinupair_type, type)
secondinupair = (∀xx: $i, xy: $i: (in@xy@(setadjoin@xx@(setadjoin@xy@emptyset)))) thf(secondinupair, definition)
setunionI ⇒ (secondinupair ⇒ ∀xx: $i, xy: $i: (in@xy@(setunion@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setad

SEU619∧2.p Ordered Pairs - Kuratowski Pairs
(! x:i.! y:i.iskpair (setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset)))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
iskpair: $i → $o thf(iskpair_type, type)
iskpair = (λa: $i: ∃xx: $i: (in@xx@(setunion@$a$) and ∃xy: $i: (in@xy@(setunion@$a$) and a = (setadjoin@(setadjoin@xx@empt
setukpairIL: $o thf(setukpairIL_type, type)
setukpairIL = (∀xx: $i, xy: $i: (in@xx@(setunion@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjo
setukpairIR: $o thf(setukpairIR_type, type)
setukpairIR = (∀xx: $i, xy: $i: (in@xy@(setunion@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjo
setukpairIL ⇒ (setukpairIR ⇒ ∀xx: $i, xy: $i: (iskpair@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(

SEU620∧2.p Ordered Pairs - Kuratowski Pairs
(! x:i.! y:i.iskpair (kpair x y))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
iskpair: $i → $o thf(iskpair_type, type)
iskpair = (λa: $i: ∃xx: $i: (in@xx@(setunion@$a$) and ∃xy: $i: (in@xy@(setunion@$a$) and a = (setadjoin@(setadjoin@xx@empt
kpairiskpair: $o thf(kpairiskpair_type, type)
kpairiskpair = (∀xx: $i, xy: $i: (iskpair@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@en
kpair: $i → $i → $i thf(kpair_type, type)
kpair = (λxx: $i, xy: $i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
kpairiskpair ⇒ ∀xx: $i, xy: $i: (iskpair@(kpair@xx@xy)) thf(kpairp, conjecture)

SEU621∧2.p Ordered Pairs - Cartesian Products
(! A:i.! x:i.in x A → subset (setadjoin x emptyset) A)
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
uniqinunit: $o thf(uniqinunit_type, type)
uniqinunit = (∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
⊆ : $i → $i → $o thf(subset_type, type)
subsetI$_2$: $o thf(subsetI2_type, type)
subsetI$_2$ = (∀a: $i, b: $i: (∀xx: $i: ((in@xx@$a$) ⇒ (in@xx@$b$)) ⇒ (⊆ @$a$@$b$))) thf(subsetI$_2$, definition)
uniqinunit ⇒ (subsetI$_2$ ⇒ ∀a: $i, xx: $i: ((in@xx@$a$) ⇒ (⊆ @(setadjoin@xx@emptyset)@a))) thf(singletonsubset, conje

SEU622∧2.p Ordered Pairs - Cartesian Products
(! A:i.! x:i.in x A → in (setadjoin x emptyset) (powerset A))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)

setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
powerset: \$i → \$i thf(powerset_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
powersetI_1: \$o thf(powersetI1_type, type)
powersetI_1 = (∀*a*: \$i, *b*: \$i: ((⊆ @*b*@*a*) ⇒ (in@*b*@(powerset@*a*)))) thf(powersetI_1, definition)
singletonsubset: \$o thf(singletonsubset_type, type)
singletonsubset = (∀*a*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (⊆ @(setadjoin@xx@emptyset)@*a*))) thf(singletonsubset, definition)
powersetI_1 ⇒ (singletonsubset ⇒ ∀*a*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (in@(setadjoin@xx@emptyset)@(powerset@*a*)))) thf(sin

SEU623∧2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! x:i.in x A → in (setadjoin x emptyset) (powerset (binunion A B)))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
powerset: \$i → \$i thf(powerset_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
subsetE: \$o thf(subsetE_type, type)
subsetE = (∀*a*: \$i, *b*: \$i, xx: \$i: ((⊆ @*a*@*b*) ⇒ ((in@xx@*a*) ⇒ (in@xx@*b*)))) thf(subsetE, definition)
powersetsubset: \$o thf(powersetsubset_type, type)
powersetsubset = (∀*a*: \$i, *b*: \$i: ((⊆ @*a*@*b*) ⇒ (⊆ @(powerset@*a*)@(powerset@*b*)))) thf(powersetsubset, definition)
binunion: \$i → \$i → \$i thf(binunion_type, type)
binunionLsub: \$o thf(binunionLsub_type, type)
binunionLsub = (∀*a*: \$i, *b*: \$i: (⊆ @*a*@(binunion@*a*@*b*))) thf(binunionLsub, definition)
singletoninpowerset: \$o thf(singletoninpowerset_type, type)
singletoninpowerset = (∀*a*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (in@(setadjoin@xx@emptyset)@(powerset@*a*)))) thf(singletoninpow
subsetE ⇒ (powersetsubset ⇒ (binunionLsub ⇒ (singletoninpowerset ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ (in@(setadjoin@xx@emptyset)@(powerset@(binunion@*a*@*b*)))))) thf(singletoninpowunion, conjecture)

SEU625∧2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → subset (setadjoin x (setadjoin y emptyset)) (binunion A B)))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
subsetI_2: \$o thf(subsetI2_type, type)
subsetI_2 = (∀*a*: \$i, *b*: \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@xx@*b*)) ⇒ (⊆ @*a*@*b*))) thf(subsetI_2, definition)
subsetE: \$o thf(subsetE_type, type)
subsetE = (∀*a*: \$i, *b*: \$i, xx: \$i: ((⊆ @*a*@*b*) ⇒ ((in@xx@*a*) ⇒ (in@xx@*b*)))) thf(subsetE, definition)
binunion: \$i → \$i → \$i thf(binunion_type, type)
binunionLsub: \$o thf(binunionLsub_type, type)
binunionLsub = (∀*a*: \$i, *b*: \$i: (⊆ @*a*@(binunion@*a*@*b*))) thf(binunionLsub, definition)
binunionRsub: \$o thf(binunionRsub_type, type)
binunionRsub = (∀*a*: \$i, *b*: \$i: (⊆ @*b*@(binunion@*a*@*b*))) thf(binunionRsub, definition)
upairset2E: \$o thf(upairset2E_type, type)
upairset2E = (∀xx: \$i, xy: \$i, xz: \$i: ((in@xz@(setadjoin@xx@(setadjoin@xy@emptyset))) ⇒ (xz = xx or xz = xy))) thf(upairset2E, definition)
subsetI_2 ⇒ (subsetE ⇒ (binunionLsub ⇒ (binunionRsub ⇒ (upairset2E ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*b*) ⇒ (⊆ @(setadjoin@xx@(setadjoin@xy@emptyset))@(binunion@*a*@*b*)))))))) thf(upairsubunion, conje

SEU626∧2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → in (setadjoin x (setadjoin y emptyset)) (powerset (binunion A B))))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
powerset: \$i → \$i thf(powerset_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
powersetI_1: \$o thf(powersetI1_type, type)
powersetI_1 = (∀*a*: \$i, *b*: \$i: ((⊆ @*b*@*a*) ⇒ (in@*b*@(powerset@*a*)))) thf(powersetI_1, definition)
binunion: \$i → \$i → \$i thf(binunion_type, type)
upairsubunion: \$o thf(upairsubunion_type, type)
upairsubunion = (∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*b*) ⇒ (⊆ @(setadjoin@xx@(setadjoin@xy@emptyset))

powersetI$_1$ ⇒ (upairsubunion ⇒ ∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ (in@(setadjoin@xx@(setadjoin

SEU627^2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → subset (setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset)) (powerset (binunion A B))))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i   thf(setadjoin_type, type)
powerset: \$i → \$i   thf(powerset_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
subsetI$_2$: \$o   thf(subsetI2_type, type)
subsetI$_2$ = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (⊆ @a@b)))   thf(subsetI$_2$, definition)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
singletoninpowunion: \$o   thf(singletoninpowunion_type, type)
singletoninpowunion = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ (in@(setadjoin@xx@emptyset)@(powerset@(binunion@a@b)))))
upairset2E: \$o   thf(upairset2E_type, type)
upairset2E = (∀xx: \$i, xy: \$i, xz: \$i: ((in@xz@(setadjoin@xx@(setadjoin@xy@emptyset))) ⇒ (xz = xx or xz = xy)))   thf(upairset2E, definition)
upairinpowunion: \$o   thf(upairinpowunion_type, type)
upairinpowunion = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ (in@(setadjoin@xx@(setadjoin@xy@emptyse
subsetI$_2$ ⇒ (singletoninpowunion ⇒ (upairset2E ⇒ (upairinpowunion ⇒ ∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ (⊆ @(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@

SEU628^2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → in (setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset)) (powerset (powerset (binunion A B)))))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i   thf(setadjoin_type, type)
powerset: \$i → \$i   thf(powerset_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
powersetI$_1$: \$o   thf(powersetI1_type, type)
powersetI$_1$ = (∀a: \$i, b: \$i: ((⊆ @b@a) ⇒ (in@b@(powerset@a))))   thf(powersetI$_1$, definition)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
ubforcartprodlem$_1$: \$o   thf(ubforcartprodlem1_type, type)
ubforcartprodlem$_1$ = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ (⊆ @(setadjoin@(setadjoin@xx@emptyset)
powersetI$_1$ ⇒ (ubforcartprodlem$_1$ ⇒ ∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ (in@(setadjoin@(setadjoin

SEU629^2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → in (kpair x y) (powerset (powerset (binunion A B)))))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i   thf(setadjoin_type, type)
powerset: \$i → \$i   thf(powerset_type, type)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
kpair = (λxx: \$i, xy: \$i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
ubforcartprodlem$_2$: \$o   thf(ubforcartprodlem2_type, type)
ubforcartprodlem$_2$ = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ (in@(setadjoin@(setadjoin@xx@emptyset)@
ubforcartprodlem$_2$ ⇒ ∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ (in@(kpair@xx@xy)@(powerset@(powerset

SEU630^2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → in (kpair x y) (cartprod A B)))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i   thf(setadjoin_type, type)
powerset: \$i → \$i   thf(powerset_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrI: \$o   thf(dsetconstrI_type, type)
dsetconstrI = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))))))
binunion: \$i → \$i → \$i   thf(binunion_type, type)

kpair: $i → $i → $i   thf(kpair_type, type)
kpair = (λxx: $i, xy: $i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
cartprod: $i → $i → $i   thf(cartprod_type, type)
cartprod = (λ*a*: $i, *b*: $i: (dsetconstr@(powerset@(powerset@(binunion@*a*@*b*)))@λxx: $i: ∃xy: $i: (in@xy@*a* and ∃xz: $i: (in@x
(kpair@xy@xz)))))   thf(cartprod, definition)
ubforcartprodlem$_3$: $o   thf(ubforcartprodlem3_type, type)
ubforcartprodlem$_3$ = (∀*a*: $i, *b*: $i, xx: $i: ((in@xx@*a*) ⇒ ∀xy: $i: ((in@xy@*b*) ⇒ (in@(kpair@xx@xy)@(powerset@(powerset@
dsetconstrI ⇒ (ubforcartprodlem$_3$ ⇒ ∀*a*: $i, *b*: $i, xx: $i: ((in@xx@*a*) ⇒ ∀xy: $i: ((in@xy@*b*) ⇒ (in@(kpair@xx@xy)@(car

SEU631∧2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! u:i.in u (cartprod A B) → (? x:i.in x A & (? y:i.in y B & u = kpair x y)))
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
powerset: $i → $i   thf(powerset_type, type)
dsetconstr: $i → ($i → $o) → $i   thf(dsetconstr_type, type)
dsetconstrER: $o   thf(dsetconstrER_type, type)
dsetconstrER = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@*a*@λxy: $i: (xphi@xy))) ⇒ (xphi@xx)))   thf(dsetcons
binunion: $i → $i → $i   thf(binunion_type, type)
kpair: $i → $i → $i   thf(kpair_type, type)
kpair = (λxx: $i, xy: $i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
cartprod: $i → $i → $i   thf(cartprod_type, type)
cartprod = (λ*a*: $i, *b*: $i: (dsetconstr@(powerset@(powerset@(binunion@*a*@*b*)))@λxx: $i: ∃xy: $i: (in@xy@*a* and ∃xz: $i: (in@x
(kpair@xy@xz)))))   thf(cartprod, definition)
dsetconstrER ⇒ ∀*a*: $i, *b*: $i, xu: $i: ((in@xu@(cartprod@*a*@*b*)) ⇒ ∃xx: $i: (in@xx@*a* and ∃xy: $i: (in@xy@*b* and xu =
(kpair@xx@xy))))   thf(cartprodmempair$_1$, conjecture)

SEU632∧2.p Ordered Pairs - Cartesian Products
(! A:i.! B:i.! u:i.in u (cartprod A B) → iskpair u)
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setunion: $i → $i   thf(setunion_type, type)
iskpair: $i → $o   thf(iskpair_type, type)
iskpair = (λ*a*: $i: ∃xx: $i: (in@xx@(setunion@*a*) and ∃xy: $i: (in@xy@(setunion@*a*) and *a* = (setadjoin@(setadjoin@xx@empt
kpair: $i → $i → $i   thf(kpair_type, type)
kpair = (λxx: $i, xy: $i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
kpairp: $o   thf(kpairp_type, type)
kpairp = (∀xx: $i, xy: $i: (iskpair@(kpair@xx@xy)))   thf(kpairp, definition)
cartprod: $i → $i → $i   thf(cartprod_type, type)
cartprodmempair$_1$: $o   thf(cartprodmempair1_type, type)
cartprodmempair$_1$ = (∀*a*: $i, *b*: $i, xu: $i: ((in@xu@(cartprod@*a*@*b*)) ⇒ ∃xx: $i: (in@xx@*a* and ∃xy: $i: (in@xy@*b* and xu =
(kpair@xx@xy)))))   thf(cartprodmempair$_1$, definition)
kpairp ⇒ (cartprodmempair$_1$ ⇒ ∀*a*: $i, *b*: $i, xu: $i: ((in@xu@(cartprod@*a*@*b*)) ⇒ (iskpair@xu)))   thf(cartprodmempair

SEU633∧2.p Ordered Pairs - Singletons
(! A:i.! x:i.in x (setunion A) → (? X:i.in X A & in x X))
in: $i → $i → $o   thf(in_type, type)
setunion: $i → $i   thf(setunion_type, type)
setunionE: $o   thf(setunionE_type, type)
setunionE = (∀*a*: $i, xx: $i: ((in@xx@(setunion@*a*)) ⇒ ∀xphi: $o: (∀*b*: $i: ((in@xx@*b*) ⇒ ((in@*b*@*a*) ⇒ xphi)) ⇒
xphi)))   thf(setunionE, definition)
setunionE ⇒ ∀*a*: $i, xx: $i: ((in@xx@(setunion@*a*)) ⇒ ∃*x*: $i: (in@*x*@*a* and in@xx@*x*))   thf(setunionE$_2$, conjecture)

SEU634∧2.p Ordered Pairs - Singletons
(! A:i.subset (setunion (setadjoin A emptyset)) A)
in: $i → $i → $o   thf(in_type, type)
emptyset: $i   thf(emptyset_type, type)
setadjoin: $i → $i → $i   thf(setadjoin_type, type)
setunion: $i → $i   thf(setunion_type, type)
uniqinunit: $o   thf(uniqinunit_type, type)

$\mathrm{uniqinunit} = (\forall xx{:}\ \$i, xy{:}\ \$i{:}\ ((in@xx@(setadjoin@xy@emptyset)) \Rightarrow xx = xy))$ thf(uniqinunit, definition)
$\subseteq{:}\ \$i \to \$i \to \$o$ thf(subset_type, type)
$subsetI_2{:}\ \$o$ thf(subsetI2_type, type)
$subsetI_2 = (\forall a{:}\ \$i, b{:}\ \$i{:}\ (\forall xx{:}\ \$i{:}\ ((in@xx@a) \Rightarrow (in@xx@b)) \Rightarrow (\subseteq @a@b)))$ thf(subsetI$_2$, definition)
$setunionE_2{:}\ \$o$ thf(setunionE2_type, type)
$setunionE_2 = (\forall a{:}\ \$i, xx{:}\ \$i{:}\ ((in@xx@(setunion@a)) \Rightarrow \exists x{:}\ \$i{:}\ (in@x@a \text{ and } in@xx@x)))$ thf(setunionE$_2$, definition)
$uniqinunit \Rightarrow (subsetI_2 \Rightarrow (setunionE_2 \Rightarrow \forall a{:}\ \$i{:}\ (\subseteq @(setunion@(setadjoin@a@emptyset))@a)))$ thf(setunionsingleton

SEU635^2.p Ordered Pairs - Singletons
(! A:i.subset A (setunion (setadjoin A emptyset)))
$in{:}\ \$i \to \$i \to \$o$ thf(in_type, type)
$emptyset{:}\ \$i$ thf(emptyset_type, type)
$setadjoin{:}\ \$i \to \$i \to \$i$ thf(setadjoin_type, type)
$setunion{:}\ \$i \to \$i$ thf(setunion_type, type)
$setadjoinIL{:}\ \$o$ thf(setadjoinIL_type, type)
$setadjoinIL = (\forall xx{:}\ \$i, xy{:}\ \$i{:}\ (in@xx@(setadjoin@xx@xy)))$ thf(setadjoinIL, definition)
$setunionI{:}\ \$o$ thf(setunionI_type, type)
$setunionI = (\forall a{:}\ \$i, xx{:}\ \$i, b{:}\ \$i{:}\ ((in@xx@b) \Rightarrow ((in@b@a) \Rightarrow (in@xx@(setunion@a)))))$ thf(setunionI, definition)
$\subseteq{:}\ \$i \to \$i \to \$o$ thf(subset_type, type)
$subsetI_2{:}\ \$o$ thf(subsetI2_type, type)
$subsetI_2 = (\forall a{:}\ \$i, b{:}\ \$i{:}\ (\forall xx{:}\ \$i{:}\ ((in@xx@a) \Rightarrow (in@xx@b)) \Rightarrow (\subseteq @a@b)))$ thf(subsetI$_2$, definition)
$setadjoinIL \Rightarrow (setunionI \Rightarrow (subsetI_2 \Rightarrow \forall a{:}\ \$i{:}\ (\subseteq @a@(setunion@(setadjoin@a@emptyset)))))$ thf(setunionsingleton

SEU636^2.p Ordered Pairs - Singletons
(! x:i.setunion (setadjoin x emptyset) = x)
$emptyset{:}\ \$i$ thf(emptyset_type, type)
$setadjoin{:}\ \$i \to \$i \to \$i$ thf(setadjoin_type, type)
$setunion{:}\ \$i \to \$i$ thf(setunion_type, type)
$\subseteq{:}\ \$i \to \$i \to \$o$ thf(subset_type, type)
$setextsub{:}\ \$o$ thf(setextsub_type, type)
$setextsub = (\forall a{:}\ \$i, b{:}\ \$i{:}\ ((\subseteq @a@b) \Rightarrow ((\subseteq @b@a) \Rightarrow a = b)))$ thf(setextsub, definition)
$setunionsingleton_1{:}\ \$o$ thf(setunionsingleton1_type, type)
$setunionsingleton_1 = (\forall a{:}\ \$i{:}\ (\subseteq @(setunion@(setadjoin@a@emptyset))@a))$ thf(setunionsingleton$_1$, definition)
$setunionsingleton_2{:}\ \$o$ thf(setunionsingleton2_type, type)
$setunionsingleton_2 = (\forall a{:}\ \$i{:}\ (\subseteq @a@(setunion@(setadjoin@a@emptyset))))$ thf(setunionsingleton$_2$, definition)
$setextsub \Rightarrow (setunionsingleton_1 \Rightarrow (setunionsingleton_2 \Rightarrow \forall xx{:}\ \$i{:}\ (setunion@(setadjoin@xx@emptyset)) = xx))$ thf(setunionsingleton, conjecture)

SEU637^2.p Ordered Pairs - Singletons
(! A:i.! phi:i>o.(! x:i.in x A → (! y:i.in y A → phi x → phi y → x = y)) → (? x:i.in x A & phi x) → singleton (dsetconstr A (^ x:i.phi x)))
$in{:}\ \$i \to \$i \to \$o$ thf(in_type, type)
$emptyset{:}\ \$i$ thf(emptyset_type, type)
$setadjoin{:}\ \$i \to \$i \to \$i$ thf(setadjoin_type, type)
$dsetconstr{:}\ \$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$dsetconstrI{:}\ \$o$ thf(dsetconstrI_type, type)
$dsetconstrI = (\forall a{:}\ \$i, xphi{:}\ \$i \to \$o, xx{:}\ \$i{:}\ ((in@xx@a) \Rightarrow ((xphi@xx) \Rightarrow (in@xx@(dsetconstr@a@\lambda xy{:}\ \$i{:}\ (xphi@xy))))))$
$dsetconstrEL{:}\ \$o$ thf(dsetconstrEL_type, type)
$dsetconstrEL = (\forall a{:}\ \$i, xphi{:}\ \$i \to \$o, xx{:}\ \$i{:}\ ((in@xx@(dsetconstr@a@\lambda xy{:}\ \$i{:}\ (xphi@xy))) \Rightarrow (in@xx@a)))$ thf(dsetcons
$dsetconstrER{:}\ \$o$ thf(dsetconstrER_type, type)
$dsetconstrER = (\forall a{:}\ \$i, xphi{:}\ \$i \to \$o, xx{:}\ \$i{:}\ ((in@xx@(dsetconstr@a@\lambda xy{:}\ \$i{:}\ (xphi@xy))) \Rightarrow (xphi@xx)))$ thf(dsetcons
$setext{:}\ \$o$ thf(setext_type, type)
$setext = (\forall a{:}\ \$i, b{:}\ \$i{:}\ (\forall xx{:}\ \$i{:}\ ((in@xx@a) \Rightarrow (in@xx@b)) \Rightarrow (\forall xx{:}\ \$i{:}\ ((in@xx@b) \Rightarrow (in@xx@a)) \Rightarrow a = b)))$ thf(setext, definition)
$uniqinunit{:}\ \$o$ thf(uniqinunit_type, type)
$uniqinunit = (\forall xx{:}\ \$i, xy{:}\ \$i{:}\ ((in@xx@(setadjoin@xy@emptyset)) \Rightarrow xx = xy))$ thf(uniqinunit, definition)
$eqinunit{:}\ \$o$ thf(eqinunit_type, type)
$eqinunit = (\forall xx{:}\ \$i, xy{:}\ \$i{:}\ (xx = xy \Rightarrow (in@xx@(setadjoin@xy@emptyset))))$ thf(eqinunit, definition)
$singleton{:}\ \$i \to \$o$ thf(singleton_type, type)
$singleton = (\lambda a{:}\ \$i{:}\ \exists xx{:}\ \$i{:}\ (in@xx@a \text{ and } a = (setadjoin@xx@emptyset)))$ thf(singleton, definition)

dsetconstrI ⇒ (dsetconstrEL ⇒ (dsetconstrER ⇒ (setext ⇒ (uniqinunit ⇒ (eqinunit ⇒ ∀a: $i, xphi: $i → $o: (∀xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ ((xphi@xx) ⇒ ((xphi@xy) ⇒ xx = xy)))) ⇒ (∃xx: $i: (in@xx@a and xphi@xx) ⇒ (singleton@(dsetconstr@a@λxx: $i: (xphi@xx))))))))) thf(singletonprop, conjectur

SEU638∧2.p Ordered Pairs - Singletons
(! A:i.! phi:i>o.ex1 A (∧ x:i.phi x) → (? x:i.in x A & phi x))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrEL: $o thf(dsetconstrEL_type, type)
dsetconstrEL = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (in@xx@a))) thf(dsetcons
dsetconstrER: $o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex1: $i → ($i → $o) → $o thf(ex1_type, type)
ex1 = (λa: $i, xphi: $i → $o: (singleton@(dsetconstr@a@λxx: $i: (xphi@xx)))) thf(ex1, definition)
dsetconstrEL ⇒ (dsetconstrER ⇒ ∀a: $i, xphi: $i → $o: ((ex1@a@λxx: $i: (xphi@xx)) ⇒ ∃xx: $i: (in@xx@a and xphi@xx)

SEU639∧2.p Ordered Pairs - Singletons
(! A:i.! phi:i>o.! x:i.in x A → phi x → (! y:i.in y A → phi y → y = x) → ex1 A (∧ y:i.phi y))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrI: $o thf(dsetconstrI_type, type)
dsetconstrI = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))))))
dsetconstrEL: $o thf(dsetconstrEL_type, type)
dsetconstrEL = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (in@xx@a))) thf(dsetcons
dsetconstrER: $o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
setext: $o thf(setext_type, type)
setext = (∀a: $i, b: $i: (∀xx: $i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (∀xx: $i: ((in@xx@b) ⇒ (in@xx@a)) ⇒ a = b))) thf(setext, definition)
uniqinunit: $o thf(uniqinunit_type, type)
uniqinunit = (∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
eqinunit: $o thf(eqinunit_type, type)
eqinunit = (∀xx: $i, xy: $i: (xx = xy ⇒ (in@xx@(setadjoin@xy@emptyset)))) thf(eqinunit, definition)
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex1: $i → ($i → $o) → $o thf(ex1_type, type)
ex1 = (λa: $i, xphi: $i → $o: (singleton@(dsetconstr@a@λxx: $i: (xphi@xx)))) thf(ex1, definition)
dsetconstrI ⇒ (dsetconstrEL ⇒ (dsetconstrER ⇒ (setext ⇒ (uniqinunit ⇒ (eqinunit ⇒ ∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (∀xy: $i: ((in@xy@a) ⇒ ((xphi@xy) ⇒ xy = xx)) ⇒ (ex1@a@λxy: $i: (xphi@xy)

SEU640∧2.p Ordered Pairs - Singletons
(! A:i.! phi:i>o.(! x:i.in x A → (! y:i.in y A → phi x → phi y → x = y)) → (? x:i.in x A & phi x) → ex1 A (∧ x:i.phi x))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
singletonprop: $o thf(singletonprop_type, type)
singletonprop = (∀a: $i, xphi: $i → $o: (∀xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ ((xphi@xx) ⇒ ((xphi@xy) ⇒ xx = xy)))) ⇒ (∃xx: $i: (in@xx@a and xphi@xx) ⇒ (singleton@(dsetconstr@a@λxx: $i: (xphi@xx))))))
ex1: $i → ($i → $o) → $o thf(ex1_type, type)
ex1 = (λa: $i, xphi: $i → $o: (singleton@(dsetconstr@a@λxx: $i: (xphi@xx)))) thf(ex1, definition)

singletonprop ⇒ ∀a: \$i, xphi: \$i → \$o: (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((xphi@xx) ⇒ ((xphi@xy) ⇒ xx = xy)))) ⇒ (∃xx: \$i: (in@xx@a and xphi@xx) ⇒ (ex$_1$@a@λxx: \$i: (xphi@xx)))) thf(ex1I$_2$, conjecture

SEU641∧2.p Ordered Pairs - Singletons
(! x:i.! y:i.setadjoin x emptyset = setadjoin y emptyset → x = y)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setadjoinIL: \$o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: \$i, xy: \$i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
uniqinunit: \$o thf(uniqinunit_type, type)
uniqinunit = (∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
setadjoinIL ⇒ (uniqinunit ⇒ ∀xx: \$i, xy: \$i: ((setadjoin@xx@emptyset) = (setadjoin@xy@emptyset) ⇒ xx = xy)) thf(singletonsuniq, conjecture)

SEU642∧2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.! z:i.in (setadjoin z emptyset) (setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset)) → x = z)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setadjoinIL: \$o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: \$i, xy: \$i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
uniqinunit: \$o thf(uniqinunit_type, type)
uniqinunit = (∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
upairset2E: \$o thf(upairset2E_type, type)
upairset2E = (∀xx: \$i, xy: \$i, xz: \$i: ((in@xz@(setadjoin@xx@(setadjoin@xy@emptyset))) ⇒ (xz = xx or xz = xy))) thf(upairset2E, definition)
singletonsuniq: \$o thf(singletonsuniq_type, type)
singletonsuniq = (∀xx: \$i, xy: \$i: ((setadjoin@xx@emptyset) = (setadjoin@xy@emptyset) ⇒ xx = xy)) thf(singletonsuni
setadjoinIL ⇒ (uniqinunit ⇒ (upairset2E ⇒ (singletonsuniq ⇒ ∀xx: \$i, xy: \$i, xz: \$i: ((in@(setadjoin@xz@emptyset)@(se
xx = xz)))) thf(setukpairinjL$_1$, conjecture)

SEU643∧2.p Ordered Pairs - Properties of Pairs
(! u:i.iskpair u → singleton (dsetconstr (setunion u) (∧ x:i.in (setadjoin x emptyset) u)))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setunion: \$i → \$i thf(setunion_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
setadjoinIL: \$o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: \$i, xy: \$i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
iskpair: \$i → \$o thf(iskpair_type, type)
iskpair = (λa: \$i: ∃xx: \$i: (in@xx@(setunion@a) and ∃xy: \$i: (in@xy@(setunion@a) and a = (setadjoin@(setadjoin@xx@empt
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex$_1$: \$i → (\$i → \$o) → \$o thf(ex1_type, type)
ex$_1$ = (λa: \$i, xphi: \$i → \$o: (singleton@(dsetconstr@a@λxx: \$i: (xphi@xx)))) thf(ex$_1$, definition)
ex1I: \$o thf(ex1I_type, type)
ex1I = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (∀xy: \$i: ((in@xy@a) ⇒ ((xphi@xy) ⇒ xy = xx)) ⇒ (ex$_1$@a@λxy: \$i: (xphi@xy)))))) thf(ex1I, definition)
setukpairinjL$_1$: \$o thf(setukpairinjL1_type, type)
setukpairinjL$_1$ = (∀xx: \$i, xy: \$i, xz: \$i: ((in@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(s
xx = xz)) thf(setukpairinjL$_1$, definition)
setadjoinIL ⇒ (ex1I ⇒ (setukpairinjL$_1$ ⇒ ∀xu: \$i: ((iskpair@xu) ⇒ (singleton@(dsetconstr@(setunion@xu)@λxx: \$i: (in@

SEU644∧2.p Ordered Pairs - Properties of Pairs
(! X:i.singleton X → in (setunion X) X)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)

setunion: \$i → \$i thf(setunion_type, type)
eqinunit: \$o thf(eqinunit_type, type)
eqinunit = (∀xx: \$i, xy: \$i: (xx = xy ⇒ (in@xx@(setadjoin@xy@emptyset)))) thf(eqinunit, definition)
setunionsingleton: \$o thf(setunionsingleton_type, type)
setunionsingleton = (∀xx: \$i: (setunion@(setadjoin@xx@emptyset)) = xx) thf(setunionsingleton, definition)
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
eqinunit ⇒ (setunionsingleton ⇒ ∀x: \$i: ((singleton@x) ⇒ (in@(setunion@x)@x))) thf(theprop, conjecture)

SEU645∧2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.kfst (kpair x y) = x)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setunion: \$i → \$i thf(setunion_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrER: \$o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
iskpair: \$i → \$o thf(iskpair_type, type)
iskpair = (λa: \$i: ∃xx: \$i: (in@xx@(setunion@a) and ∃xy: \$i: (in@xy@(setunion@a) and a = (setadjoin@(setadjoin@xx@empt
kpair: \$i → \$i → \$i thf(kpair_type, type)
kpair = (λxx: \$i, xy: \$i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
kpairp: \$o thf(kpairp_type, type)
kpairp = (∀xx: \$i, xy: \$i: (iskpair@(kpair@xx@xy))) thf(kpairp, definition)
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
setukpairinjL_1: \$o thf(setukpairinjL1_type, type)
setukpairinjL_1 = (∀xx: \$i, xy: \$i, xz: \$i: ((in@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(s
xx = xz)) thf(setukpairinjL_1, definition)
kfstsingleton: \$o thf(kfstsingleton_type, type)
kfstsingleton = (∀xu: \$i: ((iskpair@xu) ⇒ (singleton@(dsetconstr@(setunion@xu)@λxx: \$i: (in@(setadjoin@xx@emptyset)@x
theprop: \$o thf(theprop_type, type)
theprop = (∀x: \$i: ((singleton@x) ⇒ (in@(setunion@x)@x))) thf(theprop, definition)
kfst: \$i → \$i thf(kfst_type, type)
kfst = (λxu: \$i: (setunion@(dsetconstr@(setunion@xu)@λxx: \$i: (in@(setadjoin@xx@emptyset)@xu)))) thf(kfst, definitio
dsetconstrER ⇒ (kpairp ⇒ (setukpairinjL_1 ⇒ (kfstsingleton ⇒ (theprop ⇒ ∀xx: \$i, xy: \$i: (kfst@(kpair@xx@xy)) =
xx)))) thf(kfstpairEq, conjecture)

SEU646∧2.p Ordered Pairs - Properties of Pairs
(! A:i.! B:i.! u:i.in u (cartprod A B) → in (kfst u) A)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
kpair = (λxx: \$i, xy: \$i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
cartprodmempair$_1$: \$o thf(cartprodmempair1_type, type)
cartprodmempair$_1$ = (∀a: \$i, b: \$i, xu: \$i: ((in@xu@(cartprod@a@b)) ⇒ ∃xx: \$i: (in@xx@a and ∃xy: \$i: (in@xy@b and xu =
(kpair@xx@xy))))) thf(cartprodmempair$_1$, definition)
kfst: \$i → \$i thf(kfst_type, type)
kfstpairEq: \$o thf(kfstpairEq_type, type)
kfstpairEq = (∀xx: \$i, xy: \$i: (kfst@(kpair@xx@xy)) = xx) thf(kfstpairEq, definition)
cartprodmempair$_1$ ⇒ (kfstpairEq ⇒ ∀a: \$i, b: \$i, xu: \$i: ((in@xu@(cartprod@a@b)) ⇒ (in@(kfst@xu)@a))) thf(cartpro

SEU647∧2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.! z:i.! u:i.setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset) =
setadjoin (setadjoin z emptyset) (setadjoin (setadjoin z (setadjoin u emptyset)) emptyset) → x = z)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setadjoinIL: \$o thf(setadjoinIL_type, type)

setadjoinIL = (∀xx: \$i, xy: \$i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
setukpairinjL$_1$: \$o thf(setukpairinjL1_type, type)
setukpairinjL$_1$ = (∀xx: \$i, xy: \$i, xz: \$i: ((in@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(s
xx = xz)) thf(setukpairinjL$_1$, definition)
setadjoinIL ⇒ (setukpairinjL$_1$ ⇒ ∀xx: \$i, xy: \$i, xz: \$i, xu: \$i: ((setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin
(setadjoin@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xz@(setadjoin@xu@emptyset))@emptyset)) ⇒ xx =
xz)) thf(setukpairinjL$_2$, conjecture)

SEU648∧2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.! z:i.! u:i.kpair x y = kpair z u → x = z)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
kpair = (λxx: \$i, xy: \$i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
setukpairinjL$_2$: \$o thf(setukpairinjL2_type, type)
setukpairinjL$_2$ = (∀xx: \$i, xy: \$i, xz: \$i, xu: \$i: ((setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@
(setadjoin@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xz@(setadjoin@xu@emptyset))@emptyset)) ⇒ xx =
xz)) thf(setukpairinjL$_2$, definition)
setukpairinjL$_2$ ⇒ ∀xx: \$i, xy: \$i, xz: \$i, xu: \$i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xx = xz) thf(setukpairinjL, conjectu

SEU649∧2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.x = y → setadjoin x (setadjoin y emptyset) = setadjoin x emptyset)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setext: \$o thf(setext_type, type)
setext = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (∀xx: \$i: ((in@xx@b) ⇒ (in@xx@a)) ⇒ a =
b))) thf(setext, definition)
setadjoinIL: \$o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: \$i, xy: \$i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
uniqinunit: \$o thf(uniqinunit_type, type)
uniqinunit = (∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
eqinunit: \$o thf(eqinunit_type, type)
eqinunit = (∀xx: \$i, xy: \$i: (xx = xy ⇒ (in@xx@(setadjoin@xy@emptyset)))) thf(eqinunit, definition)
upairset2E: \$o thf(upairset2E_type, type)
upairset2E = (∀xx: \$i, xy: \$i, xz: \$i: ((in@xz@(setadjoin@xx@(setadjoin@xy@emptyset))) ⇒ (xz = xx or xz =
xy))) thf(upairset2E, definition)
setext ⇒ (setadjoinIL ⇒ (uniqinunit ⇒ (eqinunit ⇒ (upairset2E ⇒ ∀xx: \$i, xy: \$i: (xx = xy ⇒
(setadjoin@xx@(setadjoin@xy@emptyset)) = (setadjoin@xx@emptyset)))))) thf(setukpairinjR$_{11}$, conjecture)

SEU650∧2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.x = y → setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset) =
setadjoin (setadjoin x emptyset) emptyset)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setukpairinjR$_{11}$: \$o thf(setukpairinjR11_type, type)
setukpairinjR$_{11}$ = (∀xx: \$i, xy: \$i: (xx = xy ⇒ (setadjoin@xx@(setadjoin@xy@emptyset)) = (setadjoin@xx@emptyset)))
setukpairinjR$_{11}$ ⇒ ∀xx: \$i, xy: \$i: (xx = xy ⇒ (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin
(setadjoin@(setadjoin@xx@emptyset)@emptyset)) thf(setukpairinjR$_{12}$, conjecture)

SEU651∧2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.! z:i.! u:i.setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset) =
setadjoin (setadjoin z emptyset) (setadjoin (setadjoin z (setadjoin u emptyset)) emptyset) → z = u → y = u)
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
uniqinunit: \$o thf(uniqinunit_type, type)
uniqinunit = (∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
secondinupair: \$o thf(secondinupair_type, type)
secondinupair = (∀xx: \$i, xy: \$i: (in@xy@(setadjoin@xx@(setadjoin@xy@emptyset)))) thf(secondinupair, definition)
setukpairinjR$_{12}$: \$o thf(setukpairinjR12_type, type)

setukpairinjR$_{12}$ = (∀xx: $i, xy: $i: (xx = xy ⇒ (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin
(setadjoin@(setadjoin@xx@emptyset)@emptyset))) thf(setukpairinjR$_{12}$, definition)
uniqinunit ⇒ (secondinupair ⇒ (setukpairinjR$_{12}$ ⇒ ∀xx: $i, xy: $i, xz: $i, xu: $i: ((setadjoin@(setadjoin@xx@emptyset)@(s
(setadjoin@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xz@(setadjoin@xu@emptyset))@emptyset)) ⇒ (xz =
xu ⇒ xy = xu)))) thf(setukpairinjR$_1$, conjecture)

SEU652^2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.! z:i.setadjoin x (setadjoin y emptyset) = setadjoin z emptyset → x = y)
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setadjoinIL: $o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
uniqinunit: $o thf(uniqinunit_type, type)
uniqinunit = (∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
secondinupair: $o thf(secondinupair_type, type)
secondinupair = (∀xx: $i, xy: $i: (in@xy@(setadjoin@xx@(setadjoin@xy@emptyset)))) thf(secondinupair, definition)
setadjoinIL ⇒ (uniqinunit ⇒ (secondinupair ⇒ ∀xx: $i, xy: $i, xz: $i: ((setadjoin@xx@(setadjoin@xy@emptyset)) =
(setadjoin@xz@emptyset) ⇒ xx = xy))) thf(upairequniteq, conjecture)

SEU653^2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.! z:i.! u:i.setadjoin (setadjoin x emptyset) (setadjoin (setadjoin x (setadjoin y emptyset)) emptyset) =
setadjoin (setadjoin z emptyset) (setadjoin (setadjoin z (setadjoin u emptyset)) emptyset) → y = u)
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
secondinupair: $o thf(secondinupair_type, type)
secondinupair = (∀xx: $i, xy: $i: (in@xy@(setadjoin@xx@(setadjoin@xy@emptyset)))) thf(secondinupair, definition)
upairset2E: $o thf(upairset2E_type, type)
upairset2E = (∀xx: $i, xy: $i, xz: $i: ((in@xz@(setadjoin@xx@(setadjoin@xy@emptyset))) ⇒ (xz = xx or xz =
xy))) thf(upairset2E, definition)
setukpairinjL$_2$: $o thf(setukpairinjL2_type, type)
setukpairinjL$_2$ = (∀xx: $i, xy: $i, xz: $i, xu: $i: ((setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin
(setadjoin@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xz@(setadjoin@xu@emptyset))@emptyset)) ⇒ xx =
xz)) thf(setukpairinjL$_2$, definition)
setukpairinjR$_1$: $o thf(setukpairinjR1_type, type)
setukpairinjR$_1$ = (∀xx: $i, xy: $i, xz: $i, xu: $i: ((setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin
(setadjoin@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xz@(setadjoin@xu@emptyset))@emptyset)) ⇒ (xz =
xu ⇒ xy = xu))) thf(setukpairinjR$_1$, definition)
upairequniteq: $o thf(upairequniteq_type, type)
upairequniteq = (∀xx: $i, xy: $i, xz: $i: ((setadjoin@xx@(setadjoin@xy@emptyset)) = (setadjoin@xz@emptyset) ⇒
xx = xy)) thf(upairequniteq, definition)
secondinupair ⇒ (upairset2E ⇒ (setukpairinjL$_2$ ⇒ (setukpairinjR$_1$ ⇒ (upairequniteq ⇒ ∀xx: $i, xy: $i, xz: $i, xu: $i: ((se
(setadjoin@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xz@(setadjoin@xu@emptyset))@emptyset)) ⇒ xy =
xu))))) thf(setukpairinjR$_2$, conjecture)

SEU654^2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.! z:i.! u:i.kpair x y = kpair z u → y = u)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
kpair: $i → $i → $i thf(kpair_type, type)
kpair = (λxx: $i, xy: $i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
setukpairinjR$_2$: $o thf(setukpairinjR2_type, type)
setukpairinjR$_2$ = (∀xx: $i, xy: $i, xz: $i, xu: $i: ((setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin
(setadjoin@(setadjoin@xz@emptyset)@(setadjoin@(setadjoin@xz@(setadjoin@xu@emptyset))@emptyset)) ⇒ xy =
xu)) thf(setukpairinjR$_2$, definition)
setukpairinjR$_2$ ⇒ ∀xx: $i, xy: $i, xz: $i, xu: $i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xy = xu) thf(setukpairinjR, conject

SEU655^2.p Ordered Pairs - Properties of Pairs
(! u:i.iskpair u → singleton (dsetconstr (setunion u) (^ x:i.u = kpair (kfst u) x)))
in: $i → $i → $o thf(in_type, type)

emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
iskpair: $i → $o thf(iskpair_type, type)
iskpair = (λ*a*: $i: ∃xx: $i: (in@xx@(setunion@*a*) and ∃xy: $i: (in@xy@(setunion@*a*) and *a* = (setadjoin@(setadjoin@xx@empt
kpair: $i → $i → $i thf(kpair_type, type)
kpair = (λxx: $i, xy: $i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
singleton: $i → $o thf(singleton_type, type)
singleton = (λ*a*: $i: ∃xx: $i: (in@xx@*a* and *a* = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex$_1$: $i → ($i → $o) → $o thf(ex1_type, type)
ex$_1$ = (λ*a*: $i, xphi: $i → $o: (singleton@(dsetconstr@*a*@λxx: $i: (xphi@xx)))) thf(ex$_1$, definition)
ex1I: $o thf(ex1I_type, type)
ex1I = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@*a*) ⇒ ((xphi@xx) ⇒ (∀xy: $i: ((in@xy@*a*) ⇒ ((xphi@xy) ⇒ xy = xx)) ⇒ (ex$_1$@*a*@λxy: $i: (xphi@xy)))))) thf(ex1I, definition)
kfst: $i → $i thf(kfst_type, type)
kfstpairEq: $o thf(kfstpairEq_type, type)
kfstpairEq = (∀xx: $i, xy: $i: (kfst@(kpair@xx@xy)) = xx) thf(kfstpairEq, definition)
setukpairinjR: $o thf(setukpairinjR_type, type)
setukpairinjR = (∀xx: $i, xy: $i, xz: $i, xu: $i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xy = xu)) thf(setukpairinjR, definitio
ex1I ⇒ (kfstpairEq ⇒ (setukpairinjR ⇒ ∀xu: $i: ((iskpair@xu) ⇒ (singleton@(dsetconstr@(setunion@xu)@λxx: $i: xu = (kpair@(kfst@xu)@xx)))))) thf(ksndsingleton, conjecture)

SEU656^2.p Ordered Pairs - Properties of Pairs
(! x:i.! y:i.ksnd (kpair x y) = y)
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrER: $o thf(dsetconstrER_type, type)
dsetconstrER = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@*a*@λxy: $i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
iskpair: $i → $o thf(iskpair_type, type)
iskpair = (λ*a*: $i: ∃xx: $i: (in@xx@(setunion@*a*) and ∃xy: $i: (in@xy@(setunion@*a*) and *a* = (setadjoin@(setadjoin@xx@empt
kpair: $i → $i → $i thf(kpair_type, type)
kpair = (λxx: $i, xy: $i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
kpairp: $o thf(kpairp_type, type)
kpairp = (∀xx: $i, xy: $i: (iskpair@(kpair@xx@xy))) thf(kpairp, definition)
singleton: $i → $o thf(singleton_type, type)
singleton = (λ*a*: $i: ∃xx: $i: (in@xx@*a* and *a* = (setadjoin@xx@emptyset))) thf(singleton, definition)
theprop: $o thf(theprop_type, type)
theprop = (∀*x*: $i: ((singleton@*x*) ⇒ (in@(setunion@*x*)@*x*))) thf(theprop, definition)
kfst: $i → $i thf(kfst_type, type)
setukpairinjR: $o thf(setukpairinjR_type, type)
setukpairinjR = (∀xx: $i, xy: $i, xz: $i, xu: $i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xy = xu)) thf(setukpairinjR, definitio
ksndsingleton: $o thf(ksndsingleton_type, type)
ksndsingleton = (∀xu: $i: ((iskpair@xu) ⇒ (singleton@(dsetconstr@(setunion@xu)@λxx: $i: xu = (kpair@(kfst@xu)@xx)))))
ksnd: $i → $i thf(ksnd_type, type)
ksnd = (λxu: $i: (setunion@(dsetconstr@(setunion@xu)@λxx: $i: xu = (kpair@(kfst@xu)@xx)))) thf(ksnd, definition)
dsetconstrER ⇒ (kpairp ⇒ (theprop ⇒ (setukpairinjR ⇒ (ksndsingleton ⇒ ∀xx: $i, xy: $i: (ksnd@(kpair@xx@xy)) = xy)))) thf(ksndpairEq, conjecture)

SEU657^2.p Ordered Pairs - Properties of Pairs
(! u:i.iskpair u → kpair (kfst u) (ksnd u) = u)
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
iskpair: $i → $o thf(iskpair_type, type)
iskpair = (λ*a*: $i: ∃xx: $i: (in@xx@(setunion@*a*) and ∃xy: $i: (in@xy@(setunion@*a*) and *a* = (setadjoin@(setadjoin@xx@empt

kpair: \$i → \$i → \$i thf(kpair_type, type)
kpair = (λxx: \$i, xy: \$i: (setadjoin@(setadjoin@xx@emptyset)@(setadjoin@(setadjoin@xx@(setadjoin@xy@emptyset))@empty
kfst: \$i → \$i thf(kfst_type, type)
kfstpairEq: \$o thf(kfstpairEq_type, type)
kfstpairEq = (∀xx: \$i, xy: \$i: (kfst@(kpair@xx@xy)) = xx) thf(kfstpairEq, definition)
ksnd: \$i → \$i thf(ksnd_type, type)
ksndpairEq: \$o thf(ksndpairEq_type, type)
ksndpairEq = (∀xx: \$i, xy: \$i: (ksnd@(kpair@xx@xy)) = xy) thf(ksndpairEq, definition)
kfstpairEq ⇒ (ksndpairEq ⇒ ∀xu: \$i: ((iskpair@xu) ⇒ (kpair@(kfst@xu)@(ksnd@xu)) = xu)) thf(kpairsurjEq, conject

SEU658∧2.p Ordered Pairs - Properties of Pairs
(! A:i.! B:i.! u:i.in u (cartprod A B) → in (ksnd u) B)
in: \$i → \$i → \$o thf(in_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
cartprodmempair_1: \$o thf(cartprodmempair1_type, type)
cartprodmempair_1 = (∀*a*: \$i, *b*: \$i, xu: \$i: ((in@xu@(cartprod@*a*@*b*)) ⇒ ∃xx: \$i: (in@xx@*a* and ∃xy: \$i: (in@xy@*b* and xu = (kpair@xx@xy))))) thf(cartprodmempair_1, definition)
ksnd: \$i → \$i thf(ksnd_type, type)
ksndpairEq: \$o thf(ksndpairEq_type, type)
ksndpairEq = (∀xx: \$i, xy: \$i: (ksnd@(kpair@xx@xy)) = xy) thf(ksndpairEq, definition)
cartprodmempair_1 ⇒ (ksndpairEq ⇒ ∀*a*: \$i, *b*: \$i, xu: \$i: ((in@xu@(cartprod@*a*@*b*)) ⇒ (in@(ksnd@xu)@*b*))) thf(cartpr

