

SYO axioms

SYO problems

SYO001 \wedge 1.p Leibniz equality is transitive

```
leibeq: $i → $i → $o      thf(leibeq_decl, type)
leibeq = (λx: $i, y: $i: ∀p: $i → $o: ((p@x) ⇒ (p@y)))      thf(leibeq, definition)
∀x: $i, y: $i, z: $i: ((leibeq@x@y and leibeq@y@z) ⇒ (leibeq@x@z))      thf(conj, conjecture)
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SYO002 \wedge 1.p Leibniz equality obeys the congruence property under functions

```
leibeq: $i → $i → $o      thf(leibeq_decl, type)
leibeq = (λx: $i, y: $i: ∀p: $i → $o: ((p@x) ⇒ (p@y)))      thf(leibeq, definition)
∀x: $i, y: $i, f: $i → $i: ((leibeq@x@y) ⇒ (leibeq@(f@x)@(f@y)))      thf(conj, conjecture)
```

SYO003 \wedge 1.p Leibniz equality obeys the congruence property under predicates

```
leibeq: $i → $i → $o      thf(leibeq_decl, type)
leibeq = (λx: $i, y: $i: ∀p: $i → $o: ((p@x) ⇒ (p@y)))      thf(leibeq, definition)
∀x: $i, y: $i, p: $i → $o: ((leibeq@x@y and p@x) ⇒ (p@y))      thf(conj, conjecture)
```

SYO004 \wedge 1.p Relating Leibniz equality to primitive equality

```
leibeq: $i → $i → $o      thf(leibeq_decl, type)
leibeq = (λx: $i, y: $i: ∀p: $i → $o: ((p@x) ⇒ (p@y)))      thf(leibeq, definition)
∀x: $i, y: $i: ((leibeq@x@y) ⇒ x = y)      thf(conj, conjecture)
```

SYO005 \wedge 1.p The trivial direction of functional extensionality

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leibeq1: $i → $i → $o      thf(leibeq1_type, type)
leibeq1 = (λu: $i, v: $i: ∀q: $i → $o: ((q@u) ⇒ (q@v)))      thf(leibeq1, definition)
leibeq2: ($i → $i) → ($i → $i) → $o      thf(leibeq2_type, type)
leibeq2 = (λx: $i → $i, y: $i → $i: ∀p: ($i → $i) → $o: ((p@x) ⇒ (p@y)))      thf(leibeq2, definition)
∀f: $i → $i, g: $i → $i: ((leibeq2@f@g) ⇒ ∀x: $i: (leibeq1@(f@x)@(g@x)))      thf(conj, conjecture)
```

SYO006 \wedge 1.p The trivial direction of Boolean extensionality

```
leibeq: $o → $o → $o      thf(leibeq1_type, type)
leibeq = (λu: $o, v: $o: ∀q: $o → $o: ((q@u) ⇒ (q@v)))      thf(leibeq1, definition)
∀a: $o, b: $o: ((leibeq@a@b) ⇒ (a ⇐⇒ b))      thf(conj, conjecture)
```

SYO007 \wedge 1.p The non-trivial direction of Boolean extensionality

```
leibeq: $o → $o → $o      thf(leibeq1_type, type)
leibeq = (λu: $o, v: $o: ∀q: $o → $o: ((q@u) ⇒ (q@v)))      thf(leibeq1, definition)
∀a: $o, b: $o: ((a ⇐⇒ b) ⇒ (leibeq@a@b))      thf(conj, conjecture)
```

SYO008 \wedge 1.p The non-trivial direction of functional extensionality

```
leibeq1: $i → $i → $o      thf(leibeq1_type, type)
leibeq1 = (λu: $i, v: $i: ∀q: $i → $o: ((q@u) ⇒ (q@v)))      thf(leibeq1, definition)
leibeq2: ($i → $i) → ($i → $i) → $o      thf(leibeq2_type, type)
leibeq2 = (λx: $i → $i, y: $i → $i: ∀p: ($i → $i) → $o: ((p@x) ⇒ (p@y)))      thf(leibeq2, definition)
∀f: $i → $i, g: $i → $i: (∀x: $i: (leibeq1@(f@x)@(g@x)) ⇒ (leibeq2@f@g))      thf(conj, conjecture)
```

SYO009 \wedge 1.p Eta-equality using Leibniz equality

```
p: ($i → $i) → $o      thf(p, type)
f: $i → $i      thf(f, type)
(p@λx: $i: (f@x)) ⇒ (p@f)      thf(conj, conjecture)
```

SYO010 \wedge 1.p Something requiring Xi but not Eta

```
leibeq: $i → $i → $o      thf(leibeq_type, type)
leibeq = (λu: $i, v: $i: ∀q: $i → $o: ((q@u) ⇒ (q@v)))      thf(leibeq, definition)
p: ($i → $i) → $o      thf(p_type, type)
f: $i → $i      thf(f_type, type)
(∀x: $i: (leibeq@(f@x)@x) and p@λx: $i: x) ⇒ (p@λx: $i: (f@x))      thf(conj, conjecture)
```

SYO011 \wedge 1.p Invalid formula in model classes not requiring f

```
leibeq: $i → $i → $o      thf(leibeq_type, type)
leibeq = (λu: $i, v: $i: ∀q: $i → $o: ((q@u) ⇒ (q@v)))      thf(leibeq, definition)
p: ($i → $i) → $o      thf(p_type, type)
f: $i → $i      thf(f_type, type)
```

$(\forall x: \$i: (\text{leibeq}@(f@x)@x) \text{ and } p@\lambda x: \$i: x) \Rightarrow (p@f) \quad \text{thf(conj, conjecture)}$

SYO012 \wedge 1.p Formula valid with Boolean extentionality 1

$a: \$o \quad \text{thf}(a, \text{type})$

$b: \$o \quad \text{thf}(b, \text{type})$

$p: \$o \rightarrow \$o \quad \text{thf}(p, \text{type})$

$(p@(a \text{ and } b)) \Rightarrow (p@(b \text{ and } a)) \quad \text{thf(conj, conjecture)}$

SYO013 \wedge 1.p Formula valid with Boolean extentionality 2

$a: \$o \quad \text{thf}(a_type, \text{type})$

$b: \$o \quad \text{thf}(b_type, \text{type})$

$p: \$o \rightarrow \$o \quad \text{thf}(p_type, \text{type})$

$(a \text{ and } b \text{ and } p@a) \Rightarrow (p@b) \quad \text{thf(conj, conjecture)}$

SYO015 \wedge 1.p A is not equal to not A

$a: \$o \quad \text{thf}(a, \text{type})$

$a \neq \neg a \quad \text{thf(conj, conjecture)}$

SYO016 \wedge 1.p Formula valid in MBb, but not in model classes not requiring b

$\text{leibeq}: \$o \rightarrow \$o \rightarrow \$o \quad \text{thf}(\text{leibeq_decl}, \text{type})$

$\text{leibeq} = (\lambda x: \$o, y: \$o: \forall p: \$o \rightarrow \$o: ((p@x) \Rightarrow (p@y))) \quad \text{thf}(\text{leibeq}, \text{definition})$

$h: \$o \rightarrow \$o \quad \text{thf}(h, \text{type})$

$\text{leibeq}@(h@(h@(\text{leibeq}@(h@\$true)@(h@\$false))))@(h@\$false) \quad \text{thf(conj, conjecture)}$

SYO017 \wedge 1.p Formula valid in MBb, but not in model classes not requiring b

$h: \$o \rightarrow \$o \quad \text{thf}(h, \text{type})$

$(h@(h@\$true)) = (h@\$false) \quad \text{thf(conj, conjecture)}$

SYO018 \wedge 1.p Formula requiring b and Eta

$f: \$o \rightarrow \$i \rightarrow \$i \quad \text{thf}(f, \text{type})$

$p: (\$i \rightarrow \$i) \rightarrow \$o \quad \text{thf}(p, \text{type})$

$a: (\$i \rightarrow \$i) \rightarrow \$o \quad \text{thf}(a, \text{type})$

$b: \$o \quad \text{thf}(b, \text{type})$

$(p@\lambda x: \$i: (f@(a@\lambda x: \$i: (f@b@x) \text{ and } b)@x)) \Rightarrow (p@(f@(b \text{ and } a@(f@b)))) \quad \text{thf(conj, conjecture)}$

SYO019 \wedge 1.p De Morgan by equivalance

$\forall x: \$o, y: \$o: ((x \text{ and } y) \iff \neg \neg x \text{ or } \neg y) \quad \text{thf(conj, conjecture)}$

SYO020 \wedge 1.p De Morgan by Leibnitz

$\text{leibeq}: \$o \rightarrow \$o \rightarrow \$o \quad \text{thf}(\text{leibeq_decl}, \text{type})$

$\text{leibeq} = (\lambda x: \$o, y: \$o: \forall p: \$o \rightarrow \$o: ((p@x) \Rightarrow (p@y))) \quad \text{thf}(\text{leibeq}, \text{definition})$

$\forall x: \$o, y: \$o: (\text{leibeq}@(x \text{ and } y)@\neg \neg x \text{ or } \neg y) \quad \text{thf(conj, conjecture)}$

SYO021 \wedge 1.p De Morgan by equality

$\forall x: \$o, y: \$o: (x \text{ and } y) = \neg \neg x \text{ or } \neg y \quad \text{thf(conj, conjecture)}$

