

# SEV axioms

## SEV problems

### SEV000 $\wedge$ 5.p TPS problem MODULAR-EQUIV-THM

The equivalence of two definitions of modularity.

$a: \$tType \quad thf(a\_type, type)$

$\forall jOIN: a \rightarrow a \rightarrow a, mEET: a \rightarrow a \rightarrow a: ((\forall xx: a: (jOIN@xx@xx) = xx \text{ and } \forall xx: a: (mEET@xx@xx) = xx \text{ and } \forall xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) \text{ and } \forall xx: a, xy: a, xz: a: (mEET@(mEET@xx@(mEET@xy@xz)) = (mEET@xx@(mEET@xy@xz)) \text{ and } \forall xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) \text{ and } \forall xx: a, xy: a: (mEET@xx@xy) = (mEET@xy@xx) \text{ and } \forall xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy \text{ and } \forall xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) = xy) \Rightarrow (\forall xx: a, xy: a, xz: a: ((jOIN@xx@xz) = xz \Rightarrow (jOIN@xx@(mEET@xy@xz)) = (mEET@(jOIN@xx@xy)@xz)) \iff \forall xx: a, xy: a, xz: a: (jOIN@xx@(mEET@xy@(jOIN@xx@xz))) = (mEET@(jOIN@xx@xy)@(jOIN@xx@xz)))) \quad thf(cMODULAR\_THM\_DEF2\_pme, conjecture)$

### SEV001 $\wedge$ 5.p TPS problem DISTRIB-THM

In a lattice, join distributes over meet iff meet distributes over join.

$a: \$tType \quad thf(a\_type, type)$

$\forall jOIN: a \rightarrow a \rightarrow a, mEET: a \rightarrow a \rightarrow a: ((\forall xx: a: (jOIN@xx@xx) = xx \text{ and } \forall xx: a: (mEET@xx@xx) = xx \text{ and } \forall xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) \text{ and } \forall xx: a, xy: a, xz: a: (mEET@(mEET@xx@(mEET@xy@xz)) = (mEET@xx@(mEET@xy@xz)) \text{ and } \forall xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) \text{ and } \forall xx: a, xy: a: (mEET@xx@xy) = (mEET@xy@xx) \text{ and } \forall xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy \text{ and } \forall xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) = xy) \Rightarrow (\forall xx: a, xy: a, xz: a: (jOIN@xx@(mEET@xy@xz)) = (mEET@(jOIN@xx@xy)@(jOIN@xx@xz)) \iff \forall xx: a, xy: a, xz: a: (mEET@xx@(jOIN@xy@xz)) = (jOIN@(mEET@xx@xy)@(mEET@xx@xz)))) \quad thf(cDISTRIB\_THM\_DEF2\_pme, conjecture)$

### SEV002 $\wedge$ 5.p TPS problem from LATTICES-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall jOIN: a \rightarrow a \rightarrow a, mEET: a \rightarrow a \rightarrow a: ((\forall xx: a: (jOIN@xx@xx) = xx \text{ and } \forall xx: a: (mEET@xx@xx) = xx \text{ and } \forall xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) \text{ and } \forall xx: a, xy: a, xz: a: (mEET@(mEET@xx@(mEET@xy@xz)) = (mEET@xx@(mEET@xy@xz)) \text{ and } \forall xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) \text{ and } \forall xx: a, xy: a: (mEET@xx@xy) = (mEET@xy@xx) \text{ and } \forall xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy \text{ and } \forall xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) = xy \text{ and } \forall xx: a, xy: a, xz: a: (jOIN@xx@(mEET@xy@xz)) = (mEET@(jOIN@xx@xy)@(jOIN@xx@xz)) \text{ and } \forall xx: a, xy: a, xz: a: (jOIN@(mEET@xx@xy)@(mEET@xx@xz)) \Rightarrow \forall xx: a, xy: a, xz: a: (jOIN@xx@(mEET@xy@(jOIN@xx@xz))) = (mEET@(jOIN@xx@xy)@(jOIN@xx@xz))) \quad thf(cMODULAR\_THM\_DEF2\_pme, conjecture)$

### SEV003 $\wedge$ 5.p TPS problem from LATTICES-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall jOIN: a \rightarrow a \rightarrow a, mEET: a \rightarrow a \rightarrow a: ((\forall xx: a: (jOIN@xx@xx) = xx \text{ and } \forall xx: a: (mEET@xx@xx) = xx \text{ and } \forall xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) \text{ and } \forall xx: a, xy: a, xz: a: (mEET@(mEET@xx@(mEET@xy@xz)) = (mEET@xx@(mEET@xy@xz)) \text{ and } \forall xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) \text{ and } \forall xx: a, xy: a: (mEET@xx@xy) = (mEET@xy@xx) \text{ and } \forall xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy \text{ and } \forall xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) = xy) \Rightarrow (\neg \forall xx: a, xy: a, xz: a: (jOIN@xx@(mEET@xy@(jOIN@xx@xz))) = (mEET@(jOIN@xx@xy)@(jOIN@xx@xz)) \Rightarrow \exists xx: a, xy: a, xa: a, xb: a, xc: a: (xa \neq xb \text{ and } xa \neq xc \text{ and } xa \neq xx \text{ and } xa \neq xy \text{ and } xb \neq xc \text{ and } xb \neq xx \text{ and } xb \neq xy \text{ and } xc \neq xx \text{ and } xc \neq xy \text{ and } xx \neq xy \text{ and } (mEET@xx@xy) = xy \text{ and } (jOIN@xx@xy) = xx \text{ and } (mEET@xx@xa) = xa \text{ and } (jOIN@xx@xa) = xx \text{ and } (mEET@xx@xb) = xb \text{ and } (jOIN@xx@xb) = xx \text{ and } (mEET@xx@xc) = xc \text{ and } (jOIN@xx@xc) = xx \text{ and } (mEET@xa@xb) = xy \text{ and } (jOIN@xa@xb) = xx \text{ and } (mEET@xa@xc) = xa \text{ and } (jOIN@xa@xc) = xc \text{ and } (mEET@xa@xy) = xy \text{ and } (jOIN@xa@xy) = xa \text{ and } (mEET@xb@xc) = xy \text{ and } (jOIN@xb@xc) = xx \text{ and } (mEET@xb@xy) = xy \text{ and } (jOIN@xb@xy) = xx \text{ and } (mEET@xc@xy) = xy \text{ and } (jOIN@xc@xy) = xc))) \quad thf(cPENTAGON\_THM2C\_pme, conjecture)$

### SEV004 $\wedge$ 5.p TPS problem from LATTICES-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall jOIN: a \rightarrow a \rightarrow a, mEET: a \rightarrow a \rightarrow a: ((\forall xx: a: (jOIN@xx@xx) = xx \text{ and } \forall xx: a: (mEET@xx@xx) = xx \text{ and } \forall xx: a, xy: a, xz: a: (jOIN@(jOIN@xx@xy)@xz) = (jOIN@xx@(jOIN@xy@xz)) \text{ and } \forall xx: a, xy: a, xz: a: (mEET@(mEET@xx@(mEET@xy@xz)) = (mEET@xx@(mEET@xy@xz)) \text{ and } \forall xx: a, xy: a: (jOIN@xx@xy) = (jOIN@xy@xx) \text{ and } \forall xx: a, xy: a: (mEET@xx@xy) = (mEET@xy@xx) \text{ and } \forall xx: a, xy: a: (jOIN@(mEET@xx@xy)@xy) = xy \text{ and } \forall xx: a, xy: a: (mEET@(jOIN@xx@xy)@xy) = xy) \Rightarrow (\forall xx: a, xy: a, xz: a: (jOIN@xx@(mEET@xy@(jOIN@xx@xz))) = (mEET@(jOIN@xx@xy)@(jOIN@xx@xz)) \iff \neg \exists xx: a, xy: a, xa: a, xb: a, xc: a: (xa \neq xb \text{ and } xa \neq xc \text{ and } xa \neq xx \text{ and } xa \neq xy \text{ and } xb \neq xc \text{ and } xb \neq xx \text{ and } xb \neq xy \text{ and } xc \neq xx \text{ and } xc \neq xy \text{ and } xx \neq xy \text{ and } (mEET@xx@xy) = xy \text{ and } (jOIN@xx@xy) = xx \text{ and } (mEET@xx@xa) = xa \text{ and } (jOIN@xx@xa) = xx \text{ and } (mEET@xx@xb) = xb \text{ and } (jOIN@xx@xb) = xx \text{ and } (mEET@xx@xc) = xc \text{ and } (jOIN@xx@xc) = xx \text{ and } (mEET@xa@xb) = xy \text{ and } (jOIN@xa@xb) = xx \text{ and } (mEET@xa@xc) = xa \text{ and } (jOIN@xa@xc) = xc \text{ and } (mEET@xa@xy) = xy \text{ and } (jOIN@xa@xy) = xa \text{ and } (mEET@xb@xc) = xy \text{ and } (jOIN@xb@xc) = xx \text{ and } (mEET@xb@xy) = xy \text{ and } (jOIN@xb@xy) = xx \text{ and } (mEET@xc@xy) = xy \text{ and } (jOIN@xc@xy) = xc))) \quad thf(cMODULAR\_THM2\_DEF2\_pme, conjecture)$

**SEV005** $\wedge$ **5.p** TPS problem from LATTICES-THMS

$a$ : \$tType thf(a\_type, type)

$\forall j\text{OIN}: a \rightarrow a \rightarrow a, \text{mEET}: a \rightarrow a \rightarrow a: ((\forall xx: a: (j\text{OIN}@xx@xx) = xx \text{ and } \forall xx: a: (\text{mEET}@xx@xx) = xx \text{ and } \forall xx: a, xy: a, xz: a: (j\text{OIN}@(j\text{OIN}@xx@xy)@xz) = (j\text{OIN}@xx@(j\text{OIN}@xy@xz)) \text{ and } \forall xx: a, xy: a, xz: a: (\text{mEET}@(j\text{OIN}@xx@xy)@xz) = (\text{mEET}@xx@(\text{mEET}@xy@xz)) \text{ and } \forall xx: a, xy: a: (j\text{OIN}@xx@xy) = (j\text{OIN}@xy@xx) \text{ and } \forall xx: a, xy: a: (\text{mEET}@xx@xy) = (\text{mEET}@xy@xx) \text{ and } \forall xx: a, xy: a: (j\text{OIN}@(m\text{EET}@xx@xy)@xy) = xy \text{ and } \forall xx: a, xy: a: (\text{mEET}@(j\text{OIN}@xx@xy)@xy) = xy) \Rightarrow (\forall xx: a, xy: a, xz: a: ((j\text{OIN}@xx@xz) = xz \Rightarrow (j\text{OIN}@xx@(m\text{EET}@xy@xz)) = (\text{mEET}@(j\text{OIN}@xx@xy)@xz))) \iff \neg \exists xx: a, xy: a, xa: a, xb: a, xc: a: (xa \neq xb \text{ and } xa \neq xc \text{ and } xa \neq xx \text{ and } xa \neq xy \text{ and } xb \neq xc \text{ and } xb \neq xx \text{ and } xb \neq xy \text{ and } xc \neq xx \text{ and } xc \neq xy \text{ and } xx \neq xy \text{ and } (\text{mEET}@xx@xy) = xy \text{ and } (j\text{OIN}@xx@xy) = xx \text{ and } (\text{mEET}@xx@xa) = xa \text{ and } (j\text{OIN}@xx@xa) = xx \text{ and } (\text{mEET}@xx@xb) = xb \text{ and } (j\text{OIN}@xx@xb) = xx \text{ and } (\text{mEET}@xx@xc) = xc \text{ and } (j\text{OIN}@xx@xc) = xx \text{ and } (\text{mEET}@xa@xb) = xy \text{ and } (j\text{OIN}@xa@xb) = xx \text{ and } (\text{mEET}@xa@xc) = xa \text{ and } (j\text{OIN}@xa@xc) = xc \text{ and } (\text{mEET}@xy@xy) = xy \text{ and } (j\text{OIN}@xa@xy) = xa \text{ and } (\text{mEET}@xb@xc) = xy \text{ and } (j\text{OIN}@xb@xc) = xx \text{ and } (\text{mEET}@xb@xy) = xy \text{ and } (j\text{OIN}@xb@xb) = xb \text{ and } (\text{mEET}@xc@xy) = xy \text{ and } (j\text{OIN}@xc@xy) = xc)))$  thf(cMODULAR\_THM2\_pme, conjecture)

**SEV006** $\wedge$ **5.p** TPS problem from LATTICES-THMS

$a$ : \$tType thf(a\_type, type)

$\forall j\text{OIN}: a \rightarrow a \rightarrow a, \text{mEET}: a \rightarrow a \rightarrow a: ((\forall xx: a: (j\text{OIN}@xx@xx) = xx \text{ and } \forall xx: a: (\text{mEET}@xx@xx) = xx \text{ and } \forall xx: a, xy: a, xz: a: (j\text{OIN}@(j\text{OIN}@xx@xy)@xz) = (j\text{OIN}@xx@(j\text{OIN}@xy@xz)) \text{ and } \forall xx: a, xy: a, xz: a: (\text{mEET}@(j\text{OIN}@xx@xy)@xz) = (\text{mEET}@xx@(\text{mEET}@xy@xz)) \text{ and } \forall xx: a, xy: a: (j\text{OIN}@xx@xy) = (j\text{OIN}@xy@xx) \text{ and } \forall xx: a, xy: a: (\text{mEET}@xx@xy) = (\text{mEET}@xy@xx) \text{ and } \forall xx: a, xy: a: (j\text{OIN}@(m\text{EET}@xx@xy)@xy) = xy \text{ and } \forall xx: a, xy: a: (\text{mEET}@(j\text{OIN}@xx@xy)@xy) = xy \text{ and } \exists xx: a, xy: a, xa: a, xb: a, xc: a: (xa \neq xb \text{ and } xa \neq xc \text{ and } xa \neq xx \text{ and } xa \neq xy \text{ and } xb \neq xc \text{ and } xb \neq xx \text{ and } xb \neq xy \text{ and } xc \neq xx \text{ and } xc \neq xy \text{ and } xx \neq xy) \text{ and } \neg \forall xx: a, xy: a, xz: a: (j\text{OIN}@xx@(m\text{EET}@xy@(j\text{OIN}@xx@xz))) = (\text{mEET}@(j\text{OIN}@xx@xy)@(j\text{OIN}@xx@xz))) \Rightarrow \exists xx: a, xy: a, xa: a, xb: a, xc: a: ((\text{mEET}@xx@xy) = xy \text{ and } (j\text{OIN}@xx@xy) = xx \text{ and } (\text{mEET}@xx@xa) = xa \text{ and } (j\text{OIN}@xx@xa) = xx \text{ and } (\text{mEET}@xx@xb) = xb \text{ and } (j\text{OIN}@xx@xb) = xx \text{ and } (\text{mEET}@xx@xc) = xc \text{ and } (j\text{OIN}@xx@xc) = xx \text{ and } (\text{mEET}@xa@xb) = xy \text{ and } (j\text{OIN}@xa@xb) = xx \text{ and } (\text{mEET}@xa@xc) = xa \text{ and } (j\text{OIN}@xa@xc) = xc \text{ and } (\text{mEET}@xa@xy) = xy \text{ and } (j\text{OIN}@xa@xy) = xa \text{ and } (\text{mEET}@xb@xc) = xy \text{ and } (j\text{OIN}@xb@xc) = xx \text{ and } (\text{mEET}@xb@xy) = xy \text{ and } (j\text{OIN}@xb@xb) = xb \text{ and } (\text{mEET}@xc@xy) = xy \text{ and } (j\text{OIN}@xc@xy) = xc)))$  thf(cPENTAGON\_THM2D\_pme, conjecture)

**SEV008** $\wedge$ **5.p** TPS problem THM261

A partition defines an equivalence relation.

$a$ : \$tType thf(a\_type, type)

$\forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow (\forall xx: a: \exists s: a \rightarrow \$o: (p@s \text{ and } s@xx \text{ and } s@xx) \text{ and } \forall xx: a, xy: a: (\exists s: a \rightarrow \$o: (p@s \text{ and } s@xx \text{ and } s@xy) \Rightarrow \exists s: a \rightarrow \$o: (p@s \text{ and } s@xy \text{ and } s@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((\exists s: a \rightarrow \$o: (p@s \text{ and } s@xx \text{ and } s@xy) \text{ and } \exists s: a \rightarrow \$o: (p@s \text{ and } s@xy \text{ and } s@xz)) \Rightarrow \exists s: a \rightarrow \$o: (p@s \text{ and } s@xx \text{ and } s@xz))))$

**SEV009** $\wedge$ **5.p** TPS problem THM261-B

A partition defines an equivalence relation.

$a$ : \$tType thf(a\_type, type)

$\forall p: (a \rightarrow \$o) \rightarrow \$o: (\forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp)) \Rightarrow (\forall xx: a: \exists s: a \rightarrow \$o: (p@s \text{ and } s@xx \text{ and } s@xx) \text{ and } \forall xx: a, xy: a: (\exists s: a \rightarrow \$o: (p@s \text{ and } s@xx \text{ and } s@xy) \Rightarrow \exists s: a \rightarrow \$o: (p@s \text{ and } s@xy \text{ and } s@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((\exists s: a \rightarrow \$o: (p@s \text{ and } s@xx \text{ and } s@xy) \text{ and } \exists s: a \rightarrow \$o: (p@s \text{ and } s@xy \text{ and } s@xz)) \Rightarrow \exists s: a \rightarrow \$o: (p@s \text{ and } s@xx \text{ and } s@xz))))$  thf(cTHM261\_B\_pme, conjecture)

**SEV010** $\wedge$ **5.p** TPS problem THM260

An equivalence relation defines a partition.

$a$ : \$tType thf(a\_type, type)

$\forall r: a \rightarrow a \rightarrow \$o: ((\forall xx: a: (r@xx@xx) \text{ and } \forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((r@xx@xy \text{ and } r@xx@xz)) \Rightarrow (\forall xp: a \rightarrow \$o: ((\exists xz: a: (xp@xz) \text{ and } \forall xx: a: ((xp@xx) \Rightarrow \forall xy: a: ((xp@xy) \iff (r@xx@xy)))) \Rightarrow \exists xz: a: (xp@xz) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (\exists xz: a: (xp@xz) \text{ and } \forall xx_0: a: ((xp@xx_0) \Rightarrow \forall xy: a: ((xp@xy) \iff (r@xx_0@xy))) \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((\exists xz: a: (xq@xz) \text{ and } \forall xx_0: a: ((xq@xx_0) \Rightarrow \forall xy: a: ((xq@xy) \iff (r@xx_0@xy))) \text{ and } xq@xx) \Rightarrow xq = xp))))$  thf(cTHM260\_pme, conjecture)

**SEV011** $\wedge$ **5.p** TPS problem THM260-B

An equivalence relation defines a partition.

$a$ : \$tType thf(a\_type, type)

$\forall r: a \rightarrow a \rightarrow \$o: ((\forall xx: a: (r@xx@xx) \text{ and } \forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((r@xx@xy \text{ and } r@xx@xz)) \Rightarrow \forall xx: a: \exists xp: a \rightarrow \$o: (\forall xx_0: a: ((xp@xx_0) \Rightarrow \forall xy: a: ((xp@xy) \iff (r@xx_0@xy))) \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((\forall xx_0: a: ((xq@xx_0) \Rightarrow \forall xy: a: ((xq@xy) \iff (r@xx_0@xy))) \text{ and } xq@xx) \Rightarrow xq = xp))))$  thf(cTHM260\_B\_pme, conjecture)

**SEV012** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$\forall xx: \$o, xy: \$o: (\$true \Rightarrow \$true) \text{ and } \forall xx: \$o, xy: \$o, xz: \$o: ((\$true \text{ and } \$true) \Rightarrow \$true) \text{ and } (\lambda xx: \$o, xy: \$o: \$true) = (\lambda xx: \$o, xy: \$o: \$true)$  thf(cTHM519\_pme, conjecture)

**SEV013** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall xx: a: xx = xx \text{ and } \forall xx: a, xy: a: (xx = xy \Rightarrow xy = xx) \text{ and } \forall xx: a, xy: a, xz: a: ((xx = xy \text{ and } xy = xz) \Rightarrow xx = xz) \quad thf(cTHM511\_pme, conjecture)$

**SEV014** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o: ((\forall xx: a: (xr@xx@xx) \text{ and } \forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xr@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((xr@xx@xy) \Rightarrow (xr@xx@xz))) \Rightarrow \forall xx: a: (xr@xx@xx)) \quad thf(cTHM513\_pme, conjecture)$

**SEV015** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((xp@xx@xy \text{ and } xp@xy@xz) \Rightarrow (xp@xx@xz)) \text{ and } xp = xp) \Rightarrow \forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xx@xx))) \quad thf(cTHM520\_pme, conjecture)$

**SEV016** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\lambda xs: a \rightarrow \$o: \exists xz: a: xs = (\lambda xx: a: (q@xx@xz))) = p) \quad thf(cTHM262\_NEW\_pme, conjecture)$

**SEV017** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o: ((\forall xx: a: (xr@xx@xx) \text{ and } \forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xr@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((xr@xx@xy) \Rightarrow (xr@xx@xz))) \Rightarrow \forall xx: a, xy: a, xz: a: ((xr@xx@xy) \Rightarrow ((xr@xy@xz) \Rightarrow (xr@xx@xz)))) \quad thf(cTHM514\_pme, conjecture)$

**SEV018** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (q@xx@xy)))) = p) \quad thf(cTHM262A\_pme, conjecture)$

**SEV019** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$cQ: a \rightarrow a \rightarrow \$o \quad thf(cQ, type)$

$\forall xx: a: \exists xp: a \rightarrow \$o: (\exists xz: a: (xp@xz) \text{ and } \forall xx_0: a: ((xp@xx_0) \Rightarrow \forall xy: a: ((xp@xy) \iff (cQ@xx_0@xy))) \text{ and } xp@xx) \Rightarrow (\forall xx: a: (cQ@xx@xx) \text{ and } \forall xx: a, xy: a: ((cQ@xx@xy) \Rightarrow (cQ@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((cQ@xx@xy \text{ and } cQ@xy@xx) \Rightarrow (cQ@xx@xz))) \quad thf(cTHM559\_pme, conjecture)$

**SEV020** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall p: (a \rightarrow \$o) \rightarrow \$o: (\forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp)) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\forall xx: a: (q@xx@xx) \text{ and } \forall xx: a, xy: a: ((q@xx@xy) \Rightarrow (q@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((q@xx@xy) \Rightarrow (q@xx@xz)) \text{ and } (\lambda xs: a \rightarrow \$o: \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (q@xx@xy)))) = p) \quad thf(cTHM262\_B\_pme, conjecture)$

**SEV021** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$cP: (a \rightarrow \$o) \rightarrow \$o \quad thf(cP, type)$

$\forall xq_1: a \rightarrow \$o, xq_2: a \rightarrow \$o: ((xq_1 = xq_2 \text{ and } cP@xq_1) \Rightarrow (cP@xq_2)) \Rightarrow ((\forall xp: a \rightarrow \$o: ((cP@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (cP@xp \text{ and } xp@xx) \text{ and } \forall xx: a, xy: a, xp: a \rightarrow \$o, xq: a \rightarrow \$o: ((cP@xp \text{ and } cP@xy) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (q@xx@xy)))) = cP) \quad thf(cTHM262\_D\_EXT2\_pme, conjecture)$

**SEV021** $\wedge$ **6.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

$cP: (a \rightarrow \$o) \rightarrow \$o \quad thf(cP, type)$

$cQ: a \rightarrow a \rightarrow \$o \quad thf(cQ, type)$

$cQ = (\lambda x: a, y: a: \exists s: a \rightarrow \$o: (cP@s \text{ and } s@x \text{ and } s@y)) \quad thf(cQ\_def, definition)$

$\forall xq_1: a \rightarrow \$o, xq_2: a \rightarrow \$o: ((xq_1 = xq_2 \text{ and } cP@xq_1) \Rightarrow (cP@xq_2)) \Rightarrow ((\forall xp: a \rightarrow \$o: ((cP@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (cP@xp \text{ and } xp@xx) \text{ and } \forall xx: a, xy: a, xp: a \rightarrow \$o, xq: a \rightarrow \$o: ((cP@xp \text{ and } cP@xy) \Rightarrow (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (cQ@xx@xy)))) = cP) \quad thf(cTHM262\_D\_EXT2\_pme, conjecture)$

**SEV021** $\wedge$ **7.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType \quad thf(a\_type, type)$

cP:  $(a \rightarrow \$o) \rightarrow \$o$  thf(cP, type)

cQ:  $a \rightarrow a \rightarrow \$o$  thf(cQ, type)

cQ =  $(\lambda x: a, y: a: \forall s: a \rightarrow \$o: ((cP@s) \Rightarrow ((s@x) \iff (s@y))))$  thf(cQ\_def, definition)

$\forall xq_1: a \rightarrow \$o, xq_2: a \rightarrow \$o: ((xq_1 = xq_2 \text{ and } cP@xq_1) \Rightarrow (cP@xq_2)) \Rightarrow ((\forall xp: a \rightarrow \$o: ((cP@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (cP@xp \text{ and } xp@xx) \text{ and } \forall xx: a, xy: a, xp: a \rightarrow \$o, xq: a \rightarrow \$o: ((cP@xp \text{ and } cP@xq) \Rightarrow (xq@xy)))) \Rightarrow (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (cQ@xx@xy)))))) = cP$  thf(cTHM262\_D\_EXT2\_pme, conjecture)

**SEV022** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

a: \$tType thf(a\_type, type)

cP:  $(a \rightarrow \$o) \rightarrow \$o$  thf(cP, type)

$(\forall xp: a \rightarrow \$o: ((cP@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a, xp: a \rightarrow \$o, xq: a \rightarrow \$o: ((cP@xp \text{ and } cP@xq \text{ and } xp@xx \text{ and } xq@xx) \Rightarrow xp = xq)) \Rightarrow \exists r: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz)) \text{ and } (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (r@xx@xy)))))) = cP$  thf(cTHM556\_pme, conjecture)

**SEV023** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

a: \$tType thf(a\_type, type)

$\forall xp: a \rightarrow \$o, xa: a \rightarrow \$o: (\forall xb: a: (xp@xb) = (xa@xb) \Rightarrow \forall p: (a \rightarrow \$o) \rightarrow \$o: ((p@xp) \Rightarrow (p@xa))) \Rightarrow \forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx) \text{ and } \forall xx: a, xy: a, xp: a \rightarrow \$o, xq: a \rightarrow \$o: ((p@xp \text{ and } p@xq \text{ and } xp@xx \text{ and } xq@xx \text{ and } xp@xy) \Rightarrow (xq@xy))) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (q@xx@xy)))))) = p$  thf(cTHM262\_D\_EXT\_pme, conjecture)

**SEV024** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

a: \$tType thf(a\_type, type)

$\forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx) \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\forall xx: a: (q@xx@xx) \text{ and } \forall xx: a, xy: a: ((q@xx@xy) \Rightarrow (q@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((q@xx@xy \text{ and } q@xy@xz) \Rightarrow (q@xx@xz)) \text{ and } (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (q@xx@xy)))))) = p$  thf(cTHM262\_pme, conjecture)

**SEV025** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

a: \$tType thf(a\_type, type)

cP:  $(a \rightarrow \$o) \rightarrow \$o$  thf(cP, type)

$\forall xq_1: a \rightarrow \$o, xq_2: a \rightarrow \$o: ((xq_1 = xq_2 \text{ and } cP@xq_1) \Rightarrow (cP@xq_2)) \Rightarrow ((\forall xp: a \rightarrow \$o: ((cP@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (cP@xp \text{ and } xp@xx) \text{ and } \forall xq: a \rightarrow \$o: ((cP@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\forall xx: a: (q@xx@xx) \text{ and } \forall xx: a, xy: a: ((q@xx@xy) \Rightarrow (q@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((q@xx@xy \text{ and } q@xy@xz) \Rightarrow (q@xx@xz)) \text{ and } (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (q@xx@xy)))))) = cP$  thf(cTHM262\_EXT2\_pme, conjecture)

**SEV026** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$\forall xr: \$i \rightarrow \$i \rightarrow \$o: \exists xs: \$i \rightarrow \$i \rightarrow \$o: (\forall xa: \$i, xb: \$i: ((xr@xa@xb) \Rightarrow (xs@xa@xb)) \text{ and } \forall xx: \$i: (xs@xx@xx) \text{ and } \forall xx: \$i, xy: \$i: (xs@xy@xx)) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i: ((xs@xx@xy \text{ and } xs@xy@xz) \Rightarrow (xs@xx@xz)) \text{ and } \forall xt: \$i \rightarrow \$i \rightarrow \$o: ((\forall xa: \$i, xb: \$i: (xt@xa@xb)) \text{ and } \forall xx: \$i: (xt@xx@xx) \text{ and } \forall xx: \$i, xy: \$i: ((xt@xx@xy) \Rightarrow (xt@xy@xx)) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i: ((xt@xx@xy \text{ and } xt@xy@xz) \Rightarrow (xt@xx@xz)) \Rightarrow \forall xa: \$i, xb: \$i: ((xs@xa@xb) \Rightarrow (xt@xa@xb))))$  thf(cTHM601\_pme, conjecture)

**SEV027** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

a: \$tType thf(a\_type, type)

$\forall xp: a \rightarrow \$o, xa: a \rightarrow \$o: (\forall xb: a: (xp@xb) = (xa@xb) \Rightarrow \forall p: (a \rightarrow \$o) \rightarrow \$o: ((p@xp) \Rightarrow (p@xa))) \Rightarrow \forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx) \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\forall xx: a: (q@xx@xx) \text{ and } \forall xx: a, xy: a: ((q@xx@xy) \Rightarrow (q@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((q@xx@xy \text{ and } q@xy@xz) \Rightarrow (q@xx@xz)) \text{ and } (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (q@xx@xy)))))) = p$  thf(cTHM262\_EXT\_pme, conjecture)

**SEV028** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

a: \$tType thf(a\_type, type)

cQ:  $a \rightarrow a \rightarrow \$o$  thf(cQ, type)

$(\forall xp: a \rightarrow \$o: ((\exists xz: a: (xp@xz) \text{ and } \forall xx: a: ((xp@xx) \Rightarrow \forall xy: a: ((xp@xy) \iff (cQ@xx@xy)))) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (\exists xz: a: (xp@xz) \text{ and } \forall xx_0: a: ((xp@xx_0) \Rightarrow \forall xy: a: ((xp@xy) \iff (cQ@xx_0@xy)))) \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((\exists xz: a: (xq@xz) \text{ and } \forall xx_0: a: ((xq@xx_0) \Rightarrow \forall xy: a: ((xq@xy) \iff (cQ@xx_0@xy)))) \text{ and } xq@xx) \Rightarrow xq = xp)) \Rightarrow (\forall xx: a: (cQ@xx@xx) \text{ and } \forall xx: a, xy: a: ((cQ@xx@xy) \Rightarrow (cQ@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((cQ@xx@xy \text{ and } cQ@xy@xz) \Rightarrow (cQ@xx@xz)))$  thf(cTHM558\_pme, conjecture)

**SEV029** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

a: \$tType thf(a\_type, type)

$\forall x: a \rightarrow a \rightarrow \$o, xs: a \rightarrow \$o: ((\forall xx: a: ((xs@xx) \Rightarrow (xr@xx@xx)) \text{ and } \forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xr@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((xr@xx@xy \text{ and } xr@xy@xz) \Rightarrow (xr@xx@xz)))) \Rightarrow (\forall xa: a \rightarrow \$o: (\exists xx: a: \forall xx_1: a: ((xa@xx_1) \Rightarrow \exists xx: a: (xa@xx)) \text{ and } \forall xx: a: ((xs@xx) \iff \exists s: a \rightarrow \$o: (\exists xx_0: a: \forall xx_2: a: ((s@xx_2) \iff (xr@xx_0@xx_2)) \text{ and } s@xx)) \text{ and } \forall xb: a \rightarrow \$o, xc: a \rightarrow \$o: ((\exists xx: a: \forall xx_3: a: ((xb@xx_3) \iff (xr@xx@xx_3)) \text{ and } \exists xx: a: \forall xx_4: (xr@xx@xx_4)) \text{ and } \exists xx: a: (xb@xx \text{ and } xc@xx)) \Rightarrow xb = xc)))$  thf(cTHM260A\_pme, conjecture)

**SEV030** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType$  thf(a\_type, type)  
 $cP: (a \rightarrow \$o) \rightarrow \$o$  thf(cP, type)  
 $\forall xq_1: a \rightarrow \$o, xq_2: a \rightarrow \$o: ((xq_1 = xq_2 \text{ and } cP@xq_1) \Rightarrow (cP@xq_2)) \Rightarrow ((\forall xp: a \rightarrow \$o: ((cP@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (cP@xp \text{ and } xp@xx) \text{ and } \forall xx: a, xy: a, xp: a \rightarrow \$o, xq: a \rightarrow \$o: ((cP@xp \text{ and } cP@xq) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\forall xx: a: (q@xx@xx) \text{ and } \forall xx: a, xy: a: ((q@xx@xy) \Rightarrow (q@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((q@xx@xz) \text{ and } (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \Rightarrow \forall xy: a: ((xs@xy) \iff (q@xx@xy)))))) = cP))$  thf(cTHM262\_C\_EXT2\_pme, conjecture)

**SEV031** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType$  thf(a\_type, type)  
 $b: \$tType$  thf(b\_type, type)  
 $\forall xp: a \rightarrow \$o, xe: a \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow b) \rightarrow \$o: (\forall xx: a: ((xp@xx) \Rightarrow (\forall xx_0: a \rightarrow b: (xe@xx@xx_0@xx_0) \text{ and } \forall xx_0: a \rightarrow b, xy: a \rightarrow b: ((xe@xx@xx_0@xy) \Rightarrow (xe@xx@xy@xx_0)) \text{ and } \forall xx_0: a \rightarrow b, xy: a \rightarrow b, xz: a \rightarrow b: ((xe@xx@xx_0@xy \text{ and } xe@xx_0@xx_0@xz))) \Rightarrow (\forall xx: a \rightarrow b, xx_0: a: ((xp@xx_0) \Rightarrow (xe@xx_0@xx@xx)) \text{ and } \forall xx: a \rightarrow b, xy: a \rightarrow b: (\forall xx_0: a: ((xp@xx_0) \Rightarrow (xe@xx_0@xx@xy)) \Rightarrow \forall xx_0: a: ((xp@xx_0) \Rightarrow (xe@xx_0@xy@xx))) \text{ and } \forall xx: a \rightarrow b, xy: a \rightarrow b, xz: a \rightarrow b: ((\forall xx_0: a: ((xp@xx_0) \Rightarrow (xe@xx_0@xx@xy)) \text{ and } \forall xx_0: a: ((xp@xx_0) \Rightarrow (xe@xx_0@xy@xz))) \Rightarrow \forall xx_0: a: ((xp@xx_0) \Rightarrow (xe@xx_0@xx@xz))))$  thf(cTHM512\_pme, conjecture)

**SEV032** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType$  thf(a\_type, type)  
 $\forall t: (a \rightarrow \$o) \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow \$o: ((t \neq u \text{ and } \forall xp: a \rightarrow \$o: ((t@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (t@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((t@xq \text{ and } xq@xx) \Rightarrow xq = xp)) \text{ and } \forall xp: a \rightarrow \$o: ((u@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (u@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((u@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow \exists xx: a, xy: a: ((\exists xs: a \rightarrow \$o: (t@xs \text{ and } xs@xx \text{ and } xs@xy) \text{ and } \forall xq: a \rightarrow \$o: ((u@xq) \Rightarrow (\neg xq@xx \text{ or } \neg xq@xy))) \text{ and } \forall xs: a \rightarrow \$o: (u@xs \text{ and } xs@xx \text{ and } xs@xy) \text{ and } \forall xq: a \rightarrow \$o: ((t@xq) \Rightarrow (\neg xq@xx \text{ or } \neg xq@xy))))$  thf(cTHM266\_LEMMA\_pme, conjecture)

**SEV033** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType$  thf(a\_type, type)  
 $\forall xp: a \rightarrow \$o, xa: a \rightarrow \$o: (\forall xb: a: (xp@xb) = (xa@xb) \Rightarrow \forall p: (a \rightarrow \$o) \rightarrow \$o: ((p@xp) \Rightarrow (p@xa))) \Rightarrow \forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx) \text{ and } \forall xx: a, xy: a, xp: a \rightarrow \$o, xq: a \rightarrow \$o: ((p@xp \text{ and } p@xq \text{ and } xp@xx \text{ and } xq@xx \text{ and } xp@xy) \Rightarrow (xq@xy))) \Rightarrow \exists q: a \rightarrow a \rightarrow \$o: (\forall xx: a: (q@xx@xx) \text{ and } \forall xx: a, xy: a, xz: a: ((q@xx@xy \text{ and } q@xy@xz) \Rightarrow (q@xx@xz)) \text{ and } (\lambda xs: a \rightarrow \$o: (\exists xz: a: (xs@xz) \text{ and } \forall xx: a: ((xs@xx) \iff (q@xx@xy)))) = p))$  thf(cTHM262\_C\_EXT\_pme, conjecture)

**SEV034** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType$  thf(a\_type, type)  
 $b: \$tType$  thf(b\_type, type)  
 $\forall xp: a \rightarrow a \rightarrow \$o, xq: a \rightarrow b \rightarrow b \rightarrow \$o, xf: a \rightarrow b, xg: a \rightarrow b: (\forall xx: a: ((xp@xx@xx) \Rightarrow (xq@xx@(xf@xx)@(xg@xx))) \Rightarrow (\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xq@xx@(xf@xx)@(xg@xy))) \Rightarrow ((\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx)) \text{ and } \forall xx: a, (xp@xx@xz) \text{ and } xp = xp) \Rightarrow (\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (\forall xx_0: b, xy_0: b: ((xq@xx@xx_0@xy_0) \Rightarrow (xq@xx@xy_0@xx_0)) \text{ and } \forall xx_0: b, xy_0: b, xz: b: ((xq@xx@xx_0@xy_0 \text{ and } xq@xx@xy_0@xz) \Rightarrow (xq@xx@xx_0@xz)) \text{ and } (xq@xx) = (xq@xy))) \Rightarrow \forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xq@xx@(xf@xx)@(xg@xy))))$  thf(cTHM518\_pme, conjecture)

**SEV035** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a: \$tType$  thf(a\_type, type)  
 $\forall xx: a \rightarrow \$o: \exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xx@(xs@xx_0))) \text{ and } \forall xy: a: ((xx@xy) \Rightarrow \exists xy_{20}: a: (\lambda xx_0: a: (xx@xx_0 \text{ and } (xs@xx_0)) = (\lambda xx: a, xy: a: xx = xy@xy_{20}))) \text{ and } \forall xx: a \rightarrow \$o, xy: a \rightarrow \$o: (\exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xy@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xy@xy_0) \Rightarrow \exists xy_{21}: a: (\lambda xx_0: a: (xx@xx_0 \text{ and } xy_0 = (xs@xx_0)) = (\lambda xx: a, xy: a: xx = xy@xy_{21}))) \Rightarrow \exists xs: a \rightarrow a: (\forall xx_0: a: ((xy@xx_0) \Rightarrow (xx@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xx@xy_0) \Rightarrow \exists xy_{22}: a: (\lambda xx_0: a: (xy@xx_0) \text{ and } (xs@xx_0)) = (\lambda xx: a, xy: a: xx = xy@xy_{22})))) \text{ and } \forall xx: a \rightarrow \$o, xy: a \rightarrow \$o, xz: a \rightarrow \$o: ((\exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xy@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xy@xy_0) \Rightarrow \exists xy_{23}: a: (\lambda xx_0: a: (xx@xx_0 \text{ and } xy_0 = (xs@xx_0)) = (\lambda xx: a, xy: a: xx = xy@xy_{23}))) \text{ and } \exists xs: a \rightarrow a: (\forall xx_0: a: ((xy@xx_0) \Rightarrow (xz@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xz@xy_0) \Rightarrow \exists xy_{24}: a: (\lambda xx_0: a: (xy@xx_0) \text{ and } (xs@xx_0)) = (\lambda xx: a, xy: a: xx = xy@xy_{24})))) \Rightarrow \exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xz@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xz@xy_0) \Rightarrow \exists xy_{25}: a: (\lambda xx_0: a: (xx@xx_0 \text{ and } xy_0 = (xs@xx_0)) = (\lambda xx: a, xy: a: xx = xy@xy_{25}))))$  thf(cEQP1.1\_pme, conjecture)

**SEV037** $\wedge$ **5.p** TPS problem from EQUIVALENCE-RELATIONS-THMS

$a$ : \$tType    thf(a\_type, type)

$b$ : \$tType    thf(b\_type, type)

$\forall xp: a \rightarrow a \rightarrow \$o, xq: a \rightarrow a \rightarrow \$o, xr: a \rightarrow b \rightarrow b \rightarrow \$o: ((\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((xp@xx@xz) \text{ and } xp = xq) \Rightarrow (\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (\forall xx_0: b, xy_0: b: ((xr@xx@xx_0@xy_0) \Rightarrow (xr@xx@xy_0@xx_0)) \text{ and } \forall xx_0: b, xy_0: b, xz: b: ((xr@xx@xx_0@xy_0 \text{ and } xr@xx@xy_0@xz) \Rightarrow (xr@xx@xx_0@xz)) \text{ and } (xr@xx) = (xr@xy)))) \Rightarrow (\forall xx: a \rightarrow b, xy: a \rightarrow b: (\forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xx@xx_0)@(xy@xy_0))) \Rightarrow \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xy@xx_0)@(xx@xy_0)))) \text{ and } \forall xx: a \rightarrow b, xy: a \rightarrow b, xz: a \rightarrow b: ((\forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xx@xx_0)@(xy@xy_0))) \text{ and } \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xy@xx_0)@(xz@xy_0)))) \Rightarrow \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xx@xx_0)@(xz@xy_0)))) \text{ and } (\lambda xf: a \rightarrow b, xg: a \rightarrow b: \forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xr@xx@(xf@xx)@(xg@xy)))) = (\lambda xf: a \rightarrow b, xg: a \rightarrow b: \forall xx: a, xy: a: ((xq@xx@xy) \Rightarrow (xr@xx@(xf@xx)@(xg@xy))))))$     thf(cTHM516\_pme, conjecture)

### SEV038 $\wedge$ 5.p TPS problem from EQUIVALENCE-RELATIONS-THMS

$a$ : \$tType    thf(a\_type, type)

$\exists f: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow a \rightarrow a \rightarrow \$o: (\forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow (\forall xx: a: (f@p@xx@xx) \text{ and } \forall xx: a, xy: a: ((f@p@xy@xx) \text{ and } \forall xx: a, xy: a, xz: a: ((f@p@xx@xy \text{ and } f@p@xy@xz) \Rightarrow (f@p@xx@xz)))) \text{ and } \forall r: a \rightarrow a \rightarrow \$o: ((\forall xx: a: (r@xx@xx) \text{ and } \forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \Rightarrow \exists p: (a \rightarrow \$o) \rightarrow \$o: (\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp)) \text{ and } r = (f@p))) \text{ and } \forall t: (a \rightarrow \$o) \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow \$o: ((t \neq u \text{ and } \forall xp: a \rightarrow \$o: ((t@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (t@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((t@xq \text{ and } xq@xx) \Rightarrow xq = xp)) \text{ and } \forall xp: a \rightarrow \$o: ((u@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (u@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((u@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow (f@t) \neq (f@u)))$     thf(cTHM266\_pme, conjecture)

### SEV039 $\wedge$ 5.p TPS problem from EQUIVALENCE-RELATIONS-THMS

$a$ : \$tType    thf(a\_type, type)

$\exists f: (a \rightarrow a \rightarrow \$o) \rightarrow (a \rightarrow \$o) \rightarrow \$o: (\forall r: a \rightarrow a \rightarrow \$o: ((\forall xx: a: (r@xx@xx) \text{ and } \forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \Rightarrow (\forall xp: a \rightarrow \$o: ((f@r@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (f@r@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((f@r@xq \text{ and } xq@xx) \Rightarrow xq = xp)))) \text{ and } \forall p: (a \rightarrow \$o) \rightarrow \$o: ((\forall xp: a \rightarrow \$o: ((p@xp) \Rightarrow \exists xz: a: (xp@xz)) \text{ and } \forall xx: a: \exists xp: a \rightarrow \$o: (p@xp \text{ and } xp@xx \text{ and } \forall xq: a \rightarrow \$o: ((p@xq \text{ and } xq@xx) \Rightarrow xq = xp))) \Rightarrow \exists s: a \rightarrow a \rightarrow \$o: (\forall xx: a: (s@xx@xx) \text{ and } \forall xx: a, xy: a: (s@xy@xx) \text{ and } \forall xx: a, xy: a, xz: a: ((s@xx@xy \text{ and } s@xy@xz) \Rightarrow (s@xx@xz)) \text{ and } p = (f@s))) \text{ and } \forall t: a \rightarrow a \rightarrow \$o, u: a \rightarrow a \rightarrow \$o: ((t \neq u \text{ and } \forall xx: a: (t@xx@xx) \text{ and } \forall xx: a, xy: a: ((t@xx@xy) \Rightarrow (t@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((t@xx@xy \text{ and } t@xy@xz) \Rightarrow (t@xx@xz)) \text{ and } \forall xx: a: (u@xx@xx) \text{ and } \forall xx: a, xy: a: ((u@xx@xy) \Rightarrow (u@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((u@xx@xy \text{ and } u@xy@xz) \Rightarrow (f@t) \neq (f@u))))$     thf(cTHM265\_pme, conjecture)

### SEV040 $\wedge$ 5.p TPS problem from EQUIVALENCE-RELATIONS-THMS

$a$ : \$tType    thf(a\_type, type)

$\forall xx: a \rightarrow a \rightarrow \$o, xy: a \rightarrow a \rightarrow \$o: ((\forall xx_0: a, xy_0: a: ((xx@xx_0@xy_0) \Rightarrow (xx@xy_0@xx_0)) \text{ and } \forall xx_0: a, xy_0: a, xz: a: ((xx@xx_0@xy_0) \Rightarrow (xx@xx_0@xz)) \text{ and } xx = xy) \Rightarrow (\forall xx_0: a, xy_0: a: ((xy@xx_0@xy_0) \Rightarrow (xy@xy_0@xx_0)) \text{ and } \forall xx_0: a, xy_0: a, xz: a: ((xy@xx_0@xy_0) \Rightarrow (xy@xx_0@xz)) \text{ and } xy = xx)) \text{ and } \forall xx: a \rightarrow a \rightarrow \$o, xy: a \rightarrow a \rightarrow \$o, xz: a \rightarrow a \rightarrow \$o: ((\forall xx_0: a, xy_0: a: ((xx@xx_0@xy_0) \Rightarrow (xx@xy_0@xx_0)) \text{ and } \forall xx_0: a, xy_0: a, xz_0: a: ((xx@xx_0@xy_0 \text{ and } xx@xy_0@xz_0) \Rightarrow (xx@xx_0@xz_0)) \text{ and } xx = xy \text{ and } \forall xx_0: a, xy_0: a, xz_0: a: ((xy@xx_0@xy_0) \Rightarrow (xy@xy_0@xx_0)) \text{ and } \forall xx_0: a, xy_0: a, xz_0: a: ((xy@xx_0@xy_0 \text{ and } xy@xy_0@xz_0) \Rightarrow (xy@xx_0@xz_0)) \text{ and } xy = xz) \Rightarrow (\forall xx_0: a, xy_0: a: ((xx@xx_0@xy_0) \Rightarrow (xx@xy_0@xx_0)) \text{ and } \forall xx_0: a, xy_0: a, xz_0: a: ((xx@xx_0@xy_0 \text{ and } xx@xy_0@xz_0) \Rightarrow (xx@xx_0@xz_0)) \text{ and } xx = xz)) \text{ and } (\lambda xp: a \rightarrow a \rightarrow \$o, xq: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((xp@xx@xy \text{ and } xp@xy@xz) \Rightarrow (xp@xx@xz)) \text{ and } xp = xq)) = (\lambda xp: a \rightarrow a \rightarrow \$o, xq: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((xp@xx@xy \text{ and } xp@xy@xz) \Rightarrow (xp@xx@xz)) \text{ and } xp = xq))$     thf(cTHM515\_pme, conjecture)

### SEV041 $\wedge$ 5.p TPS problem from EQUIVALENCE-RELATIONS-THMS

$a$ : \$tType    thf(a\_type, type)

$b$ : \$tType    thf(b\_type, type)

$\forall xp: a \rightarrow a \rightarrow \$o, xq: a \rightarrow a \rightarrow \$o, xr: a \rightarrow b \rightarrow b \rightarrow \$o, xs: a \rightarrow b \rightarrow b \rightarrow \$o: ((\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx)) \text{ and } \forall xx: a, xy: a, xz: a: ((xp@xx@xy \text{ and } xp@xy@xz) \Rightarrow (xp@xx@xz)) \text{ and } xp = xq) \Rightarrow (\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx)) \text{ and } \forall xx_0: b, xy_0: b: ((xr@xx@xx_0@xy_0) \Rightarrow (xr@xx@xy_0@xx_0)) \text{ and } \forall xx_0: b, xy_0: b, xz: b: ((xr@xx@xx_0@xy_0 \text{ and } xr@xx@xy_0@xz) \Rightarrow (xr@xx@xx_0@xz)) \text{ and } (xr@xx) = (xr@xy))) \Rightarrow (\forall xx: a: ((xp@xx@xx) \Rightarrow (\forall xx_0: b, xy: b: ((xr@xx@xx_0@xy) \Rightarrow (xr@xx@xy@xx_0)) \text{ and } \forall xx_0: b, xy: b, xz: b: ((xr@xx@xx_0@xy \text{ and } xr@xx@xy@xz) \Rightarrow (xr@xx@xx_0@xz)) \text{ and } (xr@xx) = (xs@xx)))) \Rightarrow (\forall xx: a \rightarrow b, xy: a \rightarrow b: (\forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xx@xx_0)@(xy@xy_0))) \Rightarrow \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xy@xx_0)@(xx@xy_0)))) \text{ and } \forall xx: a \rightarrow b, xy: a \rightarrow b, xz: a \rightarrow b: ((\forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xx@xx_0)@(xy@xy_0))) \text{ and } \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xy@xx_0)@(xz@xy_0)))) \Rightarrow \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xx@xx_0)@(xz@xy_0))))$     thf(cTHM515\_pme, conjecture)

$(xr@xx_0@(xy@xx_0)@(xz@xy_0))) \Rightarrow \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xr@xx_0@(xx@xx_0)@(xz@xy_0)))$  and  $(\lambda x f: a \rightarrow b, xg: a \rightarrow b: \forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xr@xx@(xf@xx)@(xg@xy))) = (\lambda x f: a \rightarrow b, xg: a \rightarrow b: \forall xx: a, xy: a: ((xq@xx@xy) \Rightarrow (xs@xx@(xf@xx)@(xg@xy))))))$  thf(cTHM517\_pme, conjecture)

**SEV042**<sup>5.p</sup> TPS problem THM600

Existence of a symmetric, transitive closure (PER closure).