SEU659∧2.p Ordered Pairs - Properties of Pairs
(! A:i.! B:i.! x:i.! y:i.in (kpair x y) (cartprod A B) → in x A)
in: \$i → \$i → \$o thf(in_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
cartprodmempair_1: \$o thf(cartprodmempair1_type, type)
cartprodmempair_1 = (∀*a*: \$i, *b*: \$i, xu: \$i: ((in@xu@(cartprod@*a*@*b*)) ⇒ ∃xx: \$i: (in@xx@*a* and ∃xy: \$i: (in@xy@*b* and xu = (kpair@xx@xy))))) thf(cartprodmempair_1, definition)
setukpairinjL: \$o thf(setukpairinjL_type, type)
setukpairinjL = (∀xx: \$i, xy: \$i, xz: \$i, xu: \$i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xx = xz)) thf(setukpairinjL, definitio
cartprodmempair_1 ⇒ (setukpairinjL ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i, xy: \$i: ((in@(kpair@xx@xy)@(cartprod@*a*@*b*)) ⇒ (in@xx@*a*))) thf(cartprodpairmemEL, conjecture)

SEU660∧2.p Ordered Pairs - Properties of Pairs
(! A:i.! B:i.! x:i.! y:i.in (kpair x y) (cartprod A B) → in y B)
in: \$i → \$i → \$o thf(in_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
cartprodmempair_1: \$o thf(cartprodmempair1_type, type)
cartprodmempair_1 = (∀*a*: \$i, *b*: \$i, xu: \$i: ((in@xu@(cartprod@*a*@*b*)) ⇒ ∃xx: \$i: (in@xx@*a* and ∃xy: \$i: (in@xy@*b* and xu = (kpair@xx@xy))))) thf(cartprodmempair_1, definition)
setukpairinjR: \$o thf(setukpairinjR_type, type)
setukpairinjR = (∀xx: \$i, xy: \$i, xz: \$i, xu: \$i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xy = xu)) thf(setukpairinjR, definitio
cartprodmempair_1 ⇒ (setukpairinjR ⇒ ∀*a*: \$i, *b*: \$i, xx: \$i, xy: \$i: ((in@(kpair@xx@xy)@(cartprod@*a*@*b*)) ⇒ (in@xy@*b*))) thf(cartprodpairmemER, conjecture)

SEU661∧2.p Ordered Pairs - Properties of Pairs
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → kpair x y = kpair x y))
in: \$i → \$i → \$o thf(in, type)
kpair: \$i → \$i → \$i thf(kpair, type)
∀*a*: \$i, *b*: \$i, xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*b*) ⇒ (kpair@xx@xy) = (kpair@xx@xy))) thf(cartprodmempaircEq,

SEU662∧2.p Ordered Pairs - Properties of Pairs
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → kfst (kpair x y) = x))
in: \$i → \$i → \$o thf(in_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
kfst: \$i → \$i thf(kfst_type, type)
kfstpairEq: \$o thf(kfstpairEq_type, type)
kfstpairEq = (∀xx: \$i, xy: \$i: (kfst@(kpair@xx@xy)) = xx) thf(kfstpairEq, definition)

cartprodmempaircEq: $o   thf(cartprodmempaircEq_type, type)
cartprodmempaircEq = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ (kpair@xx@xy) = (kpair@xx@xy))))
kfstpairEq ⇒ (cartprodmempaircEq ⇒ ∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ (kfst@(kpair@xx@xy)) = xx)))   thf(cartprodfstpairEq, conjecture)

SEU663∧2.p Ordered Pairs - Properties of Pairs
(! A:i.! B:i.! x:i.in x A → (! y:i.in y B → ksnd (kpair x y) = y))
in: $i → $i → $o   thf(in_type, type)
kpair: $i → $i → $i   thf(kpair_type, type)
ksnd: $i → $i   thf(ksnd_type, type)
ksndpairEq: $o   thf(ksndpairEq_type, type)
ksndpairEq = (∀xx: $i, xy: $i: (ksnd@(kpair@xx@xy)) = xy)   thf(ksndpairEq, definition)
cartprodmempaircEq: $o   thf(cartprodmempaircEq_type, type)
cartprodmempaircEq = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ (kpair@xx@xy) = (kpair@xx@xy))))
ksndpairEq ⇒ (cartprodmempaircEq ⇒ ∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ (ksnd@(kpair@xx@xy)) = xy)))   thf(cartprodsndpairEq, conjecture)

SEU664∧2.p Ordered Pairs - Properties of Pairs
(! A:i.! B:i.! u:i.in u (cartprod A B) → kpair (kfst u) (ksnd u) = u)
in: $i → $i → $o   thf(in_type, type)
kpair: $i → $i → $i   thf(kpair_type, type)
cartprod: $i → $i → $i   thf(cartprod_type, type)
cartprodmempair$_1$: $o   thf(cartprodmempair1_type, type)
cartprodmempair$_1$ = (∀a: $i, b: $i, xu: $i: ((in@xu@(cartprod@a@b)) ⇒ ∃xx: $i: (in@xx@a and ∃xy: $i: (in@xy@b and xu = (kpair@xx@xy)))))   thf(cartprodmempair$_1$, definition)
kfst: $i → $i   thf(kfst_type, type)
ksnd: $i → $i   thf(ksnd_type, type)
cartprodfstpairEq: $o   thf(cartprodfstpairEq_type, type)
cartprodfstpairEq = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ (kfst@(kpair@xx@xy)) = xx)))   thf(cartprodfstpairEq, definition)
cartprodsndpairEq: $o   thf(cartprodsndpairEq_type, type)
cartprodsndpairEq = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ (ksnd@(kpair@xx@xy)) = xy)))   thf(cartprodsndpairEq, definition)
cartprodmempair$_1$ ⇒ (cartprodfstpairEq ⇒ (cartprodsndpairEq ⇒ ∀a: $i, b: $i, xu: $i: ((in@xu@(cartprod@a@b)) ⇒ (kpair@(kfst@xu)@(ksnd@xu)) = xu)))   thf(cartprodpairsurjEq, conjecture)

SEU665∧2.p Ordered Pairs - Sets of Pairs
(! A:i.! B:i.! phi:i>(i>o).! x:i.in x A → (! y:i.in y B → phi x y → in (kpair x y) (dpsetconstr A B (∧ z,u:i.phi z u))))
in: $i → $i → $o   thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i   thf(dsetconstr_type, type)
dsetconstrI: $o   thf(dsetconstrI_type, type)
dsetconstrI = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))))))
setext: $o   thf(setext_type, type)
setext = (∀a: $i, b: $i: (∀xx: $i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (∀xx: $i: ((in@xx@b) ⇒ (in@xx@a)) ⇒ a = b)))   thf(setext, definition)
kpair: $i → $i → $i   thf(kpair_type, type)
cartprod: $i → $i → $i   thf(cartprod_type, type)
cartprodpairin: $o   thf(cartprodpairin_type, type)
cartprodpairin = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ (in@(kpair@xx@xy)@(cartprod@a@b)))))   t
dpsetconstr: $i → $i → ($i → $i → $o) → $i   thf(dpsetconstr_type, type)
dpsetconstr = (λa: $i, b: $i, xphi: $i → $i → $o: (dsetconstr@(cartprod@a@b)@λxu: $i: ∃xx: $i: (in@xx@a and ∃xy: $i: (in@xy (kpair@xx@xy)))))   thf(dpsetconstr, definition)
dsetconstrI ⇒ (setext ⇒ (cartprodpairin ⇒ ∀a: $i, b: $i, xphi: $i → $i → $o, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ ((xphi@xx@xy) ⇒ (in@(kpair@xx@xy)@(dpsetconstr@a@b@λxz: $i, xu: $i: (xphi@xz@xu))))))))

SEU666∧2.p Ordered Pairs - Sets of Pairs
(! A:i.! B:i.! phi:i>(i>o).subset (dpsetconstr A B (∧ x,y:i.phi x y)) (cartprod A B))
in: $i → $i → $o   thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i   thf(dsetconstr_type, type)
dsetconstrEL: $o   thf(dsetconstrEL_type, type)

dsetconstrEL = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (in@xx@a))) thf(dsetcons
⊆ : $i → $i → $o thf(subset_type, type)
subsetI_2: $o thf(subsetI2_type, type)
subsetI_2 = (∀a: $i, b: $i: (∀xx: $i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (⊆ @a@b))) thf(subsetI_2, definition)
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
dpsetconstr: $i → $i → ($i → $i → $o) → $i thf(dpsetconstr_type, type)
dpsetconstr = (λa: $i, b: $i, xphi: $i → $i → $o: (dsetconstr@(cartprod@a@b)@λxu: $i: ∃xx: $i: (in@xx@a and ∃xy: $i: (in@xy
(kpair@xx@xy))))) thf(dpsetconstr, definition)
dsetconstrEL ⇒ (subsetI_2 ⇒ ∀a: $i, b: $i, xphi: $i → $i → $o: (⊆ @(dpsetconstr@a@b@λxx: $i, xy: $i: (xphi@xx@xy))@(cart

SEU667∧2.p Ordered Pairs - Sets of Pairs
(! A:i.! B:i.! phi:i>(i>o).breln A B (dpsetconstr A B (∧ x,y:i.phi x y)))
in: $i → $i → $o thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
breln: $i → $i → $i → $o thf(breln_type, type)
breln = (λa: $i, b: $i, c: $i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
dpsetconstr: $i → $i → ($i → $i → $o) → $i thf(dpsetconstr_type, type)
dpsetconstr = (λa: $i, b: $i, xphi: $i → $i → $o: (dsetconstr@(cartprod@a@b)@λxu: $i: ∃xx: $i: (in@xx@a and ∃xy: $i: (in@xy
(kpair@xx@xy))))) thf(dpsetconstr, definition)
dpsetconstrSub: $o thf(dpsetconstrSub_type, type)
dpsetconstrSub = (∀a: $i, b: $i, xphi: $i → $i → $o: (⊆ @(dpsetconstr@a@b@λxx: $i, xy: $i: (xphi@xx@xy))@(cartprod@a@b))
dpsetconstrSub ⇒ ∀a: $i, b: $i, xphi: $i → $i → $o: (breln@a@b@(dpsetconstr@a@b@λxx: $i, xy: $i: (xphi@xx@xy))) thf(

SEU668∧2.p Ordered Pairs - Sets of Pairs
(! A:i.! B:i.! phi:i>(i>o).! x:i.in x A → (! y:i.in y B → in (kpair x y) (dpsetconstr A B (∧ z,u:i.phi z u)) → phi x
y))
in: $i → $i → $o thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrER: $o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
setukpairinjL: $o thf(setukpairinjL_type, type)
setukpairinjL = (∀xx: $i, xy: $i, xz: $i, xu: $i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xx = xz)) thf(setukpairinjL, definitio
setukpairinjR: $o thf(setukpairinjR_type, type)
setukpairinjR = (∀xx: $i, xy: $i, xz: $i, xu: $i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xy = xu)) thf(setukpairinjR, definitio
dpsetconstr: $i → $i → ($i → $i → $o) → $i thf(dpsetconstr_type, type)
dpsetconstr = (λa: $i, b: $i, xphi: $i → $i → $o: (dsetconstr@(cartprod@a@b)@λxu: $i: ∃xx: $i: (in@xx@a and ∃xy: $i: (in@xy
(kpair@xx@xy))))) thf(dpsetconstr, definition)
dsetconstrER ⇒ (setukpairinjL ⇒ (setukpairinjR ⇒ ∀a: $i, b: $i, xphi: $i → $i → $o, xx: $i: ((in@xx@a) ⇒
∀xy: $i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@(dpsetconstr@a@b@λxz: $i, xu: $i: (xphi@xz@xu))) ⇒ (xphi@xx@xy))))))

SEU669∧2.p Ordered Pairs - Sets of Pairs
(! A:i.! B:i.! phi:i>(i>o).! x:i.! y:i.in (kpair x y) (dpsetconstr A B (∧ z,u:i.phi z u)) → in x A)
in: $i → $i → $o thf(in_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrEL: $o thf(dsetconstrEL_type, type)
dsetconstrEL = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (in@xx@a))) thf(dsetcons
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
cartprodpairmemEL: $o thf(cartprodpairmemEL_type, type)
cartprodpairmemEL = (∀a: $i, b: $i, xx: $i, xy: $i: ((in@(kpair@xx@xy)@(cartprod@a@b)) ⇒ (in@xx@a))) thf(cartprodp
dpsetconstr: $i → $i → ($i → $i → $o) → $i thf(dpsetconstr_type, type)
dpsetconstr = (λa: $i, b: $i, xphi: $i → $i → $o: (dsetconstr@(cartprod@a@b)@λxu: $i: ∃xx: $i: (in@xx@a and ∃xy: $i: (in@xy
(kpair@xx@xy))))) thf(dpsetconstr, definition)
dsetconstrEL ⇒ (cartprodpairmemEL ⇒ ∀a: $i, b: $i, xphi: $i → $i → $o, xx: $i, xy: $i: ((in@(kpair@xx@xy)@(dpsetconstr@
(in@xx@a))) thf(dpsetconstrEL_1, conjecture)

SEU670^2.p Ordered Pairs - Sets of Pairs
(! A:i.! B:i.! phi:i>(i>o).! x:i.! y:i.in (kpair x y) (dpsetconstr A B (^ z,u:i.phi z u)) → in y B)
in: \$i → \$i → \$o   thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrEL: \$o   thf(dsetconstrEL_type, type)
dsetconstrEL = (∀*a*: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@*a*@λxy: \$i: (xphi@xy))) ⇒ (in@xx@*a*)))   thf(dsetcons
kpair: \$i → \$i → \$i   thf(kpair_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
cartprodpairmemER: \$o   thf(cartprodpairmemER_type, type)
cartprodpairmemER = (∀*a*: \$i, *b*: \$i, xx: \$i, xy: \$i: ((in@(kpair@xx@xy)@(cartprod@*a*@*b*)) ⇒ (in@xy@*b*)))   thf(cartprodp
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i   thf(dpsetconstr_type, type)
dpsetconstr = (λ*a*: \$i, *b*: \$i, xphi: \$i → \$i → \$o: (dsetconstr@(cartprod@*a*@*b*)@λxu: \$i: ∃xx: \$i: (in@xx@*a* and ∃xy: \$i: (in@xy
(kpair@xx@xy)))))   thf(dpsetconstr, definition)
dsetconstrEL ⇒ (cartprodpairmemER ⇒ ∀*a*: \$i, *b*: \$i, xphi: \$i → \$i → \$o, xx: \$i, xy: \$i: ((in@(kpair@xx@xy)@(dpsetconstr@
(in@xy@*b*)))   thf(dpsetconstrEL$_2$, conjecture)

SEU671^2.p Ordered Pairs - Sets of Pairs
(! A:i.! B:i.! phi:i>(i>o).! x:i.! y:i.in (kpair x y) (dpsetconstr A B (^ z,u:i.phi z u)) → phi x y)
in: \$i → \$i → \$o   thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrER: \$o   thf(dsetconstrER_type, type)
dsetconstrER = (∀*a*: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@*a*@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx)))   thf(dsetcons
kpair: \$i → \$i → \$i   thf(kpair_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
setukpairinjL: \$o   thf(setukpairinjL_type, type)
setukpairinjL = (∀xx: \$i, xy: \$i, xz: \$i, xu: \$i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xx = xz))   thf(setukpairinjL, definitio
setukpairinjR: \$o   thf(setukpairinjR_type, type)
setukpairinjR = (∀xx: \$i, xy: \$i, xz: \$i, xu: \$i: ((kpair@xx@xy) = (kpair@xz@xu) ⇒ xy = xu))   thf(setukpairinjR, definitio
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i   thf(dpsetconstr_type, type)
dpsetconstr = (λ*a*: \$i, *b*: \$i, xphi: \$i → \$i → \$o: (dsetconstr@(cartprod@*a*@*b*)@λxu: \$i: ∃xx: \$i: (in@xx@*a* and ∃xy: \$i: (in@xy
(kpair@xx@xy)))))   thf(dpsetconstr, definition)
dsetconstrER ⇒ (setukpairinjL ⇒ (setukpairinjR ⇒ ∀*a*: \$i, *b*: \$i, xphi: \$i → \$i → \$o, xx: \$i, xy: \$i: ((in@(kpair@xx@xy)@(
(xphi@xx@xy))))   thf(dpsetconstrER, conjecture)

SEU672^2.p Functions
(! A:i.! B:i.! f:i.func A B f → (! x:i.in x A → singleton (dsetconstr B (^ y:i.in (kpair x y) f))))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i   thf(setadjoin_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
singleton: \$i → \$o   thf(singleton_type, type)
singleton = (λ*a*: \$i: ∃xx: \$i: (in@xx@*a* and *a* = (setadjoin@xx@emptyset)))   thf(singleton, definition)
ex_1: \$i → (\$i → \$o) → \$o   thf(ex1_type, type)
ex_1 = (λ*a*: \$i, xphi: \$i → \$o: (singleton@(dsetconstr@*a*@λxx: \$i: (xphi@xx))))   thf(ex_1, definition)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λ*a*: \$i, *b*: \$i, *c*: \$i: (⊆ @*c*@(cartprod@*a*@*b*)))   thf(breln, definition)
func: \$i → \$i → \$i → \$o   thf(func_type, type)
func = (λ*a*: \$i, *b*: \$i, *r*: \$i: (breln@*a*@*b*@*r* and ∀xx: \$i: ((in@xx@*a*) ⇒ (ex_1@*b*@λxy: \$i: (in@(kpair@xx@xy)@*r*)))))   thf(fu
∀*a*: \$i, *b*: \$i, xf: \$i: ((func@*a*@*b*@xf) ⇒ ∀xx: \$i: ((in@xx@*a*) ⇒ (singleton@(dsetconstr@*b*@λxy: \$i: (in@(kpair@xx@xy)@xf))

SEU673^2.p Functions - Application
(! A:i.! B:i.! f:i.func A B f → (! x:i.in x A → in (setunion (dsetconstr B (^ y:i.in (kpair x y) f))) B))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i   thf(setadjoin_type, type)
setunion: \$i → \$i   thf(setunion_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrEL: \$o   thf(dsetconstrEL_type, type)

dsetconstrEL = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (in@xx@a))) thf(dsetcons
⊆ : $i → $i → $o thf(subset_type, type)
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex_1: $i → ($i → $o) → $o thf(ex1_type, type)
ex_1 = (λa: $i, xphi: $i → $o: (singleton@(dsetconstr@a@λxx: $i: (xphi@xx)))) thf($\mathrm{ex}_1$, definition)
theprop: $o thf(theprop_type, type)
theprop = (∀x: $i: ((singleton@x) ⇒ (in@(setunion@x)@x))) thf(theprop, definition)
breln: $i → $i → $i → $o thf(breln_type, type)
breln = (λa: $i, b: $i, c: $i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
func: $i → $i → $i → $o thf(func_type, type)
func = (λa: $i, b: $i, r: $i: (breln@a@b@r and ∀xx: $i: ((in@xx@a) ⇒ ($\mathrm{ex}_1$@b@λxy: $i: (in@(kpair@xx@xy)@r))))) thf(fu
funcImageSingleton: $o thf(funcImageSingleton_type, type)
funcImageSingleton = (∀a: $i, b: $i, xf: $i: ((func@a@b@xf) ⇒ ∀xx: $i: ((in@xx@a) ⇒ (singleton@(dsetconstr@b@λxy: $i: (in
dsetconstrEL ⇒ (theprop ⇒ (funcImageSingleton ⇒ ∀a: $i, b: $i, xf: $i: ((func@a@b@xf) ⇒ ∀xx: $i: ((in@xx@a) ⇒
(in@(setunion@(dsetconstr@b@λxy: $i: (in@(kpair@xx@xy)@xf)))@b))))) thf(apProp, conjecture)

SEU674^2.p Functions - Application
(! A:i.! B:i.! f:i.func A B f → (! x:i.in x A → in (ap A B f x) B))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex_1: $i → ($i → $o) → $o thf(ex1_type, type)
ex_1 = (λa: $i, xphi: $i → $o: (singleton@(dsetconstr@a@λxx: $i: (xphi@xx)))) thf($\mathrm{ex}_1$, definition)
breln: $i → $i → $i → $o thf(breln_type, type)
breln = (λa: $i, b: $i, c: $i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
func: $i → $i → $i → $o thf(func_type, type)
func = (λa: $i, b: $i, r: $i: (breln@a@b@r and ∀xx: $i: ((in@xx@a) ⇒ ($\mathrm{ex}_1$@b@λxy: $i: (in@(kpair@xx@xy)@r))))) thf(fu
apProp: $o thf(apProp_type, type)
apProp = (∀a: $i, b: $i, xf: $i: ((func@a@b@xf) ⇒ ∀xx: $i: ((in@xx@a) ⇒ (in@(setunion@(dsetconstr@b@λxy: $i: (in@(kpair
ap: $i → $i → $i → $i → $i thf(ap_type, type)
ap = (λa: $i, b: $i, xf: $i, xx: $i: (setunion@(dsetconstr@b@λxy: $i: (in@(kpair@xx@xy)@xf)))) thf(ap, definition)
apProp ⇒ ∀a: $i, b: $i, xf: $i: ((func@a@b@xf) ⇒ ∀xx: $i: ((in@xx@a) ⇒ (in@(ap@a@b@xf@xx)@b))) thf(app, conjectu

SEU675^2.p Functions - Application
(! A:i.! B:i.! f:i.in f (funcSet A B) → func A B f)
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
powerset: $i → $i thf(powerset_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrER: $o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@a@λxy: $i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
⊆ : $i → $i → $o thf(subset_type, type)
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex_1: $i → ($i → $o) → $o thf(ex1_type, type)
ex_1 = (λa: $i, xphi: $i → $o: (singleton@(dsetconstr@a@λxx: $i: (xphi@xx)))) thf($\mathrm{ex}_1$, definition)
breln: $i → $i → $i → $o thf(breln_type, type)

breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
func: \$i → \$i → \$i → \$o thf(func_type, type)
func = (λa: \$i, b: \$i, r: \$i: (breln@a@b@r and ∀xx: \$i: ((in@xx@a) ⇒ (ex$_1$@b@λxy: \$i: (in@(kpair@xx@xy)@r))))) thf(fu
funcSet: \$i → \$i → \$i thf(funcSet_type, type)
funcSet = (λa: \$i, b: \$i: (dsetconstr@(powerset@(cartprod@a@b))@λxf: \$i: (func@a@b@xf))) thf(funcSet, definition)
dsetconstrER ⇒ ∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ (func@a@b@xf)) thf(infuncsetfunc, conjecture)

SEU676∧2.p Functions - Application
(! A:i.! B:i.! f:i.in f (funcSet A B) → (! x:i.in x A → in (ap A B f x) B))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
powerset: \$i → \$i thf(powerset_type, type)
setunion: \$i → \$i thf(setunion_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex$_1$: \$i → (\$i → \$o) → \$o thf(ex1_type, type)
ex$_1$ = (λa: \$i, xphi: \$i → \$o: (singleton@(dsetconstr@a@λxx: \$i: (xphi@xx)))) thf(ex$_1$, definition)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
func: \$i → \$i → \$i → \$o thf(func_type, type)
func = (λa: \$i, b: \$i, r: \$i: (breln@a@b@r and ∀xx: \$i: ((in@xx@a) ⇒ (ex$_1$@b@λxy: \$i: (in@(kpair@xx@xy)@r))))) thf(fu
funcSet: \$i → \$i → \$i thf(funcSet_type, type)
funcSet = (λa: \$i, b: \$i: (dsetconstr@(powerset@(cartprod@a@b))@λxf: \$i: (func@a@b@xf))) thf(funcSet, definition)
ap: \$i → \$i → \$i → \$i → \$i thf(ap_type, type)
ap = (λa: \$i, b: \$i, xf: \$i, xx: \$i: (setunion@(dsetconstr@b@λxy: \$i: (in@(kpair@xx@xy)@xf)))) thf(ap, definition)
app: \$o thf(app_type, type)
app = (∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (in@(ap@a@b@xf@xx)@b)))) thf(app, definition)
infuncsetfunc: \$o thf(infuncsetfunc_type, type)
infuncsetfunc = (∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ (func@a@b@xf))) thf(infuncsetfunc, definition)
app ⇒ (infuncsetfunc ⇒ ∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (in@(ap@a@b@xf@xx)@b

SEU677∧2.p Functions - Lambda Abstraction
(! A:i.! B:i.! f:i.func A B f → in f (funcSet A B))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
powerset: \$i → \$i thf(powerset_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrI: \$o thf(dsetconstrI_type, type)
dsetconstrI = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))))))
⊆ : \$i → \$i → \$o thf(subset_type, type)
powersetI$_1$: \$o thf(powersetI1_type, type)
powersetI$_1$ = (∀a: \$i, b: \$i: ((⊆ @b@a) ⇒ (in@b@(powerset@a)))) thf(powersetI$_1$, definition)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex$_1$: \$i → (\$i → \$o) → \$o thf(ex1_type, type)
ex$_1$ = (λa: \$i, xphi: \$i → \$o: (singleton@(dsetconstr@a@λxx: \$i: (xphi@xx)))) thf(ex$_1$, definition)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
func: \$i → \$i → \$i → \$o thf(func_type, type)
func = (λa: \$i, b: \$i, r: \$i: (breln@a@b@r and ∀xx: \$i: ((in@xx@a) ⇒ (ex$_1$@b@λxy: \$i: (in@(kpair@xx@xy)@r))))) thf(fu
funcSet: \$i → \$i → \$i thf(funcSet_type, type)
funcSet = (λa: \$i, b: \$i: (dsetconstr@(powerset@(cartprod@a@b))@λxf: \$i: (func@a@b@xf))) thf(funcSet, definition)

dsetconstrI ⇒ (powersetI$_1$ ⇒ ∀a: $i, b: $i, xf: $i: ((func@a@b@xf) ⇒ (in@xf@(funcSet@a@b)))) thf(funcinfuncset, conje

SEU679^2.p Functions - Lambda Abstraction
(! A:i.! B:i.! f:i>i.(! x:i.in x A → in (f x) B) → func A B (lam A B (^ x:i.f x)))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex$_1$: $i → ($i → $o) → $o thf(ex1_type, type)
ex$_1$ = (λa: $i, xphi: $i → $o: (singleton@(dsetconstr@a@λxx: $i: (xphi@xx)))) thf(ex$_1$, definition)
breln: $i → $i → $i → $o thf(breln_type, type)
breln = (λa: $i, b: $i, c: $i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
dpsetconstr: $i → $i → ($i → $i → $o) → $i thf(dpsetconstr_type, type)
func: $i → $i → $i → $o thf(func_type, type)
func = (λa: $i, b: $i, r: $i: (breln@a@b@r and ∀xx: $i: ((in@xx@a) ⇒ (ex$_1$@b@λxy: $i: (in@(kpair@xx@xy)@r))))) thf(fu
lamProp: $o thf(lamProp_type, type)
lamProp = (∀a: $i, b: $i, xf: $i → $i: (∀xx: $i: ((in@xx@a) ⇒ (in@(xf@xx)@b)) ⇒ (func@a@b@(dpsetconstr@a@b@λxx: $i, x
xy)))) thf(lamProp, definition)
lam: $i → $i → ($i → $i) → $i thf(lam_type, type)
lam = (λa: $i, b: $i, xf: $i → $i: (dpsetconstr@a@b@λxx: $i, xy: $i: (xf@xx) = xy)) thf(lam, definition)
lamProp ⇒ ∀a: $i, b: $i, xf: $i → $i: (∀xx: $i: ((in@xx@a) ⇒ (in@(xf@xx)@b)) ⇒ (func@a@b@(lam@a@b@λxx: $i: (xf@xx)

SEU680^2.p Functions - Lambda Abstraction
(! A:i.! B:i.! f:i>i.(! x:i.in x A → in (f x) B) → in (lam A B (^ x:i.f x)) (funcSet A B))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
kpair: $i → $i → $i thf(kpair_type, type)
cartprod: $i → $i → $i thf(cartprod_type, type)
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex$_1$: $i → ($i → $o) → $o thf(ex1_type, type)
ex$_1$ = (λa: $i, xphi: $i → $o: (singleton@(dsetconstr@a@λxx: $i: (xphi@xx)))) thf(ex$_1$, definition)
breln: $i → $i → $i → $o thf(breln_type, type)
breln = (λa: $i, b: $i, c: $i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
dpsetconstr: $i → $i → ($i → $i → $o) → $i thf(dpsetconstr_type, type)
func: $i → $i → $i → $o thf(func_type, type)
func = (λa: $i, b: $i, r: $i: (breln@a@b@r and ∀xx: $i: ((in@xx@a) ⇒ (ex$_1$@b@λxy: $i: (in@(kpair@xx@xy)@r))))) thf(fu
funcSet: $i → $i → $i thf(funcSet_type, type)
funcinfuncset: $o thf(funcinfuncset_type, type)
funcinfuncset = (∀a: $i, b: $i, xf: $i: ((func@a@b@xf) ⇒ (in@xf@(funcSet@a@b)))) thf(funcinfuncset, definition)
lam: $i → $i → ($i → $i) → $i thf(lam_type, type)
lam = (λa: $i, b: $i, xf: $i → $i: (dpsetconstr@a@b@λxx: $i, xy: $i: (xf@xx) = xy)) thf(lam, definition)
lamp: $o thf(lamp_type, type)
lamp = (∀a: $i, b: $i, xf: $i → $i: (∀xx: $i: ((in@xx@a) ⇒ (in@(xf@xx)@b)) ⇒ (func@a@b@(lam@a@b@λxx: $i: (xf@xx)))))
funcinfuncset ⇒ (lamp ⇒ ∀a: $i, b: $i, xf: $i → $i: (∀xx: $i: ((in@xx@a) ⇒ (in@(xf@xx)@b)) ⇒ (in@(lam@a@b@λxx: $i: (

SEU681^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! R:i.breln A B R → (! phi:i>o.(! x:i.in x A → (! y:i.in y B → in (kpair x y) R → phi (kpair x y))) →
(! x:i.in x R → phi x)))
in: $i → $i → $o thf(in_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
subsetE: $o thf(subsetE_type, type)
subsetE = (∀a: $i, b: $i, xx: $i: ((⊆ @a@b) ⇒ ((in@xx@a) ⇒ (in@xx@b)))) thf(subsetE, definition)

kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
cartprodmempair$_1$: \$o thf(cartprodmempair1_type, type)
cartprodmempair$_1$ = (∀a: \$i, b: \$i, xu: \$i: ((in@xu@(cartprod@a@b)) ⇒ ∃xx: \$i: (in@xx@a and ∃xy: \$i: (in@xy@b and xu = (kpair@xx@xy))))) thf(cartprodmempair$_1$, definition)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
subsetE ⇒ (cartprodmempair$_1$ ⇒ ∀a: \$i, b: \$i, r: \$i: ((breln@a@b@r) ⇒ ∀xphi: \$i → \$o: (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (xphi@(kpair@xx@xy))))) ⇒ ∀xx: \$i: ((in@xx@r) ⇒ (xphi@xx))))) thf(brelnall$_1$, conjecture)

SEU682∧2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! R:i.breln A B R → (! phi:i>o.(! x:i.in x A → (! y:i.in y B → in (kpair x y) R → phi (kpair x y))) → (! x:i.in x R → phi x)))
in: \$i → \$i → \$o thf(in_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
brelnall$_1$: \$o thf(brelnall1_type, type)
brelnall$_1$ = (∀a: \$i, b: \$i, r: \$i: ((breln@a@b@r) ⇒ ∀xphi: \$i → \$o: (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (xphi@(kpair@xx@xy))))) ⇒ ∀xx: \$i: ((in@xx@r) ⇒ (xphi@xx))))) thf(brelnall$_1$, definitio
brelnall$_1$ ⇒ ∀a: \$i, b: \$i, r: \$i: ((breln@a@b@r) ⇒ ∀xphi: \$i → \$o: (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (xphi@(kpair@xx@xy))))) ⇒ ∀xx: \$i: ((in@xx@r) ⇒ (xphi@xx)))) thf(brelnall$_2$, conjectur

SEU683∧2.p Functions - Extensionality and Beta Reduction
(! A:i.! phi:i>o.ex1 A (∧ x:i.phi x) → (! x:i.in x A → (! y:i.in y A → phi x → phi y → x = y)))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrI: \$o thf(dsetconstrI_type, type)
dsetconstrI = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))))))
uniqinunit: \$o thf(uniqinunit_type, type)
uniqinunit = (∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex$_1$: \$i → (\$i → \$o) → \$o thf(ex1_type, type)
ex$_1$ = (λa: \$i, xphi: \$i → \$o: (singleton@(dsetconstr@a@λxx: \$i: (xphi@xx)))) thf(ex$_1$, definition)
dsetconstrI ⇒ (uniqinunit ⇒ ∀a: \$i, xphi: \$i → \$o: ((ex$_1$@a@λxx: \$i: (xphi@xx)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((xphi@xx) ⇒ ((xphi@xy) ⇒ xx = xy)))))) thf(ex1E$_2$, conjecture)

SEU684∧2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.func A B f → (! x:i.in x A → in (kpair x (ap A B f x)) f))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setunion: \$i → \$i thf(setunion_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrER: \$o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
⊆ : \$i → \$i → \$o thf(subset_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
ex$_1$: \$i → (\$i → \$o) → \$o thf(ex1_type, type)
ex$_1$ = (λa: \$i, xphi: \$i → \$o: (singleton@(dsetconstr@a@λxx: \$i: (xphi@xx)))) thf(ex$_1$, definition)
theprop: \$o thf(theprop_type, type)
theprop = (∀x: \$i: ((singleton@x) ⇒ (in@(setunion@x)@x))) thf(theprop, definition)

breln: $\$i \to \$i \to \$i \to \$o$ thf(breln_type, type)
breln $= (\lambda a\colon \$i, b\colon \$i, c\colon \$i\colon (\subseteq @c@(\text{cartprod}@a@b)))$ thf(breln, definition)
func: $\$i \to \$i \to \$i \to \$o$ thf(func_type, type)
func $= (\lambda a\colon \$i, b\colon \$i, r\colon \$i\colon (\text{breln}@a@b@r \text{ and } \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow (\text{ex}_1@b@\lambda \text{xy}\colon \$i\colon (\text{in}@(\text{kpair}@\text{xx}@\text{xy})@r)))))$ thf(fu
funcImageSingleton: $\$o$ thf(funcImageSingleton_type, type)
funcImageSingleton $= (\forall a\colon \$i, b\colon \$i, \text{xf}\colon \$i\colon ((\text{func}@a@b@\text{xf}) \Rightarrow \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow (\text{singleton}@(\text{dsetconstr}@b@\lambda \text{xy}\colon \$i\colon (\text{in}$
ap: $\$i \to \$i \to \$i \to \$i \to \$i$ thf(ap_type, type)
ap $= (\lambda a\colon \$i, b\colon \$i, \text{xf}\colon \$i, \text{xx}\colon \$i\colon (\text{setunion}@(\text{dsetconstr}@b@\lambda \text{xy}\colon \$i\colon (\text{in}@(\text{kpair}@\text{xx}@\text{xy})@\text{xf}))))$ thf(ap, definition)
dsetconstrER $\Rightarrow$ (theprop $\Rightarrow$ (funcImageSingleton $\Rightarrow \forall a\colon \$i, b\colon \$i, \text{xf}\colon \$i\colon ((\text{func}@a@b@\text{xf}) \Rightarrow \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow$
$(\text{in}@(\text{kpair}@\text{xx}@(\text{ap}@a@b@\text{xf}@\text{xx}))@\text{xf})))))$ thf(funcGraphProp$_1$, conjecture)

SEU685^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.in f (funcSet A B) → (! x:i.in x A → in (kpair x (ap A B f x)) f))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
emptyset: $\$i$ thf(emptyset_type, type)
setadjoin: $\$i \to \$i \to \$i$ thf(setadjoin_type, type)
dsetconstr: $\$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$\subseteq$: $\$i \to \$i \to \$o$ thf(subset_type, type)
kpair: $\$i \to \$i \to \$i$ thf(kpair_type, type)
cartprod: $\$i \to \$i \to \$i$ thf(cartprod_type, type)
singleton: $\$i \to \$o$ thf(singleton_type, type)
singleton $= (\lambda a\colon \$i\colon \exists \text{xx}\colon \$i\colon (\text{in}@\text{xx}@a \text{ and } a = (\text{setadjoin}@\text{xx}@\text{emptyset})))$ thf(singleton, definition)
ex$_1$: $\$i \to (\$i \to \$o) \to \$o$ thf(ex1_type, type)
ex$_1$ $= (\lambda a\colon \$i, \text{xphi}\colon \$i \to \$o\colon (\text{singleton}@(\text{dsetconstr}@a@\lambda \text{xx}\colon \$i\colon (\text{xphi}@\text{xx}))))$ thf(ex$_1$, definition)
breln: $\$i \to \$i \to \$i \to \$o$ thf(breln_type, type)
breln $= (\lambda a\colon \$i, b\colon \$i, c\colon \$i\colon (\subseteq @c@(\text{cartprod}@a@b)))$ thf(breln, definition)
func: $\$i \to \$i \to \$i \to \$o$ thf(func_type, type)
func $= (\lambda a\colon \$i, b\colon \$i, r\colon \$i\colon (\text{breln}@a@b@r \text{ and } \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow (\text{ex}_1@b@\lambda \text{xy}\colon \$i\colon (\text{in}@(\text{kpair}@\text{xx}@\text{xy})@r)))))$ thf(fu
funcSet: $\$i \to \$i \to \$i$ thf(funcSet_type, type)
ap: $\$i \to \$i \to \$i \to \$i \to \$i$ thf(ap_type, type)
infuncsetfunc: $\$o$ thf(infuncsetfunc_type, type)
infuncsetfunc $= (\forall a\colon \$i, b\colon \$i, \text{xf}\colon \$i\colon ((\text{in}@\text{xf}@(\text{funcSet}@a@b)) \Rightarrow (\text{func}@a@b@\text{xf})))$ thf(infuncsetfunc, definition)
funcGraphProp$_1$: $\$o$ thf(funcGraphProp1_type, type)
funcGraphProp$_1$ $= (\forall a\colon \$i, b\colon \$i, \text{xf}\colon \$i\colon ((\text{func}@a@b@\text{xf}) \Rightarrow \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@(\text{kpair}@\text{xx}@(\text{ap}@a@b@\text{xf}@\text{xx}))@\text{xf}))$
infuncsetfunc $\Rightarrow$ (funcGraphProp$_1$ $\Rightarrow \forall a\colon \$i, b\colon \$i, \text{xf}\colon \$i\colon ((\text{in}@\text{xf}@(\text{funcSet}@a@b)) \Rightarrow \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow$
$(\text{in}@(\text{kpair}@\text{xx}@(\text{ap}@a@b@\text{xf}@\text{xx}))@\text{xf}))))$ thf(funcGraphProp$_3$, conjecture)

SEU686^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.func A B f → (! x:i.in x A → (! y:i.in y B → in (kpair x y) f → ap A B f x = y)))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
emptyset: $\$i$ thf(emptyset_type, type)
setadjoin: $\$i \to \$i \to \$i$ thf(setadjoin_type, type)
dsetconstr: $\$i \to (\$i \to \$o) \to \$i$ thf(dsetconstr_type, type)
$\subseteq$: $\$i \to \$i \to \$o$ thf(subset_type, type)
kpair: $\$i \to \$i \to \$i$ thf(kpair_type, type)
cartprod: $\$i \to \$i \to \$i$ thf(cartprod_type, type)
singleton: $\$i \to \$o$ thf(singleton_type, type)
singleton $= (\lambda a\colon \$i\colon \exists \text{xx}\colon \$i\colon (\text{in}@\text{xx}@a \text{ and } a = (\text{setadjoin}@\text{xx}@\text{emptyset})))$ thf(singleton, definition)
ex$_1$: $\$i \to (\$i \to \$o) \to \$o$ thf(ex1_type, type)
ex$_1$ $= (\lambda a\colon \$i, \text{xphi}\colon \$i \to \$o\colon (\text{singleton}@(\text{dsetconstr}@a@\lambda \text{xx}\colon \$i\colon (\text{xphi}@\text{xx}))))$ thf(ex$_1$, definition)
breln: $\$i \to \$i \to \$i \to \$o$ thf(breln_type, type)
breln $= (\lambda a\colon \$i, b\colon \$i, c\colon \$i\colon (\subseteq @c@(\text{cartprod}@a@b)))$ thf(breln, definition)
func: $\$i \to \$i \to \$i \to \$o$ thf(func_type, type)
func $= (\lambda a\colon \$i, b\colon \$i, r\colon \$i\colon (\text{breln}@a@b@r \text{ and } \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow (\text{ex}_1@b@\lambda \text{xy}\colon \$i\colon (\text{in}@(\text{kpair}@\text{xx}@\text{xy})@r)))))$ thf(fu
ap: $\$i \to \$i \to \$i \to \$i \to \$i$ thf(ap_type, type)
app: $\$o$ thf(app_type, type)
app $= (\forall a\colon \$i, b\colon \$i, \text{xf}\colon \$i\colon ((\text{func}@a@b@\text{xf}) \Rightarrow \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@(\text{ap}@a@b@\text{xf}@\text{xx})@b))))$ thf(app, definition)
ex1E$_2$: $\$o$ thf(ex1E2_type, type)
ex1E$_2$ $= (\forall a\colon \$i, \text{xphi}\colon \$i \to \$o\colon ((\text{ex}_1@a@\lambda \text{xx}\colon \$i\colon (\text{xphi}@\text{xx})) \Rightarrow \forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow \forall \text{xy}\colon \$i\colon ((\text{in}@\text{xy}@a) \Rightarrow$
$((\text{xphi}@\text{xx}) \Rightarrow ((\text{xphi}@\text{xy}) \Rightarrow \text{xx} = \text{xy}))))))$ thf(ex1E$_2$, definition)

funcGraphProp$_1$: \$o   thf(funcGraphProp1_type, type)
funcGraphProp$_1$ = (∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (in@(kpair@xx@(ap@a@b@xf@xx))@xf))
app ⇒ (ex1E$_2$ ⇒ (funcGraphProp$_1$ ⇒ ∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xx: \$i: ((in@xx@a) ⇒
∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@xf) ⇒ (ap@a@b@xf@xx) = xy))))))   thf(funcGraphProp$_2$, conjecture)

SEU687^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.func A B f → (! g:i.func A B g → (! x:i.in x A → ap A B f x = ap A B g x) → (! x:i.in x A → (!
y:i.in y B → in (kpair x y) g → in (kpair x y) f))))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i   thf(setadjoin_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
singleton: \$i → \$o   thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset)))   thf(singleton, definition)
ex$_1$: \$i → (\$i → \$o) → \$o   thf(ex1_type, type)
ex$_1$ = (λa: \$i, xphi: \$i → \$o: (singleton@(dsetconstr@a@λxx: \$i: (xphi@xx))))   thf(ex$_1$, definition)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b)))   thf(breln, definition)
func: \$i → \$i → \$i → \$o   thf(func_type, type)
func = (λa: \$i, b: \$i, r: \$i: (breln@a@b@r and ∀xx: \$i: ((in@xx@a) ⇒ (ex$_1$@b@λxy: \$i: (in@(kpair@xx@xy)@r)))))   thf(fun
ap: \$i → \$i → \$i → \$i → \$i   thf(ap_type, type)
funcGraphProp$_1$: \$o   thf(funcGraphProp1_type, type)
funcGraphProp$_1$ = (∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (in@(kpair@xx@(ap@a@b@xf@xx))@xf))
funcGraphProp$_2$: \$o   thf(funcGraphProp2_type, type)
funcGraphProp$_2$ = (∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒
((in@(kpair@xx@xy)@xf) ⇒ (ap@a@b@xf@xx) = xy)))))   thf(funcGraphProp$_2$, definition)
funcGraphProp$_1$ ⇒ (funcGraphProp$_2$ ⇒ ∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xg: \$i: ((func@a@b@xg) ⇒
(∀xx: \$i: ((in@xx@a) ⇒ (ap@a@b@xf@xx) = (ap@a@b@xg@xx)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒
((in@(kpair@xx@xy)@xg) ⇒ (in@(kpair@xx@xy)@xf))))))))   thf(funcextLem, conjecture)

SEU688^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.in f (funcSet A B) → (! x:i.in x A → (! y:i.in y B → in (kpair x y) f → ap A B f x = y)))
in: \$i → \$i → \$o   thf(in_type, type)
emptyset: \$i   thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i   thf(setadjoin_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
singleton: \$i → \$o   thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset)))   thf(singleton, definition)
ex$_1$: \$i → (\$i → \$o) → \$o   thf(ex1_type, type)
ex$_1$ = (λa: \$i, xphi: \$i → \$o: (singleton@(dsetconstr@a@λxx: \$i: (xphi@xx))))   thf(ex$_1$, definition)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b)))   thf(breln, definition)
func: \$i → \$i → \$i → \$o   thf(func_type, type)
func = (λa: \$i, b: \$i, r: \$i: (breln@a@b@r and ∀xx: \$i: ((in@xx@a) ⇒ (ex$_1$@b@λxy: \$i: (in@(kpair@xx@xy)@r)))))   thf(fun
funcSet: \$i → \$i → \$i   thf(funcSet_type, type)
ap: \$i → \$i → \$i → \$i → \$i   thf(ap_type, type)
infuncsetfunc: \$o   thf(infuncsetfunc_type, type)
infuncsetfunc = (∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ (func@a@b@xf)))   thf(infuncsetfunc, definition)
funcGraphProp$_2$: \$o   thf(funcGraphProp2_type, type)
funcGraphProp$_2$ = (∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒
((in@(kpair@xx@xy)@xf) ⇒ (ap@a@b@xf@xx) = xy)))))   thf(funcGraphProp$_2$, definition)
infuncsetfunc ⇒ (funcGraphProp$_2$ ⇒ ∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒
∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@xf) ⇒ (ap@a@b@xf@xx) = xy)))))   thf(funcGraphProp$_4$, conjecture)

SEU689^2.p Functions - Extensionality and Beta Reduction

(! A:i.! B:i.! R:i.breln A B R → (! S:i.breln A B S → (! x:i.in x A → (! y:i.in y B → in (kpair x y) R → in (kpair x y) S)) → subset R S))
in: \$i → \$i → \$o thf(in_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
subsetI$_1$: \$o thf(subsetI1_type, type)
subsetI$_1$ = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (⊆ @a@b))) thf(subsetI$_1$, definition)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
brelnall$_1$: \$o thf(brelnall1_type, type)
brelnall$_1$ = (∀a: \$i, b: \$i, r: \$i: ((breln@a@b@r) ⇒ ∀xphi: \$i → \$o: (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (xphi@(kpair@xx@xy))))) ⇒ ∀xx: \$i: ((in@xx@r) ⇒ (xphi@xx))))) thf(brelnall$_1$, definitio
subsetI$_1$ ⇒ (brelnall$_1$ ⇒ ∀a: \$i, b: \$i, r: \$i: ((breln@a@b@r) ⇒ ∀s: \$i: ((breln@a@b@s) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@s)))) ⇒ (⊆ @r@s))))) thf(subbreln, conjectu

SEU690^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! R:i.breln A B R → (! S:i.breln A B S → (! x:i.in x A → (! y:i.in y B → in (kpair x y) R → in (kpair x y) S)) → (! x:i.in x A → (! y:i.in y B → in (kpair x y) S → in (kpair x y) R)) → R = S))
in: \$i → \$i → \$o thf(in_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
setextsub: \$o thf(setextsub_type, type)
setextsub = (∀a: \$i, b: \$i: ((⊆ @a@b) ⇒ ((⊆ @b@a) ⇒ $a = b$))) thf(setextsub, definition)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b))) thf(breln, definition)
subbreln: \$o thf(subbreln_type, type)
subbreln = (∀a: \$i, b: \$i, r: \$i: ((breln@a@b@r) ⇒ ∀s: \$i: ((breln@a@b@s) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@s)))) ⇒ (⊆ @r@s))))) thf(subbreln, definitio
setextsub ⇒ (subbreln ⇒ ∀a: \$i, b: \$i, r: \$i: ((breln@a@b@r) ⇒ ∀s: \$i: ((breln@a@b@s) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@s)))) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@s) ⇒ (in@(kpair@xx@xy)@r)))) ⇒ $r = s$))))) thf(eqbreln, conjecture)