SYO022 \wedge 1.p De Morgan lambda terms by Leibnitz

$\text{leibeq}: (\$o \rightarrow \$o \rightarrow \$o) \rightarrow (\$o \rightarrow \$o \rightarrow \$o) \rightarrow \$o \quad \text{thf}(\text{leibeq_decl}, \text{type})$

$\text{leibeq} = (\lambda x: \$o \rightarrow \$o \rightarrow \$o, y: \$o \rightarrow \$o \rightarrow \$o: \forall p: (\$o \rightarrow \$o \rightarrow \$o) \rightarrow \$o: ((p@x) \Rightarrow (p@y))) \quad \text{thf}(\text{leibeq}, \text{definition})$

$\text{leibeq}@\lambda u: \$o, v: \$o: (u \text{ and } v)@\lambda x: \$o, y: \$o: \neg \neg x \text{ or } \neg y \quad \text{thf(conj, conjecture)}$

SYO023 \wedge 1.p De Morgan lambda terms by Leibnitz

$(\lambda u: \$o, v: \$o: (u \text{ and } v)) = (\lambda x: \$o, y: \$o: \neg \neg x \text{ or } \neg y) \quad \text{thf(conj, conjecture)}$

SYO024 \wedge 1.p De Morgan by connectives and Leibnitz

$\text{leibeq}: (\$o \rightarrow \$o \rightarrow \$o) \rightarrow (\$o \rightarrow \$o \rightarrow \$o) \rightarrow \$o \quad \text{thf}(\text{leibeq_decl}, \text{type})$

$\text{leibeq} = (\lambda x: \$o \rightarrow \$o \rightarrow \$o, y: \$o \rightarrow \$o \rightarrow \$o: \forall p: (\$o \rightarrow \$o \rightarrow \$o) \rightarrow \$o: ((p@x) \Rightarrow (p@y))) \quad \text{thf}(\text{leibeq}, \text{definition})$

$\text{leibeq}@\text{and}@(\lambda x: \$o, y: \$o: \neg \neg x \text{ or } \neg y) \quad \text{thf(conj, conjecture)}$

SYO025 \wedge 1.p De Morgan by connectives and equality

$\text{and} = (\lambda x: \$o, y: \$o: \neg \neg x \text{ or } \neg y) \quad \text{thf(conj, conjecture)}$

SYO026 \wedge 1.p Four functions from truth values to truth values

In Henkin semantics there are exactly four functions from truth values to truth values.

$p: (\$o \rightarrow \$o) \rightarrow \$o \quad \text{thf}(p_decl, \text{type})$

$(p@\lambda x: \$o: x \text{ and } p@\lambda x: \$o: \neg x \text{ and } p@\lambda x: \$o: \$false \text{ and } p@\lambda x: \$o: \$true) \Rightarrow \forall y: \$o \rightarrow \$o: (p@y) \quad \text{thf(conj, conjecture)}$

SYO027 \wedge 1.p Something is true

$\exists p: \$o: p \quad \text{thf(conj, conjecture)}$

SYO028^1.p Not all things are false

$\neg \forall p: \$o: p \quad \text{thf(conj, conjecture)}$

SYO029^1.p There is an identity unary connective

$\exists n: \$o \rightarrow \$o: \$o: ((n@p) \iff \neg p) \quad \text{thf(conj, conjecture)}$

SYO030^1.p Not every unary connective is the identity

$\text{leibeq}: \$o \rightarrow \$o \rightarrow \$o \quad \text{thf(leibeq_decl, type)}$

$\text{leibeq} = (\lambda x: \$o, y: \$o: \forall p: \$o \rightarrow \$o: ((p@x) \Rightarrow (p@y))) \quad \text{thf(leibeq, definition)}$

$\neg \forall f: \$o \rightarrow \$o: \exists x: \$o: (\text{leibeq}@((f@x)@x)) \quad \text{thf(conj, conjecture)}$

SYO031^1.p Not every unary connective is the identity

$\neg \forall f: \$o \rightarrow \$o: \exists x: \$o: (f@x) = x \quad \text{thf(conj, conjecture)}$

SYO032^1.p There is a disjunction connective

$\exists d: \$o \rightarrow \$o \rightarrow \$o: \forall p: \$o, q: \$o: ((d@p@q) \iff (p \text{ or } q)) \quad \text{thf(conj, conjecture)}$

SYO033^1.p There is a universal quantifier

$\exists q: (\$i \rightarrow \$o) \rightarrow \$o: \forall p: \$i \rightarrow \$o: ((q@p) \iff \forall x: \$i: (p@x)) \quad \text{thf(conj, conjecture)}$

SYO034^1.p Formula not making use of projection

$\exists n: \$o \rightarrow \$o: \forall p: \$o: ((n@p) \iff p) \quad \text{thf(conj, conjecture)}$

SYO035^1.p Higher-order unification does not always provide projection terms

$\text{leibeq}_1: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf(leibeq1_type, type)}$

$\text{leibeq}_1 = (\lambda x: \$i, y: \$i: \forall p: \$i \rightarrow \$o: ((p@x) \Rightarrow (p@y))) \quad \text{thf(leibeq}_1\text{, definition)}$

$\text{leibeq}_2: (\$i \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf(leibeq2_type, type)}$

$\text{leibeq}_2 = (\lambda x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: \forall p: (\$i \rightarrow \$o) \rightarrow \$o: ((p@x) \Rightarrow (p@y))) \quad \text{thf(leibeq}_2\text{, definition)}$

$\forall x: \$i, y: \$i: ((\text{leibeq}_2@\lambda z: \$i: z = x@\lambda z: \$i: z = y) \Rightarrow (\text{leibeq}_1@x@y)) \quad \text{thf(conj, conjecture)}$

SYO037^1.p Injective Cantor theorem

$\neg \exists h: (\$i \rightarrow \$o) \rightarrow \$i: \forall p: \$i \rightarrow \$o, q: \$i \rightarrow \$o: ((h@p) = (h@q) \Rightarrow p = q) \quad \text{thf(conj, conjecture)}$

SYO038-1.002.304.p Boolos' Curious Inference, size f(2,f(3,4))

$f(n, 1) = s(1) \quad \text{cnf(ax}_1\text{, axiom)}$

$f(1, s(x)) = s(s(f(1, x))) \quad \text{cnf(ax}_2\text{, axiom)}$

$f(s(n), s(x)) = f(n, f(s(n), x)) \quad \text{cnf(ax}_3\text{, axiom)}$

$d(1) \quad \text{cnf(ax}_4\text{, axiom)}$

$d(x) \Rightarrow d(s(x)) \quad \text{cnf(ax}_5\text{, axiom)}$

$\neg d(f(s(1)), f(s(s(1))), s(s(s(1))))) \quad \text{cnf(conj, negated_conjecture)}$

SYO038-1.003.003.p Boolos' Curious Inference, size f(3,3)

$f(n, 1) = s(1) \quad \text{cnf(ax}_1\text{, axiom)}$

$f(1, s(x)) = s(s(f(1, x))) \quad \text{cnf(ax}_2\text{, axiom)}$

$f(s(n), s(x)) = f(n, f(s(n), x)) \quad \text{cnf(ax}_3\text{, axiom)}$

$d(1) \quad \text{cnf(ax}_4\text{, axiom)}$

$d(x) \Rightarrow d(s(x)) \quad \text{cnf(ax}_5\text{, axiom)}$

$\neg d(f(s(s(1))), s(s(s(1)))) \quad \text{cnf(conj, negated_conjecture)}$

SYO038-1.003.004.p Boolos' Curious Inference, size f(3,4)

$f(n, 1) = s(1) \quad \text{cnf(ax}_1\text{, axiom)}$

$f(1, s(x)) = s(s(f(1, x))) \quad \text{cnf(ax}_2\text{, axiom)}$

$f(s(n), s(x)) = f(n, f(s(n), x)) \quad \text{cnf(ax}_3\text{, axiom)}$

$d(1) \quad \text{cnf(ax}_4\text{, axiom)}$

$d(x) \Rightarrow d(s(x)) \quad \text{cnf(ax}_5\text{, axiom)}$

$\neg d(f(s(s(1))), s(s(s(1)))) \quad \text{cnf(conj, negated_conjecture)}$

SYO038-1.004.004.p Boolos' Curious Inference, size f(4,4)

$f(n, 1) = s(1) \quad \text{cnf(ax}_1\text{, axiom)}$

$f(1, s(x)) = s(s(f(1, x))) \quad \text{cnf(ax}_2\text{, axiom)}$

$f(s(n), s(x)) = f(n, f(s(n), x)) \quad \text{cnf(ax}_3\text{, axiom)}$

$d(1) \quad \text{cnf(ax}_4\text{, axiom)}$

$d(x) \Rightarrow d(s(x)) \quad \text{cnf(ax}_5\text{, axiom)}$

$\neg d(f(s(s(s(1)))), s(s(s(1)))) \quad \text{cnf(conj, negated_conjecture)}$

SYO038-1.005.005.p Boolos' Curious Inference, size f(5,5)

$f(n, 1) = s(1) \quad \text{cnf(ax}_1\text{, axiom)}$

$f(1, s(x)) = s(s(f(1, x))) \quad \text{cnf(ax}_2\text{, axiom)}$
 $f(s(n), s(x)) = f(n, f(s(n), x)) \quad \text{cnf(ax}_3\text{, axiom)}$
 $d(1) \quad \text{cnf(ax}_4\text{, axiom)}$
 $d(x) \Rightarrow d(s(x)) \quad \text{cnf(ax}_5\text{, axiom)}$
 $\neg d(f(s(s(s(s(1))))), s(s(s(s(1)))))) \quad \text{cnf(conj, negated_conjecture)}$