$\forall xr: \$i \rightarrow \$i \rightarrow \$o: \exists xs: \$i \rightarrow \$i \rightarrow \$o: (\forall xa: \$i, xb: \$i: ((xr@xa@xb) \Rightarrow (xs@xa@xb))$  and  $\forall xx: \$i, xy: \$i: ((xs@xx@xy) \Rightarrow (xs@xy@xx))$  and  $\forall xx: \$i, xy: \$i, xz: \$i: ((xs@xx@xy$  and  $xs@xy@xz) \Rightarrow (xs@xx@xz))$  and  $\forall xt: \$i \rightarrow \$i \rightarrow \$o: ((\forall xa: \$i, xb: \$i: ((xr@xa@xb) \Rightarrow (xt@xa@xb))$  and  $\forall xx: \$i, xy: \$i: ((xt@xx@xy) \Rightarrow (xt@xy@xx))$  and  $\forall xx: \$i, xy: \$i, xz: \$i: ((xt@xx@xy$  and  $xt@xy@xz) \Rightarrow (xt@xx@xz)) \Rightarrow \forall xa: \$i, xb: \$i: ((xs@xa@xb) \Rightarrow (xt@xa@xb)))$  thf(cTHM600\_pme, conjecture)

**SEV043**<sup>5.p</sup> TPS problem from PERS-THMS

$a: \$tType$  thf(a\_type, type)

$\forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx))$  and  $\forall xx: a, xy: a, xz: a: ((xp@xx@xy$  and  $xp@xy@xz) \Rightarrow (xp@xx@xz)) \Rightarrow \forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xx@xx)))$  thf(cTHM510\_pme, conjecture)

**SEV044**<sup>5.p</sup> TPS problem from PERS-THMS

$b: \$tType$  thf(b\_type, type)

$a: \$tType$  thf(a\_type, type)

$\forall xs: b \rightarrow \$o, xp: b \rightarrow a \rightarrow a \rightarrow \$o: (\forall xx: b: ((xs@xx) \Rightarrow (\forall xx_0: a, xy: a: ((xp@xx@xx_0@xy) \Rightarrow (xp@xx@xy@xx_0))$  and  $\forall xx_0: (xp@xx@xx_0@xz))) \Rightarrow (\forall xx: b \rightarrow a, xy: b \rightarrow a: (\forall xx_0: b: ((xs@xx_0) \Rightarrow (xp@xx_0@(xx@xx_0)@(xy@xx_0))) \Rightarrow \forall xx_0: b: ((xs@xx_0) \Rightarrow (xp@xx_0@(xy@xx_0)@(xx@xx_0))))$  and  $\forall xx: b \rightarrow a, xy: b \rightarrow a, xz: b \rightarrow a: ((\forall xx_0: b: ((xs@xx_0) \Rightarrow (xp@xx_0@(xx@xx_0)@(xy@xx_0)))$  and  $\forall xx_0: b: ((xs@xx_0) \Rightarrow (xp@xx_0@(xy@xx_0)@(xz@xx_0)))) \Rightarrow \forall xx_0: b: ((xs@xx_0) \Rightarrow (xp@xx_0@(xx@xx_0)@(xz@xx_0))))$  thf(cTHM506\_pme, conjecture)

**SEV045**<sup>5.p</sup> TPS problem from PERS-THMS

$a: \$tType$  thf(a\_type, type)

$b: \$tType$  thf(b\_type, type)

$g: a \rightarrow b$  thf(g, type)

$f: a \rightarrow b$  thf(f, type)

$cQ: a \rightarrow b \rightarrow b \rightarrow \$o$  thf(cQ, type)

$cP: a \rightarrow a \rightarrow \$o$  thf(cP, type)

$\forall xx: a: ((cP@xx@xx) \Rightarrow (cQ@xx@(f@xx)@(g@xx))) \Rightarrow (\forall xx: a, xy: a: ((cP@xx@xy) \Rightarrow (cQ@xx@(f@xx)@(g@xy))) \Rightarrow ((\forall xx: a, xy: a: ((cP@xx@xy) \Rightarrow (cP@xy@xx))$  and  $\forall xx: a, xy: a, xz: a: ((cP@xx@xy$  and  $cP@xy@xz) \Rightarrow (cP@xx@xz)) \Rightarrow ((\forall xx: a: ((cP@xx@xx) \Rightarrow (\forall xx_0: b, xy: b: ((cQ@xx@xx_0@xy) \Rightarrow (cQ@xx@xy@xx_0))$  and  $\forall xx_0: b, xy: b, xz: b: ((cQ@xx@xx_0@xz)))$  and  $\forall xx: a, xy: a: ((cP@xx@xy) \Rightarrow (cQ@xx) = (cQ@xy))) \Rightarrow \forall xx: a, xy: a: ((cP@xx@xy) \Rightarrow (cQ@xx@(f@xx)@(g@xy))))$  thf(cTHM509\_pme, conjecture)

**SEV046**<sup>5.p</sup> TPS problem from PERS-THMS

$a: \$tType$  thf(a\_type, type)

$b: \$tType$  thf(b\_type, type)

$\forall xp: a \rightarrow a \rightarrow \$o, xp_2: a \rightarrow b \rightarrow b \rightarrow \$o: ((\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp@xy@xx))$  and  $\forall xx: a, xy: a, xz: a: ((xp@xx@xy) \Rightarrow (xp@xx@xz))$  and  $\forall xx: a: ((xp@xx@xx) \Rightarrow (\forall xx_0: b, xy: b: ((xp_2@xx@xx_0@xy) \Rightarrow (xp_2@xx@xy@xx_0))$  and  $\forall xx_0: b, xy: b, xz: b: ((xp_2@xx@xx_0@xz)))$  and  $\forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xp_2@xx) = (xp_2@xy))) \Rightarrow (\forall xx: a \rightarrow b, xy: a \rightarrow b: (\forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xp_2@xx_0@(xx@xx_0)@(xy@xy_0))) \Rightarrow \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xp_2@xx_0@(xy@xx_0)@(xx@xy_0))))$  and  $\forall xx: a \rightarrow b, xy: a \rightarrow b, xz: a \rightarrow b: ((\forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xp_2@xx_0@(xx@xx_0)@(xy@xy_0)))$  and  $\forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xp_2@xx_0@(xy@xx_0)@(xz@xy_0)))) \Rightarrow \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xp_2@xx_0@(xx@xx_0)@(xz@xy_0))))$  thf(cTHM507\_pme, conjecture)

**SEV047**<sup>5.p</sup> TPS problem THM175

Reflexivity of subrelation.

$\forall xs: \$i \rightarrow \$i \rightarrow \$o, xx: \$i, xy: \$i: ((xs@xx@xy) \Rightarrow (xs@xx@xy))$  thf(cTHM175\_pme, conjecture)

**SEV048**<sup>5.p</sup> TPS problem THM120

There exists a transitive relation on sets.

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy$  and  $r@xy@xz) \Rightarrow (r@xx@xz))$  thf(cTHM120\_pme, conjecture)

**SEV049**<sup>5.p</sup> TPS problem THM120A

Variant of THM120 designed to eliminate trivial proof. Subset is one such relation.

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\neg r@lxx: \$i: \$false$  and  $\forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy$  and  $r@xy@xz) \Rightarrow (r@xx@xz)))$  thf(cTHM120A\_pme, conjecture)

**SEV050**<sup>5.p</sup> TPS problem THM599

Existence of reflexive closure.

$a: \$tType \quad thf(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o: \exists xs: a \rightarrow a \rightarrow \$o: (\forall xa: a, xb: a: ((xr@xa@xb) \Rightarrow (xs@xa@xb)) \text{ and } \forall xx: a: (xs@xx@xx) \text{ and } \forall xt: a \rightarrow a \rightarrow \$o: ((\forall xa: a, xb: a: ((xr@xa@xb) \Rightarrow (xt@xa@xb)) \text{ and } \forall xx: a: (xt@xx@xx)) \Rightarrow \forall xa: a, xb: a: ((xs@xa@xb) \Rightarrow (xt@xa@xb)))) \quad thf(cTHM599\_pme, conjecture)$

**SEV051** $\wedge$ **5.p** TPS problem THM557

Equality is an LC-relation.

$a: \$tType \quad thf(a\_type, type)$

$\forall xx: a: xx = xx \text{ and } \forall xu: a, xv: a, xw: a: ((xu = xw \text{ and } xv = xw) \Rightarrow xu = xv) \quad thf(cTHM557\_pme, conjecture)$

**SEV052** $\wedge$ **5.p** TPS problem THM120B

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\neg r@\lambda xx: \$i: \$true@\lambda xx: \$i: \$false \text{ and } \forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \quad thf(cTHM120B\_pme, conjecture)$

**SEV053** $\wedge$ **5.p** TPS problem THM89B

$b: \$tType \quad thf(b\_type, type)$

$a: \$tType \quad thf(a\_type, type)$

$cF: b \rightarrow b \quad thf(cF, type)$

$cA: b \rightarrow a \quad thf(cA, type)$

$cL: a \rightarrow a \rightarrow \$o \quad thf(cL, type)$

$(\forall xx: a, xy: a, xz: a: ((cL@xx@xy \text{ and } cL@xy@xz) \Rightarrow (cL@xx@xz)) \text{ and } \forall x: b: (cL@(cA@x)@(cA@(cF@x)))) \Rightarrow \forall y: b: (cL@(cA@y)@(cA@(cF@y))) \quad thf(cTHM89B\_pme, conjecture)$

**SEV054** $\wedge$ **5.p** TPS problem THM403

$a: \$tType \quad thf(a\_type, type)$

$\forall r: a \rightarrow a \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow a: ((\forall xx: a, xy: a, xz: a: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz)) \text{ and } \forall xs: a \rightarrow \$o: (\forall xz: a: ((xs@xz) \Rightarrow (r@xz@(u@xs))) \text{ and } \forall xj: a: (\forall xk: a: ((xs@xk) \Rightarrow (r@xk@xj)) \Rightarrow (r@(u@xs)@xj)))) \Rightarrow \forall xf: a \rightarrow a: (\forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@(xf@xx)@(xf@xy))) \Rightarrow \exists xw: a: (r@(xf@xw)@xw))) \quad thf(cTHM403\_pme, conjecture)$

**SEV055** $\wedge$ **5.p** TPS problem THM402

$a: \$tType \quad thf(a\_type, type)$

$\forall r: a \rightarrow a \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow a: ((\forall xx: a, xy: a, xz: a: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz)) \text{ and } \forall xs: a \rightarrow \$o: (\forall xz: a: ((xs@xz) \Rightarrow (r@xz@(u@xs))) \text{ and } \forall xj: a: (\forall xk: a: ((xs@xk) \Rightarrow (r@xk@xj)) \Rightarrow (r@(u@xs)@xj)))) \Rightarrow \forall xf: a \rightarrow a: (\forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@(xf@xx)@(xf@xy))) \Rightarrow \exists xw: a: (r@xw@(xf@xw))) \quad thf(cTHM402\_pme, conjecture)$

**SEV056** $\wedge$ **5.p** TPS problem THM275

$a: \$tType \quad thf(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o: \exists xp: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xp@xx@xy)) \text{ and } \forall xx: a, xy: a, xz: a: ((xp@xx@xy \text{ and } xp@xx@xz)) \Rightarrow (xp@xx@xz)) \quad thf(cTHM275\_pme, conjecture)$

**SEV057** $\wedge$ **5.p** TPS problem EQP1-1A

$a: \$tType \quad thf(a\_type, type)$

$\forall xx: a \rightarrow \$o: \exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xx@(xs@xx_0))) \text{ and } \forall xy: a: ((xx@xy) \Rightarrow \exists xx_0: a: (xx@xx_0 \text{ and } xy = (xs@xx_0) \text{ and } \forall xz: a: ((xx@xz \text{ and } xy = (xs@xz)) \Rightarrow xz = xx_0))) \quad thf(cEQP1\_1A\_pme, conjecture)$

**SEV058** $\wedge$ **5.p** TPS problem THM122

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\forall xu: \$i \rightarrow \$o, xv: \$i \rightarrow \$o: ((r@xu@xv) \Rightarrow \forall xz: \$i: ((xu@xz) \Rightarrow (xv@xz))) \text{ and } \forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \quad thf(cTHM122\_pme, conjecture)$

**SEV059** $\wedge$ **5.p** TPS problem THM89A

$b: \$tType \quad thf(b\_type, type)$

$a: \$tType \quad thf(a\_type, type)$

$cG: b \rightarrow b \quad thf(cG, type)$

$cA: b \rightarrow a \quad thf(cA, type)$

$c\_less.: a \rightarrow a \rightarrow \$o \quad thf(c\_less., type)$

$cF: b \rightarrow b \quad thf(cF, type)$

$(\forall xx: a, xy: a, xz: a: ((c\_less\_@xx@xy \text{ and } c\_less\_@xy@xz) \Rightarrow (c\_less\_@xx@xz)) \text{ and } \forall x: b: (c\_less\_@(cA@x)@(cA@(cF@x))) \text{ and } (\lambda z: b: (cF@(cF@z)))) \Rightarrow \forall y: b: (c\_less\_@(cA@y)@(cA@(cG@y))) \quad thf(cTHM89A\_pme, conjecture)$

**SEV060** $\wedge$ **5.p** TPS problem THM173

$b: \$tType \quad thf(b\_type, type)$

$a: \$tType \quad thf(a\_type, type)$

$\forall xx: b, xy: a, xs: b \rightarrow a \rightarrow \$o, xk: b \rightarrow a \rightarrow \$o: ((\forall xx_0: b, xy_0: a: ((xk@xx_0@xy_0) \Rightarrow (xs@xx_0@xy_0 \text{ or } (xx_0 = xx \text{ and } xy_0 = xy))) \text{ and } \neg xk@xx@xy) \Rightarrow \forall xx_0: b, xy_0: a: ((xk@xx_0@xy_0) \Rightarrow (xs@xx_0@xy_0))) \quad thf(cTHM173\_pme, conjecture)$



**SEV061** $\wedge$ **5.p** TPS problem THM176

$b$ : \$tType thf(b\_type, type)

$a$ : \$tType thf(a\_type, type)

$\forall xx: b, xy: a, xs: b \rightarrow a \rightarrow \$o, xk: b \rightarrow a \rightarrow \$o: (\forall xx_2: b, xy_{47}: a: ((xk@xx_2@xy_{47}) \Rightarrow (xs@xx_2@xy_{47} \text{ or } (xx_2 = xx \text{ and } xy_{47} = xy))) \Rightarrow \forall xx_3: b, xy_{48}: a: ((xk@xx_3@xy_{48} \text{ and } \neg xx_3 = xx \text{ and } xy_{48} = xy) \Rightarrow (xs@xx_3@xy_{48})))$  thf(cTHM176\_pme, conjecture)

**SEV062** $\wedge$ **5.p** TPS problem T146A

$\forall xr: \$i \rightarrow \$i \rightarrow \$o, xx: \$i, xy: \$i: (\forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xu: \$i, xv: \$i: ((xp@xu@xw) \Rightarrow (xp@xx@xy)) \Rightarrow (xp@xx@xy))) \Rightarrow \forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xx_0: \$i, (xp@xx_0@xz))) \Rightarrow (xp@xx@xy)))$  thf(cT146A\_pme, conjecture)

**SEV063** $\wedge$ **5.p** TPS problem THM136

The transitive closure of a relation is transitive.

$a$ : \$tType thf(a\_type, type)

$\forall xr: a \rightarrow a \rightarrow \$o, xx: a, xy: a, xz: a: ((\forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx_0: a, xy_0: a: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xx_0: a, xy_0: a: ((xp@xx_0@xz_0) \Rightarrow (xp@xx@xy)) \text{ and } \forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx_0: a, xy_0: a: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xx_0: a, xy_0: a: ((xp@xx_0@xz_0) \Rightarrow (xp@xy@xz))) \Rightarrow \forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx_0: a, xy_0: a: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xx_0: a, xy_0: a: ((xp@xx_0@xz_0) \Rightarrow (xp@xx@xz)))$  thf(cTHM136\_pme, conjecture)

**SEV064** $\wedge$ **5.p** TPS problem THM120C

$\exists xr_{28}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o, xr_{27}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\neg \forall xw_2: \$i: (xr_{27}@xw_2@xw_2 \text{ or } xr_{28}@xw_2@xw_2) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((\forall xw_2: \$i: (xr_{27}@xx@xy@xw_2 \text{ or } \forall xw_2: \$i: (xr_{27}@xx@xz@xw_2 \text{ or } xr_{28}@xx@xz@xw_2)))$  thf(cTHM120C\_pme, conjecture)

**SEV065** $\wedge$ **5.p** TPS problem THM177

$b$ : \$tType thf(b\_type, type)

$a$ : \$tType thf(a\_type, type)

$\forall xx: b, xy: a, xk: b \rightarrow a \rightarrow \$o: ((xk@xx@xy) \Rightarrow (\lambda xu: b, xv: a: ((xk@xu@xv \text{ and } \neg xu = xx \text{ and } xv = xy) \text{ or } (xu = xx \text{ and } xv = xy))) = xk)$  thf(cTHM177\_pme, conjecture)

**SEV066** $\wedge$ **5.p** TPS problem THM120D

$\exists xr_{28}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o, xr_{27}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\forall xx: \$i \rightarrow \$o, xw_2: \$i: (xr_{27}@xx@xx@xw_2 \text{ or } xr_{28}@xx@xx@xw_2) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((\forall xw_2: \$i: (xr_{27}@xx@xy@xw_2 \text{ or } xr_{28}@xx@xy@xw_2) \text{ and } \forall xw_2: \$i: (xr_{27}@xy@xz@xw_2 \text{ or } xr_{28}@xy@xz@xw_2) \text{ and } \neg \forall xw_2: \$i: (xr_{27}@xw_2@xw_2 \text{ or } xr_{28}@xw_2@xw_2)))$  thf(cTHM120D\_pme, conjecture)

**SEV067** $\wedge$ **5.p** TPS problem THM553

Downward closed subsets of a linear order are comparable.

$a$ : \$tType thf(a\_type, type)

$cS: a \rightarrow \$o$  thf(cS, type)

$cT: a \rightarrow \$o$  thf(cT, type)

$cR: a \rightarrow a \rightarrow \$o$  thf(cR, type)

$(\forall xx: a, xy: a, xz: a: ((cR@xx@xy \text{ and } cR@xy@xz) \Rightarrow (cR@xx@xz)) \text{ and } \forall xx: a: (cR@xx@xx) \text{ and } \forall xx: a, xy: a: ((cR@xx@xy \text{ and } xx = xy) \text{ and } \forall xx: a, xy: a: (cR@xx@xy \text{ or } cR@xy@xx) \text{ and } \forall xu: a, xv: a: ((cR@xu@xv \text{ and } cS@xv) \Rightarrow (cS@xu)) \text{ and } \forall xu: a, xv: a: ((cT@xu) \Rightarrow (cS@xu))) \Rightarrow (\forall xx: a: ((cS@xx) \Rightarrow (cT@xx)) \text{ or } \forall xx: a: ((cT@xx) \Rightarrow (cS@xx)))$  thf(cTHM553\_pme, conjecture)

**SEV068** $\wedge$ **5.p** TPS problem THM275A-1

$a$ : \$tType thf(a\_type, type)

$\forall xr: a \rightarrow a \rightarrow \$o: \exists xp: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xp@xx@xy)) \text{ and } \forall xx: a, xy: a, xz: a: ((xp@xx@xy \text{ and } xp@xx@xz) \Rightarrow (xp@xx@xy)) \text{ and } \forall xq: a \rightarrow a \rightarrow \$o: ((\forall xx: a, xy: a, xz: a: ((xq@xx@xy \text{ and } xq@xy@xz) \Rightarrow (xq@xx@xz)) \text{ and } \forall xx: a, xy: a: ((xq@xx@xy) \Rightarrow \forall xx: a, xy: a: ((xp@xx@xy) \Rightarrow (xq@xx@xy))))$  thf(cTHM275A\_1\_pme, conjecture)

**SEV069** $\wedge$ **6.p** TPS problem THM575

Existence of transitive closure.

$\forall xr: \$i \rightarrow \$i \rightarrow \$o: \exists xs: \$i \rightarrow \$i \rightarrow \$o: (\forall xa: \$i, xb: \$i: ((xr@xa@xb) \Rightarrow (xs@xa@xb)) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i: ((xs@xx@xy \text{ and } xs@xx@xz) \Rightarrow (xs@xx@xz)) \text{ and } \forall xt: \$i \rightarrow \$i \rightarrow \$o: ((\forall xa: \$i, xb: \$i: ((xr@xa@xb) \Rightarrow (xt@xa@xb)) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i: ((xt@xx@xy \text{ and } xt@xx@xz) \Rightarrow \forall xa: \$i, xb: \$i: ((xs@xa@xb) \Rightarrow (xt@xa@xb))))$  thf(cTHM575\_pme, conjecture)

**SEV070** $\wedge$ **5.p** TPS problem THM577

Inductive defn of  $\leq$  on naturals is transitive.

$cS: \$i \rightarrow \$i$  thf(cS, type)

$\forall xx: \$i, xy: \$i, xz: \$i: ((\forall xp: \$i \rightarrow \$o: ((xp@xx \text{ and } \forall xn: \$i: ((xp@xn) \Rightarrow (xp@(cS@xn)))) \Rightarrow (xp@xy)) \text{ and } \forall xp: \$i \rightarrow \$o: ((xp@xy \text{ and } \forall xn: \$i: ((xp@xn) \Rightarrow (xp@(cS@xn)))) \Rightarrow (xp@xz))) \Rightarrow \forall xp: \$i \rightarrow \$o: ((xp@xx \text{ and } \forall xn: \$i: ((xp@xn) \Rightarrow (xp@(cS@xn)))) \Rightarrow (xp@xz)))$  thf(cTHM577\_pme, conjecture)

**SEV071** $\wedge$ **5.p** TPS problem THM576

Existence of symmetric closure of a relation.

$a: \$tType \quad thf(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o: \exists xs: a \rightarrow a \rightarrow \$o: (\forall xa: a, xb: a: ((xr@xa@xb) \Rightarrow (xs@xa@xb)) \text{ and } \forall xx: a, xy: a: ((xs@xx@xy) \Rightarrow (xs@xy@xx)) \text{ and } \forall xt: a \rightarrow a \rightarrow \$o: ((\forall xa: a, xb: a: ((xr@xa@xb) \Rightarrow (xt@xa@xb)) \text{ and } \forall xx: a, xy: a: ((xt@xx@xy) \Rightarrow (xt@xy@xx))) \Rightarrow \forall xa: a, xb: a: ((xs@xa@xb) \Rightarrow (xt@xa@xb)))) \quad thf(cTHM576\_pme, conjecture)$

**SEV072** $\wedge$ **5.p** TPS problem THM522

Theorem about symmetric closure of relations.

$a: \$tType \quad thf(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o, xx: a, xy: a: ((xr@xx@xy \text{ or } xr@xy@xx) \iff \forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx_0: a, xy_0: a: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xx_0: a, xy_0: a: ((xp@xx_0@xy_0) \Rightarrow (xp@xy_0@xx_0))) \Rightarrow (xp@xx@xy))) \quad thf(cTHM522\_pme, conjecture)$

**SEV074** $\wedge$ **5.p** TPS problem THM523

Theorem about reflexive closure of relations.

$a: \$tType \quad thf(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o, xx: a, xy: a: ((xr@xx@xy \text{ or } xx = xy) \iff \forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx_0: a, xy_0: a: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xx_0: a: (xp@xx_0@xx_0) \Rightarrow (xp@xx@xy))) \quad thf(cTHM523\_pme, conjecture)$

**SEV075** $\wedge$ **5.p** TPS problem THM152

Equivalence of two definitions of transitive closure.

$\forall xr: \$i \rightarrow \$i \rightarrow \$o, xx: \$i, xy: \$i: (\forall xq: \$i \rightarrow \$o: ((\forall xw: \$i: ((xr@xx@xw) \Rightarrow (xq@xw)) \text{ and } \forall xv: \$i, xw: \$i: ((xq@xv \text{ and } xr@xv) \Rightarrow (xq@xw))) \Rightarrow (xq@xy)) \iff \forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xx_0: \$i, xy_0: \$i: (xp@xx_0@xz)) \Rightarrow (xp@xx@xy))) \quad thf(cTHM152\_pme, conjecture)$

**SEV076** $\wedge$ **5.p** TPS problem THM401B

In a complete lattice, every set has an upper bound.

$a: \$tType \quad thf(a\_type, type)$

$\forall rRR: a \rightarrow a \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow a: ((\forall xx: a, xy: a, xz: a: ((rRR@xx@xy \text{ and } rRR@xy@xz) \Rightarrow (rRR@xx@xz)) \text{ and } \forall xs: a \rightarrow \$o: (\forall xz: a: ((xs@xz) \Rightarrow (rRR@xz@(u@xs))) \text{ and } \forall xj: a: (\forall xk: a: ((xs@xk) \Rightarrow (rRR@xk@xj)) \Rightarrow (rRR@(u@xs)@xj)))) \Rightarrow \forall xs: a \rightarrow \$o: \exists xb: a: \forall xz: a: ((xs@xz) \Rightarrow (rRR@xz@xb))) \quad thf(cTHM401B\_pme, conjecture)$

**SEV079** $\wedge$ **5.p** TPS problem from RELN-THMS

$\forall x: \$i, y: \$i, z: \$i: ((x = y \text{ and } y = z) \Rightarrow x = z) \quad thf(cTRANS\_ID\_pme, conjecture)$

**SEV080** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall xx: a \rightarrow \$o: \exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xx@(xs@xx_0))) \text{ and } \forall xy: a: ((xx@xy) \Rightarrow \exists xy_{28}: a: (\lambda xx_0: a: (xx@xx_0 \text{ and } (xs@xx_0))) = (\lambda xx: a, xy: a: xx = xy@xy_{28}))) \quad thf(cEQP\_1A\_pme, conjecture)$

**SEV081** $\wedge$ **5.p** TPS problem from RELN-THMS

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\exists x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (r@x@y) \text{ and } \forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \quad thf(cTHM120\_1\_pme, conjecture)$

**SEV082** $\wedge$ **5.p** TPS problem from RELN-THMS

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\neg r@lxx: \$i: \$true@lxx: \$i: \$false \text{ and } \forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \text{ or } \$false \quad thf(cTHM120\_4\_pme, conjecture)$

**SEV083** $\wedge$ **5.p** TPS problem from RELN-THMS

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\neg r@lxx: \$i: \$true@lxx: \$i: \$false \text{ and } \forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \text{ and } \$true \quad thf(cTHM120\_3\_pme, conjecture)$

**SEV084** $\wedge$ **5.p** TPS problem from RELN-THMS

$cP: \$o \quad thf(cP, type)$

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\neg r@lxx: \$i: (cP \text{ or } \neg cP)@lxx: \$i: (cP \text{ and } \neg cP) \text{ and } \forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \quad thf(cTHM120J\_pme, conjecture)$

**SEV085** $\wedge$ **5.p** TPS problem from RELN-THMS

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\exists x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (r@x@y) \text{ and } \exists u: \$i \rightarrow \$o, v: \$i \rightarrow \$o: \neg r@u@v \text{ and } \forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz))) \quad thf(cTHM120\_2\_pme, conjecture)$

**SEV086** $\wedge$ **5.p** TPS problem from RELN-THMS

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz)) \text{ and } \forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: ((r@x@y \text{ and } r@y@x) \Rightarrow \forall xx: \$i: ((x@xx) \iff (y@xx)))) \quad thf(cTHM120L\_pme, conjecture)$

**SEV087** $\wedge$ **5.p** TPS problem from RELN-THMS

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\neg r@_{\lambda xx}: \$i: \$true@_{\lambda xx}: \$i: \$false \text{ and } \forall xx: \$i \rightarrow \$o: (r@_{xx@xx}) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@_{xx@xy} \text{ and } r@_{xy@xz}) \Rightarrow (r@_{xx@xz})) \text{ and } \forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: ((r@_x@y \text{ and } r@_y@x) \Rightarrow (x = y)))$  thf(cTHM120H\_pme, conjecture)

**SEV088** $\wedge$ **5.p** TPS problem from RELN-THMS

$\exists xr_{28}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o, xr_{27}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((\forall xw_2: \$i: (xr_{27}@_{xx@xy@xw_2} \text{ or } xr_{28}@_{xx@xy@xw_2}) \text{ and } \forall xw_2: \$i: (xr_{27}@_{xy@xz@xw_2} \text{ or } xr_{28}@_{xy@xz@xw_2}) \Rightarrow (\forall xw_2: \$i: (xr_{27}@_{xx@xz@xw_2} \text{ or } xr_{28}@_{xx@xz@xw_2}))) \text{ and } \forall xx: \$i \rightarrow \$o, xw_2: \$i: (xr_{27}@_{xx@xx@xw_2} \text{ or } xr_{28}@_{xx@xx@xw_2})))$

**SEV089** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$  thf(a\_type, type)  
 $\forall xx: a \rightarrow \$o, xy: a \rightarrow \$o: (\exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xy@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xy@xy_0) \Rightarrow (\exists xy_{38}: a: (\lambda xx_0: a: (xx@xx_0 \text{ and } xy_0 = (xs@xx_0))) = (\lambda xx: a, xy: a: xx = xy@xy_{38}))) \Rightarrow \exists xs: a \rightarrow a: (\forall xx_0: a: ((xy@xx_0) \Rightarrow (xx@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xx@xy_0) \Rightarrow (\exists xy_{39}: a: (\lambda xx_0: a: (xy@xx_0 \text{ and } xy_0 = (xs@xx_0))) = (\lambda xx: a, xy: a: xx = xy@xy_{39}))))))$  thf(cEQP\_1B\_pme, conjecture)

**SEV090** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$  thf(a\_type, type)  
 $\exists xr: a \rightarrow a \rightarrow \$o: (\forall xx: a: \exists xw: a: (xr@_{xx@xw}) \text{ and } \forall xx: a: \neg xr@_{xx@xx} \text{ and } \forall xx: a, xy: a, xz: a: ((xr@_{xx@xy} \text{ and } xr@_{xy@xz}) \Rightarrow (xr@_{xx@xz}))) \Rightarrow \exists r: (a \rightarrow \$o) \rightarrow (a \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow \$o: \exists xw: a \rightarrow \$o: (r@_{xx@xw}) \text{ and } \forall xx: a \rightarrow \$o: \neg r@_{xx@xx} \text{ and } \forall xx: a \rightarrow \$o, xy: a \rightarrow \$o, xz: a \rightarrow \$o: ((r@_{xx@xy} \text{ and } r@_{xy@xz}) \Rightarrow (r@_{xx@xz})))$  thf(cTHM165\_pme)

**SEV091** $\wedge$ **5.p** TPS problem from RELN-THMS

$cQ: \$i \rightarrow \$i \rightarrow \$o$  thf(cQ, type)  
 $cP: \$i \rightarrow \$i \rightarrow \$o$  thf(cP, type)  
 $(\forall xx: \$i, xy: \$i, xz: \$i: ((cP@_{xx@xy} \text{ and } cP@_{xy@xz}) \Rightarrow (cP@_{xx@xz})) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i: ((cQ@_{xx@xy} \text{ and } cQ@_{xy@xz}) \Rightarrow (cQ@_{xx@xz})) \text{ and } \forall xx: \$i, xy: \$i: ((cQ@_{xx@xy}) \Rightarrow (cQ@_{xy@xx})) \text{ and } \forall xx: \$i, xy: \$i: (cP@_{xx@xy} \text{ or } cQ@_{xx@xy}) \Rightarrow (\forall xx: \$i, xy: \$i: (cP@_{xx@xy}) \text{ or } \forall xx: \$i, xy: \$i: (cQ@_{xx@xy})))$  thf(cCADE13\_pme, conjecture)

**SEV092** $\wedge$ **5.p** TPS problem from RELN-THMS

$\forall xr: \$i \rightarrow \$i \rightarrow \$o, xx: \$i, xy: \$i: (\forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@_{xx_0@xy_0}) \Rightarrow (xp@_{xx_0@xy_0})) \text{ and } \forall xx_0: \$i, xy_0: \$i: (xp@_{xx_0@xz})) \Rightarrow (xp@_{xx@xy})) \Rightarrow \forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@_{xx_0@xy_0}) \Rightarrow (xp@_{xx_0@xy_0})) \text{ and } \forall xu: \$i, xv: \$i: (xp@_{xu@xw})) \Rightarrow (xp@_{xx@xy})))$  thf(cT146B\_pme, conjecture)

**SEV093** $\wedge$ **5.p** TPS problem from RELN-THMS

$\forall xr: \$i \rightarrow \$i \rightarrow \$o, xx: \$i, xy: \$i: (\forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@_{xx_0@xy_0}) \Rightarrow (xp@_{xx_0@xy_0})) \text{ and } \forall xu: \$i, xv: \$i: (xp@_{xu@xw})) \Rightarrow (xp@_{xx@xy})) \iff \forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@_{xx_0@xy_0}) \Rightarrow (xp@_{xx_0@xy_0})) \text{ and } \forall xx_0: \$i, xy_0: \$i: (xp@_{xx_0@xz})) \Rightarrow (xp@_{xx@xy})))$  thf(cT146\_pme, conjecture)

**SEV094** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$  thf(a\_type, type)  
 $\forall xx: a \rightarrow \$o, xy: a \rightarrow \$o: (\exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xy@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xy@xy_0) \Rightarrow (\exists xx_0: a: (xx@xx_0 \text{ and } xy_0 = (xs@xx_0) \text{ and } \forall xz: a: ((xx@xz \text{ and } xy_0 = (xs@xz)) \Rightarrow xz = xx_0)))))) \Rightarrow \exists xs: a \rightarrow a: (\forall xx_0: a: ((xy@xx_0) \Rightarrow (xx@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xx@xy_0) \Rightarrow (\exists xx_0: a: (xy@xx_0 \text{ and } xy_0 = (xs@xx_0) \text{ and } \forall xz: a: ((xs@xz) \Rightarrow xz = xx_0))))))$  thf(cEQP1\_1B\_pme, conjecture)

**SEV095** $\wedge$ **5.p** TPS problem from RELN-THMS

$\exists xr_{28}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o, xr_{27}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\forall xu: \$i \rightarrow \$o, xv: \$i \rightarrow \$o: (\forall xw_2: \$i: (xr_{27}@_{xu@xv@xw_2} \text{ or } xr_{28}@_{xu@xv@xw_2}) \Rightarrow \forall xz: \$i: ((xu@xz) \Rightarrow (xv@xz))) \text{ and } \forall xx: \$i \rightarrow \$o, xw_2: \$i: (xr_{27}@_{xx@xx@xw_2} \text{ or } xr_{28}@_{xx@xx@xw_2}) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((\forall xw_2: \$i: (xr_{27}@_{xx@xy@xw_2} \text{ or } xr_{28}@_{xx@xy@xw_2}) \Rightarrow (\forall xw_2: \$i: (xr_{27}@_{xx@xz@xw_2} \text{ or } xr_{28}@_{xx@xz@xw_2})))$  thf(cTHM122C\_pme, conjecture)

**SEV096** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$  thf(a\_type, type)  
 $b: \$tType$  thf(b\_type, type)  
 $z: a$  thf(z, type)  
 $cR: a \rightarrow a \rightarrow \$o$  thf(cR, type)  
 $f: a \rightarrow b \rightarrow \$o$  thf(f, type)  
 $cS: b \rightarrow b \rightarrow \$o$  thf(cS, type)  
 $(\forall xu: a, xv: a, xw: a: ((cR@_{xu@xv} \text{ and } cR@_{xw@xv}) \Rightarrow (cR@_{xu@xw})) \text{ and } \forall xx: a: (cR@_{xx@xx}) \Rightarrow ((\forall xx: a: \exists xy: b: (f@_{xx@xy} \text{ and } cS@_{xy_1@xy_2})) \text{ and } \forall xx_1: a, xx_2: a, xy: b: ((f@_{xx_1@xy} \text{ and } f@_{xx_2@xy}) \Rightarrow (cR@_{xx_1@xx_2}))) \Rightarrow \forall xy: b: \exists xx: a: \forall xw: a: (f@_{xx@xy} \text{ and } cS@_{xy_1@xy_2}))$

**SEV097** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$  thf(a\_type, type)  
 $b: \$tType$  thf(b\_type, type)

$z: a$      $\text{thf}(z, \text{type})$

$cR: a \rightarrow a \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$f: a \rightarrow b \rightarrow \$o$      $\text{thf}(f, \text{type})$

$cS: b \rightarrow b \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$(\forall xu: a, xv: a, xw: a: ((cR@xu@xv \text{ and } cR@xw@xv) \Rightarrow (cR@xu@xw)) \text{ and } \forall xx: a: (cR@xx@xx)) \Rightarrow ((\forall xx: a: \exists xy: b: (f@xx@xy)) \text{ and } (\forall xy_1@xy_2: (cS@xy_1@xy_2))) \text{ and } \forall xx_1: a, xx_2: a, xy: b: ((f@xx_1@xy \text{ and } f@xx_2@xy) \Rightarrow (cR@xx_1@xx_2))) \Rightarrow \forall xx: a: \exists xy: b: \forall xw: a: (f@xx@xy)$

**SEV098** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$\forall xr: a \rightarrow a \rightarrow \$o, xs: a, xt: a: ((xs \neq xt \text{ and } \forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xp@xx@xy)) \text{ and } \forall xx: a, xy: a: (xp@xx@xz))) \Rightarrow (xp@xs@xt))) \Rightarrow \exists xz: a: (xr@xs@xz \text{ and } \forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xp@xx@xy)) \text{ and } \forall xx: a, xy: a, xz_0: a: ((xp@xx@xy \text{ and } xp@xy@xz_0) \Rightarrow (xp@xx@xz_0))) \Rightarrow (xp@xz@xt))))$      $\text{thf}(cTC\_IN)$

**SEV099** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$\forall xr: a \rightarrow a \rightarrow \$o, xs: a, xt: a: ((\neg xr@xs@xt \text{ and } \forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xp@xx@xy)) \text{ and } \forall xx: a, xy: a: (xp@xx@xz))) \Rightarrow (xp@xs@xt))) \Rightarrow \exists xz: a: (xr@xs@xz \text{ and } \forall xp: a \rightarrow a \rightarrow \$o: ((\forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow (xp@xx@xy)) \text{ and } \forall xx: a, xy: a, xz_0: a: ((xp@xx@xy \text{ and } xp@xy@xz_0) \Rightarrow (xp@xx@xz_0))) \Rightarrow (xp@xz@xt))))$      $\text{thf}(cTC\_IN)$

**SEV100** $\wedge$ **5.p** TPS problem from RELN-THMS

$p: \$o$      $\text{thf}(p, \text{type})$

$\exists xr_{28}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o, xr_{27}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((\forall xw_2: \$i: (xr_{27}@xx@xy@xw_2 \text{ or } xr_{28}@xx@xy@xw_2)) \text{ and } \forall xw_2: \$i: (xr_{27}@xy@xz@xw_2 \text{ or } xr_{28}@xy@xz@xw_2)) \text{ and } \forall xw_2: \$i: (xr_{27}@xx@xz@xw_2 \text{ or } xr_{28}@xx@xz@xw_2)) \text{ and } \forall xx: \$i \rightarrow \$o, xw_2: \$i: (xr_{27}@xx@xx@xw_2 \text{ or } xr_{28}@xx@xx@xw_2)) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((\forall xw_2: \$i: (xr_{27}@xx@xy@xw_2 \text{ or } xr_{28}@xx@xy@xw_2)) \text{ and } \forall xw_2: \$i: (xr_{27}@xy@xz@xw_2 \text{ or } xr_{28}@xy@xz@xw_2)) \text{ and } \forall xw_2: \$i: (xr_{27}@xx@xz@xw_2 \text{ or } xr_{28}@xx@xz@xw_2)) \text{ and } \neg \forall xw_2: \$i: (xr_{27}@xw_2: \$i: (p \text{ or } \neg p)@xw_2: \$i: (p \text{ and } \neg p)@xw_2 \text{ or } x))$