SEU692^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.in f (funcSet A B) → (! g:i.in g (funcSet A B) → (! x:i.in x A → ap A B f x = ap A B g x) → f = g))
in: \$i → \$i → \$o thf(in_type, type)
func: \$i → \$i → \$i → \$o thf(func_type, type)
funcSet: \$i → \$i → \$i thf(funcSet_type, type)
ap: \$i → \$i → \$i → \$i → \$i thf(ap_type, type)
infuncsetfunc: \$o thf(infuncsetfunc_type, type)
infuncsetfunc = (∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ (func@a@b@xf))) thf(infuncsetfunc, definition)
funcext: \$o thf(funcext_type, type)
funcext = (∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xg: \$i: ((func@a@b@xg) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ (ap@a@b@xf@xx) = (ap@a@b@xg@xx)) ⇒ xf = xg)))) thf(funcext, definition)
infuncsetfunc ⇒ (funcext ⇒ ∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ ∀xg: \$i: ((in@xg@(funcSet@a@b)) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ (ap@a@b@xf@xx) = (ap@a@b@xg@xx)) ⇒ xf = xg)))) thf(funcext$_2$, conjecture)

SEU693^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.in f (funcSet A B) → (! x:i.in x A → ap A B f x = ap A B f x))
in: \$i → \$i → \$o thf(in, type)
funcSet: \$i → \$i → \$i thf(funcSet, type)
ap: \$i → \$i → \$i → \$i → \$i thf(ap, type)
∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (ap@a@b@xf@xx) = (ap@a@b@xf@xx))) thf(ap2

SEU694^2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.func A B f → (! x:i.in x A → ap A B f x = ap A B f x))
in: \$i → \$i → \$o thf(in, type)
func: \$i → \$i → \$i → \$o thf(func, type)
ap: \$i → \$i → \$i → \$i → \$i thf(ap, type)
∀a: \$i, b: \$i, xf: \$i: ((func@a@b@xf) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (ap@a@b@xf@xx) = (ap@a@b@xf@xx))) thf(ap2apEq$_2$, co

SEU695∧2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i>i.(! x:i.in x A → in (f x) B) → (! x:i.in x A → ap A B (lam A B (∧ y:i.f y)) x = f x))
in: \$i → \$i → \$o   thf(in_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i   thf(dpsetconstr_type, type)
dpsetconstrI: \$o   thf(dpsetconstrI_type, type)
dpsetconstrI = (∀*a*: \$i, *b*: \$i, xphi: \$i → \$i → \$o, xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*b*) ⇒ ((xphi@xx@xy) ⇒ (in@(kpair@xx@xy)@(dpsetconstr@*a*@*b*@λxz: \$i, xu: \$i: (xphi@xz@xu))))))   thf(dpsetconstrI, definition)
func: \$i → \$i → \$i → \$o   thf(func_type, type)
ap: \$i → \$i → \$i → \$i → \$i   thf(ap_type, type)
lam: \$i → \$i → (\$i → \$i) → \$i   thf(lam_type, type)
lam = (λ*a*: \$i, *b*: \$i, xf: \$i → \$i: (dpsetconstr@*a*@*b*@λxx: \$i, xy: \$i: (xf@xx) = xy))   thf(lam, definition)
lamp: \$o   thf(lamp_type, type)
lamp = (∀*a*: \$i, *b*: \$i, xf: \$i → \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@(xf@xx)@*b*)) ⇒ (func@*a*@*b*@(lam@*a*@*b*@λxx: \$i: (xf@xx)))))
funcGraphProp$_2$: \$o   thf(funcGraphProp2_type, type)
funcGraphProp$_2$ = (∀*a*: \$i, *b*: \$i, xf: \$i: ((func@*a*@*b*@xf) ⇒ ∀xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*b*) ⇒ ((in@(kpair@xx@xy)@xf) ⇒ (ap@*a*@*b*@xf@xx) = xy)))))   thf(funcGraphProp$_2$, definition)
dpsetconstrI ⇒ (lamp ⇒ (funcGraphProp$_2$ ⇒ ∀*a*: \$i, *b*: \$i, xf: \$i → \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@(xf@xx)@*b*)) ⇒ ∀xx: \$i: ((in@xx@*a*) ⇒ (ap@*a*@*b*@(lam@*a*@*b*@λxy: \$i: (xf@xy))@xx) = (xf@xx)))))   thf(beta$_1$, conjecture)

SEU696∧2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.func A B f → lam A B (∧ x:i.ap A B f x) = f)
in: \$i → \$i → \$o   thf(in_type, type)
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i   thf(dpsetconstr_type, type)
func: \$i → \$i → \$i → \$o   thf(func_type, type)
ap: \$i → \$i → \$i → \$i → \$i   thf(ap_type, type)
app: \$o   thf(app_type, type)
app = (∀*a*: \$i, *b*: \$i, xf: \$i: ((func@*a*@*b*@xf) ⇒ ∀xx: \$i: ((in@xx@*a*) ⇒ (in@(ap@*a*@*b*@xf@xx)@*b*))))   thf(app, definition)
lam: \$i → \$i → (\$i → \$i) → \$i   thf(lam_type, type)
lam = (λ*a*: \$i, *b*: \$i, xf: \$i → \$i: (dpsetconstr@*a*@*b*@λxx: \$i, xy: \$i: (xf@xx) = xy))   thf(lam, definition)
lamp: \$o   thf(lamp_type, type)
lamp = (∀*a*: \$i, *b*: \$i, xf: \$i → \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@(xf@xx)@*b*)) ⇒ (func@*a*@*b*@(lam@*a*@*b*@λxx: \$i: (xf@xx)))))
funcext: \$o   thf(funcext_type, type)
funcext = (∀*a*: \$i, *b*: \$i, xf: \$i: ((func@*a*@*b*@xf) ⇒ ∀xg: \$i: ((func@*a*@*b*@xg) ⇒ (∀xx: \$i: ((in@xx@*a*) ⇒ (ap@*a*@*b*@xf@xx) = (ap@*a*@*b*@xg@xx)) ⇒ xf = xg))))   thf(funcext, definition)
beta$_1$: \$o   thf(beta1_type, type)
beta$_1$ = (∀*a*: \$i, *b*: \$i, xf: \$i → \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@(xf@xx)@*b*)) ⇒ ∀xx: \$i: ((in@xx@*a*) ⇒ (ap@*a*@*b*@(lam@*a*@*b*@λxy: \$i: (xf@xy))@xx) = (xf@xx))))   thf(beta$_1$, definition)
app ⇒ (lamp ⇒ (funcext ⇒ (beta$_1$ ⇒ ∀*a*: \$i, *b*: \$i, xf: \$i: ((func@*a*@*b*@xf) ⇒ (lam@*a*@*b*@λxx: \$i: (ap@*a*@*b*@xf@xx)) = xf))))   thf(eta$_1$, conjecture)

SEU697∧2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i>i.(! x:i.in x A → in (f x) B) → lam A B (∧ x:i.f x) = lam A B (∧ x:i.f x))
in: \$i → \$i → \$o   thf(in_type, type)
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i   thf(dpsetconstr_type, type)
lam: \$i → \$i → (\$i → \$i) → \$i   thf(lam_type, type)
lam = (λ*a*: \$i, *b*: \$i, xf: \$i → \$i: (dpsetconstr@*a*@*b*@λxx: \$i, xy: \$i: (xf@xx) = xy))   thf(lam, definition)
∀*a*: \$i, *b*: \$i, xf: \$i → \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@(xf@xx)@*b*)) ⇒ (lam@*a*@*b*@λxx: \$i: (xf@xx)) = (lam@*a*@*b*@λxx: \$i: (xf@xx)))   thf(lam2lamEq, conjecture)

SEU699∧2.p Functions - Extensionality and Beta Reduction
(! A:i.! B:i.! f:i.in f (funcSet A B) → lam A B (∧ x:i.ap A B f x) = f)
in: \$i → \$i → \$o   thf(in_type, type)
funcSet: \$i → \$i → \$i   thf(funcSet_type, type)
ap: \$i → \$i → \$i → \$i → \$i   thf(ap_type, type)
ap2p: \$o   thf(ap2p_type, type)
ap2p = (∀*a*: \$i, *b*: \$i, xf: \$i: ((in@xf@(funcSet@*a*@*b*)) ⇒ ∀xx: \$i: ((in@xx@*a*) ⇒ (in@(ap@*a*@*b*@xf@xx)@*b*))))   thf(ap2p, c
lam: \$i → \$i → (\$i → \$i) → \$i   thf(lam_type, type)
lam2p: \$o   thf(lam2p_type, type)
lam2p = (∀*a*: \$i, *b*: \$i, xf: \$i → \$i: (∀xx: \$i: ((in@xx@*a*) ⇒ (in@(xf@xx)@*b*)) ⇒ (in@(lam@*a*@*b*@λxx: \$i: (xf@xx))@(funcSet
funcext$_2$: \$o   thf(funcext2_type, type)

funcext$_2$ = (∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ ∀xg: \$i: ((in@xg@(funcSet@a@b)) ⇒ (∀xx: \$i: ((in@xx@a) ⇒
(ap@a@b@xf@xx) = (ap@a@b@xg@xx)) ⇒ xf = xg)))) thf(funcext$_2$, definition)
beta$_2$: \$o thf(beta2_type, type)
beta$_2$ = (∀a: \$i, b: \$i, xf: \$i → \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@(xf@xx)@b)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒
(ap@a@b@(lam@a@b@λxy: \$i: (xf@xy))@xx) = (xf@xx)))) thf(beta$_2$, definition)
ap2p ⇒ (lam2p ⇒ (funcext$_2$ ⇒ (beta$_2$ ⇒ ∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ (lam@a@b@λxx: \$i: (ap@a@b@
xf)))) thf(eta$_2$, conjecture)

SEU700∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → phi → in y (dsetconstr A (∧ z:i.phi & z = x — phi & z = y))))
in: \$i → \$i → \$o thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrI: \$o thf(dsetconstrI_type, type)
dsetconstrI = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))))))
dsetconstrI ⇒ ∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (¬ xphi ⇒ (in@xy@(dsetconstr@a@λxz: \$i: ((
xx) or (¬ xphi and xz = xy))))))) thf(iffalseProp$_1$, conjecture)

SEU701∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → phi → dsetconstr A (∧ z:i.phi & z = x — phi & z = y) = setadjoin y emptyset))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrER: \$o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
setext: \$o thf(setext_type, type)
setext = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (∀xx: \$i: ((in@xx@b) ⇒ (in@xx@a)) ⇒ a =
b))) thf(setext, definition)
uniqinunit: \$o thf(uniqinunit_type, type)
uniqinunit = (∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
eqinunit: \$o thf(eqinunit_type, type)
eqinunit = (∀xx: \$i, xy: \$i: (xx = xy ⇒ (in@xx@(setadjoin@xy@emptyset)))) thf(eqinunit, definition)
in__Cong: \$o thf(in__Cong_type, type)
in__Cong = (∀a: \$i, b: \$i: ($a = b$ ⇒ ∀xx: \$i, xy: \$i: (xx = xy ⇒ ((in@xx@a) ⟺ (in@xy@b))))) thf(in__Cong, definition)
iffalseProp$_1$: \$o thf(iffalseProp1_type, type)
iffalseProp$_1$ = (∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (¬ xphi ⇒ (in@xy@(dsetconstr@a@λxz: \$i: ((
xx) or (¬ xphi and xz = xy)))))))) thf(iffalseProp$_1$, definition)
dsetconstrER ⇒ (setext ⇒ (uniqinunit ⇒ (eqinunit ⇒ (in__Cong ⇒ (iffalseProp$_1$ ⇒ ∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a)
∀xy: \$i: ((in@xy@a) ⇒ (¬ xphi ⇒ (dsetconstr@a@λxz: \$i: ((xphi and xz = xx) or (¬ xphi and xz = xy))) =
(setadjoin@xy@emptyset))))))))) thf(iffalseProp$_2$, conjecture)

SEU702∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → phi → in x (dsetconstr A (∧ z:i.phi & z = x — phi & z = y))))
in: \$i → \$i → \$o thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrI: \$o thf(dsetconstrI_type, type)
dsetconstrI = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))))))
dsetconstrI ⇒ ∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (xphi ⇒ (in@xx@(dsetconstr@a@λxz: \$i: ((x
xx) or (¬ xphi and xz = xy))))))) thf(iftrueProp$_1$, conjecture)

SEU703∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → phi → dsetconstr A (∧ z:i.phi & z = x — phi & z = y) = setadjoin x emptyset))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
dsetconstrER: \$o thf(dsetconstrER_type, type)
dsetconstrER = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
setext: \$o thf(setext_type, type)

setext = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (∀xx: \$i: ((in@xx@b) ⇒ (in@xx@a)) ⇒ a = b))) thf(setext, definition)
uniqinunit: \$o thf(uniqinunit_type, type)
uniqinunit = (∀xx: \$i, xy: \$i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
eqinunit: \$o thf(eqinunit_type, type)
eqinunit = (∀xx: \$i, xy: \$i: (xx = xy ⇒ (in@xx@(setadjoin@xy@emptyset)))) thf(eqinunit, definition)
in__Cong: \$o thf(in__Cong_type, type)
in__Cong = (∀a: \$i, b: \$i: (a = b ⇒ ∀xx: \$i, xy: \$i: (xx = xy ⇒ ((in@xx@a) ⟺ (in@xy@b))))) thf(in__Cong, definition)
iftrueProp$_1$: \$o thf(iftrueProp1_type, type)
iftrueProp$_1$ = (∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (xphi ⇒ (in@xx@(dsetconstr@a@λxz: \$i: ((xp
xx) or (¬ xphi and xz = xy)))))))) thf(iftrueProp$_1$, definition)
dsetconstrER ⇒ (setext ⇒ (uniqinunit ⇒ (eqinunit ⇒ (in__Cong ⇒ (iftrueProp$_1$ ⇒ ∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a)
∀xy: \$i: ((in@xy@a) ⇒ (xphi ⇒ (dsetconstr@a@λxz: \$i: ((xphi and xz = xx) or (¬ xphi and xz = xy))) = (setadjoin@xx@emptyset))))))))) thf(iftrueProp$_2$, conjecture)

SEU704∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → singleton (dsetconstr A (∧ z:i.phi & z = x — phi & z = y))))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
iffalseProp$_1$: \$o thf(iffalseProp1_type, type)
iffalseProp$_1$ = (∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (¬ xphi ⇒ (in@xy@(dsetconstr@a@λxz: \$i: ((
xx) or (¬ xphi and xz = xy)))))))) thf(iffalseProp$_1$, definition)
iffalseProp$_2$: \$o thf(iffalseProp2_type, type)
iffalseProp$_2$ = (∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (¬ xphi ⇒ (dsetconstr@a@λxz: \$i: ((xphi and
xx) or (¬ xphi and xz = xy))) = (setadjoin@xy@emptyset))))) thf(iffalseProp$_2$, definition)
iftrueProp$_1$: \$o thf(iftrueProp1_type, type)
iftrueProp$_1$ = (∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (xphi ⇒ (in@xx@(dsetconstr@a@λxz: \$i: ((xp
xx) or (¬ xphi and xz = xy)))))))) thf(iftrueProp$_1$, definition)
iftrueProp$_2$: \$o thf(iftrueProp2_type, type)
iftrueProp$_2$ = (∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (xphi ⇒ (dsetconstr@a@λxz: \$i: ((xphi and x
xx) or (¬ xphi and xz = xy))) = (setadjoin@xx@emptyset))))) thf(iftrueProp$_2$, definition)
iffalseProp$_1$ ⇒ (iffalseProp$_2$ ⇒ (iftrueProp$_1$ ⇒ (iftrueProp$_2$ ⇒ ∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒
∀xy: \$i: ((in@xy@a) ⇒ (singleton@(dsetconstr@a@λxz: \$i: ((xphi and xz = xx) or (¬ xphi and xz = xy))))))))) thf(ifSin

SEU705∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → in (if A phi x y) A))
in: \$i → \$i → \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
setadjoin: \$i → \$i → \$i thf(setadjoin_type, type)
setunion: \$i → \$i thf(setunion_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i thf(dsetconstr_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
subsetE: \$o thf(subsetE_type, type)
subsetE = (∀a: \$i, b: \$i, xx: \$i: ((⊆ @a@b) ⇒ ((in@xx@a) ⇒ (in@xx@b)))) thf(subsetE, definition)
sepSubset: \$o thf(sepSubset_type, type)
sepSubset = (∀a: \$i, xphi: \$i → \$o: (⊆ @(dsetconstr@a@λxx: \$i: (xphi@xx))@a)) thf(sepSubset, definition)
singleton: \$i → \$o thf(singleton_type, type)
singleton = (λa: \$i: ∃xx: \$i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
theprop: \$o thf(theprop_type, type)
theprop = (∀x: \$i: ((singleton@x) ⇒ (in@(setunion@x)@x))) thf(theprop, definition)
ifSingleton: \$o thf(ifSingleton_type, type)
ifSingleton = (∀a: \$i, xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ (singleton@(dsetconstr@a@λxz: \$i: ((xphi and
xx) or (¬ xphi and xz = xy))))))) thf(ifSingleton, definition)
if: \$i → \$o → \$i → \$i → \$i thf(if_type, type)
if = (λa: \$i, xphi: \$o, xx: \$i, xy: \$i: (setunion@(dsetconstr@a@λxz: \$i: ((xphi and xz = xx) or (¬ xphi and xz = xy))))) thf(if, definition)

subsetE ⇒ (sepSubset ⇒ (theprop ⇒ (ifSingleton ⇒ ∀a: $i, xphi: $o, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ (in@(if@a@xphi@xx@xy)@a)))))) thf(ifp, conjecture)

SEU706∧2.p Conditionals
(! X:i.singleton X → (! x:i.in x X → setunion X = x))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setunion: $i → $i thf(setunion_type, type)
uniqinunit: $o thf(uniqinunit_type, type)
uniqinunit = (∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
in__Cong: $o thf(in__Cong_type, type)
in__Cong = (∀a: $i, b: $i: (a = b ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@a) ⟺ (in@xy@b))))) thf(in__Cong, definition)
setadjoin__Cong: $o thf(setadjoin__Cong_type, type)
setadjoin__Cong = (∀xx: $i, xy: $i: (xx = xy ⇒ ∀xz: $i, xu: $i: (xz = xu ⇒ (setadjoin@xx@xz) = (setadjoin@xy@xu)))) t
setunion__Cong: $o thf(setunion__Cong_type, type)
setunion__Cong = (∀a: $i, b: $i: (a = b ⇒ (setunion@a) = (setunion@b))) thf(setunion__Cong, definition)
setunionsingleton: $o thf(setunionsingleton_type, type)
setunionsingleton = (∀xx: $i: (setunion@(setadjoin@xx@emptyset)) = xx) thf(setunionsingleton, definition)
singleton: $i → $o thf(singleton_type, type)
singleton = (λa: $i: ∃xx: $i: (in@xx@a and a = (setadjoin@xx@emptyset))) thf(singleton, definition)
uniqinunit ⇒ (in__Cong ⇒ (setadjoin__Cong ⇒ (setunion__Cong ⇒ (setunionsingleton ⇒ ∀x: $i: ((singleton@x) ⇒ ∀xx: $i: ((in@xx@x) ⇒ (setunion@x) = xx)))))) thf(theeq, conjecture)

SEU707∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → phi → if A phi x y = x))
in: $i → $i → $o thf(in_type, type)
setunion: $i → $i thf(setunion_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
singleton: $i → $o thf(singleton_type, type)
iftrueProp$_1$: $o thf(iftrueProp1_type, type)
iftrueProp$_1$ = (∀a: $i, xphi: $o, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ (xphi ⇒ (in@xx@(dsetconstr@a@λxz: $i: ((x
xx) or (¬ xphi and xz = xy)))))))) thf(iftrueProp$_1$, definition)
ifSingleton: $o thf(ifSingleton_type, type)
ifSingleton = (∀a: $i, xphi: $o, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ (singleton@(dsetconstr@a@λxz: $i: ((xphi and
xx) or (¬ xphi and xz = xy))))))) thf(ifSingleton, definition)
if: $i → $o → $i → $i → $i thf(if_type, type)
if = (λa: $i, xphi: $o, xx: $i, xy: $i: (setunion@(dsetconstr@a@λxz: $i: ((xphi and xz = xx) or (¬ xphi and xz = xy))))) thf(if, definition)
theeq: $o thf(theeq_type, type)
theeq = (∀x: $i: ((singleton@x) ⇒ ∀xx: $i: ((in@xx@x) ⇒ (setunion@x) = xx))) thf(theeq, definition)
iftrueProp$_1$ ⇒ (ifSingleton ⇒ (theeq ⇒ ∀a: $i, xphi: $o, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ (xphi ⇒ (if@a@xphi@xx@xy) = xx))))) thf(iftrue, conjecture)

SEU708∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → phi → if A phi x y = y))
in: $i → $i → $o thf(in_type, type)
setunion: $i → $i thf(setunion_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
singleton: $i → $o thf(singleton_type, type)
iffalseProp$_1$: $o thf(iffalseProp1_type, type)
iffalseProp$_1$ = (∀a: $i, xphi: $o, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ (¬ xphi ⇒ (in@xy@(dsetconstr@a@λxz: $i: ((
xx) or (¬ xphi and xz = xy)))))))) thf(iffalseProp$_1$, definition)
ifSingleton: $o thf(ifSingleton_type, type)
ifSingleton = (∀a: $i, xphi: $o, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ (singleton@(dsetconstr@a@λxz: $i: ((xphi and
xx) or (¬ xphi and xz = xy))))))) thf(ifSingleton, definition)
if: $i → $o → $i → $i → $i thf(if_type, type)
if = (λa: $i, xphi: $o, xx: $i, xy: $i: (setunion@(dsetconstr@a@λxz: $i: ((xphi and xz = xx) or (¬ xphi and xz = xy))))) thf(if, definition)
theeq: $o thf(theeq_type, type)
theeq = (∀x: $i: ((singleton@x) ⇒ ∀xx: $i: ((in@xx@x) ⇒ (setunion@x) = xx))) thf(theeq, definition)

$\mathrm{iffalseProp}_1$ ⇒ (ifSingleton ⇒ (theeq ⇒ ∀*a*: $i, xphi: $o, xx: $i: ((in@xx@*a*) ⇒ ∀xy: $i: ((in@xy@*a*) ⇒ (¬ xphi ⇒ (if@*a*@xphi@xx@xy) = xy))))) thf(iffalse, conjecture)

SEU709∧2.p Conditionals
(! A:i.! phi:o.! x:i.in x A → (! y:i.in y A → in (if A phi x y) (setadjoin x (setadjoin y emptyset))))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
setadjoinIL: $o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
in__Cong: $o thf(in__Cong_type, type)
in__Cong = (∀*a*: $i, *b*: $i: (*a* = *b* ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@*a*) ⟺ (in@xy@*b*))))) thf(in__Cong, definition)
secondinupair: $o thf(secondinupair_type, type)
secondinupair = (∀xx: $i, xy: $i: (in@xy@(setadjoin@xx@(setadjoin@xy@emptyset)))) thf(secondinupair, definition)
if: $i → $o → $i → $i → $i thf(if_type, type)
iftrue: $o thf(iftrue_type, type)
iftrue = (∀*a*: $i, xphi: $o, xx: $i: ((in@xx@*a*) ⇒ ∀xy: $i: ((in@xy@*a*) ⇒ (xphi ⇒ (if@*a*@xphi@xx@xy) = xx)))) thf(iftrue, definition)
iffalse: $o thf(iffalse_type, type)
iffalse = (∀*a*: $i, xphi: $o, xx: $i: ((in@xx@*a*) ⇒ ∀xy: $i: ((in@xy@*a*) ⇒ (¬ xphi ⇒ (if@*a*@xphi@xx@xy) = xy)))) thf(iffalse, definition)
setadjoinIL ⇒ (in__Cong ⇒ (secondinupair ⇒ (iftrue ⇒ (iffalse ⇒ ∀*a*: $i, xphi: $o, xx: $i: ((in@xx@*a*) ⇒ ∀xy: $i: ((in@xy@*a*) ⇒ (in@(if@*a*@xphi@xx@xy)@(setadjoin@xx@(setadjoin@xy@emptyset))))))))) thf(iftrueorfalse, con

SEU711∧2.p Typed Set Theory - Types of Set Operators
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → in (binunion X Y) (powerset A)))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
powersetI: $o thf(powersetI_type, type)
powersetI = (∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*b*) ⇒ (in@xx@*a*)) ⇒ (in@*b*@(powerset@*a*)))) thf(powersetI, definition)
powersetE: $o thf(powersetE_type, type)
powersetE = (∀*a*: $i, *b*: $i, xx: $i: ((in@*b*@(powerset@*a*)) ⇒ ((in@xx@*b*) ⇒ (in@xx@*a*)))) thf(powersetE, definition)
binunion: $i → $i → $i thf(binunion_type, type)
binunionEcases: $o thf(binunionEcases_type, type)
binunionEcases = (∀*a*: $i, *b*: $i, xx: $i, xphi: $o: ((in@xx@(binunion@*a*@*b*)) ⇒ (((in@xx@*a*) ⇒ xphi) ⇒ (((in@xx@*b*) ⇒ xphi) ⇒ xphi)))) thf(binunionEcases, definition)
powersetI ⇒ (powersetE ⇒ (binunionEcases ⇒ ∀*a*: $i, *x*: $i: ((in@*x*@(powerset@*a*)) ⇒ ∀*y*: $i: ((in@*y*@(powerset@*a*)) ⇒ (in@(binunion@*x*@*y*)@(powerset@*a*)))))) thf(binunionT_lem, conjecture)

SEU712∧2.p Typed Set Theory - Types of Set Operators
(! A:i.! X:i.in X (powerset A) → in (powerset X) (powerset (powerset A)))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
powersetI: $o thf(powersetI_type, type)
powersetI = (∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*b*) ⇒ (in@xx@*a*)) ⇒ (in@*b*@(powerset@*a*)))) thf(powersetI, definition)
powersetE: $o thf(powersetE_type, type)
powersetE = (∀*a*: $i, *b*: $i, xx: $i: ((in@*b*@(powerset@*a*)) ⇒ ((in@xx@*b*) ⇒ (in@xx@*a*)))) thf(powersetE, definition)
powersetI ⇒ (powersetE ⇒ ∀*a*: $i, *x*: $i: ((in@*x*@(powerset@*a*)) ⇒ (in@(powerset@*x*)@(powerset@(powerset@*a*))))) th

SEU713∧2.p Typed Set Theory - Types of Set Operators
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → in (setminus X Y) (powerset A)))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
powersetI: $o thf(powersetI_type, type)
powersetI = (∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*b*) ⇒ (in@xx@*a*)) ⇒ (in@*b*@(powerset@*a*)))) thf(powersetI, definition)
powersetE: $o thf(powersetE_type, type)
powersetE = (∀*a*: $i, *b*: $i, xx: $i: ((in@*b*@(powerset@*a*)) ⇒ ((in@xx@*b*) ⇒ (in@xx@*a*)))) thf(powersetE, definition)
setminus: $i → $i → $i thf(setminus_type, type)
setminusEL: $o thf(setminusEL_type, type)
setminusEL = (∀*a*: $i, *b*: $i, xx: $i: ((in@xx@(setminus@*a*@*b*)) ⇒ (in@xx@*a*))) thf(setminusEL, definition)

powersetI $\Rightarrow$ (powersetE $\Rightarrow$ (setminusEL $\Rightarrow \forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow \forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ (in@(setminus@x@y)@(powerset@a)))))) thf(setminusT_lem, conjecture)

SEU714^2.p Typed Set Theory - Types of Set Operators
(! A:i.! X:i.in X (powerset A) → in (setminus A X) (powerset A))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
powersetI: \$o thf(powersetI_type, type)
powersetI = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@b) $\Rightarrow$ (in@xx@a)) $\Rightarrow$ (in@b@(powerset@a)))) thf(powersetI, definition)
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusEL: \$o thf(setminusEL_type, type)
setminusEL = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@(setminus@a@b)) $\Rightarrow$ (in@xx@a))) thf(setminusEL, definition)
powersetI $\Rightarrow$ (setminusEL $\Rightarrow \forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow$ (in@(setminus@a@x)@(powerset@a)))) thf(compl

SEU715^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x X → in x Y) → (! x:i.in x A → in x Y → in x X) → X = Y))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
setext: \$o thf(setext_type, type)
setext = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@b) $\Rightarrow$ (in@xx@a)) $\Rightarrow$ $a = b$))) thf(setext, definition)
powersetE: \$o thf(powersetE_type, type)
powersetE = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@b@(powerset@a)) $\Rightarrow$ ((in@xx@b) $\Rightarrow$ (in@xx@a)))) thf(powersetE, definition)
setext $\Rightarrow$ (powersetE $\Rightarrow \forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow \forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@x) $\Rightarrow$ (in@xx@y))) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@y) $\Rightarrow$ (in@xx@x))) $\Rightarrow$ $x = y$))))) thf(setextT, conje

SEU716^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x X → in x Y) → subset X Y))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
powersetE: \$o thf(powersetE_type, type)
powersetE = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@b@(powerset@a)) $\Rightarrow$ ((in@xx@b) $\Rightarrow$ (in@xx@a)))) thf(powersetE, definition)
$\subseteq$: \$i → \$i → \$o thf(subset_type, type)
subsetI_2: \$o thf(subsetI2_type, type)
subsetI_2 = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\subseteq$ @a@b))) thf(subsetI_2, definition)
powersetE $\Rightarrow$ (subsetI_2 $\Rightarrow \forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow \forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@x) $\Rightarrow$ (in@xx@y))) $\Rightarrow$ ($\subseteq$ @x@y))))) thf(subsetTI, conjecture)

SEU717^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x X → in x Y) → in X (powerset Y)))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
$\subseteq$: \$i → \$i → \$o thf(subset_type, type)
powersetI_1: \$o thf(powersetI1_type, type)
powersetI_1 = ($\forall a$: \$i, b: \$i: (($\subseteq$ @b@a) $\Rightarrow$ (in@b@(powerset@a)))) thf(powersetI_1, definition)
subsetTI: \$o thf(subsetTI_type, type)
subsetTI = ($\forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow \forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@x) $\Rightarrow$ (in@xx@y))) $\Rightarrow$ ($\subseteq$ @x@y))))) thf(subsetTI, definition)
powersetI_1 $\Rightarrow$ (subsetTI $\Rightarrow \forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow \forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@x) $\Rightarrow$ (in@xx@y))) $\Rightarrow$ (in@x@(powerset@y)))))) thf(powersetTI_1, conjecture)

SEU718^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in X (powerset Y) → in x X → in x Y)))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
$\subseteq$: \$i → \$i → \$o thf(subset_type, type)
subsetE: \$o thf(subsetE_type, type)
subsetE = ($\forall a$: \$i, b: \$i, xx: \$i: (($\subseteq$ @a@b) $\Rightarrow$ ((in@xx@a) $\Rightarrow$ (in@xx@b)))) thf(subsetE, definition)
powersetE_1: \$o thf(powersetE1_type, type)

$\mathrm{powersetE}_1 = (\forall a\colon \$i, b\colon \$i\colon ((\mathrm{in}@b@(\mathrm{powerset}@a)) \Rightarrow (\subseteq @b@a)))$ thf(powersetE$_1$, definition)
$\mathrm{subsetE} \Rightarrow (\mathrm{powersetE}_1 \Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@a)) \Rightarrow \forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow ((\mathrm{in}@x@(\mathrm{powerset}@y)) \Rightarrow ((\mathrm{in}@\mathrm{xx}@x) \Rightarrow (\mathrm{in}@\mathrm{xx}@y)))))))$ thf(powersetTE$_1$, conjecture)

SEU719^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! x:i.in x A → in x X → (in x (setminus A X))))
$\mathrm{in}\colon \$i \rightarrow \$i \rightarrow \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \rightarrow \$i$ thf(powerset_type, type)
$\mathrm{setminus}\colon \$i \rightarrow \$i \rightarrow \$i$ thf(setminus_type, type)
$\mathrm{setminusER}\colon \$o$ thf(setminusER_type, type)
$\mathrm{setminusER} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)) \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@b))$ thf(setminusER, definition)
$\mathrm{setminusER} \Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow ((\mathrm{in}@\mathrm{xx}@x) \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@x))$

SEU720^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! x:i.in x A → (in x (setminus A X)) → in x X))
$\mathrm{in}\colon \$i \rightarrow \$i \rightarrow \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \rightarrow \$i$ thf(powerset_type, type)
$\mathrm{setminus}\colon \$i \rightarrow \$i \rightarrow \$i$ thf(setminus_type, type)
$\mathrm{setminusI}\colon \$o$ thf(setminusI_type, type)
$\mathrm{setminusI} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@b \Rightarrow (\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)))))$ thf(setminusI, definition)
$\mathrm{setminusI} \Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@x) \Rightarrow (\mathrm{in}@\mathrm{xx}@x))))$ thf(complementTE$_1$, conjecture)

SEU721^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → (in x X) → (in x (binintersect X Y)))))
$\mathrm{in}\colon \$i \rightarrow \$i \rightarrow \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \rightarrow \$i$ thf(powerset_type, type)
$\mathrm{binintersect}\colon \$i \rightarrow \$i \rightarrow \$i$ thf(binintersect_type, type)
$\mathrm{binintersectEL}\colon \$o$ thf(binintersectEL_type, type)
$\mathrm{binintersectEL} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@a@b)) \Rightarrow (\mathrm{in}@\mathrm{xx}@a)))$ thf(binintersectEL, definition)
$\mathrm{binintersectEL} \Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@a)) \Rightarrow \forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@x \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@x@y)))))$ thf(binintersectTELcontra, conjecture)

SEU722^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → (in x Y) → (in x (binintersect X Y)))))
$\mathrm{in}\colon \$i \rightarrow \$i \rightarrow \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \rightarrow \$i$ thf(powerset_type, type)
$\mathrm{binintersect}\colon \$i \rightarrow \$i \rightarrow \$i$ thf(binintersect_type, type)
$\mathrm{binintersectER}\colon \$o$ thf(binintersectER_type, type)
$\mathrm{binintersectER} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@a@b)) \Rightarrow (\mathrm{in}@\mathrm{xx}@b)))$ thf(binintersectER, definition)
$\mathrm{binintersectER} \Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@a)) \Rightarrow \forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@y \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@(\mathrm{binintersect}@x@y)))))$ thf(binintersectTERcontra, conjecture)

SEU723^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → subset X (setminus A Y) → in x Y → (in x X))))
$\mathrm{in}\colon \$i \rightarrow \$i \rightarrow \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \rightarrow \$i$ thf(powerset_type, type)
$\subseteq\colon \$i \rightarrow \$i \rightarrow \$o$ thf(subset_type, type)
$\mathrm{subsetE}\colon \$o$ thf(subsetE_type, type)
$\mathrm{subsetE} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b))))$ thf(subsetE, definition)
$\mathrm{setminus}\colon \$i \rightarrow \$i \rightarrow \$i$ thf(setminus_type, type)
$\mathrm{setminusER}\colon \$o$ thf(setminusER_type, type)
$\mathrm{setminusER} = (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)) \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@b))$ thf(setminusER, definition)
$\mathrm{subsetE} \Rightarrow (\mathrm{setminusER} \Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@a)) \Rightarrow \forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow ((\subseteq @x@(\mathrm{setminus}@a@y)) \Rightarrow ((\mathrm{in}@\mathrm{xx}@y) \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@x))))))$ thf(contrasubsetT, conjecture)

SEU724^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → subset X Y → (in x Y) → (in x X))))
$\mathrm{in}\colon \$i \rightarrow \$i \rightarrow \$o$ thf(in_type, type)
$\mathrm{powerset}\colon \$i \rightarrow \$i$ thf(powerset_type, type)
$\subseteq\colon \$i \rightarrow \$i \rightarrow \$o$ thf(subset_type, type)
$\mathrm{subsetE}\colon \$o$ thf(subsetE_type, type)

subsetE $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b))))$ thf(subsetE, definition)
subsetE $\Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@a)) \Rightarrow \forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow ((\subseteq @x@y) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@y \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@x)))))$ thf(contrasubsetT_1, conjecture)

SEU725^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → subset X Y → subset (setminus A Y) (setminus A X)))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
powerset: $\$i \to \$i$ thf(powerset_type, type)
$\subseteq$: $\$i \to \$i \to \$o$ thf(subset_type, type)
subsetE: \$o thf(subsetE_type, type)
subsetE $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b))))$ thf(subsetE, definition)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@b \Rightarrow (\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)))))$ thf(setminusI, definition)
setminusER: \$o thf(setminusER_type, type)
setminusER $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)) \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@b))$ thf(setminusER, definition)
complementT_lem: \$o thf(complementT_lem_type, type)
complementT_lem $= (\forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow (\mathrm{in}@(\mathrm{setminus}@a@x)@(\mathrm{powerset}@a))))$ thf(complementT_l
subsetTI: \$o thf(subsetTI_type, type)
subsetTI $= (\forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@a)) \Rightarrow (\forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow ((\mathrm{in}@\mathrm{xx}@x) \Rightarrow (\mathrm{in}@\mathrm{xx}@y))) \Rightarrow (\subseteq @x@y)))))$ thf(subsetTI, definition)
subsetE $\Rightarrow$ (setminusI $\Rightarrow$ (setminusER $\Rightarrow$ (complementT_lem $\Rightarrow$ (subsetTI $\Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@a)) \Rightarrow ((\subseteq @x@y) \Rightarrow (\subseteq @(\mathrm{setminus}@a@y)@(\mathrm{setminus}@a@x))))))))$ thf(contrasubsetT_2, co

SEU726^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → subset (setminus A Y) (setminus A X) → subset X Y))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
powerset: $\$i \to \$i$ thf(powerset_type, type)
$\subseteq$: $\$i \to \$i \to \$o$ thf(subset_type, type)
subsetE: \$o thf(subsetE_type, type)
subsetE $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\mathrm{in}@\mathrm{xx}@b))))$ thf(subsetE, definition)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@b \Rightarrow (\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)))))$ thf(setminusI, definition)
setminusER: \$o thf(setminusER_type, type)
setminusER $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)) \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@b))$ thf(setminusER, definition)
subsetTI: \$o thf(subsetTI_type, type)
subsetTI $= (\forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@a)) \Rightarrow (\forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow ((\mathrm{in}@\mathrm{xx}@x) \Rightarrow (\mathrm{in}@\mathrm{xx}@y))) \Rightarrow (\subseteq @x@y)))))$ thf(subsetTI, definition)
subsetE $\Rightarrow$ (setminusI $\Rightarrow$ (setminusER $\Rightarrow$ (subsetTI $\Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall y\colon \$i\colon ((\mathrm{in}@y@(\mathrm{powerset}@$ $((\subseteq @(\mathrm{setminus}@a@y)@(\mathrm{setminus}@a@x)) \Rightarrow (\subseteq @x@y)))))))$ thf(contrasubsetT_3, conjecture)

SEU727^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! x:i.in x A → in x X → in x (setminus A (setminus A X))))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
powerset: $\$i \to \$i$ thf(powerset_type, type)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@b \Rightarrow (\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)))))$ thf(setminusI, definition)
setminusER: \$o thf(setminusER_type, type)
setminusER $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)) \Rightarrow \neg\, \mathrm{in}@\mathrm{xx}@b))$ thf(setminusER, definition)
setminusI $\Rightarrow$ (setminusER $\Rightarrow \forall a\colon \$i, x\colon \$i\colon ((\mathrm{in}@x@(\mathrm{powerset}@a)) \Rightarrow \forall \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow ((\mathrm{in}@\mathrm{xx}@x) \Rightarrow (\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@(\mathrm{setminus}@a@x)))))))$ thf(doubleComplementI_1, conjecture)

SEU728^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! x:i.in x A → in x (setminus A (setminus A X)) → in x X))
in: $\$i \to \$i \to \$o$ thf(in_type, type)
powerset: $\$i \to \$i$ thf(powerset_type, type)
setminus: $\$i \to \$i \to \$i$ thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI $= (\forall a\colon \$i, b\colon \$i, \mathrm{xx}\colon \$i\colon ((\mathrm{in}@\mathrm{xx}@a) \Rightarrow (\neg\, \mathrm{in}@\mathrm{xx}@b \Rightarrow (\mathrm{in}@\mathrm{xx}@(\mathrm{setminus}@a@b)))))$ thf(setminusI, definition)

setminusER: $o   thf(setminusER_type, type)
setminusER = (∀a: $i, b: $i, xx: $i: ((in@xx@(setminus@a@b)) ⇒ ¬ in@xx@b))   thf(setminusER, definition)
setminusI ⇒ (setminusER ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(setr
(in@xx@x)))))   thf(doubleComplementE$_1$, conjecture)

SEU729^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → subset X (setminus A (setminus A X)))
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
setminus: $i → $i → $i   thf(setminus_type, type)
complementT_lem: $o   thf(complementT_lem_type, type)
complementT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ (in@(setminus@a@x)@(powerset@a))))   thf(complementT_l
subsetTI: $o   thf(subsetTI_type, type)
subsetTI = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ((in@xx@x) ⇒ (in@xx@y))) ⇒ (⊆ @x@y)))))   thf(subsetTI, definition)
doubleComplementI$_1$: $o   thf(doubleComplementI1_type, type)
doubleComplementI$_1$ = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@x) ⇒ (in@xx@(setminus@a@(setminus@a@x)))))))   thf(doubleComplementI$_1$, definition)
complementT_lem ⇒ (subsetTI ⇒ (doubleComplementI$_1$ ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ (⊆ @x@(setminus@a@(setminus@a@x))))))   thf(doubleComplementSub$_1$, conjecture)

SEU730^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → subset (setminus A (setminus A X)) X)
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
setminus: $i → $i → $i   thf(setminus_type, type)
complementT_lem: $o   thf(complementT_lem_type, type)
complementT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ (in@(setminus@a@x)@(powerset@a))))   thf(complementT_l
subsetTI: $o   thf(subsetTI_type, type)
subsetTI = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ((in@xx@x) ⇒ (in@xx@y))) ⇒ (⊆ @x@y)))))   thf(subsetTI, definition)
doubleComplementE$_1$: $o   thf(doubleComplementE1_type, type)
doubleComplementE$_1$ = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(setminus@
(in@xx@x)))))   thf(doubleComplementE$_1$, definition)
complementT_lem ⇒ (subsetTI ⇒ (doubleComplementE$_1$ ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ (⊆ @(setminus@a@(setminus@a@x))@x))))   thf(doubleComplementSub$_2$, conjecture)

SEU731^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → X = setminus A (setminus A X))
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
setminus: $i → $i → $i   thf(setminus_type, type)
complementT_lem: $o   thf(complementT_lem_type, type)
complementT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ (in@(setminus@a@x)@(powerset@a))))   thf(complementT_l
setextT: $o   thf(setextT_type, type)
setextT = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ((in@xx@x) ⇒ (in@xx@y))) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ((in@xx@y) ⇒ (in@xx@x))) ⇒ x = y)))))   thf(setextT, defin
doubleComplementI$_1$: $o   thf(doubleComplementI1_type, type)
doubleComplementI$_1$ = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@x) ⇒ (in@xx@(setminus@a@(setminus@a@x)))))))   thf(doubleComplementI$_1$, definition)
doubleComplementE$_1$: $o   thf(doubleComplementE1_type, type)
doubleComplementE$_1$ = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(setminus@
(in@xx@x)))))   thf(doubleComplementE$_1$, definition)
complementT_lem ⇒ (setextT ⇒ (doubleComplementI$_1$ ⇒ (doubleComplementE$_1$ ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a))
x = (setminus@a@(setminus@a@x))))))   thf(doubleComplementEq, conjecture)

SEU732^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (setminus A X) → (in x (binintersect X Y)))))

in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
binintersect: $i → $i → $i   thf(binintersect_type, type)
setminus: $i → $i → $i   thf(setminus_type, type)
setminusER: $o   thf(setminusER_type, type)
setminusER = (∀a: $i, b: $i, xx: $i: ((in@xx@(setminus@a@b)) ⇒ ¬ in@xx@b))   thf(setminusER, definition)
binintersectTELcontra: $o   thf(binintersectTELcontra_type, type)
binintersectTELcontra = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒
(¬ in@xx@x ⇒ ¬ in@xx@(binintersect@x@y))))))   thf(binintersectTELcontra, definition)
setminusER ⇒ (binintersectTELcontra ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@x)) ⇒ ¬ in@xx@(binintersect@x@y))))))   thf(complementTnotintersectT

SEU733^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (setminus A X) → in x (setminus A (binintersect X Y)))))
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
binintersect: $i → $i → $i   thf(binintersect_type, type)
setminus: $i → $i → $i   thf(setminus_type, type)
setminusI: $o   thf(setminusI_type, type)
setminusI = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ (¬ in@xx@b ⇒ (in@xx@(setminus@a@b)))))   thf(setminusI, definition)
complementTnotintersectT: $o   thf(complementTnotintersectT_type, type)
complementTnotintersectT = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a)
((in@xx@(setminus@a@x)) ⇒ ¬ in@xx@(binintersect@x@y))))))   thf(complementTnotintersectT, definition)
setminusI ⇒ (complementTnotintersectT ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@x)) ⇒ (in@xx@(setminus@a@(binintersect@x@y))))))))   thf(complemen

SEU734^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → subset (setminus A X) (setminus A (binintersect X Y))))
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
binintersect: $i → $i → $i   thf(binintersect_type, type)
setminus: $i → $i → $i   thf(setminus_type, type)
binintersectT_lem: $o   thf(binintersectT_lem_type, type)
binintersectT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (in@(binintersect@x@y)@(po
complementT_lem: $o   thf(complementT_lem_type, type)
complementT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ (in@(setminus@a@x)@(powerset@a))))   thf(complementT_l
subsetTI: $o   thf(subsetTI_type, type)
subsetTI = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (∀xx: $i: ((in@xx@a) ⇒
((in@xx@x) ⇒ (in@xx@y))) ⇒ (⊆ @x@y)))))   thf(subsetTI, definition)
complementImpComplementIntersect: $o   thf(complementImpComplementIntersect_type, type)
complementImpComplementIntersect = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@x)) ⇒ (in@xx@(setminus@a@(binintersect@x@y))))))))   thf(complemen
binintersectT_lem ⇒ (complementT_lem ⇒ (subsetTI ⇒ (complementImpComplementIntersect ⇒ ∀a: $i, x: $i: ((in@x@(
∀y: $i: ((in@y@(powerset@a)) ⇒ (⊆ @(setminus@a@x)@(setminus@a@(binintersect@x@y))))))))   thf(complementSubse

SEU735^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → in (setminus A X) (powerset (setminus A (binintersect X Y)))))
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
$\mathrm{powersetI}_1$: $o   thf(powersetI1_type, type)
$\mathrm{powersetI}_1$ = (∀a: $i, b: $i: ((⊆ @b@a) ⇒ (in@b@(powerset@a))))   thf($\mathrm{powersetI}_1$, definition)
binintersect: $i → $i → $i   thf(binintersect_type, type)
setminus: $i → $i → $i   thf(setminus_type, type)
complementSubsetComplementIntersect: $o   thf(complementSubsetComplementIntersect_type, type)
complementSubsetComplementIntersect = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
(⊆ @(setminus@a@x)@(setminus@a@(binintersect@x@y))))))   thf(complementSubsetComplementIntersect, definition)

powersetI1 ⇒ (complementSubsetComplementIntersect ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerse
(in@(setminus@a@x)@(powerset@(setminus@a@(binintersect@x@y))))))) thf(complementInPowersetComplementInterse

SEU736^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → subset X (setminus A Y) → (! x:i.in x A → in x Y → in x (setminus A X))))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
setminus: $i → $i → $i thf(setminus_type, type)
setminusI: $o thf(setminusI_type, type)
setminusI = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ (¬ in@xx@b ⇒ (in@xx@(setminus@a@b))))) thf(setminusI, definition)
contrasubsetT: $o thf(contrasubsetT_type, type)
contrasubsetT = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((⊆ @x@(setminus@a@y)) ⇒ ((in@xx@y) ⇒ ¬ in@xx@x)))))) thf(contrasubsetT, definition)
setminusI ⇒ (contrasubsetT ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ((⊆ @x@(setminus@a@y)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@y) ⇒ (in@xx@(setminus@a@x)))))))) thf(contraSubsetCon

SEU737^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → subset X (setminus A Y) → subset Y (setminus A X)))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
setminus: $i → $i → $i thf(setminus_type, type)
complementT_lem: $o thf(complementT_lem_type, type)
complementT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ (in@(setminus@a@x)@(powerset@a)))) thf(complementT_l
subsetTI: $o thf(subsetTI_type, type)
subsetTI = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ((in@xx@x) ⇒ (in@xx@y))) ⇒ (⊆ @x@y))))) thf(subsetTI, definition)
contraSubsetComplement: $o thf(contraSubsetComplement_type, type)
contraSubsetComplement = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ((⊆ @x@(setminus@a@y)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@y) ⇒ (in@xx@(setminus@a@x)))))))) thf(contraSubsetCon
complementT_lem ⇒ (subsetTI ⇒ (contraSubsetComplement ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ((⊆ @x@(setminus@a@y)) ⇒ (⊆ @y@(setminus@a@x))))))) thf(complementTcontraS