SYO038^1.002.304.p Boolos' Curious Inference, size f(2,f(3,4))

$1: \$i \quad \text{thf(one, type)}$
 $s: \$i \rightarrow \$i \quad \text{thf}(s, \text{type})$
 $f: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(f, \text{type})$
 $d: \$i \rightarrow \$o \quad \text{thf}(d, \text{type})$
 $\forall n: \$i: (f @ n @ 1) = (s @ 1) \quad \text{thf(ax}_1\text{, axiom)}$
 $\forall x: \$i: (f @ 1 @ (s @ x)) = (s @ (s @ (f @ 1 @ x))) \quad \text{thf(ax}_2\text{, axiom)}$
 $\forall n: \$i, x: \$i: (f @ (s @ n) @ (s @ x)) = (f @ n @ (f @ (s @ n) @ x)) \quad \text{thf(ax}_3\text{, axiom)}$
 $d @ 1 \quad \text{thf(ax}_4\text{, axiom)}$
 $\forall x: \$i: ((d @ x) \Rightarrow (d @ (s @ x))) \quad \text{thf(ax}_5\text{, axiom)}$
 $d @ (f @ (s @ 1) @ (f @ (s @ (s @ 1)) @ (s @ (s @ (s @ 1)))))) \quad \text{thf(conj, conjecture)}$

SYO038^1.003.003.p Boolos' Curious Inference, size f(3,3)

$1: \$i \quad \text{thf(one, type)}$
 $s: \$i \rightarrow \$i \quad \text{thf}(s, \text{type})$
 $f: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(f, \text{type})$
 $d: \$i \rightarrow \$o \quad \text{thf}(d, \text{type})$
 $\forall n: \$i: (f @ n @ 1) = (s @ 1) \quad \text{thf(ax}_1\text{, axiom)}$
 $\forall x: \$i: (f @ 1 @ (s @ x)) = (s @ (s @ (f @ 1 @ x))) \quad \text{thf(ax}_2\text{, axiom)}$
 $\forall n: \$i, x: \$i: (f @ (s @ n) @ (s @ x)) = (f @ n @ (f @ (s @ n) @ x)) \quad \text{thf(ax}_3\text{, axiom)}$
 $d @ 1 \quad \text{thf(ax}_4\text{, axiom)}$
 $\forall x: \$i: ((d @ x) \Rightarrow (d @ (s @ x))) \quad \text{thf(ax}_5\text{, axiom)}$
 $d @ (f @ (s @ (s @ 1)) @ (s @ (s @ 1)))) \quad \text{thf(conj, conjecture)}$

SYO038^1.003.004.p Boolos' Curious Inference, size f(3,4)

$1: \$i \quad \text{thf(one, type)}$
 $s: \$i \rightarrow \$i \quad \text{thf}(s, \text{type})$
 $f: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(f, \text{type})$
 $d: \$i \rightarrow \$o \quad \text{thf}(d, \text{type})$
 $\forall n: \$i: (f @ n @ 1) = (s @ 1) \quad \text{thf(ax}_1\text{, axiom)}$
 $\forall x: \$i: (f @ 1 @ (s @ x)) = (s @ (s @ (f @ 1 @ x))) \quad \text{thf(ax}_2\text{, axiom)}$
 $\forall n: \$i, x: \$i: (f @ (s @ n) @ (s @ x)) = (f @ n @ (f @ (s @ n) @ x)) \quad \text{thf(ax}_3\text{, axiom)}$
 $d @ 1 \quad \text{thf(ax}_4\text{, axiom)}$
 $\forall x: \$i: ((d @ x) \Rightarrow (d @ (s @ x))) \quad \text{thf(ax}_5\text{, axiom)}$
 $d @ (f @ (s @ (s @ 1)) @ (s @ (s @ (s @ 1)))) \quad \text{thf(conj, conjecture)}$

SYO038^1.004.004.p Boolos' Curious Inference, size f(4,4)

$1: \$i \quad \text{thf(one, type)}$
 $s: \$i \rightarrow \$i \quad \text{thf}(s, \text{type})$
 $f: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(f, \text{type})$
 $d: \$i \rightarrow \$o \quad \text{thf}(d, \text{type})$
 $\forall n: \$i: (f @ n @ 1) = (s @ 1) \quad \text{thf(ax}_1\text{, axiom)}$
 $\forall x: \$i: (f @ 1 @ (s @ x)) = (s @ (s @ (f @ 1 @ x))) \quad \text{thf(ax}_2\text{, axiom)}$
 $\forall n: \$i, x: \$i: (f @ (s @ n) @ (s @ x)) = (f @ n @ (f @ (s @ n) @ x)) \quad \text{thf(ax}_3\text{, axiom)}$
 $d @ 1 \quad \text{thf(ax}_4\text{, axiom)}$
 $\forall x: \$i: ((d @ x) \Rightarrow (d @ (s @ x))) \quad \text{thf(ax}_5\text{, axiom)}$
 $d @ (f @ (s @ (s @ (s @ 1)) @ (s @ (s @ (s @ 1)))))) \quad \text{thf(conj, conjecture)}$

SYO038^1.005.005.p Boolos' Curious Inference, size f(5,5)

$1: \$i \quad \text{thf(one, type)}$
 $s: \$i \rightarrow \$i \quad \text{thf}(s, \text{type})$
 $f: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(f, \text{type})$
 $d: \$i \rightarrow \$o \quad \text{thf}(d, \text{type})$
 $\forall n: \$i: (f @ n @ 1) = (s @ 1) \quad \text{thf(ax}_1\text{, axiom)}$
 $\forall x: \$i: (f @ 1 @ (s @ x)) = (s @ (s @ (f @ 1 @ x))) \quad \text{thf(ax}_2\text{, axiom)}$
 $\forall n: \$i, x: \$i: (f @ (s @ n) @ (s @ x)) = (f @ n @ (f @ (s @ n) @ x)) \quad \text{thf(ax}_3\text{, axiom)}$

*d@1 thf(ax₄, axiom)
 ∀x: \$i: ((d@x) ⇒ (d@(s@x))) thf(ax₅, axiom)
 d@(f@(s@(s@(s@s@1))))@((s@(s@(s@s@1)))) thf(conj, conjecture)*

SYO039^1.p Unsatisfiable basic formula 1

*h: \$o → \$i thf(h, type)
 (h@(h@\$false)) = (h@¬\$false)) ≠ (h@\$false) thf(1, axiom)*

SYO039^2.p Unsatisfiable basic formula 1

*h: \$o → \$i thf(h, type)
 (h@(h@\$false)) = (h@¬\$false)) = (h@\$false) thf(1, conjecture)*

SYO040^1.p Unsatisfiable basic formula 2

*f: \$o → \$o thf(f, type)
 h: \$o → \$i thf(h, type)
 x: \$o thf(x, type)
 (h@(f@(f@(f@x)))) ≠ (h@(f@x)) thf(2, axiom)*

SYO040^2.p Unsatisfiable basic formula 2

Variant of the Kaminski equation.

*f: \$o → \$o thf(f, type)
 h: \$o → \$i thf(h, type)
 x: \$o thf(x, type)
 (h@(f@(f@(f@x)))) = (h@(f@x)) thf(2, conjecture)*

SYO041^1.p Unsatisfiable basic formula 3

*a: \$o thf(a, type)
 f: \$o → \$o thf(f, type)
 g: \$o → \$o thf(g, type)
 x: \$o thf(x, type)
 y: \$o thf(y, type)
 x ≠ y and (g@x) = y and (g@y) = x and (f@(f@(f@x))) = (g@(f@x)) thf(3, axiom)*

SYO041^2.p Unsatisfiable basic formula 3

Variant of the Kaminski equation.

*a: \$o thf(a, type)
 f: \$o → \$o thf(f, type)
 g: \$o → \$o thf(g, type)
 x: \$o thf(x, type)
 y: \$o thf(y, type)
 ¬x ≠ y and (g@x) = y and (g@y) = x and (f@(f@(f@x))) = (g@(f@x)) thf(3, conjecture)*

SYO042^1.p Unsatisfiable basic formula 4

*g: \$o → \$o thf(g, type)
 p: (\$o → \$o) → \$o thf(p, type)
 x: \$o thf(x, type)
 y: \$o thf(y, type)
 x ≠ y and (g@x) = y and (g@y) = x and p@g and ¬p@¬ thf(4, axiom)*

SYO042^2.p Unsatisfiable basic formula 4

Negation is the unique function g such that g x = y and g y = x for x,y:o distinct.

*g: \$o → \$o thf(g, type)
 p: (\$o → \$o) → \$o thf(p, type)
 x: \$o thf(x, type)
 y: \$o thf(y, type)
 ¬x ≠ y and (g@x) = y and (g@y) = x and p@g and ¬p@¬ thf(4, conjecture)*

SYO043^1.p Unsatisfiable basic formula 5

*f: \$o → \$o thf(f, type)
 q: (\$o → \$o) → \$o → \$o thf(q, type)
 x: \$o thf(x, type)
 q@f@x and f@(f@x) and (f@(q@f@x)) ≠ (f@x) thf(5, axiom)*

SYO043^2.p Unsatisfiable basic formula 5

Variant of the Kaminski equation.