**SEV101** $\wedge$ **5.p** TPS problem from RELN-THMS

$p: \$o$      $\text{thf}(p, \text{type})$

$\exists xr_{28}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o, xr_{27}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\forall xx: \$i \rightarrow \$o, xw_2: \$i: (xr_{27}@xx@xx@xw_2 \text{ or } xr_{28}@xx@xx@xw_2)) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((\forall xw_2: \$i: (xr_{27}@xx@xy@xw_2 \text{ or } xr_{28}@xx@xy@xw_2)) \text{ and } \forall xw_2: \$i: (xr_{27}@xy@xz@xw_2 \text{ or } xr_{28}@xy@xz@xw_2)) \text{ and } \forall xw_2: \$i: (xr_{27}@xx@xz@xw_2 \text{ or } xr_{28}@xx@xz@xw_2)) \text{ and } \neg \forall xw_2: \$i: (xr_{27}@xw_2: \$i: (p \text{ or } \neg p)@xw_2: \$i: (p \text{ and } \neg p)@xw_2 \text{ or } x))$

**SEV102** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$\forall xx: a \rightarrow \$o, xy: a \rightarrow \$o, xz: a \rightarrow \$o: ((\exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xy@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xy@xy_0) \Rightarrow \exists xy_{52}: a: (\lambda xx_0: a: (xx@xx_0 \text{ and } xy_0 = (xs@xx_0))) = (\lambda xx: a, xy: a: xx = xy@xy_{52}))) \text{ and } \exists xs: a \rightarrow a: (\forall xx_0: a: ((xy@xx_0) \Rightarrow (xz@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xz@xy_0) \Rightarrow \exists xy_{53}: a: (\lambda xx_0: a: (xy@xx_0 \text{ and } xy_0 = (xs@xx_0))) = (\lambda xx: a, xy: a: xx = xy@xy_{53})))) \Rightarrow \exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xz@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xz@xy_0) \Rightarrow \exists xy_{55}: a: (\lambda xx_0: a: (xx@xx_0) \text{ and } xy_0 = (xs@xx_0))) = (\lambda xx: a, xy: a: xx = xy@xy_{55}))))$      $\text{thf}(cEQP\_1C\_pme, \text{conjecture})$

**SEV103** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$b: \$tType$      $\text{thf}(b\_type, \text{type})$

$cR: a \rightarrow a \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$cS: b \rightarrow b \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$(\forall xu: a, xv: a, xw: a: ((cR@xu@xv \text{ and } cR@xw@xv) \Rightarrow (cR@xu@xw)) \text{ and } \forall xx: a: (cR@xx@xx)) \Rightarrow (\exists f: a \rightarrow b \rightarrow \$o: (\forall xx: a: \exists xy: b: (f@xx@xy) \text{ and } \forall xx: a, xy_1: b, xy_2: b: ((f@xx@xy_1 \text{ and } f@xx@xy_2) \Rightarrow (cS@xy_1@xy_2)) \text{ and } \forall xx_1: a, xx_2: a: (cR@xx_1@xx_2))) \Rightarrow \exists xg: b \rightarrow a \rightarrow \$o: (\forall xx: a: \exists xy: b: (xg@xy@xx) \text{ and } \forall xy: b, xx_1: a, xx_2: a: ((xg@xy@xx_1 \text{ and } xg@xy@xx_2) \Rightarrow (cR@xx_1@xx_2)) \text{ and } \forall xy: b: \exists xx: a: (xg@xy@xx)))$      $\text{thf}(cTHM552A\_pme, \text{conjecture})$

**SEV104** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$b: \$tType$      $\text{thf}(b\_type, \text{type})$

$cR: a \rightarrow a \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$z: a$      $\text{thf}(z, \text{type})$

$f: a \rightarrow b \rightarrow \$o$      $\text{thf}(f, \text{type})$

$cS: b \rightarrow b \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$(\forall xu: a, xv: a, xw: a: ((cR@xu@xv \text{ and } cR@xw@xv) \Rightarrow (cR@xu@xw)) \text{ and } \forall xx: a: (cR@xx@xx)) \Rightarrow ((\forall xx: a: \exists xy: b: (f@xx@xy)) \text{ and } (\forall xy_1@xy_2: (cS@xy_1@xy_2))) \text{ and } \forall xx_1: a, xx_2: a, xy: b: ((f@xx_1@xy \text{ and } f@xx_2@xy) \Rightarrow (cR@xx_1@xx_2))) \Rightarrow \forall xy: b, xx_1: a, xx_2: a: ((\forall xx: a: (cR@xx@xx)) \Rightarrow (f@xx@xy))$

**SEV105** $\wedge$ **5.p** TPS problem from RELN-THMS

$a\_type: \$tType$      $\text{thf}(a\_type, \text{type})$

$a: a\_type$      $\text{thf}(a, \text{type})$

$b: a\_type$      $\text{thf}(b, \text{type})$

$\forall x: \text{atype} \rightarrow \text{atype} \rightarrow \$o, t: \text{atype} \rightarrow \text{atype} \rightarrow \$o: ((\forall xx: \text{atype}: (t@xx@xx) \text{ and } \forall xx: \text{atype}, xy: \text{atype}, xz: \text{atype}: ((t@xx@xy) \text{ and } (t@xx@xz)) \text{ and } \forall xx: \text{atype}, xy: \text{atype}: ((xr@xx@xy) \Rightarrow (t@xx@xy)) \text{ and } \forall s: \text{atype} \rightarrow \text{atype} \rightarrow \$o: ((\forall xx: \text{atype}: (s@xx@xx) (s@xx@xz)) \text{ and } \forall xx: \text{atype}, xy: \text{atype}: ((xr@xx@xy) \Rightarrow (s@xx@xy)))) \Rightarrow \forall xx: \text{atype}, xy: \text{atype}: ((t@xx@xy) \Rightarrow (s@xx@xy)))) \Rightarrow ((a \neq b \text{ and } t@a@b) \Rightarrow \exists xc: \text{atype}: (xr@a@xc \text{ and } t@xc@b)) \quad \text{thf}(cTC\_INTERP\_OTHER\_pme, \text{conjecture})$

**SEV106** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$

$\forall xx: a \rightarrow \$o, xy: a \rightarrow \$o, xz: a \rightarrow \$o: ((\exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xy@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xy@xy_0) \Rightarrow \exists xx_0: a: (xx@xx_0 \text{ and } xy_0 = (xs@xx_0) \text{ and } \forall xz_0: a: ((xx@xz_0 \text{ and } xy_0 = (xs@xz_0)) \Rightarrow xz_0 = xx_0)))) \text{ and } \exists xs: a \rightarrow a: (\forall xx_0: a: ((xy@xx_0) \Rightarrow (xz@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xz@xy_0) \Rightarrow \exists xx_0: a: (xy@xx_0 \text{ and } xy_0 = (xs@xx_0) \text{ and } \forall xz_0: a: ((xs@xz_0) \Rightarrow xz_0 = xx_0)))))) \Rightarrow \exists xs: a \rightarrow a: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (xz@(xs@xx_0))) \text{ and } \forall xy_0: a: ((xz@xy_0) \Rightarrow \exists xx_0: a: (xx@xx_0 \text{ and } xy_0 = (xs@xx_0) \text{ and } \forall xz_0: a: ((xx@xz_0 \text{ and } xy_0 = (xs@xz_0)) \Rightarrow xz_0 = xx_0)))) \quad \text{thf}(cEQP1\_1C\_pme)$

**SEV107** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$

$z: a \quad \text{thf}(z, type)$

$cR: a \rightarrow a \rightarrow \$o \quad \text{thf}(cR, type)$

$f: a \rightarrow \$i \rightarrow \$o \quad \text{thf}(f, type)$

$cS: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cS, type)$

$(\forall xu: a, xv: a, xw: a: ((cR@xu@xv \text{ and } cR@xw@xv) \Rightarrow (cR@xu@xw)) \text{ and } \forall xx: a: (cR@xx@xx) \Rightarrow ((\forall xx: a: \exists xy: \$i: (f@xx@xy) (cS@xy_1@xy_2)) \text{ and } \forall xx_1: a, xx_2: a, xy: \$i: ((f@xx_1@xy \text{ and } f@xx_2@xy) \Rightarrow (cR@xx_1@xx_2))) \Rightarrow (\forall xx: a: \exists xy: \$i: \forall xw: a: (f@xx@xw) (cR@xx_1@xx_2)) \text{ and } \forall xy: \$i: \exists xx: a: \forall xw: a: (f@xx@xy \text{ or } (\neg f@xw@xy \text{ and } cR@xx@z)))) \quad \text{thf}(cTHM552B\_pme, \text{conjecture})$

**SEV108** $\wedge$ **5.p** TPS problem from RELN-THMS

$\forall r: \$i \rightarrow \$i \rightarrow \$o, a: \$i, b: \$i, c: \$i, d: \$i, e: \$i, f: \$i: ((\forall xx: \$i, xy: \$i: ((r@xx@xy) \Rightarrow (r@xy@xx)) \text{ and } a \neq b \text{ and } a \neq c \text{ and } a \neq d \text{ and } a \neq e \text{ and } a \neq f \text{ and } b \neq c \text{ and } b \neq d \text{ and } b \neq e \text{ and } b \neq f \text{ and } c \neq d \text{ and } c \neq e \text{ and } c \neq f \text{ and } d \neq e \text{ and } d \neq f \text{ and } e \neq f) \Rightarrow \exists xa: \$i, xb: \$i, xc: \$i: (xa \neq xb \text{ and } xa \neq xc \text{ and } xb \neq xc \text{ and } ((r@xa@xb \text{ and } r@xa@xc \text{ and } r@xb@xc) \text{ or } (\neg r@xa@xb \text{ and } \neg r@xa@xc \text{ and } \neg r@xb@xc)))) \quad \text{thf}(cSIX\_THEOREM, \text{conjecture})$

**SEV109** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$

$\forall r: a \rightarrow a \rightarrow \$o, s: a \rightarrow a \rightarrow \$o, xx: a, xy: a: (\forall xq_1: a \rightarrow a \rightarrow \$o: ((\forall xs: a, xt: a: ((r@xs@xt \text{ or } s@xs@xt) \Rightarrow (xq_1@xs@xt)) \text{ and } \forall xx_0: a, xy_0: a, xz: a: ((xq_1@xx_0@xy_0 \text{ and } xq_1@xy_0@xz) \Rightarrow (xq_1@xx_0@xz))) \Rightarrow (xq_1@xx@xy)) \Rightarrow \forall xp_1: a \rightarrow a \rightarrow \$o: ((\forall xss: a, xtt: a: ((\forall xq_2: a \rightarrow a \rightarrow \$o: ((\forall xssss: a, xtttt: a: ((r@xssss@xttt) \Rightarrow (xq_2@xssss@xttt)) \text{ and } \forall xx_0: a: (xq_2@xx_0@xz))) \Rightarrow (xq_2@xss@xtt)) \text{ or } \forall xq_3: a \rightarrow a \rightarrow \$o: ((\forall xsssss: a, xtttt: a: ((s@xsssss@xtttt) \Rightarrow (xq_3@xsssss@xtttt)) \text{ and } \forall xx_0: a: (xq_3@xx_0@xz))) \Rightarrow (xq_3@xss@xtt))) \Rightarrow (xp_1@xss@xtt)) \text{ and } \forall xx_0: a, xy_0: a, xz: a: ((xp_1@xx_0@xy_0 \text{ and } xp_1@xy_0@xz) \Rightarrow (xp_1@xx_0@xz))) \Rightarrow (xp_1@xx@xy)) \quad \text{thf}(cTHM251F\_pme, \text{conjecture})$

**SEV113** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$

$\forall k: (a \rightarrow a \rightarrow \$o) \rightarrow a \rightarrow a \rightarrow \$o: (\forall xr_1: a \rightarrow a \rightarrow \$o, xr_2: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((xr_1@xx@xy) \Rightarrow (xr_2@xx@xy)) \Rightarrow \forall xx: a, xy: a: ((k@xr_1@xx@xy) \Rightarrow (k@xr_2@xx@xy))) \Rightarrow \exists l: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((l@xx@xy) \Rightarrow (k@l@xx@xy)) \text{ and } \forall xx: a, xy: a: ((k@l@xx@xy) \Rightarrow (l@xx@xy)) \text{ and } \forall t: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((k@t@xx@xy) \Rightarrow (t@xx@xy)) \Rightarrow \forall xx: a, xy: a: ((l@xx@xy) \Rightarrow (t@xx@xy)))))) \Rightarrow \forall xr: a \rightarrow a \rightarrow \$o: \exists xs: a \rightarrow a \rightarrow \$o: (\forall xa: a, xb: a: ((xr@xa@xb) \Rightarrow (xs@xa@xb)) \text{ and } \forall xx: a, xy: a: ((xs@xx@xy) \Rightarrow (xs@xy@xx)) \text{ and } \forall xt: a \rightarrow a \rightarrow \$o: ((\forall xa: a, xb: a: ((xr@xa@xb) \Rightarrow (xt@xa@xb)) \text{ and } \forall xx: a, xy: a: ((xt@xx@xy) \Rightarrow (xt@xy@xx))) \Rightarrow \forall xa: a, xb: a: ((xs@xa@xb) \Rightarrow (xt@xa@xb)))) \quad \text{thf}(cTHM576\_LFP\_pme, \text{conjecture})$

**SEV114** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$

$\forall x_2: (a \rightarrow \$o) \rightarrow \$o: \exists m: (a \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow \$o: ((m@xx) \Rightarrow (x_2@xx)) \text{ and } \forall u: a \rightarrow \$o, v: a \rightarrow \$o: ((m@u \text{ and } m@v) \Rightarrow (\forall xx: a: ((u@xx) \Rightarrow (v@xx)) \text{ or } \forall xx: a: ((v@xx) \Rightarrow (u@xx)))) \text{ and } \forall xy: (a \rightarrow \$o) \rightarrow \$o: ((\forall xx: a \rightarrow \$o: ((xy@xx) \Rightarrow (x_2@xx)) \text{ and } \forall u: a \rightarrow \$o, v: a \rightarrow \$o: ((xy@u \text{ and } xy@v) \Rightarrow (\forall xx: a: ((u@xx) \Rightarrow (v@xx)) \text{ or } \forall xx: a: ((v@xx) \Rightarrow (u@xx)))) \text{ and } \forall xx: a \rightarrow \$o: ((m@xx) \Rightarrow (xy@xx))) \Rightarrow \forall xx: a \rightarrow \$o: ((xy@xx) \Rightarrow (m@xx))) \Rightarrow \forall r: a \rightarrow a \rightarrow \$o: ((\forall xx: a, xy: a, xz: a: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz)) \text{ and } \forall xx: a: (r@xx@xx) \text{ and } xx = xy) \Rightarrow \exists s: a \rightarrow \$o: (\forall xx: a, xy: a: ((s@xx \text{ and } s@xy) \Rightarrow (r@xx@xy \text{ or } r@xy@xx)) \text{ and } \forall xy: a \rightarrow \$o: ((\forall xx: a, xy_0: a: ((xy@xx \text{ and } xy@xy_0) \Rightarrow (r@xx@xy_0 \text{ or } r@xy_0@xx)) \text{ and } \forall xx: a: ((s@xx) \Rightarrow (xy@xx))) \Rightarrow \forall xx: a: ((xy@xx) \Rightarrow (s@xx)))) \quad \text{thf}(cTHM540\_pme, \text{conjecture})$

**SEV115** $\wedge$ **5.p** TPS problem from RELN-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$

$\forall r: (a \rightarrow \$o) \rightarrow (a \rightarrow \$o) \rightarrow \$o: ((\forall xx: a \rightarrow \$o, xy: a \rightarrow \$o, xz: a \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz)) \text{ and } \forall xx: a \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: a \rightarrow \$o, xy: a \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xx) \Rightarrow xx =$





$\forall x r: a \rightarrow a \rightarrow \$o, xx: a, xy: a: ((x r @ x x @ x y) \Rightarrow \forall x x_0: a \rightarrow \$o: (\forall x y_0: a, x z: a: ((x r @ x y_0 @ x z \text{ and } x x_0 @ x y_0) \Rightarrow (x x_0 @ x z)) \Rightarrow ((x x_0 @ x x) \Rightarrow (x x_0 @ x y))))$  thf(cTHM202\_pme, conjecture)

**SEV132** $\wedge$ **5.p** TPS problem from TC-THMS

$a: \$tType$  thf(a\_type, type)

$\forall x r: a \rightarrow a \rightarrow \$o, x s: a, x t: a: ((x s \neq x t \text{ and } \forall x x: a \rightarrow \$o: (\forall x y: a, x z: a: ((x r @ x y @ x z \text{ and } x x @ x y) \Rightarrow (x x @ x z)) \Rightarrow ((x x @ x s) \Rightarrow (x x @ x t)))) \Rightarrow \exists x c: a: (x r @ x s @ x c \text{ and } \forall x x: a \rightarrow \$o: (\forall x y: a, x z: a: ((x r @ x y @ x z \text{ and } x x @ x y) \Rightarrow (x x @ x z)) \Rightarrow ((x x @ x c) \Rightarrow (x x @ x t))))$  thf(cTC\_INTERP\_BBP\_OLD\_pme, conjecture)

**SEV133** $\wedge$ **5.p** TPS problem from TC-THMS

$a: type$  thf(a\_type, type)

$a: atype$  thf(a, type)

$b: atype$  thf(b, type)

$cSTAR: (atype \rightarrow atype \rightarrow \$o) \rightarrow atype \rightarrow atype \rightarrow \$o$  thf(cSTAR, type)

$\forall x r: atype \rightarrow atype \rightarrow \$o: ((\forall x x: atype \rightarrow \$o: (\forall x y: atype, x z: atype: ((x r @ x y @ x z \text{ and } x x @ x y) \Rightarrow (x x @ x z)) \iff \forall x y: atype, x z: atype: ((x r @ x y @ x z \text{ and } x x @ x y) \Rightarrow (x x @ x z))) \text{ and } \forall x a_0: atype, x b_0: atype: ((cSTAR @ x r @ x a_0 @ x b_0) \iff \forall x x: atype \rightarrow \$o: (\forall x y: atype, x z: atype: ((x r @ x y @ x z \text{ and } x x @ x y) \Rightarrow (x x @ x z)) \Rightarrow ((x x @ x a_0) \Rightarrow (x x @ x b_0)))) \text{ and } a \neq b \text{ and } cSTAR @ x r @ a @ b) \Rightarrow \exists x c: atype: (x r @ a @ x c \text{ and } cSTAR @ x r @ x c @ b)$  thf(cTC\_INTERP\_THIRD\_pme, conjecture)

**SEV134** $\wedge$ **5.p** TPS problem THM201

$a: \$tType$  thf(a\_type, type)

$\forall x r: a \rightarrow a \rightarrow \$o, xx: a, xx_0: a \rightarrow \$o: (\forall x y: a, x z: a: ((x r @ x y @ x z \text{ and } x x_0 @ x y) \Rightarrow (x x_0 @ x z)) \Rightarrow ((x x_0 @ x x) \Rightarrow (x x_0 @ x x)))$  thf(cTHM201\_pme, conjecture)

**SEV135** $\wedge$ **5.p** TPS problem THM151

$a: \$tType$  thf(a\_type, type)

$\forall x r: a \rightarrow a \rightarrow \$o, xx: a, xy: a: ((x r @ x x @ x y) \Rightarrow \forall x q: a \rightarrow \$o: ((\forall x w: a: ((x r @ x x @ x w) \Rightarrow (x q @ x w)) \text{ and } \forall x u: a, x v: a: ((x q @ x u) \Rightarrow (x q @ x v))) \Rightarrow (x q @ x y)))$  thf(cTHM151\_pme, conjecture)

**SEV136** $\wedge$ **5.p** TPS problem THM203

B&B-P's defn of TRCL is the minimal transitive reflexive relation containing r.

$a: \$tType$  thf(a\_type, type)

$\forall x r: a \rightarrow a \rightarrow \$o, t: (a \rightarrow a \rightarrow \$o) \rightarrow a \rightarrow a \rightarrow \$o: ((\forall x x: a: (t @ x r @ x x @ x x) \text{ and } \forall x x: a, x y: a, x z: a: ((t @ x r @ x x @ x y \text{ and } t @ x r @ x y @ x z) \text{ and } \forall x x: a, x y: a: ((x r @ x x @ x y) \Rightarrow (t @ x r @ x x @ x y))) \Rightarrow \forall x x: a, x y: a: (\forall x x_0: a \rightarrow \$o: (\forall x y_0: a, x z: a: ((x r @ x y_0 @ x z) \Rightarrow ((x x_0 @ x x) \Rightarrow (x x_0 @ x y))) \Rightarrow (t @ x r @ x x @ x y)))$  thf(cTHM203\_pme, conjecture)

**SEV137** $\wedge$ **5.p** TPS problem THM204

$a: \$tType$  thf(a\_type, type)

$\forall x r: a \rightarrow a \rightarrow \$o, xx: a, xy: a, xz: a: ((\forall x x_0: a \rightarrow \$o: (\forall x y_0: a, x z_0: a: ((x r @ x y_0 @ x z_0 \text{ and } x x_0 @ x y_0) \Rightarrow (x x_0 @ x z_0)) \Rightarrow ((x x_0 @ x x) \Rightarrow (x x_0 @ x y))) \text{ and } \forall x x_0: a \rightarrow \$o: (\forall x y_0: a, x z_0: a: ((x r @ x y_0 @ x z_0 \text{ and } x x_0 @ x y_0) \Rightarrow (x x_0 @ x z_0)) \Rightarrow ((x x_0 @ x y) \Rightarrow (x x_0 @ x z)))) \Rightarrow \forall x x_0: a \rightarrow \$o: (\forall x y_0: a, x z_0: a: ((x r @ x y_0 @ x z_0 \text{ and } x x_0 @ x y_0) \Rightarrow (x x_0 @ x z_0)) \Rightarrow ((x x_0 @ x x) \Rightarrow (x x_0 @ x z))))$  thf(cTHM204\_pme, conjecture)

**SEV138** $\wedge$ **5.p** TPS problem THM150

The transitive closure TC2 of a relation is transitive.

$a: \$tType$  thf(a\_type, type)

$\forall x r: a \rightarrow a \rightarrow \$o, xx: a, xy: a, xz: a: ((\forall x q: a \rightarrow \$o: ((\forall x w: a: ((x r @ x x @ x w) \Rightarrow (x q @ x w)) \text{ and } \forall x u: a, x v: a: ((x q @ x u \text{ and } x r @ x u @ x v) \Rightarrow (x q @ x v))) \Rightarrow (x q @ x y)) \text{ and } \forall x q: a \rightarrow \$o: ((\forall x w: a: ((x r @ x y @ x w) \Rightarrow (x q @ x w)) \text{ and } \forall x u: a, x v: a: ((x q @ x u \text{ and } x r @ x u @ x v) \Rightarrow (x q @ x v))) \Rightarrow (x q @ x z))) \Rightarrow \forall x q: a \rightarrow \$o: ((\forall x w: a: ((x r @ x x @ x w) \Rightarrow (x q @ x w)) \text{ and } \forall x u: a, x v: a: ((x q @ x u \text{ and } x r @ x u @ x v) \Rightarrow (x q @ x v))) \Rightarrow (x q @ x z)))$  thf(cTHM150\_pme, conjecture)

**SEV140** $\wedge$ **5.p** TPS problem THM250C

$a: \$tType$  thf(a\_type, type)

$\forall r: a \rightarrow a \rightarrow \$o, s: a \rightarrow a \rightarrow \$o, xx: a, xy: a: (\forall x p_1: a \rightarrow a \rightarrow \$o: ((\forall x x_0: a, x y_0: a: ((\forall x p_{10}: a \rightarrow a \rightarrow \$o: ((\forall x x_1: a, x y_1: a: ((r @ x x_1 @ x y_1) \Rightarrow (x p_{10} @ x x_1 @ x y_1)) \text{ and } \forall x x_1: a, x y_1: a, x z: a: ((x p_{10} @ x x_1 @ x y_1 \text{ and } x p_{10} @ x y_1 @ x z) \Rightarrow (x p_{10} @ x x_1 @ x z))) \Rightarrow (x p_{10} @ x x_0 @ x y_0)) \text{ or } \forall x p_{10}: a \rightarrow a \rightarrow \$o: ((\forall x x_1: a, x y_1: a: ((s @ x x_1 @ x y_1) \Rightarrow (x p_{10} @ x x_1 @ x y_1)) \text{ and } \forall x x_1: a, x y_1: a, x z: a: ((x p_{10} @ x x_1 @ x z) \Rightarrow (x p_{10} @ x x_0 @ x y_0))) \Rightarrow (x p_{10} @ x x_0 @ x y_0)) \text{ and } \forall x x_0: a, x y_0: a, x z: a: ((x p_{10} @ x x_0 @ x y_0 \text{ and } x p_{10} @ x y_0 @ x z) \Rightarrow (x p_{10} @ x x_0 @ x z))) \Rightarrow (x p_{10} @ x x @ x y))) \Rightarrow \forall x p_1: a \rightarrow a \rightarrow \$o: ((\forall x x_0: a, x y_0: a: ((r @ x x_0 @ x y_0 \text{ or } s @ x x_0 @ x y_0) \Rightarrow (x p_1 @ x x_0 @ x y_0)) \text{ and } \forall x x_0: a, x y_0: a, x z: a: ((x p_1 @ x x_0 @ x y_0 \text{ and } x p_1 @ x y_0 @ x z) \Rightarrow (x p_1 @ x x_0 @ x z))) \Rightarrow (x p_1 @ x x @ x y)))$  thf(cTHM250C\_pme, conjecture)

**SEV141** $\wedge$ **5.p** TPS problem THM250

$a: \$tType$  thf(a\_type, type)

$\forall r: a \rightarrow a \rightarrow \$o, s: a \rightarrow a \rightarrow \$o: (\lambda x x_1: a, x y_1: a: \forall x p_1: a \rightarrow a \rightarrow \$o: ((\forall x x: a, x y: a: ((r @ x x @ x y \text{ or } s @ x x @ x y) \Rightarrow (x p_1 @ x x @ x y)) \text{ and } \forall x x: a, x y: a, x z: a: ((x p_1 @ x x @ x y \text{ and } x p_1 @ x y @ x z) \Rightarrow (x p_1 @ x x @ x z))) \Rightarrow (x p_1 @ x x_1 @ x y_1))) = (\lambda x x_1: a, x y_1: a: \forall x p_1: a \rightarrow a \rightarrow \$o: ((\forall x x: a, x y: a: ((\forall x p_{10}: a \rightarrow a \rightarrow \$o: ((\forall x x_0: a, x y_0: a: ((r @ x x_0 @ x y_0) \Rightarrow (x p_{10} @ x x_0 @ x y_0)) \text{ and } \forall x x_0: a, x y_0: a, x z: a: ((x p_{10} @ x x_0 @ x y_0 \text{ and } x p_{10} @ x y_0 @ x z) \Rightarrow (x p_{10} @ x x_0 @ x z))) \Rightarrow (x p_{10} @ x x @ x y)))$  thf(cTHM250\_pme, conjecture)



$(xp_{10}@xx_0@xy_0))$  and  $\forall xx_0: a, xy_0: a, xz: a: ((xp_{10}@xx_0@xy_0 \text{ and } xp_{10}@xy_0@xz) \Rightarrow (xp_{10}@xx_0@xz))) \Rightarrow (xp_{10}@xx@xy))$  or  $a \rightarrow \$o: ((\forall xx_0: a, xy_0: a: ((s@xx_0@xy_0) \Rightarrow (xp_{10}@xx_0@xy_0)) \text{ and } \forall xx_0: a, xy_0: a, xz: a: ((xp_{10}@xx_0@xy_0 \text{ and } xp_{10}@xy_0@xz) \Rightarrow (xp_{10}@xx_0@xz))) \Rightarrow (xp_{10}@xx@xy))) \Rightarrow (xp_1@xx@xy))$  and  $\forall xx: a, xy: a, xz: a: ((xp_1@xx@xy \text{ and } xp_1@xy@xz) \Rightarrow (xp_1@xx@xz))) \Rightarrow (xp_1@xx_1@xy_1)))$  thf(cTHM250\_pme, conjecture)

**SEV143** $\wedge$ **5.p** TPS problem THM146

Equivalence of two definitions of transitive closure.

cTCLOSED:  $(\$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$  thf(cTCLOSED\_type, type)

cTCLOSED =  $(\lambda xp: \$i \rightarrow \$i \rightarrow \$o, xs: \$i \rightarrow \$i \rightarrow \$o: \forall xu: \$i, xv: \$i, xw: \$i: ((xp@xu@xv \text{ and } xs@xv@xw) \Rightarrow (xp@xu@xw)))$  thf(cTCLOSED\_def, definition)

$\forall xr: \$i \rightarrow \$i \rightarrow \$o, xx: \$i, xy: \$i: (\forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } cTCLOSED@xp@xx@xy)) \iff \forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } \forall xx_0: \$i, xy_0: \$i, xz: \$i: ((xp@xx_0@xz)) \Rightarrow (xp@xx@xy))))$  thf(cTHM146\_pme, conjecture)

**SEV144** $\wedge$ **5.p** TPS problem from TRANSITIVE-CLOSURE

cTCLOSED:  $(\$i \rightarrow \$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$o$  thf(cTCLOSED\_type, type)

cTCLOSED =  $(\lambda xp: \$i \rightarrow \$i \rightarrow \$o, xs: \$i \rightarrow \$i \rightarrow \$o: \forall xu: \$i, xv: \$i, xw: \$i: ((xp@xu@xv \text{ and } xs@xv@xw) \Rightarrow (xp@xu@xw)))$  thf(cTCLOSED\_def, definition)

$\forall xr: \$i \rightarrow \$i \rightarrow \$o, xx: \$i, xy: \$i: (\forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } cTCLOSED@xp@xx@xy)) \iff \forall xp: \$i \rightarrow \$i \rightarrow \$o: ((\forall xx_0: \$i, xy_0: \$i: ((xr@xx_0@xy_0) \Rightarrow (xp@xx_0@xy_0)) \text{ and } cTCLOSED@xp@xp) \Rightarrow (xp@xx@xy))))$  thf(cTHM146B\_pme, conjecture)

**SEV146** $\wedge$ **5.p** TPS problem from TRANSITIVE-CLOSURE

$a: \$tType$  thf(a\_type, type)

$\forall xr: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((xr@xx@xy) \Rightarrow \forall xq: a \rightarrow \$o: ((\forall xw: a: ((xr@xx@xw) \Rightarrow (xq@xw)) \text{ and } \forall xv: a, xw: a: ((xq@xw) \Rightarrow (xq@xy))) \text{ and } \forall xx: a, xy: a, xz: a: ((\forall xq: a \rightarrow \$o: ((\forall xw: a: ((xr@xx@xw) \Rightarrow (xq@xw)) \text{ and } \forall xv: a, xw: a: ((xq@xw) \Rightarrow (xq@xy)) \text{ and } \forall xq: a \rightarrow \$o: ((\forall xw: a: ((xr@xy@xw) \Rightarrow (xq@xw)) \text{ and } \forall xv: a, xw: a: ((xq@xv \text{ and } xr@xv@xw) \Rightarrow (xq@xz))) \Rightarrow \forall xq: a \rightarrow \$o: ((\forall xw: a: ((xr@xx@xw) \Rightarrow (xq@xw)) \text{ and } \forall xv: a, xw: a: ((xq@xv \text{ and } xr@xv@xw) \Rightarrow (xq@xz))))))$  thf(cTHM525\_pme, conjecture)

**SEV148** $\wedge$ **5.p** TPS problem from TRANSITIVE-CLOSURE

$a: \$tType$  thf(a\_type, type)

$\forall r: a \rightarrow a \rightarrow \$o, s: a \rightarrow a \rightarrow \$o, xx: a, xy: a: (\forall xq: a \rightarrow \$o: ((\forall xw: a: ((\forall xq_0: a \rightarrow \$o: ((\forall xw_0: a: ((r@xx@xw_0) \Rightarrow (xq_0@xw_0)) \text{ and } \forall xu: a, xv: a: ((xq_0@xu \text{ and } r@xu@xv) \Rightarrow (xq_0@xv))) \Rightarrow (xq_0@xw)) \text{ or } \forall xq_0: a \rightarrow \$o: ((\forall xw_0: a: ((s@xx@xw_0) \Rightarrow (xq_0@xw_0)) \text{ and } \forall xu: a, xv: a: ((xq_0@xu \text{ and } s@xu@xv) \Rightarrow (xq_0@xv))) \Rightarrow (xq_0@xw))) \Rightarrow (xq@xw)) \text{ and } \forall xu: a, xv: a: ((xq@xu \text{ and } r@xu@xv) \Rightarrow (xq@xv)) \text{ and } \forall xu_0: a, xv_0: a: ((xq_0@xu_0 \text{ and } r@xu_0@xv_0) \Rightarrow (xq_0@xv_0)) \Rightarrow (xq_0@xv)) \text{ or } \forall xq_0: a \rightarrow \$o: ((\forall xw: a: ((s@xu@xw) \Rightarrow (xq_0@xw)) \text{ and } \forall xu_0: a, xv_0: a: ((xq_0@xu_0 \text{ and } s@xu_0@xv_0) \Rightarrow (xq_0@xv_0)) \Rightarrow (xq_0@xv)) \text{ or } \forall xq_0: a \rightarrow \$o: ((\forall xw: a: ((s@xu@xw) \Rightarrow (xq_0@xw)) \text{ and } \forall xu_0: a, xv_0: a: ((xq_0@xu_0 \text{ and } s@xu_0@xv_0) \Rightarrow (xq_0@xv_0)) \Rightarrow (xq_0@xv)))))) \Rightarrow (xq@xv)) \Rightarrow (xq@xy)) \Rightarrow \forall xq: a \rightarrow \$o: ((\forall xw: a: ((r@xx@xw \text{ or } s@xx@xw) \Rightarrow (xq@xw)) \text{ and } \forall xu: a, xv: a: ((xq@xu \text{ and } r@xu@xv \text{ or } s@xu@xv) \Rightarrow (xq@xv)) \Rightarrow (xq@xy))))$  thf(cTHM251C\_pme, conjecture)

**SEV149** $\wedge$ **5.p** TPS problem from TRANSITIVE-CLOSURE

$a: \$tType$  thf(a\_type, type)

$\forall r: a \rightarrow a \rightarrow \$o, s: a \rightarrow a \rightarrow \$o, xx: a, xy: a: (\forall xq: a \rightarrow \$o: ((\forall xw: a: ((r@xx@xw \text{ or } s@xx@xw) \Rightarrow (xq@xw)) \text{ and } \forall xu: a, xv: a: ((xq@xu \text{ and } r@xu@xv) \Rightarrow (xq@xv)) \Rightarrow (xq@xy)) \Rightarrow \forall xq: a \rightarrow \$o: ((\forall xw: a: ((\forall xq_0: a \rightarrow \$o: ((\forall xw_0: a: ((r@xx@xw_0) \Rightarrow (xq_0@xw_0)) \text{ and } \forall xu: a, xv: a: ((xq_0@xu \text{ and } r@xu@xv) \Rightarrow (xq_0@xv)) \Rightarrow (xq_0@xw)) \text{ or } \forall xq_0: a \rightarrow \$o: ((\forall xw_0: a: ((s@xx@xw_0) \Rightarrow (xq_0@xw_0)) \text{ and } \forall xu: a, xv: a: ((xq_0@xu \text{ and } s@xu@xv) \Rightarrow (xq_0@xv)) \Rightarrow (xq_0@xw))) \Rightarrow (xq@xw)) \text{ and } \forall xu: a, xv: a: ((xq@xu \text{ and } (r@xu@xv \text{ or } s@xu@xv) \Rightarrow (xq@xv)) \Rightarrow (xq@xy)) \Rightarrow \forall xq_0: a \rightarrow \$o: ((\forall xw: a: ((r@xu@xw) \Rightarrow (xq_0@xw)) \text{ and } \forall xu_0: a, xv_0: a: ((xq_0@xu_0 \text{ and } r@xu_0@xv_0) \Rightarrow (xq_0@xv_0)) \Rightarrow (xq_0@xv)) \text{ or } \forall xq_0: a \rightarrow \$o: ((\forall xw: a: ((s@xu@xw) \Rightarrow (xq_0@xw)) \text{ and } \forall xu_0: a, xv_0: a: ((xq_0@xu_0 \text{ and } s@xu_0@xv_0) \Rightarrow (xq_0@xv_0)) \Rightarrow (xq_0@xv)))))) \Rightarrow (xq@xv)) \Rightarrow (xq@xy))$  thf(cTHM251B\_pme, conjecture)

**SEV150** $\wedge$ **5.p** TPS problem from TRANSITIVE-CLOSURE

$a: \$tType$  thf(a\_type, type)

$\forall r: a \rightarrow a \rightarrow \$o, s: a \rightarrow a \rightarrow \$o: (\lambda xp: a, xq: a: \forall xq_0: a \rightarrow \$o: ((\forall xw: a: ((r@xp@xw \text{ or } s@xp@xw) \Rightarrow (xq_0@xw)) \text{ and } \forall xu: a, xv: a: ((xq_0@xu \text{ and } (r@xu@xv \text{ or } s@xu@xv) \Rightarrow (xq_0@xv)) \Rightarrow (xq_0@xq))) = (\lambda xp: a, xq: a: \forall xq_0: a \rightarrow \$o: ((\forall xw: a: ((\forall xq_1: a \rightarrow \$o: ((\forall xw_0: a: ((r@xp@xw_0) \Rightarrow (xq_1@xw_0)) \text{ and } \forall xu: a, xv: a: ((xq_1@xu \text{ and } r@xu@xv) \Rightarrow (xq_1@xv)) \Rightarrow (xq_1@xw)) \text{ or } \forall xq_1: a \rightarrow \$o: ((\forall xw_0: a: ((s@xp@xw_0) \Rightarrow (xq_1@xw_0)) \text{ and } \forall xu: a, xv: a: ((xq_1@xu \text{ and } s@xu@xv) \Rightarrow (xq_1@xv)) \Rightarrow (xq_1@xw))) \Rightarrow (xq_0@xw)) \text{ and } \forall xu: a, xv: a: ((xq_0@xu \text{ and } (\forall xq_1: a \rightarrow \$o: ((\forall xw: a: ((r@xu@xw) \Rightarrow (xq_1@xw)) \text{ and } \forall xu_0: a, xv_0: a: ((xq_1@xu_0 \text{ and } r@xu_0@xv_0) \Rightarrow (xq_1@xv_0)) \Rightarrow (xq_1@xv)) \text{ or } \forall xq_1: a \rightarrow \$o: ((\forall xw: a: ((s@xu@xw) \Rightarrow (xq_1@xw)) \text{ and } \forall xu_0: a, xv_0: a: ((xq_1@xu_0 \text{ and } s@xu_0@xv_0) \Rightarrow (xq_1@xv_0)) \Rightarrow (xq_1@xv)))))) \Rightarrow (xq_0@xv)) \Rightarrow (xq_0@xq)))$  thf(cTHM251A\_pme, conjecture)

**SEV158** $\wedge$ **5.p** TPS problem THM120I-1

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: (\forall xx: \$i \rightarrow \$o: (r@xx@xx) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o, xz: \$i \rightarrow \$o: ((r@xx@xy \text{ and } r@xy@xz) \Rightarrow (r@xx@xz)) \text{ and } \forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: ((r@x@y \text{ and } r@y@x) \Rightarrow \forall xx: \$i: (x@xx) = (y@xx)))$      $\text{thf}(\text{cTHM120L1\_pme, conjecture})$

**SEV159** $\wedge$ **5.p** TPS problem THM181

Basic theorem about pairing.

$a: \$tType$      $\text{thf}(a\_type, type)$

$\forall xx: a, xy: a: xy = xy$      $\text{thf}(\text{cTHM181\_pme, conjecture})$

**SEV160** $\wedge$ **5.p** TPS problem THM186

Basic theorem about pairing.

$a: \$tType$      $\text{thf}(a\_type, type)$

$y: a$      $\text{thf}(y, type)$

$x: a$      $\text{thf}(x, type)$

$(\lambda xg: a \rightarrow a \rightarrow a: (xg@x@y)) = (\lambda xg: a \rightarrow a \rightarrow a: (xg@x@y))$      $\text{thf}(\text{cTHM186\_pme, conjecture})$

**SEV161** $\wedge$ **5.p** TPS problem THM183

Basic theorem about pairing.

$a: \$tType$      $\text{thf}(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o, xx: a, xy: a: ((xr@xx@xy) \iff (xr@xx@xy))$      $\text{thf}(\text{cTHM183\_pme, conjecture})$

**SEV162** $\wedge$ **5.p** TPS problem THM184

Theorem about representing relations in terms of ordered pairs.

$a: \$tType$      $\text{thf}(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: (xr@xx@xy) \iff \forall xp: (a \rightarrow a \rightarrow a) \rightarrow a: (xr@(xp@\lambda xx: a, xy: a: xx)@(xp@\lambda xx: a, xy: a: xy))$

**SEV163** $\wedge$ **5.p** TPS problem THM187

Basic theorem about pairing.

$a: \$tType$      $\text{thf}(a\_type, type)$

$\forall xp: (a \rightarrow a \rightarrow a) \rightarrow a: (xp = (\lambda xg: a \rightarrow a \rightarrow a: (xg@(xp@\lambda xx: a, xy: a: xx)@(xp@\lambda xx: a, xy: a: xy))) \Rightarrow (\lambda xg: a \rightarrow a \rightarrow a: (xg@(xp@\lambda xx: a, xy: a: xx)@(xp@\lambda xx: a, xy: a: xy))) = xp)$      $\text{thf}(\text{cTHM187\_pme, conjecture})$

**SEV164** $\wedge$ **5.p** TPS problem THM185

Basic theorem about representing relations in terms of ordered pairs.

$a: \$tType$      $\text{thf}(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow \$o: (\exists xx: a, xy: a: (xr@xx@xy) \iff \exists xp: (a \rightarrow a \rightarrow a) \rightarrow a: (xr@(xp@\lambda xx: a, xy: a: xx)@(xp@\lambda xx: a, xy: a: xy))$

**SEV165** $\wedge$ **5.p** TPS problem EXISTS-CART-SET-PROD

$\exists \text{cROSS}: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow ((\$i \rightarrow \$i \rightarrow \$i) \rightarrow \$i) \rightarrow \$o: \forall a: \$i \rightarrow \$o, b: \$i \rightarrow \$o, xa: \$i, xb: \$i: ((\text{cROSS}@a@b@\lambda g: \$i \rightarrow \$i \rightarrow \$i: (g@xa@xb)) \iff (a@xa \text{ and } b@xb))$      $\text{thf}(\text{cEXISTS\_CART\_SET\_PROD\_pme, conjecture})$

**SEV166** $\wedge$ **5.p** TPS problem THM182

Basic theorem about pairing.

$a: \$tType$      $\text{thf}(a\_type, type)$

$\forall xx: a, xy: a, xu: a, xv: a: ((\lambda xg: a \rightarrow a \rightarrow a: (xg@xx@xy)) = (\lambda xg: a \rightarrow a \rightarrow a: (xg@xu@xv)) \iff (xx = xu \text{ and } xy = xv))$      $\text{thf}(\text{cTHM182\_pme, conjecture})$

**SEV167** $\wedge$ **5.p** TPS problem THM189

Basic theorem about pairing.