SEU738^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → (in x (binunion X Y)) → (in x X))))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
binunion: $i → $i → $i thf(binunion_type, type)
binunionIL: $o thf(binunionIL_type, type)
binunionIL = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ (in@xx@(binunion@a@b)))) thf(binunionIL, definition)
binunionIL ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ (¬ in@xx@(binunion@x@y) ⇒ ¬ in@xx@x)))) thf(binunionTILcontra, conjecture)

SEU739^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → (in x (binunion X Y)) → (in x Y))))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
binunion: $i → $i → $i thf(binunion_type, type)
binunionIR: $o thf(binunionIR_type, type)
binunionIR = (∀a: $i, b: $i, xx: $i: ((in@xx@b) ⇒ (in@xx@(binunion@a@b)))) thf(binunionIR, definition)
binunionIR ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ (¬ in@xx@(binunion@x@y) ⇒ ¬ in@xx@y)))) thf(binunionTIRcontra, conjecture)

SEU740^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → (! x:i.in x A → in x (binintersect X Y) → in x (binunion X Z)))))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
binunion: $i → $i → $i thf(binunion_type, type)
binunionIL: $o thf(binunionIL_type, type)

binunionIL = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ (in@xx@(binunion@a@b)))) thf(binunionIL, definition)
binintersect: $i → $i → $i thf(binintersect_type, type)
binintersectEL: $o thf(binintersectEL_type, type)
binintersectEL = (∀a: $i, b: $i, xx: $i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@a))) thf(binintersectEL, definition)
binunionIL ⇒ (binintersectEL ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
∀z: $i: ((in@z@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(binintersect@x@y)) ⇒ (in@xx@(binunion@x@z)))))))

SEU741^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → (! x:i.in x A → in x (binintersect X Y) → in x (binunion Y Z)))))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
binunion: $i → $i → $i thf(binunion_type, type)
binunionIL: $o thf(binunionIL_type, type)
binunionIL = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ (in@xx@(binunion@a@b)))) thf(binunionIL, definition)
binintersect: $i → $i → $i thf(binintersect_type, type)
binintersectER: $o thf(binintersectER_type, type)
binintersectER = (∀a: $i, b: $i, xx: $i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@b))) thf(binintersectER, definition)
binunionIL ⇒ (binintersectER ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
∀z: $i: ((in@z@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(binintersect@x@y)) ⇒ (in@xx@(binunion@y@z)))))))

SEU742^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → (! x:i.in x A → in x (binintersect X Y) → in x (binintersect (binunion X Z) (binunion Y Z))))))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
binunion: $i → $i → $i thf(binunion_type, type)
binintersect: $i → $i → $i thf(binintersect_type, type)
binintersectI: $o thf(binintersectI_type, type)
binintersectI = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ((in@xx@b) ⇒ (in@xx@(binintersect@a@b))))) thf(binintersectI, defi
inIntersectImpInUnion: $o thf(inIntersectImpInUnion_type, type)
inIntersectImpInUnion = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀z: $i: ((in@z@(powerse
∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(binintersect@x@y)) ⇒ (in@xx@(binunion@x@z)))))))) thf(inIntersectImpInUnion, de
inIntersectImpInUnion$_2$: $o thf(inIntersectImpInUnion2_type, type)
inIntersectImpInUnion$_2$ = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀z: $i: ((in@z@(powers
∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(binintersect@x@y)) ⇒ (in@xx@(binunion@y@z)))))))) thf(inIntersectImpInUnion$_2$, c
binintersectI ⇒ (inIntersectImpInUnion ⇒ (inIntersectImpInUnion$_2$ ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒
∀y: $i: ((in@y@(powerset@a)) ⇒ ∀z: $i: ((in@z@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(binintersect@x@y)) ⇒
(in@xx@(binintersect@(binunion@x@z)@(binunion@y@z))))))))) thf(inIntersectImpInIntersectUnions, conjecture)

SEU743^2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → in (binintersect X Y) (powerset (binintersect (binunion X Z) (binunion Y Z))))))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
binunion: $i → $i → $i thf(binunion_type, type)
binintersect: $i → $i → $i thf(binintersect_type, type)
binintersectT_lem: $o thf(binintersectT_lem_type, type)
binintersectT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (in@(binintersect@x@y)@(po
binunionT_lem: $o thf(binunionT_lem_type, type)
binunionT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (in@(binunion@x@y)@(powerse
powersetTI$_1$: $o thf(powersetTI1_type, type)
powersetTI$_1$ = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ((in@xx@x) ⇒ (in@xx@y))) ⇒ (in@x@(powerset@y)))))) thf(powersetTI$_1$, definition)
inIntersectImpInIntersectUnions: $o thf(inIntersectImpInIntersectUnions_type, type)
inIntersectImpInIntersectUnions = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
∀z: $i: ((in@z@(powerset@a)) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ((in@xx@(binintersect@x@y)) ⇒ (in@xx@(binintersect@(binunio
binintersectT_lem ⇒ (binunionT_lem ⇒ (powersetTI$_1$ ⇒ (inIntersectImpInIntersectUnions ⇒ ∀a: $i, x: $i: ((in@x@(powe
∀y: $i: ((in@y@(powerset@a)) ⇒ ∀z: $i: ((in@z@(powerset@a)) ⇒ (in@(binintersect@x@y)@(powerset@(binintersect@(bin

SEU744^2.p Typed Set Theory - Laws for Typed Sets

(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (setminus A (binunion X Y)) → (in x X))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusER: \$o thf(setminusER_type, type)
setminusER = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(setminus@a@b)) ⇒ ¬ in@xx@b)) thf(setminusER, definition)
binunionTILcontra: \$o thf(binunionTILcontra_type, type)
binunionTILcontra = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (¬ in@xx@(binunion@x@y) ⇒ ¬ in@xx@x))))) thf(binunionTILcontra, definition)
setminusER ⇒ (binunionTILcontra ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(binunion@x@y))) ⇒ ¬ in@xx@x))))) thf(inComplementUnionImpNotI

SEU745∧2.p Typed Set Theory - Laws for Typed Sets - DeMorgan Laws
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (setminus A (binunion X Y)) → in x (setminus A X))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ (¬ in@xx@b ⇒ (in@xx@(setminus@a@b))))) thf(setminusI, definition)
inComplementUnionImpNotIn$_1$: \$o thf(inComplementUnionImpNotIn1_type, type)
inComplementUnionImpNotIn$_1$ = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(binunion@x@y))) ⇒ ¬ in@xx@x))))) thf(inComplementUnionImpNotI
setminusI ⇒ (inComplementUnionImpNotIn$_1$ ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(binunion@x@y))) ⇒ (in@xx@(setminus@a@x))))))) thf(inComplemen

SEU746∧2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! phi:o.! x:i.in x A → in x (binunion X Y) → (in x X → phi) → (in x Y → phi) → phi)))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
binunionE: \$o thf(binunionE_type, type)
binunionE = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(binunion@a@b)) ⇒ (in@xx@a or in@xx@b))) thf(binunionE, definition)
binunionE ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xphi: \$o, xx: \$i: ((in@xx@a) ⇒ ((in@xx@(binunion@x@y)) ⇒ (((in@xx@x) ⇒ xphi) ⇒ (((in@xx@y) ⇒ xphi) ⇒ xphi)))))) thf(binunionTE, conject

SEU747∧2.p Typed Set Theory - Laws for Typed Sets
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → (in x X) → (in x Y) → (in x (binunion X Y)))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
binunionE: \$o thf(binunionE_type, type)
binunionE = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(binunion@a@b)) ⇒ (in@xx@a or in@xx@b))) thf(binunionE, definition)
binunionE ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (¬ in@xx@x ⇒ (¬ in@xx@y ⇒ ¬ in@xx@(binunion@x@y)))))) thf(binunionTEcontra, conjecture)

SEU749∧2.p Typed Set Theory - Laws for Typed Sets - DeMorgan Laws
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → in (setminus A (binunion X Y)) (powerset (setminus A X))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
setminus: \$i → \$i → \$i thf(setminus_type, type)
binunionT_lem: \$o thf(binunionT_lem_type, type)
binunionT_lem = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ (in@(binunion@x@y)@(powerse
complementT_lem: \$o thf(complementT_lem_type, type)

complementT_lem = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ (in@(setminus@a@x)@(powerset@a)))) thf(complementT_l
powersetTI$_1$: \$o thf(powersetTI1_type, type)
powersetTI$_1$ = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@x) ⇒ (in@xx@y))) ⇒ (in@x@(powerset@y)))))) thf(powersetTI$_1$, definition)
demorgan2a$_1$: \$o thf(demorgan2a1_type, type)
demorgan2a$_1$ = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(binunion@x@y))) ⇒ (in@xx@(setminus@a@x))))))) thf(demorgan2a$_1$, definition)
binunionT_lem ⇒ (complementT_lem ⇒ (powersetTI$_1$ ⇒ (demorgan2a$_1$ ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ (in@(setminus@a@(binunion@x@y))@(powerset@(setminus@a@x)))))))) thf(compleme

SEU750^2.p Typed Set Theory - Laws for Typed Sets - DeMorgan Laws
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (setminus A (binunion X Y)) → in x (setminus A Y))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ (¬ in@xx@b ⇒ (in@xx@(setminus@a@b))))) thf(setminusI, definition)
setminusER: \$o thf(setminusER_type, type)
setminusER = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(setminus@a@b)) ⇒ ¬ in@xx@b)) thf(setminusER, definition)
binunionTIRcontra: \$o thf(binunionTIRcontra_type, type)
binunionTIRcontra = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (¬ in@xx@(binunion@x@y) ⇒ ¬ in@xx@y))))) thf(binunionTIRcontra, definition)
setminusI ⇒ (setminusER ⇒ (binunionTIRcontra ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a
∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(binunion@x@y))) ⇒ (in@xx@(setminus@a@y)))))))) thf(demorgan2a$_2$

SEU751^2.p Typed Set Theory - Laws for Typed Sets - DeMorgan Laws
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (setminus A (binintersect X Y)) → in x (binunion (setminus A X) (setminus A Y)))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
binintersect: \$i → \$i → \$i thf(binintersect_type, type)
binintersectI: \$o thf(binintersectI_type, type)
binintersectI = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ ((in@xx@b) ⇒ (in@xx@(binintersect@a@b))))) thf(binintersectI, defi
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusER: \$o thf(setminusER_type, type)
setminusER = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(setminus@a@b)) ⇒ ¬ in@xx@b)) thf(setminusER, definition)
complementT_lem: \$o thf(complementT_lem_type, type)
complementT_lem = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ (in@(setminus@a@x)@(powerset@a)))) thf(complementT_l
complementTE$_1$: \$o thf(complementTE1_type, type)
complementTE$_1$ = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (¬ in@xx@(setminus@a@x) ⇒ (in@xx@x))))) thf(complementTE$_1$, definition)
binunionTILcontra: \$o thf(binunionTILcontra_type, type)
binunionTILcontra = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (¬ in@xx@(binunion@x@y) ⇒ ¬ in@xx@x))))) thf(binunionTILcontra, definition)
binunionTIRcontra: \$o thf(binunionTIRcontra_type, type)
binunionTIRcontra = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ (¬ in@xx@(binunion@x@y) ⇒ ¬ in@xx@y))))) thf(binunionTIRcontra, definition)
binintersectI ⇒ (setminusER ⇒ (complementT_lem ⇒ (complementTE$_1$ ⇒ (binunionTILcontra ⇒ (binunionTIRcontra ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@(binintersect@x@y))) ⇒ (in@xx@(binunion@(setminus@a@x)@(setminus@a@y)))))))))))) thf(d

SEU752^2.p Typed Set Theory - Laws for Typed Sets - DeMorgan Laws
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (binunion (setminus A X) (setminus A Y)) → in x (setminus A (binintersect X Y)))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
binunionE: \$o thf(binunionE_type, type)

binunionE = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@(binunion@a@b)) $\Rightarrow$ (in@xx@a or in@xx@b))) thf(binunionE, definition)
binintersect: \$i → \$i → \$i thf(binintersect_type, type)
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@a) $\Rightarrow$ ($\neg$ in@xx@b $\Rightarrow$ (in@xx@(setminus@a@b))))) thf(setminusI, definition)
setminusER: \$o thf(setminusER_type, type)
setminusER = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@(setminus@a@b)) $\Rightarrow$ $\neg$ in@xx@b)) thf(setminusER, definition)
binintersectTELcontra: \$o thf(binintersectTELcontra_type, type)
binintersectTELcontra = ($\forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow$ $\forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ $\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ($\neg$ in@xx@x $\Rightarrow$ $\neg$ in@xx@(binintersect@x@y)))))) thf(binintersectTELcontra, definition)
binintersectTERcontra: \$o thf(binintersectTERcontra_type, type)
binintersectTERcontra = ($\forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow$ $\forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ $\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ($\neg$ in@xx@y $\Rightarrow$ $\neg$ in@xx@(binintersect@x@y)))))) thf(binintersectTERcontra, definition)
binunionE $\Rightarrow$ (setminusI $\Rightarrow$ (setminusER $\Rightarrow$ (binintersectTELcontra $\Rightarrow$ (binintersectTERcontra $\Rightarrow$ $\forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow$ $\forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ $\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@(binunion@(setm
(in@xx@(setminus@a@(binintersect@x@y)))))))))) thf(demorgan1b, conjecture)

SEU754^2.p Typed Set Theory - Laws for Typed Sets - DeMorgan Laws
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (setminus A (binunion X Y)) → in x (binintersect (setminus A X) (setminus A Y)))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
binintersect: \$i → \$i → \$i thf(binintersect_type, type)
binintersectI: \$o thf(binintersectI_type, type)
binintersectI = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@b) $\Rightarrow$ (in@xx@(binintersect@a@b))))) thf(binintersectI, defi
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@a) $\Rightarrow$ ($\neg$ in@xx@b $\Rightarrow$ (in@xx@(setminus@a@b))))) thf(setminusI, definition)
setminusER: \$o thf(setminusER_type, type)
setminusER = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@(setminus@a@b)) $\Rightarrow$ $\neg$ in@xx@b)) thf(setminusER, definition)
binunionTILcontra: \$o thf(binunionTILcontra_type, type)
binunionTILcontra = ($\forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow$ $\forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ $\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ($\neg$ in@xx@(binunion@x@y) $\Rightarrow$ $\neg$ in@xx@x))))) thf(binunionTILcontra, definition)
binunionTIRcontra: \$o thf(binunionTIRcontra_type, type)
binunionTIRcontra = ($\forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow$ $\forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ $\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ($\neg$ in@xx@(binunion@x@y) $\Rightarrow$ $\neg$ in@xx@y))))) thf(binunionTIRcontra, definition)
binintersectI $\Rightarrow$ (setminusI $\Rightarrow$ (setminusER $\Rightarrow$ (binunionTILcontra $\Rightarrow$ (binunionTIRcontra $\Rightarrow$ $\forall a$: \$i, x: \$i: ((in@x@(pow
$\forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ $\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@(setminus@a@(binunion@x@y))) $\Rightarrow$ (in@xx@(binintersе

SEU755^2.p Typed Set Theory - Laws for Typed Sets - DeMorgan Laws
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (setminus A X) → in x (setminus A Y) → in x (setminus A (binunion X Y)))))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
setminus: \$i → \$i → \$i thf(setminus_type, type)
setminusI: \$o thf(setminusI_type, type)
setminusI = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@a) $\Rightarrow$ ($\neg$ in@xx@b $\Rightarrow$ (in@xx@(setminus@a@b))))) thf(setminusI, definition)
setminusER: \$o thf(setminusER_type, type)
setminusER = ($\forall a$: \$i, b: \$i, xx: \$i: ((in@xx@(setminus@a@b)) $\Rightarrow$ $\neg$ in@xx@b)) thf(setminusER, definition)
binunionTEcontra: \$o thf(binunionTEcontra_type, type)
binunionTEcontra = ($\forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow$ $\forall y$: \$i: ((in@y@(powerset@a)) $\Rightarrow$ $\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ($\neg$ in@xx@x $\Rightarrow$ ($\neg$ in@xx@y $\Rightarrow$ $\neg$ in@xx@(binunion@x@y))))))) thf(binunionTEcontra, definition)
setminusI $\Rightarrow$ (setminusER $\Rightarrow$ (binunionTEcontra $\Rightarrow$ $\forall a$: \$i, x: \$i: ((in@x@(powerset@a)) $\Rightarrow$ $\forall y$: \$i: ((in@y@(powerset@a))
$\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ ((in@xx@(setminus@a@x)) $\Rightarrow$ ((in@xx@(setminus@a@y)) $\Rightarrow$ (in@xx@(setminus@a@(binunion@x@

SEU756^2.p Typed Set Theory - Laws for Typed Sets - DeMorgan Laws
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x A → in x (binintersect (setminus A X) (setminus A Y)) → in x (setminus A (binunion X Y)))))
in: \$i → \$i → \$o thf(in_type, type)

powerset: \$i → \$i thf(powerset_type, type)
binunion: \$i → \$i → \$i thf(binunion_type, type)
binintersect: \$i → \$i → \$i thf(binintersect_type, type)
binintersectEL: \$o thf(binintersectEL_type, type)
binintersectEL = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@a))) thf(binintersectEL, definition)
binintersectER: \$o thf(binintersectER_type, type)
binintersectER = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@b))) thf(binintersectER, definition)
setminus: \$i → \$i → \$i thf(setminus_type, type)
demorgan2b$_2$: \$o thf(demorgan2b2_type, type)
demorgan2b$_2$ = (∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@(setminus@a@x)) ⇒ ((in@xx@(setminus@a@y)) ⇒ (in@xx@(setminus@a@(binunion@x@y))))))))) thf(demor
binintersectEL ⇒ (binintersectER ⇒ (demorgan2b$_2$ ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@
∀xx: \$i: ((in@xx@a) ⇒ ((in@xx@(binintersect@(setminus@a@x)@(setminus@a@y))) ⇒ (in@xx@(setminus@a@(binunion@

SEU758∧2.p Typed Set Theory - First Wizard of Oz Examples - WoZ1 Lemmas
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! x:i.in x (binintersect X Y) → in x A)))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
powersetE: \$o thf(powersetE_type, type)
powersetE = (∀a: \$i, b: \$i, xx: \$i: ((in@b@(powerset@a)) ⇒ ((in@xx@b) ⇒ (in@xx@a)))) thf(powersetE, definition)
binintersect: \$i → \$i → \$i thf(binintersect_type, type)
binintersectEL: \$o thf(binintersectEL_type, type)
binintersectEL = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@a))) thf(binintersectEL, definition)
powersetE ⇒ (binintersectEL ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀xx: \$i: ((in@xx@(binintersect@x@y)) ⇒ (in@xx@a))))) thf(woz13rule$_0$, conjecture)

SEU759∧2.p Typed Set Theory - First Wizard of Oz Examples - WoZ1 Lemmas
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → subset X Z → subset (binintersect X Y) Z)))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
subsetI$_1$: \$o thf(subsetI1_type, type)
subsetI$_1$ = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (⊆ @a@b))) thf(subsetI$_1$, definition)
subsetE: \$o thf(subsetE_type, type)
subsetE = (∀a: \$i, b: \$i, xx: \$i: ((⊆ @a@b) ⇒ ((in@xx@a) ⇒ (in@xx@b)))) thf(subsetE, definition)
binintersect: \$i → \$i → \$i thf(binintersect_type, type)
binintersectEL: \$o thf(binintersectEL_type, type)
binintersectEL = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@a))) thf(binintersectEL, definition)
subsetI$_1$ ⇒ (subsetE ⇒ (binintersectEL ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀z: \$i: ((in@z@(powerset@a)) ⇒ ((⊆ @x@z) ⇒ (⊆ @(binintersect@x@y)@z))))))) thf(woz13rule$_1$, conjecture)

SEU760∧2.p Typed Set Theory - First Wizard of Oz Examples - WoZ1 Lemmas
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → subset Y Z → subset (binintersect X Y) Z)))
in: \$i → \$i → \$o thf(in_type, type)
powerset: \$i → \$i thf(powerset_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
subsetI$_1$: \$o thf(subsetI1_type, type)
subsetI$_1$ = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (⊆ @a@b))) thf(subsetI$_1$, definition)
subsetE: \$o thf(subsetE_type, type)
subsetE = (∀a: \$i, b: \$i, xx: \$i: ((⊆ @a@b) ⇒ ((in@xx@a) ⇒ (in@xx@b)))) thf(subsetE, definition)
binintersect: \$i → \$i → \$i thf(binintersect_type, type)
binintersectER: \$o thf(binintersectER_type, type)
binintersectER = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@b))) thf(binintersectER, definition)
subsetI$_1$ ⇒ (subsetE ⇒ (binintersectER ⇒ ∀a: \$i, x: \$i: ((in@x@(powerset@a)) ⇒ ∀y: \$i: ((in@y@(powerset@a)) ⇒ ∀z: \$i: ((in@z@(powerset@a)) ⇒ ((⊆ @y@z) ⇒ (⊆ @(binintersect@x@y)@z))))))) thf(woz13rule$_2$, conjecture)

SEU761∧2.p Typed Set Theory - First Wizard of Oz Examples - WoZ1 Lemmas
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → subset X Y → subset X Z → subset X (binintersect Y Z))))

in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
subsetI_1: $o   thf(subsetI1_type, type)
subsetI_1 = (∀a: $i, b: $i: (∀xx: $i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (⊆ @a@b)))   thf(subsetI_1, definition)
subsetE: $o   thf(subsetE_type, type)
subsetE = (∀a: $i, b: $i, xx: $i: ((⊆ @a@b) ⇒ ((in@xx@a) ⇒ (in@xx@b))))   thf(subsetE, definition)
binintersect: $i → $i → $i   thf(binintersect_type, type)
binintersectI: $o   thf(binintersectI_type, type)
binintersectI = (∀a: $i, b: $i, xx: $i: ((in@xx@a) ⇒ ((in@xx@b) ⇒ (in@xx@(binintersect@a@b)))))   thf(binintersectI, defi
subsetI_1 ⇒ (subsetE ⇒ (binintersectI ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
∀z: $i: ((in@z@(powerset@a)) ⇒ ((⊆ @x@y) ⇒ ((⊆ @x@z) ⇒ (⊆ @x@(binintersect@y@z)))))))))   thf(woz13rule_3, conj

SEU762∧2.p Typed Set Theory - First Wizard of Oz Examples - WoZ1 Lemmas
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → (! W:i.in W (powerset A)
→ subset X Z → subset Y W → subset (binintersect X Y) (binintersect Z W)))))
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
binintersect: $i → $i → $i   thf(binintersect_type, type)
binintersectT_lem: $o   thf(binintersectT_lem_type, type)
binintersectT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (in@(binintersect@x@y)@(po
woz13rule_1: $o   thf(woz13rule1_type, type)
woz13rule_1 = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀z: $i: ((in@z@(powerset@a)) ⇒
((⊆ @x@z) ⇒ (⊆ @(binintersect@x@y)@z))))))   thf(woz13rule_1, definition)
woz13rule_2: $o   thf(woz13rule2_type, type)
woz13rule_2 = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀z: $i: ((in@z@(powerset@a)) ⇒
((⊆ @y@z) ⇒ (⊆ @(binintersect@x@y)@z))))))   thf(woz13rule_2, definition)
woz13rule_3: $o   thf(woz13rule3_type, type)
woz13rule_3 = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ ∀z: $i: ((in@z@(powerset@a)) ⇒
((⊆ @x@y) ⇒ ((⊆ @x@z) ⇒ (⊆ @x@(binintersect@y@z))))))))   thf(woz13rule_3, definition)
binintersectT_lem ⇒ (woz13rule_1 ⇒ (woz13rule_2 ⇒ (woz13rule_3 ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒
∀y: $i: ((in@y@(powerset@a)) ⇒ ∀z: $i: ((in@z@(powerset@a)) ⇒ ∀w: $i: ((in@w@(powerset@a)) ⇒ ((⊆
@x@z) ⇒ ((⊆ @y@w) ⇒ (⊆ @(binintersect@x@y)@(binintersect@z@w))))))))))   thf(woz13rule_4, conjecture)

SEU764∧2.p Typed Set Theory - First Wizard of Oz Examples - WoZ1 Problems
(! A:i.! X:i.in X (powerset A) → (! Y:i.in Y (powerset A) → (! Z:i.in Z (powerset A) → (! W:i.in W (powerset A)
→ setminus A (binintersect (binunion X Y) (binunion Z W)) = binunion (binintersect (setminus A X) (setminus A
Y)) (binintersect (setminus A Z) (setminus A W))))))
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
binunion: $i → $i → $i   thf(binunion_type, type)
binintersect: $i → $i → $i   thf(binintersect_type, type)
setminus: $i → $i → $i   thf(setminus_type, type)
binunionT_lem: $o   thf(binunionT_lem_type, type)
binunionT_lem = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (in@(binunion@x@y)@(powerse
demorgan_1: $o   thf(demorgan1_type, type)
demorgan_1 = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (setminus@a@(binintersect@x@y))
(binunion@(setminus@a@x)@(setminus@a@y)))))   thf(demorgan_1, definition)
demorgan_2: $o   thf(demorgan2_type, type)
demorgan_2 = (∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒ (setminus@a@(binunion@x@y)) =
(binintersect@(setminus@a@x)@(setminus@a@y)))))   thf(demorgan_2, definition)
binunionT_lem ⇒ (demorgan_1 ⇒ (demorgan_2 ⇒ ∀a: $i, x: $i: ((in@x@(powerset@a)) ⇒ ∀y: $i: ((in@y@(powerset@a)) ⇒
∀z: $i: ((in@z@(powerset@a)) ⇒ ∀w: $i: ((in@w@(powerset@a)) ⇒ (setminus@a@(binintersect@(binunion@x@y)@(binunio
(binunion@(binintersect@(setminus@a@x)@(setminus@a@y))@(binintersect@(setminus@a@z)@(setminus@a@w)))))))))

SEU768∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! phi:i>o.(! x:i.in x A → (! y:i.in y A → in (kpair x y) R → phi (kpair x y))) → (! x:i.in
x R → phi x)))
in: $i → $i → $o   thf(in_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)

kpair: \$i → \$i → \$i   thf(kpair_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λ*a*: \$i, *b*: \$i, *c*: \$i: (⊆ @*c*@(cartprod@*a*@*b*)))   thf(breln, definition)
brelnall$_2$: \$o   thf(brelnall2_type, type)
brelnall$_2$ = (∀*a*: \$i, *b*: \$i, *r*: \$i: ((breln@*a*@*b*@*r*) ⇒ ∀xphi: \$i → \$o: (∀xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*b*) ⇒ ((in@(kpair@xx@xy)@*r*) ⇒ (xphi@(kpair@xx@xy))))) ⇒ ∀xx: \$i: ((in@xx@*r*) ⇒ (xphi@xx)))))   thf(brelnall$_2$, definitio
breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)
breln$_1$ = (λ*a*: \$i, *r*: \$i: (breln@*a*@*a*@*r*))   thf(breln$_1$, definition)
brelnall$_2$ ⇒ ∀*a*: \$i, *r*: \$i: ((breln$_1$@*a*@*r*) ⇒ ∀xphi: \$i → \$o: (∀xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*a*) ⇒ ((in@(kpair@xx@xy)@*r*) ⇒ (xphi@(kpair@xx@xy))))) ⇒ ∀xx: \$i: ((in@xx@*r*) ⇒ (xphi@xx))))   thf(breln1all$_2$, conjectu

SEU769∧2.p Binary Relations on a Set
(! A:i.! R:i.in R (breln1Set A) → breln1 A R)
in: \$i → \$i → \$o   thf(in_type, type)
powerset: \$i → \$i   thf(powerset_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrER: \$o   thf(dsetconstrER_type, type)
dsetconstrER = (∀*a*: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@*a*@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx)))   thf(dsetcons
⊆ : \$i → \$i → \$o   thf(subset_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λ*a*: \$i, *b*: \$i, *c*: \$i: (⊆ @*c*@(cartprod@*a*@*b*)))   thf(breln, definition)
breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)
breln$_1$ = (λ*a*: \$i, *r*: \$i: (breln@*a*@*a*@*r*))   thf(breln$_1$, definition)
breln1Set: \$i → \$i   thf(breln1Set_type, type)
breln1Set = (λ*a*: \$i: (dsetconstr@(powerset@(cartprod@*a*@*a*))@λ*r*: \$i: (breln$_1$@*a*@*r*)))   thf(breln1Set, definition)
dsetconstrER ⇒ ∀*a*: \$i, *r*: \$i: ((in@*r*@(breln1Set@*a*)) ⇒ (breln$_1$@*a*@*r*))   thf(breln1SetBreln$_1$, conjecture)

SEU771∧2.p Binary Relations on a Set
(! A:i.! phi:i>(i>o).breln1 A (dpsetconstr A A (∧ x,y:i.phi x y)))
⊆ : \$i → \$i → \$o   thf(subset_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λ*a*: \$i, *b*: \$i, *c*: \$i: (⊆ @*c*@(cartprod@*a*@*b*)))   thf(breln, definition)
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i   thf(dpsetconstr_type, type)
setOfPairsIsBReln: \$o   thf(setOfPairsIsBReln_type, type)
setOfPairsIsBReln = (∀*a*: \$i, *b*: \$i, xphi: \$i → \$i → \$o: (breln@*a*@*b*@(dpsetconstr@*a*@*b*@λxx: \$i, xy: \$i: (xphi@xx@xy))))   t
breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)
breln$_1$ = (λ*a*: \$i, *r*: \$i: (breln@*a*@*a*@*r*))   thf(breln$_1$, definition)
setOfPairsIsBReln ⇒ ∀*a*: \$i, xphi: \$i → \$i → \$o: (breln$_1$@*a*@(dpsetconstr@*a*@*a*@λxx: \$i, xy: \$i: (xphi@xx@xy)))   thf(setC

SEU773∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) R → in (kpair x y) S)) → subset R S))
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λ*a*: \$i, *b*: \$i, *c*: \$i: (⊆ @*c*@(cartprod@*a*@*b*)))   thf(breln, definition)
subbreln: \$o   thf(subbreln_type, type)
subbreln = (∀*a*: \$i, *b*: \$i, *r*: \$i: ((breln@*a*@*b*@*r*) ⇒ ∀*s*: \$i: ((breln@*a*@*b*@*s*) ⇒ (∀xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*b*) ⇒ ((in@(kpair@xx@xy)@*r*) ⇒ (in@(kpair@xx@xy)@*s*)))) ⇒ (⊆ @*r*@*s*)))))   thf(subbreln, definitio
breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)
breln$_1$ = (λ*a*: \$i, *r*: \$i: (breln@*a*@*a*@*r*))   thf(breln$_1$, definition)
subbreln ⇒ ∀*a*: \$i, *r*: \$i: ((breln$_1$@*a*@*r*) ⇒ ∀*s*: \$i: ((breln$_1$@*a*@*s*) ⇒ (∀xx: \$i: ((in@xx@*a*) ⇒ ∀xy: \$i: ((in@xy@*a*) ⇒ ((in@(kpair@xx@xy)@*r*) ⇒ (in@(kpair@xx@xy)@*s*)))) ⇒ (⊆ @*r*@*s*))))   thf(subbreln$_1$, conjecture)

SEU774∧2.p Binary Relations on a Set

(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) R → in (kpair x y) S)) → (! x:i.in x A → (! y:i.in y A → in (kpair x y) S → in (kpair x y) R)) → R = S))
in: $i → $i → $o   thf(in_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
kpair: $i → $i → $i   thf(kpair_type, type)
cartprod: $i → $i → $i   thf(cartprod_type, type)
breln: $i → $i → $i → $o   thf(breln_type, type)
breln = (λa: $i, b: $i, c: $i: (⊆ @c@(cartprod@a@b)))   thf(breln, definition)
eqbreln: $o   thf(eqbreln_type, type)
eqbreln = (∀a: $i, b: $i, r: $i: ((breln@a@b@r) ⇒ ∀s: $i: ((breln@a@b@s) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@s)))) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ ((in@(kpair@xx@xy)@s) ⇒ (in@(kpair@xx@xy)@r)))) ⇒ r = s)))))   thf(eqbreln, definition)
breln$_1$: $i → $i → $o   thf(breln1_type, type)
breln$_1$ = (λa: $i, r: $i: (breln@a@a@r))   thf(breln$_1$, definition)
eqbreln ⇒ ∀a: $i, r: $i: ((breln$_1$@a@r) ⇒ ∀s: $i: ((breln$_1$@a@s) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@s)))) ⇒ (∀xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@s) ⇒ (in@(kpair@xx@xy)@r)))) ⇒ r = s))))   thf(eqbreln$_1$, conjecture)

SEU775∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → breln1 A (breln1invset A R))
in: $i → $i → $o   thf(in_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
kpair: $i → $i → $i   thf(kpair_type, type)
cartprod: $i → $i → $i   thf(cartprod_type, type)
breln: $i → $i → $i → $o   thf(breln_type, type)
breln = (λa: $i, b: $i, c: $i: (⊆ @c@(cartprod@a@b)))   thf(breln, definition)
dpsetconstr: $i → $i → ($i → $i → $o) → $i   thf(dpsetconstr_type, type)
breln$_1$: $i → $i → $o   thf(breln1_type, type)
breln$_1$ = (λa: $i, r: $i: (breln@a@a@r))   thf(breln$_1$, definition)
setOfPairsIsBReln$_1$: $o   thf(setOfPairsIsBReln1_type, type)
setOfPairsIsBReln$_1$ = (∀a: $i, xphi: $i → $i → $o: (breln$_1$@a@(dpsetconstr@a@a@λxx: $i, xy: $i: (xphi@xx@xy))))   thf(set
breln1invset: $i → $i → $i   thf(breln1invset_type, type)
breln1invset = (λa: $i, r: $i: (dpsetconstr@a@a@λxx: $i, xy: $i: (in@(kpair@xy@xx)@r)))   thf(breln1invset, definition)
setOfPairsIsBReln$_1$ ⇒ ∀a: $i, r: $i: ((breln$_1$@a@r) ⇒ (breln$_1$@a@(breln1invset@a@r)))   thf(breln1invprop, conjecture)

SEU776∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! x:i.in x A → (! y:i.in y A → in (kpair x y) R → in (kpair y x) (breln1invset A R))))
in: $i → $i → $o   thf(in_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
kpair: $i → $i → $i   thf(kpair_type, type)
cartprod: $i → $i → $i   thf(cartprod_type, type)
breln: $i → $i → $i → $o   thf(breln_type, type)
breln = (λa: $i, b: $i, c: $i: (⊆ @c@(cartprod@a@b)))   thf(breln, definition)
dpsetconstr: $i → $i → ($i → $i → $o) → $i   thf(dpsetconstr_type, type)
dpsetconstrI: $o   thf(dpsetconstrI_type, type)
dpsetconstrI = (∀a: $i, b: $i, xphi: $i → $i → $o, xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@b) ⇒ ((xphi@xx@xy) ⇒ (in@(kpair@xx@xy)@(dpsetconstr@a@b@λxz: $i, xu: $i: (xphi@xz@xu)))))))   thf(dpsetconstrI, definition)
breln$_1$: $i → $i → $o   thf(breln1_type, type)
breln$_1$ = (λa: $i, r: $i: (breln@a@a@r))   thf(breln$_1$, definition)
breln1invset: $i → $i → $i   thf(breln1invset_type, type)
breln1invset = (λa: $i, r: $i: (dpsetconstr@a@a@λxx: $i, xy: $i: (in@(kpair@xy@xx)@r)))   thf(breln1invset, definition)
dpsetconstrI ⇒ ∀a: $i, r: $i: ((breln$_1$@a@r) ⇒ ∀xx: $i: ((in@xx@a) ⇒ ∀xy: $i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xy@xx)@(breln1invset@a@r))))))   thf(breln1invI, conjecture)

SEU777∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! x:i.in x A → (! y:i.in y A → in (kpair y x) (breln1invset A R) → in (kpair x y) R)))
in: $i → $i → $o   thf(in_type, type)
⊆ : $i → $i → $o   thf(subset_type, type)
kpair: $i → $i → $i   thf(kpair_type, type)
cartprod: $i → $i → $i   thf(cartprod_type, type)
breln: $i → $i → $i → $o   thf(breln_type, type)

breln = (λa: \$i, b: \$i, c: \$i: ($\subseteq$ @c@(cartprod@a@b))) thf(breln, definition)
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i thf(dpsetconstr_type, type)
dpsetconstrER: \$o thf(dpsetconstrER_type, type)
dpsetconstrER = ($\forall a$: \$i, b: \$i, xphi: \$i → \$i → \$o, xx: \$i, xy: \$i: ((in@(kpair@xx@xy)@(dpsetconstr@a@b@λxz: \$i, xu: \$i: (xph
(xphi@xx@xy))) thf(dpsetconstrER, definition)
breln$_1$: \$i → \$i → \$o thf(breln1_type, type)
breln$_1$ = (λa: \$i, r: \$i: (breln@a@a@r)) thf(breln$_1$, definition)
breln1invset: \$i → \$i → \$i thf(breln1invset_type, type)
breln1invset = (λa: \$i, r: \$i: (dpsetconstr@a@a@λxx: \$i, xy: \$i: (in@(kpair@xy@xx)@r))) thf(breln1invset, definition)
dpsetconstrER ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xy@xx)@(br
(in@(kpair@xx@xy)@r))))) thf(breln1invE, conjecture)

SEU778∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → breln1 A (breln1compset A R S)))
in: \$i → \$i → \$o thf(in_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: ($\subseteq$ @c@(cartprod@a@b))) thf(breln, definition)
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i thf(dpsetconstr_type, type)
breln$_1$: \$i → \$i → \$o thf(breln1_type, type)
breln$_1$ = (λa: \$i, r: \$i: (breln@a@a@r)) thf(breln$_1$, definition)
setOfPairsIsBReln$_1$: \$o thf(setOfPairsIsBReln1_type, type)
setOfPairsIsBReln$_1$ = (∀a: \$i, xphi: \$i → \$i → \$o: (breln$_1$@a@(dpsetconstr@a@a@λxx: \$i, xy: \$i: (xphi@xx@xy)))) thf(set
breln1compset: \$i → \$i → \$i → \$i thf(breln1compset_type, type)
breln1compset = (λa: \$i, r: \$i, s: \$i: (dpsetconstr@a@a@λxx: \$i, xy: \$i: ∃xz: \$i: (in@xz@a and in@(kpair@xx@xz)@r and in@(
setOfPairsIsBReln$_1$ ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ (breln$_1$@a@(breln1compset@a@r@s))))

SEU779∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → (! z:i.in z A → in (kpair x z) R → in (kpair z y) S → in (kpair x y) (breln1compset A R S))))))
in: \$i → \$i → \$o thf(in_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: ($\subseteq$ @c@(cartprod@a@b))) thf(breln, definition)
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i thf(dpsetconstr_type, type)
dpsetconstrI: \$o thf(dpsetconstrI_type, type)
dpsetconstrI = (∀a: \$i, b: \$i, xphi: \$i → \$i → \$o, xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@b) ⇒ ((xphi@xx@xy) ⇒ (in@(kpair@xx@xy)@(dpsetconstr@a@b@λxz: \$i, xu: \$i: (xphi@xz@xu))))))) thf(dpsetconstrI, definition)
breln$_1$: \$i → \$i → \$o thf(breln1_type, type)
breln$_1$ = (λa: \$i, r: \$i: (breln@a@a@r)) thf(breln$_1$, definition)
breln1compset: \$i → \$i → \$i → \$i thf(breln1compset_type, type)
breln1compset = (λa: \$i, r: \$i, s: \$i: (dpsetconstr@a@a@λxx: \$i, xy: \$i: ∃xz: \$i: (in@xz@a and in@(kpair@xx@xz)@r and in@(
dpsetconstrI ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒
∀xz: \$i: ((in@xz@a) ⇒ ((in@(kpair@xx@xz)@r) ⇒ ((in@(kpair@xz@xy)@s) ⇒ (in@(kpair@xx@xy)@(breln1compset@a@r

SEU780∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) (breln1compset A R S) → (? z:i.in z A & (in (kpair x z) R & in (kpair z y) S))))))
in: \$i → \$i → \$o thf(in_type, type)
⊆ : \$i → \$i → \$o thf(subset_type, type)
kpair: \$i → \$i → \$i thf(kpair_type, type)
cartprod: \$i → \$i → \$i thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: ($\subseteq$ @c@(cartprod@a@b))) thf(breln, definition)
dpsetconstr: \$i → \$i → (\$i → \$i → \$o) → \$i thf(dpsetconstr_type, type)
dpsetconstrER: \$o thf(dpsetconstrER_type, type)

dpsetconstrER = (∀a: \$i, b: \$i, xphi: \$i → \$i → \$o, xx: \$i, xy: \$i: ((in@(kpair@xx@xy)@(dpsetconstr@a@b@λxz: \$i, xu: \$i: (xph
(xphi@xx@xy)))   thf(dpsetconstrER, definition)
breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)
breln$_1$ = (λa: \$i, r: \$i: (breln@a@a@r))   thf(breln$_1$, definition)
breln1compset: \$i → \$i → \$i → \$i   thf(breln1compset_type, type)
breln1compset = (λa: \$i, r: \$i, s: \$i: (dpsetconstr@a@a@λxx: \$i, xy: \$i: ∃xz: \$i: (in@xz@a and in@(kpair@xx@xz)@r and in@(
dpsetconstrER ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒
∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@(breln1compset@a@r@s)) ⇒ ∃xz: \$i: (in@xz@a and in@(kpair@xx@xz)@r ar

SEU781∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) (breln1compset A R S)
→ (! phi:o.(! z:i.in z A → in (kpair x z) R → in (kpair z y) S → phi) → phi)))))
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b)))   thf(breln, definition)
breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)
breln$_1$ = (λa: \$i, r: \$i: (breln@a@a@r))   thf(breln$_1$, definition)
breln1compset: \$i → \$i → \$i → \$i   thf(breln1compset_type, type)
breln1compE: \$o   thf(breln1compE_type, type)
breln1compE = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒
((in@(kpair@xx@xy)@(breln1compset@a@r@s)) ⇒ ∃xz: \$i: (in@xz@a and in@(kpair@xx@xz)@r and in@(kpair@xz@xy)@s
breln1compE ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒
((in@(kpair@xx@xy)@(breln1compset@a@r@s)) ⇒ ∀xphi: \$o: (∀xz: \$i: ((in@xz@a) ⇒ ((in@(kpair@xx@xz)@r) ⇒
((in@(kpair@xz@xy)@s) ⇒ xphi))) ⇒ xphi))))))   thf(breln1compEex, conjecture)

SEU782∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → breln1 A (binunion R S)))
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
subsetI$_1$: \$o   thf(subsetI1_type, type)
subsetI$_1$ = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (⊆ @a@b)))   thf(subsetI$_1$, definition)
subsetE: \$o   thf(subsetE_type, type)
subsetE = (∀a: \$i, b: \$i, xx: \$i: ((⊆ @a@b) ⇒ ((in@xx@a) ⇒ (in@xx@b))))   thf(subsetE, definition)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
binunionEcases: \$o   thf(binunionEcases_type, type)
binunionEcases = (∀a: \$i, b: \$i, xx: \$i, xphi: \$o: ((in@xx@(binunion@a@b)) ⇒ (((in@xx@a) ⇒ xphi) ⇒
(((in@xx@b) ⇒ xphi) ⇒ xphi))))   thf(binunionEcases, definition)
cartprod: \$i → \$i → \$i   thf(cartprod_type, type)
breln: \$i → \$i → \$i → \$o   thf(breln_type, type)
breln = (λa: \$i, b: \$i, c: \$i: (⊆ @c@(cartprod@a@b)))   thf(breln, definition)
breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)
breln$_1$ = (λa: \$i, r: \$i: (breln@a@a@r))   thf(breln$_1$, definition)
subsetI$_1$ ⇒ (subsetE ⇒ (binunionEcases ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒
(breln$_1$@a@(binunion@r@s))))))   thf(breln1unionprop, conjecture)

SEU783∧2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) R → in (kpair x y)
(binunion R S)))))
in: \$i → \$i → \$o   thf(in_type, type)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
binunionIL: \$o   thf(binunionIL_type, type)
binunionIL = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@a) ⇒ (in@xx@(binunion@a@b))))   thf(binunionIL, definition)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)
binunionIL ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒
((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@(binunion@r@s)))))))   thf(breln1unionIL, conjecture)

SEU784∧2.p Binary Relations on a Set

(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) S → in (kpair x y) (binunion R S)))))
in: \$i → \$i → \$o   thf(in_type, type)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
binunionIR: \$o   thf(binunionIR_type, type)
binunionIR = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@b) ⇒ (in@xx@(binunion@a@b))))   thf(binunionIR, definition)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
breln_1: \$i → \$i → \$o   thf(breln1_type, type)
binunionIR ⇒ ∀a: \$i, r: \$i: ((breln_1@a@r) ⇒ ∀s: \$i: ((breln_1@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@s) ⇒ (in@(kpair@xx@xy)@(binunion@r@s)))))))   thf(breln1unionIR, conjecture)

SEU785^2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) R — in (kpair x y) S → in (kpair x y) (binunion R S)))))
in: \$i → \$i → \$o   thf(in_type, type)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
breln_1: \$i → \$i → \$o   thf(breln1_type, type)
breln1unionIL: \$o   thf(breln1unionIL_type, type)
breln1unionIL = (∀a: \$i, r: \$i: ((breln_1@a@r) ⇒ ∀s: \$i: ((breln_1@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@(binunion@r@s))))))))   thf(breln1unionIL, definition)
breln1unionIR: \$o   thf(breln1unionIR_type, type)
breln1unionIR = (∀a: \$i, r: \$i: ((breln_1@a@r) ⇒ ∀s: \$i: ((breln_1@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@s) ⇒ (in@(kpair@xx@xy)@(binunion@r@s))))))))   thf(breln1unionIR, definition)
breln1unionIL ⇒ (breln1unionIR ⇒ ∀a: \$i, r: \$i: ((breln_1@a@r) ⇒ ∀s: \$i: ((breln_1@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@r or in@(kpair@xx@xy)@s) ⇒ (in@(kpair@xx@xy)@(binunion@r@s))))))))

SEU786^2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) (binunion R S) → in (kpair x y) R — in (kpair x y) S))))
in: \$i → \$i → \$o   thf(in_type, type)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
binunionE: \$o   thf(binunionE_type, type)
binunionE = (∀a: \$i, b: \$i, xx: \$i: ((in@xx@(binunion@a@b)) ⇒ (in@xx@a or in@xx@b)))   thf(binunionE, definition)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
breln_1: \$i → \$i → \$o   thf(breln1_type, type)
binunionE ⇒ ∀a: \$i, r: \$i: ((breln_1@a@r) ⇒ ∀s: \$i: ((breln_1@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@(binunion@r@s)) ⇒ (in@(kpair@xx@xy)@r or in@(kpair@xx@xy)@s))))))   thf(breln1unionE, con

SEU787^2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! x:i.in x A → (! y:i.in y A → in (kpair x y) (binunion R S) → (! phi:o.(in (kpair x y) R → phi) → (in (kpair x y) S → phi) → phi)))))
in: \$i → \$i → \$o   thf(in_type, type)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)
breln_1: \$i → \$i → \$o   thf(breln1_type, type)
breln1unionE: \$o   thf(breln1unionE_type, type)
breln1unionE = (∀a: \$i, r: \$i: ((breln_1@a@r) ⇒ ∀s: \$i: ((breln_1@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@(binunion@r@s)) ⇒ (in@(kpair@xx@xy)@r or in@(kpair@xx@xy)@s)))))))   thf(breln1unionE, def
breln1unionE ⇒ ∀a: \$i, r: \$i: ((breln_1@a@r) ⇒ ∀s: \$i: ((breln_1@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@(binunion@r@s)) ⇒ ∀xphi: \$o: (((in@(kpair@xx@xy)@r) ⇒ xphi) ⇒ (((in@(kpair@xx@xy)@s) ⇒ xphi) ⇒ xphi)))))))   thf(breln1unionEcases, conjecture)

SEU788^2.p Binary Relations on a Set
(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → binunion R S = binunion S R))
in: \$i → \$i → \$o   thf(in_type, type)
⊆ : \$i → \$i → \$o   thf(subset_type, type)
setextsub: \$o   thf(setextsub_type, type)
setextsub = (∀a: \$i, b: \$i: ((⊆ @a@b) ⇒ ((⊆ @b@a) ⇒ $a = b$)))   thf(setextsub, definition)
binunion: \$i → \$i → \$i   thf(binunion_type, type)
kpair: \$i → \$i → \$i   thf(kpair_type, type)

breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)

subbreln$_1$: \$o   thf(subbreln1_type, type)

subbreln$_1$ = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@s)))) ⇒ (⊆ @r@s)))))   thf(subbreln$_1$, definition)

breln1unionprop: \$o   thf(breln1unionprop_type, type)

breln1unionprop = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ (breln$_1$@a@(binunion@r@s)))))   thf(breln1u

breln1unionIL: \$o   thf(breln1unionIL_type, type)

breln1unionIL = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@(binunion@r@s))))))))   thf(breln1unionIL, definition)

breln1unionIR: \$o   thf(breln1unionIR_type, type)

breln1unionIR = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@s) ⇒ (in@(kpair@xx@xy)@(binunion@r@s))))))))   thf(breln1unionIR, definition)

breln1unionE: \$o   thf(breln1unionE_type, type)

breln1unionE = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@(binunion@r@s)) ⇒ (in@(kpair@xx@xy)@r or in@(kpair@xx@xy)@s)))))))   thf(breln1unionE, def

setextsub ⇒ (subbreln$_1$ ⇒ (breln1unionprop ⇒ (breln1unionIL ⇒ (breln1unionIR ⇒ (breln1unionE ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ (binunion@r@s) = (binunion@s@r))))))))   thf(breln1unionComm