$f: \$o \rightarrow \$o \quad \text{thf}(f, \text{type})$
 $q: (\$o \rightarrow \$o) \rightarrow \$o \rightarrow \$o \quad \text{thf}(q, \text{type})$
 $x: \$o \quad \text{thf}(x, \text{type})$
 $q @ f @ x \text{ and } f @ (q @ x) \text{ and } (f @ (q @ f @ x)) = (f @ x) \quad \text{thf}(5, \text{conjecture})$

SYO044^1.p Simple textbook example 1

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mbox@r@mtrue))      thf(conj, conjecture)
```

SYO045^1.p Simple textbook example 2

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mforall_prop@λa: $i → $o: (mforall_prop@λb: $i → $o: (mimplies@(mbox@r@(mimplies@a@b))@a)))
```

SYO046^1.p Simple textbook example 3

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mforall_prop@λa: $i → $o: (mforall_prop@λb: $i → $o: (mimplies@(mdia@r@(mimplies@a@b))@a)))
```

SYO047^1.p Simple textbook example 4

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mforall_prop@λa: $i → $o: (mforall_prop@λb: $i → $o: (mimplies@(mbox@r@(mimplies@a@b))@a)))
```

SYO048^1.p Simple textbook example 5

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mforall_prop@λa: $i → $o: (mforall_prop@λb: $i → $o: (mequiv@(mbox@r@(mand@a@b))@a)))
```

SYO049^1.p Simple textbook example 6

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@r@a)@a)))      thf(conj, conjecture)
```

SYO050^2.p Simple textbook example 7

```
include('Axioms/LCL013^0.ax')
∃r: $i → $i → $o: ¬mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@r@a)@a))      thf(conj, conjecture)
```

SYO051^1.p Simple textbook example 8

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@r@a)@(mbox@r@(mbox@r@a)))))      thf(conj, co)
```

SYO051^2.p Simple textbook example 8

```
include('Axioms/LCL013^0.ax')
∃r: $i → $i → $o: ¬mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mbox@r@a)@(mbox@r@(mbox@r@a))))      thf(con, co)
```

SYO052^1.p Simple textbook example 9

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mforall_prop@λa: $i → $o: (mforall_prop@λb: $i → $o: (mimplies@(mbox@r@(mimplies@a@b))@a)))
```

SYO052^2.p Simple textbook example 9

```
include('Axioms/LCL013^0.ax')
r: $i → $i → $o      thf(r, type)
a: $i → $o      thf(a, type)
b: $i → $o      thf(b, type)
```

∃r: \$i → \$i → \$o: ¬mvalid@(mforall_prop@λa: \$i → \$o: (mforall_prop@λb: \$i → \$o: (mimplies@(mbox@r@(mimplies@a@b))@a)))

SYO053^1.p Simple textbook example 10

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mdia@r@mtrue))      thf(conj, conjecture)
```

SYO053^2.p Simple textbook example 10

```
include('Axioms/LCL013^0.ax')
∃r: $i → $i → $o: ¬mvalid@(mdia@r@mtrue)      thf(conj, conjecture)
```

SYO054^1.p Simple textbook example 11

```
include('Axioms/LCL013^0.ax')
∀r: $i → $i → $o: (mvalid@(mforall_prop@λa: $i → $o: (mimplies@(mdia@r@a)@(mbox@r@a))))      thf(conj, conjecture)
```

SYO054^2.p Simple textbook example 11

```

include('Axioms/LCL013^0.ax')
 $\exists r: \$i \rightarrow \$i \rightarrow \$o: \neg \text{mvalid} @ (\text{mforall\_prop} @ \lambda a: \$i \rightarrow \$o: (\text{mimplies} @ (\text{mdia} @ r @ a) @ (\text{mbox} @ r @ a)))$  thf(conj, conjecture)

SYO055^1.p Simple textbook example 12
include('Axioms/LCL013^0.ax')
 $\forall r: \$i \rightarrow \$i \rightarrow \$o: (\text{mvalid} @ (\text{mforall\_prop} @ \lambda a: \$i \rightarrow \$o: (\text{mforall\_prop} @ \lambda b: \$i \rightarrow \$o: (\text{mor} @ (\text{mbox} @ r @ (\text{mimplies} @ (\text{mbox} @ r @ a) @ (\text{mbox} @ r @ b))))))$  thf(conj, conjecture)

SYO055^2.p Simple textbook example 12
include('Axioms/LCL013^0.ax')
 $\exists r: \$i \rightarrow \$i \rightarrow \$o: \neg \text{mvalid} @ (\text{mforall\_prop} @ \lambda a: \$i \rightarrow \$o: (\text{mforall\_prop} @ \lambda b: \$i \rightarrow \$o: (\text{mor} @ (\text{mbox} @ r @ (\text{mimplies} @ (\text{mbox} @ r @ a) @ (\text{mbox} @ r @ b))))))$  thf(conj, conjecture)

SYO056^1.p Simple textbook example 13
include('Axioms/LCL013^0.ax')
 $\forall r: \$i \rightarrow \$i \rightarrow \$o: (\text{mvalid} @ (\text{mforall\_prop} @ \lambda a: \$i \rightarrow \$o: (\text{mforall\_prop} @ \lambda b: \$i \rightarrow \$o: (\text{mimplies} @ (\text{mbox} @ r @ (\text{mor} @ a @ b)) @ (\text{mbox} @ r @ a))))))$  thf(conj, conjecture)

SYO056^2.p Simple textbook example 13
include('Axioms/LCL013^0.ax')
 $\exists r: \$i \rightarrow \$i \rightarrow \$o: \neg \text{mvalid} @ (\text{mforall\_prop} @ \lambda a: \$i \rightarrow \$o: (\text{mforall\_prop} @ \lambda b: \$i \rightarrow \$o: (\text{mimplies} @ (\text{mbox} @ r @ (\text{mor} @ a @ b)) @ (\text{mbox} @ r @ a))))$  thf(conj, conjecture)

SYO057^1.p Simple textbook example 14
include('Axioms/LCL013^0.ax')
 $\forall r: \$i \rightarrow \$i \rightarrow \$o: (\text{mvalid} @ (\text{mforall\_prop} @ \lambda a: \$i \rightarrow \$o: (\text{mimplies} @ (\text{mbox} @ r @ (\text{mimplies} @ (\text{mbox} @ r @ a) @ a)) @ (\text{mbox} @ r @ a))))$  thf(conj, conjecture)

SYO057^2.p Simple textbook example 14
include('Axioms/LCL013^0.ax')
 $\exists r: \$i \rightarrow \$i \rightarrow \$o: \neg \text{mvalid} @ (\text{mforall\_prop} @ \lambda a: \$i \rightarrow \$o: (\text{mimplies} @ (\text{mbox} @ r @ (\text{mimplies} @ (\text{mbox} @ r @ a) @ a)) @ (\text{mbox} @ r @ a))))$  thf(conj, conjecture)

SYO058^4.p ILTP problem SYJ101+1
include('Axioms/LCL010^0.ax')
 $a: \$i \rightarrow \$o \quad \text{thf(a\_type, type)}$ 
 $\text{invalid} @ (\text{iatom} @ a) \quad \text{thf(axiom}_1, \text{axiom})$ 
 $\text{invalid} @ (\text{iatom} @ a) \quad \text{thf(con, conjecture)}$ 

SYO059^4.p ILTP problem SYJ102+1
include('Axioms/LCL010^0.ax')
 $a: \$i \rightarrow \$o \quad \text{thf(a\_type, type)}$ 
 $\text{invalid} @ (\text{iatom} @ a) \quad \text{thf(axiom}_1, \text{axiom})$ 
 $\text{invalid} @ (\text{inot} @ (\text{inot} @ (\text{iatom} @ a))) \quad \text{thf(con, conjecture)}$ 

SYO060^4.p ILTP problem SYJ103+1
include('Axioms/LCL010^0.ax')
 $a: \$i \rightarrow \$o \quad \text{thf(a\_type, type)}$ 
 $b: \$i \rightarrow \$o \quad \text{thf(b\_type, type)}$ 
 $\text{invalid} @ (\text{ior} @ (\text{inot} @ (\text{iatom} @ a)) @ (\text{inot} @ (\text{iatom} @ b))) \quad \text{thf(axiom}_1, \text{axiom})$ 
 $\text{invalid} @ (\text{ior} @ (\text{inot} @ (\text{iatom} @ b)) @ (\text{inot} @ (\text{iatom} @ a))) \quad \text{thf(con, conjecture)}$ 

SYO061^4.p ILTP Problem SYJ104+1
include('Axioms/LCL010^0.ax')
 $a: \$i \rightarrow \$o \quad \text{thf(a\_type, type)}$ 
 $b: \$i \rightarrow \$o \quad \text{thf(b\_type, type)}$ 
 $\text{invalid} @ (\text{implies} @ (\text{iatom} @ a) @ (\text{iatom} @ b)) \quad \text{thf(axiom}_1, \text{axiom})$ 
 $\text{invalid} @ (\text{implies} @ (\text{iatom} @ a) @ (\text{iatom} @ b)) \quad \text{thf(con, conjecture)}$ 

SYO062^4.002.p ILTP Problem SYJ105+1.002
include('Axioms/LCL010^0.ax')
 $a: \$i \rightarrow \$o \quad \text{thf(a\_type, type)}$ 
 $\text{invalid} @ (\text{inot} @ (\text{inot} @ (\text{ior} @ (\text{iatom} @ a) @ (\text{inot} @ (\text{iatom} @ a))))) \quad \text{thf(con, conjecture)}$ 