$a: \$tType$      $\text{thf}(a\_type, type)$

$\forall xr: a \rightarrow a \rightarrow a \rightarrow a \rightarrow \$o: (\forall xp: (a \rightarrow a \rightarrow a) \rightarrow a, xq: (a \rightarrow a \rightarrow a) \rightarrow a: ((xr@(xp@\lambda xx: a, xy: a: xx)@(xp@\lambda xx: a, xy: a: xy)) \text{ and } xp = xq) \Rightarrow \forall xx_1: a, xy_1: a, xx_2: a, xy_2: a: ((xr@xx_1@xy_1@xx_2@xy_2) \Rightarrow (xx_1 = xx_2 \text{ and } xy_1 = xy_2)))$      $\text{thf}(\text{cTHM189\_pme, conjecture})$

**SEV168** $\wedge$ **5.p** TPS problem from PAIRS-THMS

$a: \$tType$      $\text{thf}(a\_type, type)$

$q: (a \rightarrow a \rightarrow a) \rightarrow a$      $\text{thf}(q, type)$

$p: (a \rightarrow a \rightarrow a) \rightarrow a$      $\text{thf}(p, type)$

$(q = (\lambda xg: a \rightarrow a \rightarrow a: (xg@(q@\lambda xx: a, xy: a: xx)@(q@\lambda xx: a, xy: a: xy))) \text{ and } p = (\lambda xg: a \rightarrow a \rightarrow a: (xg@(q@\lambda xx: a, xy: a: xy)))$      $p = q$      $\text{thf}(\text{cTHM188\_PARTIAL\_pme, conjecture})$

**SEV169** $\wedge$ **5.p** TPS problem from PAIRS-THMS

$a: \$tType$      $\text{thf}(a\_type, type)$

$\forall xp: (a \rightarrow a \rightarrow a) \rightarrow a, xq: (a \rightarrow a \rightarrow a) \rightarrow a: ((xp = (\lambda xg: a \rightarrow a \rightarrow a: (xg@(xp@\lambda xx: a, xy: a: xx)@(xp@\lambda xx: a, xy: a: xy))) \text{ and } (\lambda xg: a \rightarrow a \rightarrow a: (xg@(xq@\lambda xx: a, xy: a: xx)@(xq@\lambda xx: a, xy: a: xy))) \text{ and } (xp@\lambda xx: a, xy: a: xx) = (xq@\lambda xx: a, xy: a: xx)) \Rightarrow (xp = xq))$      $\text{thf}(\text{cTHM188\_pme, conjecture})$

**SEV170** $\wedge$ **5.p** TPS problem from PAIRS-THMS

$a$ : \$tType    thf(a\_type, type)

$\forall x: a \rightarrow a \rightarrow a \rightarrow a \rightarrow \$o$ : ( $\forall xp: (a \rightarrow a \rightarrow a) \rightarrow a, xq: (a \rightarrow a \rightarrow a) \rightarrow a$ : (( $xp = (\lambda xg: a \rightarrow a \rightarrow a$ : ( $xg@(\lambda xx: a, xy: a: xx)@(\lambda xx: a, xy: a: xy))$ ) and  $xq = (\lambda xg: a \rightarrow a \rightarrow a$ : ( $xg@(\lambda xq: a, xy: a: xx)@(\lambda xx: a, xy: a: xy)$ )  $\iff \forall xx_1: a, xy_1: a, xx_2: a, xy_2: a$ : (( $xr@xx_1@xy_1@xx_2@xy_2$ )  $\implies$  ( $xx_1 = xx_2$  and  $xy_1 = xy_2$ )))    thf(cTHM190\_pme, conjecture)

**SEV171** $\wedge$ **5.p** TPS problem from PAIRS-FUNS-THMS

$a$ : \$tType    thf(a\_type, type)

$\exists f: a \rightarrow (a \rightarrow a \rightarrow a) \rightarrow a$ : ( $\forall xx: a, xy: a$ : (( $f@xx = f@xy$ )  $\implies$   $xx = xy$ ) and  $\forall x: a$ :  $\exists y: a, z: a$ : ( $f@x = (\lambda g: a \rightarrow a \rightarrow a$ : ( $g@x@y$ )))    thf(cTHM33\_pme, conjecture)

**SEV172** $\wedge$ **5.p** TPS problem from SETPAIRS-THMS

$b$ : \$tType    thf(b\_type, type)

$cZ: b \rightarrow \$o$     thf(cZ, type)

$cR: b \rightarrow \$o$     thf(cR, type)

$cS: b \rightarrow \$o$     thf(cS, type)

$\forall xx: (b \rightarrow b \rightarrow b) \rightarrow b$ : ( $\exists x: b, y: b$ : (( $cR@x$  or  $cS@x$ ) and ( $cR@y$  or  $cS@y$ ) and  $xx = (\lambda g: b \rightarrow b \rightarrow b$ : ( $g@x@y$ )))  $\implies \exists x: b, y: b$ : (( $cR@x$  or  $cZ@x$ ) and ( $cR@y$  or  $cZ@y$ ) and  $xx = (\lambda g: b \rightarrow b \rightarrow b$ : ( $g@x@y$ )))    thf(cTHM32A\_pme, conjecture)

**SEV173** $\wedge$ **5.p** TPS problem from SETPAIRS-THMS

$b$ : \$tType    thf(b\_type, type)

$cZ: b \rightarrow \$o$     thf(cZ, type)

$cR: b \rightarrow \$o$     thf(cR, type)

$cS: b \rightarrow \$o$     thf(cS, type)

$\forall xx: (b \rightarrow b \rightarrow b) \rightarrow b$ : ( $\exists x: b, y: b$ : (( $cR@x$  or  $cS@x$ ) and ( $cR@y$  or  $cS@y$ ) and  $xx = (\lambda g: b \rightarrow b \rightarrow b$ : ( $g@x@y$ )))  $\iff \exists x: b, y: b$ : (( $cR@x$  or  $cZ@x$ ) and ( $cR@y$  or  $cZ@y$ ) and  $xx = (\lambda g: b \rightarrow b \rightarrow b$ : ( $g@x@y$ )))    thf(cTHM32\_pme, conjecture)

**SEV174** $\wedge$ **5.p** TPS problem from SETS-OF-SETS

$a$ : \$tType    thf(a\_type, type)

$cP: (a \rightarrow \$o) \rightarrow \$o$     thf(cP, type)

( $\forall xp: a \rightarrow \$o$ : (( $cP@xp$ )  $\implies \exists xz: a$ : ( $xp@xz$ )) and  $\forall xx: a, xp: a \rightarrow \$o, xq: a \rightarrow \$o$ : (( $cP@xp$  and  $cP@xq$  and  $xp@xx$  and  $xq@xx$  and  $xp = xq$ )  $\implies \exists s: a \rightarrow \$o$ : ( $\forall xa: a \rightarrow \$o$ : (( $cP@xa$ )  $\implies \exists xx: a$ : ( $xa@xx$ )) and  $\forall xx: a$ : (( $s@xx$ )  $\iff \exists s_0: a \rightarrow \$o$ : ( $cP@s_0$  and  $s_0@xx$ )) and  $\forall xb: a \rightarrow \$o, xc: a \rightarrow \$o$ : (( $cP@xb$  and  $cP@xc$  and  $\exists xx: a$ : ( $xb@xx$  and  $xc@xx$ ))  $\implies xb = xc$ ))    thf(cTHM555\_pme, conjecture)

**SEV175** $\wedge$ **5.p** TPS problem THM144A

A lemma for the Injective Cantor Theorem X5309.

$\forall xh: (\$i \rightarrow \$o) \rightarrow \$i$ :  $\exists xt: \$i \rightarrow \$o$ : ( $\neg xt@(xh@xt)$  and ( $xh@x\lambda z: \$i$ :  $\exists xt_0: \$i \rightarrow \$o$ : ( $\neg xt_0@(xh@xt_0)$  and  $xz = (xh@xt_0)$ )) = ( $xh@xt$ ))    thf(cTHM144A\_pme, conjecture)

**SEV176** $\wedge$ **5.p** TPS problem THM25

Quine's modification of Russell's paradox.

$cR: \$i \rightarrow \$i \rightarrow \$o$     thf(cR, type)

$\neg \exists y: \$i$ :  $\forall x: \$i$ : (( $cR@x@y$ )  $\iff \neg \exists z: \$i$ : ( $cR@x@z$  and  $cR@z@x$ ))    thf(cTHM25, conjecture)

**SEV177** $\wedge$ **5.p** TPS problem THM144

A lemma for the Injective Cantor Theorem X5309.

$\forall xh: (\$i \rightarrow \$o) \rightarrow \$i, xd: \$i \rightarrow \$o$ : ( $xd = (\lambda xz: \$i$ :  $\exists xt: \$i \rightarrow \$o$ : ( $\neg xt@(xh@xt)$  and  $xz = (xh@xt)$ ))  $\implies (xd@(xh@xd))$ )    thf(cTHM144\_pme, conjecture)

**SEV179** $\wedge$ **5.p** TPS problem from CANTOR-THMS

$cD\_FOR\_X5309: ((\$i \rightarrow \$o) \rightarrow \$i) \rightarrow \$i \rightarrow \$o$     thf(cD\_FOR\_X5309\_type, type)

$cD\_FOR\_X5309 = (\lambda xh: (\$i \rightarrow \$o) \rightarrow \$i, xz: \$i$ :  $\exists xt: \$i \rightarrow \$o$ : ( $\neg xt@(xh@xt)$  and  $xz = (xh@xt)$ ))    thf(cD\_FOR\_X5309\_def, conjecture)

$\forall xh: (\$i \rightarrow \$o) \rightarrow \$i$ : ( $cD\_FOR\_X5309@xh@(xh@(cD\_FOR\_X5309@xh))$ )    thf(cTHM144C\_pme, conjecture)

**SEV180** $\wedge$ **5.p** TPS problem from CANTOR-THMS

$cF: \$i \rightarrow \$o$     thf(cF, type)

$cQ: (\$i \rightarrow \$o) \rightarrow \$o$     thf(cQ, type)

$cJ: (\$i \rightarrow \$o) \rightarrow \$i$     thf(cJ, type)

$cG: \$i \rightarrow \$i \rightarrow \$o$     thf(cG, type)

$\neg cQ@(cG@(cJ@cF))$  or  $cQ@cF$     thf(cCANTOR\_PROBLEM, conjecture)

**SEV181** $\wedge$ **5.p** TPS problem from CANTOR-THMS

$cD\_FOR\_X5309: ((\$i \rightarrow \$o) \rightarrow \$i) \rightarrow \$i \rightarrow \$o$     thf(cD\_FOR\_X5309\_type, type)

$cD\_FOR\_X5309 = (\lambda xh: (\$i \rightarrow \$o) \rightarrow \$i, xz: \$i$ :  $\exists xt: \$i \rightarrow \$o$ : ( $\neg xt@(xh@xt)$  and  $xz = (xh@xt)$ ))    thf(cD\_FOR\_X5309\_def, conjecture)

$\forall xh: (\$i \rightarrow \$o) \rightarrow \$i$ : ( $\forall xp: \$i \rightarrow \$o, xq: \$i \rightarrow \$o$ : (( $xh@xp = xh@xq$ )  $\implies xp = xq$ )  $\implies \neg cD\_FOR\_X5309@xh@(xh@(cD\_FOR\_X5309@xh))$ )    thf(cTHM144C\_pme, conjecture)

**SEV182** $\wedge$ **5.p** TPS problem from CANTOR-THMS

$\forall s: \$i \rightarrow \$o: \neg \exists z: \$i \rightarrow \$o: (\forall xx: \$i: ((z@xx) \Rightarrow (s@xx))$  and  $\exists xs: (\$i \rightarrow \$o) \rightarrow \$i: (\forall xx: \$i \rightarrow \$o: (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (s@xx_0)) \Rightarrow (z@(xs@xx)))$  and  $\forall xy: \$i: ((z@xy) \Rightarrow \exists xy_0: \$i \rightarrow \$o: (\lambda xx: \$i \rightarrow \$o: (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (s@xx_0))$  and  $xy = (xs@xx))) = (\lambda xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o: xx = xy@xy_0))$  thf(cTHM110\_pme, conjecture)

**SEV185** $\wedge$ **5.p** TPS problem THM564

$b: \$tType$  thf(b\_type, type)

$\forall p: (b \rightarrow \$o) \rightarrow b \rightarrow \$o, s: (b \rightarrow \$o) \rightarrow \$o: (\forall xx: b \rightarrow \$o: ((s@xx) \Rightarrow \forall x: b \rightarrow \$o, xy: b: ((\forall xx_0: b: ((x@xx_0) \Rightarrow (xx@xx_0))$  and  $p@x@xy) \Rightarrow (xx@xy))) \Rightarrow \forall x: b \rightarrow \$o, xy: b: ((\forall xx: b: ((x@xx) \Rightarrow \forall s_0: b \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xx)))$  and  $p@x@xy) \Rightarrow \forall s_0: b \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xy)))$  thf(cTHM564\_pme, conjecture)

**SEV186** $\wedge$ **5.p** TPS problem THM565

$b: \$tType$  thf(b\_type, type)

$\forall p: (b \rightarrow \$o) \rightarrow (b \rightarrow \$o) \rightarrow \$o, s: (b \rightarrow \$o) \rightarrow \$o: (\forall xx: b \rightarrow \$o: ((s@xx) \Rightarrow \forall x: b \rightarrow \$o, y: b \rightarrow \$o: ((\forall xx_0: b: ((x@xx_0) \Rightarrow (xx@xx_0))$  and  $p@x@y) \Rightarrow \forall xx_0: b: ((y@xx_0) \Rightarrow (xx@xx_0)))) \Rightarrow \forall x: b \rightarrow \$o, y: b \rightarrow \$o: ((\forall xx: b: ((x@xx) \Rightarrow \forall s_0: b \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xx)))$  and  $p@x@y) \Rightarrow \forall xx: b: ((y@xx) \Rightarrow \forall s_0: b \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xx))))$  thf(cTHM565\_pme, conjecture)

**SEV187** $\wedge$ **5.p** TPS problem from CLOS-SYS-THMS

$a: \$tType$  thf(a\_type, type)

$b: \$tType$  thf(b\_type, type)

$c: \$tType$  thf(c\_type, type)

$\forall s: (a \rightarrow b \rightarrow c \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow b \rightarrow c \rightarrow \$o: ((s@xx) \Rightarrow \$true) \Rightarrow \$true)$  thf(cCS3\_ALL\_THM\_pme, conjecture)

**SEV188** $\wedge$ **5.p** TPS problem from CLOS-SYS-THMS

$a: \$tType$  thf(a\_type, type)

$b: \$tType$  thf(b\_type, type)

$\forall s: (a \rightarrow b \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow b \rightarrow \$o: ((s@xx) \Rightarrow \$true) \Rightarrow \$true)$  thf(cCS2\_ALL\_THM\_pme, conjecture)

**SEV189** $\wedge$ **5.p** TPS problem from CLOS-SYS-THMS

$b: \$tType$  thf(b\_type, type)

$cQ: (b \rightarrow \$o) \rightarrow \$o$  thf(cQ, type)

$cP: (b \rightarrow \$o) \rightarrow \$o$  thf(cP, type)

$(\forall s: (b \rightarrow \$o) \rightarrow \$o: (\forall xx: b \rightarrow \$o: ((s@xx) \Rightarrow (cP@xx)) \Rightarrow (cP@\lambda xx: b: \forall s_0: b \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xx))))$  and  $\forall s: (b \rightarrow \$o) \rightarrow \$o: (\forall xx: b \rightarrow \$o: ((s@xx) \Rightarrow (cQ@xx)) \Rightarrow (cQ@\lambda xx: b: \forall s_0: b \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xx)))) \Rightarrow \forall s: (b \rightarrow \$o) \rightarrow \$o: (\forall xx: b \rightarrow \$o: ((s@xx) \Rightarrow (cP@xx$  and  $cQ@xx)) \Rightarrow (cP@\lambda xx: b: \forall s_0: b \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xx))$  and  $cQ@\lambda xx: b: \forall s_0: b \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xx))))$  thf(cTHM567\_pme, conjecture)

**SEV190** $\wedge$ **5.p** TPS problem THM580

The join (in the initial pairing algebra  $\$$ ) of  $x$  and  $x$  is  $x$ .

$iS: \$tType$  thf(iS\_type, type)

$c_0: iS$  thf(c0\_type, type)

$cJOIN: iS \rightarrow iS \rightarrow iS \rightarrow \$o$  thf(cJOIN\_type, type)

$cP: iS \rightarrow iS \rightarrow iS$  thf(cP\_type, type)

$cS\_JOIN\_CLOS: iS \rightarrow (iS \rightarrow iS \rightarrow iS) \rightarrow (iS \rightarrow iS \rightarrow iS \rightarrow \$o) \rightarrow \$o$  thf(cS\_JOIN\_CLOS\_type, type)

$cS\_JOIN\_CLOS = (\lambda x_0: iS, p: iS \rightarrow iS \rightarrow iS, jOIN: iS \rightarrow iS \rightarrow iS \rightarrow \$o: (\forall xx: iS: (jOIN@xx@x_0@xx)$  and  $\forall xy: iS: (jOIN@x_0@xx@xy) \Rightarrow (jOIN@(p@xx@xu)@(p@xy@xv)@(p@xz@xw))))$  thf(cS\_JOIN\_CLOS\_def, definition)

$(\forall xx: iS, xy: iS: (cP@xx@xy) \neq c_0$  and  $\forall xx: iS, xy: iS, xu: iS, xv: iS: ((cP@xx@xu) = (cP@xy@xv) \Rightarrow (xx = xy$  and  $xu = xv))$  and  $\forall x: iS \rightarrow \$o: ((x@c_0$  and  $\forall xx: iS, xy: iS: ((x@xx$  and  $x@xy) \Rightarrow (x@(cP@xx@xy)))) \Rightarrow \forall xx: iS: (x@xx)$  and  $cS\_JOIN\_CLOS@c_0@cP@cJOIN) \Rightarrow \forall xx: iS: (cJOIN@xx@xx@xx)$  thf(cTHM580\_pme, conjecture)

**SEV191** $\wedge$ **5.p** TPS problem S-JOINFN-MONOTONE

The function used to define JOIN as a LFP is monotone.

$a: \$tType$  thf(a\_type, type)

$cP: a \rightarrow a \rightarrow a$  thf(cP, type)

$c_0: a$  thf(c0, type)

$\forall r: a \rightarrow a \rightarrow a \rightarrow \$o: (\$true \Rightarrow \$true)$  and  $\forall r: a \rightarrow a \rightarrow a \rightarrow \$o, s: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true$  and  $\$true$  and  $\forall xa: a, xb: a, xc: a: ((xa = c_0$  and  $xb = xc)$  or  $(xb = c_0$  and  $xa = xc)$  or  $\exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xc: a: (cP@xx_1@xx_2)$  and  $xb = (cP@xy_1@xy_2)$  and  $xc = (cP@xz_1@xz_2)$  and  $r@xx_1@xy_1@xz_1$  and  $r@xx_2@xy_2@xz_2)) \Rightarrow ((xa = c_0$  and  $xb = xc)$  or  $(xb = c_0$  and  $xa = xc)$  or  $\exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2)$  and  $xb = (cP@xy_1@xy_2)$  and  $xc = (cP@xz_1@xz_2)$  and  $s@xx_1@xy_1@xz_1$  and  $s@xx_2@xy_2@xz_2))))$  thf(cS\_JOINFN\_MONOTONE\_pme, conjecture)

**SEV193** $\wedge$ **5.p** TPS problem from S-THMS

$cT: (\$i \rightarrow \$o) \rightarrow \$o$  thf(cT, type)

$\forall s: (\$i \rightarrow \$o) \rightarrow \$o: (\forall xx: \$i \rightarrow \$o: ((s@xx) \Rightarrow (cT@xx)) \Rightarrow (cT@\lambda xx: \$i: \exists s_0: \$i \rightarrow \$o: (s@s_0 \text{ and } s_0@xx))) \Rightarrow$   
 $\forall u: \$i \rightarrow \$o, v: \$i \rightarrow \$o: ((cT@u \text{ and } cT@v) \Rightarrow (cT@\lambda xz: \$i: (u@xz \text{ or } v@xz))) \quad \text{thf}(cTHM501\_pme, \text{conjecture})$

**SEV194** $\wedge$ **5.p** TPS problem from S-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$

$x: a \quad \text{thf}(x, \text{type})$

$c_0: a \quad \text{thf}(c_0, \text{type})$

$cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, \text{type})$

$\forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@c_0@x@x)) \quad \text{thf}(cS\_INCL\_LEM2\_pme, \text{conjecture})$

**SEV195** $\wedge$ **5.p** TPS problem from S-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$

$cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, \text{type})$

$cZ: a \quad \text{thf}(cZ, \text{type})$

$(\forall xx: a, xy: a: (cP@xx@xy) \neq cZ \text{ and } \forall xx: a, xy: a, xu: a, xv: a: ((cP@xx@xu) = (cP@xy@xv) \Rightarrow (xx = xy \text{ and } xu = xv)) \text{ and } \forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx: a, xy: a: ((x@xx \text{ and } x@xy) \Rightarrow (x@(cP@xx@xy)))) \Rightarrow \forall xx: a: (x@xx))) \Rightarrow \forall xx: a: (xx = cZ \text{ or } \exists xy: a, xz: a: xx = (cP@xy@xz)) \quad \text{thf}(cS\_LEM1D\_pme, \text{conjecture})$

**SEV196** $\wedge$ **5.p** TPS problem from S-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$

$y: a \quad \text{thf}(y, \text{type})$

$x: a \quad \text{thf}(x, \text{type})$

$cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, \text{type})$

$c_0: a \quad \text{thf}(c_0, \text{type})$

$\forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@x@c_0)@(cP@c_0@y)@(cP@x@y))) \quad \text{thf}(cS\_JOIN\_LEM2\_pme, \text{conjecture})$

**SEV197** $\wedge$ **5.p** TPS problem from S-THMS

$iS: \$tType \quad \text{thf}(iS\_type, \text{type})$

$c_0: iS \quad \text{thf}(c_0, \text{type})$

$cP: iS \rightarrow iS \rightarrow iS \quad \text{thf}(cP, \text{type})$

$(\forall xx: iS, xy: iS: (cP@xx@xy) \neq c_0 \text{ and } \forall xx: iS, xy: iS, xu: iS, xv: iS: ((cP@xx@xu) = (cP@xy@xv) \Rightarrow (xx = xy \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx: iS, xy: iS: ((x@xx \text{ and } x@xy) \Rightarrow (x@(cP@xx@xy)))) \Rightarrow \forall xx: iS: (x@xx))) \Rightarrow (\forall xx: iS, xy: iS, xu: iS, xv: iS: ((cP@xx@xu) = (cP@xy@xv) \Rightarrow (xx = xy \text{ and } xu = xv)) \text{ and } \forall xx: iS, xy: iS: (cP@xx@xy) \neq c_0) \quad \text{thf}(cS\_ALG02\_pme, \text{conjecture})$

**SEV198** $\wedge$ **5.p** TPS problem from S-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$

$c_0: a \quad \text{thf}(c_0, \text{type})$

$x: a \quad \text{thf}(x, \text{type})$

$cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, \text{type})$

$(\forall xx_0: a, xy: a, xu: a, xv: a: ((cP@xx_0@xu) = (cP@xy@xv) \Rightarrow (xx_0 = xy \text{ and } xu = xv)) \text{ and } \forall xx_0: a, xy: a: (cP@xx_0@xy) \neq c_0) \Rightarrow (\forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@x@c_0@c_0)) \Rightarrow x = c_0) \quad \text{thf}(cS\_INCL\_LEM6\_pme, \text{conjecture})$

**SEV199** $\wedge$ **5.p** TPS problem from S-THMS

$iS: \$tType \quad \text{thf}(iS\_type, \text{type})$

$x: iS \quad \text{thf}(x, \text{type})$

$cP: iS \rightarrow iS \rightarrow iS \quad \text{thf}(cP, \text{type})$

$c_0: iS \quad \text{thf}(c_0, \text{type})$

$(\forall xx_0: iS, xy: iS: (cP@xx_0@xy) \neq c_0 \text{ and } \forall xx_0: iS, xy: iS, xu: iS, xv: iS: ((cP@xx_0@xu) = (cP@xy@xv) \Rightarrow (xx_0 = xy \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx_0: iS, xy: iS: ((x@xx_0 \text{ and } x@xy) \Rightarrow (x@(cP@xx_0@xy)))) \Rightarrow \forall xx_0: iS: (x@xx_0))) \Rightarrow (\forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@x@x@x)) \quad \text{thf}(cS\_INCL\_LEM3\_pme, \text{conjecture})$

**SEV200** $\wedge$ **5.p** TPS problem from S-THMS

$a$ :  $\text{\$tType}$      $\text{thf}(a\_type, \text{type})$

$x$ :  $a$      $\text{thf}(x, \text{type})$

$cZ$ :  $a$      $\text{thf}(cZ, \text{type})$

$cP$ :  $a \rightarrow a \rightarrow a$      $\text{thf}(cP, \text{type})$

$(\forall xx_0: a, xy: a: (cP@xx_0@xy) \neq cZ \text{ and } \forall xx_0: a, xy: a, xu: a, xv: a: ((cP@xx_0@xu) = (cP@xy@xv) \Rightarrow (xx_0 = xy \text{ and } xu = xv)) \text{ and } \forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx_0: a, xy: a: ((x@xx_0 \text{ and } x@xy) \Rightarrow (x@(cP@xx_0@xy)))) \Rightarrow \forall xx_0: a: (x@xx_0))) \Rightarrow \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = cZ \text{ and } xb = xc) \text{ or } (xb = cZ \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@cZ@x@x))$      $\text{thf}(cS\_LEM1C\_pme)$

### SEV203 $\wedge$ 5.p TPS problem from S-THMS

$iS$ :  $\text{\$tType}$      $\text{thf}(iS\_type, \text{type})$

$y$ :  $iS$      $\text{thf}(y, \text{type})$

$x$ :  $iS$      $\text{thf}(x, \text{type})$

$cP$ :  $iS \rightarrow iS \rightarrow iS$      $\text{thf}(cP, \text{type})$

$c_0$ :  $iS$      $\text{thf}(c_0, \text{type})$

$(\forall xx_0: iS, xy_0: iS: (cP@xx_0@xy_0) \neq c_0 \text{ and } \forall xx_0: iS, xy_0: iS, xu: iS, xv: iS: ((cP@xx_0@xu) = (cP@xy_0@xv) \Rightarrow (xx_0 = xy_0 \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx_0: iS, xy_0: iS: ((x@xx_0 \text{ and } x@xy_0) \Rightarrow (x@(cP@xx_0@xy_0)))) \Rightarrow \forall xx_0: iS: (x@xx_0))) \Rightarrow \forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@c_0@y)@(cP@x@y)@(cP@x@y)))$      $\text{thf}(cS\_INCL\_LEM5\_pme, \text{conjecture})$

### SEV204 $\wedge$ 5.p TPS problem from S-THMS

$iS$ :  $\text{\$tType}$      $\text{thf}(iS\_type, \text{type})$

$y$ :  $iS$      $\text{thf}(y, \text{type})$

$x$ :  $iS$      $\text{thf}(x, \text{type})$

$cP$ :  $iS \rightarrow iS \rightarrow iS$      $\text{thf}(cP, \text{type})$

$c_0$ :  $iS$      $\text{thf}(c_0, \text{type})$

$(\forall xx_0: iS, xy_0: iS: (cP@xx_0@xy_0) \neq c_0 \text{ and } \forall xx_0: iS, xy_0: iS, xu: iS, xv: iS: ((cP@xx_0@xu) = (cP@xy_0@xv) \Rightarrow (xx_0 = xy_0 \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx_0: iS, xy_0: iS: ((x@xx_0 \text{ and } x@xy_0) \Rightarrow (x@(cP@xx_0@xy_0)))) \Rightarrow \forall xx_0: iS: (x@xx_0))) \Rightarrow \forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@x@c_0)@(cP@x@y)@(cP@x@y)))$      $\text{thf}(cS\_INCL\_LEM4\_pme, \text{conjecture})$

### SEV205 $\wedge$ 5.p TPS problem from S-THMS

$b$ :  $\text{\$tType}$      $\text{thf}(b\_type, \text{type})$

$iS$ :  $\text{\$tType}$      $\text{thf}(iS\_type, \text{type})$

$cP_2$ :  $b \rightarrow b \rightarrow b$      $\text{thf}(cP_2, \text{type})$

$cP$ :  $iS \rightarrow iS \rightarrow iS$      $\text{thf}(cP, \text{type})$

$c_{02}$ :  $b$      $\text{thf}(c_{02}, \text{type})$

$c_0$ :  $iS$      $\text{thf}(c_0, \text{type})$

$(\forall xx: iS, xy: iS: (cP@xx@xy) \neq c_0 \text{ and } \forall xx: iS, xy: iS, xu: iS, xv: iS: ((cP@xx@xu) = (cP@xy@xv) \Rightarrow (xx = xy \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx: iS, xy: iS: ((x@xx \text{ and } x@xy) \Rightarrow (x@(cP@xx@xy)))) \Rightarrow \forall xx: iS: (x@xx)) \text{ and } \forall xx: b, xy: b, xu: b, xv: b: ((cP_2@xx@xu) = (cP_2@xy@xv) \Rightarrow (xx = xy \text{ and } xu = xv)) \text{ and } \forall xx: b, xy: b: (c_{02}) \Rightarrow \exists xf: iS \rightarrow b: ((xf@c_0) = c_{02} \text{ and } \forall xx: iS, xy: iS: (xf@(cP@xx@xy)) = (cP_2@(xf@xx)@(xf@xy)) \text{ and } \forall xg: iS \rightarrow b: (((xg@c_0) = c_{02} \text{ and } \forall xx: iS, xy: iS: (xg@(cP@xx@xy)) = (cP_2@(xg@xx)@(xg@xy))) \Rightarrow xf = xg))$      $\text{thf}(cTHM\_S\_INIT\_p)$

### SEV206 $\wedge$ 5.p TPS problem from S-THMS

$iS$ :  $\text{\$tType}$      $\text{thf}(iS\_type, \text{type})$

$y$ :  $iS$      $\text{thf}(y, \text{type})$

$z$ :  $iS$      $\text{thf}(z, \text{type})$

$cP$ :  $iS \rightarrow iS \rightarrow iS$      $\text{thf}(cP, \text{type})$

$x$ :  $iS$      $\text{thf}(x, \text{type})$

$c_0$ :  $iS$      $\text{thf}(c_0, \text{type})$

$(\forall xx_0: iS, xy_0: iS: (cP@xx_0@xy_0) \neq c_0 \text{ and } \forall xx_0: iS, xy_0: iS, xu: iS, xv: iS: ((cP@xx_0@xu) = (cP@xy_0@xv) \Rightarrow (xx_0 = xy_0 \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx_0: iS, xy_0: iS: ((x@xx_0 \text{ and } x@xy_0) \Rightarrow (x@(cP@xx_0@xy_0)))) \Rightarrow \forall xx_0: iS: (x@xx_0))) \Rightarrow (\forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow$

$(r@x@y@y) \Rightarrow \forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2))) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@z@x)@(cP@z@y)@(cP@z@y))$

**SEV207** $\wedge$ **5.p** TPS problem from S-THMS

$iS: \$tType \quad thf(iS\_type, type)$

$z: iS \quad thf(z, type)$

$y: iS \quad thf(y, type)$

$cP: iS \rightarrow iS \rightarrow iS \quad thf(cP, type)$

$x: iS \quad thf(x, type)$

$c_0: iS \quad thf(c_0, type)$

$(\forall xx_0: iS, xy_0: iS: (cP@xx_0@xy_0) \neq c_0 \text{ and } \forall xx_0: iS, xy_0: iS, xu: iS, xv: iS: ((cP@xx_0@xu) = (cP@xy_0@xv) \Rightarrow (xx_0 = xy_0 \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx_0: iS, xy_0: iS: ((x@xx_0 \text{ and } x@xy_0) \Rightarrow (x@(cP@xx_0@xy_0)))) \Rightarrow \forall xx_0: iS: (x@xx_0))) \Rightarrow (\forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2))) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@x@y@y) \Rightarrow \forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2))) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@x@z)@(cP@y@z)@(cP@y@z))$

**SEV208** $\wedge$ **5.p** TPS problem from S-THMS

$a: \$tType \quad thf(a\_type, type)$

$z: a \quad thf(z, type)$

$y: a \quad thf(y, type)$

$cP: a \rightarrow a \rightarrow a \quad thf(cP, type)$

$w: a \quad thf(w, type)$

$x: a \quad thf(x, type)$

$c_0: a \quad thf(c_0, type)$

$(\forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2))) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@x@y@y) \text{ and } \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2))) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@w@z@z)) \Rightarrow \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2))) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@x@w)@(cP@y@z)@(cP@y@z))$

**SEV209** $\wedge$ **5.p** TPS problem from S-THMS

$a: \$tType \quad thf(a\_type, type)$

$cP: a \rightarrow a \rightarrow a \quad thf(cP, type)$

$c_0: a \quad thf(c_0, type)$

$(\lambda xa: a, xb: a, xc: a: \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa_0: a, xb_0: a, xc_0: a: (((xa_0 = c_0 \text{ and } xb_0 = xc_0) \text{ or } (xb_0 = c_0 \text{ and } xa_0 = xc_0) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa_0 = (cP@xx_1@xx_2) \text{ and } xb_0 = (cP@xy_1@xy_2) \text{ and } xc_0 = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2))) \Rightarrow (r@xa_0@xb_0@xc_0))) \Rightarrow (r@xa@xb@xc)) = (\lambda xx: a, xy: a, xz: a: ((xx = c_0 \text{ and } xy = xz) \text{ or } (xy = c_0 \text{ and } xx = xz) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xx = (cP@xx_1@xx_2) \text{ and } xy = (cP@xy_1@xy_2) \text{ and } xz = (cP@xz_1@xz_2) \text{ and } \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: ((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_{10}: a, xx_{20}: a, xy_{10}: a, xy_{20}: a, xz_{10}: a, xz_{20}: a: (xa = (cP@xx_{10}@xx_{20}) \text{ and } xb = (cP@xy_{10}@xy_{20}) \text{ and } xc = (cP@xz_{10}@xz_{20}) \text{ and } r@xx_{10}@xy_{10}@xz_{10} \text{ and } r@xx_{20}@xy_{20}@xz_{20})) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@xx_1@xy_1@xz_1)) \text{ and } \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_{10}: a, xx_{20}: a, xy_{10}: a, xy_{20}: a, xz_{10}: a, xz_{20}: a: (xa = (cP@xx_{10}@xx_{20}) \text{ and } xb = (cP@xy_{10}@xy_{20}) \text{ and } xc = (cP@xz_{10}@xz_{20}) \text{ and } r@xx_{10}@xy_{10}@xz_{10} \text{ and } r@xx_{20}@xy_{20}@xz_{20})) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@xx_2@xy_2@xz_2)))) \quad thf(c$

**SEV210** $\wedge$ **5.p** TPS problem from S-THMS

$a: \$tType \quad thf(a\_type, type)$

$v: a \quad thf(v, type)$

$u: a \quad thf(u, type)$

$cP: a \rightarrow a \rightarrow a \quad thf(cP, type)$

$y: a \quad thf(y, type)$

$x: a \quad thf(x, type)$

$cZ: a \quad thf(cZ, type)$

$(\forall xx_0: a, xy_0: a: (cP@xx_0@xy_0) \neq cZ \text{ and } \forall xx_0: a, xy_0: a, xu_0: a, xv_0: a: ((cP@xx_0@xu_0) = (cP@xy_0@xv_0) \Rightarrow (xx_0 = xy_0 \text{ and } xu_0 = xv_0))) \text{ and } \forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx_0: a, xy_0: a: ((x@xx_0 \text{ and } x@xy_0) \Rightarrow (x@(cP@xx_0@xy_0)))) \Rightarrow \forall xx_0: a: (x@xx_0))) \Rightarrow (\forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = cZ \text{ and } xb = xc) \text{ or } (xb = cZ \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@x@u@u)) \Rightarrow (\forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = cZ \text{ and } xb = xc) \text{ or } (xb = cZ \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@y@v@v)) \Rightarrow \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = cZ \text{ and } xb = xc) \text{ or } (xb = cZ \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@y@v@v)) \Rightarrow \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = cZ \text{ and } xb = xc) \text{ or } (xb = cZ \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@x@y)@(cP@u@v)@(cP@u@v)))))) \quad \text{thf}(cS\_LEM1E\_pme, \text{conjecture})$

### SEV211 $\wedge$ 5.p TPS problem from S-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$

$y: a \quad \text{thf}(y, type)$

$cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, type)$

$cZ: a \quad \text{thf}(cZ, type)$

$x: a \quad \text{thf}(x, type)$

$z: a \quad \text{thf}(z, type)$

$(\forall xx_0: a, xy_0: a: (cP@xx_0@xy_0) \neq cZ \text{ and } \forall xx_0: a, xy_0: a, xu: a, xv: a: ((cP@xx_0@xu) = (cP@xy_0@xv) \Rightarrow (xx_0 = xy_0 \text{ and } xu = xv)) \text{ and } \forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx_0: a, xy_0: a: ((x@xx_0 \text{ and } x@xy_0) \Rightarrow (x@(cP@xx_0@xy_0)))) \Rightarrow \forall xx_0: a: (x@xx_0))) \Rightarrow (\forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = cZ \text{ and } xb = xc) \text{ or } (xb = cZ \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@x@y)@z@z)) \Rightarrow \exists xu: a, xv: a: (z = (cP@xu@xv) \text{ and } \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = cZ \text{ and } xb = xc) \text{ or } (xb = cZ \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@x@xu@xu)) \text{ and } \forall r: a \rightarrow a \rightarrow a \rightarrow \$o: ((\$true \text{ and } \forall xa: a, xb: a, xc: a: (((xa = cZ \text{ and } xb = xc) \text{ or } (xb = cZ \text{ and } xa = xc) \text{ or } \exists xx_1: a, xx_2: a, xy_1: a, xy_2: a, xz_1: a, xz_2: a: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@y@xv@xv)))))) \quad \text{thf}(cS\_LEM1F\_p, \text{conjecture})$

### SEV212 $\wedge$ 5.p TPS problem from S-THMS

$iS: \$tType \quad \text{thf}(iS\_type, type)$

$cP: iS \rightarrow iS \rightarrow iS \quad \text{thf}(cP, type)$

$c_0: iS \quad \text{thf}(c_0, type)$

$(\forall xx: iS, xy: iS: (cP@xx@xy) \neq c_0 \text{ and } \forall xx: iS, xy: iS, xu: iS, xv: iS: ((cP@xx@xu) = (cP@xy@xv) \Rightarrow (xx = xy \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx: iS, xy: iS: ((x@xx \text{ and } x@xy) \Rightarrow (x@(cP@xx@xy)))) \Rightarrow \forall xx: iS: (x@xx))) \Rightarrow \forall xx: iS, xy: iS, xu: iS, xv: iS: ((\forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@xx@xy@xy)) \text{ and } \forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@xu@xv@xv))) \Rightarrow \forall r: iS \rightarrow iS \rightarrow iS \rightarrow \$o: ((\$true \text{ and } \forall xa: iS, xb: iS, xc: iS: (((xa = c_0 \text{ and } xb = xc) \text{ or } (xb = c_0 \text{ and } xa = xc) \text{ or } \exists xx_1: iS, xx_2: iS, xy_1: iS, xy_2: iS, xz_1: iS, xz_2: iS: (xa = (cP@xx_1@xx_2) \text{ and } xb = (cP@xy_1@xy_2) \text{ and } xc = (cP@xz_1@xz_2) \text{ and } r@xx_1@xy_1@xz_1 \text{ and } r@xx_2@xy_2@xz_2)) \Rightarrow (r@xa@xb@xc))) \Rightarrow (r@(cP@xx@xu)@(cP@xy@xv)@(cP@xy@xv)))))) \quad \text{thf}(cS\_LEM2\_pme, \text{conjecture})$

### SEV214 $\wedge$ 5.p TPS problem from S-T-THMS

$iS: \$tType \quad \text{thf}(iS\_type, type)$

$c_0: iS \quad \text{thf}(c_0, type)$

$cP: iS \rightarrow iS \rightarrow iS \quad \text{thf}(cP, type)$

$(\forall xx: iS, xy: iS: (cP@xx@xy) \neq c_0 \text{ and } \forall xx: iS, xy: iS, xu: iS, xv: iS: ((cP@xx@xu) = (cP@xy@xv) \Rightarrow (xx = xy \text{ and } xu = xv)) \text{ and } \forall x: iS \rightarrow \$o: ((x@c_0 \text{ and } \forall xx: iS, xy: iS: ((x@xx \text{ and } x@xy) \Rightarrow (x@(cP@xx@xy)))) \Rightarrow \forall xx: iS: (x@xx))) \Rightarrow ((\lambda xx: iS: (xx = c_0 \text{ or } \exists xy: iS: c_0 = (cP@xx@xy))) = (\lambda xx: iS, xy: iS: xx = xy@c_0) \text{ and } (\lambda xy: iS: (xy = c_0 \text{ or } \exists xx: iS: c_0 = (cP@xx@xy))) = (\lambda xx: iS, xy: iS: xx = xy@c_0))) \quad \text{thf}(cS\_T\_LR\_LEM2\_pme, \text{conjecture})$

### SEV218 $\wedge$ 5.p TPS problem from CHOICE-COVER-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$

$cQ: a \rightarrow a \rightarrow \$o \quad \text{thf}(cQ, type)$



$\exists x f: a \rightarrow a \rightarrow \$o: \forall xx: a: (\exists xz: a: (xf@xx@xz) \text{ and } \forall xx_0: a: ((xf@xx@xx_0) \Rightarrow \forall xy: a: ((xf@xx@xy) \iff (cQ@xx_0@xy)))) \text{ and } xf@xx@xx) \Rightarrow (\forall xx: a: (cQ@xx@xx) \text{ and } \forall xx: a, xy: a: ((cQ@xx@xy) \Rightarrow (cQ@xy@xx))) \text{ and } \forall xx: a, xy: (cQ@xx@xz)))$  thf(cTHM559A\_pme, conjecture)

**SEV220** $\wedge$ **5.p** TPS problem X5205

$b: \$tType$  thf(b\_type, type)

$a: \$tType$  thf(a\_type, type)

$f: b \rightarrow a$  thf(f, type)

$w: (b \rightarrow \$o) \rightarrow \$o$  thf(w, type)

$\forall xx: a: (\exists xt: b: (\forall s: b \rightarrow \$o: ((w@s) \Rightarrow (s@xt)) \text{ and } xx = (f@xt)) \Rightarrow \forall s: a \rightarrow \$o: (\exists xt: b \rightarrow \$o: (w@xt \text{ and } s = (\lambda xz: a: \exists xt_0: b: (xt@xt_0 \text{ and } xz = (f@xt_0)))) \Rightarrow (s@xx)))$  thf(cX5205\_pme, conjecture)

**SEV221** $\wedge$ **5.p** TPS problem THM61

$a: \$tType$  thf(a\_type, type)

$cZ: a \rightarrow \$o$  thf(cZ, type)

$cW: (a \rightarrow \$o) \rightarrow \$o$  thf(cW, type)

$\forall xx: a: ((\exists s: a \rightarrow \$o: (cW@s \text{ and } s@xx) \text{ and } cZ@xx) \iff \exists s: a \rightarrow \$o: (\exists xt: a \rightarrow \$o: (cW@xt \text{ and } s = (\lambda xx_0: a: (cZ@xx_0 \text{ and } xt@xx_0))) \text{ and } s@xx)))$  thf(cTHM61\_pme, conjecture)

**SEV222** $\wedge$ **5.p** TPS problem THM60

$a: \$tType$  thf(a\_type, type)

$cZ: a \rightarrow \$o$  thf(cZ, type)

$cW: (a \rightarrow \$o) \rightarrow \$o$  thf(cW, type)

$\forall xx: a: ((\forall s: a \rightarrow \$o: ((cW@s) \Rightarrow (s@xx)) \text{ or } cZ@xx) \iff \forall s: a \rightarrow \$o: (\exists xt: a \rightarrow \$o: (cW@xt \text{ and } s = (\lambda xz: a: (cZ@xz \text{ or } xt@xz))) \Rightarrow (s@xx)))$  thf(cTHM60\_pme, conjecture)

**SEV223** $\wedge$ **5.p** TPS problem X5204

$b: \$tType$  thf(b\_type, type)

$a: \$tType$  thf(a\_type, type)

$f: b \rightarrow a$  thf(f, type)

$w: (b \rightarrow \$o) \rightarrow \$o$  thf(w, type)

$(\lambda xz: a: \exists xt: b: (\exists s: b \rightarrow \$o: (w@s \text{ and } s@xt) \text{ and } xz = (f@xt))) = (\lambda xx: a: \exists s: a \rightarrow \$o: (\exists xt: b \rightarrow \$o: (w@xt \text{ and } s = (\lambda xz: a: \exists xt_0: b: (xt@xt_0 \text{ and } xz = (f@xt_0)))) \text{ and } s@xx))$  thf(cX5204\_pme, conjecture)

**SEV224** $\wedge$ **5.p** TPS problem from FUNS-AND-SETS-OF-SETS-THMS

$b: \$tType$  thf(b\_type, type)

$a: \$tType$  thf(a\_type, type)

$\forall xa: b \rightarrow a \rightarrow \$o, xy: a, xr: b \rightarrow \$o: (xr = (\lambda xj: b: (xa@xj@xy)) \Rightarrow \exists xp: (b \rightarrow \$o) \rightarrow (b \rightarrow \$o) \rightarrow \$o: \forall xs: b \rightarrow \$o: (\forall s: a \rightarrow \$o: (\exists xt: b: (xs@xt \text{ and } s = (xa@xt)) \Rightarrow (s@xy)) \iff (xp@xr@xs)))$  thf(cTHM142.1\_pme, conjecture)

**SEV225** $\wedge$ **5.p** TPS problem from REALS-THMS

$r: \$tType$  thf(r\_type, type)

$c_0: r$  thf(c0\_type, type)

less:  $r \rightarrow r \rightarrow \$o$  thf(less\_type, type)

cIRREFLEXIVE\_LAW:  $\$o$  thf(cIRREFLEXIVE\_LAW\_type, type)

cIRREFLEXIVE\_LAW =  $(\forall xx: r: \neg \text{less}@xx@xx)$  thf(cIRREFLEXIVE\_LAW\_def, definition)

cIRREFLEXIVE\_LAW  $\Rightarrow \forall xx: r: \neg \text{less}@xx@c_0 \text{ and } xx = c_0$  thf(cPARNAS\_FIG3-A, conjecture)

**SEV226** $\wedge$ **5.p** TPS problem from REALS-THMS

c\_less.:  $\$i \rightarrow \$i \rightarrow \$o$  thf(c\_less., type)

$b: \$i$  thf(b, type)

$a: \$i$  thf(a, type)

$\forall xf: \$i \rightarrow \$i, x_0: \$i: ((\forall a: \$i \rightarrow \$o: ((\exists xl: \$i: (a@xl) \text{ and } \exists xu: \$i: \forall xx: \$i: ((a@xx) \Rightarrow (c\_less\_@xx@xu \text{ or } xx = xu))) \Rightarrow \exists xl: \$i: (\forall xx: \$i: ((a@xx) \Rightarrow (c\_less\_@xx@xl \text{ or } xx = xl)) \text{ and } \forall xy: \$i: (\forall xx: \$i: ((a@xx) \Rightarrow (c\_less\_@xx@xy \text{ or } xx = xy)) \Rightarrow (c\_less\_@xl@xy \text{ or } xl = xy)))) \text{ and } \forall xx: \$i: ((c\_less\_@x_0@(xf@xx)) \Rightarrow \exists xt: \$i: (c\_less\_@xt@x_0 @ (c\_less\_@x_0@(xf@xs)))) \text{ and } \forall xx: \$i: ((c\_less\_@(xf@xx)@x_0) \Rightarrow \exists xt: \$i: (c\_less\_@xx@xt \text{ and } \forall xs: \$i: ((c\_less\_@xs@xt \text{ and } c\_less\_@xt@x_0) \Rightarrow (c\_less\_@xs@x_0)))) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i: ((c\_less\_@xx@xy \text{ and } c\_less\_@xy@xz) \Rightarrow (c\_less\_@xx@xz)) \text{ and } \forall xx: \$i: \neg c\_less\_@a@b \text{ and } c\_less\_@a@b \text{ and } c\_less\_@(xf@a)@x_0 \text{ and } c\_less\_@x_0@(xf@b)) \Rightarrow \exists xx: \$i: (c\_less\_@a@xx \text{ and } c\_less\_@xx@b \text{ and } \neg c\_less\_@a@b))$

**SEV227** $\wedge$ **5.p** TPS problem X5200

$a: \$tType$  thf(a\_type, type)

$y: a \rightarrow \$o$  thf(y, type)

$x: a \rightarrow \$o$  thf(x, type)

$(\lambda xz: a: (x@xz \text{ or } y@xz)) = (\lambda xx_0: a: \exists s: a \rightarrow \$o: ((s = x \text{ or } s = y) \text{ and } s@xx_0))$  thf(cX5200\_pme, conjecture)

**SEV228**<sup>5.p</sup> TPS problem THM91A

$a: \$tType \quad thf(a\_type, type)$

$cS: a \rightarrow \$o \quad thf(cS, type)$

$\forall xx: a: ((cS@xx) \Rightarrow \exists s_{11}: a \rightarrow \$o: (\forall xx_0: a: ((s_{11}@xx_0) \Rightarrow (cS@xx_0)) \text{ and } s_{11}@xx)) \quad thf(cTHM91A\_pme, conjecture)$

**SEV229**<sup>5.p</sup> TPS problem X5209

$a: \$tType \quad thf(a\_type, type)$

$cE: a \rightarrow \$o \quad thf(cE, type)$

$cD: a \rightarrow \$o \quad thf(cD, type)$

$(\lambda r: a \rightarrow \$o: \forall xx: a: ((r@xx) \Rightarrow (cD@xx \text{ and } cE@xx))) = (\lambda xx: a \rightarrow \$o: (\forall xx_0: a: ((xx@xx_0) \Rightarrow (cD@xx_0)) \text{ and } \forall xx_0: a: ((xx@xx_0) \Rightarrow (cE@xx_0)))) \quad thf(cX5209\_pme, conjecture)$

**SEV230**<sup>5.p</sup> TPS problem THM88

$a: \$tType \quad thf(a\_type, type)$

$\forall u: a \rightarrow \$o, v: a \rightarrow \$o: (\forall xx: a: ((u@xx) \Rightarrow (v@xx)) \Rightarrow \forall xx: (a \rightarrow \$o) \rightarrow \$o: (\forall xx_0: a \rightarrow \$o: ((xx@xx_0) \Rightarrow$

$\forall xx_1: a: ((xx_0@xx_1) \Rightarrow (u@xx_1))) \Rightarrow \forall xx_0: a \rightarrow \$o: ((xx@xx_0) \Rightarrow \forall xx_1: a: ((xx_0@xx_1) \Rightarrow (v@xx_1)))) \quad thf(cTHM88\_pme, conjecture)$

**SEV231**<sup>5.p</sup> TPS problem X5201

$a: \$tType \quad thf(a\_type, type)$

$y: a \rightarrow \$o \quad thf(y, type)$

$x: a \rightarrow \$o \quad thf(x, type)$

$(\lambda xx_0: a: (x@xx_0 \text{ and } y@xx_0)) = (\lambda xx_0: a: \forall s: a \rightarrow \$o: ((s = x \text{ or } s = y) \Rightarrow (s@xx_0))) \quad thf(cX5201\_pme, conjecture)$

**SEV232**<sup>5.p</sup> TPS problem X6007

$cS: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cS, type)$

$c_0: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(c_0, type)$

$(\lambda n: (\$i \rightarrow \$o) \rightarrow \$o: \forall p: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((p@c_0 \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o: ((p@x) \Rightarrow (p@(cS@x)))) \Rightarrow (p@n))) = (\lambda xx: (\$i \rightarrow \$o) \rightarrow \$o: \forall s_0: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((s_0@c_0 \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o: ((s_0@x) \Rightarrow (s_0@(cS@x)))) \Rightarrow (s_0@xx))) \quad thf(cX6007\_pme, conjecture)$

**SEV233**<sup>5.p</sup> TPS problem THM46

$cE: \$i \rightarrow \$o \quad thf(cE, type)$

$cD: \$i \rightarrow \$o \quad thf(cD, type)$

$\forall xx: \$i \rightarrow \$o: (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (cD@xx_0 \text{ and } cE@xx_0)) \Rightarrow (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (cD@xx_0)) \text{ and } \forall xx_0: \$i: ((xx@xx_0) \Rightarrow (cE@xx_0)))) \quad thf(cTHM46\_pme, conjecture)$

**SEV234**<sup>5.p</sup> TPS problem BLEDSOE-FENG-SV-10

If a set B has the property that every x in B has a nbhd D subset B, then B is open.