SEU789∧2.p Binary Relations on a Set - Second Wizard of Oz Examples

(! A:i.! R:i.breln1 A R → R = breln1invset A (breln1invset A R))

in: \$i → \$i → \$o   thf(in_type, type)

⊆ : \$i → \$i → \$o   thf(subset_type, type)

setextsub: \$o   thf(setextsub_type, type)

setextsub = (∀a: \$i, b: \$i: ((⊆ @a@b) ⇒ ((⊆ @b@a) ⇒ $a = b$)))   thf(setextsub, definition)

kpair: \$i → \$i → \$i   thf(kpair_type, type)

breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)

subbreln$_1$: \$o   thf(subbreln1_type, type)

subbreln$_1$ = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ (∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xx@xy)@s)))) ⇒ (⊆ @r@s)))))   thf(subbreln$_1$, definition)

breln1invset: \$i → \$i → \$i   thf(breln1invset_type, type)

breln1invprop: \$o   thf(breln1invprop_type, type)

breln1invprop = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ (breln$_1$@a@(breln1invset@a@r))))   thf(breln1invprop, definition)

breln1invI: \$o   thf(breln1invI_type, type)

breln1invI = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xx@xy)@r) ⇒ (in@(kpair@xy@xx)@(breln1invset@a@r)))))))   thf(breln1invI, definition)

breln1invE: \$o   thf(breln1invE_type, type)

breln1invE = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀xx: \$i: ((in@xx@a) ⇒ ∀xy: \$i: ((in@xy@a) ⇒ ((in@(kpair@xy@xx)@(breln1 (in@(kpair@xx@xy)@r))))))   thf(breln1invE, definition)

setextsub ⇒ (subbreln$_1$ ⇒ (breln1invprop ⇒ (breln1invI ⇒ (breln1invE ⇒ ∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ r = (breln1invset@a@(breln1invset@a@r)))))))   thf(woz2Ex, conjecture)

SEU792∧2.p Binary Relations on a Set - Second Wizard of Oz Examples

(! A:i.! R:i.breln1 A R → (! S:i.breln1 A S → (! T:i.breln1 A T → breln1compset A (binunion R S) T = binunion (breln1invset A (breln1compset A (breln1invset A T) (breln1invset A S))) (breln1invset A (breln1compset A (breln1invset A T) (breln1invset A R))))))

binunion: \$i → \$i → \$i   thf(binunion_type, type)

breln$_1$: \$i → \$i → \$o   thf(breln1_type, type)

breln1invset: \$i → \$i → \$i   thf(breln1invset_type, type)

breln1invprop: \$o   thf(breln1invprop_type, type)

breln1invprop = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ (breln$_1$@a@(breln1invset@a@r))))   thf(breln1invprop, definition)

breln1compset: \$i → \$i → \$i → \$i   thf(breln1compset_type, type)

breln1compprop: \$o   thf(breln1compprop_type, type)

breln1compprop = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ (breln$_1$@a@(breln1compset@a@r@s)))))   th

breln1unionCommutes: \$o   thf(breln1unionCommutes_type, type)

breln1unionCommutes = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ (binunion@r@s) = (binunion@s@r))))

woz2Ex: \$o   thf(woz2Ex_type, type)

woz2Ex = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ r = (breln1invset@a@(breln1invset@a@r))))   thf(woz2Ex, definition)

woz2W: \$o   thf(woz2W_type, type)

woz2W = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ (breln1invset@a@(breln1compset@a@r@s)) = (breln1compset@a@(breln1invset@a@s)@(breln1invset@a@r)))))   thf(woz2W, definition)

woz2A: \$o thf(woz2A_type, type)
woz2A = (∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀t: \$i: ((breln$_1$@a@t) ⇒ (breln1compset@a@(binunion@
(binunion@(breln1compset@a@r@t)@(breln1compset@a@s@t)))))) thf(woz2A, definition)
breln1invprop ⇒ (breln1compprop ⇒ (breln1unionCommutes ⇒ (woz2Ex ⇒ (woz2W ⇒ (woz2A ⇒
∀a: \$i, r: \$i: ((breln$_1$@a@r) ⇒ ∀s: \$i: ((breln$_1$@a@s) ⇒ ∀t: \$i: ((breln$_1$@a@t) ⇒ (breln1compset@a@(binunion@r@s)@t) =
(binunion@(breln1invset@a@(breln1compset@a@(breln1invset@a@t)@(breln1invset@a@s)))@(breln1invset@a@(breln1comps

SEU793∧2.p More about Functions - Images of Functions
(! A:i.! f:i>i.? B:i.! x:i.in x B <−→ (? y:i.in y A & x = f y))
in: \$i → \$i → \$o thf(in_type, type)
exu: (\$i → \$o) → \$o thf(exu_type, type)
exu = (λxphi: \$i → \$o: ∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy))) thf(exu, definition)
replAx: \$o thf(replAx_type, type)
replAx = (∀xphi: \$i → \$i → \$o, a: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (exu@λxy: \$i: (xphi@xx@xy))) ⇒ ∃b: \$i: ∀xx: \$i: ((in@xx@b)
∃xy: \$i: (in@xy@a and xphi@xy@xx)))) thf(replAx, definition)
replAx ⇒ ∀a: \$i, xf: \$i → \$i: ∃b: \$i: ∀xx: \$i: ((in@xx@b) ⟺ ∃xy: \$i: (in@xy@a and xx = (xf@xy))) thf(image1Ex, conje

SEU794∧2.p More about Functions - Images of Functions
(! A:i.! f:i>i.exu (∧ B:i.! x:i.in x B <−→ (? y:i.in y A & x = f y)))
in: \$i → \$i → \$o thf(in_type, type)
exu: (\$i → \$o) → \$o thf(exu_type, type)
exu = (λxphi: \$i → \$o: ∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy))) thf(exu, definition)
⊆ : \$i → \$i → \$o thf(subset_type, type)
subsetI$_1$: \$o thf(subsetI1_type, type)
subsetI$_1$ = (∀a: \$i, b: \$i: (∀xx: \$i: ((in@xx@a) ⇒ (in@xx@b)) ⇒ (⊆ @a@b))) thf(subsetI$_1$, definition)
setextsub: \$o thf(setextsub_type, type)
setextsub = (∀a: \$i, b: \$i: ((⊆ @a@b) ⇒ ((⊆ @b@a) ⇒ $a = b$))) thf(setextsub, definition)
image1Ex: \$o thf(image1Ex_type, type)
image1Ex = (∀a: \$i, xf: \$i → \$i: ∃b: \$i: ∀xx: \$i: ((in@xx@b) ⟺ ∃xy: \$i: (in@xy@a and xx = (xf@xy)))) thf(image1Ex, d
subsetI$_1$ ⇒ (setextsub ⇒ (image1Ex ⇒ ∀a: \$i, xf: \$i → \$i: (exu@λb: \$i: ∀xx: \$i: ((in@xx@b) ⟺
∃xy: \$i: (in@xy@a and xx = (xf@xy)))))) thf(image1Ex$_1$, conjecture)

SEU795∧2.p More about Functions - Images of Functions
(! A:i.! f:i>i.! x:i.in x (image1 A (∧ y:i.f y)) <−→ (? y:i.in y A & x = f y))
in: \$i → \$i → \$o thf(in_type, type)
exu: (\$i → \$o) → \$o thf(exu_type, type)
exu = (λxphi: \$i → \$o: ∃xx: \$i: (xphi@xx and ∀xy: \$i: ((xphi@xy) ⇒ xx = xy))) thf(exu, definition)
descr: (\$i → \$o) → \$i thf(descr_type, type)
descrp: \$o thf(descrp_type, type)
descrp = (∀xphi: \$i → \$o: ((exu@λxx: \$i: (xphi@xx)) ⇒ (xphi@(descr@λxx: \$i: (xphi@xx))))) thf(descrp, definition)
in__Cong: \$o thf(in__Cong_type, type)
in__Cong = (∀a: \$i, b: \$i: ($a = b$ ⇒ ∀xx: \$i, xy: \$i: (xx = xy ⇒ ((in@xx@a) ⟺ (in@xy@b))))) thf(in__Cong, definition)
image1Ex$_1$: \$o thf(image1Ex1_type, type)
image1Ex$_1$ = (∀a: \$i, xf: \$i → \$i: (exu@λb: \$i: ∀xx: \$i: ((in@xx@b) ⟺ ∃xy: \$i: (in@xy@a and xx = (xf@xy))))) thf(ima
image$_1$: \$i → (\$i → \$i) → \$i thf(image1_type, type)
image$_1$ = (λa: \$i, xf: \$i → \$i: (descr@λb: \$i: ∀xx: \$i: ((in@xx@b) ⟺ ∃xy: \$i: (in@xy@a and xx = (xf@xy))))) thf(image
descrp ⇒ (in__Cong ⇒ (image1Ex$_1$ ⇒ ∀a: \$i, xf: \$i → \$i, xx: \$i: ((in@xx@(image$_1$@a@λxy: \$i: (xf@xy))) ⟺
∃xy: \$i: (in@xy@a and xx = (xf@xy))))) thf(image1Equiv, conjecture)

SEU796∧2.p More about Functions - Images of Functions
(! A:i.! f:i>i.! x:i.in x (image1 A (∧ y:i.f y)) → (? y:i.in y A & x = f y))
in: \$i → \$i → \$o thf(in_type, type)
image$_1$: \$i → (\$i → \$i) → \$i thf(image1_type, type)
image1Equiv: \$o thf(image1Equiv_type, type)
image1Equiv = (∀a: \$i, xf: \$i → \$i, xx: \$i: ((in@xx@(image$_1$@a@λxy: \$i: (xf@xy))) ⟺ ∃xy: \$i: (in@xy@a and xx =
(xf@xy)))) thf(image1Equiv, definition)
image1Equiv ⇒ ∀a: \$i, xf: \$i → \$i, xx: \$i: ((in@xx@(image$_1$@a@λxy: \$i: (xf@xy))) ⇒ ∃xy: \$i: (in@xy@a and xx =
(xf@xy))) thf(image1E, conjecture)

SEU797∧2.p More about Functions - Images of Functions
(! A:i.! f:i>i.! x:i.(? y:i.in y A & x = f y) → in x (image1 A (∧ y:i.f y)))
in: \$i → \$i → \$o thf(in_type, type)

image_1: \$i → (\$i → \$i) → \$i   thf(image1_type, type)
image1Equiv: \$o   thf(image1Equiv_type, type)
image1Equiv = (∀a: \$i, xf: \$i → \$i, xx: \$i: ((in@xx@(image_1@a@λxy: \$i: (xf@xy))) ⟺ ∃xy: \$i: (in@xy@a and xx = (xf@xy))))   thf(image1Equiv, definition)
image1Equiv ⇒ ∀a: \$i, xf: \$i → \$i, xx: \$i: (∃xy: \$i: (in@xy@a and xx = (xf@xy)) ⇒ (in@xx@(image_1@a@λxy: \$i: (xf@xy)))

SEU798∧2.p More about Functions - Injective Functions
(! A:i.! B:i.! f:i.in f (funcSet A B) → injective A B f → in f (injFuncSet A B))
in: \$i → \$i → \$o   thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrI: \$o   thf(dsetconstrI_type, type)
dsetconstrI = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@a) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))))))
funcSet: \$i → \$i → \$i   thf(funcSet_type, type)
injective: \$i → \$i → \$i → \$o   thf(injective_type, type)
injFuncSet: \$i → \$i → \$i   thf(injFuncSet_type, type)
injFuncSet = (λa: \$i, b: \$i: (dsetconstr@(funcSet@a@b)@λxf: \$i: (injective@a@b@xf)))   thf(injFuncSet, definition)
dsetconstrI ⇒ ∀a: \$i, b: \$i, xf: \$i: ((in@xf@(funcSet@a@b)) ⇒ ((injective@a@b@xf) ⇒ (in@xf@(injFuncSet@a@b))))   thf

SEU799∧2.p More about Functions - Injective Functions
(! A:i.! B:i.! f:i.in f (injFuncSet A B) → in f (funcSet A B))
in: \$i → \$i → \$o   thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrEL: \$o   thf(dsetconstrEL_type, type)
dsetconstrEL = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) ⇒ (in@xx@a)))   thf(dsetcons
funcSet: \$i → \$i → \$i   thf(funcSet_type, type)
injective: \$i → \$i → \$i → \$o   thf(injective_type, type)
injFuncSet: \$i → \$i → \$i   thf(injFuncSet_type, type)
injFuncSet = (λa: \$i, b: \$i: (dsetconstr@(funcSet@a@b)@λxf: \$i: (injective@a@b@xf)))   thf(injFuncSet, definition)
dsetconstrEL ⇒ ∀a: \$i, b: \$i, xf: \$i: ((in@xf@(injFuncSet@a@b)) ⇒ (in@xf@(funcSet@a@b)))   thf(injFuncSetFuncIn, conj

SEU800∧2.p More about Functions - Injective Functions
(! x:i.! y:i.! f:i.in f (injFuncSet x y) → injective x y f)
in: \$i → \$i → \$o   thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrER: \$o   thf(dsetconstrER_type, type)
dsetconstrER = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx)))   thf(dsetcons
funcSet: \$i → \$i → \$i   thf(funcSet_type, type)
injective: \$i → \$i → \$i → \$o   thf(injective_type, type)
injFuncSet: \$i → \$i → \$i   thf(injFuncSet_type, type)
injFuncSet = (λa: \$i, b: \$i: (dsetconstr@(funcSet@a@b)@λxf: \$i: (injective@a@b@xf)))   thf(injFuncSet, definition)
dsetconstrER ⇒ ∀xx: \$i, xy: \$i, xf: \$i: ((in@xf@(injFuncSet@xx@xy)) ⇒ (injective@xx@xy@xf))   thf(injFuncSetFuncInj,

SEU801∧2.p More about Functions - Surjective Functions
(! A:i.! B:i.! f:i.in f (surjFuncSet A B) → in f (funcSet A B))
in: \$i → \$i → \$o   thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrEL: \$o   thf(dsetconstrEL_type, type)
dsetconstrEL = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) ⇒ (in@xx@a)))   thf(dsetcons
funcSet: \$i → \$i → \$i   thf(funcSet_type, type)
ap: \$i → \$i → \$i → \$i → \$i   thf(ap_type, type)
surjective: \$i → \$i → \$i → \$o   thf(surjective_type, type)
surjective = (λa: \$i, b: \$i, xf: \$i: ∀xx: \$i: ((in@xx@b) ⇒ ∃xy: \$i: (in@xy@a and (ap@a@b@xf@xy) = xx)))   thf(surjective, d
surjFuncSet: \$i → \$i → \$i   thf(surjFuncSet_type, type)
surjFuncSet = (λa: \$i, b: \$i: (dsetconstr@(funcSet@a@b)@λxf: \$i: (surjective@a@b@xf)))   thf(surjFuncSet, definition)
dsetconstrEL ⇒ ∀a: \$i, b: \$i, xf: \$i: ((in@xf@(surjFuncSet@a@b)) ⇒ (in@xf@(funcSet@a@b)))   thf(surjFuncSetFuncIn, co

SEU802∧2.p More about Functions - Surjective Functions
(! x:i.! y:i.! f:i.in f (surjFuncSet x y) → surjective x y f)
in: \$i → \$i → \$o   thf(in_type, type)
dsetconstr: \$i → (\$i → \$o) → \$i   thf(dsetconstr_type, type)
dsetconstrER: \$o   thf(dsetconstrER_type, type)
dsetconstrER = (∀a: \$i, xphi: \$i → \$o, xx: \$i: ((in@xx@(dsetconstr@a@λxy: \$i: (xphi@xy))) ⇒ (xphi@xx)))   thf(dsetcons

funcSet: $i → $i → $i thf(funcSet_type, type)
ap: $i → $i → $i → $i → $i thf(ap_type, type)
surjective: $i → $i → $i → $o thf(surjective_type, type)
surjective = (λ*a*: $i, *b*: $i, xf: $i: ∀xx: $i: ((in@xx@*b*) ⇒ ∃xy: $i: (in@xy@*a* and (ap@*a*@*b*@xf@xy) = xx))) thf(surjective, d
surjFuncSet: $i → $i → $i thf(surjFuncSet_type, type)
surjFuncSet = (λ*a*: $i, *b*: $i: (dsetconstr@(funcSet@*a*@*b*)@λxf: $i: (surjective@*a*@*b*@xf))) thf(surjFuncSet, definition)
dsetconstrER ⇒ ∀xx: $i, xy: $i, xf: $i: ((in@xf@(surjFuncSet@xx@xy)) ⇒ (surjective@xx@xy@xf)) thf(surjFuncSetFunc

SEU803∧2.p More about Functions - Surjective Functions
(! A:i.! B:i.! f:i>i.(! x:i.in x A → in (f x) B) → (! g:i.in g (funcSet B A) → (! x:i.in x A → ap B A g (f x) = x) → surjective B A g))
in: $i → $i → $o thf(in_type, type)
funcSet: $i → $i → $i thf(funcSet_type, type)
ap: $i → $i → $i → $i → $i thf(ap_type, type)
surjective: $i → $i → $i → $o thf(surjective_type, type)
surjective = (λ*a*: $i, *b*: $i, xf: $i: ∀xx: $i: ((in@xx@*b*) ⇒ ∃xy: $i: (in@xy@*a* and (ap@*a*@*b*@xf@xy) = xx))) thf(surjective, d
∀*a*: $i, *b*: $i, xf: $i → $i: (∀xx: $i: ((in@xx@*a*) ⇒ (in@(xf@xx)@*b*)) ⇒ ∀xg: $i: ((in@xg@(funcSet@*b*@*a*)) ⇒ (∀xx: $i: ((in@xx@*a*) ⇒ (ap@*b*@*a*@xg@(xf@xx)) = xx) ⇒ (surjective@*b*@*a*@xg)))) thf(leftInvIsSurj, conjecture)

SEU804∧2.p More Functions - Surjective Functions - Surjective Cantor Theorem
(! A:i.! f:i.in f (funcSet A (powerset A)) → (surjective A (powerset A) f))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
dsetconstr: $i → ($i → $o) → $i thf(dsetconstr_type, type)
dsetconstrI: $o thf(dsetconstrI_type, type)
dsetconstrI = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@*a*) ⇒ ((xphi@xx) ⇒ (in@xx@(dsetconstr@*a*@λxy: $i: (xphi@xy))))))
dsetconstrEL: $o thf(dsetconstrEL_type, type)
dsetconstrEL = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@*a*@λxy: $i: (xphi@xy))) ⇒ (in@xx@*a*))) thf(dsetcons
dsetconstrER: $o thf(dsetconstrER_type, type)
dsetconstrER = (∀*a*: $i, xphi: $i → $o, xx: $i: ((in@xx@(dsetconstr@*a*@λxy: $i: (xphi@xy))) ⇒ (xphi@xx))) thf(dsetcons
powersetI: $o thf(powersetI_type, type)
powersetI = (∀*a*: $i, *b*: $i: (∀xx: $i: ((in@xx@*b*) ⇒ (in@xx@*a*)) ⇒ (in@*b*@(powerset@*a*)))) thf(powersetI, definition)
funcSet: $i → $i → $i thf(funcSet_type, type)
ap: $i → $i → $i → $i → $i thf(ap_type, type)
surjective: $i → $i → $i → $o thf(surjective_type, type)
surjective = (λ*a*: $i, *b*: $i, xf: $i: ∀xx: $i: ((in@xx@*b*) ⇒ ∃xy: $i: (in@xy@*a* and (ap@*a*@*b*@xf@xy) = xx))) thf(surjective, d
dsetconstrI ⇒ (dsetconstrEL ⇒ (dsetconstrER ⇒ (powersetI ⇒ ∀*a*: $i, xf: $i: ((in@xf@(funcSet@*a*@(powerset@*a*))) ⇒ ¬ surjective@*a*@(powerset@*a*)@xf)))) thf(surjCantorThm, conjecture)

SEU805∧2.p The Foundation Axiom
(! A:i.nonempty A → (? X:i.in X A & binintersect X A = emptyset))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
foundationAx: $o thf(foundationAx_type, type)
foundationAx = (∀*a*: $i: (∃xx: $i: (in@xx@*a*) ⇒ ∃*b*: $i: (in@*b*@*a* and ¬ ∃xx: $i: (in@xx@*b* and in@xx@*a*)))) thf(foundatio
nonempty: $i → $o thf(nonempty_type, type)
nonempty = (λxx: $i: xx ≠ emptyset) thf(nonempty, definition)
nonemptyE₁: $o thf(nonemptyE1_type, type)
nonemptyE₁ = (∀*a*: $i: ((nonempty@*a*) ⇒ ∃xx: $i: (in@xx@*a*))) thf(nonemptyE₁, definition)
binintersect: $i → $i → $i thf(binintersect_type, type)
disjointsetsI₁: $o thf(disjointsetsI1_type, type)
disjointsetsI₁ = (∀*a*: $i, *b*: $i: (¬ ∃xx: $i: (in@xx@*a* and in@xx@*b*) ⇒ (binintersect@*a*@*b*) = emptyset)) thf(disjointsetsI₁,
foundationAx ⇒ (nonemptyE₁ ⇒ (disjointsetsI₁ ⇒ ∀*a*: $i: ((nonempty@*a*) ⇒ ∃*x*: $i: (in@*x*@*a* and (binintersect@*x*@*a*) = emptyset)))) thf(foundation₂, conjecture)

SEU806∧2.p The Foundation Axiom
(! A:i. (in A A))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
foundationAx: $o thf(foundationAx_type, type)

foundationAx = (∀*a*: $i: (∃xx: $i: (in@xx@*a*) ⇒ ∃*b*: $i: (in@*b*@*a* and ¬ ∃xx: $i: (in@xx@*b* and in@xx@*a*)))) thf(foundatio
setadjoinIL: $o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
uniqinunit: $o thf(uniqinunit_type, type)
uniqinunit = (∀xx: $i, xy: $i: ((in@xx@(setadjoin@xy@emptyset)) ⇒ xx = xy)) thf(uniqinunit, definition)
in__Cong: $o thf(in__Cong_type, type)
in__Cong = (∀*a*: $i, *b*: $i: (*a* = *b* ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@*a*) ⟺ (in@xy@*b*))))) thf(in__Cong, definition)
foundationAx ⇒ (setadjoinIL ⇒ (uniqinunit ⇒ (in__Cong ⇒ ∀*a*: $i: ¬ in@*a*@*a*))) thf(notinself, conjecture)

SEU807∧2.p The Foundation Axiom
(! A:i.! B:i.in A B → (in B A))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
foundationAx: $o thf(foundationAx_type, type)
foundationAx = (∀*a*: $i: (∃xx: $i: (in@xx@*a*) ⇒ ∃*b*: $i: (in@*b*@*a* and ¬ ∃xx: $i: (in@xx@*b* and in@xx@*a*)))) thf(foundatio
setadjoinIL: $o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
setadjoinIR: $o thf(setadjoinIR_type, type)
setadjoinIR = (∀xx: $i, *a*: $i, xy: $i: ((in@xy@*a*) ⇒ (in@xy@(setadjoin@xx@*a*)))) thf(setadjoinIR, definition)
in__Cong: $o thf(in__Cong_type, type)
in__Cong = (∀*a*: $i, *b*: $i: (*a* = *b* ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@*a*) ⟺ (in@xy@*b*))))) thf(in__Cong, definition)
upairset2E: $o thf(upairset2E_type, type)
upairset2E = (∀xx: $i, xy: $i, xz: $i: ((in@xz@(setadjoin@xx@(setadjoin@xy@emptyset))) ⇒ (xz = xx or xz = xy))) thf(upairset2E, definition)
foundationAx ⇒ (setadjoinIL ⇒ (setadjoinIR ⇒ (in__Cong ⇒ (upairset2E ⇒ ∀*a*: $i, *b*: $i: ((in@*a*@*b*) ⇒ ¬ in@*b*@*a*))))) thf(notinself$_2$, conjecture)

SEU808∧2.p Omega and Peano
(! x:i.in x omega → in (omegaS x) omega)
in: $i → $i → $o thf(in_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
omega: $i thf(omega_type, type)
omegaSAx: $o thf(omegaSAx_type, type)
omegaSAx = (∀xx: $i: ((in@xx@omega) ⇒ (in@(setadjoin@xx@xx)@omega))) thf(omegaSAx, definition)
omegaS: $i → $i thf(omegaS_type, type)
omegaS = (λxx: $i: (setadjoin@xx@xx)) thf(omegaS, definition)
omegaSAx ⇒ ∀xx: $i: ((in@xx@omega) ⇒ (in@(omegaS@xx)@omega)) thf(omegaSp, conjecture)

SEU809∧2.p Omega and Peano
(! x:i.in x omega → in (setadjoin x x) omega)
in: $i → $i → $o thf(in_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
omega: $i thf(omega_type, type)
omegaSAx: $o thf(omegaSAx_type, type)
omegaSAx = (∀xx: $i: ((in@xx@omega) ⇒ (in@(setadjoin@xx@xx)@omega))) thf(omegaSAx, definition)
omegaSAx ⇒ ∀xx: $i: ((in@xx@omega) ⇒ (in@(setadjoin@xx@xx)@omega)) thf(omegaSclos, conjecture)

SEU810∧2.p Omega and Peano
(! x:i.in x omega → (omegaS x = emptyset))
in: $i → $i → $o thf(in_type, type)
emptyset: $i thf(emptyset_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
omega: $i thf(omega_type, type)
emptysetE: $o thf(emptysetE_type, type)
emptysetE = (∀xx: $i: ((in@xx@emptyset) ⇒ ∀xphi: $o: xphi)) thf(emptysetE, definition)
setadjoinIL: $o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
in__Cong: $o thf(in__Cong_type, type)
in__Cong = (∀*a*: $i, *b*: $i: (*a* = *b* ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@*a*) ⟺ (in@xy@*b*))))) thf(in__Cong, definition)
omegaS: $i → $i thf(omegaS_type, type)

omegaS = (λxx: $i: (setadjoin@xx@xx)) thf(omegaS, definition)
emptysetE ⇒ (setadjoinIL ⇒ (in__Cong ⇒ ∀xx: $i: ((in@xx@omega) ⇒ (omegaS@xx) ≠ emptyset))) thf(peano0notS

SEU811∧2.p Omega and Peano
(! x:i.in x omega → (! y:i.in y omega → omegaS x = omegaS y → x = y))
in: $i → $i → $o thf(in_type, type)
setadjoin: $i → $i → $i thf(setadjoin_type, type)
omega: $i thf(omega_type, type)
setadjoinIL: $o thf(setadjoinIL_type, type)
setadjoinIL = (∀xx: $i, xy: $i: (in@xx@(setadjoin@xx@xy))) thf(setadjoinIL, definition)
setadjoinE: $o thf(setadjoinE_type, type)
setadjoinE = (∀xx: $i, a: $i, xy: $i: ((in@xy@(setadjoin@xx@a)) ⇒ ∀xphi: $o: ((xy = xx ⇒ xphi) ⇒ (((in@xy@a) ⇒ xphi) ⇒ xphi)))) thf(setadjoinE, definition)
in__Cong: $o thf(in__Cong_type, type)
in__Cong = (∀a: $i, b: $i: ($a = b$ ⇒ ∀xx: $i, xy: $i: (xx = xy ⇒ ((in@xx@$a$) ⟺ (in@xy@$b$))))) thf(in__Cong, definition)
notinself$_2$: $o thf(notinself2_type, type)
notinself$_2$ = (∀a: $i, b: $i: ((in@$a$@$b$) ⇒ ¬ in@$b$@$a$)) thf(notinself$_2$, definition)
omegaS: $i → $i thf(omegaS_type, type)
omegaS = (λxx: $i: (setadjoin@xx@xx)) thf(omegaS, definition)
setadjoinIL ⇒ (setadjoinE ⇒ (in__Cong ⇒ (notinself$_2$ ⇒ ∀xx: $i: ((in@xx@omega) ⇒ ∀xy: $i: ((in@xy@omega) ⇒ ((omegaS@xx) = (omegaS@xy) ⇒ xx = xy)))))) thf(peanoSinj, conjecture)

SEU812∧2.p Transitive Sets
(! X:i.transitiveset X → (! A:i.in A X → subset A X))
in: $i → $i → $o thf(in_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
transitiveset: $i → $o thf(transitiveset_type, type)
transitiveset = (λa: $i: ∀$x$: $i: ((in@$x$@$a$) ⇒ (⊆ @$x$@$a$))) thf(transitiveset, definition)
∀x: $i: ((transitiveset@$x$) ⇒ ∀$a$: $i: ((in@$a$@$x$) ⇒ (⊆ @$a$@$x$))) thf(transitivesetOp$_1$, conjecture)

SEU813∧2.p Transitive Sets
(! X:i.transitiveset X → (! Y:i.transitiveset Y → transitiveset (binintersect X Y)))
in: $i → $i → $o thf(in_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
binintersect: $i → $i → $i thf(binintersect_type, type)
binintersectSubset$_5$: $o thf(binintersectSubset5_type, type)
binintersectSubset$_5$ = (∀a: $i, b: $i, c: $i: ((⊆ @$c$@$a$) ⇒ ((⊆ @$c$@$b$) ⇒ (⊆ @$c$@(binintersect@$a$@$b$))))) thf(binintersectSu
binintersectEL: $o thf(binintersectEL_type, type)
binintersectEL = (∀a: $i, b: $i, xx: $i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@a))) thf(binintersectEL, definition)
binintersectER: $o thf(binintersectER_type, type)
binintersectER = (∀a: $i, b: $i, xx: $i: ((in@xx@(binintersect@a@b)) ⇒ (in@xx@b))) thf(binintersectER, definition)
transitiveset: $i → $o thf(transitiveset_type, type)
transitiveset = (λa: $i: ∀$x$: $i: ((in@$x$@$a$) ⇒ (⊆ @$x$@$a$))) thf(transitiveset, definition)
transitivesetOp$_1$: $o thf(transitivesetOp1_type, type)
transitivesetOp$_1$ = (∀x: $i: ((transitiveset@$x$) ⇒ ∀$a$: $i: ((in@$a$@$x$) ⇒ (⊆ @$a$@$x$)))) thf(transitivesetOp$_1$, definition)
binintersectSubset$_5$ ⇒ (binintersectEL ⇒ (binintersectER ⇒ (transitivesetOp$_1$ ⇒ ∀x: $i: ((transitiveset@$x$) ⇒ ∀$y$: $i: ((transitiveset@$y$) ⇒ (transitiveset@(binintersect@x@y))))))) thf(binintTransitive, conjecture)

SEU814∧2.p Transitive Sets
(! X:i.transitiveset X → (! A:i.! x:i.in A X → in x A → in x X))
in: $i → $i → $o thf(in_type, type)
⊆ : $i → $i → $o thf(subset_type, type)
subsetE: $o thf(subsetE_type, type)
subsetE = (∀a: $i, b: $i, xx: $i: ((⊆ @$a$@$b$) ⇒ ((in@xx@$a$) ⇒ (in@xx@$b$)))) thf(subsetE, definition)
transitiveset: $i → $o thf(transitiveset_type, type)
transitiveset = (λa: $i: ∀$x$: $i: ((in@$x$@$a$) ⇒ (⊆ @$x$@$a$))) thf(transitiveset, definition)
transitivesetOp$_1$: $o thf(transitivesetOp1_type, type)
transitivesetOp$_1$ = (∀x: $i: ((transitiveset@$x$) ⇒ ∀$a$: $i: ((in@$a$@$x$) ⇒ (⊆ @$a$@$x$)))) thf(transitivesetOp$_1$, definition)
subsetE ⇒ (transitivesetOp$_1$ ⇒ ∀x: $i: ((transitiveset@$x$) ⇒ ∀$a$: $i, xx: $i: ((in@$a$@$x$) ⇒ ((in@xx@$a$) ⇒ (in@xx@$x$))))) thf(transitivesetOp$_2$, conjecture)

SEU815∧2.p Transitive Sets

(! X:i.(! x:i.in x X → transitiveset x) → transitiveset (setunion X))
$\text{in}: \$i \to \$i \to \$o$ thf(in_type, type)
$\text{setunion}: \$i \to \$i$ thf(setunion_type, type)
$\text{setunionI}: \$o$ thf(setunionI_type, type)
$\text{setunionI} = (\forall a\colon \$i, \text{xx}\colon \$i, b\colon \$i\colon ((\text{in}@\text{xx}@b) \Rightarrow ((\text{in}@b@a) \Rightarrow (\text{in}@\text{xx}@(\text{setunion}@a)))))$ thf(setunionI, definition)
$\text{setunionE}: \$o$ thf(setunionE_type, type)
$\text{setunionE} = (\forall a\colon \$i, \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@(\text{setunion}@a)) \Rightarrow \forall \text{xphi}\colon \$o\colon (\forall b\colon \$i\colon ((\text{in}@\text{xx}@b) \Rightarrow ((\text{in}@b@a) \Rightarrow \text{xphi})) \Rightarrow \text{xphi})))$ thf(setunionE, definition)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\text{subsetI}_1\colon \$o$ thf(subsetI1_type, type)
$\text{subsetI}_1 = (\forall a\colon \$i, b\colon \$i\colon (\forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@\text{xx}@b)) \Rightarrow (\subseteq @a@b)))$ thf(subsetI_1, definition)
$\text{transitiveset}\colon \$i \to \$o$ thf(transitiveset_type, type)
$\text{transitiveset} = (\lambda a\colon \$i\colon \forall x\colon \$i\colon ((\text{in}@x@a) \Rightarrow (\subseteq @x@a)))$ thf(transitiveset, definition)
$\text{transitivesetOp}_2\colon \$o$ thf(transitivesetOp2_type, type)
$\text{transitivesetOp}_2 = (\forall x\colon \$i\colon ((\text{transitiveset}@x) \Rightarrow \forall a\colon \$i, \text{xx}\colon \$i\colon ((\text{in}@a@x) \Rightarrow ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@\text{xx}@x)))))$ thf(transit
$\text{setunionI} \Rightarrow (\text{setunionE} \Rightarrow (\text{subsetI}_1 \Rightarrow (\text{transitivesetOp}_2 \Rightarrow \forall x\colon \$i\colon (\forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@x) \Rightarrow (\text{transitiveset}@\text{xx})) \Rightarrow (\text{transitiveset}@(\text{setunion}@x))))))$ thf(setunionTransitive, conjecture)

SEU816∧2.p Ordinals
(! X:i.ordinal X → (! Y:i.ordinal Y → transitiveset (binintersect X Y)))
$\text{in}: \$i \to \$i \to \$o$ thf(in_type, type)
$\text{emptyset}: \$i$ thf(emptyset_type, type)
$\text{powerset}: \$i \to \$i$ thf(powerset_type, type)
$\text{nonempty}: \$i \to \$o$ thf(nonempty_type, type)
$\text{nonempty} = (\lambda \text{xx}\colon \$i\colon \text{xx} \neq \text{emptyset})$ thf(nonempty, definition)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\text{binintersect}: \$i \to \$i \to \$i$ thf(binintersect_type, type)
$\text{transitiveset}\colon \$i \to \$o$ thf(transitiveset_type, type)
$\text{transitiveset} = (\lambda a\colon \$i\colon \forall x\colon \$i\colon ((\text{in}@x@a) \Rightarrow (\subseteq @x@a)))$ thf(transitiveset, definition)
$\text{binintTransitive}\colon \$o$ thf(binintTransitive_type, type)
$\text{binintTransitive} = (\forall x\colon \$i\colon ((\text{transitiveset}@x) \Rightarrow \forall y\colon \$i\colon ((\text{transitiveset}@y) \Rightarrow (\text{transitiveset}@(\text{binintersect}@x@y)))))$ thf
$\text{stricttotalorderedByIn}\colon \$i \to \$o$ thf(stricttotalorderedByIn_type, type)
$\text{stricttotalorderedByIn} = (\lambda a\colon \$i\colon (\forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow \forall x\colon \$i\colon ((\text{in}@x@a) \Rightarrow \forall y\colon \$i\colon ((\text{in}@y@a) \Rightarrow ((\text{in}@\text{xx}@x \text{ and in}@x@y) \Rightarrow (\text{in}@\text{xx}@y))))) \text{ and } \forall x\colon \$i\colon ((\text{in}@x@a) \Rightarrow \forall y\colon \$i\colon ((\text{in}@y@a) \Rightarrow (x = y \text{ or in}@x@y \text{ or in}@y@x)$ $\neg\, \text{in}@x@x)))$ thf(stricttotalorderedByIn, definition)
$\text{wellorderedByIn}\colon \$i \to \$o$ thf(wellorderedByIn_type, type)
$\text{wellorderedByIn} = (\lambda a\colon \$i\colon (\text{stricttotalorderedByIn}@a \text{ and } \forall x\colon \$i\colon ((\text{in}@x@(\text{powerset}@a)) \Rightarrow ((\text{nonempty}@x) \Rightarrow \exists \text{xx}\colon \$i\colon (\text{in}@\text{xx}@x \text{ and } \forall y\colon \$i\colon ((\text{in}@y@x) \Rightarrow (\text{xx} = y \text{ or in}@\text{xx}@y)))))))$ thf(wellorderedByIn, definition)
$\text{ordinal}\colon \$i \to \$o$ thf(ordinal_type, type)
$\text{ordinal} = (\lambda \text{xx}\colon \$i\colon (\text{transitiveset}@\text{xx} \text{ and wellorderedByIn}@\text{xx}))$ thf(ordinal, definition)
$\text{binintTransitive} \Rightarrow \forall x\colon \$i\colon ((\text{ordinal}@x) \Rightarrow \forall y\colon \$i\colon ((\text{ordinal}@y) \Rightarrow (\text{transitiveset}@(\text{binintersect}@x@y))))$ thf(ordinalMi

SEU817∧2.p Ordinals
(! X:i.ordinal X → (! x:i.! A:i.in A X → in x A → in x X))
$\text{in}: \$i \to \$i \to \$o$ thf(in_type, type)
$\text{emptyset}: \$i$ thf(emptyset_type, type)
$\text{powerset}: \$i \to \$i$ thf(powerset_type, type)
$\text{nonempty}: \$i \to \$o$ thf(nonempty_type, type)
$\text{nonempty} = (\lambda \text{xx}\colon \$i\colon \text{xx} \neq \text{emptyset})$ thf(nonempty, definition)
$\subseteq\colon \$i \to \$i \to \$o$ thf(subset_type, type)
$\text{subsetE}\colon \$o$ thf(subsetE_type, type)
$\text{subsetE} = (\forall a\colon \$i, b\colon \$i, \text{xx}\colon \$i\colon ((\subseteq @a@b) \Rightarrow ((\text{in}@\text{xx}@a) \Rightarrow (\text{in}@\text{xx}@b))))$ thf(subsetE, definition)
$\text{transitiveset}\colon \$i \to \$o$ thf(transitiveset_type, type)
$\text{transitiveset} = (\lambda a\colon \$i\colon \forall x\colon \$i\colon ((\text{in}@x@a) \Rightarrow (\subseteq @x@a)))$ thf(transitiveset, definition)
$\text{stricttotalorderedByIn}\colon \$i \to \$o$ thf(stricttotalorderedByIn_type, type)
$\text{stricttotalorderedByIn} = (\lambda a\colon \$i\colon (\forall \text{xx}\colon \$i\colon ((\text{in}@\text{xx}@a) \Rightarrow \forall x\colon \$i\colon ((\text{in}@x@a) \Rightarrow \forall y\colon \$i\colon ((\text{in}@y@a) \Rightarrow ((\text{in}@\text{xx}@x \text{ and in}@x@y) \Rightarrow (\text{in}@\text{xx}@y))))) \text{ and } \forall x\colon \$i\colon ((\text{in}@x@a) \Rightarrow \forall y\colon \$i\colon ((\text{in}@y@a) \Rightarrow (x = y \text{ or in}@x@y \text{ or in}@y@x)$ $\neg\, \text{in}@x@x)))$ thf(stricttotalorderedByIn, definition)
$\text{wellorderedByIn}\colon \$i \to \$o$ thf(wellorderedByIn_type, type)

wellorderedByIn = (λa: \$i: (stricttotalorderedByIn@a and $\forall x$: \$i: ((in@x@(powerset@a)) $\Rightarrow$ ((nonempty@x) $\Rightarrow$ $\exists$xx: \$i: (in@xx@x and $\forall y$: \$i: ((in@y@x) $\Rightarrow$ (xx = y or in@xx@y))))))) thf(wellorderedByIn, definition)
ordinal: \$i $\rightarrow$ \$o thf(ordinal_type, type)
ordinal = (λxx: \$i: (transitiveset@xx and wellorderedByIn@xx)) thf(ordinal, definition)
subsetE $\Rightarrow$ $\forall x$: \$i: ((ordinal@x) $\Rightarrow$ $\forall$xx: \$i, a: \$i: ((in@a@x) $\Rightarrow$ ((in@xx@a) $\Rightarrow$ (in@xx@x)))) thf(ordinalTransSet, conj

SEU818^2.p Ordinals
(! X:i.ordinal X $\rightarrow$ (! A:i.in A X $\rightarrow$ subset A X))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
powerset: \$i $\rightarrow$ \$i thf(powerset_type, type)
nonempty: \$i $\rightarrow$ \$o thf(nonempty_type, type)
nonempty = (λxx: \$i: xx $\neq$ emptyset) thf(nonempty, definition)
$\subseteq$: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(subset_type, type)
subsetI_1: \$o thf(subsetI1_type, type)
subsetI_1 = ($\forall a$: \$i, b: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ (in@xx@b)) $\Rightarrow$ ($\subseteq$ @a@b))) thf(subsetI_1, definition)
transitiveset: \$i $\rightarrow$ \$o thf(transitiveset_type, type)
transitiveset = (λa: \$i: $\forall x$: \$i: ((in@x@a) $\Rightarrow$ ($\subseteq$ @x@a))) thf(transitiveset, definition)
stricttotalorderedByIn: \$i $\rightarrow$ \$o thf(stricttotalorderedByIn_type, type)
stricttotalorderedByIn = (λa: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ $\forall x$: \$i: ((in@x@a) $\Rightarrow$ $\forall y$: \$i: ((in@y@a) $\Rightarrow$ ((in@xx@x and in@x@y) $\Rightarrow$ (in@xx@y))))) and $\forall x$: \$i: ((in@x@a) $\Rightarrow$ $\forall y$: \$i: ((in@y@a) $\Rightarrow$ ($x = y$ or in@x@y or in@y@x)) $\neg$ in@x@x))) thf(stricttotalorderedByIn, definition)
wellorderedByIn: \$i $\rightarrow$ \$o thf(wellorderedByIn_type, type)
wellorderedByIn = (λa: \$i: (stricttotalorderedByIn@a and $\forall x$: \$i: ((in@x@(powerset@a)) $\Rightarrow$ ((nonempty@x) $\Rightarrow$ $\exists$xx: \$i: (in@xx@x and $\forall y$: \$i: ((in@y@x) $\Rightarrow$ (xx = y or in@xx@y))))))) thf(wellorderedByIn, definition)
ordinal: \$i $\rightarrow$ \$o thf(ordinal_type, type)
ordinal = (λxx: \$i: (transitiveset@xx and wellorderedByIn@xx)) thf(ordinal, definition)
ordinalTransSet: \$o thf(ordinalTransSet_type, type)
ordinalTransSet = ($\forall x$: \$i: ((ordinal@x) $\Rightarrow$ $\forall$xx: \$i, a: \$i: ((in@a@x) $\Rightarrow$ ((in@xx@a) $\Rightarrow$ (in@xx@x))))) thf(ordinalTrans
subsetI_1 $\Rightarrow$ (ordinalTransSet $\Rightarrow$ $\forall x$: \$i: ((ordinal@x) $\Rightarrow$ $\forall a$: \$i: ((in@a@x) $\Rightarrow$ ($\subseteq$ @a@x)))) thf(ordinalTransSet$_1$, conje

SEU819^2.p Ordinals
(! X:i.(! x:i.in x X $\rightarrow$ ordinal x) $\rightarrow$ transitiveset (setunion X))
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
powerset: \$i $\rightarrow$ \$i thf(powerset_type, type)
setunion: \$i $\rightarrow$ \$i thf(setunion_type, type)
nonempty: \$i $\rightarrow$ \$o thf(nonempty_type, type)
nonempty = (λxx: \$i: xx $\neq$ emptyset) thf(nonempty, definition)
$\subseteq$: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(subset_type, type)
transitiveset: \$i $\rightarrow$ \$o thf(transitiveset_type, type)
transitiveset = (λa: \$i: $\forall x$: \$i: ((in@x@a) $\Rightarrow$ ($\subseteq$ @x@a))) thf(transitiveset, definition)
setunionTransitive: \$o thf(setunionTransitive_type, type)
setunionTransitive = ($\forall x$: \$i: ($\forall$xx: \$i: ((in@xx@x) $\Rightarrow$ (transitiveset@xx)) $\Rightarrow$ (transitiveset@(setunion@x)))) thf(setunio
stricttotalorderedByIn: \$i $\rightarrow$ \$o thf(stricttotalorderedByIn_type, type)
stricttotalorderedByIn = (λa: \$i: ($\forall$xx: \$i: ((in@xx@a) $\Rightarrow$ $\forall x$: \$i: ((in@x@a) $\Rightarrow$ $\forall y$: \$i: ((in@y@a) $\Rightarrow$ ((in@xx@x and in@x@y) $\Rightarrow$ (in@xx@y))))) and $\forall x$: \$i: ((in@x@a) $\Rightarrow$ $\forall y$: \$i: ((in@y@a) $\Rightarrow$ ($x = y$ or in@x@y or in@y@x)) $\neg$ in@x@x))) thf(stricttotalorderedByIn, definition)
wellorderedByIn: \$i $\rightarrow$ \$o thf(wellorderedByIn_type, type)
wellorderedByIn = (λa: \$i: (stricttotalorderedByIn@a and $\forall x$: \$i: ((in@x@(powerset@a)) $\Rightarrow$ ((nonempty@x) $\Rightarrow$ $\exists$xx: \$i: (in@xx@x and $\forall y$: \$i: ((in@y@x) $\Rightarrow$ (xx = y or in@xx@y))))))) thf(wellorderedByIn, definition)
ordinal: \$i $\rightarrow$ \$o thf(ordinal_type, type)
ordinal = (λxx: \$i: (transitiveset@xx and wellorderedByIn@xx)) thf(ordinal, definition)
setunionTransitive $\Rightarrow$ $\forall x$: \$i: ($\forall$xx: \$i: ((in@xx@x) $\Rightarrow$ (ordinal@xx)) $\Rightarrow$ (transitiveset@(setunion@x))) thf(setunionOrd

SEU820^2.p Ordinals
ordinal emptyset
in: \$i $\rightarrow$ \$i $\rightarrow$ \$o thf(in_type, type)
emptyset: \$i thf(emptyset_type, type)
powerset: \$i $\rightarrow$ \$i thf(powerset_type, type)
nonempty: \$i $\rightarrow$ \$o thf(nonempty_type, type)

nonempty = (λxx: $i: xx ≠ emptyset) thf(nonempty, definition)
vacuousDall: $o thf(vacuousDall_type, type)
vacuousDall = (∀xphi: $i → $o, xx: $i: ((in@xx@emptyset) ⇒ (xphi@xx))) thf(vacuousDall, definition)
⊆ : $i → $i → $o thf(subset_type, type)
subsetemptysetimpeq: $o thf(subsetemptysetimpeq_type, type)
subsetemptysetimpeq = (∀*a*: $i: ((⊆ @*a*@emptyset) ⇒ *a* = emptyset)) thf(subsetemptysetimpeq, definition)
$\mathrm{powersetE}_1$: $o thf(powersetE1_type, type)
$\mathrm{powersetE}_1$ = (∀*a*: $i, *b*: $i: ((in@*b*@(powerset@*a*)) ⇒ (⊆ @*b*@*a*))) thf($\mathrm{powersetE}_1$, definition)
transitiveset: $i → $o thf(transitiveset_type, type)
transitiveset = (λ*a*: $i: ∀*x*: $i: ((in@*x*@*a*) ⇒ (⊆ @*x*@*a*))) thf(transitiveset, definition)
stricttotalorderedByIn: $i → $o thf(stricttotalorderedByIn_type, type)
stricttotalorderedByIn = (λ*a*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ ∀*x*: $i: ((in@*x*@*a*) ⇒ ∀*y*: $i: ((in@*y*@*a*) ⇒ ((in@xx@*x* and in@*x*@*y*) ⇒ (in@xx@*y*))))) and ∀*x*: $i: ((in@*x*@*a*) ⇒ ∀*y*: $i: ((in@*y*@*a*) ⇒ (*x* = *y* or in@*x*@*y* or in@*y*@*x*)) ¬ in@*x*@*x*))) thf(stricttotalorderedByIn, definition)
wellorderedByIn: $i → $o thf(wellorderedByIn_type, type)
wellorderedByIn = (λ*a*: $i: (stricttotalorderedByIn@*a* and ∀*x*: $i: ((in@*x*@(powerset@*a*)) ⇒ ((nonempty@*x*) ⇒ ∃xx: $i: (in@xx@*x* and ∀*y*: $i: ((in@*y*@*x*) ⇒ (xx = *y* or in@xx@*y*))))))) thf(wellorderedByIn, definition)
ordinal: $i → $o thf(ordinal_type, type)
ordinal = (λxx: $i: (transitiveset@xx and wellorderedByIn@xx)) thf(ordinal, definition)
vacuousDall ⇒ (subsetemptysetimpeq ⇒ ($\mathrm{powersetE}_1$ ⇒ (ordinal@emptyset))) thf(emptysetOrdinal, conjecture)