SYO062^4.003.p ILTP Problem SYJ105+1.003
include('Axioms/LCL010^0.ax')
 $a: \$i \rightarrow \$o \quad \text{thf(a\_type, type)}$ 
 $b: \$i \rightarrow \$o \quad \text{thf(b\_type, type)}$ 
 $\text{invalid} @ (\text{inot} @ (\text{inot} @ (\text{ior} @ (\text{iand} @ (\text{iatom} @ a) @ (\text{iatom} @ b)) @ (\text{ior} @ (\text{inot} @ (\text{iatom} @ a)) @ (\text{inot} @ (\text{iatom} @ b))))))) \quad \text{thf(con, conjecture)}$ 

SYO062^4.004.p ILTP Problem SYJ105+1.004
include('Axioms/LCL010^0.ax')
 $a: \$i \rightarrow \$o \quad \text{thf(a\_type, type)}$ 

```

```

b: $i → $o      thf(b_type, type)
c: $i → $o      thf(c_type, type)
invalid@(inot@(inot@(ior@(iand@(iatom@a)@)(iand@(iatom@b)@)(iatom@c)))@)(ior@(inot@(iatom@a))@)(ior@(inot@(iatom@b)

SYO063^4.p ILTP Problem SYJ106+1
include('Axioms/LCL010^0.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
s: $i → $o      thf(s_type, type)
t: $i → $o      thf(t_type, type)
invalid@(iatom@s)      thf(axiom1, axiom)
invalid@(iimplies@(inot@(iimplies@(iatom@t)@(iatom@r))@)(iatom@p))      thf(axiom2, axiom)
invalid@(iimplies@(inot@(iand@(iimplies@(iatom@p)@(iatom@q))@)(iimplies@(iatom@t)@(iatom@r))))@)(iand@(inot@(

```

SYO064^4.001.p ILTP Problem SYJ107+1.001

```

include('Axioms/LCL010^0.ax')
a: $i → $o      thf(a_type, type)
b: $i → $o      thf(b_type, type)
c: $i → $o      thf(c_type, type)
invalid@((atom@c)      thf(axiom1, axiom)
invalid@((or@(or@((atom@a)@((atom@b)@((atom@c))))@((atom@b)))      thf(axiom2, axiom)
invalid@((or@((atom@a)@((and@((atom@b)@((atom@c))))@((atom@c))))      thf(con, conjecture)

```

SYO064\4.002.p ILTP Problem SYJ107+1.002

```

include('Axioms/LCL010^0.ax')
a: $i → $o      thf(a_type, type)
a1: $i → $o      thf(a1_type, type)
a2: $i → $o      thf(a2_type, type)
b: $i → $o      thf(b_type, type)
b1: $i → $o      thf(b1_type, type)
invalid@((atom@a2)      thf(axiom1, axiom))
invalid@((implies@((atom@b)@(ior@(ior@((atom@b1)@(atom@a1))@(atom@b1))))      thf(axiom2, axiom))
invalid@((ior@((ior@((atom@b)@(atom@a))@(atom@b)))      thf(axiom3, axiom))
invalid@((ior@((atom@a)@(ior@((iand@((atom@b)@(atom@a1))@(iand@((atom@b1)@(atom@a2))))))      thf(con, conjecture))

```

SYO064^4.003.p ILTP Problem SYJ107+1.003

```

include('Axioms/LCL010^0.ax')
a: $i → $o      thf(a_type, type)
a1: $i → $o      thf(a1_type, type)
a2: $i → $o      thf(a2_type, type)
a3: $i → $o      thf(a3_type, type)
b: $i → $o      thf(b_type, type)
b1: $i → $o      thf(b1_type, type)
b2: $i → $o      thf(b2_type, type)
invalid@(@atom@a3)      thf(axiom1, axiom)
invalid@(@implies@(@atom@b1)@(@or@(@or@(@atom@b2)@(@atom@a2))@(@atom@b2)))      thf(axiom2, axiom)
invalid@(@implies@(@atom@b)@(@or@(@or@(@atom@b1)@(@atom@a1))@(@atom@b1)))      thf(axiom3, axiom)
invalid@(@or@(@or@(@atom@b)@(@atom@a))@(@atom@b))      thf(axiom4, axiom)
invalid@(@or@(@atom@a)@(@or@(@and@(@atom@b)@(@atom@a1))@(@or@(@and@(@atom@b1)@(@atom@a2))@(@and@(@atom@b2)@(@atom@a3))))      thf(axiom5, axiom)

```

SYO064\4.004.p ILTP Problem SYJ107+1.004

```

include('Axioms/LCL010^0.ax')
a: $i → $o      thf(a_type, type)
a1: $i → $o     thf(a1_type, type)
a2: $i → $o     thf(a2_type, type)
a3: $i → $o     thf(a3_type, type)
a4: $i → $o     thf(a4_type, type)
b: $i → $o      thf(b_type, type)
b1: $i → $o     thf(b1_type, type)
b2: $i → $o     thf(b2_type, type)
b3: $i → $o     thf(b3_type, type)

```

```

invalid@((atom@a_4))      thf(axiom1, axiom)
invalid@((implies@((atom@b_2))@((or@((or@((atom@b_3))@((atom@a_3))@((atom@b_3)))))))      thf(axiom2, axiom)
invalid@((implies@((atom@b_1))@((or@((or@((atom@b_2))@((atom@a_2))@((atom@b_2)))))))      thf(axiom3, axiom)
invalid@((implies@((atom@b))@((or@((or@((atom@b_1))@((atom@a_1))@((atom@b_1)))))))      thf(axiom4, axiom)
invalid@((or@((or@((atom@b))@((atom@a))@((atom@b)))))      thf(axiom5, axiom)
invalid@((or@((atom@a))@((or@((and@((atom@b))@((atom@a_1))@((atom@b_1))))@((or@((and@((atom@b_1))@((atom@a_2))@((atom@b_2))))@((or@((and@((atom@b_1))@((atom@a_3))@((atom@b_3)))))))))))      thf(axiom6, axiom)

```

SYO065^4.001.p ILTP Problem SYJ201+1.001

```

include('Axioms/LCL010^0.ax')


p1: $i → $o      thf(p1_type, type)



p2: $i → $o      thf(p2_type, type)



p3: $i → $o      thf(p3_type, type)


invalid@((implies@((equiv@((atom@p1))@((atom@p2))@((and@((atom@p1))@((and@((atom@p2))@((atom@p3))))))))))      thf(axiom1, axiom)
invalid@((implies@((equiv@((atom@p2))@((atom@p3))@((and@((atom@p1))@((and@((atom@p2))@((atom@p3))))))))))      thf(axiom2, axiom)
invalid@((implies@((equiv@((atom@p3))@((atom@p1))@((and@((atom@p1))@((and@((atom@p2))@((atom@p3))))))))))      thf(axiom3, axiom)
invalid@((and@((atom@p1))@((and@((atom@p2))@((atom@p3))))))      thf(con, conjecture)

```

SYO065^4.002.p ILTP Problem SYJ201+1.002

```

include('Axioms/LCL010^0.ax')
p1: $i → $o      thf(p1_type, type)
p2: $i → $o      thf(p2_type, type)
p3: $i → $o      thf(p3_type, type)
p4: $i → $o      thf(p4_type, type)
p5: $i → $o      thf(p5_type, type)
invalid@(!implies(@(iequiv(@(iatom@p1)@(iatom@p2))@(!and(@(iatom@p1)@(iatom@p2)@(!and(@(iatom@p3)@(!and(@(iatom@p4)@(!and(@(iatom@p5)@(!and(@(iatom@p1)@(!and(@(iatom@p2)@(!and(@(iatom@p3)@(!and(@(iatom@p4)@(!and(@(iatom@p5)@(!and(@(iatom@p1)@(!and(@(iatom@p2)@(!and(@(iatom@p3)@(!and(@(iatom@p4)@(!and(@(iatom@p5)))))))      thf(con, conjecture)

```

SYO065^4.003.p ILTP Problem SYJ201+1.003

SYO066^4.001.p ILTP Problem SYJ202+1.001

```

include('Axioms/LCL010^0.ax')
o11: $i → $o      thf(o11_type, type)
o21: $i → $o      thf(o21_type, type)
invalid@((atom@o11))    thf(axiom1, axiom)
invalid@((atom@o21))    thf(axiom2, axiom)
invalid@((or@((atom@o11)@((atom@o21))))  thf(con, conjecture)

```

SYO066^4.002.p ILTP Problem SYJ202+1.002

```

include('Axioms/LCL010^0.ax')
o11: $i → $o      thf(o11_type, type)
o12: $i → $o      thf(o12_type, type)
o21: $i → $o      thf(o21_type, type)

```

```

o22: $i → $o      thf(o22_type, type)
o31: $i → $o      thf(o31_type, type)
o32: $i → $o      thf(o32_type, type)
invalid@(ior@(iatom@o11)@(iatom@o12))      thf(axiom1, axiom)
invalid@(ior@(iatom@o21)@(iatom@o22))      thf(axiom2, axiom)
invalid@(ior@(iatom@o31)@(iatom@o32))      thf(axiom3, axiom)
invalid@(ior@(iand@(iatom@o11)@(iatom@o21))@(ior@(iand@(iatom@o11)@(iatom@o31))@

```

SYO066^4.003.p ILTP Problem SYJ202+1.003