$cOPEN: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cOPEN, type)$

$\forall d: \$i \rightarrow \$o, g: (\$i \rightarrow \$o) \rightarrow \$o: ((\forall xx: \$i \rightarrow \$o: ((g@xx) \Rightarrow (cOPEN@xx)) \text{ and } d = (\lambda xx: \$i: \exists s: \$i \rightarrow \$o: (g@s \text{ and } s@xx))) \Rightarrow (cOPEN@d)) \Rightarrow \forall b: \$i \rightarrow \$o: (\forall xx: \$i: ((b@xx) \Rightarrow \exists d: \$i \rightarrow \$o: (cOPEN@d \text{ and } d@xx \text{ and } \forall xx_0: \$i: (b@xx_0))) \Rightarrow (cOPEN@b)) \quad thf(cBLEDSOE\_FENG\_SV\_10\_pme, conjecture)$

**SEV235**<sup>5.p</sup> TPS problem THM46A

$cE: \$i \rightarrow \$o \quad thf(cE, type)$

$cD: \$i \rightarrow \$o \quad thf(cD, type)$

$\forall xx: \$i \rightarrow \$o: (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (cD@xx_0 \text{ and } cE@xx_0)) \iff (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (cD@xx_0)) \text{ and } \forall xx_0: \$i: ((xx@xx_0) \Rightarrow (cE@xx_0)))) \quad thf(cTHM46A\_pme, conjecture)$

**SEV236**<sup>5.p</sup> TPS problem THM91

$a: \$tType \quad thf(a\_type, type)$

$cS: a \rightarrow \$o \quad thf(cS, type)$

$cK: (a \rightarrow \$o) \rightarrow a \rightarrow \$o \quad thf(cK, type)$

$\forall x: a \rightarrow \$o, y: a \rightarrow \$o: (\forall xx: a: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: a: ((cK@x@xx) \Rightarrow (cK@y@xx))) \Rightarrow$

$\forall xx: a: ((cK@cS@xx) \Rightarrow (cK@\lambda xx_0: a: \exists s_3: a \rightarrow \$o: (\forall xx_1: a: ((s_3@xx_1) \Rightarrow (cS@xx_1)) \text{ and } s_3@xx_0)@xx)) \quad thf(cTHM91\_pme, conjecture)$

**SEV237**<sup>5.p</sup> TPS problem THM616

$cOPEN: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cOPEN, type)$

$\forall g: (\$i \rightarrow \$o) \rightarrow \$o: (\forall xx: \$i \rightarrow \$o: ((g@xx) \Rightarrow (cOPEN@xx)) \Rightarrow (cOPEN@\lambda xx: \$i: \exists s: \$i \rightarrow \$o: (g@s \text{ and } s@xx))) \Rightarrow$

$\forall b: \$i \rightarrow \$o: (\forall xx: \$i: ((b@xx) \Rightarrow \exists d: \$i \rightarrow \$o: (cOPEN@d \text{ and } d@xx \text{ and } \forall xx_0: \$i: ((d@xx_0) \Rightarrow (b@xx_0)))) \Rightarrow (cOPEN@b)) \quad thf(cTHM616\_pme, conjecture)$

**SEV238**<sup>5.p</sup> TPS problem THM2D

$\forall k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: ((\forall xx: \$i \rightarrow \$o: (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow \exists s: \$i \rightarrow \$o: (\forall xx_1: \$i: ((s@xx_1) \Rightarrow (k@s@xx_1)) \text{ and } s@xx_0)) \Rightarrow \forall xx_0: \$i: ((k@xx@xx_0) \Rightarrow (k@\lambda xx_1: \$i: \exists s: \$i \rightarrow \$o: (\forall xx_2: \$i: ((s@xx_2) \Rightarrow$



**cK:**  $(a \rightarrow \$o) \rightarrow a \rightarrow \$o$      $\text{thf}(\text{cK}, \text{type})$

$\forall x: a \rightarrow \$o, y: a \rightarrow \$o: (\forall xx: a: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: a: ((\text{cK}@x@xx) \Rightarrow (\text{cK}@y@xx))) \Rightarrow$   
 $\forall xx: a: ((\text{cK}@lxx_0: a: \exists s: a \rightarrow \$o: (\forall xx_1: a: ((s@xx_1) \Rightarrow (\text{cK}@s@xx_1)) \text{ and } s@xx_0)@xx) \Rightarrow \exists s: a \rightarrow$   
 $\$o: (\forall xx_0: a: ((s@xx_0) \Rightarrow (\text{cK}@s@xx_0)) \text{ and } s@xx))$      $\text{thf}(\text{cTHM116\_1S}, \text{conjecture})$

**SEV248^5.p** TPS problem from SETS-OF-SETS-THMS

**a:**  $\text{\$tType}$      $\text{thf}(\text{a\_type}, \text{type})$

$\forall x: a, y: a, u: a, v: a: (\forall xx: a \rightarrow \$o: ((xx = (\lambda xy: a: x = xy) \text{ or } \forall w: a: ((xx@w) \iff (w = x \text{ or } w = y))) \iff$   
 $(xx = (\lambda xy: a: u = xy) \text{ or } \forall w: a: ((xx@w) \iff (w = u \text{ or } w = v)))) \iff (x = u \text{ and } y =$   
 $v))$      $\text{thf}(\text{cTHM103\_pme}, \text{conjecture})$

**SEV249^5.p** TPS problem from SETS-OF-SETS-THMS

**cX:**  $\text{\$i} \rightarrow \$o$      $\text{thf}(\text{cX}, \text{type})$

$\forall xw: (\text{\$i} \rightarrow \$o) \rightarrow \$o: ((xw@lxx: \text{\$i}: \$\text{false} \text{ and } \forall xr: \text{\$i} \rightarrow \$o, xx: \text{\$i}: ((xw@xr) \Rightarrow (xw@lxt: \text{\$i}: (xr@xt \text{ or } xt =$   
 $xx)))) \Rightarrow (xw@cX)) \Rightarrow \forall xw: ((\text{\$i} \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xw@lxx: \text{\$i} \rightarrow \$o: \$\text{false} \text{ and } \forall xr: (\text{\$i} \rightarrow \$o) \rightarrow$   
 $\$o, xx: \text{\$i} \rightarrow \$o: ((xw@xr) \Rightarrow (xw@lxt: \text{\$i} \rightarrow \$o: (xr@xt \text{ or } xt = xx)))) \Rightarrow (xw@lr: \text{\$i} \rightarrow \$o: \forall xx: \text{\$i}: ((r@xx) \Rightarrow$   
 $(cX@xx))))$      $\text{thf}(\text{cTHM626\_pme}, \text{conjecture})$

**SEV250^5.p** TPS problem from SETS-OF-SETS-THMS

**cOPEN:**  $(\text{\$i} \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cOPEN}, \text{type})$

$\forall d: \text{\$i} \rightarrow \$o, g: (\text{\$i} \rightarrow \$o) \rightarrow \$o: ((\forall xx: \text{\$i} \rightarrow \$o: ((g@xx) \Rightarrow (\text{cOPEN}@xx)) \text{ and } d = (\lambda xx: \text{\$i}: \exists s: \text{\$i} \rightarrow$   
 $\$o: (g@s \text{ and } s@xx))) \Rightarrow (\text{cOPEN}@d)) \Rightarrow \forall a: \text{\$i} \rightarrow \$o: \exists b: \text{\$i} \rightarrow \$o: (\text{cOPEN}@b \text{ and } \forall xx: \text{\$i}: ((b@xx) \Rightarrow$   
 $(a@xx)) \text{ and } \forall c: \text{\$i} \rightarrow \$o: ((\text{cOPEN}@c \text{ and } \forall xx: \text{\$i}: ((c@xx) \Rightarrow (a@xx))) \Rightarrow \forall xx: \text{\$i}: ((c@xx) \Rightarrow (b@xx))))$      $\text{thf}(\text{cEXISTS}, \text{conjecture})$

**SEV251^5.p** TPS problem from SETS-OF-SETS-THMS

**cC:**  $(\text{\$i} \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cC}, \text{type})$

**cL:**  $(\text{\$i} \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cL}, \text{type})$

$(\forall x: \text{\$i} \rightarrow \$o, y: \text{\$i} \rightarrow \$o: ((cL@x \text{ and } cL@y) \Rightarrow (\forall xx: \text{\$i}: ((x@xx) \Rightarrow (y@xx)) \text{ or } \forall xx: \text{\$i}: ((y@xx) \Rightarrow$   
 $(x@xx)))) \text{ and } \forall xx: \text{\$i} \rightarrow \$o: ((cC@xx) \Rightarrow (cL@xx)) \text{ and } \forall xw: ((\text{\$i} \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xw@lxx: \text{\$i} \rightarrow$   
 $\$o: \$\text{false} \text{ and } \forall xr: (\text{\$i} \rightarrow \$o) \rightarrow \$o, xx: \text{\$i} \rightarrow \$o: ((xw@xr) \Rightarrow (xw@lxt: \text{\$i} \rightarrow \$o: (xr@xt \text{ or } xt = xx)))) \Rightarrow$   
 $(xw@cC)) \Rightarrow \exists u: \text{\$i} \rightarrow \$o: (cC@u \text{ and } \forall v: \text{\$i} \rightarrow \$o: ((cC@v) \Rightarrow \forall xx: \text{\$i}: ((u@xx) \Rightarrow (v@xx))))$      $\text{thf}(\text{cTHM629\_pme}, \text{conjecture})$

**SEV252^5.p** TPS problem from SETS-OF-SETS-THMS

$\forall k: (\text{\$i} \rightarrow \$o) \rightarrow \text{\$i} \rightarrow \$o: ((\forall xx: \text{\$i} \rightarrow \$o, xy: \text{\$i} \rightarrow \$o: (\forall xx_0: \text{\$i}: ((xx@xx_0) \Rightarrow (xy@xx_0)) \Rightarrow \forall xx_0: \text{\$i}: ((k@xx@xx_0) \Rightarrow$   
 $(k@xy@xx_0))) \text{ and } \forall xx: \text{\$i} \rightarrow \$o, xy: \text{\$i} \rightarrow \$o: (\forall xx_0: \text{\$i}: ((xx@xx_0) \Rightarrow (xy@xx_0)) \Rightarrow \forall xx_0: \text{\$i}: ((k@xx@xx_0) \Rightarrow$   
 $(k@xy@xx_0)))) \Rightarrow \forall xx: \text{\$i}: ((k@lxx_0: \text{\$i}: \exists s: \text{\$i} \rightarrow \$o: (\forall xx_1: \text{\$i}: ((s@xx_1) \Rightarrow (k@s@xx_1)) \text{ and } s@xx_0)@xx) \iff$   
 $\exists s: \text{\$i} \rightarrow \$o: (\forall xx_0: \text{\$i}: ((s@xx_0) \Rightarrow (k@s@xx_0)) \text{ and } s@xx))$      $\text{thf}(\text{cTHM2A\_EXPANDED\_pme}, \text{conjecture})$

**SEV253^5.p** TPS problem from SETS-OF-SETS-THMS

**cL:**  $(\text{\$i} \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cL}, \text{type})$

**cG:**  $(\text{\$i} \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cG}, \text{type})$

**cF:**  $((\text{\$i} \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cF}, \text{type})$

$(\forall c: (\text{\$i} \rightarrow \$o) \rightarrow \$o: ((\forall xx: \text{\$i} \rightarrow \$o: ((c@xx) \Rightarrow (\text{cG}@xx)) \text{ and } \forall xx: \text{\$i}: \exists y: \text{\$i} \rightarrow \$o: (c@y \text{ and } y@xx)) \Rightarrow$   
 $\exists d: (\text{\$i} \rightarrow \$o) \rightarrow \$o: (\text{cF}@d \text{ and } \forall xx: \text{\$i} \rightarrow \$o: ((d@xx) \Rightarrow (c@xx)) \text{ and } \forall xx: \text{\$i}: \exists y: \text{\$i} \rightarrow \$o: (d@y \text{ and } y@xx))) \text{ and } \forall c: (\text{\$i} \rightarrow$   
 $\$o) \rightarrow \$o: ((cF@c) \Rightarrow (\text{cF}@lu: \text{\$i} \rightarrow \$o: (c@lxx: \text{\$i}: \neg u@xx))) \text{ and } \forall b: (\text{\$i} \rightarrow \$o) \rightarrow \$o: ((cF@b \text{ and } \forall xx: \text{\$i} \rightarrow$   
 $\$o: ((b@xx) \Rightarrow (\text{cL}@xx))) \Rightarrow \exists xm: \text{\$i}: \forall z: \text{\$i} \rightarrow \$o: ((b@z) \Rightarrow (z@xm))) \text{ and } \forall z: \text{\$i} \rightarrow \$o: ((cL@z) \Rightarrow$   
 $(\text{cG}@lxx: \text{\$i}: \neg z@xx)) \Rightarrow \exists xa: \text{\$i}: \forall z: \text{\$i} \rightarrow \$o: ((cL@z) \Rightarrow (z@xa))$      $\text{thf}(\text{cTHM630\_pme}, \text{conjecture})$

**SEV254^5.p** TPS problem from SETS-OF-SETS-THMS

$\forall k: (\text{\$i} \rightarrow \$o) \rightarrow \text{\$i} \rightarrow \$o: ((\forall xx: \text{\$i} \rightarrow \$o: (\forall xx_0: \text{\$i}: ((xx@xx_0) \Rightarrow \exists s: \text{\$i} \rightarrow \$o: (\forall xx_1: \text{\$i}: ((s@xx_1) \Rightarrow$   
 $(k@s@xx_1)) \text{ and } s@xx_0)) \Rightarrow \forall xx_0: \text{\$i}: ((k@xx@xx_0) \Rightarrow (k@lxx_1: \text{\$i}: \exists s: \text{\$i} \rightarrow \$o: (\forall xx_2: \text{\$i}: ((s@xx_2) \Rightarrow$   
 $(k@s@xx_2)) \text{ and } s@xx_1)@xx_0))) \text{ and } (\forall xx: \text{\$i}: (\exists s: \text{\$i} \rightarrow \$o: (\forall xx_0: \text{\$i}: ((s@xx_0) \Rightarrow (k@s@xx_0)) \text{ and } s@xx) \Rightarrow$   
 $(k@lxx_0: \text{\$i}: \exists s: \text{\$i} \rightarrow \$o: (\forall xx_1: \text{\$i}: ((s@xx_1) \Rightarrow (k@s@xx_1)) \text{ and } s@xx_0)@xx) \Rightarrow \forall xx: \text{\$i}: ((k@lxx_0: \text{\$i}: \exists s: \text{\$i} \rightarrow$   
 $\$o: (\forall xx_1: \text{\$i}: ((s@xx_1) \Rightarrow (k@s@xx_1)) \text{ and } s@xx_0)@xx) \Rightarrow (k@(k@lxx_0: \text{\$i}: \exists s: \text{\$i} \rightarrow \$o: (\forall xx_1: \text{\$i}: ((s@xx_1) \Rightarrow$   
 $(k@s@xx_1)) \text{ and } s@xx_0)@xx)))) \Rightarrow \forall xx: \text{\$i}: ((k@lxx_0: \text{\$i}: \exists s: \text{\$i} \rightarrow \$o: (\forall xx_1: \text{\$i}: ((s@xx_1) \Rightarrow (k@s@xx_1)) \text{ and } s@xx_0)@xx) \iff$   
 $\exists s: \text{\$i} \rightarrow \$o: (\forall xx_0: \text{\$i}: ((s@xx_0) \Rightarrow (k@s@xx_0)) \text{ and } s@xx))$      $\text{thf}(\text{cTHM2C\_pme}, \text{conjecture})$

**SEV256^5.p** TPS problem THM625A

THM625 without assuming full topology axioms.

**a:**  $\text{\$tType}$      $\text{thf}(\text{a\_type}, \text{type})$

**cOPEN:**  $(a \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cOPEN}, \text{type})$

$\forall k: (a \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow (\text{cOPEN}@xx)) \Rightarrow (\text{cOPEN}@lxx: a: \exists s: a \rightarrow \$o: (k@s \text{ and } s@xx))) \Rightarrow$   
 $(\text{cOPEN}@lxx: a: \$\text{false})$      $\text{thf}(\text{cTHM625A\_pme}, \text{conjecture})$

**SEV257^5.p** TPS problem THM625

Empty sets are open in any topology.

$a: \text{\$tType} \quad \text{thf}(a\_type, type)$

$cOPEN: (a \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cOPEN, type)$

$(cOPEN@l\lambda xy: a: \$true \text{ and } \forall k: (a \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow (cOPEN@xx)) \Rightarrow (cOPEN@l\lambda xx: a: \exists s: a \rightarrow \$o: (k@s \text{ and } s@xx))) \text{ and } \forall y: a \rightarrow \$o, z: a \rightarrow \$o: ((cOPEN@y \text{ and } cOPEN@z) \Rightarrow (cOPEN@l\lambda xx: a: (y@xx \text{ and } z@xx)))) \Rightarrow (cOPEN@l\lambda xy: a: \$false) \quad \text{thf}(cTHM625\_pme, conjecture)$

### SEV258^5.p TPS problem DISCRETE-TOPOLOGY

The discrete topology really is a topology.

$a: \text{\$tType} \quad \text{thf}(a\_type, type)$

$\forall r: a \rightarrow \$o: (r = (l\lambda xx: a: \$false) \Rightarrow \$true) \text{ and } \forall r: a \rightarrow \$o: (r = (l\lambda xx: a: \neg \$false) \Rightarrow \$true) \text{ and } \forall k: (a \rightarrow \$o) \rightarrow \$o, r: a \rightarrow \$o: ((\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow \$true) \text{ and } r = (l\lambda xx: a: \exists s: a \rightarrow \$o: (k@s \text{ and } s@xx))) \Rightarrow \$true) \text{ and } \forall y: a \rightarrow \$o, z: a \rightarrow \$o, s: a \rightarrow \$o: ((\$true \text{ and } \$true \text{ and } s = (l\lambda xx: a: (y@xx \text{ and } z@xx))) \Rightarrow \$true) \quad \text{thf}(cDISC$

### SEV259^5.p TPS problem CLOSURE-THM0

$b: \text{\$tType} \quad \text{thf}(b\_type, type)$

$\forall s: (b \rightarrow \$o) \rightarrow \$o: ((\forall r: b \rightarrow \$o: (r = (l\lambda xx: b: \$false) \Rightarrow (s@r)) \text{ and } \forall r: b \rightarrow \$o: (r = (l\lambda xx: b: \neg \$false) \Rightarrow (s@r)) \text{ and } \forall k: (b \rightarrow \$o) \rightarrow \$o, r: b \rightarrow \$o: ((\forall xx: b \rightarrow \$o: ((k@xx) \Rightarrow (s@xx)) \text{ and } r = (l\lambda xx: b: \exists s_0: b \rightarrow \$o: (k@s_0 \text{ and } s_0@xx))) \Rightarrow (s@r)) \text{ and } \forall y: b \rightarrow \$o, z: b \rightarrow \$o, s_0: b \rightarrow \$o: ((s@y \text{ and } s@z \text{ and } s_0 = (l\lambda xx: b: (y@xx \text{ and } z@xx) \text{ and } s@s_0)) \Rightarrow \forall w: b \rightarrow \$o, xx: b: ((w@xx) \Rightarrow \forall s_0: b \rightarrow \$o: ((\forall xx_0: b: ((w@xx_0) \Rightarrow (s_0@xx_0)) \text{ and } \forall r: b \rightarrow \$o: (r = (l\lambda xx_0: b: \neg s_0@xx_0) \Rightarrow (s@r))) \Rightarrow (s_0@xx)))) \quad \text{thf}(cCLOSURE\_THM0\_pme, conjecture)$

### SEV260^5.p TPS problem CLOSED-THM1

The inverse image of a closed set under a continuous function is closed.

$a: \text{\$tType} \quad \text{thf}(a\_type, type)$

$b: \text{\$tType} \quad \text{thf}(b\_type, type)$

$\forall t: (a \rightarrow \$o) \rightarrow \$o, s: (b \rightarrow \$o) \rightarrow \$o, xf: a \rightarrow b: ((\forall r: a \rightarrow \$o: (r = (l\lambda xx: a: \$false) \Rightarrow (t@r)) \text{ and } \forall r: a \rightarrow \$o: (r = (l\lambda xx: a: \neg \$false) \Rightarrow (t@r)) \text{ and } \forall k: (a \rightarrow \$o) \rightarrow \$o, r: a \rightarrow \$o: ((\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow (t@xx)) \text{ and } r = (l\lambda xx: a: \exists s_0: a \rightarrow \$o: (k@s_0 \text{ and } s_0@xx))) \Rightarrow (t@r)) \text{ and } \forall y: a \rightarrow \$o, z: a \rightarrow \$o, s_0: a \rightarrow \$o: ((t@y \text{ and } t@z \text{ and } s_0 = (l\lambda xx: a: (y@xx \text{ and } z@xx))) \Rightarrow (t@s_0)) \text{ and } \forall r: b \rightarrow \$o: (r = (l\lambda xx: b: \$false) \Rightarrow (s@r)) \text{ and } \forall r: b \rightarrow \$o: (r = (l\lambda xx: b: \neg \$false) \Rightarrow (s@r)) \text{ and } \forall k: (b \rightarrow \$o) \rightarrow \$o, r: b \rightarrow \$o: ((\forall xx: b \rightarrow \$o: ((k@xx) \Rightarrow (s@xx)) \text{ and } r = (l\lambda xx: b: \exists s_0: b \rightarrow \$o: (k@s_0 \text{ and } s_0@xx))) \Rightarrow (s@r)) \text{ and } \forall y: b \rightarrow \$o, z: b \rightarrow \$o, s_0: b \rightarrow \$o: ((s@y \text{ and } s@z \text{ and } s_0 = (l\lambda xx: b: (y@xx \text{ and } z@xx))) \Rightarrow (s@s_0)) \text{ and } \forall x: b \rightarrow \$o: ((s@x) \Rightarrow \forall y: a \rightarrow \$o: (y = (l\lambda xb: a: (x@(xf@xb))) \Rightarrow (t@y)))) \Rightarrow \forall x: b \rightarrow \$o: (\forall r: b \rightarrow \$o: (r = (l\lambda xx: b: \neg x@xx) \Rightarrow (s@r)) \Rightarrow \forall y: a \rightarrow \$o: (y = (l\lambda xb: a: (x@(xf@xb))) \Rightarrow \forall r: a \rightarrow \$o: (r = (l\lambda xx: a: \neg y@xx) \Rightarrow (t@r)))) \quad \text{thf}(cCLOSED\_THM1\_pme, conjecture)$

### SEV261^5.p TPS problem INDISCRETE-TOPOLOGY

$a: \text{\$tType} \quad \text{thf}(a\_type, type)$

$\forall r: a \rightarrow \$o: (r = (l\lambda xx: a: \$false) \Rightarrow (r = (l\lambda xy: a: \$false) \text{ or } r = (l\lambda xy: a: \neg \$false))) \text{ and } \forall r: a \rightarrow \$o: (r = (l\lambda xx: a: \neg \$false) \Rightarrow (r = (l\lambda xy: a: \$false) \text{ or } r = (l\lambda xy: a: \neg \$false))) \text{ and } \forall k: (a \rightarrow \$o) \rightarrow \$o, r: a \rightarrow \$o: ((\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow (xx = (l\lambda xy: a: \$false) \text{ or } xx = (l\lambda xy: a: \neg \$false))) \text{ and } r = (l\lambda xx: a: \exists s: a \rightarrow \$o: (k@s \text{ and } s@xx))) \Rightarrow (r = (l\lambda xy: a: \$false) \text{ or } r = (l\lambda xy: a: \neg \$false))) \text{ and } \forall y: a \rightarrow \$o, z: a \rightarrow \$o, s: a \rightarrow \$o: (((y = (l\lambda xy: a: \$false) \text{ or } y = (l\lambda xy: a: \neg \$false)) \text{ and } (z = (l\lambda xy: a: \$false) \text{ or } z = (l\lambda xy: a: \neg \$false)) \text{ and } s = (l\lambda xx: a: (y@xx \text{ and } z@xx))) \Rightarrow (s = (l\lambda xy: a: \$false) \text{ or } s = (l\lambda xy: a: \neg \$false))) \quad \text{thf}(cINDISCRETE\_TOPOLOGY\_pme, conjecture)$

### SEV262^5.p TPS problem NBHD-THM2

$a: \text{\$tType} \quad \text{thf}(a\_type, type)$

$\forall t: (a \rightarrow \$o) \rightarrow \$o: (\forall k: (a \rightarrow \$o) \rightarrow \$o, r: a \rightarrow \$o: ((\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow (t@xx)) \text{ and } r = (l\lambda xx: a: \exists s: a \rightarrow \$o: (k@s \text{ and } s@xx))) \Rightarrow (t@r)) \Rightarrow \forall s: a \rightarrow \$o: ((t@s) \iff \forall xx: a: ((s@xx) \Rightarrow \exists r: a \rightarrow \$o: (\exists n: a \rightarrow \$o: (t@n \text{ and } \forall xx_0: a: ((n@xx_0) \Rightarrow (r@xx_0)) \text{ and } n@xx) \text{ and } \forall xx_0: a: ((r@xx_0) \Rightarrow (s@xx_0)))))) \quad \text{thf}(cNBHD\_THM2\_pme, conjecture)$

### SEV263^5.p TPS problem from TOPOLOGY-THMS

$cOPEN: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cOPEN, type)$

$\forall g: (\$i \rightarrow \$o) \rightarrow \$o: (\forall xx: \$i \rightarrow \$o: ((g@xx) \Rightarrow (cOPEN@xx)) \Rightarrow (cOPEN@l\lambda xx: \$i: \exists s: \$i \rightarrow \$o: (g@s \text{ and } s@xx))) \Rightarrow \forall a: \$i \rightarrow \$o: \exists b: \$i \rightarrow \$o: (cOPEN@b \text{ and } \forall xx: \$i: ((b@xx) \Rightarrow (a@xx)) \text{ and } \forall c: \$i \rightarrow \$o: ((cOPEN@c \text{ and } \forall xx: \$i: ((c@xx) \Rightarrow (a@xx))) \Rightarrow \forall xx: \$i: ((c@xx) \Rightarrow (b@xx)))) \quad \text{thf}(cEXISTS\_INTERIOR\_EXT\_pme, conjecture)$

### SEV264^5.p TPS problem from TOPOLOGY-THMS

$a: \text{\$tType} \quad \text{thf}(a\_type, type)$

$\forall t: (a \rightarrow \$o) \rightarrow \$o: (\forall k: (a \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow (t@xx)) \Rightarrow (t@l\lambda xx: a: \exists s: a \rightarrow \$o: (k@s \text{ and } s@xx))) \Rightarrow \forall s: a \rightarrow \$o: ((t@s) \iff \forall xx: a: ((s@xx) \Rightarrow \exists r: a \rightarrow \$o: (\exists n: a \rightarrow \$o: (t@n \text{ and } \forall xx_0: a: ((n@xx_0) \Rightarrow (r@xx_0)) \text{ and } n@xx) \text{ and } \forall xx_0: a: ((r@xx_0) \Rightarrow (s@xx_0)))))) \quad \text{thf}(cNBHD\_THM2A\_pme, conjecture)$

### SEV265^5.p TPS problem from TOPOLOGY-THMS

$\forall t: (\$i \rightarrow \$o) \rightarrow \$o: ((\forall s: (\$i \rightarrow \$o) \rightarrow \$o: (\forall xx: \$i \rightarrow \$o: ((s@xx) \Rightarrow (t@xx)) \Rightarrow (t@\lambda xx: \$i: \exists s_0: \$i \rightarrow \$o: (s@s_0 \text{ and } s_0@xx)))) \text{ and } \forall p: \$i \rightarrow \$o, q: \$i \rightarrow \$o: ((t@p \text{ and } t@q) \Rightarrow (t@\lambda xx: \$i: (p@xx \text{ and } q@xx)))))) \Rightarrow \forall u: \$i \rightarrow \$o, v: \$i \rightarrow \$o: ((t@u \text{ and } t@v) \Rightarrow (t@\lambda xz: \$i: (u@xz \text{ or } v@xz)))) \quad \text{thf(cTHM614\_pme, conjecture)}$

**SEV266** $\wedge$ **5.p** TPS problem from TOPOLOGY-THMS

$\forall p: \$i \rightarrow \$o, q: \$i \rightarrow \$o: (\forall x: \$i: ((p@x) \iff (q@x)) \Rightarrow p = q) \Rightarrow \forall t: (\$i \rightarrow \$o) \rightarrow \$o: ((\forall s: (\$i \rightarrow \$o) \rightarrow \$o: (\forall xx: \$i \rightarrow \$o: ((s@xx) \Rightarrow (t@xx)) \Rightarrow (t@\lambda xx: \$i: \exists s_0: \$i \rightarrow \$o: (s@s_0 \text{ and } s_0@xx)))) \text{ and } \forall p: \$i \rightarrow \$o, q: \$i \rightarrow \$o: ((t@p \text{ and } t@q) \Rightarrow (t@\lambda xx: \$i: (p@xx \text{ and } q@xx)))))) \Rightarrow \forall u: \$i \rightarrow \$o, v: \$i \rightarrow \$o: ((t@u \text{ and } t@v) \Rightarrow (t@\lambda xz: \$i: (u@xz \text{ or } v@xz)))) \quad \text{thf(cTHM4\_pme, conjecture)}$

**SEV267** $\wedge$ **5.p** TPS problem from TOPOLOGY-THMS

$a: \$tType \quad \text{thf(a\_type, type)}$

$\forall t: (a \rightarrow \$o) \rightarrow \$o: (\forall s: a \rightarrow \$o, r: a \rightarrow \$o: ((\forall xx: a \rightarrow \$o: ((t@xx \text{ and } \forall xx_0: a: ((xx@xx_0) \Rightarrow (s@xx_0))) \Rightarrow (t@xx)) \text{ and } r = (\lambda xx: a: \exists s_{19}: a \rightarrow \$o: (t@s_{19} \text{ and } \forall xx_0: a: ((s_{19}@xx_0) \Rightarrow (s@xx_0)) \text{ and } s_{19}@xx)))) \Rightarrow (t@r)) \Rightarrow \forall s: a \rightarrow \$o: ((t@s) \iff \forall xx: a: ((s@xx) \Rightarrow \exists r: a \rightarrow \$o: (\exists n: a \rightarrow \$o: (t@n \text{ and } \forall xx_0: a: ((n@xx_0) \Rightarrow (r@xx_0)) \text{ and } n@xx) \text{ and } \forall xx_0: a: ((r@xx_0) \Rightarrow (s@xx_0)))))) \quad \text{thf(cNBHD\_THM1\_pme, conjecture)}$

**SEV268** $\wedge$ **5.p** TPS problem from TOPOLOGY-THMS

$a: \$tType \quad \text{thf(a\_type, type)}$

$\forall t: (a \rightarrow \$o) \rightarrow \$o: ((\forall r: a \rightarrow \$o: (r = (\lambda xx: a: \$false) \Rightarrow (t@r)) \text{ and } \forall r: a \rightarrow \$o: (r = (\lambda xx: a: \neg \$false) \Rightarrow (t@r)) \text{ and } \forall k: (a \rightarrow \$o) \rightarrow \$o, r: a \rightarrow \$o: ((\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow (t@xx)) \text{ and } r = (\lambda xx: a: \exists s: a \rightarrow \$o: (k@s \text{ and } s@xx)))) \Rightarrow (t@r)) \text{ and } \forall y: a \rightarrow \$o, z: a \rightarrow \$o, s: a \rightarrow \$o: ((t@y \text{ and } t@z \text{ and } s = (\lambda xx: a: (y@xx \text{ and } z@xx)))) \Rightarrow (t@s))) \Rightarrow \forall s: a \rightarrow \$o: ((t@s) \iff \forall xx: a: ((s@xx) \Rightarrow \exists r: a \rightarrow \$o: (\exists n: a \rightarrow \$o: (t@n \text{ and } \forall xx_0: a: ((n@xx_0) \Rightarrow (r@xx_0)) \text{ and } n@xx) \text{ and } \forall xx_0: a: ((r@xx_0) \Rightarrow (s@xx_0)))))) \quad \text{thf(cNBHD\_THM\_pme, conjecture)}$

**SEV269** $\wedge$ **5.p** TPS problem from TOPOLOGY-THMS

$b: \$tType \quad \text{thf(b\_type, type)}$

$\forall s: (b \rightarrow \$o) \rightarrow \$o: ((\forall r: b \rightarrow \$o: (r = (\lambda xx: b: \$false) \Rightarrow (s@r)) \text{ and } \forall r: b \rightarrow \$o: (r = (\lambda xx: b: \neg \$false) \Rightarrow (s@r)) \text{ and } \forall k: (b \rightarrow \$o) \rightarrow \$o, r: b \rightarrow \$o: ((\forall xx: b \rightarrow \$o: ((k@xx) \Rightarrow (s@xx)) \text{ and } r = (\lambda xx: b: \exists s_0: b \rightarrow \$o: (k@s_0 \text{ and } s_0@xx)))) \Rightarrow (s@r)) \text{ and } \forall y: b \rightarrow \$o, z: b \rightarrow \$o, s_0: b \rightarrow \$o: ((s@y \text{ and } s@z \text{ and } s_0 = (\lambda xx: b: (y@xx \text{ and } z@xx)))) \Rightarrow (s@s_0))) \Rightarrow \forall w: b \rightarrow \$o, r: b \rightarrow \$o: (r = (\lambda xx: b: \neg \forall s_0: b \rightarrow \$o: ((\forall xx_0: b: ((w@xx_0) \Rightarrow (s_0@xx_0)) \text{ and } \forall r_0: b \rightarrow \$o: (r_0 = (\lambda xx_0: b: \neg s_0@xx_0) \Rightarrow (s@r_0)))) \Rightarrow (s_0@xx))) \Rightarrow (s@r)) \quad \text{thf(cCLOSURE\_THM1\_pme, conjecture)}$

**SEV270** $\wedge$ **5.p** TPS problem from TOPOLOGY-THMS

$cL: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf(cL, type)}$

$cG: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf(cG, type)}$

$(\forall c: (\$i \rightarrow \$o) \rightarrow \$o: ((\forall xx: \$i \rightarrow \$o: ((c@xx) \Rightarrow (cG@xx)) \text{ and } \forall xx: \$i: \exists y: \$i \rightarrow \$o: (c@y \text{ and } y@xx)) \Rightarrow \exists d: (\$i \rightarrow \$o) \rightarrow \$o: (\forall xw: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xw@\lambda xx: \$i \rightarrow \$o: \$false \text{ and } \forall xr: (\$i \rightarrow \$o) \rightarrow \$o, xx: \$i \rightarrow \$o: ((xw@xr) \Rightarrow (xw@\lambda xt: \$i \rightarrow \$o: (xr@xt \text{ or } xt = xx)))) \Rightarrow (xw@d)) \text{ and } \forall xx: \$i \rightarrow \$o: ((d@xx) \Rightarrow (c@xx)) \text{ and } \forall xx: \$i: \exists y: \$i \rightarrow \$o: (d@y \text{ and } y@xx)) \text{ and } \forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: ((cL@x \text{ and } cL@y) \Rightarrow (\forall xx: \$i: ((x@xx) = (y@xx)) \text{ or } \forall xx: \$i: ((y@xx) \Rightarrow (x@xx)))) \text{ and } \forall y: \$i \rightarrow \$o: ((cL@y) \Rightarrow \exists xx: \$i: (y@xx)) \text{ and } \forall y: \$i \rightarrow \$o: ((cL@y) \Rightarrow (cG@\lambda xx: \$i: \neg y@xx))) \Rightarrow \exists xa: \$i: \forall y: \$i \rightarrow \$o: ((cL@y) \Rightarrow (y@xa))) \quad \text{thf(cTHM628\_pme, conjecture)}$

**SEV271** $\wedge$ **5.p** TPS problem from TOPOLOGY-THMS

$b: \$tType \quad \text{thf(b\_type, type)}$

$a: \$tType \quad \text{thf(a\_type, type)}$

$\forall s: (b \rightarrow \$o) \rightarrow \$o, t: (a \rightarrow \$o) \rightarrow \$o: ((\forall r: a \rightarrow \$o: (r = (\lambda xx: a: \$false) \Rightarrow (t@r)) \text{ and } \forall r: a \rightarrow \$o: (r = (\lambda xx: a: \neg \$false) \Rightarrow (t@r)) \text{ and } \forall k: (a \rightarrow \$o) \rightarrow \$o, r: a \rightarrow \$o: ((\forall xx: a \rightarrow \$o: ((k@xx) \Rightarrow (t@xx)) \text{ and } r = (\lambda xx: a: \exists s_0: a \rightarrow \$o: (k@s_0 \text{ and } s_0@xx)))) \Rightarrow (t@r)) \text{ and } \forall y: a \rightarrow \$o, z: a \rightarrow \$o, s_0: a \rightarrow \$o: ((t@y \text{ and } t@z \text{ and } s_0 = (\lambda xx: a: (y@xx \text{ and } z@xx)))) \Rightarrow (t@s_0)) \text{ and } \forall r: b \rightarrow \$o: (r = (\lambda xx: b: \$false) \Rightarrow (s@r)) \text{ and } \forall r: b \rightarrow \$o: (r = (\lambda xx: b: \neg \$false) \Rightarrow (s@r)) \text{ and } \forall k: (b \rightarrow \$o) \rightarrow \$o, r: b \rightarrow \$o: ((\forall xx: b \rightarrow \$o: ((k@xx) \Rightarrow (s@xx)) \text{ and } r = (\lambda xx: b: \exists s_0: b \rightarrow \$o: (k@s_0 \text{ and } s_0@xx)))) \Rightarrow (s@r)) \text{ and } \forall y: b \rightarrow \$o, z: b \rightarrow \$o, s_0: b \rightarrow \$o: ((s@y \text{ and } s@z \text{ and } s_0 = (\lambda xx: b: (y@xx \text{ and } z@xx)))) \Rightarrow (s@s_0))) \Rightarrow \forall f: b \rightarrow a: (\forall x: a \rightarrow \$o: ((t@x) \Rightarrow \forall y: b \rightarrow \$o: (y = (\lambda xb: b: (x@(f@xb)))) \Rightarrow (s@y))) \iff \forall x: b \rightarrow \$o, xx: a: (\exists xt: b: (\forall s_0: b \rightarrow \$o: ((\forall xx_0: b: ((x@xx_0) \Rightarrow (s_0@xx_0)) \text{ and } \forall r: b \rightarrow \$o: (r = (\lambda xx_0: b: \neg s_0@xx_0) \Rightarrow (s@r)))) \Rightarrow (s_0@xt)) \text{ and } xx = (f@xt)) \Rightarrow \forall s_0: a \rightarrow \$o: ((\forall xx_0: a: (\exists xt: b: (x@xt \text{ and } xx_0 = (f@xt)) \Rightarrow (s_0@xx_0)) \text{ and } \forall r: a \rightarrow \$o: (r = (\lambda xx_0: a: \neg s_0@xx_0) \Rightarrow (t@r))) \Rightarrow (s_0@xx)))) \quad \text{thf(cCLOSURE\_THM2\_pme, conjecture)}$

**SEV272** $\wedge$ **5.p** TPS problem X6007A

$cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf(cSUCC\_type, type)}$

$cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf(cZERO\_type, type)}$

$cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf(cSUCC\_def, def)}$

$cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad \text{thf(cZERO\_def, definition)}$

$(\lambda n: (\$i \rightarrow \$o) \rightarrow \$o: \forall p: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((p@cZERO \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o: ((p@x) \Rightarrow (p@(cSUCC@x)))) \Rightarrow (p@n))) = (\lambda xx: (\$i \rightarrow \$o) \rightarrow \$o: \forall s: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((s@cZERO \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o: ((s@x) \Rightarrow (s@(cSUCC@x)))) \Rightarrow (s@xx)))$  thf(cX6007A\_pme, conjecture)

**SEV273** $\wedge$ **5.p** TPS problem THM542

A well-ordering is reflexive.