SEU821∧2.p Ordinals
(! X:i.ordinal X → (! A:i.in A X → (in A A)))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
nonempty: $i → $o thf(nonempty_type, type)
transitiveset: $i → $o thf(transitiveset_type, type)
stricttotalorderedByIn: $i → $o thf(stricttotalorderedByIn_type, type)
stricttotalorderedByIn = (λ*a*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ ∀*x*: $i: ((in@*x*@*a*) ⇒ ∀*y*: $i: ((in@*y*@*a*) ⇒ ((in@xx@*x* and in@*x*@*y*) ⇒ (in@xx@*y*))))) and ∀*x*: $i: ((in@*x*@*a*) ⇒ ∀*y*: $i: ((in@*y*@*a*) ⇒ (*x* = *y* or in@*x*@*y* or in@*y*@*x*)) ¬ in@*x*@*x*))) thf(stricttotalorderedByIn, definition)
wellorderedByIn: $i → $o thf(wellorderedByIn_type, type)
wellorderedByIn = (λ*a*: $i: (stricttotalorderedByIn@*a* and ∀*x*: $i: ((in@*x*@(powerset@*a*)) ⇒ ((nonempty@*x*) ⇒ ∃xx: $i: (in@xx@*x* and ∀*y*: $i: ((in@*y*@*x*) ⇒ (xx = *y* or in@xx@*y*))))))) thf(wellorderedByIn, definition)
ordinal: $i → $o thf(ordinal_type, type)
ordinal = (λxx: $i: (transitiveset@xx and wellorderedByIn@xx)) thf(ordinal, definition)
∀*x*: $i: ((ordinal@*x*) ⇒ ∀*a*: $i: ((in@*a*@*x*) ⇒ ¬ in@*a*@*a*)) thf(ordinalIrrefl, conjecture)

SEU822∧2.p Ordinals
(! X:i.ordinal X → (in X X))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)
nonempty: $i → $o thf(nonempty_type, type)
transitiveset: $i → $o thf(transitiveset_type, type)
stricttotalorderedByIn: $i → $o thf(stricttotalorderedByIn_type, type)
stricttotalorderedByIn = (λ*a*: $i: (∀xx: $i: ((in@xx@*a*) ⇒ ∀*x*: $i: ((in@*x*@*a*) ⇒ ∀*y*: $i: ((in@*y*@*a*) ⇒ ((in@xx@*x* and in@*x*@*y*) ⇒ (in@xx@*y*))))) and ∀*x*: $i: ((in@*x*@*a*) ⇒ ∀*y*: $i: ((in@*y*@*a*) ⇒ (*x* = *y* or in@*x*@*y* or in@*y*@*x*)) ¬ in@*x*@*x*))) thf(stricttotalorderedByIn, definition)
wellorderedByIn: $i → $o thf(wellorderedByIn_type, type)
wellorderedByIn = (λ*a*: $i: (stricttotalorderedByIn@*a* and ∀*x*: $i: ((in@*x*@(powerset@*a*)) ⇒ ((nonempty@*x*) ⇒ ∃xx: $i: (in@xx@*x* and ∀*y*: $i: ((in@*y*@*x*) ⇒ (xx = *y* or in@xx@*y*))))))) thf(wellorderedByIn, definition)
ordinal: $i → $o thf(ordinal_type, type)
ordinal = (λxx: $i: (transitiveset@xx and wellorderedByIn@xx)) thf(ordinal, definition)
ordinalIrrefl: $o thf(ordinalIrrefl_type, type)
ordinalIrrefl = (∀*x*: $i: ((ordinal@*x*) ⇒ ∀*a*: $i: ((in@*a*@*x*) ⇒ ¬ in@*a*@*a*))) thf(ordinalIrrefl, definition)
ordinalIrrefl ⇒ ∀*x*: $i: ((ordinal@*x*) ⇒ ¬ in@*x*@*x*) thf($\mathrm{ordinalIrrefl}_2$, conjecture)

SEU823∧2.p Ordinals
(! X:i.ordinal X → (! A:i.in X A → (in A X)))
in: $i → $i → $o thf(in_type, type)
powerset: $i → $i thf(powerset_type, type)

nonempty: $i → $o   thf(nonempty_type, type)
transitiveset: $i → $o   thf(transitiveset_type, type)
stricttotalorderedByIn: $i → $o   thf(stricttotalorderedByIn_type, type)
stricttotalorderedByIn = (λa: $i: (∀xx: $i: ((in@xx@a) ⇒ ∀x: $i: ((in@x@a) ⇒ ∀y: $i: ((in@y@a) ⇒ ((in@xx@x and in@x@y) ⇒ (in@xx@y))))) and ∀x: $i: ((in@x@a) ⇒ ∀y: $i: ((in@y@a) ⇒ (x = y or in@x@y or in@y@x)
¬ in@x@x)))   thf(stricttotalorderedByIn, definition)
wellorderedByIn: $i → $o   thf(wellorderedByIn_type, type)
wellorderedByIn = (λa: $i: (stricttotalorderedByIn@a and ∀x: $i: ((in@x@(powerset@a)) ⇒ ((nonempty@x) ⇒ ∃xx: $i: (in@xx@x and ∀y: $i: ((in@y@x) ⇒ (xx = y or in@xx@y)))))))   thf(wellorderedByIn, definition)
ordinal: $i → $o   thf(ordinal_type, type)
ordinal = (λxx: $i: (transitiveset@xx and wellorderedByIn@xx))   thf(ordinal, definition)
ordinalTransSet: $o   thf(ordinalTransSet_type, type)
ordinalTransSet = (∀x: $i: ((ordinal@x) ⇒ ∀xx: $i, a: $i: ((in@a@x) ⇒ ((in@xx@a) ⇒ (in@xx@x)))))   thf(ordinalTrans
ordinalIrrefl: $o   thf(ordinalIrrefl_type, type)
ordinalIrrefl = (∀x: $i: ((ordinal@x) ⇒ ∀a: $i: ((in@a@x) ⇒ ¬ in@a@a)))   thf(ordinalIrrefl, definition)
ordinalTransSet ⇒ (ordinalIrrefl ⇒ ∀x: $i: ((ordinal@x) ⇒ ∀a: $i: ((in@x@a) ⇒ ¬ in@a@x)))   thf(ordinalNoCycle, cor

SEU824∧2.p Ordinals
(! X:i.ordinal X → (! x:i.! A:i.! B:i.in x A → in A B → in B X → in x B))
in: $i → $i → $o   thf(in_type, type)
powerset: $i → $i   thf(powerset_type, type)
nonempty: $i → $o   thf(nonempty_type, type)
transitiveset: $i → $o   thf(transitiveset_type, type)
stricttotalorderedByIn: $i → $o   thf(stricttotalorderedByIn_type, type)
stricttotalorderedByIn = (λa: $i: (∀xx: $i: ((in@xx@a) ⇒ ∀x: $i: ((in@x@a) ⇒ ∀y: $i: ((in@y@a) ⇒ ((in@xx@x and in@x@y) ⇒ (in@xx@y))))) and ∀x: $i: ((in@x@a) ⇒ ∀y: $i: ((in@y@a) ⇒ (x = y or in@x@y or in@y@x)
¬ in@x@x)))   thf(stricttotalorderedByIn, definition)
wellorderedByIn: $i → $o   thf(wellorderedByIn_type, type)
wellorderedByIn = (λa: $i: (stricttotalorderedByIn@a and ∀x: $i: ((in@x@(powerset@a)) ⇒ ((nonempty@x) ⇒ ∃xx: $i: (in@xx@x and ∀y: $i: ((in@y@x) ⇒ (xx = y or in@xx@y)))))))   thf(wellorderedByIn, definition)
ordinal: $i → $o   thf(ordinal_type, type)
ordinal = (λxx: $i: (transitiveset@xx and wellorderedByIn@xx))   thf(ordinal, definition)
ordinalTransSet: $o   thf(ordinalTransSet_type, type)
ordinalTransSet = (∀x: $i: ((ordinal@x) ⇒ ∀xx: $i, a: $i: ((in@a@x) ⇒ ((in@xx@a) ⇒ (in@xx@x)))))   thf(ordinalTrans
ordinalTransSet ⇒ ∀x: $i: ((ordinal@x) ⇒ ∀xx: $i, a: $i, b: $i: ((in@xx@a) ⇒ ((in@a@b) ⇒ ((in@b@x) ⇒ (in@xx@b)))))   thf(ordinalTransIn, conjecture)

SEU825∧3.p setextAx and powersetAx and notinemptyset are consistent
in: $i → $i → $o   thf(in_type, type)
setextAx: $o   thf(setextAx_type, type)
setextAx = (∀a: $i, b: $i: (∀xx: $i: ((in@xx@a) ⟺ (in@xx@b)) ⇒ a = b))   thf(setextAx, definition)
emptyset: $i   thf(emptyset_type, type)
powerset: $i → $i   thf(powerset_type, type)
powersetAx: $o   thf(powersetAx_type, type)
powersetAx = (∀a: $i, b: $i: ((in@b@(powerset@a)) ⟺ ∀xx: $i: ((in@xx@b) ⇒ (in@xx@a))))   thf(powersetAx, definitio
notinemptyset: $o   thf(notinemptyset_type, type)
notinemptyset = (∀xx: $i: ¬ in@xx@emptyset)   thf(notinemptyset, definition)
setextAx ⇒ (powersetAx ⇒ (notinemptyset ⇒ $false))   thf(setext, conjecture)

SEU826∧1.p About sets 1
seteq: ($i → $o) → ($i → $o) → $o   thf(seteq_type, type)
seteq = (λx: $i → $o, y: $i → $o: ∀u: $i: ((x@u) ⟺ (y@u)))   thf(seteq, definition)
u: ($i → $o) → ($i → $o) → $i → $o   thf(u_type, type)
u = (λx: $i → $o, y: $i → $o, u: $i: (x@u or y@u))   thf(u, definition)
n: ($i → $o) → ($i → $o) → $i → $o   thf(n_type, type)
n = (λx: $i → $o, y: $i → $o, u: $i: (x@u and y@u))   thf(n, definition)
a: $i → $o   thf(a_type, type)
b: $i → $o   thf(b_type, type)
c: $i → $o   thf(c_type, type)
seteq@(u@a@(n@b@c))@(n@(u@a@b)@(u@a@c))   thf(conj, conjecture)

SEU827^1.p About sets 2
$\mathrm{leibeq}\colon (\$i \to \$o) \to (\$i \to \$o) \to \$o$ thf(leibeq_type, type)
$\mathrm{leibeq} = (\lambda x\colon \$i \to \$o, y\colon \$i \to \$o\colon \forall p\colon (\$i \to \$o) \to \$o\colon ((p@x) \Rightarrow (p@y)))$ thf(leibeq, definition)
$u\colon (\$i \to \$o) \to (\$i \to \$o) \to \$i \to \$o$ thf(u_type, type)
$u = (\lambda x\colon \$i \to \$o, y\colon \$i \to \$o, u\colon \$i\colon (x@u \text{ or } y@u))$ thf(u, definition)
$n\colon (\$i \to \$o) \to (\$i \to \$o) \to \$i \to \$o$ thf(n_type, type)
$n = (\lambda x\colon \$i \to \$o, y\colon \$i \to \$o, u\colon \$i\colon (x@u \text{ and } y@u))$ thf(n, definition)
$a\colon \$i \to \$o$ thf(a_type, type)
$b\colon \$i \to \$o$ thf(b_type, type)
$c\colon \$i \to \$o$ thf(c_type, type)
$\mathrm{leibeq}@(u@a@(n@b@c))@(n@(u@a@b)@(u@a@c))$ thf(conj, conjecture)

SEU828^1.p About powersets 1
$\mathrm{seteq}\colon ((\$i \to \$o) \to \$o) \to ((\$i \to \$o) \to \$o) \to \$o$ thf(seteq_type, type)
$\mathrm{seteq} = (\lambda x\colon (\$i \to \$o) \to \$o, y\colon (\$i \to \$o) \to \$o\colon \forall u\colon \$i \to \$o\colon ((x@u) \iff (y@u)))$ thf(seteq, definition)
$\mathrm{subseteq}\colon (\$i \to \$o) \to (\$i \to \$o) \to \$o$ thf(subseteq_type, type)
$\mathrm{subseteq} = (\lambda x\colon \$i \to \$o, y\colon \$i \to \$o\colon \forall u\colon \$i\colon ((x@u) \Rightarrow (y@u)))$ thf(subseteq, definition)
$\mathrm{powerset}\colon (\$i \to \$o) \to (\$i \to \$o) \to \$o$ thf(powerset_type, type)
$\mathrm{powerset} = (\lambda x\colon \$i \to \$o, y\colon \$i \to \$o\colon (\mathrm{subseteq}@y@x))$ thf(poserset, definition)
$\mathrm{emptyset}\colon \$i \to \$o$ thf(emptyset_type, type)
$\mathrm{emptyset} = (\lambda x\colon \$i\colon \$\mathrm{false})$ thf(emptyset, definition)
$\mathrm{seteq}@(\mathrm{powerset}@\mathrm{emptyset})@\lambda x\colon \$i \to \$o\colon x = \mathrm{emptyset}$ thf(conj, conjecture)

SEU829^1.p About powersets 2
$\mathrm{subseteq}\colon (\$i \to \$o) \to (\$i \to \$o) \to \$o$ thf(subseteq_type, type)
$\mathrm{subseteq} = (\lambda x\colon \$i \to \$o, y\colon \$i \to \$o\colon \forall u\colon \$i\colon ((x@u) \Rightarrow (y@u)))$ thf(subseteq, definition)
$\mathrm{powerset}\colon (\$i \to \$o) \to (\$i \to \$o) \to \$o$ thf(powerset_type, type)
$\mathrm{powerset} = (\lambda x\colon \$i \to \$o, y\colon \$i \to \$o\colon (\mathrm{subseteq}@y@x))$ thf(poserset, definition)
$\mathrm{emptyset}\colon \$i \to \$o$ thf(emptyset_type, type)
$\mathrm{emptyset} = (\lambda x\colon \$i\colon \$\mathrm{false})$ thf(emptyset, definition)
$(\mathrm{powerset}@\mathrm{emptyset}) = (\lambda x\colon \$i \to \$o\colon x = \mathrm{emptyset})$ thf(conj, conjecture)

SEU831^5.p TPS problem GAZING-THM32
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall s\colon a \to \$o\colon (\lambda \mathrm{xz}\colon a\colon (s@\mathrm{xz} \text{ or } s@\mathrm{xz})) = s$ thf(cGAZING_THM32_pme, conjecture)

SEU832^5.p TPS problem GAZING-THM31
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall s\colon a \to \$o\colon (\lambda \mathrm{xx}\colon a\colon (s@\mathrm{xx} \text{ and } s@\mathrm{xx})) = s$ thf(cGAZING_THM31_pme, conjecture)

SEU833^5.p TPS problem GAZING-THM24
Trybulec's 61st Boolean property of sets
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall s\colon a \to \$o\colon (\lambda \mathrm{xx}\colon a\colon (s@\mathrm{xx} \text{ and } \$\mathrm{false})) = (\lambda \mathrm{xx}\colon a\colon \$\mathrm{false})$ thf(cGAZING_THM24_pme, conjecture)

SEU834^5.p TPS problem GAZING-THM21
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, \mathrm{xx}\colon a\colon ((s@\mathrm{xx} \text{ and } t@\mathrm{xx}) \Rightarrow (t@\mathrm{xx}))$ thf(cGAZING_THM21_pme, conjecture)

SEU836^5.p TPS problem GAZING-THM19
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, \mathrm{xx}\colon a\colon ((s@\mathrm{xx}) \Rightarrow (t@\mathrm{xx} \text{ or } s@\mathrm{xx}))$ thf(cGAZING_THM19_pme, conjecture)

SEU839^5.p TPS problem GAZING-THM26
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o\colon (\lambda \mathrm{xz}\colon a\colon (s@\mathrm{xz} \text{ or } t@\mathrm{xz})) = (\lambda \mathrm{xz}\colon a\colon (t@\mathrm{xz} \text{ or } s@\mathrm{xz}))$ thf(cGAZING_THM26_pme, conjecture)

SEU841^5.p TPS problem GAZING-THM7
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, u\colon a \to \$o\colon ((s = t \text{ and } t = u) \Rightarrow s = u)$ thf(cGAZING_THM$_7$, conjecture)

SEU842^5.p TPS problem GAZING-THM25
$a\colon \$\mathrm{tType}$ thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o\colon (\lambda \mathrm{xx}\colon a\colon (s@\mathrm{xx} \text{ and } t@\mathrm{xx})) = (\lambda \mathrm{xx}\colon a\colon (t@\mathrm{xx} \text{ and } s@\mathrm{xx}))$ thf(cGAZING_THM25_pme, conjecture)

SEU843^5.p TPS problem GAZING-THM23

a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o\colon ((\lambda xz\colon a\colon (s@xz \text{ or } t@xz)) = s \Rightarrow \forall xx\colon a\colon ((t@xx) \Rightarrow (s@xx)))$ thf(cGAZING_THM23_pme, conjec

SEU844^5.p TPS problem GAZING-THM8
a: \$tType thf(a_type, type)
cS: $a \to \$o$ thf(cS, type)
cT: $a \to \$o$ thf(cT, type)
$\forall s_0\colon a \to \$o, t_0\colon a \to \$o\colon s_0 = t_0 \Rightarrow (\forall xx\colon a\colon ((cS@xx) \Rightarrow (cT@xx)) \text{ and } \forall xx\colon a\colon ((cT@xx) \Rightarrow (cS@xx)))$ thf(cGAZING_T

SEU845^5.p TPS problem GAZING-THM12
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o\colon (\forall xx\colon a\colon ((s@xx) \Rightarrow (t@xx)) \Rightarrow s = (\lambda xx\colon a\colon (t@xx \text{ and } \neg t@xx \text{ and } \neg s@xx)))$ thf(cGAZING_THM

SEU846^5.p TPS problem GAZING-THM11
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, u\colon a \to \$o\colon ((\forall xx\colon a\colon ((s@xx) \Rightarrow (u@xx)) \text{ and } \forall xx\colon a\colon ((t@xx) \Rightarrow (u@xx)) \text{ and } \forall xx\colon a\colon ((s@xx) \Rightarrow (t@xx))) \Rightarrow \forall xx\colon a\colon ((u@xx \text{ and } \neg t@xx) \Rightarrow (u@xx \text{ and } \neg s@xx)))$ thf(cGAZING_THM11_pme, conjecture)

SEU847^5.p TPS problem GAZING-THM41
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o\colon (\lambda xz\colon a\colon ((s@xz \text{ and } \neg \$false) \text{ or } (\$false \text{ and } \neg s@xz))) = s$ thf(cGAZING_THM41_pme, conjecture)

SEU848^5.p TPS problem GAZING-THM38
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o\colon ((\lambda xz\colon a\colon (s@xz \text{ or } t@xz)) = s \iff \forall xx\colon a\colon ((t@xx) \Rightarrow (s@xx)))$ thf(cGAZING_THM38_pme, conj

SEU849^5.p TPS problem GAZING-THM39
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o\colon ((\lambda xx\colon a\colon (s@xx \text{ and } t@xx)) = s \iff \forall xx\colon a\colon ((s@xx) \Rightarrow (t@xx)))$ thf(cGAZING_THM39_pme, co

SEU850^5.p TPS problem GAZING-THM9
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, u\colon a \to \$o\colon ((s = t \text{ and } t = u) \Rightarrow (\forall xx\colon a\colon ((s@xx) \Rightarrow (t@xx)) \text{ and } \forall xx\colon a\colon ((t@xx) \Rightarrow (u@xx)) \text{ and } \forall xx\colon a\colon ((u@xx) \Rightarrow (s@xx))))$ thf(cGAZING_THM9_pme, conjecture)

SEU851^5.p TPS problem GAZING-THM42
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o\colon (\lambda xz\colon a\colon ((s@xz \text{ and } \neg t@xz) \text{ or } (t@xz \text{ and } \neg s@xz))) = (\lambda xz\colon a\colon ((t@xz \text{ and } \neg s@xz) \text{ or } (s@xz \text{ and } \neg t@$

SEU852^5.p TPS problem GAZING-THM36
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, u\colon a \to \$o\colon ((\forall xx\colon a\colon ((s@xx) \Rightarrow (u@xx)) \text{ and } \forall xx\colon a\colon ((t@xx) \Rightarrow (u@xx))) \Rightarrow (\forall xx\colon a\colon ((s@xx) \Rightarrow (t@xx)) \iff \forall xx\colon a\colon ((s@xx \text{ and } u@xx \text{ and } \neg t@xx) \Rightarrow (t@xx))))$ thf(cGAZING_THM36_pme, con

SEU853^5.p TPS problem GAZING-THM35
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, u\colon a \to \$o\colon ((\forall xx\colon a\colon ((s@xx) \Rightarrow (u@xx)) \text{ and } \forall xx\colon a\colon ((t@xx) \Rightarrow (u@xx))) \Rightarrow (\forall xx\colon a\colon ((s@xx) \Rightarrow (t@xx)) \iff \forall xx\colon a\colon ((s@xx \text{ and } u@xx \text{ and } \neg t@xx) \Rightarrow (u@xx \text{ and } \neg s@xx))))$ thf(cGAZING_THM

SEU854^5.p TPS problem GAZING-THM34
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, u\colon a \to \$o\colon ((\forall xx\colon a\colon ((s@xx) \Rightarrow (u@xx)) \text{ and } \forall xx\colon a\colon ((t@xx) \Rightarrow (u@xx))) \Rightarrow (\forall xx\colon a\colon ((s@xx) \Rightarrow (t@xx)) \iff (\lambda xz\colon a\colon ((u@xz \text{ and } \neg s@xz) \text{ or } t@xz)) = u))$ thf(cGAZING_THM34_pme, conjecture)

SEU855^5.p TPS problem GAZING-THM33
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, u\colon a \to \$o\colon ((\forall xx\colon a\colon ((s@xx) \Rightarrow (u@xx)) \text{ and } \forall xx\colon a\colon ((t@xx) \Rightarrow (u@xx))) \Rightarrow (\forall xx\colon a\colon ((s@xx) \Rightarrow (t@xx)) \iff (\lambda xx\colon a\colon (s@xx \text{ and } u@xx \text{ and } \neg t@xx)) = (\lambda xx\colon a\colon \$false)))$ thf(cGAZING_THM33_pr

SEU856^5.p TPS problem GAZING-THM46
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o\colon (s = t \iff (\lambda xz\colon a\colon ((t@xz \text{ and } \neg s@xz) \text{ or } (s@xz \text{ and } \neg t@xz))) = (\lambda xx\colon a\colon \$false))$ thf(cGAZING

SEU857^5.p TPS problem GAZING-THM43
a: \$tType thf(a_type, type)
$\forall s\colon a \to \$o, t\colon a \to \$o, u\colon a \to \$o\colon (\lambda xz\colon a\colon ((((s@xz \text{ and } \neg t@xz) \text{ or } (t@xz \text{ and } \neg s@xz)) \text{ and } \neg u@xz) \text{ or } (u@xz \text{ and } \neg (s@xz \text{ an}$
$(\lambda xz\colon a\colon ((s@xz \text{ and } \neg (t@xz \text{ and } \neg u@xz) \text{ or } (u@xz \text{ and } \neg t@xz)) \text{ or } (((t@xz \text{ and } \neg u@xz) \text{ or } (u@xz \text{ and } \neg t@xz)) \text{ and } \neg s@xz$

SEU858^5.p TPS problem THM163

A direct consequence of the definition of FINITE1.
a: $tType   thf(a_type, type)
∀xw: (*a* → $o) → $o: ((xw@λxx: *a*: $false and ∀xr: *a* → $o, xx: *a*: ((xw@xr) ⇒ (xw@λxt: *a*: (xr@xt or xt = xx)))) ⇒ (xw@λxx: *a*: $false))   thf(cTHM163_pme, conjecture)

SEU859∧5.p TPS problem THM164
Direct consequence of the definition of FINITE1.
a: $tType   thf(a_type, type)
∀xr: *a* → $o, xx: *a*: (∀xw: (*a* → $o) → $o: ((xw@λ$xx_0$: *a*: $false and ∀$xr_0$: *a* → $o, xx_0: *a*: ((xw@xr_0) ⇒ (xw@λxt: *a*: (xr_0@xt or xt = xx_0)))) ⇒ (xw@xr)) ⇒ ∀xw: (*a* → $o) → $o: ((xw@λ$xx_0$: *a*: $false and ∀$xr_0$: *a* → $o, xx_0: *a*: ((xw@xr_0) ⇒ (xw@λxt: *a*: (xr_0@xt or xt = xx_0)))) ⇒ (xw@λxt: *a*: (xr@xt or xt = xx))))   thf(cTHM164_pme

SEU860∧5.p TPS problem from FINITE-SET-THMS
a: $tType   thf(a_type, type)
∀xp: *a* → $o, xq: *a* → $o: ((∀xw: (*a* → $o) → $o: ((xw@λxx: *a*: $false and ∀xr: *a* → $o, xx: *a*: ((xw@xr) ⇒ (xw@λxt: *a*: (xr@xt or xt = xx)))) ⇒ (xw@xp)) and ∃xt: *a* → $o: (λxz: *a*: (xq@xz or xt@xz)) = xp) ⇒ ∀xw: (*a* → $o) → $o: ((xw@λxx: *a*: $false and ∀xr: *a* → $o, xx: *a*: ((xw@xr) ⇒ (xw@λxt: *a*: (xr@xt or xt = xx)))) ⇒ (xw@xq)))   thf(cTHM160B_pme, conjecture)

SEU861∧5.p TPS problem THM531E
Subset of a finite set is finite.
a: $tType   thf(a_type, type)
cB: *a* → $o   thf(cB, type)
cC: *a* → $o   thf(cC, type)
(∀*p*: (*a* → $o) → $o: ((∀*e*: *a* → $o: (¬∃xt: *a*: (*e*@xt) ⇒ (*p*@*e*)) and ∀*y*: *a* → $o, xx: *a*, *z*: *a* → $o: ((*p*@*y* and ∀$xx_{27}$: *a*: ((*z*@xx2
(*y*@xx_{27} or xx_{27} = xx))) ⇒ (*p*@*z*))) ⇒ (*p*@cC)) and ∀xx: *a*: ((cB@xx) ⇒ (cC@xx))) ⇒ ∀*p*: (*a* → $o) → $o: ((∀*e*: *a* → $o: (¬∃xt: *a*: (*e*@xt) ⇒ (*p*@*e*)) and ∀*y*: *a* → $o, xx: *a*, *z*: *a* → $o: ((*p*@*y* and ∀$xx_{28}$: *a*: ((*z*@$xx_{28}$) ⇒ (*y*@$xx_{28}$ or xx_{28} = xx))) ⇒ (*p*@*z*))) ⇒ (*p*@cB))   thf(cTHM531E_pme, conjecture)

SEU862∧5.p TPS problem from FINITE-FINITE1-EQUIV
cA: $i → $o   thf(cA_type, type)
cFINITE: ($i → $o) → $o   thf(cFINITE_type, type)
cNAT: (($i → $o) → $o) → $o   thf(cNAT_type, type)
cSUCC: (($i → $o) → $o) → ($i → $o) → $o   thf(cSUCC_type, type)
cZERO: ($i → $o) → $o   thf(cZERO_type, type)
cZERO = (λxp: $i → $o: ¬∃xx: $i: (xp@xx))   thf(cZERO_def, definition)
cSUCC = (λxn: ($i → $o) → $o, xp: $i → $o: ∃xx: $i: (xp@xx and xn@λxt: $i: (xt ≠ xx and xp@xt)))   thf(cSUCC_def, def
cNAT = (λxn: ($i → $o) → $o: ∀xp: (($i → $o) → $o) → $o: ((xp@cZERO and ∀xx: ($i → $o) → $o: ((xp@xx) ⇒ (xp@(cSUCC@xx)))) ⇒ (xp@xn)))   thf(cNAT_def, definition)
cFINITE = (λxp: $i → $o: ∃xn: ($i → $o) → $o: (cNAT@xn and xn@xp))   thf(cFINITE_def, definition)
(cFINITE@cA) ⇒ ∀xw: ($i → $o) → $o: ((xw@λxx: $i: $false and ∀xr: $i → $o, xx: $i: ((xw@xr) ⇒ (xw@λxt: $i: (xr@xt or
xx)))) ⇒ (xw@cA))   thf(cTHM538_pme, conjecture)

SEU863∧5.p TPS problem from FINITE-FINITE1-EQUIV
cA: $i → $o   thf(cA_type, type)
cFINITE: ($i → $o) → $o   thf(cFINITE_type, type)
cNAT: (($i → $o) → $o) → $o   thf(cNAT_type, type)
cSUCC: (($i → $o) → $o) → ($i → $o) → $o   thf(cSUCC_type, type)
cZERO: ($i → $o) → $o   thf(cZERO_type, type)
cZERO = (λxp: $i → $o: ¬∃xx: $i: (xp@xx))   thf(cZERO_def, definition)
cSUCC = (λxn: ($i → $o) → $o, xp: $i → $o: ∃xx: $i: (xp@xx and xn@λxt: $i: (xt ≠ xx and xp@xt)))   thf(cSUCC_def, def
cNAT = (λxn: ($i → $o) → $o: ∀xp: (($i → $o) → $o) → $o: ((xp@cZERO and ∀xx: ($i → $o) → $o: ((xp@xx) ⇒ (xp@(cSUCC@xx)))) ⇒ (xp@xn)))   thf(cNAT_def, definition)
cFINITE = (λxp: $i → $o: ∃xn: ($i → $o) → $o: (cNAT@xn and xn@xp))   thf(cFINITE_def, definition)
∀xw: ($i → $o) → $o: ((xw@λxx: $i: $false and ∀xr: $i → $o, xx: $i: ((xw@xr) ⇒ (xw@λxt: $i: (xr@xt or xt = xx)))) ⇒ (xw@cA)) ⇒ (cFINITE@cA)   thf(cTHM537_pme, conjecture)

SEU864∧5.p TPS problem from FINITE-SET-THMS
a: $tType   thf(a_type, type)
t: *a*   thf(*t*, type)
∀*x*: (*a* → $o) → $o: ((*x*@λxy: *a*: $false and ∀xx: *a* → $o: ((*x*@xx) ⇒ ∀$xt_0$: *a*: (*t* = xt_0 ⇒ (*x*@λxz: *a*: (xx@xz or xt_0 = xz))))) ⇒ (*x*@λxy: *a*: *t* = xy))   thf(cDOMLEMMA1_pme, conjecture)

SEU865^5.p TPS problem from FINITE-SET-THMS
$a: \$tType$ thf(a_type, type)
$cB: a \to \$o$ thf(cB, type)
$cC: a \to \$o$ thf(cC, type)
$(\forall xw: (a \to \$o) \to \$o: ((xw@\lambda xx: a: \$false \text{ and } \forall xr: a \to \$o, xx: a: ((xw@xr) \Rightarrow (xw@\lambda xt: a: (xr@xt \text{ or } xt = xx)))) \Rightarrow (xw@cC)) \text{ and } \forall xx: a: ((cB@xx) \Rightarrow (cC@xx))) \Rightarrow \forall xw: (a \to \$o) \to \$o: ((xw@\lambda xx: a: \$false \text{ and } \forall xr: a \to \$o, xx: a: ((xw@xr) \Rightarrow (xw@\lambda xt: a: (xr@xt \text{ or } xt = xx)))) \Rightarrow (xw@cB))$ thf(cTHM531_pme, conjecture)

SEU866^5.p TPS problem from PIGEON-HOLE
$a: \$tType$ thf(a_type, type)
$\forall xp: a \to \$o: (\forall xw: (a \to \$o) \to \$o: ((xw@\lambda xx: a: \$false \text{ and } \forall xr: a \to \$o, xx: a: ((xw@xr) \Rightarrow (xw@\lambda xt: a: (xr@xt \text{ or } xt = xx)))) \Rightarrow (xw@xp)) \Rightarrow \neg \exists xq: a \to \$o: (\forall xx: a: ((xq@xx) \Rightarrow (xp@xx)) \text{ and } \exists xx: a: (\neg xq@xx \text{ and } xp@xx) \text{ and } \exists xf: a \to a: \forall xy: a: ((xp@xy) \Rightarrow \exists xx: a: (xq@xx \text{ and } xy = (xf@xx)))))$ thf(cTHM161_pme, conjecture)

SEU867^5.p TPS problem from FINITE-SET-THMS
$a: \$tType$ thf(a_type, type)
$\forall xp: a \to \$o, xq: a \to \$o: ((\forall xw: (a \to \$o) \to \$o: ((xw@\lambda xx: a: \$false \text{ and } \forall xr: a \to \$o, xx: a: ((xw@xr) \Rightarrow (xw@\lambda xt: a: (xr@xt \text{ or } xt = xx)))) \Rightarrow (xw@xp)) \text{ and } \forall xx: a: ((xq@xx) \Rightarrow (xp@xx))) \Rightarrow \forall xw: (a \to \$o) \to \$o: ((xw@\lambda xx: a: \$false \text{ and } \forall xr: a \to \$o, xx: a: ((xw@xr) \Rightarrow (xw@\lambda xt: a: (xr@xt \text{ or } xt = xx)))) \Rightarrow (xw@xq)))$ thf(cTHM160_pme, conjecture)

SEU868^5.p TPS problem from FINITE-SET-THMS
$a: \$tType$ thf(a_type, type)
$cC: a \to \$o$ thf(cC, type)
$\forall xw: (a \to \$o) \to \$o: ((xw@\lambda xx: a: \$false \text{ and } \forall xr: a \to \$o, xx: a: ((xw@xr) \Rightarrow (xw@\lambda xt: a: (xr@xt \text{ or } xt = xx)))) \Rightarrow (xw@cC)) \Rightarrow \forall p: (a \to \$o) \to \$o: ((\forall e: a \to \$o: (\neg \exists xt: a: (e@xt) \Rightarrow (p@e)) \text{ and } \forall y: a \to \$o, xx: a, z: a \to \$o: ((p@y \text{ and } \forall xx_{16}: a: ((z@xx_{16}) \iff (y@xx_{16} \text{ or } xx_{16} = xx))) \Rightarrow (p@z))) \Rightarrow (p@cC))$ thf(cTHM55

SEU869^5.p TPS problem from FINITE-SET-THMS
$a: \$tType$ thf(a_type, type)
$cB: a \to \$o$ thf(cB, type)
$cC: a \to \$o$ thf(cC, type)
$(\forall x: (a \to \$o) \to \$o: ((x@\lambda xy: a: \$false \text{ and } \forall xx: a \to \$o: ((x@xx) \Rightarrow \forall xt: a: ((cC@xt) \Rightarrow (x@\lambda xz: a: (xx@xz \text{ or } xt = xz))))) \Rightarrow (x@cC)) \text{ and } \forall xx: a: ((cB@xx) \Rightarrow (cC@xx))) \Rightarrow \forall x: (a \to \$o) \to \$o: ((x@\lambda xy: a: \$false \text{ and } \forall xx: a \to \$o: ((x@xx) \Rightarrow \forall xt: a: ((cB@xt) \Rightarrow (x@\lambda xz: a: (xx@xz \text{ or } xt = xz))))) \Rightarrow (x@cB))$ thf(cTHM531C_pme, conjecture)

SEU871^5.p TPS problem from FINITE-SET-THMS
$a: \$tType$ thf(a_type, type)
$cB: a \to \$o$ thf(cB, type)
$cC: a \to \$o$ thf(cC, type)
$(\forall p: (a \to \$o) \to \$o: ((\forall e: a \to \$o: (\neg \exists xt: a: (e@xt) \Rightarrow (p@e)) \text{ and } \forall y: a \to \$o, xx: a, z: a \to \$o: ((p@y \text{ and } \forall xx_{30}: a: ((z@xx_3$
$(y@xx_{30} \text{ or } xx_{30} = xx))) \Rightarrow (p@z))) \Rightarrow (p@cC)) \text{ and } \forall xx: a: ((cB@xx) \Rightarrow (cC@xx))) \Rightarrow \forall p: (a \to \$o) \to \$o: ((\forall e: a \to \$o: (\neg \exists xt: a: (e@xt) \Rightarrow (p@e)) \text{ and } \forall y: a \to \$o, xx: a, z: a \to \$o: ((p@y \text{ and } \forall xx_{31}: a: ((z@xx_{31}) \iff (y@xx_{31} \text{ or } xx_{31} = xx))) \Rightarrow (p@z))) \Rightarrow (p@cB))$ thf(cTHM531D_pme, conjecture)

SEU872^5.p TPS problem from FINITE-SET-THMS
$a: \$tType$ thf(a_type, type)
$cF: a \to \$o$ thf(cF, type)
$cE: a \to \$o$ thf(cE, type)
$(\forall x: (a \to \$o) \to \$o: ((x@\lambda xy: a: \$false \text{ and } \forall xx: a \to \$o: ((x@xx) \Rightarrow \forall xt: a: ((cE@xt) \Rightarrow (x@\lambda xz: a: (xx@xz \text{ or } xt = xz))))) \Rightarrow (x@cE)) \text{ and } \forall x: (a \to \$o) \to \$o: ((x@\lambda xy: a: \$false \text{ and } \forall xx: a \to \$o: ((x@xx) \Rightarrow \forall xt: a: ((cF@xt) \Rightarrow (x@\lambda xz: a: (xx@xz \text{ or } xt = xz))))) \Rightarrow (x@cF))) \Rightarrow \forall x: (a \to \$o) \to \$o: ((x@\lambda xy: a: \$false \text{ and } \forall xx: a \to \$o: ((x@xx) \Rightarrow \forall xt: a: ((cE@xt \text{ or } cF@xt) \Rightarrow (x@\lambda xz: a: (xx@xz \text{ or } xt = xz))))) \Rightarrow (x@\lambda xz: a: (cE@xz \text{ or } cF@xz)))$ thf

SEU873^5.p TPS problem from FINITE-SET-THMS
$a: \$tType$ thf(a_type, type)
$cC: a \to \$o$ thf(cC, type)
$cB: a \to \$o$ thf(cB, type)
$(\forall p: (a \to \$o) \to \$o: ((\forall e: a \to \$o: (\neg \exists xt: a: (e@xt) \Rightarrow (p@e)) \text{ and } \forall y: a \to \$o, xx: a, z: a \to \$o: ((p@y \text{ and } \forall xx_{20}: a: ((z@xx_2$
$(y@xx_{20} \text{ or } xx_{20} = xx))) \Rightarrow (p@z))) \Rightarrow (p@cB)) \text{ and } \forall p: (a \to \$o) \to \$o: ((\forall e: a \to \$o: (\neg \exists xt: a: (e@xt) \Rightarrow (p@e)) \text{ and } \forall y: a \to \$o, xx: a, z: a \to \$o: ((p@y \text{ and } \forall xx_{21}: a: ((z@xx_{21}) \iff (y@xx_{21} \text{ or } xx_{21} = xx))) \Rightarrow (p@z))) \Rightarrow (p@cC))) \Rightarrow \forall p: (a \to \$o) \to \$o: ((\forall e: a \to \$o: (\neg \exists xt: a: (e@xt) \Rightarrow (p@e)) \text{ and } \forall y: a \to \$o, xx: a, z: a \to \$o: ((p@y \text{ and } \forall xx_{22}: a: ((z@xx_{22}) \iff (y@xx_{22} \text{ or } xx_{22} = xx))) \Rightarrow (p@z))) \Rightarrow (p@\lambda xz: a: (cB@xz \text{ or } cC@$

SEU874^5.p TPS problem from SET-TOPOLOGY-THMS
a: \$tType thf(a_type, type)
cX: $a \to$ \$o thf(cX, type)
cX $= (\lambda$xx: a: $\exists s$: $a \to$ \$o: $(\forall x_0$: $(a \to$ \$o$) \to$ \$o: $((x_0$@λxy: a: \$false and $\forall$xx$_0$: $a \to$ \$o: $((x_0$@xx$_0) \Rightarrow$ $\forall$xt: a: $((s$@xt$) \Rightarrow (x_0$@λxz: a: (xx$_0$@xz or xt = xz$)))))$ $\Rightarrow (x_0$@$s))$ and $\forall$xx$_0$: a: $((s$@xx$_0) \Rightarrow$ (cX@xx$_0$)) and s@xx)) th

SEU875^5.p TPS problem from SET-TOPOLOGY-THMS
a: \$tType thf(a_type, type)
t: a thf(t, type)
cA: $(a \to$ \$o$) \to$ \$o thf(cA, type)
$\forall$xx: $a \to$ \$o: ((cA@xx and xx@t) $\Rightarrow$ (cA@xx)) and $\forall$xx: $a \to$ \$o: ((cA@xx and xx@t) $\Rightarrow$ $\exists$xe: $a \to$ \$o: $(\forall x$: $(a \to$ \$o$) \to$ \$o: $((x$@λxy: a: \$false and $\forall$xx$_0$: $a \to$ \$o: $((x$@xx$_0) \Rightarrow$ $\forall$xt$_0$: a: ((xe@xt$_0$) $\Rightarrow$ $(x$@λxz: a: (xx$_0$@xz or xt$_0$ = xz$)))))$ $\Rightarrow$ $(x$@xe)) and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xx@xx$_0$)) and $\forall$xy: $a \to$ \$o: ((cA@xy and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xy@xx$_0$))) $\Rightarrow$ (cA@xy and xy@t)))) thf(cDOMLEMMA5_pme, conjecture)

SEU876^5.p TPS problem from SET-TOPOLOGY-THMS
a: \$tType thf(a_type, type)
cE: $a \to$ \$o thf(cE, type)
cA: $(a \to$ \$o$) \to$ \$o thf(cA, type)
$\forall x$: $(a \to$ \$o$) \to$ \$o: $((x$@λxy: a: \$false and $\forall$xx: $a \to$ \$o: $((x$@xx$) \Rightarrow$ $\forall$xt: a: ((cE@xt) $\Rightarrow$ $(x$@λxz: a: (xx@xz or xt = xz$)))))$ $\Rightarrow$ $(x$@cE)) $\Rightarrow$ ($\forall$xx: $a \to$ \$o: ((cA@xx and $\forall$xx$_0$: a: ((cE@xx$_0$) $\Rightarrow$ (xx@xx$_0$))) $\Rightarrow$ (cA@xx)) and $\forall$xx: $a \to$ \$o: ((cA@xx and $\forall$xx$_0$: a: ((cE@xx$_0$) $\Rightarrow$ (xx@xx$_0$))) $\Rightarrow$ $\exists$xe: $a \to$ \$o: $(\forall x$: $(a \to$ \$o$) \to$ \$o: $((x$@λxy: a: \$false and $\forall$xx$_0$: $a \to$ \$o: $((x$@xx$_0) \Rightarrow$ $\forall$xt: a: ((xe@xt) $\Rightarrow$ $(x$@λxz: a: (xx$_0$@xz or xt = xz$)))))$ $\Rightarrow$ $(x$@xe)) and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xx@xx$_0$)) and $\forall$xy: $a \to$ \$o: ((cA@xy and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xy@xx$_0$))) $\Rightarrow$ (cA@xy and $\forall$xx$_0$: a: ((cE@xx$_0$) $\Rightarrow$ (xy@xx$_0$))))))) thf(cDOMLEMMA4_pme, conjecture)

SEU877^5.p TPS problem from SET-TOPOLOGY-THMS
a: \$tType thf(a_type, type)
cA: $(a \to$ \$o$) \to$ \$o thf(cA, type)
cB: $(a \to$ \$o$) \to$ \$o thf(cB, type)
$\forall$xx: $a \to$ \$o: ((cA@xx) $\Rightarrow$ (cB@xx)) $\Rightarrow$ $(\lambda u$: $(a \to$ \$o$) \to$ \$o: ($\forall$xx: $a \to$ \$o: $((u$@xx$) \Rightarrow$ (cA@xx)) and $\forall$xx: $a \to$ \$o: $((u$@xx$) \Rightarrow$ $\exists$xe: $a \to$ \$o: $(\forall x$: $(a \to$ \$o$) \to$ \$o: $((x$@λxy: a: \$false and $\forall$xx$_0$: $a \to$ \$o: $((x$@xx$_0) \Rightarrow$ $\forall$xt: a: ((xe@xt) $\Rightarrow$ $(x$@λxz: a: (xx$_0$@xz or xt = xz$)))))$ $\Rightarrow$ $(x$@xe)) and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xx@xx$_0$)) and $\forall$xy: $a \to$ \$o: ((cA@xy and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xy@xx$_0$))) $\Rightarrow$ $(u$@xy$))))))$ $= (\lambda u$: $(a \to$ \$o$) \to$ \$o: $\exists v$: $(a \to$ \$o$) \to$ \$o: ($\forall$xx: $a \to$ \$o: $((v$@xx$) \Rightarrow$ (cB@xx)) and $\forall$xx: $a \to$ \$o: $((v$@xx$) \Rightarrow$ $\exists$xe: $a \to$ \$o: $(\forall x$: $(a \to$ \$o$) \to$ \$o: $((x$@λxy: a: \$false and $\forall$xx$_0$: $a \to$ \$o: $((x$@xx$_0) \Rightarrow$ $\forall$xt: a: ((xe@xt) $\Rightarrow$ $(x$@λxz: a: (xx$_0$@xz or xt = xz$)))))$ $\Rightarrow$ $(x$@xe)) and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xx@xx$_0$)) and $\forall$xy: $a \to$ \$o: ((cB@xy and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xy@xx$_0$))) $\Rightarrow$ $(v$@xy)))) and $u = (\lambda$xx: $a \to$ \$o: $(v$@xx and cA@xx)))) thf(cDOMTHM3_pme, conjecture)

SEU882^5.p TPS problem THM139
Every object is in the range of some function.
$\forall$xy: \$i: $\exists$xf: \$i $\to$ \$i, xx: \$i: (xf@xx) = xy thf(cTHM139_pme, conjecture)

SEU883^5.p TPS problem X5212
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
f: $b \to a$ thf(f, type)
g: $b \to$ \$o thf(g, type)
$(\lambda$xz: a: $\exists$xx: b: $(g$@xx and xz $= (f$@xx$)))$ $= (\lambda$xz: a: $\exists$xt: b: $(g$@xt and xz $= (f$@xt$)))$ thf(cX5212_pme, conjecture)

SEU884^5.p TPS problem THM30B
a: \$tType thf(a_type, type)
cS: $a \to$ \$o thf(cS, type)
cR: $a \to$ \$o thf(cR, type)
$\forall$xx: a: ($\exists$xt: a: (cR@xt and xx = xt) $\Rightarrow$ $\exists$xt: a: (cS@xt and xx = xt)) $\Rightarrow$ $\forall$xx: a: ((cR@xx) $\Rightarrow$ (cS@xx)) thf(cTHM30B

SEU885^5.p TPS problem THM30A
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
cF: $b \to a$ thf(cF, type)
cV: $b \to$ \$o thf(cV, type)
cU: $b \to$ \$o thf(cU, type)

$\forall \text{xx}\colon b\colon ((\text{cU@xx}) \Rightarrow (\text{cV@xx})) \Rightarrow \forall \text{xx}\colon a\colon (\exists \text{xt}\colon b\colon (\text{cU@xt and xx} = (\text{cF@xt})) \Rightarrow \exists \text{xt}\colon b\colon (\text{cV@xt and xx} = (\text{cF@xt})))$ thf(cTHM30A_pme, conjecture)

SEU886^5.p TPS problem X5203
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
$f\colon b \to a$ thf(f, type)
$y\colon b \to \$o$ thf(y, type)
$x\colon b \to \$o$ thf(x, type)
$\forall \text{xx}_0\colon a\colon (\exists \text{xt}\colon b\colon (x\text{@xt and } y\text{@xt and xx}_0 = (f\text{@xt})) \Rightarrow (\exists \text{xt}\colon b\colon (x\text{@xt and xx}_0 = (f\text{@xt})) \text{ and } \exists \text{xt}\colon b\colon (y\text{@xt and xx}_0 = (f\text{@xt}))))$ thf(cX5203_pme, conjecture)