```

include('Axioms/LCL010^0.ax')
o11: $i → $o      thf(o11_type, type)
o12: $i → $o      thf(o12_type, type)
o13: $i → $o      thf(o13_type, type)
o21: $i → $o      thf(o21_type, type)
o22: $i → $o      thf(o22_type, type)
o23: $i → $o      thf(o23_type, type)
o31: $i → $o      thf(o31_type, type)
o32: $i → $o      thf(o32_type, type)
o33: $i → $o      thf(o33_type, type)
o41: $i → $o      thf(o41_type, type)
o42: $i → $o      thf(o42_type, type)
o43: $i → $o      thf(o43_type, type)
invalid@(ior@(iatom@o11)@(ior@(iatom@o12)@(iatom@o13)))      thf(axiom1, axiom)
invalid@(ior@(iatom@o21)@(ior@(iatom@o22)@(iatom@o23)))      thf(axiom2, axiom)
invalid@(ior@(iatom@o31)@(ior@(iatom@o32)@(iatom@o33)))      thf(axiom3, axiom)
invalid@(ior@(iatom@o41)@(ior@(iatom@o42)@(iatom@o43)))      thf(axiom4, axiom)
invalid@(ior@(iand@(iatom@o11)@(iatom@o21))@(ior@(iand@(iatom@o11)@(iatom@o31))@

```

SYO067^4.001.p ILTP Problem SYJ203+1.001

```

include('Axioms/LCL010^0.ax')
f: $i → $o      thf(f_type, type)
p1: $i → $o      thf(p1_type, type)
invalid@(iimplies@(ior@(iatom@p1)@(iimplies@(iatom@p1)@(iatom@f)))@(iatom@f))      thf(axiom1, axiom)
invalid@(iatom@f)      thf(con, conjecture)

```

SYO067^4.002.p ILTP Problem SYJ203+1.002

```

include('Axioms/LCL010^0.ax')
f: $i → $o      thf(f_type, type)
p1: $i → $o      thf(p1_type, type)
p2: $i → $o      thf(p2_type, type)
invalid@(iimplies@(ior@(iand@(iatom@p1)@(iatom@p2))@(ior@(iimplies@(iatom@p1)@(iatom@f))@
invalid@(iatom@f)      thf(con, conjecture)

```

SYO067^4.003.p ILTP Problem SYJ203+1.003

```

include('Axioms/LCL010^0.ax')
f: $i → $o      thf(f_type, type)
p1: $i → $o      thf(p1_type, type)
p2: $i → $o      thf(p2_type, type)
p3: $i → $o      thf(p3_type, type)
invalid@(iimplies@(ior@(iand@(iatom@p1)@(iand@(iatom@p2)@(iatom@p3))@
invalid@(iatom@f)      thf(con, conjecture)

```

SYO067^4.004.p ILTP Problem SYJ203+1.004

```

include('Axioms/LCL010^0.ax')
f: $i → $o      thf(f_type, type)
p1: $i → $o      thf(p1_type, type)
p2: $i → $o      thf(p2_type, type)
p3: $i → $o      thf(p3_type, type)
p4: $i → $o      thf(p4_type, type)
invalid@(iimplies@(ior@(iand@(iatom@p1)@(iand@(iatom@p2)@(iand@(iatom@p3)@(iatom@p4))))@
invalid@(iatom@f)      thf(con, conjecture)

```

SYO068^4.001.p ILTP Problem SYJ204+1.001

```

include('Axioms/LCL010^0.ax')
p0: $i → $o      thf(p0_type, type)
p1: $i → $o      thf(p1_type, type)
invalid@((atom@p1))      thf(axiom1, axiom)
invalid@((implies@((atom@p1))@((implies@((atom@p1))@((atom@p0)))))      thf(axiom2, axiom)
invalid@((atom@p0))      thf(con, conjecture)

```

SYO068^4.005.p ILTP Problem SYJ204+1.005

```

include('Axioms/LCL010^0.ax')
p0: $i → $o      thf(p0_type, type)
p1: $i → $o      thf(p1_type, type)
p2: $i → $o      thf(p2_type, type)
p3: $i → $o      thf(p3_type, type)
p4: $i → $o      thf(p4_type, type)
p5: $i → $o      thf(p5_type, type)
invalid@((atom@p5))    thf(axiom1, axiom)
invalid@((implies@((atom@p1))@((implies@((atom@p1))@((atom@p0))))) thf(axiom2, axiom)
invalid@((implies@((atom@p2))@((implies@((atom@p2))@((atom@p1))))) thf(axiom3, axiom)
invalid@((implies@((atom@p3))@((implies@((atom@p3))@((atom@p2))))) thf(axiom4, axiom)
invalid@((implies@((atom@p4))@((implies@((atom@p4))@((atom@p3))))) thf(axiom5, axiom)
invalid@((implies@((atom@p5))@((implies@((atom@p5))@((atom@p4))))) thf(axiom6, axiom)
invalid@((atom@p0))    thf(con, conjecture)

```

SYO068^4.010.p ILTP Problem SYJ204+1.010

SYO069\4,001.p ILTP Problem SYJ205+1,001

SYO069^4.002.p ILTP Problem SYJ205+1.002

```
include('Axioms/LCL010^0.ax')
a0: $i → $o      thf(a0_type,type)
```



```
ivalid@(iatom@f)      thf(con, conjecture)
```

SYO070^4.003.p ILTP Problem SYJ211+1.003

```
include('Axioms/LCL010^0.ax')
```

```
a0: $i → $o      thf(a0_type, type)
a1: $i → $o      thf(a1_type, type)
a2: $i → $o      thf(a2_type, type)
a3: $i → $o      thf(a3_type, type)
b0: $i → $o      thf(b0_type, type)
b1: $i → $o      thf(b1_type, type)
b2: $i → $o      thf(b2_type, type)
b3: $i → $o      thf(b3_type, type)
f: $i → $o        thf(f_type, type)
```

```
ivalid@(iimplies@(iatom@a0)@(iatom@f))    thf(axiom1, axiom)
```

```
thf(axiom2, axiom)
```

```
ivalid@(iimplies@(iimplies@(inot@(inot@(iatom@b3))@(iatom@b0))@(iatom@a3))
```

```
thf(axiom3, axiom)
```

```
ivalid@(iimplies@(iimplies@(inot@(inot@(iatom@b0))@(iatom@a1))@(iatom@a0))
```

```
thf(axiom4, axiom)
```

```
ivalid@(iimplies@(iimplies@(inot@(inot@(iatom@b1))@(iatom@a2))@(iatom@a1))
```

```
thf(axiom5, axiom)
```

```
ivalid@(iimplies@(iimplies@(inot@(inot@(iatom@b2))@(iatom@a3))@(iatom@a2))
```

```
ivalid@(iatom@f)      thf(con, conjecture)
```

SYO070^4.004.p ILTP Problem SYJ211+1.004

```
include('Axioms/LCL010^0.ax')
```

```
a0: $i → $o      thf(a0_type, type)
a1: $i → $o      thf(a1_type, type)
a2: $i → $o      thf(a2_type, type)
a3: $i → $o      thf(a3_type, type)
a4: $i → $o      thf(a4_type, type)
b0: $i → $o      thf(b0_type, type)
b1: $i → $o      thf(b1_type, type)
b2: $i → $o      thf(b2_type, type)
b3: $i → $o      thf(b3_type, type)
b4: $i → $o      thf(b4_type, type)
f: $i → $o        thf(f_type, type)
```

```
ivalid@(iimplies@(iatom@a0)@(iatom@f))    thf(axiom1, axiom)
```

```
thf(axiom2, axiom)
```

```
ivalid@(iimplies@(iimplies@(inot@(inot@(iatom@b4))@(iatom@b0))@(iatom@a4))
```

```
thf(axiom3, axiom)
```

```
ivalid@(iimplies@(iimplies@(inot@(inot@(iatom@b0))@(iatom@a1))@(iatom@a0))
```

```
thf(axiom4, axiom)
```

```
ivalid@(iimplies@(iimplies@(inot@(inot@(iatom@b1))@(iatom@a2))@(iatom@a1))
```

```
thf(axiom5, axiom)
```

```
ivalid@(iimplies@(iimplies@(inot@(inot@(iatom@b2))@(iatom@a3))@(iatom@a2))
```

```
thf(axiom6, axiom)
```

```
ivalid@(iatom@f)      thf(con, conjecture)
```

SYO071^4.001.p ILTP Problem SYJ207+1.001

```
include('Axioms/LCL010^0.ax')
```

```
p0: $i → $o      thf(p0_type, type)
p1: $i → $o      thf(p1_type, type)
p2: $i → $o      thf(p2_type, type)
```

```
ivalid@(iimplies@(iequiv@(iatom@p1)@(iatom@p2))@(iand@(iatom@p1)@(iatom@p2)))
```

```
thf(axiom1, axiom)
```

```
ivalid@(iimplies@(iequiv@(iatom@p2)@(iatom@p1))@(iand@(iatom@p1)@(iatom@p2)))
```

```
thf(axiom2, axiom)
```

```
ivalid@(ior@(iatom@p0)@(ior@(iand@(iatom@p1)@(iatom@p2))@(inot@(iatom@p0))))
```

```
thf(con, conjecture)
```

SYO071^4.002.p ILTP Problem SYJ207+1.002

```
include('Axioms/LCL010^0.ax')
```

```
p0: $i → $o      thf(p0_type, type)
p1: $i → $o      thf(p1_type, type)
p2: $i → $o      thf(p2_type, type)
p3: $i → $o      thf(p3_type, type)
p4: $i → $o      thf(p4_type, type)
```