$a: \$tType$  thf(a\_type, type)

$cR: a \rightarrow a \rightarrow \$o$  thf(cR, type)

$\forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xy@xx))) \Rightarrow xy = xz))) \Rightarrow \forall xx: a: (cR@xx@xx)$  thf(cTHM542\_pme, conjecture)

**SEV274** $\wedge$ **5.p** TPS problem from WELL-ORD-THMS

$a: \$tType$  thf(a\_type, type)

$cR: a \rightarrow a \rightarrow \$o$  thf(cR, type)

$\forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xy@xx))) \Rightarrow xy = xz))) \Rightarrow \forall xx: a, xy: a: (cR@xx@xy \text{ or } cR@xy@xx)$  thf(cTHM543\_pme, conjecture)

**SEV275** $\wedge$ **5.p** TPS problem from WELL-ORD-THMS

$a: \$tType$  thf(a\_type, type)

$\exists xc: (a \rightarrow \$o) \rightarrow a: \forall x: a \rightarrow \$o: (\exists xt: a: (x@xt) \Rightarrow (x@(xc@x))) \Rightarrow \exists r: a \rightarrow a \rightarrow \$o: \forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (r@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } \forall xx: a: ((x@xx) \Rightarrow (r@xy@xx))) \Rightarrow xy = xz)))$  thf(cTHM550\_pme, conjecture)

**SEV276** $\wedge$ **5.p** TPS problem from WELL-ORD-THMS

$a: \$tType$  thf(a\_type, type)

$cR: a \rightarrow a \rightarrow \$o$  thf(cR, type)

$\forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xy@xx))) \Rightarrow xy = xz))) \Rightarrow \forall xx: a, xy: a: ((cR@xx@xy \text{ and } cR@xy@xx) \Rightarrow xx = xy)$  thf(cTHM544\_pme, conjecture)

**SEV277** $\wedge$ **5.p** TPS problem from WELL-ORD-THMS

$a: \$tType$  thf(a\_type, type)

$cR: a \rightarrow a \rightarrow \$o$  thf(cR, type)

$\forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xy@xx))) \Rightarrow xy = xz))) \Rightarrow \forall xx: a, xy: a, xz: a: ((cR@xx@xy \text{ and } cR@xy@xz) \Rightarrow (cR@xx@xz))$  thf(cTHM545\_pme, conjecture)

**SEV278** $\wedge$ **5.p** TPS problem from WELL-ORD-THMS

$a: \$tType$  thf(a\_type, type)

$\forall xg: ((a \rightarrow \$o) \rightarrow a) \rightarrow a \rightarrow \$o: (\forall xh: (a \rightarrow \$o) \rightarrow a: \exists xu: a: (xg@xh@xu) \Rightarrow \exists xf: (a \rightarrow \$o) \rightarrow a: (xg@xf@(xf@(xg@xf)))) \Rightarrow \exists r: a \rightarrow a \rightarrow \$o: \forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (r@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } (r@xy@xx))) \Rightarrow xy = xz)))$  thf(cTHM562\_pme, conjecture)

**SEV279** $\wedge$ **5.p** TPS problem from WELL-ORD-THMS

$b: \$tType$  thf(b\_type, type)

$a: \$tType$  thf(a\_type, type)

$h: (b \rightarrow \$o) \rightarrow a$  thf(h, type)

$cW: (b \rightarrow \$o) \rightarrow \$o$  thf(cW, type)

$(\forall u: (b \rightarrow \$o) \rightarrow \$o: ((\exists z: b \rightarrow \$o: (u@z) \text{ and } \forall xx: b \rightarrow \$o: ((u@xx) \Rightarrow (cW@xx))) \Rightarrow (cW@l\lambda xx: b: \forall s: b \rightarrow \$o: ((u@s) \Rightarrow (s@xx)))) \text{ and } \forall xx: a: \exists v: b \rightarrow \$o: (cW@v \text{ and } xx = (h@v))) \Rightarrow \exists r: a \rightarrow a \rightarrow \$o: \forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (r@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } (r@xy@xx))) \Rightarrow xy = xz)))$  thf(cLEM562A\_pme, conjecture)

**SEV280** $\wedge$ **5.p** TPS problem from WELL-ORD-THMS

$a: \$tType$  thf(a\_type, type)

$cR: a \rightarrow a \rightarrow \$o$  thf(cR, type)

$\forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } \forall xx: a: ((x@xx) \Rightarrow (cR@xy@xx))) \Rightarrow xy = xz))) \Rightarrow (\forall xx: a, xy: a, xz: a: ((cR@xx@xy \text{ and } cR@xy@xz) \Rightarrow (cR@xx@xz)) \text{ and } \forall xx: a: (cR@xx@xx = xy) \text{ and } \forall xx: a, xy: a: (cR@xx@xy \text{ or } cR@xy@xx))$  thf(cTHM546\_pme, conjecture)

**SEV281** $\wedge$ **5.p** TPS problem from WELL-ORD-THMS

$a: \$tType$  thf(a\_type, type)

$cR: a \rightarrow a \rightarrow \$o$  thf(cR, type)

$(\exists w: a \rightarrow a \rightarrow \$o: \forall x: a \rightarrow \$o: (\exists xz: a: (x@xz) \Rightarrow \exists xz: a: (x@xz \text{ and } \forall xx: a: ((x@xx) \Rightarrow (w@xz@xx)) \text{ and } \forall xy: a: ((x@xy \text{ and } (w@xy@xx))) \Rightarrow xy = xz))) \text{ and } \forall xx: a, xy: a, xz: a: ((cR@xx@xy \text{ and } cR@xy@xz) \Rightarrow (cR@xx@xz)) \text{ and } \forall xx: a: (cR@xx@xx = xy) \Rightarrow \exists s: a \rightarrow \$o: (\forall xx: a, xy: a: ((s@xx \text{ and } s@xy) \Rightarrow (cR@xx@xy \text{ or } cR@xy@xx)) \text{ and } \forall t: a \rightarrow$

$\$o: ((\forall xx: a, xy: a: ((t@xx \text{ and } t@xy) \Rightarrow (cR@xx@xy \text{ or } cR@xy@xx)) \text{ and } \forall xx: a: ((s@xx) \Rightarrow (t@xx))) \Rightarrow \forall xx: a: ((t@xx) \Rightarrow (s@xx)))$      $\text{thf}(c\text{THM548\_pme, conjecture})$

**SEV282** $\wedge$ **5.p** TPS problem TTTTP6100

$c\text{NAT}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{NAT\_type, type})$   
 $c\text{SUCC}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{SUCC\_type, type})$   
 $c\text{ZERO}: (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{ZERO\_type, type})$   
 $c\text{ZERO} = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx))$      $\text{thf}(c\text{ZERO\_def, definition})$   
 $c\text{SUCC} = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt)))$      $\text{thf}(c\text{SUCC\_def, def})$   
 $c\text{NAT} = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@c\text{ZERO} \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(c\text{SUCC}@xx)))) \Rightarrow (xp@xn)))$      $\text{thf}(c\text{NAT\_def, definition})$   
 $c\text{NAT}@c\text{ZERO}$      $\text{thf}(c\text{TTTP}_{6100}, \text{conjecture})$

**SEV285** $\wedge$ **5.p** TPS problem from TTTTP-NATS-THMS

$a: \$t\text{Type}$      $\text{thf}(a\_type, \text{type})$   
 $b: \$t\text{Type}$      $\text{thf}(b\_type, \text{type})$   
 $\forall f: a \rightarrow b, g: a \rightarrow b: (\forall a: (f@a) = (g@a) \Rightarrow f = g)$      $\text{thf}(c\text{EE\_eq\_}, \text{conjecture})$

**SEV286** $\wedge$ **5.p** TPS problem from TTTTP-NATS-THMS

$a: \$t\text{Type}$      $\text{thf}(a\_type, \text{type})$   
 $b: \$t\text{Type}$      $\text{thf}(b\_type, \text{type})$   
 $\forall f: a \rightarrow b, a: a, b: a: (a = b \Rightarrow (f@a) = (f@b))$      $\text{thf}(c\text{EC\_eq\_}, \text{conjecture})$

**SEV288** $\wedge$ **5.p** TPS problem from TTTTP-NATS-THMS

$a: \$t\text{Type}$      $\text{thf}(a\_type, \text{type})$   
 $(\lambda x: a, y: a: \forall xq: a \rightarrow \$o: ((xq@x) \Rightarrow (xq@y))) = (\lambda x: a, y: a: x = y)$      $\text{thf}(c\text{E1\_eq\_pme, conjecture})$

**SEV289** $\wedge$ **5.p** TPS problem TTTTP6101

$c\text{NAT}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{NAT\_type, type})$   
 $c\text{SUCC}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{SUCC\_type, type})$   
 $c\text{ZERO}: (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{ZERO\_type, type})$   
 $c\text{ZERO} = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx))$      $\text{thf}(c\text{ZERO\_def, definition})$   
 $c\text{SUCC} = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt)))$      $\text{thf}(c\text{SUCC\_def, def})$   
 $c\text{NAT} = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@c\text{ZERO} \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(c\text{SUCC}@xx)))) \Rightarrow (xp@xn)))$      $\text{thf}(c\text{NAT\_def, definition})$   
 $\forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((c\text{NAT}@xx) \Rightarrow (c\text{NAT}@(c\text{SUCC}@xx)))$      $\text{thf}(c\text{TTTP}_{6101}, \text{conjecture})$

**SEV290** $\wedge$ **5.p** TPS problem BLEDSOE1

$c_0: (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c_0\_type, \text{type})$   
 $c\text{SUCC}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{SUCC\_type, type})$   
 $c\_less\_eq\_ : ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\_less\_eq\_type, \text{type})$   
 $c\text{SUCC} = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt)))$      $\text{thf}(c\text{SUCC\_def, def})$   
 $c\_less\_eq\_ = (\lambda xx: (\$i \rightarrow \$o) \rightarrow \$o, xy: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@xx \text{ and } \forall xz: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xz) \Rightarrow (xp@(c\text{SUCC}@xz)))) \Rightarrow (xp@xy)))$      $\text{thf}(c\_less\_eq\_def, \text{definition})$   
 $\exists a: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((a@xx) \Rightarrow (c\_less\_eq\_@xx@c_0))$      $\text{thf}(c\text{BLEDSOE}_1, \text{conjecture})$

**SEV291** $\wedge$ **5.p** TPS problem THM130-B

$r: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$      $\text{thf}(r\_type, \text{type})$   
 $c\text{NAT}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{NAT\_type, type})$   
 $c\text{SUCC}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{SUCC\_type, type})$   
 $c\text{ZERO}: (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{ZERO\_type, type})$   
 $c\text{ZERO} = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx))$      $\text{thf}(c\text{ZERO\_def, definition})$   
 $c\text{SUCC} = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt)))$      $\text{thf}(c\text{SUCC\_def, def})$   
 $c\text{NAT} = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@c\text{ZERO} \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(c\text{SUCC}@xx)))) \Rightarrow (xp@xn)))$      $\text{thf}(c\text{NAT\_def, definition})$   
 $(r@c\text{ZERO}@c\text{ZERO} \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o, xy: (\$i \rightarrow \$o) \rightarrow \$o: ((r@xx@xy) \Rightarrow (r@(c\text{SUCC}@xx)@(c\text{SUCC}@xy)))) \Rightarrow \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((c\text{NAT}@xx) \Rightarrow \exists xy: (\$i \rightarrow \$o) \rightarrow \$o: (r@xx@xy))$      $\text{thf}(c\text{THM130\_B, conjecture})$

**SEV292** $\wedge$ **5.p** TPS problem BLEDSOE7A

$c\text{P}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{P\_type, type})$   
 $c\text{ONE}: (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{ONE\_type, type})$   
 $c\text{SUCC}: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{SUCC\_type, type})$   
 $c\text{ZERO}: (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\text{ZERO\_type, type})$   
 $c\_less\_eq\_ : ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$      $\text{thf}(c\_less\_eq\_type, \text{type})$



$cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad thf(cZERO\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad thf(cSUCC\_def, def)$   
 $cONE = (cSUCC@cZERO) \quad thf(cONE\_def, definition)$   
 $c\_less\_eq\_ = (\lambda xx: (\$i \rightarrow \$o) \rightarrow \$o, xy: (\$i \rightarrow \$o) \rightarrow \$o: \forall px: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@xx \text{ and } \forall xz: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xz) \Rightarrow (xp@(cSUCC@xz)))) \Rightarrow (xp@xy))) \quad thf(c\_less\_eq\_def, definition)$   
 $(cP@cONE) \Rightarrow \exists xx: (\$i \rightarrow \$o) \rightarrow \$o: (c\_less\_eq\_@cZERO@xx \text{ and } c\_less\_eq\_@xx@(cSUCC@cONE) \text{ and } cP@xx) \quad thf(cB$

**SEV293** $\wedge$ **5.p** TPS problem X6101

$cONE: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cONE\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cSUCC\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cZERO\_type, type)$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad thf(cZERO\_def, definition)$   
 $cONE = (cSUCC@cZERO) \quad thf(cONE\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad thf(cSUCC\_def, def)$   
 $cONE = (\lambda p: \$i \rightarrow \$o: \exists xy: \$i: p = (\lambda xx: \$i, xy: \$i: xx = xy@xy)) \quad thf(cX6101\_pme, conjecture)$

**SEV294** $\wedge$ **5.p** TPS problem TTTP6102

$cNAT: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad thf(cNAT\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cSUCC\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cZERO\_type, type)$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad thf(cZERO\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad thf(cSUCC\_def, def)$   
 $cNAT = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))) \Rightarrow (xp@xn))) \quad thf(cNAT\_def, definition)$   
 $\forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((cNAT@xx) \Rightarrow ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))))) \Rightarrow (xp@xn)) \quad thf(cNAT\_def, definition)$   
 $\forall xm: (\$i \rightarrow \$o) \rightarrow \$o: ((cNAT@xm) \Rightarrow (xp@xm)) \quad thf(cTTTP_{6102}, conjecture)$

**SEV295** $\wedge$ **5.p** TPS problem THM130-NAT

$r: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad thf(r\_type, type)$   
 $cINDUCTION: \$o \quad thf(cINDUCTION\_type, type)$   
 $cNAT: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad thf(cNAT\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cSUCC\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cZERO\_type, type)$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad thf(cZERO\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad thf(cSUCC\_def, def)$   
 $cNAT = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))) \Rightarrow (xp@xn))) \quad thf(cNAT\_def, definition)$   
 $cINDUCTION = (\forall p: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((p@cZERO \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o: ((p@x) \Rightarrow (p@(cSUCC@x)))) \Rightarrow (p@m)) \Rightarrow (p@m)) \quad thf(cINDUCTION\_def, definition)$   
 $(cINDUCTION \text{ and } r@cZERO@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o, xy: (\$i \rightarrow \$o) \rightarrow \$o: ((r@xx@xy) \Rightarrow (r@(cSUCC@xx)@(cSUCC@xy)))) \Rightarrow (r@xx@xy) \quad thf(cTHM130\_NAT, conjecture)$

**SEV296** $\wedge$ **5.p** TPS problem from TTTP-NATS-THMS

$c\_plus: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad thf(c\_plus\_type, type)$   
 $c\_star: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad thf(c\_star\_type, type)$   
 $cONE: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cONE\_type, type)$   
 $cPLUS\_AXIOMS: \$o \quad thf(cPLUS\_AXIOMS\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cSUCC\_type, type)$   
 $cTIMES\_AXIOMS: \$o \quad thf(cTIMES\_AXIOMS\_type, type)$   
 $cTWO: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cTWO\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad thf(cZERO\_type, type)$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad thf(cZERO\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad thf(cSUCC\_def, def)$   
 $cONE = (cSUCC@cZERO) \quad thf(cONE\_def, definition)$   
 $cTWO = (cSUCC@cONE) \quad thf(cTWO\_def, definition)$   
 $cPLUS\_AXIOMS = (\forall x: (\$i \rightarrow \$o) \rightarrow \$o: (c\_plus@x@cZERO) = x \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o, y: (\$i \rightarrow \$o) \rightarrow \$o: (c\_plus@x@(cSUCC@y)) = (cSUCC@(c\_plus@x@y))) \quad thf(cPLUS\_AXIOMS\_def, definition)$   
 $cTIMES\_AXIOMS = (\forall x: (\$i \rightarrow \$o) \rightarrow \$o: (c\_star@x@cZERO) = cZERO \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o, y: (\$i \rightarrow \$o) \rightarrow \$o: (c\_star@x@(cSUCC@y)) = (c\_plus@(c\_star@x@y)@y)) \quad thf(cTIMES\_AXIOMS\_def, definition)$   
 $(cPLUS\_AXIOMS \text{ and } cTIMES\_AXIOMS) \Rightarrow (c\_star@cTWO@cTWO) = (c\_plus@cTWO@cTWO) \quad thf(cFOUR\_THEOR$

**SEV297** $\wedge$ **5.p** TPS problem from TTTP-NATS-THMS

$cB: \$i \rightarrow \$o \quad \text{thf}(cB\_type, type)$   
 $cC: \$i \rightarrow \$o \quad \text{thf}(cC\_type, type)$   
 $cFINITE: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cFINITE\_type, type)$   
 $cNAT: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cNAT\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cSUCC\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cZERO\_type, type)$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad \text{thf}(cZERO\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf}(cSUCC\_def, def)$   
 $cNAT = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))) \Rightarrow (xp@xn))) \quad \text{thf}(cNAT\_def, definition)$   
 $cFINITE = (\lambda xp: \$i \rightarrow \$o: \exists xn: (\$i \rightarrow \$o) \rightarrow \$o: (cNAT@xn \text{ and } xn@xp)) \quad \text{thf}(cFINITE\_def, definition)$   
 $(cFINITE@cC \text{ and } \forall xx: \$i: ((cB@xx) \Rightarrow (cC@xx))) \Rightarrow (cFINITE@cB) \quad \text{thf}(cTHM531B\_pme, conjecture)$

**SEV298** $\wedge 5.p$  TPS problem from TTPP-NATS-THMS

$c_0: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(c_0\_type, type)$   
 $c_1: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(c_1\_type, type)$   
 $c_2: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(c_2\_type, type)$   
 $cP: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cP\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cSUCC\_type, type)$   
 $c\_less\_eq: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(c\_less\_eq\_type, type)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf}(cSUCC\_def, def)$   
 $c\_less\_eq = (\lambda xx: (\$i \rightarrow \$o) \rightarrow \$o, xy: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@xx \text{ and } \forall xz: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xz) \Rightarrow (xp@(cSUCC@xz)))) \Rightarrow (xp@xy))) \quad \text{thf}(c\_less\_eq\_def, definition)$   
 $(cP@c_1) \Rightarrow \exists xx: (\$i \rightarrow \$o) \rightarrow \$o: (c\_less\_eq\_@c_0@xx \text{ and } c\_less\_eq\_@xx@c_2 \text{ and } cP@xx) \quad \text{thf}(cBLEDSOE_7, conjecture)$

**SEV299** $\wedge 5.p$  TPS problem from TTPP-NATS-THMS

$cNAT: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cNAT\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cSUCC\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cZERO\_type, type)$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad \text{thf}(cZERO\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf}(cSUCC\_def, def)$   
 $cNAT = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))) \Rightarrow (xp@xn))) \quad \text{thf}(cNAT\_def, definition)$   
 $\forall p: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((p@cZERO \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o: ((p@x) \Rightarrow (p@(cSUCC@x)))) \Rightarrow \forall m: (\$i \rightarrow \$o) \rightarrow \$o: ((cNAT@m) \Rightarrow (p@m))) \quad \text{thf}(cINDUCTION, conjecture)$

**SEV300** $\wedge 5.p$  TPS problem from TTPP-NATS-THMS

$cONE: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cONE\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cSUCC\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cZERO\_type, type)$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad \text{thf}(cZERO\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf}(cSUCC\_def, def)$   
 $cONE = (cSUCC@cZERO) \quad \text{thf}(cONE\_def, definition)$   
 $\forall xf: \$i \rightarrow \$i, xg: \$i \rightarrow \$i: (\forall xx: \$i: (xf@xx) = (xg@xx) \Rightarrow xf = xg) \Rightarrow cONE = (\lambda p: \$i \rightarrow \$o: \exists xy: \$i: p = (\lambda xx: \$i, xy: \$i: xx = xy@xy)) \quad \text{thf}(cX6101\_EXT\_pme, conjecture)$

**SEV301** $\wedge 5.p$  TPS problem from TTPP-NATS-THMS

$cNAT: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cNAT\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cSUCC\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cZERO\_type, type)$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad \text{thf}(cZERO\_def, definition)$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf}(cSUCC\_def, def)$   
 $cNAT = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))) \Rightarrow (xp@xn))) \quad \text{thf}(cNAT\_def, definition)$   
 $\forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((cNAT@xx) \Rightarrow ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))))) \Rightarrow$   
 $\forall xm: (\$i \rightarrow \$o) \rightarrow \$o: (\forall xp_0: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp_0@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp_0@xx) \Rightarrow (xp_0@(cSUCC@xx)))) \Rightarrow (xp_0@xm)) \Rightarrow (xp@xm)) \quad \text{thf}(cX6102\_A, conjecture)$

**SEV302** $\wedge 5.p$  TPS problem from TTPP-NATS-THMS

$cNAT: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cNAT\_type, type)$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cSUCC\_type, type)$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cZERO\_type, type)$

$cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad \text{thf}(cZERO\_def, \text{definition})$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf}(cSUCC\_def, \text{definition})$   
 $cNAT = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))) \Rightarrow (xp@xn))) \quad \text{thf}(cNAT\_def, \text{definition})$   
 $\forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((cNAT@xx) \Rightarrow ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))))) \Rightarrow$   
 $\forall xm: (\$i \rightarrow \$o) \rightarrow \$o: (((cNAT@cZERO \text{ and } xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((cNAT@xx \text{ and } xp@xx) \Rightarrow (cNAT@(cSUCC@xx) \text{ and } xp@(cSUCC@xx)))) \Rightarrow (cNAT@xm \text{ and } xp@xm)) \Rightarrow (xp@xm)) \quad \text{thf}(cX6102\_B, \text{conjecture})$

**SEV303**<sup>5.p</sup> TPS problem from TTTP-NATS-THMS

$cNAT: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cNAT\_type, \text{type})$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cSUCC\_type, \text{type})$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cZERO\_type, \text{type})$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad \text{thf}(cZERO\_def, \text{definition})$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf}(cSUCC\_def, \text{definition})$   
 $cNAT = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o: \forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))) \Rightarrow (xp@xn))) \quad \text{thf}(cNAT\_def, \text{definition})$   
 $\forall xp: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((cNAT@xx) \Rightarrow ((xp@xx) \Rightarrow (xp@(cSUCC@xx)))))) \Rightarrow$   
 $\forall xm: (\$i \rightarrow \$o) \rightarrow \$o: (((\forall xp_0: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp_0@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((xp_0@xx) \Rightarrow (xp_0@(cSUCC@xx)))) \Rightarrow (xp_0@cZERO)) \text{ and } xp@cZERO \text{ and } \forall xx: (\$i \rightarrow \$o) \rightarrow \$o: ((\forall xp_0: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp_0@cZERO \text{ and } \forall xx_0: (\$i \rightarrow \$o) \rightarrow \$o: ((xp_0@xx_0) \Rightarrow (xp_0@(cSUCC@xx_0)))) \Rightarrow (xp_0@xx) \text{ and } xp@xx) \Rightarrow (\forall xp_0: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((xp_0@cZERO \text{ and } \forall xx_0: (\$i \rightarrow \$o) \rightarrow \$o: ((xp_0@xx_0) \Rightarrow (xp_0@(cSUCC@xx_0)))) \Rightarrow (xp_0@(cSUCC@xx)) \text{ and } xp@(cSUCC@xx)))) \Rightarrow (cNAT@xm \text{ and } xp@xm)) \Rightarrow (xp@xm)) \quad \text{thf}(cX6102\_C, \text{conjecture})$

**SEV304**<sup>5.p</sup> TPS problem from TTTP-NATS-THMS

$cONE: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cONE\_type, \text{type})$   
 $cSUCC: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cSUCC\_type, \text{type})$   
 $cZERO: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cZERO\_type, \text{type})$   
 $cZERO = (\lambda xp: \$i \rightarrow \$o: \neg \exists xx: \$i: (xp@xx)) \quad \text{thf}(cZERO\_def, \text{definition})$   
 $cSUCC = (\lambda xn: (\$i \rightarrow \$o) \rightarrow \$o, xp: \$i \rightarrow \$o: \exists xx: \$i: (xp@xx \text{ and } xn@\lambda xt: \$i: (xt \neq xx \text{ and } xp@xt))) \quad \text{thf}(cSUCC\_def, \text{definition})$   
 $cONE = (cSUCC@cZERO) \quad \text{thf}(cONE\_def, \text{definition})$   
 $\forall k: \$i \rightarrow \$i \rightarrow \$o, s: \$i \rightarrow \$o: ((\exists xs: \$i \rightarrow \$i \rightarrow \$o: (\forall xx: \$i: ((s@xx) \Rightarrow (cSUCC@(cSUCC@(cSUCC@(cSUCC@cONE@xs)@xy) \Rightarrow \exists xy_0: \$i: (\lambda xx: \$i: (s@xx \text{ and } xy = (xs@xx))) = (\lambda xx: \$i, xy: \$i: xx = xy@xy_0))) \text{ and } \forall xx: \$i, xy: \$i: ((k@xx@xy) \Rightarrow (k@xy@xx)) \Rightarrow \exists xx: \$i, xy: \$i, xz: \$i: (s@xx \text{ and } xy \text{ and } xy \neq xz \text{ and } xz \neq xx \text{ and } ((k@xx@xy \text{ and } k@xy@xz \text{ and } k@xx@xz) \text{ or } (\neg k@xx@xy \text{ and } \neg k@xy@xz \text{ and } \neg k@xx@xz)))))) \Rightarrow \exists xu: \$i \rightarrow \$o: \forall xx: \$i: ((k@xu@xx) \iff (xu@xx))) \quad \text{thf}(cTHM2\_pme, \text{conjecture})$

**SEV305**<sup>5.p</sup> TPS problem THM2

This is a fixed point theorem for sets. Assume K is a monotonic operator on sets with respect to inclusion and conclude that K has a fixed point. Taken from [Kol67]. Related to the Knaster-Tarski theorem.

$\forall k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o: (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (xy@xx_0)) \Rightarrow \forall xx_0: \$i: ((k@xx@xx_0) \Rightarrow (k@xy@xx_0))) \Rightarrow \exists xu: \$i \rightarrow \$o: \forall xx: \$i: ((k@xu@xx) \iff (xu@xx))) \quad \text{thf}(cTHM2\_pme, \text{conjecture})$

**SEV305**<sup>6.p</sup> TPS problem THM2E

Related to the Knaster-Tarski theorem.

$cK: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad \text{thf}(cK, \text{type})$   
 $\forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (\forall xx: \$i: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: \$i: ((cK@x@xx) \Rightarrow (cK@y@xx))) \Rightarrow \exists u: \$i \rightarrow \$o: \forall xx: \$i: ((cK@u@xx) \iff (u@xx)) \quad \text{thf}(cTHM2E\_pme, \text{conjecture})$

**SEV306**<sup>5.p</sup> TPS problem THM2F

Related to the Knaster-Tarski theorem.

$cK: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o \quad \text{thf}(cK, \text{type})$   
 $\forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (\forall xx: \$i: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: \$i: ((cK@x@xx) \Rightarrow (cK@y@xx))) \Rightarrow \exists u: \$i \rightarrow \$o: (cK@u) = u \quad \text{thf}(cTHM2F\_pme, \text{conjecture})$

**SEV308**<sup>5.p</sup> TPS problem THM1A

Related to the Knaster-Tarski theorem.

$\forall k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\exists x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (\forall xx: \$i: ((x@xx) \Rightarrow (y@xx)) \text{ and } \neg \forall xx: \$i: ((k@x@xx) \Rightarrow (k@y@xx))) \text{ or } \exists u: \$i \rightarrow \$o: (k@u) = u) \quad \text{thf}(cTHM1A\_pme, \text{conjecture})$

**SEV309**<sup>5.p</sup> TPS problem THM1

Related to the Knaster-Tarski theorem.

$\forall p: \$i \rightarrow \$o, q: \$i \rightarrow \$o: \exists x: \$i: ((p@x) \iff (q@x)) \Rightarrow p = q \Rightarrow \forall k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\exists x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (\forall xx: \$i: ((x@xx) \Rightarrow (y@xx)) \text{ and } \neg \forall xx: \$i: ((k@x@xx) \Rightarrow (k@y@xx))) \text{ or } \exists u: \$i \rightarrow \$o: (k@u) = u) \quad \text{thf}(cTHM1\_pme, \text{conjecture})$

**SEV310** $\wedge$ **5.p** TPS problem from SET-KNASTER-TARSKI-INST

Related to the Knaster-Tarski theorem.

 $a: \$tType \quad thf(a\_type, type)$  $cK: (a \rightarrow \$o) \rightarrow a \rightarrow \$o \quad thf(cK, type)$ 

$$\forall x: a \rightarrow \$o, y: a \rightarrow \$o: (\forall xx: a: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: a: ((cK@x@xx) \Rightarrow (cK@y@xx))) \Rightarrow \forall xx: a: (\forall s: a \rightarrow \$o: (\forall xx_0: a: ((cK@s@xx_0) \Rightarrow (s@xx_0)) \Rightarrow (s@xx)) \Rightarrow (cK@lxx_0: a: \forall s: a \rightarrow \$o: (\forall xx_1: a: ((cK@s@xx_1) \Rightarrow (s@xx_1)) \Rightarrow (s@xx_0))@xx)) \Rightarrow (s@xx)) \Rightarrow (s@xx_0))@xx) \quad thf(cTHM90A\_pme, conjecture)$$
**SEV311** $\wedge$ **5.p** TPS problem from SET-KNASTER-TARSKI-INST

Related to the Knaster-Tarski theorem.

 $a: \$tType \quad thf(a\_type, type)$  $cF: (a \rightarrow \$o) \rightarrow a \rightarrow \$o \quad thf(cF, type)$ 

$$\forall x: a \rightarrow \$o, y: a \rightarrow \$o: (\forall xx: a: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: a: ((cF@x@xx) \Rightarrow (cF@y@xx))) \Rightarrow \forall xx: a: ((cF@lxx_0: a: \forall s: a \rightarrow \$o: (\forall xx_1: a: ((cF@s@xx_1) \Rightarrow (s@xx_1)) \Rightarrow (s@xx_0))@xx) \Rightarrow \forall s: a \rightarrow \$o: (\forall xx_0: a: ((cF@s@xx_0) \Rightarrow (s@xx_0)) \Rightarrow (s@xx)) \Rightarrow (cF@lxx_0: a: \forall s: a \rightarrow \$o: (\forall xx_1: a: ((cF@s@xx_1) \Rightarrow (s@xx_1)) \Rightarrow (s@xx_0))@xx)) \Rightarrow (s@xx)) \Rightarrow (s@xx_0))@xx) \quad thf(cTHM521\_pme, conjecture)$$
**SEV312** $\wedge$ **5.p** TPS problem from SET-KNASTER-TARSKI

Related to the Knaster-Tarski theorem.

$$\forall k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: ((\forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o: (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (xy@xx_0)) \Rightarrow \forall xx_0: \$i: ((k@xx@xx_0) \Rightarrow (k@xy@xx_0))) \text{ and } \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o: (\forall xx_0: \$i: ((xx@xx_0) \Rightarrow (xy@xx_0)) \Rightarrow \forall xx_0: \$i: ((k@xx@xx_0) \Rightarrow (k@xy@xx_0)))) \Rightarrow \exists xu: \$i \rightarrow \$o: \forall xx: \$i: ((k@xu@xx) \iff (xu@xx))) \quad thf(cTHM2\_B\_pme, conjecture)$$
**SEV313** $\wedge$ **5.p** TPS problem from CLOS-SYS-FP-THMS

Related to the Knaster-Tarski theorem.

 $a: \$tType \quad thf(a\_type, type)$  $cF: (a \rightarrow \$o) \rightarrow a \rightarrow \$o \quad thf(cF, type)$  $cCL: (a \rightarrow \$o) \rightarrow \$o \quad thf(cCL, type)$ 

$$(\forall s: (a \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow \$o: ((s@xx) \Rightarrow (cCL@xx)) \Rightarrow (cCL@lxx: a: \forall s_0: a \rightarrow \$o: ((s@s_0) \Rightarrow (s_0@xx)))) \text{ and } \forall r: a \rightarrow \$o: ((cCL@r) \Rightarrow (cCL@(cF@r))) \text{ and } \forall r: a \rightarrow \$o, s: a \rightarrow \$o: ((cCL@r \text{ and } cCL@s \text{ and } \forall xx: a: ((r@xx) \Rightarrow (s@xx))) \Rightarrow \forall xx: a: ((cF@r@xx) \Rightarrow (cF@s@xx)))) \Rightarrow \exists x: a \rightarrow \$o: (cCL@x \text{ and } (cF@x) = x \text{ and } \forall y: a \rightarrow \$o: ((cCL@y \text{ and } (cF@y) = y) \Rightarrow \forall xx: a: ((x@xx) \Rightarrow (y@xx)))) \quad thf(cFP\_THM1\_pme, conjecture)$$
**SEV314** $\wedge$ **5.p** TPS problem from CLOS-SYS-FP-THMS

Related to the Knaster-Tarski theorem.

 $a: \$tType \quad thf(a\_type, type)$  $b: \$tType \quad thf(b\_type, type)$  $cF: (a \rightarrow b \rightarrow \$o) \rightarrow a \rightarrow b \rightarrow \$o \quad thf(cF, type)$  $cCL: (a \rightarrow b \rightarrow \$o) \rightarrow \$o \quad thf(cCL, type)$ 

$$(\forall s: (a \rightarrow b \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow b \rightarrow \$o: ((s@xx) \Rightarrow (cCL@xx)) \Rightarrow (cCL@lxx: a, xb: b: \forall r: a \rightarrow b \rightarrow \$o: ((s@r) \Rightarrow (r@xa@xb)))) \text{ and } \forall r: a \rightarrow b \rightarrow \$o: ((cCL@r) \Rightarrow (cCL@(cF@r))) \text{ and } \forall r: a \rightarrow b \rightarrow \$o, s: a \rightarrow b \rightarrow \$o: ((cCL@r \text{ and } cCL@s \text{ and } \forall xa: a, xb: b: ((r@xa@xb) \Rightarrow (s@xa@xb))) \Rightarrow \forall xa: a, xb: b: ((cF@r@xa@xb) \Rightarrow (cF@s@xa@xb)))) \Rightarrow \exists x: a \rightarrow b \rightarrow \$o: (cCL@x \text{ and } (cF@x) = x \text{ and } \forall y: a \rightarrow b \rightarrow \$o: ((cCL@y \text{ and } (cF@y) = y) \Rightarrow \forall xa: a, xb: b: ((x@xa@xb) \Rightarrow (y@xa@xb)))) \quad thf(cFP\_THM2\_pme, conjecture)$$
**SEV315** $\wedge$ **5.p** TPS problem from CLOS-SYS-FP-THMS

Related to the Knaster-Tarski theorem.

 $a: \$tType \quad thf(a\_type, type)$  $b: \$tType \quad thf(b\_type, type)$  $c: \$tType \quad thf(c\_type, type)$  $cF: (a \rightarrow b \rightarrow c \rightarrow \$o) \rightarrow a \rightarrow b \rightarrow c \rightarrow \$o \quad thf(cF, type)$  $cCL: (a \rightarrow b \rightarrow c \rightarrow \$o) \rightarrow \$o \quad thf(cCL, type)$ 

$$(\forall s: (a \rightarrow b \rightarrow c \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow b \rightarrow c \rightarrow \$o: ((s@xx) \Rightarrow (cCL@xx)) \Rightarrow (cCL@lxx: a, xb: b, xc: c: \forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((s@r) \Rightarrow (r@xa@xb@xc)))) \text{ and } \forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((cCL@r) \Rightarrow (cCL@(cF@r))) \text{ and } \forall r: a \rightarrow b \rightarrow c \rightarrow \$o, s: a \rightarrow b \rightarrow c \rightarrow \$o: ((cCL@r \text{ and } cCL@s \text{ and } \forall xa: a, xb: b, xc: c: ((r@xa@xb@xc) \Rightarrow (s@xa@xb@xc)))) \Rightarrow \forall xa: a, xb: b, xc: c: ((cF@r@xa@xb@xc) \Rightarrow (cF@s@xa@xb@xc)))) \Rightarrow \exists x: a \rightarrow b \rightarrow c \rightarrow \$o: (cCL@x \text{ and } (cF@x) = x \text{ and } \forall y: a \rightarrow b \rightarrow c \rightarrow \$o: ((cCL@y \text{ and } (cF@y) = y) \Rightarrow \forall xa: a, xb: b, xc: c: ((x@xa@xb@xc) \Rightarrow (y@xa@xb@xc)))) \quad thf(cFP\_THM3\_pme, conjecture)$$
**SEV316** $\wedge$ **5.p** TPS problem from CLOS-SYS-FP-THMS

Related to the Knaster-Tarski theorem.

 $a: \$tType \quad thf(a\_type, type)$  $b: \$tType \quad thf(b\_type, type)$  $c: \$tType \quad thf(c\_type, type)$  $cF: (a \rightarrow b \rightarrow c \rightarrow \$o) \rightarrow a \rightarrow b \rightarrow c \rightarrow \$o \quad thf(cF, type)$

**cCL:**  $(a \rightarrow b \rightarrow c \rightarrow \$o) \rightarrow \$o$      $\text{thf}(\text{cCL}, \text{type})$

$(\forall s: (a \rightarrow b \rightarrow c \rightarrow \$o) \rightarrow \$o: (\forall xx: a \rightarrow b \rightarrow c \rightarrow \$o: ((s@xx) \Rightarrow (\text{cCL}@xx)) \Rightarrow (\text{cCL}@ \lambda xa: a, xb: b, xc: c: \forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((s@r) \Rightarrow (r@xa@xb@xc))))$  and  $\forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((\text{cCL}@r) \Rightarrow (\text{cCL}@(\text{cF}@r)))$  and  $\forall r: a \rightarrow b \rightarrow c \rightarrow \$o, s: a \rightarrow b \rightarrow c \rightarrow \$o: ((\text{cCL}@r$  and  $\text{cCL}@s$  and  $\forall xa: a, xb: b, xc: c: ((r@xa@xb@xc) \Rightarrow (s@xa@xb@xc))) \Rightarrow \forall xa: a, xb: b, xc: c: ((\text{cF}@r@xa@xb@xc) \Rightarrow (\text{cF}@s@xa@xb@xc))) \Rightarrow (\text{cCL}@ \lambda xa: a, xb: b, xc: c: \forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((\text{cCL}@r$  and  $\forall xa_0: a, xb_0: b, xc_0: c: ((\text{cF}@r@xa_0@xb_0@xc_0) \Rightarrow (r@xa_0@xb_0@xc_0))) \Rightarrow (r@xa@xb@xc)$  and  $(\text{cF}@ \lambda xa: a, xb: b, xc: c: \forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((\text{cCL}@r$  and  $\forall xa_0: a, xb_0: b, xc_0: c: ((\text{cF}@r@xa_0@xb_0@xc_0) \Rightarrow (r@xa_0@xb_0@xc_0))) \Rightarrow (r@xa@xb@xc))) = (\lambda xa: a, xb: b, xc: c: \forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((\text{cCL}@r$  and  $\forall xa_0: a, xb_0: b, xc_0: c: ((\text{cF}@r@xa_0@xb_0@xc_0) \Rightarrow (r@xa_0@xb_0@xc_0))) \Rightarrow (r@xa@xb@xc))) \Rightarrow (\lambda xa: a, xb: b, xc: c: \forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((\text{cCL}@r$  and  $\forall xa_0: a, xb_0: b, xc_0: c: ((\text{cF}@r@xa_0@xb_0@xc_0) \Rightarrow (r@xa_0@xb_0@xc_0))) \Rightarrow (r@xa@xb@xc)))$  and  $\forall y: a \rightarrow b \rightarrow c \rightarrow \$o: ((\text{cCL}@y$  and  $(\text{cF}@y) = y) \Rightarrow \forall xa: a, xb: b, xc: c: (\forall r: a \rightarrow b \rightarrow c \rightarrow \$o: ((\text{cCL}@r$  and  $\forall xa_0: a, xb_0: b, xc_0: c: ((\text{cF}@r@xa_0@xb_0@xc_0) \Rightarrow (r@xa_0@xb_0@xc_0))) \Rightarrow (r@xa@xb@xc))) \Rightarrow (y@xa@xb@xc)))$      $\text{thf}(\text{cFP\_THM3\_INST\_pme}, \text{conjecture})$

**SEV317^5.p** TPS problem THM145-A

Related to the Knaster-Tarski theorem.

**a:**  $\$t\text{Type}$      $\text{thf}(\text{a\_type}, \text{type})$

$\forall r: a \rightarrow a \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow a: ((\forall xx: a, xy: a, xz: a: ((r@xx@xy$  and  $r@xy@xz) \Rightarrow (r@xx@xz))$  and  $\forall xs: a \rightarrow \$o: (\forall xz: a: ((xs@xz) \Rightarrow (r@xz@(u@xs)))$  and  $\forall xz: a: ((xs@xz) \Rightarrow (r@xz@(u@xs)))$  and  $\forall xj: a: (\forall xk: a: ((xs@xk$  and  $xs@xk) \Rightarrow (r@xk@xj)) \Rightarrow (r@(u@xs)@xj$  and  $r@(u@xs)@xj))) \Rightarrow \forall xf: a \rightarrow a: ((\forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@(xf@xx)@(xf@xy)))$  and  $\forall xx: a, xy: a: (r@(xf@xx)@(xf@xy))) \Rightarrow \exists xw: a: (r@xw@(xf@xw)$  and  $r@(xf@xw)@xw))$      $\text{thf}(\text{cTHM145\_A\_pme}, \text{conjecture})$

**SEV318^5.p** TPS problem THM145-B

Related to the Knaster-Tarski theorem.

**a:**  $\$t\text{Type}$      $\text{thf}(\text{a\_type}, \text{type})$

$\forall r: a \rightarrow a \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow a: ((\forall xx: a, xy: a, xz: a: ((r@xx@xy$  and  $r@xy@xz) \Rightarrow (r@xx@xz))$  and  $\forall xs: a \rightarrow \$o: (\forall xz: a: ((xs@xz) \Rightarrow (r@xz@(u@xs)))$  and  $\forall xz: a: ((xs@xz) \Rightarrow (r@xz@(u@xs)))$  and  $\forall xj: a: (\forall xk: a: ((xs@xk) \Rightarrow (r@xk@xj)) \Rightarrow (r@(u@xs)@xj))) \Rightarrow \forall xf: a \rightarrow a: ((\forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@(xf@xx)@(xf@xy)))$  and  $\forall xx: a, xy: a: (r@(xf@xx)@(xf@xy))) \Rightarrow \exists xw: a: (r@xw@(xf@xw)$  and  $r@(xf@xw)@xw))$      $\text{thf}(\text{cTHM145\_B\_pme}, \text{conjecture})$

**SEV319^5.p** TPS problem THM145L

Tarski's (actually Knaster's) Fixed Point Theorem for lattices: In a complete lattice, every monotone function has a fixed point.

**a:**  $\$t\text{Type}$      $\text{thf}(\text{a\_type}, \text{type})$

$\forall l: a \rightarrow a \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow a: ((\forall xx: a, xy: a, xz: a: ((l@xx@xy$  and  $l@xy@xz) \Rightarrow (l@xx@xz))$  and  $\forall xs: a \rightarrow \$o: (\forall xz: a: ((xs@xz) \Rightarrow (l@xz@(u@xs)))$  and  $\forall xj: a: (\forall xk: a: ((xs@xk) \Rightarrow (l@xk@xj)) \Rightarrow (l@(u@xs)@xj))) \Rightarrow \forall xf: a \rightarrow a: ((l@xx@xy) \Rightarrow (l@(xf@xx)@(xf@xy))) \Rightarrow \exists xw: a: (l@xw@(xf@xw)$  and  $l@(xf@xw)@xw))$      $\text{thf}(\text{cTHM145\_L\_pme}, \text{conjecture})$

**SEV319^6.p** TPS problem THM145L1

Related to the Knaster-Tarski theorem.