SEU887^5.p TPS problem THM28
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
c: \$tType thf(c_type, type)
$\forall f\colon b \to a, g\colon c \to b, s\colon c \to \$o, \text{xx}\colon a\colon (\exists \text{xt}\colon c\colon (s\text{@xt and xx} = (f\text{@}(g\text{@xt}))) \Rightarrow \exists \text{xt}\colon b\colon (\exists \text{xt}_0\colon c\colon (s\text{@xt}_0 \text{ and xt} = (g\text{@xt}_0)) \text{ and xx} = (f\text{@xt})))$ thf(cTHM28_pme, conjecture)

SEU888^5.p TPS problem THM500C-WFF
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
$g\colon b \to a$ thf(g, type)
$z\colon a$ thf(z, type)
$y\colon b$ thf(y, type)
$x\colon b$ thf(x, type)
$(z = (g\text{@}x) \text{ or } z = (g\text{@}y)) \Rightarrow \exists \text{xt}\colon b\colon ((\text{xt} = x \text{ or xt} = y) \text{ and } z = (g\text{@xt}))$ thf(cTHM500C_WFF_pme, conjecture)

SEU889^5.p TPS problem THM29A
c: \$tType thf(c_type, type)
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
cG: $c \to b$ thf(cG, type)
cF: $b \to a$ thf(cF, type)
cS: $c \to \$o$ thf(cS, type)
$\forall \text{xx}\colon a\colon (\exists \text{xt}\colon b\colon (\exists \text{xt}_0\colon c\colon (\text{cS@xt}_0 \text{ and xt} = (\text{cG@xt}_0)) \text{ and xx} = (\text{cF@xt})) \Rightarrow \exists \text{xt}\colon c\colon (\text{cS@xt and xx} = (\text{cF@(cG@xt)})))$ th

SEU890^5.p TPS problem THM29
c: \$tType thf(c_type, type)
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
cG: $c \to b$ thf(cG, type)
cF: $b \to a$ thf(cF, type)
cS: $c \to \$o$ thf(cS, type)
$\forall \text{xx}\colon a\colon (\exists \text{xt}\colon b\colon (\exists \text{xt}_0\colon c\colon (\text{cS@xt}_0 \text{ and xt} = (\text{cG@xt}_0)) \text{ and xx} = (\text{cF@xt})) \iff \exists \text{xt}\colon c\colon (\text{cS@xt and xx} = (\text{cF@(cG@xt)})))$ thf(cTHM29_pme, conjecture)

SEU891^5.p TPS problem THM34B
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
cF: $b \to a$ thf(cF, type)
cS: $b \to \$o$ thf(cS, type)
cR: $b \to \$o$ thf(cR, type)
$\forall \text{xx}\colon a\colon ((\exists \text{xt}\colon b\colon (\text{cR@xt and xx} = (\text{cF@xt})) \text{ or } \exists \text{xt}\colon b\colon (\text{cS@xt and xx} = (\text{cF@xt}))) \Rightarrow \exists \text{xt}\colon b\colon ((\text{cR@xt or cS@xt}) \text{ and xx} = (\text{cF@xt})))$ thf(cTHM34B_pme, conjecture)

SEU892^5.p TPS problem X6104
a: \$tType thf(a_type, type)
$\exists \text{xi}\colon (a \to a) \to (a \to a) \to \$o\colon (\forall \text{xg}\colon a \to a\colon (\text{xi@xg@}\lambda \text{xx}\colon a\colon \text{xx and xi@xg@}\lambda \text{xx}\colon a\colon (\text{xg@(xg@xx)})) \text{ and } \forall \text{xf}\colon a \to a, \text{xy}\colon a\colon ((\text{xi@}\lambda \text{xx}\colon a\colon \text{xy@xf}) \Rightarrow (\text{xf@xy}) = \text{xy}))$ thf(cX_{6104}, conjecture)

SEU893^5.p TPS problem THM34A
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)

cF: $b \to a$ thf(cF, type)
cS: $b \to \$o$ thf(cS, type)
cR: $b \to \$o$ thf(cR, type)
$\forall xx\colon a\colon (\exists xt\colon b\colon ((cR@xt \text{ or } cS@xt) \text{ and } xx = (cF@xt)) \Rightarrow (\exists xt\colon b\colon (cR@xt \text{ and } xx = (cF@xt)) \text{ or } \exists xt\colon b\colon (cS@xt \text{ and } xx = (cF@xt))))$ thf(cTHM34A_pme, conjecture)

SEU894^5.p TPS problem THM15-0
$\forall f\colon \$i \to \$i\colon \exists g\colon \$i \to \$i\colon ((\forall p\colon (\$i \to \$i) \to \$o\colon ((p@f \text{ and } \forall h\colon \$i \to \$i\colon ((p@h) \Rightarrow (p@\lambda t\colon \$i\colon (f@(h@t))))) \Rightarrow (p@g)) \text{ and } \exists x\colon \$i\colon ((g@x) = x \text{ and } \forall y\colon \$i\colon ((g@y) = y \Rightarrow x = y))) \Rightarrow \exists y\colon \$i\colon (f@y) = y)$ thf(cTHM15_0_pme, conjecture

SEU895^5.p TPS problem X5202
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
f: $b \to a$ thf(f, type)
y: $b \to \$o$ thf(y, type)
x: $b \to \$o$ thf(x, type)
$(\lambda xz\colon a\colon \exists xt\colon b\colon ((x@xt \text{ or } y@xt) \text{ and } xz = (f@xt))) = (\lambda xz\colon a\colon (\exists xt\colon b\colon (x@xt \text{ and } xz = (f@xt)) \text{ or } \exists xt\colon b\colon (y@xt \text{ and } xz = (f@xt))))$ thf(cX5202_pme, conjecture)

SEU896^5.p TPS problem THM500
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
y: b thf(y, type)
g: $b \to a$ thf(g, type)
x: b thf(x, type)
$(\lambda xz\colon a\colon \exists xt\colon b\colon ((xt = x \text{ or } xt = y) \text{ and } xz = (g@xt))) = (\lambda xv\colon a\colon (xv = (g@x) \text{ or } xv = (g@y)))$ thf(cTHM500_pme, conjec

SEU897^5.p TPS problem THM30
a: \$tType thf(a_type, type)
cS: $a \to \$o$ thf(cS, type)
cR: $a \to \$o$ thf(cR, type)
$\forall xx\colon a\colon ((cR@xx) \Rightarrow (cS@xx)) \iff \forall f\colon a \to a, xx\colon a\colon (\exists xt\colon a\colon (cR@xt \text{ and } xx = (f@xt)) \Rightarrow \exists xt\colon a\colon (cS@xt \text{ and } xx = (f@xt)))$ thf(cTHM30_pme, conjecture)

SEU898^5.p TPS problem THM132
a: \$tType thf(a_type, type)
$\forall xh\colon a \to a, xs\colon a \to \$o, xf\colon a \to a\colon ((\forall xx\colon a\colon ((xs@xx) \Rightarrow (xs@(xf@xx))) \text{ and } \forall xx\colon a\colon ((xs@xx) \Rightarrow (xs@(xf@xx))) \text{ and } \forall xx\colon a\colon$ $(xs@(xh@xx))) \text{ and } \forall xx\colon a\colon ((xs@xx) \Rightarrow (xh@(xf@xx)) = (xf@(xh@xx)))) \Rightarrow \forall xx\colon a\colon ((xs@xx) \Rightarrow (xh@(xh@(xf@xx))) = (xf@(xh@(xh@xx)))))$ thf(cTHM132_pme, conjecture)

SEU899^5.p TPS problem THM34
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
cF: $b \to a$ thf(cF, type)
cS: $b \to \$o$ thf(cS, type)
cR: $b \to \$o$ thf(cR, type)
$\forall xx\colon a\colon (\exists xt\colon b\colon ((cR@xt \text{ or } cS@xt) \text{ and } xx = (cF@xt)) \iff (\exists xt\colon b\colon (cR@xt \text{ and } xx = (cF@xt)) \text{ or } \exists xt\colon b\colon (cS@xt \text{ and } xx = (cF@xt))))$ thf(cTHM34_pme, conjecture)

SEU901^5.p TPS problem THM131D
g: \$tType thf(g_type, type)
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
$\forall xh_1\colon g \to b, xh_2\colon b \to a, xs_1\colon g \to \$o, xf_1\colon g \to g, xs_2\colon b \to \$o, xf_2\colon b \to b, xs_3\colon a \to \$o, xf_3\colon a \to a\colon ((\forall xx\colon g\colon ((xs_1@xx) \Rightarrow$ $(xs_1@(xf_1@xx))) \text{ and } \forall xx\colon b\colon ((xs_2@xx) \Rightarrow (xs_2@(xf_2@xx))) \text{ and } \forall xx\colon g\colon ((xs_1@xx) \Rightarrow (xs_2@(xh_1@xx))) \text{ and } \forall xx\colon g\colon ((xs_1@x$ $(xh_1@(xf_1@xx)) = (xf_2@(xh_1@xx))) \text{ and } \forall xx\colon b\colon ((xs_2@xx) \Rightarrow (xs_2@(xf_2@xx))) \text{ and } \forall xx\colon a\colon ((xs_3@xx) \Rightarrow (xs_3@(xf_3@xx)))$ a $(xs_3@(xh_2@xx))) \text{ and } \forall xx\colon b\colon ((xs_2@xx) \Rightarrow (xh_2@(xf_2@xx)) = (xf_3@(xh_2@xx)))) \Rightarrow \forall xx\colon g\colon ((xs_1@xx) \Rightarrow (xh_2@(xh_1@(xf_1@xx))) = (xf_3@(xh_2@(xh_1@xx)))))$ thf(cTHM131D_pme, conjecture)

SEU902^5.p TPS problem THM143
A lemma for the Injective Cantor Theorem X5309.
d: $\$i \to \$o$ thf(d, type)
$\forall xh\colon (\$i \to \$o) \to \$i\colon ((\forall xp\colon \$i \to \$o, xq\colon \$i \to \$o\colon ((xh@xp) = (xh@xq) \Rightarrow xp = xq) \text{ and } d = (\lambda xz\colon \$i\colon \exists xt\colon \$i \to \$o\colon (\neg\, xt@(xh@xt) \text{ and } xz = (xh@xt)))) \Rightarrow \neg\, d@(xh@d))$ thf(cTHM143_pme, conjecture)

SEU903^5.p TPS problem THM131
g: \$tType thf(g_type, type)
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
$\forall \mathrm{xh}_1\colon g \to b, \mathrm{xh}_2\colon b \to a, \mathrm{xs}_1\colon g \to \$\mathrm{o}, \mathrm{xf}_1\colon g \to g, \mathrm{xs}_2\colon b \to \$\mathrm{o}, \mathrm{xf}_2\colon b \to b, \mathrm{xs}_3\colon a \to \$\mathrm{o}, \mathrm{xf}_3\colon a \to a\colon ((\forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx}) \Rightarrow$
$(\mathrm{xs}_1@(\mathrm{xf}_1@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx}) \Rightarrow (\mathrm{xs}_2@(\mathrm{xf}_2@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx}) \Rightarrow (\mathrm{xs}_2@(\mathrm{xh}_1@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{x}$
$(\mathrm{xh}_1@(\mathrm{xf}_1@\mathrm{xx})) = (\mathrm{xf}_2@(\mathrm{xh}_1@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx}) \Rightarrow (\mathrm{xs}_2@(\mathrm{xf}_2@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon a\colon ((\mathrm{xs}_3@\mathrm{xx}) \Rightarrow (\mathrm{xs}_3@(\mathrm{xf}_3@\mathrm{xx})))$ a
$(\mathrm{xs}_3@(\mathrm{xh}_2@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx}) \Rightarrow (\mathrm{xh}_2@(\mathrm{xf}_2@\mathrm{xx})) = (\mathrm{xf}_3@(\mathrm{xh}_2@\mathrm{xx})))) \Rightarrow (\forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx}) \Rightarrow$
$(\mathrm{xs}_1@(\mathrm{xf}_1@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon a\colon ((\mathrm{xs}_3@\mathrm{xx}) \Rightarrow (\mathrm{xs}_3@(\mathrm{xf}_3@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx}) \Rightarrow (\mathrm{xs}_3@(\mathrm{xh}_2@(\mathrm{xh}_1@\mathrm{xx})))) \text{ and } \forall \mathrm{xx}\colon g\colon$
$(\mathrm{xh}_2@(\mathrm{xh}_1@(\mathrm{xf}_1@\mathrm{xx}))) = (\mathrm{xf}_3@(\mathrm{xh}_2@(\mathrm{xh}_1@\mathrm{xx}))))))$ thf(cTHM131_pme, conjecture)

SEU904^5.p TPS problem THM126
The composition of homomorphisms of binary operators is a homomorphism. Suggested by [BL+86].
g: \$tType thf(g_type, type)
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
$\forall \mathrm{xh}_1\colon g \to b, \mathrm{xh}_2\colon b \to a, \mathrm{xs}_1\colon g \to \$\mathrm{o}, \mathrm{xf}_1\colon g \to g \to g, \mathrm{xs}_2\colon b \to \$\mathrm{o}, \mathrm{xf}_2\colon b \to b \to b, \mathrm{xs}_3\colon a \to \$\mathrm{o}, \mathrm{xf}_3\colon a \to a \to$
$a\colon ((\forall \mathrm{xx}\colon g, \mathrm{xy}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx} \text{ and } \mathrm{xs}_1@\mathrm{xy}) \Rightarrow (\mathrm{xs}_1@(\mathrm{xf}_1@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon b, \mathrm{xy}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx} \text{ and } \mathrm{xs}_2@\mathrm{xy}) \Rightarrow$
$(\mathrm{xs}_2@(\mathrm{xf}_2@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx}) \Rightarrow (\mathrm{xs}_2@(\mathrm{xh}_1@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon g, \mathrm{xy}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx} \text{ and } \mathrm{xs}_1@\mathrm{xy}) \Rightarrow$
$(\mathrm{xh}_1@(\mathrm{xf}_1@\mathrm{xx}@\mathrm{xy})) = (\mathrm{xf}_2@(\mathrm{xh}_1@\mathrm{xx})@(\mathrm{xh}_1@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon b, \mathrm{xy}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx} \text{ and } \mathrm{xs}_2@\mathrm{xy}) \Rightarrow (\mathrm{xs}_2@(\mathrm{xf}_2@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall$
$(\mathrm{xs}_3@(\mathrm{xf}_3@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx}) \Rightarrow (\mathrm{xs}_3@(\mathrm{xh}_2@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon b, \mathrm{xy}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx} \text{ and } \mathrm{xs}_2@\mathrm{xy}) \Rightarrow$
$(\mathrm{xh}_2@(\mathrm{xf}_2@\mathrm{xx}@\mathrm{xy})) = (\mathrm{xf}_3@(\mathrm{xh}_2@\mathrm{xx})@(\mathrm{xh}_2@\mathrm{xy})))) \Rightarrow (\forall \mathrm{xx}\colon g, \mathrm{xy}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx} \text{ and } \mathrm{xs}_1@\mathrm{xy}) \Rightarrow (\mathrm{xs}_1@(\mathrm{xf}_1@\mathrm{xx}@\mathrm{xy}))) \text{ and }$
$(\mathrm{xs}_3@(\mathrm{xf}_3@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx}) \Rightarrow (\mathrm{xs}_3@(\mathrm{xh}_2@(\mathrm{xh}_1@\mathrm{xx})))) \text{ and } \forall \mathrm{xx}\colon g, \mathrm{xy}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx} \text{ and } \mathrm{xs}_1@\mathrm{xy}) \Rightarrow$
$(\mathrm{xh}_2@(\mathrm{xh}_1@(\mathrm{xf}_1@\mathrm{xx}@\mathrm{xy}))) = (\mathrm{xf}_3@(\mathrm{xh}_2@(\mathrm{xh}_1@\mathrm{xx}))@(\mathrm{xh}_2@(\mathrm{xh}_1@\mathrm{xy}))))))$ thf(cTHM126_pme, conjecture)

SEU905^5.p TPS problem THM126A
g: \$tType thf(g_type, type)
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
$\forall \mathrm{xh}_1\colon g \to b, \mathrm{xh}_2\colon b \to a, \mathrm{xs}_1\colon g \to \$\mathrm{o}, \mathrm{xf}_1\colon g \to g \to g, \mathrm{xs}_2\colon b \to \$\mathrm{o}, \mathrm{xf}_2\colon b \to b \to b, \mathrm{xs}_3\colon a \to \$\mathrm{o}, \mathrm{xf}_3\colon a \to a \to$
$a\colon (\neg \forall \mathrm{xx}\colon g, \mathrm{xy}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx} \text{ and } \mathrm{xs}_1@\mathrm{xy}) \Rightarrow (\mathrm{xs}_1@(\mathrm{xf}_1@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon a, \mathrm{xy}\colon a\colon ((\mathrm{xs}_3@\mathrm{xx} \text{ and } \mathrm{xs}_3@\mathrm{xy}) \Rightarrow$
$(\mathrm{xs}_3@(\mathrm{xf}_3@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx}) \Rightarrow (\mathrm{xs}_3@(\mathrm{xh}_2@(\mathrm{xh}_1@\mathrm{xx})))) \text{ and } \forall \mathrm{xx}\colon g, \mathrm{xy}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx} \text{ and } \mathrm{xs}_1@\mathrm{xy}) \Rightarrow$
$(\mathrm{xh}_2@(\mathrm{xh}_1@(\mathrm{xf}_1@\mathrm{xx}@\mathrm{xy}))) = (\mathrm{xf}_3@(\mathrm{xh}_2@(\mathrm{xh}_1@\mathrm{xx}))@(\mathrm{xh}_2@(\mathrm{xh}_1@\mathrm{xy})))) \Rightarrow \neg \forall \mathrm{xx}\colon g, \mathrm{xy}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx} \text{ and } \mathrm{xs}_1@\mathrm{xy}) \Rightarrow$
$(\mathrm{xs}_1@(\mathrm{xf}_1@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon b, \mathrm{xy}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx} \text{ and } \mathrm{xs}_2@\mathrm{xy}) \Rightarrow (\mathrm{xs}_2@(\mathrm{xf}_2@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx}) \Rightarrow$
$(\mathrm{xs}_2@(\mathrm{xh}_1@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon g, \mathrm{xy}\colon g\colon ((\mathrm{xs}_1@\mathrm{xx} \text{ and } \mathrm{xs}_1@\mathrm{xy}) \Rightarrow (\mathrm{xh}_1@(\mathrm{xf}_1@\mathrm{xx}@\mathrm{xy})) = (\mathrm{xf}_2@(\mathrm{xh}_1@\mathrm{xx})@(\mathrm{xh}_1@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon$
$(\mathrm{xs}_2@(\mathrm{xf}_2@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon a, \mathrm{xy}\colon a\colon ((\mathrm{xs}_3@\mathrm{xx} \text{ and } \mathrm{xs}_3@\mathrm{xy}) \Rightarrow (\mathrm{xs}_3@(\mathrm{xf}_3@\mathrm{xx}@\mathrm{xy}))) \text{ and } \forall \mathrm{xx}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx}) \Rightarrow$
$(\mathrm{xs}_3@(\mathrm{xh}_2@\mathrm{xx}))) \text{ and } \forall \mathrm{xx}\colon b, \mathrm{xy}\colon b\colon ((\mathrm{xs}_2@\mathrm{xx} \text{ and } \mathrm{xs}_2@\mathrm{xy}) \Rightarrow (\mathrm{xh}_2@(\mathrm{xf}_2@\mathrm{xx}@\mathrm{xy})) = (\mathrm{xf}_3@(\mathrm{xh}_2@\mathrm{xx})@(\mathrm{xh}_2@\mathrm{xy}))))$ thf(c

SEU906^5.p TPS problem from FUNS-AND-SETS-THMS
$f\colon \$\mathrm{i} \to \$\mathrm{i}$ thf(f, type)
$\mathrm{cS}\colon \$\mathrm{i} \to \$\mathrm{o}$ thf(cS, type)
$\forall \mathrm{xx}\colon \$\mathrm{i}, \mathrm{xy}\colon \$\mathrm{i}\colon ((f@\mathrm{xx}) = (f@\mathrm{xy}) \Rightarrow \mathrm{xx} = \mathrm{xy}) \Rightarrow \exists \mathrm{xu}\colon \$\mathrm{i} \to \$\mathrm{o}\colon \forall \mathrm{xx}\colon \$\mathrm{i}\colon ((\mathrm{cS}@\mathrm{xx}) \iff (\mathrm{xu}@(f@\mathrm{xx})))$ thf(cSV5_pme, co

SEU907^5.p TPS problem from FUNS-AND-SETS-THMS
$\forall \mathrm{xf}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\exists \mathrm{xg}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\forall \mathrm{xp}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon ((\mathrm{xp}@\mathrm{xf} \text{ and } \forall \mathrm{xj}\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\mathrm{xp}@\mathrm{xj}) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xf}@(\mathrm{xj}@\mathrm{xx}))))) \Rightarrow$
$(\mathrm{xp}@\mathrm{xg})) \text{ and } \exists \mathrm{xy}\colon \$\mathrm{i}\colon (\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xg}@\mathrm{xx}) = \mathrm{xx}) = (\lambda \mathrm{xx}\colon \$\mathrm{i}, \mathrm{xy}\colon \$\mathrm{i}\colon \mathrm{xx} = \mathrm{xy}@\mathrm{xy})) \Rightarrow \exists \mathrm{xy}\colon \$\mathrm{i}\colon (\mathrm{xf}@\mathrm{xy}) =$
$\mathrm{xy})$ thf(cTHM15_pme, conjecture)

SEU908^5.p TPS problem from MISTAKEN-LEASTCLOSEDUNDER
a: \$tType thf(a_type, type)
$f\colon a \to a$ thf(f, type)
$\forall \mathrm{xx}\colon a\colon (\forall \mathrm{xp}\colon a \to \$\mathrm{o}\colon (\forall \mathrm{xx}_0\colon a\colon ((\mathrm{xp}@\mathrm{xx}_0) \Rightarrow (\mathrm{xp}@(f@\mathrm{xx}_0))) \Rightarrow (\mathrm{xp}@\mathrm{xx})) \Rightarrow \forall \mathrm{xp}\colon a \to \$\mathrm{o}\colon (\forall \mathrm{xx}_0\colon a\colon ((\mathrm{xp}@\mathrm{xx}_0) \Rightarrow$
$(\mathrm{xp}@(f@\mathrm{xx}_0))) \Rightarrow (\mathrm{xp}@(f@\mathrm{xx}))))$ thf(cTHM527_pme, conjecture)

SEU909^5.p TPS problem from SET-TOP-CATEGORY-THMS
a: \$tType thf(a_type, type)
$\mathrm{cA}\colon (a \to \$\mathrm{o}) \to \$\mathrm{o}$ thf(cA, type)
$\forall \mathrm{xx}\colon a \to \$\mathrm{o}\colon ((\mathrm{cA}@\mathrm{xx}) \Rightarrow (\mathrm{cA}@\mathrm{xx})) \text{ and } \forall \mathrm{xe}\colon a \to \$\mathrm{o}\colon ((\forall x\colon (a \to \$\mathrm{o}) \to \$\mathrm{o}\colon ((x@\lambda \mathrm{xy}\colon a\colon \$\mathrm{false} \text{ and } \forall \mathrm{xx}\colon a \to$
$\$\mathrm{o}\colon ((x@\mathrm{xx}) \Rightarrow \forall \mathrm{xt}\colon a\colon ((\mathrm{xe}@\mathrm{xt}) \Rightarrow (x@\lambda \mathrm{xz}\colon a\colon (\mathrm{xx}@\mathrm{xz} \text{ or } \mathrm{xt} = \mathrm{xz}))))) \Rightarrow (x@\mathrm{xe})) \text{ and } \forall \mathrm{xx}\colon a\colon ((\mathrm{xe}@\mathrm{xx}) \Rightarrow$
$\exists s\colon a \to \$\mathrm{o}\colon (\mathrm{cA}@s \text{ and } s@\mathrm{xx}))) \Rightarrow (\forall \mathrm{xx}\colon a \to \$\mathrm{o}\colon ((\mathrm{cA}@\mathrm{xx} \text{ and } \forall \mathrm{xx}_0\colon a\colon ((\mathrm{xe}@\mathrm{xx}_0) \Rightarrow (\mathrm{xx}@\mathrm{xx}_0))) \Rightarrow$
$(\mathrm{cA}@\mathrm{xx})) \text{ and } \forall \mathrm{xx}\colon a \to \$\mathrm{o}\colon ((\mathrm{cA}@\mathrm{xx} \text{ and } \forall \mathrm{xx}_0\colon a\colon ((\mathrm{xe}@\mathrm{xx}_0) \Rightarrow (\mathrm{xx}@\mathrm{xx}_0))) \Rightarrow \exists \mathrm{xe}_0\colon a \to \$\mathrm{o}\colon (\forall x\colon (a \to$
$\$\mathrm{o}) \to \$\mathrm{o}\colon ((x@\lambda \mathrm{xy}\colon a\colon \$\mathrm{false} \text{ and } \forall \mathrm{xx}_0\colon a \to \$\mathrm{o}\colon ((x@\mathrm{xx}_0) \Rightarrow \forall \mathrm{xt}\colon a\colon ((\mathrm{xe}_0@\mathrm{xt}) \Rightarrow (x@\lambda \mathrm{xz}\colon a\colon (\mathrm{xx}_0@\mathrm{xz} \text{ or } \mathrm{xt} =$

xz))))) $\Rightarrow$ (x@xe$_0$)) and $\forall$xx$_0$: a: ((xe$_0$@xx$_0$) $\Rightarrow$ (xx@xx$_0$)) and $\forall$xy: $a \to$ \$o: ((cA@xy and $\forall$xx$_0$: a: ((xe$_0$@xx$_0$) $\Rightarrow$ (xy@xx$_0$))) $\Rightarrow$ (cA@xy and $\forall$xx$_0$: a: ((xe@xx$_0$) $\Rightarrow$ (xy@xx$_0$)))))))) thf(cDOMTHM8_pme, conjecture)

SEU917^5.p TPS problem THM8
$\exists f$: \$i $\to$ \$i: $\forall$xx: \$i, xy: \$i: ((f@xx) = (f@xy) $\Rightarrow$ xx = xy) thf(cTHM8_pme, conjecture)

SEU918^5.p TPS problem THM197
If there are at least two individuals, then WHAT?
b: \$i thf(b, type)
a: \$i thf(a, type)
$a \neq b$ $\Rightarrow$ $\neg\forall$xf: \$i $\to$ \$i, xg: \$i $\to$ \$i: (λxx: \$i: (xf@(xg@xx))) = (λxx: \$i: (xg@(xf@xx))) thf(cTHM197_pme, conjecture)

SEU919^5.p TPS problem THM127
a: \$i thf(a, type)
f: \$i $\to$ \$i thf(f, type)
g: \$i $\to$ \$i thf(g, type)
cP: \$i $\to$ \$o thf(cP, type)
(λxx: \$i: (f@(g@xx))) = (λxx: \$i: (g@(f@xx))) $\Rightarrow$ ((cP@(f@(g@a))) $\Rightarrow$ (cP@(g@(f@a)))) thf(cTHM127_pme, conjectu

SEU920^5.p TPS problem FN-THM-4
a: \$tType thf(a_type, type)
b: \$tType thf(b_type, type)
c: \$tType thf(c_type, type)
$\forall f$: $a \to b$, g: $b \to c$: ($\forall y$: c: $\exists x$: a: (g@(f@x)) = y $\Rightarrow$ $\forall y$: c: $\exists x$: b: (g@x) = y) thf(cFN_THM_4_pme, conjecture)

SEU921^5.p TPS problem THM588LEM2
Another possible lemma for THM588, for manipulating composite functions.
f: \$i $\to$ \$i thf(f, type)
g: \$i $\to$ \$i thf(g, type)
h: \$i $\to$ \$i thf(h, type)
$\forall$xx: \$i, xy: \$i: ((g@xx) = xy $\Rightarrow$ (h@xy) = (f@xx)) $\Rightarrow$ (λxx: \$i: (h@(g@xx))) = f thf(cTHM588LEM2_pme, conjecture)

SEU922^5.p TPS problem THM128
a: \$i thf(a, type)
f: \$i $\to$ \$i thf(f, type)
g: \$i $\to$ \$i thf(g, type)
cP: \$i $\to$ \$o thf(cP, type)
(λxx: \$i: (f@(g@xx))) = (λxx: \$i: (g@(f@xx))) $\Rightarrow$ ((cP@(f@(f@(g@a)))) $\Rightarrow$ (cP@(g@(f@(f@a))))) thf(cTHM128_pme

SEU923^5.p TPS problem THM54
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
cF: $b \to a$ thf(cF, type)
$\forall$xx: b, xy: b: ((cF@xx) = (cF@xy) $\Rightarrow$ xx = xy) $\Rightarrow$ $\forall$xx: b, xy: b: ((cF@xx) = (cF@xy) $\Rightarrow$ xx = xy) thf(cTHM54_pme, co

SEU924^5.p TPS problem THM134
Every positive iterate of a constant function is a constant function.
$\forall$xz: \$i, xg: \$i $\to$ \$i: ($\forall$xp: (\$i $\to$ \$i) $\to$ \$o: ((xp@λxx: \$i: xz and $\forall$xj: \$i $\to$ \$i: ((xp@xj) $\Rightarrow$ (xp@λxx: \$i: xz))) $\Rightarrow$ (xp@xg)) $\Rightarrow$ $\forall$xx: \$i: (xg@xx) = xz) thf(cTHM134_pme, conjecture)

SEU925^5.p TPS problem THM7-TPS2
Unitset is injective.
$\forall$xx: \$i, xy: \$i: ((λxy$_0$: \$i: xx = xy$_0$) = (λxy$_2$: \$i: xy = xy$_2$) $\Rightarrow$ xx = xy) thf(cTHM7_TPS2_pme, conjecture)

SEU926^5.p TPS problem THM113
There is a set of functions on P closed under composition.
$\forall p$: \$i $\to$ \$o: $\exists m$: (\$i $\to$ \$i) $\to$ \$o: $\forall g$: \$i $\to$ \$i: ((m@g) $\Rightarrow$ (m@λz: \$i: (g@(g@z)) and $\forall y$: \$i: ((p@y) $\Rightarrow$ (p@(g@y))))) thf(cTHM$_{113}$, conjecture)

SEU927^5.p TPS problem THM92
Trivial theorem which gives nice simple example of an expansion proof.
a: \$tType thf(a_type, type)
$\forall$xf: $a \to a$, xp: ($a \to a$) $\to$ \$o: ((xp@λxu: a: xu and $\forall$xj: $a \to a$: ((xp@xj) $\Rightarrow$ (xp@λxx: a: (xf@(xj@xx))))) $\Rightarrow$ (xp@λxx: a: (xf@(xf@xx)))) thf(cTHM92_pme, conjecture)

SEU928^5.p TPS problem THM48A

$\forall f$: \$i $\rightarrow$ \$i: ($\forall$xx: \$i, xy: \$i: ((f@xx) = (f@xy) $\Rightarrow$ xx = xy) $\Rightarrow$ $\forall$xx: \$i, xy: \$i: ((f@(f@xx)) = (f@(f@xy)) $\Rightarrow$ xx = xy)) thf(cTHM48A_pme, conjecture)

SEU929^5.p TPS problem THM170
f: \$i $\rightarrow$ \$i thf(f, type)
g: \$i $\rightarrow$ \$i thf(g, type)
$\forall$xp: (\$i $\rightarrow$ \$i) $\rightarrow$ \$o: ((xp@f and $\forall$xj: \$i $\rightarrow$ \$i: ((xp@xj) $\Rightarrow$ (xp@λxx: \$i: (f@(xj@xx))))) $\Rightarrow$ (xp@g)) $\Rightarrow$ (λxx: \$i: (f@(g@xx))) = (λxx: \$i: (g@(f@xx))) thf(cTHM170_pme, conjecture)

SEU930^5.p TPS problem THM171A
If g commutes with f, any unique fixed point of gi is a fixed point of f.
f: \$i $\rightarrow$ \$i thf(f, type)
g: \$i $\rightarrow$ \$i thf(g, type)
(λxx: \$i: (f@(g@xx))) = (λxx: \$i: (g@(f@xx))) $\Rightarrow$ $\forall$xx: \$i: ((((g@xx) = xx and $\forall$xz: \$i: ((g@xz) = xz $\Rightarrow$ xz = xx)) $\Rightarrow$ (f@xx) = xx) thf(cTHM171A_pme, conjecture)

SEU931^5.p TPS problem THM171
If g commutes with f, and g has a unique fixed point, then f has a fixed point.
f: \$i $\rightarrow$ \$i thf(f, type)
g: \$i $\rightarrow$ \$i thf(g, type)
(λxx: \$i: (f@(g@xx))) = (λxx: \$i: (g@(f@xx))) $\Rightarrow$ ($\exists$xx: \$i: ((g@xx) = xx and $\forall$xz: \$i: ((g@xz) = xz $\Rightarrow$ xz = xx)) $\Rightarrow$ $\exists$xy: \$i: (f@xy) = xy) thf(cTHM171_pme, conjecture)

SEU932^5.p TPS problem THM141
If some function which commutes with f has a unique fixed point, then f has a fixed point.
$\forall$xf: \$i $\rightarrow$ \$i: ($\exists$xg: \$i $\rightarrow$ \$i: ((λxx: \$i: (xf@(xg@xx))) = (λxx: \$i: (xg@(xf@xx))) and $\exists$xx: \$i: ((xg@xx) = xx and $\forall$xz: \$i: ((xg@xz) = xz $\Rightarrow$ xz = xx))) $\Rightarrow$ $\exists$xy: \$i: (xf@xy) = xy) thf(cTHM141_pme, conjecture)

SEU933^5.p TPS problem THM196B
It is not true that if [k COMPOSE j] is an iterate of j, then k must be an iterate of j, provided we have distinct elements a and b.
b: \$i thf(b, type)
a: \$i thf(a, type)
$a \neq b$ $\Rightarrow$ $\neg\forall$xj: \$i $\rightarrow$ \$i, xk: \$i $\rightarrow$ \$i: ($\forall$xp: (\$i $\rightarrow$ \$i) $\rightarrow$ \$o: ((xp@xj and $\forall$xj$_2$: \$i $\rightarrow$ \$i: ((xp@xj$_2$) $\Rightarrow$ (xp@λxx: \$i: (xj@(xj$_2$@xx))))) $\Rightarrow$ (xp@λxx: \$i: (xk@(xj@xx)))) $\Rightarrow$ $\forall$xp: (\$i $\rightarrow$ \$i) $\rightarrow$ \$o: ((xp@xj and $\forall$xj$_3$: \$i $\rightarrow$ \$i: ((xp@xj$_3$) $\Rightarrow$ (xp@λxx: \$i: (xj@(xj$_3$@xx))))) $\Rightarrow$ (xp@xk))) thf(cTHM196B_pme, conjecture)

SEU934^5.p TPS problem THM14
$\forall$xx: \$i, xy: \$i: ((λy: \$i: y = xx) = (λy: \$i: y = xy) $\Rightarrow$ xx = xy) thf(cTHM14_pme, conjecture)

SEU935^5.p TPS problem FN-THM-2
a: \$tType thf(a_type, type)
b: \$tType thf(b_type, type)
c: \$tType thf(c_type, type)
$\forall f$: $a \rightarrow b$, g: $b \rightarrow c$: (($\forall y$: b: $\exists x$: a: (f@x) = y and $\forall y$: c: $\exists x$: b: (g@x) = y) $\Rightarrow$ $\forall y$: c: $\exists x$: a: (g@(f@x)) = y) thf(cFN_THM_2_pme, conjecture)

SEU936^5.p TPS problem FN-THM-3
a: \$tType thf(a_type, type)
b: \$tType thf(b_type, type)
c: \$tType thf(c_type, type)
$\forall f$: $a \rightarrow b$, g: $b \rightarrow c$: ($\forall$xx: a, xy: a: ((g@(f@xx)) = (g@(f@xy)) $\Rightarrow$ xx = xy) $\Rightarrow$ $\forall$xx: a, xy: a: ((f@xx) = (f@xy) $\Rightarrow$ xx = xy)) thf(cFN_THM_3_pme, conjecture)

SEU937^5.p TPS problem THM48
The composition of injective functions is injective.
b: \$tType thf(b_type, type)
a: \$tType thf(a_type, type)
c: \$tType thf(c_type, type)
$\forall f$: $b \rightarrow a$, g: $c \rightarrow b$: (($\forall$xx: b, xy: b: ((f@xx) = (f@xy) $\Rightarrow$ xx = xy) and $\forall$xx: c, xy: c: ((g@xx) = (g@xy) $\Rightarrow$ xx = xy)) $\Rightarrow$ $\forall$xx: c, xy: c: ((f@(g@xx)) = (f@(g@xy)) $\Rightarrow$ xx = xy)) thf(cTHM48_pme, conjecture)

SEU938^5.p TPS problem THM196
It is not true that if [k COMPOSE j] is an iterate of j, provided we assume extensionality and the existence of the described function h (which can be proved if we have distinct elements a and b and descriptions).

$a\colon \$\mathrm{i} \quad \mathrm{thf}(a, \mathrm{type})$
$b\colon \$\mathrm{i} \quad \mathrm{thf}(b, \mathrm{type})$
$h\colon \$\mathrm{i} \to \$\mathrm{i} \quad \mathrm{thf}(h, \mathrm{type})$
$((h@a) = a \text{ and } (h@b) \neq a \text{ and } \forall \mathrm{xf}\colon \$\mathrm{i} \to \$\mathrm{i}, \mathrm{xg}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\forall \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xf}@\mathrm{xx}) = (\mathrm{xg}@\mathrm{xx}) \Rightarrow \mathrm{xf} = \mathrm{xg})) \Rightarrow \neg \forall \mathrm{xj}\colon \$\mathrm{i} \to \$\mathrm{i}, \mathrm{xk}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\forall \mathrm{xp}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon ((\mathrm{xp}@\mathrm{xj} \text{ and } \forall \mathrm{xj}_6\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\mathrm{xp}@\mathrm{xj}_6) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xj}@(\mathrm{xj}_6@\mathrm{xx}))))) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xk}@(\mathrm{xj}@\mathrm{xx})))) \Rightarrow \forall \mathrm{xp}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon ((\mathrm{xp}@\mathrm{xj} \text{ and } \forall \mathrm{xj}_7\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\mathrm{xp}@\mathrm{xj}_7) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xj}@(\mathrm{xj}_7@\mathrm{xx}))) (\mathrm{xp}@\mathrm{xk}))) \quad \mathrm{thf}(\mathrm{cTHM196\_pme}, \mathrm{conjecture})$

SEU939^5.p TPS problem THM112B
$\forall p\colon \$\mathrm{i} \to \$\mathrm{o}\colon \exists \mathrm{xm}_9\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{i} \to \$\mathrm{o}, \mathrm{xm}_{10}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{i} \to \$\mathrm{o}\colon (\forall \mathrm{xw}_1\colon \$\mathrm{i}\colon (\mathrm{xm}_9@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon \mathrm{xx}@\mathrm{xw}_1 \text{ or } \mathrm{xm}_{10}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon \mathrm{xx}@ \$\mathrm{i}, h\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\forall \mathrm{xw}_1\colon \$\mathrm{i}\colon (\mathrm{xm}_9@g@\mathrm{xw}_1 \text{ or } \mathrm{xm}_{10}@g@\mathrm{xw}_1) \text{ and } \forall \mathrm{xw}_1\colon \$\mathrm{i}\colon (\mathrm{xm}_9@h@\mathrm{xw}_1 \text{ or } \mathrm{xm}_{10}@h@\mathrm{xw}_1)) \Rightarrow (\forall \mathrm{xw}_1\colon \$\mathrm{i}\colon (\mathrm{xm}_9@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (g@(h@\mathrm{xx}))@\mathrm{xw}_1 \text{ or } \mathrm{xm}_{10}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (g@(h@\mathrm{xx}))@\mathrm{xw}_1) \text{ and } \forall y\colon \$\mathrm{i}\colon ((p@y) \Rightarrow (p@(g@y)))))) \quad \mathrm{th}$

SEU940^5.p TPS problem THM112A
$\forall p\colon \$\mathrm{i} \to \$\mathrm{o}\colon \exists m\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon (m@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon \mathrm{xx} \text{ and } \forall g\colon \$\mathrm{i} \to \$\mathrm{i}, h\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((m@g \text{ and } m@h) \Rightarrow (m@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (g@(h@\mathrm{xx})) (p@(g@y)))))) \quad \mathrm{thf}(\mathrm{cTHM112A\_pme}, \mathrm{conjecture})$

SEU941^5.p TPS problem FN-THM-1
$a\colon \$\mathrm{tType} \quad \mathrm{thf}(\mathrm{a\_type}, \mathrm{type})$
$b\colon \$\mathrm{tType} \quad \mathrm{thf}(\mathrm{b\_type}, \mathrm{type})$
$c\colon \$\mathrm{tType} \quad \mathrm{thf}(\mathrm{c\_type}, \mathrm{type})$
$\forall f\colon a \to b, g\colon b \to c\colon ((\forall \mathrm{xx}\colon a, \mathrm{xy}\colon a\colon ((f@\mathrm{xx}) = (f@\mathrm{xy}) \Rightarrow \mathrm{xx} = \mathrm{xy}) \text{ and } \forall \mathrm{xx}\colon b, \mathrm{xy}\colon b\colon ((g@\mathrm{xx}) = (g@\mathrm{xy}) \Rightarrow \mathrm{xx} = \mathrm{xy})) \Rightarrow \forall \mathrm{xx}\colon a, \mathrm{xy}\colon a\colon ((g@(f@\mathrm{xx})) = (g@(f@\mathrm{xy})) \Rightarrow \mathrm{xx} = \mathrm{xy})) \quad \mathrm{thf}(\mathrm{cFN\_THM\_1\_pme}, \mathrm{conjecture})$

SEU942^5.p TPS problem THM15B
$\forall \mathrm{xf}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\exists \mathrm{xg}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\forall \mathrm{xp}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon ((\mathrm{xp}@\mathrm{xf} \text{ and } \forall \mathrm{xj}\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\mathrm{xp}@\mathrm{xj}) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xf}@(\mathrm{xj}@\mathrm{xx}))))) \Rightarrow (\mathrm{xp}@\mathrm{xg})) \text{ and } \exists \mathrm{xx}\colon \$\mathrm{i}\colon ((\mathrm{xg}@\mathrm{xx}) = \mathrm{xx} \text{ and } \forall \mathrm{xz}\colon \$\mathrm{i}\colon ((\mathrm{xg}@\mathrm{xz}) = \mathrm{xz} \Rightarrow \mathrm{xz} = \mathrm{xx}))) \Rightarrow \exists \mathrm{xy}\colon \$\mathrm{i}\colon (\mathrm{xf}@\mathrm{xy}) = \mathrm{xy}) \quad \mathrm{thf}(\mathrm{cTHM15B\_pme}, \mathrm{conjecture})$

SEU943^5.p TPS problem THM172
If g is an iterate of f, and g has a unique fixed point, then f has a fixed point.
$f\colon \$\mathrm{i} \to \$\mathrm{i} \quad \mathrm{thf}(f, \mathrm{type})$
$g\colon \$\mathrm{i} \to \$\mathrm{i} \quad \mathrm{thf}(g, \mathrm{type})$
$\forall \mathrm{xp}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon ((\mathrm{xp}@f \text{ and } \forall \mathrm{xj}\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\mathrm{xp}@\mathrm{xj}) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (f@(\mathrm{xj}@\mathrm{xx}))))) \Rightarrow (\mathrm{xp}@g)) \Rightarrow (\exists \mathrm{xx}\colon \$\mathrm{i}\colon ((g@\mathrm{xx}) = \mathrm{xx} \text{ and } \forall \mathrm{xz}\colon \$\mathrm{i}\colon ((g@\mathrm{xz}) = \mathrm{xz} \Rightarrow \mathrm{xz} = \mathrm{xx})) \Rightarrow \exists \mathrm{xy}\colon \$\mathrm{i}\colon (f@\mathrm{xy}) = \mathrm{xy}) \quad \mathrm{thf}(\mathrm{cTHM172\_pme}, \mathrm{conjecture})$

SEU944^5.p TPS problem THM15C
$\forall \mathrm{xf}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\exists \mathrm{xg}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\forall \mathrm{xp}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon ((\mathrm{xp}@\lambda \mathrm{xu}\colon \$\mathrm{i}\colon \mathrm{xu} \text{ and } \forall \mathrm{xj}\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\mathrm{xp}@\mathrm{xj}) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xf}@(\mathrm{xj}@\mathrm{xx}))) (\mathrm{xp}@\mathrm{xg})) \text{ and } \exists \mathrm{xx}\colon \$\mathrm{i}\colon ((\mathrm{xg}@\mathrm{xx}) = \mathrm{xx} \text{ and } \forall \mathrm{xz}\colon \$\mathrm{i}\colon ((\mathrm{xg}@\mathrm{xz}) = \mathrm{xz} \Rightarrow \mathrm{xz} = \mathrm{xx}))) \Rightarrow \exists \mathrm{xy}\colon \$\mathrm{i}\colon (\mathrm{xf}@\mathrm{xy}) = \mathrm{xy}) \quad \mathrm{thf}(\mathrm{cTHM15C\_pme}, \mathrm{conjecture})$

SEU945^5.p TPS problem THM3
$\exists \mathrm{xf}\colon \$\mathrm{i} \to \$\mathrm{i} \to \$\mathrm{o}\colon \forall \mathrm{xx}\colon \$\mathrm{i}, \mathrm{xy}\colon \$\mathrm{i}\colon ((\mathrm{xf}@\mathrm{xx}) = (\mathrm{xf}@\mathrm{xy}) \Rightarrow \mathrm{xx} = \mathrm{xy}) \quad \mathrm{thf}(\mathrm{cTHM3\_pme}, \mathrm{conjecture})$

SEU946^5.p TPS problem THM15A
If some iterate of a function f has a unique fixed point, then f has a fixed point.
$\forall \mathrm{xf}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\exists \mathrm{xg}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\forall \mathrm{xp}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon ((\mathrm{xp}@\mathrm{xf} \text{ and } \forall \mathrm{xj}\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\mathrm{xp}@\mathrm{xj}) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xf}@(\mathrm{xj}@\mathrm{xx}))))) \Rightarrow (\mathrm{xp}@\mathrm{xg})) \text{ and } \exists \mathrm{xx}\colon \$\mathrm{i}\colon ((\mathrm{xg}@\mathrm{xx}) = \mathrm{xx} \text{ and } \forall \mathrm{xz}\colon \$\mathrm{i}\colon ((\mathrm{xg}@\mathrm{xz}) = \mathrm{xz} \Rightarrow \mathrm{xx} = \mathrm{xz}))) \Rightarrow \exists \mathrm{xy}\colon \$\mathrm{i}\colon (\mathrm{xf}@\mathrm{xy}) = \mathrm{xy}) \quad \mathrm{thf}(\mathrm{cTHM15A\_pme}, \mathrm{conjecture})$

SEU947^5.p TPS problem THM15B-V2
$\mathrm{cFIXPOINT}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{i} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{cFIXPOINT\_type}, \mathrm{type})$
$\mathrm{cUNIQUE\_FIXPOINT}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{i} \to \$\mathrm{o} \quad \mathrm{thf}(\mathrm{cUNIQUE\_FIXPOINT\_type}, \mathrm{type})$
$\mathrm{cFIXPOINT} = (\lambda \mathrm{xg}\colon \$\mathrm{i} \to \$\mathrm{i}, \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xg}@\mathrm{xx}) = \mathrm{xx}) \quad \mathrm{thf}(\mathrm{cFIXPOINT\_def}, \mathrm{definition})$
$\mathrm{cUNIQUE\_FIXPOINT} = (\lambda \mathrm{xg}\colon \$\mathrm{i} \to \$\mathrm{i}, \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{cFIXPOINT}@\mathrm{xg}@\mathrm{xx} \text{ and } \forall \mathrm{xz}\colon \$\mathrm{i}\colon ((\mathrm{cFIXPOINT}@\mathrm{xg}@\mathrm{xz}) \Rightarrow \mathrm{xx} = \mathrm{xz}))) \quad \mathrm{thf}(\mathrm{cUNIQUE\_FIXPOINT\_def}, \mathrm{definition})$
$\forall \mathrm{xf}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\exists \mathrm{xg}\colon \$\mathrm{i} \to \$\mathrm{i}\colon (\forall \mathrm{xp}\colon (\$\mathrm{i} \to \$\mathrm{i}) \to \$\mathrm{o}\colon ((\mathrm{xp}@\mathrm{xf} \text{ and } \forall \mathrm{xj}\colon \$\mathrm{i} \to \$\mathrm{i}\colon ((\mathrm{xp}@\mathrm{xj}) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{xf}@(\mathrm{xj}@\mathrm{xx}))))) \Rightarrow (\mathrm{xp}@\mathrm{xg})) \text{ and } \exists \mathrm{xx}\colon \$\mathrm{i}\colon (\mathrm{cUNIQUE\_FIXPOINT}@\mathrm{xg}@\mathrm{xx})) \Rightarrow \exists \mathrm{xy}\colon \$\mathrm{i}\colon (\mathrm{cFIXPOINT}@\mathrm{xf}@\mathrm{xy})) \quad \mathrm{thf}(\mathrm{cTHM15B\_V2\_pme}, \mathrm{con}$