```
ivalid@(iimplies@(iequiv@(iatom@p1)@(iatom@p2))@(iand@(iatom@p1)@(iand@(iatom@p2)@(iand@(iatom@p3)@(iatom@p4))))
```

```
ivalid@(iimplies@(iequiv@(iatom@p2)@(iatom@p3))@(iand@(iatom@p1)@(iand@(iatom@p2)@(iand@(iatom@p3)@(iatom@p4))))
```

```
ivalid@(iimplies@(iequiv@(iatom@p3)@(iatom@p4))@(iand@(iatom@p1)@(iand@(iatom@p2)@(iand@(iatom@p3)@(iatom@p4))))
```

```
ivalid@(iimplies@(iequiv@(iatom@p4)@(iatom@p1))@(iand@(iatom@p1)@(iand@(iatom@p2)@(iand@(iatom@p3)@(iatom@p4))))
```

SYQ072\4.001.p ILTP Problem SYJ208\1.001

```

S1C672<4.001> LCL010-1 Robein S1C208 | 4.001
include('Axioms/LCL010^0.ax')
o11: $i → $o      thf(o11_type, type)
o21: $i → $o      thf(o21_type, type)
invalid@(!not@(!not@(!atom@o11)))    thf(axiom1, axiom)
invalid@(!not@(!not@(!atom@o21)))    thf(axiom2, axiom)
invalid@(!or@(!atom@o11)@(!atom@o21))  thf(con, conjecture)

```

SYO072^4.002.p ILTP Problem SYJ208+1.002

```

S130721(1000).p LCL1 PROOF S130721 | 1.002
include('Axioms/LCL010^0.ax')
o11: $i → $o      thf(o11_type, type)
o12: $i → $o      thf(o12_type, type)
o21: $i → $o      thf(o21_type, type)
o22: $i → $o      thf(o22_type, type)
o31: $i → $o      thf(o31_type, type)
o32: $i → $o      thf(o32_type, type)
invalid@(ior@(iatom@o11)@(inot@(inot@(iatom@o12))))    thf(axiom1, axiom)
invalid@(ior@(iatom@o21)@(inot@(inot@(iatom@o22))))    thf(axiom2, axiom)
invalid@(ior@(iatom@o31)@(inot@(inot@(iatom@o32))))    thf(axiom3, axiom)
invalid@(ior@(iand@(iatom@o11)@(iatom@o21))@(ior@(iand@(iatom@o11)@(iatom@o31))@
(ior@(iand@(iatom@o21)@(iatom@o32)))))@(

```

SYO072\4.003.p ILTP Problem SYJ208+1.003

```

include('Axioms/LCL010^0.ax')

o11: $i → $o      thf(o11_type, type)
o12: $i → $o      thf(o12_type, type)
o13: $i → $o      thf(o13_type, type)
o21: $i → $o      thf(o21_type, type)
o22: $i → $o      thf(o22_type, type)
o23: $i → $o      thf(o23_type, type)
o31: $i → $o      thf(o31_type, type)
o32: $i → $o      thf(o32_type, type)
o33: $i → $o      thf(o33_type, type)
o41: $i → $o      thf(o41_type, type)
o42: $i → $o      thf(o42_type, type)
o43: $i → $o      thf(o43_type, type)

invalid@(ior@(iatom@o11)@(ior@(iatom@o12)@(inot@(inot@(iatom@o13)))))  thf(axiom1, axiom)
invalid@(ior@(iatom@o21)@(ior@(iatom@o22)@(inot@(inot@(iatom@o23)))))  thf(axiom2, axiom)
invalid@(ior@(iatom@o31)@(ior@(iatom@o32)@(inot@(inot@(iatom@o33)))))  thf(axiom3, axiom)
invalid@(ior@(iatom@o41)@(ior@(iatom@o42)@(inot@(inot@(iatom@o43)))))  thf(axiom4, axiom)
invalid@(ior@(iand@(iatom@o11)@(iatom@o21))@(ior@(iand@(iatom@o11)@(iatom@o31))@
    (ior@(iand@(iatom@o11)@(iatom@o11))@(iatom@o11)))  thf(axiom5, axiom)

```

SYO073^4.001.p ILTP Problem SYJ209+1.001