**a:**  $\$t\text{Type}$      $\text{thf}(\text{a\_type}, \text{type})$

**cLQ:**  $a \rightarrow a \rightarrow \$o$      $\text{thf}(\text{cLQ}, \text{type})$

$\forall u: (a \rightarrow \$o) \rightarrow a: ((\forall xx: a, xy: a, xz: a: ((\text{cLQ}@xx@xy$  and  $\text{cLQ}@xy@xz) \Rightarrow (\text{cLQ}@xx@xz))$  and  $\forall xs: a \rightarrow \$o: (\forall xz: a: ((xs@xz) \Rightarrow (\text{cLQ}@xz@(u@xs)))$  and  $\forall xj: a: (\forall xk: a: ((xs@xk) \Rightarrow (\text{cLQ}@xk@xj)) \Rightarrow (\text{cLQ}@(u@xs)@xj))) \Rightarrow \forall xf: a \rightarrow a: ((\text{cLQ}@xx@xy) \Rightarrow (\text{cLQ}@xf@xx)@(xf@xy))) \Rightarrow \exists xw: a: (\text{cLQ}@xw@(xf@xw)$  and  $\text{cLQ}@(xf@xw)@xw))$

**SEV321^5.p** TPS problem from KNASTER-TARSKI

Related to the Knaster-Tarski theorem.

**a:**  $\$t\text{Type}$      $\text{thf}(\text{a\_type}, \text{type})$

$\forall r: a \rightarrow a \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow a: ((\forall xx: a, xy: a, xz: a: ((r@xx@xy$  and  $r@xy@xz) \Rightarrow (r@xx@xz))$  and  $\forall xs: a \rightarrow \$o: (\forall xz: a: ((xs@xz) \Rightarrow (r@xz@(u@xs)))$  and  $\forall xj: a: (\forall xk: a: ((xs@xk$  and  $xs@xk) \Rightarrow (r@xk@xj)) \Rightarrow (r@(u@xs)@xj$  and  $r@(u@xs)@xj))) \Rightarrow \forall xf: a \rightarrow a: ((\forall xx: a, xy: a: ((r@xx@xy) \Rightarrow (r@(xf@xx)@(xf@xy)))$  and  $\forall xx: a, xy: a: (r@(xf@xx)@(xf@xy))) \Rightarrow \exists xw: a: (r@xw@(xf@xw)$  and  $r@(xf@xw)@xw))$      $\text{thf}(\text{cTHM145\_C\_pme}, \text{conjecture})$

**SEV322^5.p** TPS problem from GVB-MB-AXIOMS

**z:**  $\$i$      $\text{thf}(z, \text{type})$

**cGVB\_ZERO:**  $\$i$      $\text{thf}(\text{cGVB\_ZERO}, \text{type})$

**cGVB\_IN:**  $\$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cGVB\_IN}, \text{type})$

$\neg \text{cGVB\_IN}@z@\text{cGVB\_ZERO}$      $\text{thf}(\text{cGVB\_AX\_ZERO}, \text{conjecture})$

**SEV323^5.p** TPS problem from GVB-MB-AXIOMS

**z:**  $\$i$      $\text{thf}(z, \text{type})$

**cGVB\_M:**  $\$i \rightarrow \$o$      $\text{thf}(\text{cGVB\_M}, \text{type})$

**cGVB\_V:**  $\$i$      $\text{thf}(\text{cGVB\_V}, \text{type})$

**cGVB\_IN:**  $\$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(\text{cGVB\_IN}, \text{type})$

$(\text{cGVB\_IN}@z@\text{cGVB\_V}) \iff (\text{cGVB\_M}@z) \quad \text{thf}(\text{cGVB\_AX\_V}, \text{conjecture})$

**SEV324** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$x: \$i \quad \text{thf}(x, \text{type})$

$\text{cGVB\_NOP}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_NOP}, \text{type})$

$\text{cGVB\_SING}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_SING}, \text{type})$

$(\text{cGVB\_SING}@x) = (\text{cGVB\_NOP}@x@x) \quad \text{thf}(\text{cGVB\_AX\_SING}, \text{conjecture})$

**SEV325** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$x: \$i \quad \text{thf}(x, \text{type})$

$y: \$i \quad \text{thf}(y, \text{type})$

$\text{cGVB\_M}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_M}, \text{type})$

$\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_IN}, \text{type})$

$(\text{cGVB\_IN}@x@y) \Rightarrow (\text{cGVB\_M}@x) \quad \text{thf}(\text{cGVB\_A}_2, \text{conjecture})$

**SEV326** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$x: \$i \quad \text{thf}(x, \text{type})$

$\text{cGVB\_SING}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_SING}, \text{type})$

$\text{cGVB\_UNION}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_UNION}, \text{type})$

$\text{cGVB\_SUCC}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_SUCC}, \text{type})$

$(\text{cGVB\_SUCC}@x) = (\text{cGVB\_UNION}@x@(\text{cGVB\_SING}@x)) \quad \text{thf}(\text{cGVB\_AX\_SUCC}, \text{conjecture})$

**SEV327** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$z: \$i \quad \text{thf}(z, \text{type})$

$\text{cGVB\_SING\_VAL}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_SING\_VAL}, \text{type})$

$\text{cGVB\_RELATION}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_RELATION}, \text{type})$

$\text{cGVB\_FUNCTION}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_FUNCTION}, \text{type})$

$(\text{cGVB\_FUNCTION}@z) \iff (\text{cGVB\_RELATION}@z \text{ and } \text{cGVB\_SING\_VAL}@z) \quad \text{thf}(\text{cGVB\_AX\_FUNCTION}, \text{conjecture})$

**SEV328** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i \quad \text{thf}(y, \text{type})$

$x: \$i \quad \text{thf}(x, \text{type})$

$\text{cGVB\_V}: \$i \quad \text{thf}(\text{cGVB\_V}, \text{type})$

$\text{cGVB\_CROSS}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_CROSS}, \text{type})$

$\text{cGVB\_INTERSECT}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_INTERSECT}, \text{type})$

$\text{cGVB\_RESTRICT}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_RESTRICT}, \text{type})$

$(\text{cGVB\_RESTRICT}@x@y) = (\text{cGVB\_INTERSECT}@x@(\text{cGVB\_CROSS}@y@\text{cGVB\_V})) \quad \text{thf}(\text{cGVB\_AX\_RESTRICT}, \text{conjecture})$

**SEV329** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$f: \$i \quad \text{thf}(f, \text{type})$

$\text{cGVB\_CONVERSE}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_CONVERSE}, \text{type})$

$\text{cGVB\_FUNCTION}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_FUNCTION}, \text{type})$

$\text{cGVB\_ONE\_ONE}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_ONE\_ONE}, \text{type})$

$(\text{cGVB\_ONE\_ONE}@f) \iff (\text{cGVB\_FUNCTION}@f \text{ and } \text{cGVB\_FUNCTION}@(\text{cGVB\_CONVERSE}@f)) \quad \text{thf}(\text{cGVB\_AX\_ONE\_ONE}, \text{conjecture})$

**SEV330** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i \quad \text{thf}(y, \text{type})$

$x: \$i \quad \text{thf}(x, \text{type})$

$\text{cGVB\_COMPLEMENT}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_COMPLEMENT}, \text{type})$

$\text{cGVB\_INTERSECT}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_INTERSECT}, \text{type})$

$\text{cGVB\_UNION}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_UNION}, \text{type})$

$(\text{cGVB\_UNION}@x@y) = (\text{cGVB\_COMPLEMENT}@(\text{cGVB\_INTERSECT}@(\text{cGVB\_COMPLEMENT}@x)@\text{cGVB\_COMPLEMENT}@y)) \quad \text{thf}(\text{cGVB\_AX\_UNION}, \text{conjecture})$

**SEV331** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i \quad \text{thf}(y, \text{type})$

$x: \$i \quad \text{thf}(x, \text{type})$

$\text{cGVB\_NOP}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_NOP}, \text{type})$

$\text{cGVB\_SING}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_SING}, \text{type})$

$\text{cGVB\_OP}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_OP}, \text{type})$

$(\text{cGVB\_OP}@x@y) = (\text{cGVB\_NOP}@(\text{cGVB\_SING}@x)@\text{cGVB\_NOP}@x@y) \quad \text{thf}(\text{cGVB\_AX\_OP}, \text{conjecture})$

**SEV332** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i \quad \text{thf}(y, \text{type})$

$x: \$i \quad \text{thf}(x, \text{type})$

$f: \$i \quad \text{thf}(f, \text{type})$

$cGVB\_OP: \$i \rightarrow \$i \rightarrow \$i \quad thf(cGVB\_OP, type)$   
 $cGVB\_APPLY: \$i \rightarrow \$i \rightarrow \$i \quad thf(cGVB\_APPLY, type)$   
 $cGVB\_APP_2: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad thf(cGVB\_APP_2, type)$   
 $(cGVB\_APP_2@f@x@y) = (cGVB\_APPLY@f@(cGVB\_OP@x@y)) \quad thf(cGVB\_AX\_APP_2, conjecture)$

**SEV333** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$x: \$i \quad thf(x, type)$   
 $z: \$i \quad thf(z, type)$   
 $cGVB\_SUBSET: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_SUBSET, type)$   
 $cGVB\_M: \$i \rightarrow \$o \quad thf(cGVB\_M, type)$   
 $cGVB\_POWERSET: \$i \rightarrow \$i \quad thf(cGVB\_POWERSET, type)$   
 $cGVB\_IN: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_IN, type)$   
 $(cGVB\_IN@z@(cGVB\_POWERSET@x)) \iff (cGVB\_M@z \text{ and } cGVB\_SUBSET@z@x) \quad thf(cGVB\_C_3, conjecture)$

**SEV334** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$x: \$i \quad thf(x, type)$   
 $z: \$i \quad thf(z, type)$   
 $cGVB\_IN: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_IN, type)$   
 $cGVB\_M: \$i \rightarrow \$o \quad thf(cGVB\_M, type)$   
 $cGVB\_COMPLEMENT: \$i \rightarrow \$i \quad thf(cGVB\_COMPLEMENT, type)$   
 $(cGVB\_IN@z@(cGVB\_COMPLEMENT@x)) \iff (cGVB\_M@z \text{ and } \neg cGVB\_IN@z@x) \quad thf(cGVB\_B_3, conjecture)$

**SEV335** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i \quad thf(y, type)$   
 $x: \$i \quad thf(x, type)$   
 $cGVB\_SUBSET: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_SUBSET, type)$   
 $cGVB\_PROP\_SUBSET: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_PROP\_SUBSET, type)$   
 $(cGVB\_PROP\_SUBSET@x@y) \iff (cGVB\_SUBSET@x@y \text{ and } x \neq y) \quad thf(cGVB\_AX\_PROP\_SUBSET, conjecture)$

**SEV336** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$z: \$i \quad thf(z, type)$   
 $cGVB\_OPP: \$i \rightarrow \$o \quad thf(cGVB\_OPP, type)$   
 $cGVB\_IN: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_IN, type)$   
 $cGVB\_M: \$i \rightarrow \$o \quad thf(cGVB\_M, type)$   
 $cGVB\_RELATION: \$i \rightarrow \$o \quad thf(cGVB\_RELATION, type)$   
 $(cGVB\_RELATION@z) \iff \forall xx: \$i: ((cGVB\_M@xx) \Rightarrow ((cGVB\_IN@xx@z) \Rightarrow (cGVB\_OPP@xx))) \quad thf(cGVB\_AX\_R_1, conjecture)$

**SEV337** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$z: \$i \quad thf(z, type)$   
 $cGVB\_SECOND: \$i \rightarrow \$i \quad thf(cGVB\_SECOND, type)$   
 $cGVB\_FIRST: \$i \rightarrow \$i \quad thf(cGVB\_FIRST, type)$   
 $cGVB\_IN: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_IN, type)$   
 $cGVB\_OPP: \$i \rightarrow \$o \quad thf(cGVB\_OPP, type)$   
 $cGVB\_M: \$i \rightarrow \$o \quad thf(cGVB\_M, type)$   
 $cGVB\_ESTIN: \$i \quad thf(cGVB\_ESTIN, type)$   
 $(cGVB\_IN@z@cGVB\_ESTIN) \iff (cGVB\_M@z \text{ and } cGVB\_OPP@z \text{ and } cGVB\_IN@(cGVB\_FIRST@z)@(cGVB\_SECOND@z))$

**SEV338** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$z: \$i \quad thf(z, type)$   
 $cGVB\_SECOND: \$i \rightarrow \$i \quad thf(cGVB\_SECOND, type)$   
 $cGVB\_FIRST: \$i \rightarrow \$i \quad thf(cGVB\_FIRST, type)$   
 $cGVB\_OPP: \$i \rightarrow \$o \quad thf(cGVB\_OPP, type)$   
 $cGVB\_M: \$i \rightarrow \$o \quad thf(cGVB\_M, type)$   
 $cGVB\_IDENT: \$i \quad thf(cGVB\_IDENT, type)$   
 $cGVB\_IN: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_IN, type)$   
 $(cGVB\_IN@z@cGVB\_IDENT) \iff (cGVB\_M@z \text{ and } cGVB\_OPP@z \text{ and } (cGVB\_FIRST@z) = (cGVB\_SECOND@z))$

**SEV339** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i \quad thf(y, type)$   
 $z: \$i \quad thf(z, type)$   
 $x: \$i \quad thf(x, type)$   
 $cGVB\_IN: \$i \rightarrow \$i \rightarrow \$o \quad thf(cGVB\_IN, type)$   
 $cGVB\_M: \$i \rightarrow \$o \quad thf(cGVB\_M, type)$

cGVB.INTERSECT:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB.INTERSECT, type)  
 (cGVB.IN@z@(cGVB.INTERSECT@x@y))  $\iff$  (cGVB.M@z and cGVB.IN@z@x and cGVB.IN@z@y) thf(cGVB.B2)

**SEV340** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i$  thf( $y$ , type)  
 $x: \$i$  thf( $x$ , type)  
 cGVB.IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.IN, type)  
 cGVB.M:  $\$i \rightarrow \$o$  thf(cGVB.M, type)  
 cGVB.SUBSET:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.SUBSET, type)  
 (cGVB.SUBSET@x@y)  $\iff \forall xu: \$i: (cGVB.M@xu \text{ and } ((cGVB.IN@xu@x) \Rightarrow (cGVB.IN@xu@y)))$  thf(cGVB.AX.SU)

**SEV341** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$x: \$i$  thf( $x$ , type)  
 $f: \$i$  thf( $f$ , type)  
 cGVB.CROSS:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB.CROSS, type)  
 cGVB.MAPS:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.MAPS, type)  
 cGVB.M:  $\$i \rightarrow \$o$  thf(cGVB.M, type)  
 cGVB.CLOSED:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.CLOSED, type)  
 (cGVB.CLOSED@x@f)  $\iff (cGVB.M@x \text{ and } cGVB.M@f \text{ and } cGVB.MAPS@f@(cGVB.CROSS@x@x)@x)$  thf(cGVB

**SEV342** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i$  thf( $y$ , type)  
 $u: \$i$  thf( $u$ , type)  
 $x: \$i$  thf( $x$ , type)  
 cGVB.M:  $\$i \rightarrow \$o$  thf(cGVB.M, type)  
 cGVB.NOP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB.NOP, type)  
 cGVB.IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.IN, type)  
 (cGVB.IN@u@(cGVB.NOP@x@y))  $\iff (cGVB.M@u \text{ and } (u = x \text{ or } u = y))$  thf(cGVB.A4, conjecture)

**SEV343** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i$  thf( $y$ , type)  
 $x: \$i$  thf( $x$ , type)  
 cGVB.IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.IN, type)  
 cGVB.M:  $\$i \rightarrow \$o$  thf(cGVB.M, type)  
 $\forall xu: \$i: ((cGVB.M@xu) \Rightarrow ((cGVB.IN@xu@x) \iff (cGVB.IN@xu@y))) \Rightarrow x = y$  thf(cGVB.A3, conjecture)

**SEV344** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$x: \$i$  thf( $x$ , type)  
 cGVB.DISJOINT:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.DISJOINT, type)  
 cGVB.IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.IN, type)  
 cGVB.M:  $\$i \rightarrow \$o$  thf(cGVB.M, type)  
 cGVB.ZERO:  $\$i$  thf(cGVB.ZERO, type)  
 $x \neq cGVB.ZERO \Rightarrow \exists xu: \$i: (cGVB.M@xu \text{ and } cGVB.IN@xu@x \text{ and } cGVB.DISJOINT@xu@x)$  thf(cGVB.D, conjecture)

**SEV345** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i$  thf( $y$ , type)  
 $f: \$i$  thf( $f$ , type)  
 $x: \$i$  thf( $x$ , type)  
 cGVB.RANGE:  $\$i \rightarrow \$i$  thf(cGVB.RANGE, type)  
 cGVB.SUBSET:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.SUBSET, type)  
 cGVB.DOMAIN:  $\$i \rightarrow \$i$  thf(cGVB.DOMAIN, type)  
 cGVB.FUNCTION:  $\$i \rightarrow \$o$  thf(cGVB.FUNCTION, type)  
 cGVB.MAPS:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.MAPS, type)  
 (cGVB.MAPS@f@x@y)  $\iff (cGVB.FUNCTION@f \text{ and } (cGVB.DOMAIN@f) = x \text{ and } cGVB.SUBSET@(cGVB.RANG$

**SEV346** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$y: \$i$  thf( $y$ , type)  
 $x: \$i$  thf( $x$ , type)  
 cGVB.IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.IN, type)  
 cGVB.M:  $\$i \rightarrow \$o$  thf(cGVB.M, type)  
 cGVB.DISJOINT:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB.DISJOINT, type)  
 (cGVB.DISJOINT@x@y)  $\iff \forall xu: \$i: ((cGVB.M@xu) \Rightarrow \neg cGVB.IN@xu@x \text{ and } cGVB.IN@xu@y)$  thf(cGVB.AX.D

**SEV347** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS



$x: \$i \quad \text{thf}(x, \text{type})$   
 $z: \$i \quad \text{thf}(z, \text{type})$   
 $\text{cGVB\_FIRST}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_FIRST}, \text{type})$   
 $\text{cGVB\_SECOND}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_SECOND}, \text{type})$   
 $\text{cGVB\_OP}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_OP}, \text{type})$   
 $\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_IN}, \text{type})$   
 $\text{cGVB\_OPP}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_OPP}, \text{type})$   
 $\text{cGVB\_M}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_M}, \text{type})$   
 $\text{cGVB\_CONVERSE}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_CONVERSE}, \text{type})$   
 $(\text{cGVB\_IN}@z@(\text{cGVB\_CONVERSE}@x)) \iff (\text{cGVB\_M}@z \text{ and } \text{cGVB\_OPP}@z \text{ and } \text{cGVB\_IN}@(\text{cGVB\_OP}@(\text{cGVB\_SECO$

**SEV348** $\wedge 5.p$  TPS problem from GVB-MB-AXIOMS

$x: \$i \quad \text{thf}(x, \text{type})$   
 $\text{cGVB\_OP}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_OP}, \text{type})$   
 $\text{cGVB\_M}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_M}, \text{type})$   
 $\text{cGVB\_OPP}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_OPP}, \text{type})$   
 $(\text{cGVB\_OPP}@x) \iff \exists xy: \$i, xz: \$i: (\text{cGVB\_M}@xy \text{ and } \text{cGVB\_M}@xz \text{ and } x = (\text{cGVB\_OP}@xy@xz)) \quad \text{thf}(\text{cGVB\_AX\_OP}$

**SEV349** $\wedge 5.p$  TPS problem from GVB-MB-AXIOMS

$\text{cGVB\_SUCC}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_SUCC}, \text{type})$   
 $\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_IN}, \text{type})$   
 $\text{cGVB\_ZERO}: \$i \quad \text{thf}(\text{cGVB\_ZERO}, \text{type})$   
 $\text{cGVB\_M}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_M}, \text{type})$   
 $\exists xy: \$i: (\text{cGVB\_M}@xy \text{ and } \text{cGVB\_IN}@c\text{GVB\_ZERO}@xy \text{ and } \forall xx: \$i: ((\text{cGVB\_IN}@xx@xy) \Rightarrow (\text{cGVB\_IN}@(\text{cGVB\_SUCC}@x$

**SEV350** $\wedge 5.p$  TPS problem from GVB-MB-AXIOMS

$z: \$i \quad \text{thf}(z, \text{type})$   
 $x: \$i \quad \text{thf}(x, \text{type})$   
 $\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_IN}, \text{type})$   
 $\text{cGVB\_M}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_M}, \text{type})$   
 $\text{cGVB\_SIGMA}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_SIGMA}, \text{type})$   
 $(\text{cGVB\_IN}@z@(\text{cGVB\_SIGMA}@x)) \iff (\text{cGVB\_M}@z \text{ and } \exists xy: \$i: (\text{cGVB\_M}@xy \text{ and } \text{cGVB\_IN}@xy@x \text{ and } \text{cGVB\_IN}@z@$

**SEV351** $\wedge 5.p$  TPS problem from GVB-MB-AXIOMS

$y: \$i \quad \text{thf}(y, \text{type})$   
 $z: \$i \quad \text{thf}(z, \text{type})$   
 $x: \$i \quad \text{thf}(x, \text{type})$   
 $\text{cGVB\_SECOND}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_SECOND}, \text{type})$   
 $\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_IN}, \text{type})$   
 $\text{cGVB\_FIRST}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_FIRST}, \text{type})$   
 $\text{cGVB\_OPP}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_OPP}, \text{type})$   
 $\text{cGVB\_M}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_M}, \text{type})$   
 $\text{cGVB\_CROSS}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_CROSS}, \text{type})$   
 $(\text{cGVB\_IN}@z@(\text{cGVB\_CROSS}@x@y)) \iff (\text{cGVB\_M}@z \text{ and } \text{cGVB\_OPP}@z \text{ and } \text{cGVB\_IN}@(\text{cGVB\_FIRST}@z)@x \text{ and } \text{cGVB\_IN}@z@$

**SEV352** $\wedge 5.p$  TPS problem from GVB-MB-AXIOMS

$z: \$i \quad \text{thf}(z, \text{type})$   
 $x: \$i \quad \text{thf}(x, \text{type})$   
 $\text{cGVB\_FIRST}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_FIRST}, \text{type})$   
 $\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_IN}, \text{type})$   
 $\text{cGVB\_OPP}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_OPP}, \text{type})$   
 $\text{cGVB\_M}: \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_M}, \text{type})$   
 $\text{cGVB\_DOMAIN}: \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_DOMAIN}, \text{type})$   
 $(\text{cGVB\_IN}@z@(\text{cGVB\_DOMAIN}@x)) \iff (\text{cGVB\_M}@z \text{ and } \exists xt: \$i: (\text{cGVB\_M}@xt \text{ and } \text{cGVB\_OPP}@xt \text{ and } \text{cGVB\_IN}@xt@$   
 $(\text{cGVB\_FIRST}@xt))) \quad \text{thf}(\text{cGVB\_B}_4, \text{conjecture})$

**SEV354** $\wedge 5.p$  TPS problem from GVB-MB-AXIOMS

$g: \$i \quad \text{thf}(g, \text{type})$   
 $f: \$i \quad \text{thf}(f, \text{type})$   
 $\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_IN}, \text{type})$   
 $\text{cGVB\_COMPOSE}: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{cGVB\_COMPOSE}, \text{type})$   
 $\text{cGVB\_ITERATE}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(\text{cGVB\_ITERATE}, \text{type})$

$(\text{cGVB\_ITERATE}@f@g) \iff \forall xp: \$i: ((\text{cGVB\_IN}@f@xp \text{ and } \forall xj: \$i: ((\text{cGVB\_IN}@xj@xp) \Rightarrow (\text{cGVB\_IN}@(\text{cGVB\_COMPOSE}@xj@f@xp)))) \Rightarrow (\text{cGVB\_IN}@g@xp))$  thf(cGVB\_AX\_ITERATE, conjecture)

**SEV355** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$z: \$i$  thf( $z$ , type)

$x: \$i$  thf( $x$ , type)

$\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)

$\text{cGVB\_OP}: \$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)

$\text{cGVB\_M}: \$i \rightarrow \$o$  thf(cGVB\_M, type)

$\text{cGVB\_SECOND}: \$i \rightarrow \$i$  thf(cGVB\_SECOND, type)

$(\text{cGVB\_IN}@z@(\text{cGVB\_SECOND}@x)) \iff (\text{cGVB\_M}@z \text{ and } \exists xu: \$i, xv: \$i: (\text{cGVB\_M}@xu \text{ and } \text{cGVB\_M}@xv \text{ and } x = (\text{cGVB\_OP}@xu@xv) \text{ and } \text{cGVB\_IN}@z@xv))$  thf(cGVB\_AX\_SECOND, conjecture)

**SEV356** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$z: \$i$  thf( $z$ , type)

$x: \$i$  thf( $x$ , type)

$\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)

$\text{cGVB\_OP}: \$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)

$\text{cGVB\_M}: \$i \rightarrow \$o$  thf(cGVB\_M, type)

$\text{cGVB\_FIRST}: \$i \rightarrow \$i$  thf(cGVB\_FIRST, type)

$(\text{cGVB\_IN}@z@(\text{cGVB\_FIRST}@x)) \iff (\text{cGVB\_M}@z \text{ and } \exists xu: \$i, xv: \$i: (\text{cGVB\_M}@xu \text{ and } \text{cGVB\_M}@xv \text{ and } x = (\text{cGVB\_OP}@xu@xv) \text{ and } \text{cGVB\_IN}@z@xu))$  thf(cGVB\_AX\_FIRST, conjecture)

**SEV357** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$\text{cGVB\_OP}: \$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)

$\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)

$\text{cGVB\_M}: \$i \rightarrow \$o$  thf(cGVB\_M, type)

$\text{cGVB\_ZERO}: \$i$  thf(cGVB\_ZERO, type)

$\text{cGVB\_FUNCTION}: \$i \rightarrow \$o$  thf(cGVB\_FUNCTION, type)

$\exists xu: \$i: (\text{cGVB\_FUNCTION}@xu \text{ and } \forall xx: \$i: ((\text{cGVB\_M}@xx \text{ and } xx \neq \text{cGVB\_ZERO}) \Rightarrow \exists xy: \$i: (\text{cGVB\_M}@xy \text{ and } \text{cGVB\_OP}@xy @ xu)))$

**SEV358** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$z: \$i$  thf( $z$ , type)

$x: \$i$  thf( $x$ , type)

$f: \$i$  thf( $f$ , type)

$\text{cGVB\_SECOND}: \$i \rightarrow \$i$  thf(cGVB\_SECOND, type)

$\text{cGVB\_FIRST}: \$i \rightarrow \$i$  thf(cGVB\_FIRST, type)

$\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)

$\text{cGVB\_OPP}: \$i \rightarrow \$o$  thf(cGVB\_OPP, type)

$\text{cGVB\_M}: \$i \rightarrow \$o$  thf(cGVB\_M, type)

$\text{cGVB\_IMAGE}: \$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_IMAGE, type)

$(\text{cGVB\_IN}@z@(\text{cGVB\_IMAGE}@x@f)) \iff (\text{cGVB\_M}@z \text{ and } \exists xy: \$i: (\text{cGVB\_M}@xy \text{ and } \text{cGVB\_OPP}@xy \text{ and } \text{cGVB\_IN}@z@xy))$  thf(cGVB\_C4, conjecture)

**SEV359** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$g: \$i$  thf( $g$ , type)

$f: \$i$  thf( $f$ , type)

$\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)

$\text{cGVB\_COMPOSE}: \$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_COMPOSE, type)

$\text{cGVB\_M}: \$i \rightarrow \$o$  thf(cGVB\_M, type)

$\text{cGVB\_ITERATE}: \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_ITERATE, type)

$(\text{cGVB\_ITERATE}@f@g) \iff \forall xp: \$i: ((\text{cGVB\_IN}@f@xp \text{ and } \forall xj: \$i: ((\text{cGVB\_IN}@xj@xp \text{ and } \text{cGVB\_M}@(\text{cGVB\_COMPOSE}@xj@f@xp)))) \Rightarrow (\text{cGVB\_IN}@g@xp))$  thf(cGVB\_AX\_ITERATE2, conjecture)

**SEV360** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

$z: \$i$  thf( $z$ , type)

$y: \$i$  thf( $y$ , type)

$f: \$i$  thf( $f$ , type)

$\text{cGVB\_SECOND}: \$i \rightarrow \$i$  thf(cGVB\_SECOND, type)

$\text{cGVB\_IN}: \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)

$\text{cGVB\_FIRST}: \$i \rightarrow \$i$  thf(cGVB\_FIRST, type)

$\text{cGVB\_OPP}: \$i \rightarrow \$o$  thf(cGVB\_OPP, type)

$\text{cGVB\_M}: \$i \rightarrow \$o$  thf(cGVB\_M, type)

cGVB\_APPLY:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_APPLY, type)  
 (cGVB\_IN@z@(cGVB\_APPLY@f@y))  $\iff$  (cGVB\_M@z and  $\exists xw: \$i: (cGVB_M@xw$  and cGVB\_OPP@xw and cGVB\_IN@  
 y and cGVB\_IN@z@(cGVB\_SECOND@xw))) thf(cGVB\_AX\_APPLY, conjecture)

**SEV361** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

x:  $\$i$  thf(x, type)  
 cGVB\_OP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)  
 cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)  
 cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
 cGVB\_SING\_VAL:  $\$i \rightarrow \$o$  thf(cGVB\_SING\_VAL, type)  
 (cGVB\_SING\_VAL@x)  $\iff$   $\forall xu: \$i, xv: \$i, xw: \$i: ((cGVB_M@xu$  and cGVB\_M@xv and cGVB\_M@xw)  $\implies$   
 ((cGVB\_IN@(cGVB\_OP@xu@xv)@x and cGVB\_IN@(cGVB\_OP@xu@xw)@x)  $\implies$  xv = xw)) thf(cGVB\_AX\_SING\_VAL,

**SEV362** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

x:  $\$i$  thf(x, type)  
 z:  $\$i$  thf(z, type)  
 cGVB\_OP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)  
 cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)  
 cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
 cGVB\_FLIP\_RANGE:  $\$i \rightarrow \$i$  thf(cGVB\_FLIP\_RANGE, type)  
 (cGVB\_IN@z@(cGVB\_FLIP\_RANGE@x))  $\iff$  (cGVB\_M@z and  $\exists xu: \$i, xv: \$i, xw: \$i: (cGVB_M@xu$  and cGVB\_M@xv and  
 (cGVB\_OP@xu@(cGVB\_OP@xv@xw)) and cGVB\_IN@(cGVB\_OP@xu@(cGVB\_OP@xw@xv)@x)) thf(cGVB\_B8, conje

**SEV363** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

x:  $\$i$  thf(x, type)  
 z:  $\$i$  thf(z, type)  
 cGVB\_OP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)  
 cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)  
 cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
 cGVB\_ROT\_RIGHT:  $\$i \rightarrow \$i$  thf(cGVB\_ROT\_RIGHT, type)  
 (cGVB\_IN@z@(cGVB\_ROT\_RIGHT@x))  $\iff$  (cGVB\_M@z and  $\exists xu: \$i, xv: \$i, xw: \$i: (cGVB_M@xu$  and cGVB\_M@xv and  
 (cGVB\_OP@xu@(cGVB\_OP@xv@xw)) and cGVB\_IN@(cGVB\_OP@xv@(cGVB\_OP@xw@xu)@x)) thf(cGVB\_B7, conje

**SEV364** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

g:  $\$i$  thf(g, type)  
 f:  $\$i$  thf(f, type)  
 z:  $\$i$  thf(z, type)  
 cGVB\_OP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)  
 cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)  
 cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
 cGVB\_COMPOSE:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_COMPOSE, type)  
 (cGVB\_IN@z@(cGVB\_COMPOSE@g@f))  $\iff$  (cGVB\_M@z and  $\exists xx: \$i, xy: \$i, xw: \$i: (cGVB_M@xx$  and cGVB\_M@xy and  
 (cGVB\_OP@xx@xy) and cGVB\_IN@(cGVB\_OP@xx@xw)@f and cGVB\_IN@(cGVB\_OP@xw@xy)@g)) thf(cGVB\_AX\_C

**SEV365** $\wedge$ **5.p** TPS problem from GVB-MB-AXIOMS

h:  $\$i$  thf(h, type)  
 f<sub>2</sub>:  $\$i$  thf(f<sub>2</sub>, type)  
 f<sub>1</sub>:  $\$i$  thf(f<sub>1</sub>, type)  
 s<sub>1</sub>:  $\$i$  thf(s<sub>1</sub>, type)  
 s<sub>2</sub>:  $\$i$  thf(s<sub>2</sub>, type)  
 cGVB\_APPLY:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_APPLY, type)  
 cGVB\_APP<sub>2</sub>:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_APP<sub>2</sub>, type)  
 cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)  
 cGVB\_MAPS:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_MAPS, type)  
 cGVB\_CLOSED:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_CLOSED, type)  
 cGVB\_HOMOM:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_HOMOM, type)  
 (cGVB\_HOMOM@h@s<sub>1</sub>@f<sub>1</sub>@s<sub>2</sub>@f<sub>2</sub>)  $\iff$  (cGVB\_CLOSED@s<sub>1</sub>@f<sub>1</sub> and cGVB\_CLOSED@s<sub>2</sub>@f<sub>2</sub> and cGVB\_MAPS@h@s<sub>1</sub>  
 (cGVB\_APPLY@h@(cGVB\_APP<sub>2</sub>@f<sub>1</sub>@xx@xy)) = (cGVB\_APP<sub>2</sub>@f<sub>2</sub>@(cGVB\_APPLY@h@xx)@(cGVB\_APPLY@h@xy))))

**SEV366** $\wedge$ **5.p** TPS problem from GVB-MB-THMS

b:  $\$i$  thf(b, type)  
 a:  $\$i$  thf(a, type)  
 cGVB\_OP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)

cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
cGVB\_M@(cGVB\_OP@a@b) thf(cGVB\_OP\_PROP<sub>2</sub>, conjecture)

**SEV368**<sup>5.p</sup> TPS problem from GVB-MB-THMS

$u$ :  $\$i$  thf( $u$ , type)  
cGVB\_POWERSET:  $\$i \rightarrow \$i$  thf(cGVB\_POWERSET, type)  
cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
(cGVB\_M@ $u$ )  $\Rightarrow$  (cGVB\_M@(cGVB\_POWERSET@ $u$ )) thf(cGVB\_C3A, conjecture)

**SEV369**<sup>5.p</sup> TPS problem from GVB-MB-THMS

$f$ :  $\$i$  thf( $f$ , type)  
 $x$ :  $\$i$  thf( $x$ , type)  
cGVB\_IMAGE:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_IMAGE, type)  
cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
cGVB\_FUNCTION:  $\$i \rightarrow \$o$  thf(cGVB\_FUNCTION, type)  
(cGVB\_M@ $x$  and cGVB\_FUNCTION@ $f$ )  $\Rightarrow$  (cGVB\_M@(cGVB\_IMAGE@ $x$ @ $f$ )) thf(cGVB\_C4A, conjecture)

**SEV370**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_COMPOSE:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_COMPOSE, type)  
cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
 $\forall x f: \$i, x g: \$i: ((cGVB_M@x f$  and  $cGVB_M@x g) \Rightarrow (cGVB_M@(cGVB_COMPOSE@x f@x g)))$  thf(cGVB\_COMP\_PROP,

**SEV371**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_OP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)  
cGVB\_SECOND:  $\$i \rightarrow \$i$  thf(cGVB\_SECOND, type)  
cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
 $\forall x a: \$i, x b: \$i: ((cGVB_M@x a$  and  $cGVB_M@x b) \Rightarrow (cGVB_SECOND@(cGVB_OP@x a@x b)) = x b)$  thf(cGVB\_SND\_PR

**SEV372**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_OP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)  
cGVB\_FIRST:  $\$i \rightarrow \$i$  thf(cGVB\_FIRST, type)  
cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
 $\forall x a: \$i, x b: \$i: ((cGVB_M@x a$  and  $cGVB_M@x b) \Rightarrow (cGVB_FIRST@(cGVB_OP@x a@x b)) = x a)$  thf(cGVB\_FST\_PROF

**SEV373**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_COMPOSE:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_COMPOSE, type)  
 $\forall x f: \$i, x g: \$i, x h: \$i: (cGVB_COMPOSE@x f@(cGVB_COMPOSE@x g@x h)) = (cGVB_COMPOSE@(cGVB_COMPOSE@x f@$

**SEV374**<sup>5.p</sup> TPS problem from GVB-MB-THMS

$f$ :  $\$i$  thf( $f$ , type)  
cGVB\_COMPOSE:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_COMPOSE, type)  
cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)  
cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
 $\exists p: \$i: \forall x g: \$i: ((cGVB_M@x g) \Rightarrow ((cGVB_IN@x g@p) \iff (cGVB_COMPOSE@f@x g) = (cGVB_COMPOSE@x g@f)))$

**SEV375**<sup>5.p</sup> TPS problem from GVB-MB-THMS

$f$ :  $\$i$  thf( $f$ , type)  
cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)  
cGVB\_COMPOSE:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_COMPOSE, type)  
cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)  
 $\exists p: \$i: \forall x g: \$i: ((cGVB_IN@x g@p) \iff ((cGVB_COMPOSE@f@x g) = (cGVB_COMPOSE@x g@f) \text{ and } cGVB_M@x g))$

**SEV376**<sup>5.p</sup> TPS problem from GVB-MB-THMS

$f_3$ :  $\$i$  thf( $f_3$ , type)  
 $s_3$ :  $\$i$  thf( $s_3$ , type)  
 $f_1$ :  $\$i$  thf( $f_1$ , type)  
 $s_1$ :  $\$i$  thf( $s_1$ , type)  
 $h_1$ :  $\$i$  thf( $h_1$ , type)  
 $h_2$ :  $\$i$  thf( $h_2$ , type)  
 $f_2$ :  $\$i$  thf( $f_2$ , type)  
 $s_2$ :  $\$i$  thf( $s_2$ , type)  
cGVB\_COMPOSE:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_COMPOSE, type)  
cGVB\_HOMOM:  $\$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_HOMOM, type)  
(cGVB\_HOMOM@ $h_1$ @ $s_1$ @ $f_1$ @ $s_2$ @ $f_2$  and cGVB\_HOMOM@ $h_2$ @ $s_2$ @ $f_2$ @ $s_3$ @ $f_3$ )  $\Rightarrow$  (cGVB\_HOMOM@(cGVB\_COMPOSE@

**SEV377**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_COMPOSE:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_COMPOSE, type)

cGVB\_APPLY:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_APPLY, type)

cGVB\_DOMAIN:  $\$i \rightarrow \$i$  thf(cGVB\_DOMAIN, type)

cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)

cGVB\_FUNCTION:  $\$i \rightarrow \$o$  thf(cGVB\_FUNCTION, type)

$\forall x f: \$i, x g: \$i, x x: \$i: ((cGVB\_FUNCTION@x f \text{ and } cGVB\_IN@x x@(cGVB\_DOMAIN@x f)) \Rightarrow (cGVB\_APPLY@x g@(cGVB\_APPLY@(cGVB\_COMPOSE@x g@x f)@x x)))$  thf(cGVB\_APP\_PROP<sub>1</sub>, conjecture)

**SEV378**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_APPLY:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_APPLY, type)

cGVB\_ITERATE:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_ITERATE, type)

$\forall x f: \$i: (\exists x g: \$i: (cGVB\_ITERATE@x f@x g \text{ and } \exists x x: \$i: ((cGVB\_APPLY@x g@x x) = x x \text{ and } \forall x z: \$i: ((cGVB\_APPLY@x g@x z) \Rightarrow x z = x x))) \Rightarrow \exists x y: \$i: (cGVB\_APPLY@x f@x y) = x y)$  thf(cGVB\_THM15B, conjecture)

**SEV379**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_OP:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_OP, type)

cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)

$\forall x a: \$i, x b: \$i, x c: \$i, x d: \$i: ((cGVB\_M@x a \text{ and } cGVB\_M@x b \text{ and } cGVB\_M@x c \text{ and } cGVB\_M@x d \text{ and } (cGVB\_OP@x a@x b) (cGVB\_OP@x c@x d)) \Rightarrow (x a = x c \text{ and } x b = x d))$  thf(cGVB\_OP\_PROP<sub>1</sub>, conjecture)

**SEV380**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_APPLY:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_APPLY, type)

cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)

cGVB\_ITERATE:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_ITERATE, type)

cGVB\_FUNCTION:  $\$i \rightarrow \$o$  thf(cGVB\_FUNCTION, type)

$\forall x f: \$i: ((cGVB\_FUNCTION@x f \text{ and } \exists x g: \$i: (cGVB\_FUNCTION@x g \text{ and } cGVB\_ITERATE@x f@x g \text{ and } \exists x x: \$i: (cGVB\_M@x x \text{ and } \forall x z: \$i: ((cGVB\_M@x z \text{ and } (cGVB\_APPLY@x g@x z) = x z) \Rightarrow x z = x x)))) \Rightarrow \exists x y: \$i: (cGVB\_M@x y \text{ and } (cGVB\_APPLY@x f@x y) = x y))$  thf(cGVB\_THM15B<sub>1</sub>, conjecture)

**SEV381**<sup>5.p</sup> TPS problem from GVB-MB-THMS

cGVB\_APPLY:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cGVB\_APPLY, type)

cGVB\_DOMAIN:  $\$i \rightarrow \$i$  thf(cGVB\_DOMAIN, type)

cGVB\_IN:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_IN, type)

cGVB\_FUNCTION:  $\$i \rightarrow \$o$  thf(cGVB\_FUNCTION, type)

cGVB\_M:  $\$i \rightarrow \$o$  thf(cGVB\_M, type)

cGVB\_ITERATE:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cGVB\_ITERATE, type)

$\forall x f: \$i: ((cGVB\_M@x f \text{ and } cGVB\_FUNCTION@x f \text{ and } \exists x g: \$i: (cGVB\_ITERATE@x f@x g \text{ and } cGVB\_M@x g \text{ and } cGVB\_FUNCTION@x g \text{ and } \forall x x: \$i: ((cGVB\_APPLY@x g@x x) = x x \Rightarrow x x = x x))) \Rightarrow \exists x y: \$i: (cGVB\_APPLY@x f@x y) = x y)$  thf(cGVB\_THM15B<sub>2</sub>, conjecture)

**SEV382**<sup>5.p</sup> TPS problem TRANS-IND

Transfinite induction theorem, from [BB93].

$a: \$tType$  thf(a\_type, type)

$\forall x r: a \rightarrow a \rightarrow \$o, p: a \rightarrow \$o: ((\forall x s: a \rightarrow \$o: (\exists x z: a: (x s@x z) \Rightarrow \exists x y: a: (x s@x y \text{ and } \forall x w: a: ((x r@x w@x y) \Rightarrow \neg x s@x w))) \text{ and } \forall x x: a: (\forall x y: a: ((x r@x y@x x) \Rightarrow (p@x y)) \Rightarrow (p@x x))) \Rightarrow \forall x x: a: (p@x x))$  thf(cTRANS\_IND, conjecture)

**SEV383**<sup>5.p</sup> TPS problem BLEDSOE-FENG-7

There is some set that doesn't contain a given object a - so the empty set works.