SEU948^5.p TPS problem THM135
The composition of iterates of a function is also an iterate of that function.
$a\colon \$\mathrm{tType} \quad \mathrm{thf}(\mathrm{a\_type}, \mathrm{type})$
$\forall \mathrm{xf}\colon a \to a, \mathrm{xg}_1\colon a \to a, \mathrm{xg}_2\colon a \to a\colon ((\forall \mathrm{xp}\colon (a \to a) \to \$\mathrm{o}\colon ((\mathrm{xp}@\lambda \mathrm{xu}\colon a\colon \mathrm{xu} \text{ and } \forall \mathrm{xj}\colon a \to a\colon ((\mathrm{xp}@\mathrm{xj}) \Rightarrow (\mathrm{xp}@\lambda \mathrm{xx}\colon a\colon (\mathrm{xf}@(\mathrm{xj}@\mathrm{xx}))))) \Rightarrow (\mathrm{xp}@\mathrm{xg}_1)) \text{ and } \forall \mathrm{xp}\colon (a \to a) \to \$\mathrm{o}\colon ((\mathrm{xp}@\lambda \mathrm{xu}\colon a\colon \mathrm{xu} \text{ and } \forall \mathrm{xj}\colon a \to a\colon ((\mathrm{xp}@\mathrm{xj}) \Rightarrow$

(xp@λxx: *a*: (xf@(xj@xx))))) ⇒ (xp@xg_2))) ⇒ ∀xp: (*a* → *a*) → $o: ((xp@λxu: *a*: xu and ∀xj: *a* → *a*: ((xp@xj) ⇒ (xp@λxx: *a*: (xf@(xj@xx))))) ⇒ (xp@λxx: *a*: (xg_1@(xg_2@xx))))) thf(cTHM135_pme, conjecture)

SEU949^5.p TPS problem THM589
f: $i → $i → $o thf(*f*, type)
g: $i → $i → $o thf(*g*, type)
∀xx: $i, xy: $i: ((*g*@xx) = (*g*@xy) ⇒ (*f*@xx) = (*f*@xy)) ⇒ ∃xh: ($i → $o) → $i → $o: (λxx: $i: (xh@(*g*@xx))) = *f* thf(cTHM589_pme, conjecture)

SEU950^5.p TPS problem THM573
Challenge from Dana Scott stemming from injective Cantor Theorem.
h: ($i → $o) → $i thf(*h*, type)
∀xx: $i → $o, xy: $i → $o: ((*h*@xx) = (*h*@xy) ⇒ xx = xy) ⇒ ∃xg: $i → $i → $o: ∀*y*: $i → $o: ∃*x*: $i: (xg@*x*) = *y* thf(cTHM573_pme, conjecture)

SEU951^5.p TPS problem THM574
Challenge from Dana Scott stemming from injective Cantor Theorem.
a: $tType thf(a_type, type)
b: $tType thf(b_type, type)
h: (*a* → $o) → *b* thf(*h*, type)
∀xx: *a* → $o, xy: *a* → $o: ((*h*@xx) = (*h*@xy) ⇒ xx = xy) ⇒ ∃xg: *b* → *a* → $o: ∀*y*: *a* → $o: ∃*x*: *b*: (xg@*x*) = *y* thf(cTHM574_pme, conjecture)

SEU953^5.p TPS problem from FUNCTION-THMS
g: $i → $i thf(*g*, type)
∀xx: $i, xy: $i: ((*g*@xx) = (*g*@xy) ⇒ xx = xy) ⇒ ∃xh: $i → $i: (λxx: $i: (xh@(*g*@xx))) = (λxx: $i: xx) thf(cTHM591_pm

SEU954^5.p TPS problem from FUNCTION-THMS
f: $i → $i thf(*f*, type)
g: $i → $i thf(*g*, type)
∀xx: $i, xy: $i: ((*g*@xx) = (*g*@xy) ⇒ (*f*@xx) = (*f*@xy)) ⇒ ∃xh: $i → $i: (λxx: $i: (xh@(*g*@xx))) = *f* thf(cTHM588_pme, conjecture)

SEU955^5.p TPS problem from FUNCTION-THMS
a: $tType thf(a_type, type)
b: $tType thf(b_type, type)
∃xc: (*a* → $o) → *a*: ∀*x*: *a* → $o: (∃xt: *a*: (*x*@xt) ⇒ (*x*@(xc@*x*))) ⇒ (∃xg: *a* → *b*: ∀*y*: *b*: ∃*x*: *a*: (xg@*x*) = *y* or ∃xf: *b* → *a*: ∀*y*: *a*: ∃*x*: *b*: (xf@*x*) = *y*) thf(cTHM609_pme, conjecture)

SEU956^5.p TPS problem from FUNCTION-THMS
∀*r*: $i → $o, *s*: $i → $o: (*r* = *s* ⇒ ∀*x*: $i: ((*s*@*x*) ⇒ (*r*@*x*))) ⇒ ∀xx: $i, xy: $i: ((λ*y*: $i: xx = *y*) = (λ*y*: $i: xy = *y*) ⇒ xx = xy) thf(cTHM13_pme, conjecture)

SEU957^5.p TPS problem from FUNCTION-THMS
a: $tType thf(a_type, type)
b: $tType thf(b_type, type)
∃xc: (*a* → $o) → *a*: ∀*x*: *a* → $o: (∃xt: *a*: (*x*@xt) ⇒ (*x*@(xc@*x*))) ⇒ (∃xg: *a* → *b*: ∀*y*: *b*: ∃*x*: *a*: (xg@*x*) = *y* ⇒ ∃xf: *b* → *a*: ∀xx: *b*, xy: *b*: ((xf@xx) = (xf@xy) ⇒ xx = xy)) thf(cTHM607_pme, conjecture)

SEU958^5.p TPS problem from FUNCTION-THMS
a: $tType thf(a_type, type)
b: $tType thf(b_type, type)
∃xc: (*a* → $o) → *a*: ∀*x*: *a* → $o: (∃xt: *a*: (*x*@xt) ⇒ (*x*@(xc@*x*))) ⇒ (∃xg: *a* → *b*: ∀xx: *a*, xy: *a*: ((xg@xx) = (xg@xy) ⇒ xx = xy) or ∃xf: *b* → *a*: ∀xx: *b*, xy: *b*: ((xf@xx) = (xf@xy) ⇒ xx = xy)) thf(cTHM611_pme, conjecture)

SEU959^5.p TPS problem from FUNCTION-THMS
f: $i → $i thf(*f*, type)
g: $i → $i thf(*g*, type)
∀xr: $i → $i → $o: (∀xx: $i: ∃xy: $i: (xr@xx@xy) ⇒ ∃xh: $i → $i: ∀xx: $i: (xr@xx@(xh@xx))) ⇒ (∀xx: $i, xy: $i: ((*g*@xx) = (*g*@xy) ⇒ (*f*@xx) = (*f*@xy)) ⇒ ∃xh: $i → $i: (λxx: $i: (xh@(*g*@xx))) = *f*) thf(cTHM588AC2_pme, conjecture)

SEU960^5.p TPS problem from FUNCTION-THMS
a: $tType thf(a_type, type)
∀xf: *a* → *a*, xg_1: *a* → *a*, xg_2: *a* → *a*: ((∀xp: (*a* → *a*) → $o: ((xp@xf and ∀xj: *a* → *a*: ((xp@xj) ⇒ (xp@λxx: *a*: (xf@(xj@xx))))) (xp@xg_1)) and ∀xp: (*a* → *a*) → $o: ((xp@xf and ∀xj: *a* → *a*: ((xp@xj) ⇒ (xp@λxx: *a*: (xf@(xj@xx))))) ⇒

(xp@xg$_2$))) ⇒ ∀xp: (a → a) → \$o: ((xp@xf and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@λxx: a: (xg$_1$@(xg$_2$@xx))))) thf(cTHM135D_pme, conjecture)

SEU961^5.p TPS problem from FUNCTION-THMS
(∀xa: \$i, xb: \$i: xa = xb and ∀xf: \$i → \$i, xg: \$i → \$i: (∀xx: \$i: (xf@xx) = (xg@xx) ⇒ xf = xg)) ⇒ ∀xj: \$i → \$i, xk: \$i → \$i: (∀xp: (\$i → \$i) → \$o: ((xp@xj and ∀xj$_0$: \$i → \$i: ((xp@xj$_0$) ⇒ (xp@λxx: \$i: (xj@(xj$_0$@xx))))) ⇒ (xp@λxx: \$i: (xk@(xj@xx)))) ⇒ ∀xp: (\$i → \$i) → \$o: ((xp@xj and ∀xj$_1$: \$i → \$i: ((xp@xj$_1$) ⇒ (xp@λxx: \$i: (xj@(xj$_1$@xx))) (xp@xk))) thf(cTHM196C_pme, conjecture)

SEU963^5.p TPS problem from FUNCTION-THMS
a: \$tType thf(a_type, type)
∀xf: a → a, xg: a → a, xh: a → a: ((∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@λxx: a: (xg@(xh@xx)))) and ∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@xh))) ⇒ ∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@xg))) thf(cTHM93_pme, conjecture)

SEU965^5.p TPS problem from FUNCTION-THMS
a: \$tType thf(a_type, type)
∀xf: a → a, xg: a → a, xh: a → a: ((∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@λxx: a: (xg@(xh@xx)))) and ∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@xh))) ⇒ ∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@λxx: a: (xh@(xg@xx))))) thf(cTHM94_pme, conjecture)

SEU966^5.p TPS problem from FUNCTION-THMS
a: \$i thf(a, type)
b: \$i thf(b, type)
h: \$i → \$i thf(h, type)
((h@a) = a and (h@b) ≠ a and ∀xf: \$i → \$i, xg: \$i → \$i: (∀xx: \$i: (xf@xx) = (xg@xx) ⇒ xf = xg)) ⇒ ¬∀xj: \$i → \$i, xk: \$i → \$i: (∀xp: (\$i → \$i) → \$o: ((xp@λxu: \$i: xu and ∀xj$_4$: \$i → \$i: ((xp@xj$_4$) ⇒ (xp@λxx: \$i: (xj@(xj$_4$@xx))))) ⇒ (xp@λxx: \$i: (xk@(xj@xx)))) ⇒ ∀xp: (\$i → \$i) → \$o: ((xp@λxu: \$i: xu and ∀xj$_5$: \$i → \$i: ((xp@xj$_5$) ⇒ (xp@λxx: \$i: (xj@(xj$_5$@xx))))) ⇒ (xp@xk))) thf(cTHM196A_pme, conjecture)

SEU967^5.p TPS problem from FUNCTION-THMS
cY$_0$: ((\$i → \$i) → \$o) → \$i thf(cY$_0$, type)
cG$_0$: ((\$i → \$i) → \$o) → \$i → \$i thf(cG$_0$, type)
cP$_0$: \$i → \$o thf(cP$_0$, type)
cH$_0$: ((\$i → \$i) → \$o) → \$i → \$i thf(cH$_0$, type)
¬∀xm$_5$: (\$i → \$i) → \$i → \$o: (∃xw$_2$: \$i: (xm$_5$@(cG$_0$@λxw$_{1144}$: \$i → \$i: ∃xw$_{20}$: \$i: (xm$_5$@xw$_{1144}$@xw$_{20}$))@xw$_2$) and ∃xw$_2$: \$i: \$i: ∃xw$_{20}$: \$i: (xm$_5$@xw$_{1144}$@xw$_{20}$))@xw$_2$) and (¬∃xw$_2$: \$i: (xm$_5$@λxx: \$i: (cG$_0$@λxw$_{1144}$: \$i → \$i: ∃xw$_{20}$: \$i: (xm$_5$@xw$_{1144}$@ \$i: ∃xw$_{20}$: \$i: (xm$_5$@xw$_{1144}$@xw$_{20}$)@xx))@xw$_2$) or (cP$_0$@(cY$_0$@λxw$_{1144}$: \$i → \$i: ∃xw$_2$: \$i: (xm$_5$@xw$_{1144}$@xw$_2$)) and ¬cP$_0$@(\$i: ∃xw$_2$: \$i: (xm$_5$@xw$_{1144}$@xw$_2$)@(cY$_0$@λxw$_{1144}$: \$i → \$i: ∃xw$_2$: \$i: (xm$_5$@xw$_{1144}$@xw$_2$)))))) thf(cBUGWFFA_pme, conje

SEU968^5.p TPS problem from FUNCTION-THMS
a: \$tType thf(a_type, type)
∀xg: a → a: (λxx: a: (xg@xx)) = xg ⇒ ∀xf: a → a, xg$_1$: a → a, xg$_2$: a → a: ((∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@xg$_1$)) and ∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@xg$_2$))) ⇒ ∀xp: (a → a) → \$o: ((xp@λxu: a: xu and ∀xj: a → a: ((xp@xj) ⇒ (xp@λxx: a: (xf@(xj@xx))))) ⇒ (xp@λxx: a: (xg$_1$@(xg$_2$@xx))))) thf(cTHM135C_pme, conjecture)

SEU969^5.p TPS problem from CHECKERBOARD-RELNS
c_1: \$i thf(c1_type, type)
s: \$i → \$i thf(s_type, type)
cCKB6_BLACK: \$i → \$i → \$o thf(cCKB6_BLACK_type, type)
cCKB6_NUM: \$i → \$o thf(cCKB6_NUM_type, type)
cCKB6_BLACK = (λxu: \$i, xv: \$i: ∀xw: \$i → \$i → \$o: ((xw@c_1@c_1 and ∀xj: \$i, xk: \$i: ((xw@xj@xk) ⇒ (xw@(s@(s@xj))@xk (xw@xu@xv))) thf(cCKB6_BLACK_def, definition)
cCKB6_NUM = (λxx: \$i: ∀xp: \$i → \$o: ((xp@c_1 and ∀xw: \$i: ((xp@xw) ⇒ (xp@(s@xw)))) ⇒ (xp@xx))) thf(cCKB6_NU
∀xx: \$i, xy: \$i: ((cCKB6_BLACK@xx@xy) ⇒ (cCKB6_NUM@xx and cCKB6_NUM@xy)) thf(cCKB6_L6000_pme, conjec

SEU970^5.p TPS problem from CHECKERBOARD-RELNS
s: \$i → \$i thf(s_type, type)
cCKB_E$_2$: \$i → \$i → \$o thf(cCKB_E2_type, type)
cCKB_E$_2$ = (λxx: \$i, xy: \$i: ∀xp: \$i → \$o: ((xp@xx and ∀xu: \$i: ((xp@xu) ⇒ (xp@(s@(s@xu))))) ⇒ (xp@xy))) thf(cCK
∀xx: \$i, xy: \$i, xz: \$i: ((cCKB_E$_2$@xx@xy and cCKB_E$_2$@xy@xz) ⇒ (cCKB_E$_2$@xx@xz)) thf(cCKB_L34000_pme, conject

SEU971^5.p TPS problem from CHECKERBOARD-RELNS
cCKB_INF: ($i → $i → $o) → $o thf(cCKB_INF_type, type)
cCKB_INJ: ($i → $i → $i → $i → $o) → $o thf(cCKB_INJ_type, type)
cCKB_XPL: ($i → $i → $i → $i → $o) → ($i → $i → $o) → $i → $i → $o thf(cCKB_XPL_type, type)
cCKB_INJ = (λxh: $i → $i → $i → $i → $o: ∀$xx_1$: $i, xy_1: $i, xx_2: $i, xy_2: $i, xu: $i, xv: $i: ((xh@$xx_1$@$xy_1$@xu@xv and xh@xx
(xx_1 = xx_2 and xy_1 = xy_2))) thf(cCKB_INJ_def, definition)
cCKB_XPL = (λxh: $i → $i → $i → $i → $o, xk: $i → $i → $o, xm: $i, xn: $i: (xk@xm@xn and ∀xx: $i, xy: $i: ((xk@xx@xy) ⇒
∃xu: $i, xv: $i: (xh@xx@xy@xu@xv and xk@xu@xv and ¬ xu = xm and xv = xn)))) thf(cCKB_XPL_def, definition)
cCKB_INF = (λxk: $i → $i → $o: ∃xh: $i → $i → $i → $i → $o, xm: $i, xn: $i: (cCKB_INJ@xh and cCKB_XPL@xh@xk@xm@
∀xx: $i, xy: $i, xr: $i → $i → $o: ((cCKB_INF@λxu: $i, xv: $i: (xr@xu@xv or (xu = xx and xv = xy))) ⇒
(cCKB_INF@xr)) thf(cCKB6_L80000_pme, conjecture)

SEU972^5.p TPS problem from COINDUCTIVE-PU-ALG-THMS
a: $tType thf(a_type, type)
cR: *a* → *a* thf(cR, type)
cP: *a* → *a* → *a* thf(cP, type)
cL: *a* → *a* thf(cL, type)
cZ: *a* thf(cZ, type)
((cL@cZ) = cZ and (cR@cZ) = cZ and ∀xx: *a*, xy: *a*: (cL@(cP@xx@xy)) = xx and ∀xx: *a*, xy: *a*: (cR@(cP@xx@xy)) =
xy and ∀xt: *a*: (xt ≠ cZ ⟺ xt = (cP@(cL@xt)@(cR@xt))) and ∀*x*: *a* → $o: (∀xt: *a*, xu: *a*: ((*x*@(cP@xt@xu)) ⇒
((xt = cZ ⟺ xu = cZ) and *x*@(cP@(cL@xt)@(cL@xu)) and *x*@(cP@(cR@xt)@(cR@xu)))) ⇒ ∀xt: *a*, xu: *a*: ((*x*@(cP@xt@
xt = xu))) ⇒ ∀xt: *a*: ∃*x*: *a* → $o: (*x*@(cP@cZ@xt) and ∀$xt_0$: *a*, xu: *a*: ((*x*@(cP@xt_0@xu)) ⇒ ((xu = cZ ⇒ xt_0 =
cZ) and *x*@(cP@(cL@xt_0)@(cL@xu)) and *x*@(cP@(cR@xt_0)@(cR@xu))))) thf(cPU_LEM2E_pme, conjecture)

SEU973^5.p TPS problem from COINDUCTIVE-PU-ALG-THMS
a: $tType thf(a_type, type)
cR: *a* → *a* thf(cR, type)
cP: *a* → *a* → *a* thf(cP, type)
cL: *a* → *a* thf(cL, type)
cZ: *a* thf(cZ, type)
((cL@cZ) = cZ and (cR@cZ) = cZ and ∀xx: *a*, xy: *a*: (cL@(cP@xx@xy)) = xx and ∀xx: *a*, xy: *a*: (cR@(cP@xx@xy)) =
xy and ∀xt: *a*: (xt ≠ cZ ⟺ xt = (cP@(cL@xt)@(cR@xt))) and ∀*x*: *a* → $o: (∀xt: *a*, xu: *a*: ((*x*@(cP@xt@xu)) ⇒
((xt = cZ ⟺ xu = cZ) and *x*@(cP@(cL@xt)@(cL@xu)) and *x*@(cP@(cR@xt)@(cR@xu)))) ⇒ ∀xt: *a*, xu: *a*: ((*x*@(cP@xt@
xt = xu))) ⇒ ∀xv: *a*: (∀*x*: *a* → $o: ((*x*@cZ and ∀xx: *a*: ((*x*@xx) ⇒ (*x*@(cP@xx@cZ)))) ⇒ (*x*@xv)) ⇒ ∃*x*: *a* →
$o: (*x*@(cP@xv@(cP@xv@cZ)) and ∀xt: *a*, xu: *a*: ((*x*@(cP@xt@xu)) ⇒ ((xu = cZ ⇒ xt = cZ) and *x*@(cP@(cL@xt)@(cL@x

SEU974^5.p TPS problem from COINDUCTIVE-PU-ALG-THMS
a: $tType thf(a_type, type)
cR: *a* → *a* thf(cR, type)
cP: *a* → *a* → *a* thf(cP, type)
cL: *a* → *a* thf(cL, type)
cZ: *a* thf(cZ, type)
((cL@cZ) = cZ and (cR@cZ) = cZ and ∀xx: *a*, xy: *a*: (cL@(cP@xx@xy)) = xx and ∀xx: *a*, xy: *a*: (cR@(cP@xx@xy)) =
xy and ∀xt: *a*: (xt ≠ cZ ⟺ xt = (cP@(cL@xt)@(cR@xt))) and ∀*x*: *a* → $o: (∀xt: *a*, xu: *a*: ((*x*@(cP@xt@xu)) ⇒
((xt = cZ ⟺ xu = cZ) and *x*@(cP@(cL@xt)@(cL@xu)) and *x*@(cP@(cR@xt)@(cR@xu)))) ⇒ ∀xt: *a*, xu: *a*: ((*x*@(cP@xt@
xt = xu))) ⇒ ∀xx: *a*, xy: *a*: (∃*x*: *a* → $o: (*x*@(cP@xx@xy) and ∀xt: *a*, xu: *a*: ((*x*@(cP@xt@xu)) ⇒ ((xu = cZ ⇒
xt = cZ) and *x*@(cP@(cL@xt)@(cL@xu)) and *x*@(cP@(cR@xt)@(cR@xu))))) ⇒ ∃*x*: *a* → $o: (*x*@(cP@(cR@xx)@(cR@xy)) a
((xu = cZ ⇒ xt = cZ) and *x*@(cP@(cL@xt)@(cL@xu)) and *x*@(cP@(cR@xt)@(cR@xu)))))) thf(cPU_LEM2C_pme, conj

SEU975^5.p TPS problem from COINDUCTIVE-PU-ALG-THMS
a: $tType thf(a_type, type)
cR: *a* → *a* thf(cR, type)
cP: *a* → *a* → *a* thf(cP, type)
cL: *a* → *a* thf(cL, type)
cZ: *a* thf(cZ, type)
((cL@cZ) = cZ and (cR@cZ) = cZ and ∀xx: *a*, xy: *a*: (cL@(cP@xx@xy)) = xx and ∀xx: *a*, xy: *a*: (cR@(cP@xx@xy)) =
xy and ∀xt: *a*: (xt ≠ cZ ⟺ xt = (cP@(cL@xt)@(cR@xt))) and ∀*x*: *a* → $o: (∀xt: *a*, xu: *a*: ((*x*@(cP@xt@xu)) ⇒
((xt = cZ ⟺ xu = cZ) and *x*@(cP@(cL@xt)@(cL@xu)) and *x*@(cP@(cR@xt)@(cR@xu)))) ⇒ ∀xt: *a*, xu: *a*: ((*x*@(cP@xt@
xt = xu))) ⇒ ∀xx: *a*, xy: *a*: (∃*x*: *a* → $o: (*x*@(cP@xx@xy) and ∀xt: *a*, xu: *a*: ((*x*@(cP@xt@xu)) ⇒ ((xu = cZ ⇒
xt = cZ) and *x*@(cP@(cL@xt)@(cL@xu)) and *x*@(cP@(cR@xt)@(cR@xu))))) ⇒ ∃*x*: *a* → $o: (*x*@(cP@(cL@xx)@(cL@xy)) a
((xu = cZ ⇒ xt = cZ) and *x*@(cP@(cL@xt)@(cL@xu)) and *x*@(cP@(cR@xt)@(cR@xu)))))) thf(cPU_LEM2B_pme, conj

SEU976^5.p TPS problem from COINDUCTIVE-PU-ALG-THMS
a: \$tType thf(a_type, type)
cR: $a \to a$ thf(cR, type)
cP: $a \to a \to a$ thf(cP, type)
cL: $a \to a$ thf(cL, type)
cZ: a thf(cZ, type)
((cL@cZ) = cZ and (cR@cZ) = cZ and ∀xx: a, xy: a: (cL@(cP@xx@xy)) = xx and ∀xx: a, xy: a: (cR@(cP@xx@xy)) =
xy and ∀xt: a: (xt ≠ cZ ⟺ xt = (cP@(cL@xt)@(cR@xt))) and ∀x: $a \to$ \$o: (∀xt: a, xu: a: ((x@(cP@xt@xu)) ⇒
((xt = cZ ⟺ xu = cZ) and x@(cP@(cL@xt)@(cL@xu)) and x@(cP@(cR@xt)@(cR@xu)))) ⇒ ∀xt: a, xu: a: ((x@(cP@xt@
xt = xu))) ⇒ ∀xx: a, xy: a: ((∀x: $a \to$ \$o: ((x@cZ and ∀xx_0: a: ((x@xx_0) ⇒ (x@(cP@xx_0@cZ)))) ⇒
(x@xx)) and ∀x: $a \to$ \$o: ((x@cZ and ∀xx_0: a: ((x@xx_0) ⇒ (x@(cP@xx_0@cZ)))) ⇒ (x@xy))) ⇒ (∃x: $a \to$
\$o: (x@(cP@xx@xy) and ∀xt: a, xu: a: ((x@(cP@xt@xu)) ⇒ ((xu = cZ ⇒ xt = cZ) and x@(cP@(cL@xt)@(cL@xu)) and x@
\$o: (x@(cP@xy@xx) and ∀xt: a, xu: a: ((x@(cP@xt@xu)) ⇒ ((xu = cZ ⇒ xt = cZ) and x@(cP@(cL@xt)@(cL@xu)) and x@

SEU977^5.p TPS problem from COINDUCTIVE-PU-ALG-THMS
a: \$tType thf(a_type, type)
cR: $a \to a$ thf(cR, type)
cP: $a \to a \to a$ thf(cP, type)
cL: $a \to a$ thf(cL, type)
cZ: a thf(cZ, type)
m: a thf(m, type)
n: a thf(n, type)
((cL@cZ) = cZ and (cR@cZ) = cZ and ∀xx: a, xy: a: (cL@(cP@xx@xy)) = xx and ∀xx: a, xy: a: (cR@(cP@xx@xy)) =
xy and ∀xt: a: (xt ≠ cZ ⟺ xt = (cP@(cL@xt)@(cR@xt))) and ∀x: $a \to$ \$o: (∀xt: a, xu: a: ((x@(cP@xt@xu)) ⇒
((xt = cZ ⟺ xu = cZ) and x@(cP@(cL@xt)@(cL@xu)) and x@(cP@(cR@xt)@(cR@xu)))) ⇒ ∀xt: a, xu: a: ((x@(cP@xt@
xt = xu))) ⇒ (∀x: $a \to$ \$o: ((x@cZ and ∀xx: a: ((x@xx) ⇒ (x@(cP@xx@cZ)))) ⇒ (x@n)) ⇒ (∀x: $a \to$
\$o: ((x@cZ and ∀xx: a: ((x@xx) ⇒ (x@(cP@xx@cZ)))) ⇒ (x@m)) ⇒ (∃x: $a \to$ \$o: (x@(cP@n@m) and ∀xt: a, xu: a: ((x@
((xu = cZ ⇒ xt = cZ) and x@(cP@(cL@xt)@(cL@xu)) and x@(cP@(cR@xt)@(cR@xu))))) ⇒ ($n \neq m$ ⇒
∃x: $a \to$ \$o: (x@(cP@(cP@n@cZ)@m) and ∀xt: a, xu: a: ((x@(cP@xt@xu)) ⇒ ((xu = cZ ⇒ xt = cZ) and x@(cP@(cL@xt)@

SEU978^5.p TPS problem from COINDUCTIVE-PU-ALG-THMS
a: \$tType thf(a_type, type)
cR: $a \to a$ thf(cR, type)
cP: $a \to a \to a$ thf(cP, type)
cL: $a \to a$ thf(cL, type)
cZ: a thf(cZ, type)
((cL@cZ) = cZ and (cR@cZ) = cZ and ∀xx: a, xy: a: (cL@(cP@xx@xy)) = xx and ∀xx: a, xy: a: (cR@(cP@xx@xy)) =
xy and ∀xt: a: (xt ≠ cZ ⟺ xt = (cP@(cL@xt)@(cR@xt))) and ∀x: $a \to$ \$o: (∀xt: a, xu: a: ((x@(cP@xt@xu)) ⇒
((xt = cZ ⟺ xu = cZ) and x@(cP@(cL@xt)@(cL@xu)) and x@(cP@(cR@xt)@(cR@xu)))) ⇒ ∀xt: a, xu: a: ((x@(cP@xt@
xt = xu))) ⇒ ∀xx: a, xy: a, xu: a, xv: a: ((∃x: $a \to$ \$o: (x@(cP@xx@xy) and ∀xt: a, xu_0: a: ((x@(cP@xt@xu_0)) ⇒
((xu_0 = cZ ⇒ xt = cZ) and x@(cP@(cL@xt)@(cL@xu_0)) and x@(cP@(cR@xt)@(cR@xu_0))))) and ∃x: $a \to$
\$o: (x@(cP@xu@xv) and ∀xt: a, xu_0: a: ((x@(cP@xt@xu_0)) ⇒ ((xu_0 = cZ ⇒ xt = cZ) and x@(cP@(cL@xt)@(cL@xu_0)) an
∃x: $a \to$ \$o: (x@(cP@(cP@xx@xu)@(cP@xy@xv)) and ∀xt: a, xu_0: a: ((x@(cP@xt@xu_0)) ⇒ ((xu_0 = cZ ⇒ xt =
cZ) and x@(cP@(cL@xt)@(cL@xu_0)) and x@(cP@(cR@xt)@(cR@xu_0)))))) thf(cPU_LEM2D_pme, conjecture)

SEU984^5.p TPS problem from FINITE-SETS-CHECKERBOARD
cCKB_FIN: (\$i → \$i → \$o) → \$o thf(cCKB_FIN_type, type)
cCKB_INF: (\$i → \$i → \$o) → \$o thf(cCKB_INF_type, type)
cCKB_INJ: (\$i → \$i → \$i → \$i → \$o) → \$o thf(cCKB_INJ_type, type)
cCKB_XPL: (\$i → \$i → \$i → \$i → \$o) → (\$i → \$i → \$o) → \$i → \$i → \$o thf(cCKB_XPL_type, type)
cCKB_INJ = (λxh: \$i → \$i → \$i → \$i → \$o: ∀xx_1: \$i, xy_1: \$i, xx_2: \$i, xy_2: \$i, xu: \$i, xv: \$i: ((xh@xx_1@xy_1@xu@xv and xh@xx
(xx_1 = xx_2 and xy_1 = xy_2))) thf(cCKB_INJ_def, definition)
cCKB_XPL = (λxh: \$i → \$i → \$i → \$i → \$o, xk: \$i → \$i → \$o, xm: \$i, xn: \$i: (xk@xm@xn and ∀xx: \$i, xy: \$i: ((xk@xx@xy) ⇒
∃xu: \$i, xv: \$i: (xh@xx@xy@xu@xv and xk@xu@xv and ¬ xu = xm and xv = xn)))) thf(cCKB_XPL_def, definition)
cCKB_INF = (λxk: \$i → \$i → \$o: ∃xh: \$i → \$i → \$i → \$i → \$o, xm: \$i, xn: \$i: (cCKB_INJ@xh and cCKB_XPL@xh@xk@xm@
cCKB_FIN = (λxk: \$i → \$i → \$o: ¬ cCKB_INF@xk) thf(cCKB_FIN_def, definition)
∀xp: \$i → \$o: (∀xw: (\$i → \$o) → \$o: ((xw@λxx: \$i: \$false and ∀xr: \$i → \$o, xx: \$i: ((xw@xr) ⇒ (xw@λxt: \$i: (xr@xt or xt =
xx)))) ⇒ (xw@xp)) ⇒ ∀xq: \$i → \$o: (∀xw: (\$i → \$o) → \$o: ((xw@λxx: \$i: \$false and ∀xr: \$i → \$o, xx: \$i: ((xw@xr) ⇒
(xw@λxt: \$i: (xr@xt or xt = xx)))) ⇒ (xw@xq)) ⇒ ∀xk: \$i → \$i → \$o: (∀xx: \$i, xy: \$i: ((xk@xx@xy) ⇒
(xp@xx and xq@xy)) ⇒ (cCKB_FIN@xk)))) thf(cCKB6_L70000_pme, conjecture)

SEU985^5.p TPS problem from FINITE-SETS-RELNS-THMS
∀xp: \$i → \$o: (∀xw: (\$i → \$o) → \$o: ((xw@λxx: \$i: \$false and ∀xr: \$i → \$o, xx: \$i: ((xw@xr) ⇒ (xw@λxt: \$i: (xr@xt or xt =
xx)))) ⇒ (xw@xp)) ⇒ ∀xq: \$i → \$o: (∀xw: (\$i → \$o) → \$o: ((xw@λxx: \$i: \$false and ∀xr: \$i → \$o, xx: \$i: ((xw@xr) ⇒
(xw@λxt: \$i: (xr@xt or xt = xx)))) ⇒ (xw@xq)) ⇒ ∀xw: (\$i → \$i → \$o) → \$o: ((xw@λxu: \$i, xv: \$i: \$false and ∀xx: \$i, xy:
\$i → \$o: ((xw@xs) ⇒ (xw@λxu: \$i, xv: \$i: (xs@xu@xv or (xu = xx and xv = xy))))) ⇒ (xw@λxx: \$i, xy: \$i: (xp@xx and xq

SEU986^5.p TPS problem from FINITE-SETS-RELNS-THMS
∀xp: \$i → \$o: (∀xw: (\$i → \$o) → \$o: ((xw@λxx: \$i: \$false and ∀xr: \$i → \$o, xx: \$i: ((xw@xr) ⇒ (xw@λxt: \$i: (xr@xt or xt =
xx)))) ⇒ (xw@xp)) ⇒ ∀xq: \$i → \$o: (∀xw: (\$i → \$o) → \$o: ((xw@λxx: \$i: \$false and ∀xr: \$i → \$o, xx: \$i: ((xw@xr) ⇒
(xw@λxt: \$i: (xr@xt or xt = xx)))) ⇒ (xw@xq)) ⇒ ∀xw: (\$i → \$i → \$o) → \$o: ((xw@λxu: \$i, xv: \$i: \$false and ∀xx: \$i: ((x
∀xy: \$i: ((xq@xy) ⇒ ∀xs: \$i → \$i → \$o: ((xw@xs) ⇒ (xw@λxu: \$i, xv: \$i: (xs@xu@xv or (xu = xx and xv =
xy))))))) ⇒ (xw@λxx: \$i, xy: \$i: (xp@xx and xq@xy)))))   thf(cTHM162_pme, conjecture)

SEU987^5.p TPS problem from FUNS-AND-RELNS-THMS
b: \$tType   thf(b_type, type)
a: \$tType   thf(a_type, type)
g: *b* → *b*   thf(*g*, type)
f: *a* → *a*   thf(*f*, type)
cS: *a* → *b* → \$o   thf(cS, type)
(∀xx: *a*, xy: *a*: ((*f*@xx) = (*f*@xy) ⇒ xx = xy) and ∀xx: *b*, xy: *b*: ((*g*@xx) = (*g*@xy) ⇒ xx = xy)) ⇒ ∃xu: *a* →
b → \$o: ∀xx: *a*, xy: *b*: ((cS@xx@xy) ⟺ (xu@(*f*@xx)@(*g*@xy)))   thf(cSV7_pme, conjecture)

SEU988^5.p TPS problem from FUNS-AND-RELNS-THMS
a: \$tType   thf(a_type, type)
b: \$tType   thf(b_type, type)
f: *a* → *b* → \$o   thf(*f*, type)
∀*y*: *b* → \$o: ∃*x*: *a*: (*f*@*x*) = *y* ⇒ ∃xg: (*a* → \$o) → *b* → \$o, xh: (*b* → \$o) → *a* → \$o: (∀xx: *a* →
\$o: (∃xx_0: *a*: (λxy: *a*: (*f*@xx_0) = (*f*@xy)) = xx ⇒ \$true) and ∀xx: *b* → \$o: (\$true ⇒ ∃xx_0: *a*: (λxy: *a*: (*f*@xx_0) =
(*f*@xy)) = (xh@xx)) and ∀xy: *b* → \$o: (xg@(xh@xy)) = xy and ∀xx: *a* → \$o: (∃xx_0: *a*: (λxy: *a*: (*f*@xx_0) =
(*f*@xy)) = xx ⇒ (xh@(xg@xx)) = xx))   thf(cTHM529_pme, conjecture)

SEU989^5.p TPS problem from GRAPHS-THMS
b: \$tType   thf(b_type, type)
a: \$tType   thf(a_type, type)
∃*s*: *b* → *a* → \$o: (∀xx: *b*: ∃xy: *a*: (*s*@xx@xy) and ∀xx: *b*, xy_1: *a*, xy_2: *a*: ((*s*@xx@xy_1 and *s*@xx@xy_2) ⇒ xy_1 =
xy_2) and ∀xx_1: *b*, xx_2: *b*, xy: *a*: ((*s*@xx_1@xy and *s*@xx_2@xy) ⇒ xx_1 = xx_2)) ⇒ ∃*r*: *a* → *b* → \$o: (∀xx: *a*: ∃xy: *b*: (*r*@xx@xy)
xy_1 = xy_2))   thf(cTHM554_pme, conjecture)

SEU990^5.p TPS problem from GRAPHS-THMS
a: \$tType   thf(a_type, type)
b: \$tType   thf(b_type, type)
∃xc: (*a* → \$o) → *a*: ∀*x*: *a* → \$o: (∃xt: *a*: (*x*@xt) ⇒ (*x*@(xc@*x*))) ⇒ (∃xr: *a* → *b* → \$o: (∀xx: *a*: ∃xy: *b*: (xr@xx@xy) and ∀xy
xy_1 = xy_2)) or ∃xs: *b* → *a* → \$o: (∀xx: *b*: ∃xy: *a*: (xs@xx@xy) and ∀xy: *a*: ∃xx: *b*: (xs@xx@xy) and ∀xx: *b*, xy_1: *a*, xy_2: *a*: ((xs@
xy_1 = xy_2)))   thf(cTHM610_pme, conjecture)

SEU991^5.p TPS problem from GRAPHS-THMS
a: \$tType   thf(a_type, type)
b: \$tType   thf(b_type, type)
∃*s*: *a* → *b* → \$o: (∀xx: *a*: ∃xy: *b*: (*s*@xx@xy) and ∀xx: *a*, xy_1: *b*, xy_2: *b*: ((*s*@xx@xy_1 and *s*@xx@xy_2) ⇒ xy_1 =
xy_2) and ∀xx_1: *a*, xx_2: *a*, xy: *b*: ((*s*@xx_1@xy and *s*@xx_2@xy) ⇒ xx_1 = xx_2)) or ∃*r*: *b* → *a* → \$o: (∀xx: *b*: ∃xy: *a*: (*r*@xx@xy) a
xy_1 = xy_2) and ∀xx_1: *b*, xx_2: *b*, xy: *a*: ((*r*@xx_1@xy and *r*@xx_2@xy) ⇒ xx_1 = xx_2))   thf(cTHM612_pme, conjecture)

SEU992^5.p TPS problem from GRAPHS-THMS
a: \$tType   thf(a_type, type)
b: \$tType   thf(b_type, type)
∃xc: (*a* → \$o) → *a*: ∀*x*: *a* → \$o: (∃xt: *a*: (*x*@xt) ⇒ (*x*@(xc@*x*))) ⇒ (∃*r*: *a* → *b* → \$o: (∀xx: *a*: ∃xy: *b*: (*r*@xx@xy) and ∀xy: *b*
xy_1 = xy_2)) ⇒ ∃*s*: *b* → *a* → \$o: (∀xx: *b*: ∃xy: *a*: (*s*@xx@xy) and ∀xx: *b*, xy_1: *a*, xy_2: *a*: ((*s*@xx@xy_1 and *s*@xx@xy_2) ⇒
xy_1 = xy_2) and ∀xx_1: *b*, xx_2: *b*, xy: *a*: ((*s*@xx_1@xy and *s*@xx_2@xy) ⇒ xx_1 = xx_2)))   thf(cTHM608_pme, conjecture)

SEU993^5.p TPS problem from GRAPHS-THMS
a: \$tType   thf(a_type, type)
b: \$tType   thf(b_type, type)
cR: *a* → *a* → \$o   thf(cR, type)
cS: *b* → *b* → \$o   thf(cS, type)

(∀xx: a: (cR@xx@xx) and ∀xu: a, xv: a, xw: a: ((cR@xu@xw and cR@xv@xw) ⇒ (cR@xu@xv)) and ∃xf: a → b →
$o: (∀xx: a: ∃xy: b: (xf@xx@xy) and ∀xx: a, xy_1: b, xy_2: b: ((xf@xx@xy_1 and xf@xx@xy_2) ⇒ (cS@xy_1@xy_2)) and ∀xx_1: a, xx_2
(cR@xx_1@xx_2)))) ⇒ ∃xg: b → a → $o: (∀xx: a: ∃xy: b: (xg@xy@xx) and ∀xy: b, xx_1: a, xx_2: a: ((xg@xy@xx_1 and xg@xy@xx_2
(cR@xx_1@xx_2)) and ∀xy: b: ∃xx: a: (xg@xy@xx)) thf(cTHM552_pme, conjecture)

SEU994^5.p TPS problem from LATTICES
a: $tType thf(a_type, type)
cR: a → a → $o thf(cR, type)
∀xs: a → $o: ∃xx: a: (∀xz: a: ((xs@xz) ⇒ (cR@xz@xx)) and ∀xj: a: (∀xz: a: ((xs@xz) ⇒ (cR@xz@xj)) ⇒ (cR@xx@xj))) and ∀xs: a → $o: ∃xx: a: (∀xz: a: ((xs@xz) ⇒ (cR@xx@xz)) and ∀xj: a: (∀xz: a: ((xs@xz) ⇒ (cR@xj@xz)) ⇒ (cR@xj@xx))) and ∀xx: a, xy: a: ((cR@xx@xy and cR@xy@xx) ⇒ xx = xy) thf(cCLATTICE_pme, con

SEU995^5.p TPS problem THM24
cV: $i thf(cV, type)
cU: $i thf(cU, type)
cU ≠ cV ⇒ ∃g: ($i → $i) → ($i → $i) → $i → $i: ¬∀xx: $i → $i, xy: $i → $i: (g@xx@xy) = (g@xy@xx) thf(cTHM24_pm

SEU996^5.p TPS problem MODULAR-THM
Every distributive lattice is modular.
a: $tType thf(a_type, type)
∀jOIN: a → a → a, mEET: a → a → a: ((∀xx: a: (jOIN@xx@xx) = xx and ∀xx: a: (mEET@xx@xx) =
xx and ∀xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) and ∀xx: a, xy: a, xz: a: (mEET@(mEE
(mEET@xx@(mEET@xy@xz)) and ∀xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) and ∀xx: a, xy: a: (mEET@xx@xy) =
(mEET@xy@xx) and ∀xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy and ∀xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) =
xy and ∀xx: a, xy: a, xz: a: (jOIN@xx@(mEET@xy@xz)) = (mEET@(jOIN@xx@xy)@(jOIN@xx@xz)) and ∀xx: a, xy: a, xz: a
(jOIN@(mEET@xx@xy)@(mEET@xx@xz))) ⇒ ∀xx: a, xy: a, xz: a: ((jOIN@xx@xz) = xz ⇒ (jOIN@xx@(mEET@xy@xz))
(mEET@(jOIN@xx@xy)@xz))) thf(cMODULAR_THM_pme, conjecture)

SEU997^5.p TPS problem CD-LATTICE-THM
A complemented distributive lattice has unique complements.
a: $tType thf(a_type, type)
∀jOIN: a → a → a, mEET: a → a → a, tOP: a, bOTTOM: a: ((∀xx: a: (jOIN@xx@xx) = xx and ∀xx: a: (mEET@xx@xx) =
xx and ∀xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) and ∀xx: a, xy: a, xz: a: (mEET@(mEE
(mEET@xx@(mEET@xy@xz)) and ∀xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) and ∀xx: a, xy: a: (mEET@xx@xy) =
(mEET@xy@xx) and ∀xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy and ∀xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) =
xy and ∀xx: a, xy: a, xz: a: (mEET@xx@(jOIN@xy@xz)) = (jOIN@(mEET@xx@xy)@(mEET@xx@xz)) and ∀xx: a, xy: a, xz:
(mEET@(jOIN@xx@xy)@(jOIN@xx@xz)) and ∀xx: a: (mEET@tOP@xx) = xx and ∀xx: a: (jOIN@tOP@xx) =
tOP and ∀xx: a: (mEET@bOTTOM@xx) = bOTTOM and ∀xx: a: (jOIN@bOTTOM@xx) = xx and ∀xx: a: ∃xy: a: ((jOIN@
tOP and (mEET@xx@xy) = bOTTOM)) ⇒ ∀xx: a, xy: a, xz: a: (((jOIN@xx@xy) = tOP and (mEET@xx@xy) =
bOTTOM and (jOIN@xx@xz) = tOP and (mEET@xx@xz) = bOTTOM) ⇒ xy = xz)) thf(cCD_LATTICE_THM_pme,

SEU998^5.p TPS problem 3-DIAMOND-THM
a: $tType thf(a_type, type)
∀jOIN: a → a → a, mEET: a → a → a: ((∀xx: a: (jOIN@xx@xx) = xx and ∀xx: a: (mEET@xx@xx) =
xx and ∀xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) and ∀xx: a, xy: a, xz: a: (mEET@(mEE
(mEET@xx@(mEET@xy@xz)) and ∀xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) and ∀xx: a, xy: a: (mEET@xx@xy) =
(mEET@xy@xx) and ∀xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy and ∀xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) =
xy) ⇒ (∃xx: a, xy: a, xa: a, xb: a, xc: a: (xa ≠ xb and xa ≠ xc and xa ≠ xx and xa ≠ xy and xb ≠ xc and xb ≠
xx and xb ≠ xy and xc ≠ xx and xc ≠ xy and xx ≠ xy and (mEET@xx@xy) = xy and (jOIN@xx@xy) = xx and (mEET@xx
xa and (jOIN@xx@xa) = xx and (mEET@xx@xb) = xb and (jOIN@xx@xb) = xx and (mEET@xx@xc) = xc and (jOIN@xx
xx and (mEET@xa@xb) = xy and (jOIN@xa@xb) = xx and (mEET@xa@xc) = xy and (jOIN@xa@xc) = xx and (mEET@x
xy and (jOIN@xa@xy) = xa and (mEET@xb@xc) = xy and (jOIN@xb@xc) = xx and (mEET@xb@xy) = xy and (jOIN@xb
xb and (mEET@xc@xy) = xy and (jOIN@xc@xy) = xc) ⇒ ¬∀xx: a, xy: a, xz: a: (mEET@xx@(jOIN@xy@xz)) =
(jOIN@(mEET@xx@xy)@(mEET@xx@xz)))) thf(c3_DIAMOND_THM_pme, conjecture)

SEU999^5.p TPS problem PENTAGON-THM2B
a: $tType thf(a_type, type)
∀jOIN: a → a → a, mEET: a → a → a: ((∀xx: a: (jOIN@xx@xx) = xx and ∀xx: a: (mEET@xx@xx) =
xx and ∀xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) and ∀xx: a, xy: a, xz: a: (mEET@(mEE
(mEET@xx@(mEET@xy@xz)) and ∀xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) and ∀xx: a, xy: a: (mEET@xx@xy) =
(mEET@xy@xx) and ∀xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy and ∀xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) =
xy) ⇒ (∃xx: a, xy: a, xa: a, xb: a, xc: a: (xa ≠ xb and xa ≠ xc and xa ≠ xx and xa ≠ xy and xb ≠ xc and xb ≠
xx and xb ≠ xy and xc ≠ xx and xc ≠ xy and xx ≠ xy and (mEET@xx@xy) = xy and (jOIN@xx@xy) = xx and (mEET@xx

xa and (jOIN@xx@xa) = xx and (mEET@xx@xb) = xb and (jOIN@xx@xb) = xx and (mEET@xx@xc) = xc and (jOIN@xx
xx and (mEET@xa@xb) = xy and (jOIN@xa@xb) = xx and (mEET@xa@xc) = xa and (jOIN@xa@xc) = xc and (mEET@x
xy and (jOIN@xa@xy) = xa and (mEET@xb@xc) = xy and (jOIN@xb@xc) = xx and (mEET@xb@xy) = xy and (jOIN@xb
xb and (mEET@xc@xy) = xy and (jOIN@xc@xy) = xc) $\Rightarrow$ $\neg\forall$xx: a, xy: a, xz: a: (jOIN@xx@(mEET@xy@(jOIN@xx@xz)))
(mEET@(jOIN@xx@xy)@(jOIN@xx@xz)))) thf(cPENTAGON_THM2B_pme, conjecture)