$a: \$i$  thf(a, type)

$\exists a: \$i \rightarrow \$o: \neg a@a$  thf(cBLEDSOE\_FENG<sub>7</sub>, conjecture)

**SEV384**<sup>5.p</sup> TPS problem THM117B

If R is a well-founded relation and P is an inductive property over R restricted to s, then everything in s has property P; here R y w means y > w.

cP:  $\$i \rightarrow \$o$  thf(cP, type)

s:  $\$i \rightarrow \$o$  thf(s, type)

cR:  $\$i \rightarrow \$i \rightarrow \$o$  thf(cR, type)

$(\forall x x: \$i \rightarrow \$o, x z: \$i: ((x x@x z) \Rightarrow \exists x y: \$i: (x x@x y \text{ and } \forall x w: \$i: ((cR@x y@x w) \Rightarrow \neg x x@x w))) \text{ and } \forall x x_1: \$i: (\forall x y_1: \$i: ((s@x y_1) \Rightarrow (cP@x y_1)) \Rightarrow (cP@x x_1))) \Rightarrow \forall x x_2: \$i: ((s@x x_2) \Rightarrow (cP@x x_2))$  thf(cTHM117B, conjecture)

**SEV385**<sup>5.p</sup> TPS problem X6004

$b: \$tType$  thf(b\_type, type)

$a: \$tType$  thf(a\_type, type)

$x: b$  thf(x, type)

$y: a$      $\text{thf}(y, \text{type})$

$\exists xs: b \rightarrow a: (\forall xx_6: b: (x = xx_6 \Rightarrow y = (xs@xx_6)))$  and  $\forall xy_{56}: a: (y = xy_{56} \Rightarrow \exists xy_0: b: (\lambda xx_7: b: (x = xx_7$  and  $xy_{56} = (xs@xx_7))) = (\lambda xx: b, xy: b: xx = xy@xy_0)))$      $\text{thf}(cX6004\_pme, \text{conjecture})$

**SEV386** $\wedge$ **5.p** TPS problem TTTTP5306A

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$p: a \rightarrow \$o$      $\text{thf}(p, \text{type})$

$\exists xy: a: p = (\lambda xx: a, xy: a: xx = xy@xy) \iff \exists xy: a: (p@xy$  and  $\forall xz: a: ((p@xz) \Rightarrow xy = xz))$      $\text{thf}(cTTTP5306A\_pme, \text{conjecture})$

**SEV387** $\wedge$ **5.p** TPS problem GAZING-THM44

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$\forall s: a \rightarrow \$o, t: a \rightarrow \$o, u: a \rightarrow \$o: (\lambda xx: a: (s@xx$  and  $((t@xx$  and  $\neg u@xx)$  or  $(u@xx$  and  $\neg t@xx))) = (\lambda xz: a: ((s@xz$  and  $t@xz)))$

**SEV388** $\wedge$ **5.p** TPS problem THM36

$cS: \$i \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$cR: \$i \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$cR = cS \Rightarrow \forall xx: \$i: ((cR@xx) \Rightarrow (cS@xx))$      $\text{thf}(cTHM36\_pme, \text{conjecture})$

**SEV389** $\wedge$ **5.p** TPS problem THM37

$cS: \$i \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$cR: \$i \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$cR = (\lambda xx: \$i: (cR@xx$  and  $cS@xx)) \Rightarrow \forall xx: \$i: ((cR@xx) \Rightarrow (cS@xx))$      $\text{thf}(cTHM37\_pme, \text{conjecture})$

**SEV390** $\wedge$ **5.p** TPS problem THM35

$cS: \$i \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$cR: \$i \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$(\lambda xx: \$i: (cR@xx$  and  $cS@xx)) = cR \Rightarrow \forall xx: \$i: ((cR@xx) \Rightarrow (cS@xx))$      $\text{thf}(cTHM35\_pme, \text{conjecture})$

**SEV391** $\wedge$ **5.p** TPS problem THM87

$cP: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o$      $\text{thf}(cP, \text{type})$

$k: \$i \rightarrow \$i$      $\text{thf}(k, \text{type})$

$h: \$i \rightarrow \$i$      $\text{thf}(h, \text{type})$

$a: \$i$      $\text{thf}(a, \text{type})$

$\exists xv: \$i: \forall xj: \$i: \exists xq: \$i: ((cP@a@(h@xj)@xj$  or  $cP@xv@(k@xj)@xj) \Rightarrow (cP@xv@xq@xj))$      $\text{thf}(cTHM87\_pme, \text{conjecture})$

**SEV392** $\wedge$ **5.p** TPS problem THM38

$cS: \$i \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$cR: \$i \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$(\lambda xx: \$i: (cR@xx$  and  $cS@xx)) = (\lambda xz: \$i: (cR@xz$  or  $cS@xz)) \Rightarrow \forall xx: \$i: ((cR@xx) \Rightarrow (cS@xx))$      $\text{thf}(cTHM38\_pme, \text{conjecture})$

**SEV393** $\wedge$ **5.p** TPS problem THM39

$cS: \$i \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$cR: \$i \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$(\lambda xz: \$i: (cR@xz$  or  $cS@xz)) = (\lambda xx: \$i: (cR@xx$  and  $cS@xx)) \Rightarrow \forall xx: \$i: ((cR@xx) \Rightarrow (cS@xx))$      $\text{thf}(cTHM39\_pme, \text{conjecture})$

**SEV394** $\wedge$ **5.p** TPS problem THM269

Example for CADE-15.

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$\forall xw: a \rightarrow \$o, xy: a \rightarrow \$o, xz: a \rightarrow \$o: ((\forall xx: a: ((xw@xx$  and  $\neg xz@xx) \Rightarrow (xy@xx))$  and  $(\lambda xx: a: (xz@xx$  and  $\neg xy@xx)) = (\lambda xx: a: \$false)) \Rightarrow \forall xx: a: ((xw@xx) \Rightarrow (xy@xx))$      $\text{thf}(cTHM269\_pme, \text{conjecture})$

**SEV396** $\wedge$ **5.p** TPS problem THM31

$cR: \$i \rightarrow \$o$      $\text{thf}(cR, \text{type})$

$cS: \$i \rightarrow \$o$      $\text{thf}(cS, \text{type})$

$\forall xx: \$i: ((cR@xx) \Rightarrow (cS@xx)) \iff (\lambda xx: \$i: (cR@xx$  and  $cS@xx)) = cR$      $\text{thf}(cTHM31\_pme, \text{conjecture})$

**SEV397** $\wedge$ **5.p** TPS problem THM59

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$cZ: a \rightarrow \$o$      $\text{thf}(cZ, \text{type})$

$cY: a \rightarrow \$o$      $\text{thf}(cY, \text{type})$

$cX: a \rightarrow \$o$      $\text{thf}(cX, \text{type})$

$\forall xx: a: (((cX@xx$  and  $cY@xx) or  $cZ@xx) \iff ((cX@xx$  or  $cZ@xx) and  $(cY@xx$  or  $cZ@xx)))$      $\text{thf}(cTHM59\_pme, \text{conjecture})$$$

**SEV398** $\wedge$ **5.p** TPS problem THM67A

$a: \$tType$      $\text{thf}(a\_type, \text{type})$

$cF: (a \rightarrow \$o) \rightarrow a \rightarrow \$o$      $\text{thf}(cF, \text{type})$

$cG: (a \rightarrow \$o) \rightarrow a \rightarrow \$o$      $\text{thf}(cG, \text{type})$

$(\forall s: a \rightarrow \$o, t: a \rightarrow \$o: (\forall xx: a: ((s@xx) \Rightarrow (t@xx)) \Rightarrow \forall xx: a: ((cF@t@xx) \Rightarrow (cF@s@xx)))$  and  $\forall s: a \rightarrow \$o: (\forall xx: a: ((s@xx) \Rightarrow (cF@(cG@s@xx)))$  and  $\forall xx: a: ((s@xx) \Rightarrow (cG@(cF@s@xx))) \Rightarrow \forall s: a \rightarrow \$o, xx: a: ((cF@(cG@(cF@s))@xx) \iff (cF@s@xx))$      $\text{thf}(\text{cTHM67A\_pme, conjecture})$

**SEV399** $\wedge$ **5.p** TPS problem THM597

If K and L are order reversing, K o L has a fixed point.

$\forall k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o, l: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: ((\forall xu: \$i \rightarrow \$o, xv: \$i \rightarrow \$o: (\forall xx: \$i: ((xu@xx) \Rightarrow (xv@xx)) \Rightarrow \forall xx: \$i: ((k@xv@xx) \Rightarrow (k@xu@xx)))$  and  $\forall xu: \$i \rightarrow \$o, xv: \$i \rightarrow \$o: (\forall xx: \$i: ((xu@xx) \Rightarrow (xv@xx)) \Rightarrow \forall xx: \$i: ((l@xv@xx) \Rightarrow (l@xu@xx)))) \Rightarrow \exists xw: \$i \rightarrow \$o: (k@(l@xw)) = xw$      $\text{thf}(\text{cTHM597\_pme, conjecture})$

**SEV400** $\wedge$ **5.p** TPS problem THM590

A simple theorem about existence of intersection.

$cQ: \$i \rightarrow \$o$      $\text{thf}(cQ, \text{type})$

$cP: \$i \rightarrow \$o$      $\text{thf}(cP, \text{type})$

$\exists s: \$i \rightarrow \$o: (\forall xx: \$i: ((s@xx) \Rightarrow (cP@xx)))$  and  $\forall xx: \$i: ((s@xx) \Rightarrow (cQ@xx))$  and  $\forall r: \$i \rightarrow \$o: ((\forall xx: \$i: ((r@xx) \Rightarrow (cP@xx))$  and  $\forall xx: \$i: ((r@xx) \Rightarrow (cQ@xx))) \Rightarrow \forall xx: \$i: ((r@xx) \Rightarrow (s@xx))$      $\text{thf}(\text{cTHM590\_pme, conjecture})$

**SEV401** $\wedge$ **5.p** TPS problem THM67

$a: \$t\text{Type}$      $\text{thf}(a\_type, \text{type})$

$cF: (a \rightarrow \$o) \rightarrow a \rightarrow \$o$      $\text{thf}(cF, \text{type})$

$cG: (a \rightarrow \$o) \rightarrow a \rightarrow \$o$      $\text{thf}(cG, \text{type})$

$(\forall s: a \rightarrow \$o, t: a \rightarrow \$o: (\forall xx: a: ((s@xx) \Rightarrow (t@xx)) \Rightarrow \forall xx: a: ((cF@t@xx) \Rightarrow (cF@s@xx)))$  and  $\forall s: a \rightarrow \$o, xx: a: ((s@xx) \Rightarrow (cF@(cG@s@xx)))$  and  $\forall s: a \rightarrow \$o, xx: a: ((s@xx) \Rightarrow (cG@(cF@s@xx))) \Rightarrow \forall s: a \rightarrow \$o, xx: a: ((cF@(cG@(cF@s))@xx) \iff (cF@s@xx))$      $\text{thf}(\text{cTHM67\_pme, conjecture})$

**SEV402** $\wedge$ **5.p** TPS problem THM596

If a set function K is order reversing, then K o K has a fixed point. This is a special case of Knaster-Tarski.

$\forall k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: (\forall xu: \$i \rightarrow \$o, xv: \$i \rightarrow \$o: (\forall xx: \$i: ((xu@xx) \Rightarrow (xv@xx)) \Rightarrow \forall xx: \$i: ((k@xv@xx) \Rightarrow (k@xu@xx))) \Rightarrow \exists xw: \$i \rightarrow \$o: (k@(k@xw)) = xw$      $\text{thf}(\text{cTHM596\_pme, conjecture})$

**SEV403** $\wedge$ **5.p** TPS problem THM598

$\forall k: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o, l: (\$i \rightarrow \$o) \rightarrow \$i \rightarrow \$o: ((\forall xu: \$i \rightarrow \$o, xv: \$i \rightarrow \$o: (\forall xx: \$i: ((xu@xx) \Rightarrow (xv@xx)) \Rightarrow \forall xx: \$i: ((k@xu@xx) \Rightarrow (k@xv@xx)))$  and  $\forall xu: \$i \rightarrow \$o, xv: \$i \rightarrow \$o: (\forall xx: \$i: ((xu@xx) \Rightarrow (xv@xx)) \Rightarrow \forall xx: \$i: ((l@xu@xx) \Rightarrow (l@xv@xx)))) \Rightarrow \exists xw: \$i \rightarrow \$o: (\lambda xz: \$i: (k@xw@xz \text{ or } l@xw@xz)) = xw$      $\text{thf}(\text{cTHM598\_pme, conjecture})$

**SEV404** $\wedge$ **5.p** TPS problem THM595

Existence of a stream of P values.

$b: \$t\text{Type}$      $\text{thf}(b\_type, \text{type})$

$a: \$t\text{Type}$      $\text{thf}(a\_type, \text{type})$

$cRST: b \rightarrow b$      $\text{thf}(cRST, \text{type})$

$cFST: b \rightarrow a$      $\text{thf}(cFST, \text{type})$

$cP: a \rightarrow \$o$      $\text{thf}(cP, \text{type})$

$\exists xv: b \rightarrow \$o: (\forall xx: b: ((xv@xx) \Rightarrow (cP@(cFST@xx)))$  and  $\forall xx: b: ((xv@xx) \Rightarrow (xv@(cRST@xx)))$  and  $\forall xu: b \rightarrow \$o: ((\forall xx: b: ((xu@xx) \Rightarrow (cP@(cFST@xx)))$  and  $\forall xx: b: ((xu@xx) \Rightarrow (xu@(cRST@xx)))) \Rightarrow \forall xx: b: ((xu@xx) \Rightarrow (xv@xx))$      $\text{thf}(\text{cTHM595\_pme, conjecture})$

**SEV405** $\wedge$ **5.p** TPS problem from SETS-THMS

$cA: \$o$      $\text{thf}(cA, \text{type})$

$\exists u: \$i \rightarrow \$o: \forall v: \$i: ((u@v) \iff cA)$      $\text{thf}(\text{cCOMP}_1, \text{conjecture})$

**SEV406** $\wedge$ **5.p** TPS problem from SETS-THMS

$cA: \$i \rightarrow \$o$      $\text{thf}(cA, \text{type})$

$cB: \$i \rightarrow \$o$      $\text{thf}(cB, \text{type})$

$cP: (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(cP, \text{type})$

$(cP@\lambda xz: \$i: (cA@xz \text{ or } cB@xz)) \Rightarrow (cP@\lambda xz: \$i: (cB@xz \text{ or } cA@xz))$      $\text{thf}(\text{cTRIVEXT2\_pme, conjecture})$

**SEV408** $\wedge$ **5.p** TPS problem from SETS-THMS

$cF: (\$i \rightarrow \$o) \rightarrow \$o$      $\text{thf}(cF, \text{type})$

$\exists g: (\$i \rightarrow \$o) \rightarrow \$o: \forall a: \$i \rightarrow \$o: ((g@a) \Rightarrow \exists b: \$i \rightarrow \$o: (cF@b \text{ and } \forall xx: \$i: ((a@xx) \Rightarrow (b@xx))))$      $\text{thf}(\text{cBLEDSOE2\_pme, conjecture})$

**SEV409** $\wedge$ **5.p** TPS problem from SETS-THMS

$\exists r: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o: \forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: ((r@x@y \text{ and } r@y@x) \Rightarrow \forall xx: \$i: ((x@xx) \iff (y@xx)))$      $\text{thf}(\text{cTHM120\_BUG\_pme, conjecture})$

**SEV410** $\wedge$ **5.p** TPS problem from SETS-THMS

$cA: \$i \rightarrow \$o \quad \text{thf}(cA, \text{type})$   
 $cP: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cP, \text{type})$   
 $cB: \$i \rightarrow \$o \quad \text{thf}(cB, \text{type})$   
 $(cP@\lambda xx: \$i: (cA@xx \text{ or } cB@xx)) \Rightarrow \exists xu: \$i \rightarrow \$o: (cP@xu \text{ and } \forall xx: \$i: ((cA@xx) \Rightarrow (xu@xx))) \quad \text{thf}(cSV1\_pme, \text{conjecture})$

**SEV411** $\wedge$ **5.p** TPS problem from SETS-THMS

$cB: \$i \rightarrow \$o \quad \text{thf}(cB, \text{type})$   
 $cA: \$i \rightarrow \$o \quad \text{thf}(cA, \text{type})$   
 $(\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \text{ or } \forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx))) \Rightarrow \forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \quad \text{thf}(cDUAL\_EG1, \text{conjecture})$

**SEV412** $\wedge$ **5.p** TPS problem from SETS-THMS

$cG: \$o \quad \text{thf}(cG, \text{type})$   
 $cB: \$i \rightarrow \$o \quad \text{thf}(cB, \text{type})$   
 $cA: \$i \rightarrow \$o \quad \text{thf}(cA, \text{type})$   
 $((\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \Rightarrow cG) \text{ or } (\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \Rightarrow cG)) \Rightarrow (\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \Rightarrow cG) \quad \text{thf}(cDUAL\_EG2\_pme, \text{conjecture})$

**SEV413** $\wedge$ **5.p** TPS problem from SETS-THMS

$cB: \$i \rightarrow \$o \quad \text{thf}(cB, \text{type})$   
 $cA: \$i \rightarrow \$o \quad \text{thf}(cA, \text{type})$   
 $(\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \text{ or } \forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx))) \Rightarrow (\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \text{ and } \forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx))) \quad \text{thf}(cDUAL\_EG5\_pme, \text{conjecture})$

**SEV414** $\wedge$ **5.p** TPS problem from SETS-THMS

$cS: \$i \rightarrow \$i \quad \text{thf}(cS, \text{type})$   
 $c_0: \$i \quad \text{thf}(c_0, \text{type})$   
 $\exists xv: \$i \rightarrow \$o: (xv@c_0 \text{ and } \forall xw: \$i: ((xv@xw) \Rightarrow (xv@(cS@xw)))) \text{ and } \forall xp: \$i \rightarrow \$o: ((xp@c_0 \text{ and } \forall xw: \$i: ((xp@xw) \Rightarrow (xp@(cS@xw)))) \Rightarrow \forall xx: \$i: ((xv@xx) \Rightarrow (xp@xx))) \quad \text{thf}(cTHM594\_pme, \text{conjecture})$

**SEV416** $\wedge$ **5.p** TPS problem from SETS-THMS

$cG: \$o \quad \text{thf}(cG, \text{type})$   
 $cB: \$i \rightarrow \$o \quad \text{thf}(cB, \text{type})$   
 $cA: \$i \rightarrow \$o \quad \text{thf}(cA, \text{type})$   
 $((\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \text{ and } \forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx))) \Rightarrow cG) \text{ or } (\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \Rightarrow cG) \Rightarrow (\forall xx: \$i: ((cA@xx) \Rightarrow (cB@xx)) \Rightarrow cG) \quad \text{thf}(cDUAL\_EG3\_pme, \text{conjecture})$

**SEV417** $\wedge$ **5.p** TPS problem from SETS-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$   
 $cP: (a \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cP, \text{type})$   
 $\forall x: a \rightarrow \$o, y: a \rightarrow \$o, z: a \rightarrow \$o: ((\forall xx: a: ((x@xx) \Rightarrow (y@xx)) \text{ and } \forall xx: a: ((x@xx) \Rightarrow (z@xx)) \text{ and } (\lambda xx: a: (y@xx \text{ and } z@xx) \Rightarrow (\lambda xx: a: \$false) \text{ and } cP@\lambda xx: a: (y@xx \text{ and } z@xx)) \Rightarrow (x = (\lambda xx: a: \$false) \text{ and } cP@\lambda xx: a: \$false))) \quad \text{thf}(cTHM502\_pme, \text{conjecture})$

**SEV418** $\wedge$ **5.p** TPS problem from SETS-THMS

$b: \$tType \quad \text{thf}(b\_type, \text{type})$   
 $a: \$tType \quad \text{thf}(a\_type, \text{type})$   
 $cG: (b \rightarrow \$o) \rightarrow a \rightarrow \$o \quad \text{thf}(cG, \text{type})$   
 $cF: (a \rightarrow \$o) \rightarrow b \rightarrow \$o \quad \text{thf}(cF, \text{type})$   
 $(\forall xy: b, y: b \rightarrow \$o: ((cF@(cG@y)@xy) \iff (y@xy)) \text{ and } \forall xx: a, x: a \rightarrow \$o: ((cG@(cF@x)@xx) \iff (x@xx)) \text{ and } \forall u: a \rightarrow \$o, v: a \rightarrow \$o: (\forall xx: a: ((u@xx) \Rightarrow (v@xx)) \Rightarrow \forall xx: b: ((cF@u@xx) \Rightarrow (cF@v@xx)))) \Rightarrow \forall m: b \rightarrow \$o, n: b \rightarrow \$o: (\forall xx: b: ((m@xx) \Rightarrow (n@xx)) \Rightarrow \forall xx: a: ((cG@m@xx) \Rightarrow (cG@n@xx))) \quad \text{thf}(cTHM592\_pme, \text{conjecture})$

**SEV419** $\wedge$ **5.p** TPS problem from SETS-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$   
 $b: \$tType \quad \text{thf}(b\_type, \text{type})$   
 $\forall f: (a \rightarrow \$o) \rightarrow b \rightarrow \$o: ((\forall x: a \rightarrow \$o, y: a \rightarrow \$o: (\forall xx: a: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: b: ((f@x@xx) \Rightarrow (f@y@xx))) \text{ and } \forall x: a \rightarrow \$o, y: a \rightarrow \$o: ((f@x) = (f@y) \Rightarrow x = y) \text{ and } \forall z: b \rightarrow \$o: \exists y: a \rightarrow \$o: (f@y) = z) \Rightarrow \forall x: a \rightarrow \$o, y: a \rightarrow \$o: (\forall xx: b: ((f@x@xx) \Rightarrow (f@y@xx)) \Rightarrow \forall xx: a: ((x@xx) \Rightarrow (y@xx)))) \quad \text{thf}(cTHM593\_pme, \text{conjecture})$

**SEV420** $\wedge$ **1.p** Size of disjoint sets' union

If  $\text{---}A\text{---} = \text{---}A'\text{---} \ \& \ \text{---}B\text{---} = \text{---}B'\text{---} \ \& \ \text{---}A' \wedge B'\text{---} = 0$ , then  $\text{---}A \cup B\text{---} = \text{---}A' \cup B'\text{---}$

include('Axioms/SET008^0.ax')

is\_function:  $(\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(\text{is\_function\_type}, \text{type})$

is\_function =  $(\lambda x: \$i \rightarrow \$o, f: \$i \rightarrow \$i, y: \$i \rightarrow \$o: \forall e: \$i: ((x@e) \Rightarrow (y@(f@e)))) \quad \text{thf}(\text{is\_function}, \text{definition})$

injection:  $(\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(\text{injection\_type}, \text{type})$



injection =  $(\lambda x: \$i \rightarrow \$o, f: \$i \rightarrow \$i, y: \$i \rightarrow \$o: ((\text{is\_function}@x@f@y) \Rightarrow \forall e_1: \$i, e_2: \$i: ((x@e_1 \text{ and } x@e_2 \text{ and } (f@e_1) = (f@e_2)) \Rightarrow e_1 = e_2)))$     thf(injection, definition)  
 surjection:  $(\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$     thf(surjection\_type, type)  
 surjection =  $(\lambda x: \$i \rightarrow \$o, f: \$i \rightarrow \$i, y: \$i \rightarrow \$o: ((\text{is\_function}@x@f@y) \Rightarrow \forall e_1: \$i: ((y@e_1) \Rightarrow \exists e_2: \$i: (x@e_2 \text{ and } (f@e_2) = e_1))))$     thf(surjection, definition)  
 bijection:  $(\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$     thf(bijection\_type, type)  
 bijection =  $(\lambda x: \$i \rightarrow \$o, f: \$i \rightarrow \$i, y: \$i \rightarrow \$o: (\text{injection}@x@f@y \text{ and } \text{surjection}@x@f@y))$     thf(bijection, definition)  
 equinumerous:  $(\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$     thf(equinumerous\_type, type)  
 equinumerous =  $(\lambda x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: \exists f: \$i \rightarrow \$i: (\text{bijection}@x@f@y))$     thf(equinumerous, definition)  
 embedding:  $(\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o$     thf(embedding\_type, type)  
 embedding =  $(\lambda x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: \exists f: \$i \rightarrow \$i: (\text{injection}@x@f@y))$     thf(embedding, definition)  
 $\forall a: \$i \rightarrow \$o, \text{ap}: \$i \rightarrow \$o, b: \$i \rightarrow \$o, \text{bp}: \$i \rightarrow \$o: ((\text{equinumerous}@a@\text{ap} \text{ and } \text{equinumerous}@b@\text{bp} \text{ and } (\text{intersection}@a@\text{ap}@\text{bp})) \Rightarrow (\text{embedding}@(\text{union}@a@b)@(\text{union}@a@\text{bp})))$     thf(prove, conjecture)

### SEV421=1.p Correctness of an efficient emptiness check

Using invariant on size to prove correctness of an efficient emptiness check.

set: \$tType    tff(set\_type, type)  
 element: \$tType    tff(element\_type, type)  
 empty\_set: set    tff(empty\_set\_type, type)  
 singleton: element  $\rightarrow$  set    tff(singleton\_type, type)  
 $\in$  : (element  $\times$  set)  $\rightarrow$  \$o    tff(member\_type, type)  
 $\subseteq$  : (set  $\times$  set)  $\rightarrow$  \$o    tff(subset\_type, type)  
 intersection: (set  $\times$  set)  $\rightarrow$  set    tff(intersection\_type, type)  
 union: (set  $\times$  set)  $\rightarrow$  set    tff(union\_type, type)  
 $\setminus$ : (set  $\times$  set)  $\rightarrow$  set    tff(difference\_type, type)  
 complement: set  $\rightarrow$  set    tff(complement\_type, type)  
 cardinality: set  $\rightarrow$  \$int    tff(cardinality\_type, type)  
 $\forall s$ : set:  $(\forall x$ : element:  $\neg x \in s \iff s = \text{empty\_set})$     tff(empty\_set, axiom)  
 $\forall x$ : element,  $a$ : element:  $(x \in \text{singleton}(a) \iff x = a)$     tff(singleton, axiom)  
 $\forall a$ : set,  $b$ : set:  $(a \subseteq b \iff \forall x$ : element:  $(x \in a \Rightarrow x \in b))$     tff(subset, axiom)  
 $\forall x$ : element,  $a$ : set,  $b$ : set:  $(x \in \text{intersection}(a, b) \iff (x \in a \text{ and } x \in b))$     tff(intersection, axiom)  
 $\forall x$ : element,  $a$ : set,  $b$ : set:  $(x \in \text{union}(a, b) \iff (x \in a \text{ or } x \in b))$     tff(union, axiom)  
 $\forall b$ : element,  $a$ : set,  $e$ : set:  $(b \in (e \setminus a) \iff (b \in e \text{ and } \neg b \in a))$     tff(difference, axiom)  
 $\forall x$ : element,  $s$ : set:  $(x \in s \iff \neg x \in s')$     tff(complement, axiom)  
 $\forall s$ : set:  $(\text{cardinality}(s) = 0 \iff s = \text{empty\_set})$     tff(cardinality\_empty\_set, axiom)  
 $\forall x$ : element,  $s$ : set:  $(\text{intersection}(\text{singleton}(x), s) = \text{singleton}(x) \iff \text{cardinality}(\text{union}(\text{singleton}(x), s)) = \text{cardinality}(s))$   
 $\forall x$ : element,  $s$ : set:  $(\text{intersection}(\text{singleton}(x), s) = \text{empty\_set} \iff \text{cardinality}(\text{union}(\text{singleton}(x), s)) = \$\text{sum}(\text{cardinality}(s), \text{cardinality}(\text{empty\_set})))$   
 $\forall s$ : set,  $t$ : set:  $(\text{cardinality}(\text{intersection}(s, t)) = 0 \iff \text{intersection}(s, t) = \text{empty\_set})$     tff(cardinality\_intersection\_3, axiom)  
 $\forall a$ : set,  $b$ : set:  $(\text{intersection}(a, b) = \text{empty\_set} \iff \text{cardinality}(\text{union}(a, b)) = \$\text{sum}(\text{cardinality}(a), \text{cardinality}(b)))$     tff(cardinality\_union\_empty, axiom)  
 $\forall x$ : element,  $c$ : set, size: \$int:  $((\neg x \in c \text{ and } \text{size} = \text{cardinality}(c)) \Rightarrow (\text{size} = 0 \iff c = \text{empty\_set}))$     tff(vc1, conjecture)

### SEV422=1.p Maintaining correct size when inserting fresh element

set: \$tType    tff(set\_type, type)  
 element: \$tType    tff(element\_type, type)  
 empty\_set: set    tff(empty\_set\_type, type)  
 singleton: element  $\rightarrow$  set    tff(singleton\_type, type)  
 $\in$  : (element  $\times$  set)  $\rightarrow$  \$o    tff(member\_type, type)  
 $\subseteq$  : (set  $\times$  set)  $\rightarrow$  \$o    tff(subset\_type, type)  
 intersection: (set  $\times$  set)  $\rightarrow$  set    tff(intersection\_type, type)  
 union: (set  $\times$  set)  $\rightarrow$  set    tff(union\_type, type)  
 $\setminus$ : (set  $\times$  set)  $\rightarrow$  set    tff(difference\_type, type)  
 complement: set  $\rightarrow$  set    tff(complement\_type, type)  
 cardinality: set  $\rightarrow$  \$int    tff(cardinality\_type, type)  
 $\forall s$ : set:  $(\forall x$ : element:  $\neg x \in s \iff s = \text{empty\_set})$     tff(empty\_set, axiom)  
 $\forall x$ : element,  $a$ : element:  $(x \in \text{singleton}(a) \iff x = a)$     tff(singleton, axiom)  
 $\forall a$ : set,  $b$ : set:  $(a \subseteq b \iff \forall x$ : element:  $(x \in a \Rightarrow x \in b))$     tff(subset, axiom)  
 $\forall x$ : element,  $a$ : set,  $b$ : set:  $(x \in \text{intersection}(a, b) \iff (x \in a \text{ and } x \in b))$     tff(intersection, axiom)  
 $\forall x$ : element,  $a$ : set,  $b$ : set:  $(x \in \text{union}(a, b) \iff (x \in a \text{ or } x \in b))$     tff(union, axiom)  
 $\forall b$ : element,  $a$ : set,  $e$ : set:  $(b \in (e \setminus a) \iff (b \in e \text{ and } \neg b \in a))$     tff(difference, axiom)  
 $\forall x$ : element,  $s$ : set:  $(x \in s \iff \neg x \in s')$     tff(complement, axiom)



$\forall c: \text{set}, a: \text{set}, x_1: \text{element}, x_2: \text{element}, x_3: \text{element}: ((c \subseteq a \text{ and } \neg x_1 \in a \text{ and } \neg x_2 \in \text{union}(a, \text{singleton}(x_1))) \text{ and } \neg x_3 \in \text{union}(\text{union}(a, \text{singleton}(x_1)), \text{singleton}(x_2))) \Rightarrow \text{cardinality}(\text{union}(\text{union}(\text{union}(c, \text{singleton}(x_1)), \text{singleton}(x_2)), \text{singleton}(x_3))) = \text{\$sum}(\text{cardinality}(c), 3))$      $\text{tff}(\text{vc}_4, \text{conjecture})$

**SEV425=1.p** Allocating and inserting at least three objects

Allocating and inserting at least three objects into a container data structure.

set:  $\text{\$tType}$      $\text{tff}(\text{set\_type}, \text{type})$   
element:  $\text{\$tType}$      $\text{tff}(\text{element\_type}, \text{type})$   
empty\_set: set     $\text{tff}(\text{empty\_set\_type}, \text{type})$   
singleton: element  $\rightarrow$  set     $\text{tff}(\text{singleton\_type}, \text{type})$   
 $\in$  : (element  $\times$  set)  $\rightarrow$   $\text{\$o}$      $\text{tff}(\text{member\_type}, \text{type})$   
 $\subseteq$  : (set  $\times$  set)  $\rightarrow$   $\text{\$o}$      $\text{tff}(\text{subset\_type}, \text{type})$   
intersection: (set  $\times$  set)  $\rightarrow$  set     $\text{tff}(\text{intersection\_type}, \text{type})$   
union: (set  $\times$  set)  $\rightarrow$  set     $\text{tff}(\text{union\_type}, \text{type})$   
 $\setminus$ : (set  $\times$  set)  $\rightarrow$  set     $\text{tff}(\text{difference\_type}, \text{type})$   
complement: set  $\rightarrow$  set     $\text{tff}(\text{complement\_type}, \text{type})$   
cardinality: set  $\rightarrow$   $\text{\$int}$      $\text{tff}(\text{cardinality\_type}, \text{type})$   
 $\forall s: \text{set}: (\forall x: \text{element}: \neg x \in s \iff s = \text{empty\_set})$      $\text{tff}(\text{empty\_set}, \text{axiom})$   
 $\forall x: \text{element}, a: \text{element}: (x \in \text{singleton}(a) \iff x = a)$      $\text{tff}(\text{singleton}, \text{axiom})$   
 $\forall a: \text{set}, b: \text{set}: (a \subseteq b \iff \forall x: \text{element}: (x \in a \Rightarrow x \in b))$      $\text{tff}(\text{subset}, \text{axiom})$   
 $\forall x: \text{element}, a: \text{set}, b: \text{set}: (x \in \text{intersection}(a, b) \iff (x \in a \text{ and } x \in b))$      $\text{tff}(\text{intersection}, \text{axiom})$   
 $\forall x: \text{element}, a: \text{set}, b: \text{set}: (x \in \text{union}(a, b) \iff (x \in a \text{ or } x \in b))$      $\text{tff}(\text{union}, \text{axiom})$   
 $\forall b: \text{element}, a: \text{set}, e: \text{set}: (b \in (e \setminus a) \iff (b \in e \text{ and } \neg b \in a))$      $\text{tff}(\text{difference}, \text{axiom})$   
 $\forall x: \text{element}, s: \text{set}: (x \in s \iff \neg x \in s')$      $\text{tff}(\text{complement}, \text{axiom})$   
 $\forall s: \text{set}: (\text{cardinality}(s) = 0 \iff s = \text{empty\_set})$      $\text{tff}(\text{cardinality\_empty\_set}, \text{axiom})$   
 $\forall x: \text{element}, s: \text{set}: (\text{intersection}(\text{singleton}(x), s) = \text{singleton}(x) \iff \text{cardinality}(\text{union}(\text{singleton}(x), s)) = \text{cardinality}(s))$   
 $\forall x: \text{element}, s: \text{set}: (\text{intersection}(\text{singleton}(x), s) = \text{empty\_set} \iff \text{cardinality}(\text{union}(\text{singleton}(x), s)) = \text{\$sum}(\text{cardinality}(s), \text{cardinality}(\text{singleton}(x))))$   
 $\forall s: \text{set}, t: \text{set}: (\text{cardinality}(\text{intersection}(s, t)) = 0 \iff \text{intersection}(s, t) = \text{empty\_set})$      $\text{tff}(\text{cardinality\_intersection}_3, \text{axiom})$   
 $\forall a: \text{set}, b: \text{set}: (\text{intersection}(a, b) = \text{empty\_set} \iff \text{cardinality}(\text{union}(a, b)) = \text{\$sum}(\text{cardinality}(a), \text{cardinality}(b)))$      $\text{tff}(\text{cardinality\_union}_3, \text{axiom})$   
 $\forall c: \text{set}, a_0: \text{set}, a_1: \text{set}, a_2: \text{set}, x_1: \text{element}, x_2: \text{element}, x_3: \text{element}: ((c \subseteq a_0 \text{ and } \neg x_1 \in a_0 \text{ and } \text{union}(a_0, \text{singleton}(x_1)) \subseteq a_1 \text{ and } \neg x_2 \in a_1 \text{ and } \text{union}(a_1, \text{singleton}(x_2)) \subseteq a_2 \text{ and } \neg x_3 \in a_2) \Rightarrow \text{cardinality}(\text{union}(\text{union}(\text{union}(c, \text{singleton}(x_1)), \text{singleton}(x_2)), \text{singleton}(x_3))) = \text{\$sum}(\text{cardinality}(c), 3))$      $\text{tff}(\text{vc}_5, \text{conjecture})$

**SEV426=1.p** Bound on the number of allocated objects in a recursive function

Bound on the number of allocated objects in a recursive function that incorporates container C into another container.

set:  $\text{\$tType}$      $\text{tff}(\text{set\_type}, \text{type})$   
element:  $\text{\$tType}$      $\text{tff}(\text{element\_type}, \text{type})$   
empty\_set: set     $\text{tff}(\text{empty\_set\_type}, \text{type})$   
singleton: element  $\rightarrow$  set     $\text{tff}(\text{singleton\_type}, \text{type})$   
 $\in$  : (element  $\times$  set)  $\rightarrow$   $\text{\$o}$      $\text{tff}(\text{member\_type}, \text{type})$   
 $\subseteq$  : (set  $\times$  set)  $\rightarrow$   $\text{\$o}$      $\text{tff}(\text{subset\_type}, \text{type})$   
intersection: (set  $\times$  set)  $\rightarrow$  set     $\text{tff}(\text{intersection\_type}, \text{type})$   
union: (set  $\times$  set)  $\rightarrow$  set     $\text{tff}(\text{union\_type}, \text{type})$   
 $\setminus$ : (set  $\times$  set)  $\rightarrow$  set     $\text{tff}(\text{difference\_type}, \text{type})$   
complement: set  $\rightarrow$  set     $\text{tff}(\text{complement\_type}, \text{type})$   
cardinality: set  $\rightarrow$   $\text{\$int}$      $\text{tff}(\text{cardinality\_type}, \text{type})$   
 $\forall s: \text{set}: (\forall x: \text{element}: \neg x \in s \iff s = \text{empty\_set})$      $\text{tff}(\text{empty\_set}, \text{axiom})$   
 $\forall x: \text{element}, a: \text{element}: (x \in \text{singleton}(a) \iff x = a)$      $\text{tff}(\text{singleton}, \text{axiom})$   
 $\forall a: \text{set}, b: \text{set}: (a \subseteq b \iff \forall x: \text{element}: (x \in a \Rightarrow x \in b))$      $\text{tff}(\text{subset}, \text{axiom})$   
 $\forall x: \text{element}, a: \text{set}, b: \text{set}: (x \in \text{intersection}(a, b) \iff (x \in a \text{ and } x \in b))$      $\text{tff}(\text{intersection}, \text{axiom})$   
 $\forall x: \text{element}, a: \text{set}, b: \text{set}: (x \in \text{union}(a, b) \iff (x \in a \text{ or } x \in b))$      $\text{tff}(\text{union}, \text{axiom})$   
 $\forall b: \text{element}, a: \text{set}, e: \text{set}: (b \in (e \setminus a) \iff (b \in e \text{ and } \neg b \in a))$      $\text{tff}(\text{difference}, \text{axiom})$   
 $\forall x: \text{element}, s: \text{set}: (x \in s \iff \neg x \in s')$      $\text{tff}(\text{complement}, \text{axiom})$   
 $\forall s: \text{set}: (\text{cardinality}(s) = 0 \iff s = \text{empty\_set})$      $\text{tff}(\text{cardinality\_empty\_set}, \text{axiom})$   
 $\forall x: \text{element}, s: \text{set}: (\text{intersection}(\text{singleton}(x), s) = \text{singleton}(x) \iff \text{cardinality}(\text{union}(\text{singleton}(x), s)) = \text{cardinality}(s))$   
 $\forall x: \text{element}, s: \text{set}: (\text{intersection}(\text{singleton}(x), s) = \text{empty\_set} \iff \text{cardinality}(\text{union}(\text{singleton}(x), s)) = \text{\$sum}(\text{cardinality}(s), \text{cardinality}(\text{singleton}(x))))$   
 $\forall s: \text{set}, t: \text{set}: (\text{cardinality}(\text{intersection}(s, t)) = 0 \iff \text{intersection}(s, t) = \text{empty\_set})$      $\text{tff}(\text{cardinality\_intersection}_3, \text{axiom})$   
 $\forall a: \text{set}, b: \text{set}: (\text{intersection}(a, b) = \text{empty\_set} \iff \text{cardinality}(\text{union}(a, b)) = \text{\$sum}(\text{cardinality}(a), \text{cardinality}(b)))$      $\text{tff}(\text{cardinality\_union}_3, \text{axiom})$   
 $\forall x: \text{element}, c: \text{set}, c_1: \text{set}, a_0: \text{set}, a_1: \text{set}, a_2: \text{set}: ((x \in c \text{ and } c_1 = c \setminus \text{singleton}(x) \text{ and } \text{\$lesseq}(\text{cardinality}(a_1 \setminus \text{singleton}(x)), \text{cardinality}(c_1)) \text{ and } \text{\$lesseq}(\text{cardinality}(a_2 \setminus a_0), \text{cardinality}(c_1))) \Rightarrow \text{\$lesseq}(\text{cardinality}(a_2 \setminus a_0), \text{cardinality}(c)))$      $\text{tff}(\text{vc}_6, \text{conjecture})$

**SEV427** $\wedge$ **1.p** If two sets cover a type, a choice function must give an element

eps:  $(\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}i$     thf(eps, type)  
 $\forall p: \mathbb{S}i \rightarrow \mathbb{S}o: (\exists x: \mathbb{S}i: (p@x) \Rightarrow (p@(eps@p)))$     thf(choiceax, axiom)  
 $p: \mathbb{S}i \rightarrow \mathbb{S}o$     thf(p, type)  
 $q: \mathbb{S}i \rightarrow \mathbb{S}o$     thf(q, type)  
 $\forall x: \mathbb{S}i: (p@x \text{ or } q@x)$     thf(pq, axiom)  
 $p@(eps@p) \text{ or } q@(eps@q)$     thf(conj, conjecture)

**SEV428** $\wedge$ **1.p** If a union is nonempty we can choose a nonempty set in the set.

eps:  $(\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}i$     thf(eps, type)  
 $\forall p: \mathbb{S}i \rightarrow \mathbb{S}o: (\exists x: \mathbb{S}i: (p@x) \Rightarrow (p@(eps@p)))$     thf(choiceax, axiom)  
epsio:  $((\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}i \rightarrow \mathbb{S}o$     thf(epsio, type)  
 $\forall p: (\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}o: (\exists x: \mathbb{S}i \rightarrow \mathbb{S}o: (p@x) \Rightarrow (p@(epsio@p)))$     thf(choiceaxio, axiom)  
setunion:  $((\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}i \rightarrow \mathbb{S}o$     thf(setunion, type)  
setunion =  $(\lambda c: (\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}o, x: \mathbb{S}i: \exists y: \mathbb{S}i \rightarrow \mathbb{S}o: (c@y \text{ and } y@x))$     thf(setuniond, definition)  
choosenonempty:  $((\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}i \rightarrow \mathbb{S}o$     thf(choosenonempty, type)  
choosenonempty =  $(\lambda c: (\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}o: (epsio@\lambda y: \mathbb{S}i \rightarrow \mathbb{S}o: (c@y \text{ and } y@(eps@y))))$     thf(choosenonemptyd, definition)  
 $c: (\mathbb{S}i \rightarrow \mathbb{S}o) \rightarrow \mathbb{S}o$     thf(c, type)  
 $a: \mathbb{S}i$     thf(a, type)  
setunion@c@a    thf(ca, axiom)  
 $c@(choosenonempty@c) \text{ and } \exists x: \mathbb{S}i: (choosenonempty@c@x)$     thf(conj, conjecture)

**SEV429** $\wedge$ **1.p** Injective functions  $f: I \rightarrow I$  have left inverses

$f: \mathbb{S}i \rightarrow \mathbb{S}i$     thf(f, type)  
 $\forall x: \mathbb{S}i, y: \mathbb{S}i: ((f@x) = (f@y) \Rightarrow x = y)$     thf(finj, axiom)  
 $\exists g: \mathbb{S}i \rightarrow \mathbb{S}i: \forall x: \mathbb{S}i: (g@(f@x)) = x$     thf(invexists, conjecture)

**SEV430** $\wedge$ **1.p** Surjective functions  $f: I \rightarrow I$  have right inverses

$f: \mathbb{S}i \rightarrow \mathbb{S}i$     thf(f, type)  
 $\forall y: \mathbb{S}i: \exists x: \mathbb{S}i: (f@x) = y$     thf(fsurdj, axiom)  
 $\exists g: \mathbb{S}i \rightarrow \mathbb{S}i: \forall x: \mathbb{S}i: (f@(g@x)) = x$     thf(invexists, conjecture)

**SEV431** $\wedge$ **1.p** Injective functions  $f: A \rightarrow B$  have left inverses

$a: \mathbb{S}tType$     thf(a, type)  
 $b: \mathbb{S}tType$     thf(b, type)  
 $f: a \rightarrow b$     thf(f, type)  
 $\forall x: a, y: a: ((f@x) = (f@y) \Rightarrow x = y)$     thf(finj, axiom)  
 $\exists g: b \rightarrow a: \forall x: a: (g@(f@x)) = x$     thf(invexists, conjecture)

**SEV432** $\wedge$ **1.p** Surjective functions  $f: A \rightarrow B$  have right inverses

$a: \mathbb{S}tType$     thf(a, type)  
 $b: \mathbb{S}tType$     thf(b, type)  
 $f: a \rightarrow b$     thf(f, type)  
 $\forall y: b: \exists x: a: (f@x) = y$     thf(fsurdj, axiom)  
 $\exists g: b \rightarrow a: \forall x: b: (f@(g@x)) = x$     thf(invexists, conjecture)

**SEV433** $\wedge$ **1.p** There are at most 2 individuals if there is an injection into o

$f: \mathbb{S}i \rightarrow \mathbb{S}o$     thf(f, type)  
 $\forall x: \mathbb{S}i, y: \mathbb{S}i: ((f@x) = (f@y) \Rightarrow x = y)$     thf(finj, axiom)  
 $\forall x: \mathbb{S}i, y: \mathbb{S}i, z: \mathbb{S}i: (x = y \text{ or } x = z \text{ or } y = z)$     thf(less<sub>3</sub>, conjecture)

**SEV434** $\wedge$ **1.p** There are at most 2 individuals if there is a surjection from o

$f: \mathbb{S}o \rightarrow \mathbb{S}i$     thf(f, type)  
 $\forall y: \mathbb{S}i: \exists x: \mathbb{S}o: (f@x) = y$     thf(fsurdj, axiom)  
 $\forall x: \mathbb{S}i, y: \mathbb{S}i, z: \mathbb{S}i: (x = y \text{ or } x = z \text{ or } y = z)$     thf(less<sub>3</sub>, conjecture)

**SEV436-1.p** Membership and subsets, union, intersection, difference

include('Axioms/SET001-0.ax')  
include('Axioms/SET001-1.ax')  
include('Axioms/SET001-2.ax')  
include('Axioms/SET001-3.ax')

**SEV437+1.p** Naive set theory based on Goedel's set theory

include('Axioms/SET006+0.ax')  
include('Axioms/SET006+1.ax')

```
include('Axioms/SET006+2.ax')
```

**SEV438+1.p** Order relation (Naive set theory)

```
include('Axioms/SET006+0.ax')
```

```
include('Axioms/SET006+3.ax')
```

**SEV439+1.p** Ordinal numbers

```
include('Axioms/SET006+0.ax')
```

```
include('Axioms/SET006+4.ax')
```

**SEV440^1.p** Basic set theory, functions, relations

```
include('Axioms/SET008^0.ax')
```

```
include('Axioms/SET008^1.ax')
```

```
include('Axioms/SET008^2.ax')
```

**SEV441^1.p** Binary relations

```
include('Axioms/SET009^0.ax')
```