```
include('Axioms/LCL010^0.ax')
```



```

include('Axioms/LCL010^0.ax')
p0: $i → $o      thf(p0_type, type)
p1: $i → $o      thf(p1_type, type)
p2: $i → $o      thf(p2_type, type)
p3: $i → $o      thf(p3_type, type)
p4: $i → $o      thf(p4_type, type)
invalid@(inot@(inot@((atom@p4)))      thf(axiom1, axiom)
invalid@(iimplies@((atom@p1))@((iimplies@((atom@p1))@((atom@p0))))      thf(axiom2, axiom)
invalid@(iimplies@((atom@p2))@((iimplies@((atom@p2))@((atom@p1))))      thf(axiom3, axiom)
invalid@(iimplies@((atom@p3))@((iimplies@((atom@p3))@((atom@p2))))      thf(axiom4, axiom)
invalid@(iimplies@((atom@p4))@((iimplies@((atom@p4))@((atom@p3))))      thf(axiom5, axiom)
invalid@((atom@p0))      thf(con, conjecture)

```

SYO076^5.p TPS problem THM114

cQ: \$o thf(cQ, type)
cP: \$o thf(cP, type)
(cP and (cP ⇒ cQ)) ⇒ cQ thf(cTHM₁₁₄, conjecture)

SYO077^5.p TPS problem THM64

cP: \$i → \$o thf(cP, type)
∃x: \$i: ((cP@x) ⇒ ∀y: \$i: (cP@y)) thf(cTHM₆₄, conjecture)

SYO078^5.p TPS problem THM49

cR: \$o thf(cR, type)
cP: \$o thf(cP, type)
cQ: \$o thf(cQ, type)
(cP ⇔ cQ) or (cQ ⇔ cR) or (cP ⇔ cR) thf(cTHM₄₉, conjecture)

SYO079^5.p TPS problem THM50-A

Associativity of equivalence.
cR: \$o thf(cR, type)
cQ: \$o thf(cQ, type)
cP: \$o thf(cP, type)
((cP ⇔ cQ) ⇔ cR) ⇒ (cP ⇔ (cQ ⇔ cR)) thf(cTHM_{50_A}, conjecture)

SYO080^5.p TPS problem THM200

Nepejvoda's problem; supposedly a difficult ND problem.
cB: \$o thf(cB, type)
cA: \$o thf(cA, type)
(((cA ⇒ cB) ⇒ cA) ⇒ cB) ⇒ cB thf(cTHM₂₀₀, conjecture)

SYO081^5.p TPS problem THM137

Trivial theorem for logic lessons.
cB: \$i thf(cB, type)
cR: \$i → \$i → \$o thf(cR, type)
cA: \$i thf(cA, type)
∀x: \$i: (cR@x@cA) ⇒ ∃y: \$i: (cR@cB@y) thf(cTHM₁₃₇, conjecture)

SYO082^5.p TPS problem BAFFLER-VARIANT

f: \$i → \$i → \$i thf(f, type)
cP: \$i → \$o thf(cP, type)
∃xy: \$i: ∀xx: \$i: ((cP@xy) ⇒ (cP@(f@xy@xx))) thf(cBAFFLER_VARIANT, conjecture)

SYO083^5.p TPS problem THM62

cP: \$i → \$i → \$o thf(cP, type)
cB: \$i thf(cB, type)
cA: \$i thf(cA, type)
(∀u: \$i: (cP@cA@u) or ∀v: \$i: (cP@v@cB)) ⇒ ∃x: \$i: (cP@x@x) thf(cTHM₆₂, conjecture)

SYO084^5.p TPS problem THM75

Related to THM87, which was used for CADE-6.

cP: \$i → \$i → \$i → \$o thf(cP, type)
k: \$i → \$i thf(k, type)
h: \$i → \$i thf(h, type)
a: \$i thf(a, type)

cR: \$i → \$i → \$o thf(cR, type)
 $(\exists x: \$i: (cR@x@x) \Rightarrow \forall y: \$i: (cR@y@y)) \Rightarrow \exists u: \$i: \forall v: \$i: ((cR@u@u) \Rightarrow (cR@v@v))$ thf(cTHM55A, conjecture)

SYO095^5.p TPS problem THM81

cR: \$i → \$i → \$o thf(cR, type)
 $(\forall s: \$i, t: \$i: ((cR@s@s) \Leftrightarrow (cR@s@t)) \text{ and } \forall w: \$i, z: \$i: ((cR@w@w) \Leftrightarrow (cR@z@w))) \Rightarrow (\exists x: \$i: (cR@x@x) \Rightarrow \forall y: \$i: (cR@y@y))$ thf(cTHM₈₁, conjecture)

SYO096^5.p TPS problem LX1

cA: \$i thf(cA, type)
cQ: \$i → \$i → \$o thf(cQ, type)
cB: \$i thf(cB, type)
cR: \$i → \$i → \$o thf(cR, type)
 $(cR@cA@cB \text{ and } \forall x: \$i: (\exists y: \$i: (cR@x@y) \Rightarrow (cQ@x@x)) \text{ and } \forall u: \$i, v: \$i: ((cQ@u@v) \Rightarrow \forall z: \$i: (cR@z@v))) \Rightarrow \exists w: \$i: (cR@cB@w \text{ and } cQ@w@cA)$ thf(cLX₁, conjecture)

SYO098^5.p TPS problem THM65

cQ: \$i → \$i → \$o thf(cQ, type)
cR: \$i → \$i → \$o thf(cR, type)
 $\forall w: \$i: \neg cR@w@w \Rightarrow \exists x: \$i, y: \$i: (\neg cR@x@y \text{ and } ((cQ@y@x) \Rightarrow \forall z: \$i: (cQ@z@z)))$ thf(cTHM₆₅, conjecture)

SYO099^5.p TPS problem THM78

cG: \$i → \$o thf(cG, type)
cN: \$i → \$o thf(cN, type)
cM: \$i → \$o thf(cM, type)
 $\forall r: \$i: (cM@r) \text{ or } \exists x: \$i: \neg cG@x \text{ or } \neg \forall y: \$i: (cM@y) \text{ or } \exists s: \$i: (cN@s) \text{ or } \neg \forall z: \$i: \neg cN@z \text{ or } \neg \forall t: \$i: (cG@t)$ thf(cTHM₇₈, conjecture)

SYO101^5.p TPS problem THM83

cW: \$i thf(cW, type)
cR: \$i → \$i → \$o thf(cR, type)
cP: \$i → \$o thf(cP, type)
 $\forall x: \$i: \exists y: \$i: ((cP@x) \Rightarrow \forall z: \$i: (cR@x@y \text{ and } cP@z)) \Rightarrow \exists u: \$i: \forall v: \$i: ((cP@v) \Rightarrow (cR@cW@u))$ thf(cTHM₈₃, conjecture)

SYO102^5.p TPS problem THM101

cP: \$i → \$i → \$o thf(cP, type)
 $(\forall xx: \$i: (cP@xx@xx) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i: ((cP@xx@xy \text{ and } cP@xz@xy) \Rightarrow (cP@xx@xz))) \Rightarrow \forall xu: \$i, xv: \$i, xw: \$i: ((cP@xu@xw))$ thf(cTHM₁₀₁, conjecture)

SYO103^5.p TPS problem THM147

Theorem 211 on page 120 of [Chu56].
imp: \$i → \$i → \$i thf(imp, type)
cT: \$i → \$o thf(cT, type)
nt: \$i → \$i thf(nt, type)
 $\neg \forall xp: \$i, xq: \$i: (\neg cT@(imp@xp@xq) \text{ or } \neg cT@xp \text{ or } cT@xq) \text{ and } \forall xp: \$i, xq: \$i: (cT@(imp@xp@(imp@xq@xp))) \text{ and } \forall xp: \$i, xq: \$i: (cT@xp@xq)$

SYO104^5.p TPS problem TTTP2129

y: \$i thf(y, type)
cQ: \$i → \$i → \$o thf(cQ, type)
cP: \$i → \$i → \$o thf(cP, type)
 $\forall xx: \$i: (cP@xx@y \text{ and } cQ@xx@y) \Leftrightarrow (\forall xx: \$i: (cP@xx@y) \text{ and } \forall xx: \$i: (cQ@xx@y))$ thf(cTTTP₂₁₂₉, conjecture)

SYO105^5.p TPS problem X2201TEST

cR: \$i → \$o thf(cR, type)
y: \$i thf(y, type)
cP: \$i → \$i → \$i → \$o thf(cP, type)
cQ: \$i → \$i → \$o thf(cQ, type)
 $\neg \exists xx: \$i: (cQ@xx@y) \Rightarrow \neg \forall xz: \$i: (\forall xu: \$i: (cP@xu@y@xz) \Rightarrow \neg \exists xv: \$i: (cR@xv)) \Leftrightarrow \exists xx: \$i: \forall xz: \$i: \exists xu: \$i: \forall xv: \$i: \neg (cP@xu@y@xz) \Rightarrow \neg cR@xv$ thf(cX2201TEST, conjecture)

SYO107^5.p TPS problem THM66

cP: \$i → \$o thf(cP, type)
 $\exists x: \$i: \forall y: \$i: ((cP@x) \Rightarrow (cP@y)) \Leftrightarrow \exists x: \$i: ((cP@x) \Rightarrow \forall y: \$i: (cP@y))$ thf(cTHM₆₆, conjecture)

SYO108^5.p TPS problem THM79

cG: \$i → \$o thf(cG, type)
cN: \$i → \$o thf(cN, type)

cM: \$i → \$o thf(cM, type)
 $\forall r: \$i: (cM@r) \text{ or } \exists x: \$i: \neg cG@x \text{ or } \neg \forall y: \$i: (cM@y \text{ or } \exists s: \$i: (cN@s)) \text{ or } \neg \forall z: \$i: (\neg cN@z \text{ or } \neg \forall t: \$i: (cG@t))$ thf(cTHM₂₇₁)

SYO109^5.p TPS problem THM271
cP: \$i → \$o thf(cP, type)
cN: \$i → \$i → \$o thf(cN, type)
cM: \$i → \$i → \$o thf(cM, type)
 $(\forall xx: \$i: (\exists xy: \$i: (cM@xx@xy \text{ or } cN@xx@xy) \Rightarrow (cP@xx)) \text{ and } \forall xw: \$i: \exists xu: \$i: (\forall xv: \$i: (cM@xu@xv) \text{ or } cN@xu@xw) \text{ and } (cM@xz@xw \text{ or } cN@xz@xw \text{ or } cM@xz@xz \text{ or } cN@xz@xz)) \Rightarrow \forall xz: \$i: (cP@xz)$ thf(cTHM₂₇₁, conjecture)

SYO111^5.p TPS problem THM80
cG: \$i → \$o thf(cG, type)
cN: \$i → \$o thf(cN, type)
cM: \$i → \$o thf(cM, type)
 $(\forall r: \$i: (cM@r) \text{ or } \exists x: \$i: \neg cG@x \text{ or } \neg \forall y: \$i: (cM@y) \text{ or } \exists s: \$i: (cN@s) \text{ or } \neg \forall z: \$i: \neg cN@z \text{ or } \neg \forall t: \$i: (cG@t)) \Rightarrow (\forall r: \$i: (cM@r) \text{ or } \exists x: \$i: \neg cG@x \text{ or } \neg \forall y: \$i: (cM@y) \text{ or } \exists s: \$i: (cN@s) \text{ or } \neg \forall z: \$i: (\neg cN@z \text{ or } \neg \forall t: \$i: (cG@t)))$ thf(cTHM₈₀)

SYO112^5.p TPS problem THM53
cP: \$i → \$o thf(cP, type)
 $\forall x: \$i: ((cP@x) \iff \exists y: \$i: (cP@y)) \iff (\forall x: \$i: (cP@x) \iff \exists y: \$i: (cP@y))$ thf(cTHM₅₃, conjecture)

SYO113^5.p TPS problem THM350
cNUMBER: \$i → \$o thf(cNUMBER, type)
cODD: \$i → \$o thf(cODD, type)
cEVEN: \$i → \$o thf(cEVEN, type)
cS: \$i → \$i thf(cS, type)
c₀: \$i thf(c₀, type)
 $(cEVEN@c_0 \text{ and } \forall xn: \$i: ((cEVEN@xn) \Rightarrow (cEVEN@(cS@(cS@xn)))) \text{ and } cODD@(cS@c_0) \text{ and } \forall xn: \$i: ((cODD@xn) \Rightarrow (cODD@(cS@(cS@xn)))) \text{ and } ((cNUMBER@c_0 \text{ and } cNUMBER@(cS@c_0) \text{ and } \forall xx: \$i: ((cNUMBER@xx \text{ and } cNUMBER@(cS@xx) \text{ and } cNUMBER@(cS@xx)))) \Rightarrow \forall xx: \$i: (cNUMBER@xx \text{ and } cNUMBER@(cS@xx)) \text{ and } \forall xn: (cEVEN@xn \text{ or } cODD@xn)) \Rightarrow \forall xn: \$i: (cNUMBER@xn)$ thf(cTHM₃₅₀, conjecture)

SYO114^5.p TPS problem THM119
b: \$i thf(b, type)
cP: \$i → \$o thf(cP, type)
a: \$i thf(a, type)
d: \$i thf(d, type)
cQ: \$i → \$o thf(cQ, type)
c: \$i thf(c, type)
cR: \$i → \$o thf(cR, type)
 $\neg \forall xz: \$i: ((cP@xz \text{ or } cR@xz) \text{ and } cQ@xz) \text{ and } \forall xx: \$i: \exists xy: \$i: (cP@xx \text{ or } \neg cQ@xx \text{ or } \neg cQ@xy \text{ or } \neg cQ@c \text{ or } \neg cQ@d)$ and $\forall xx: \$i: (cP@xx \text{ or } \neg cQ@xx \text{ or } \neg cQ@xy \text{ or } \neg cQ@c \text{ or } \neg cQ@d)$

SYO118^5.p TPS problem from BASIC-FO-THMS
p: \$o thf(p, type)
q: \$o thf(q, type)
 $((p \Rightarrow q) \Rightarrow p) \Rightarrow p$ thf(cPEIRCE, conjecture)

SYO119^5.p TPS problem from BASIC-FO-THMS
a: \$i thf(a, type)
cP: \$i → \$o thf(cP, type)
 $\forall xx: \$i: (cP@xx) \Rightarrow (cP@a)$ thf(cEXX₁, conjecture)

SYO120^5.p TPS problem from BASIC-FO-THMS
cA: \$i → \$o thf(cA, type)
 $\forall z_3: \$i: ((cA@z_3) \Rightarrow (cA@z_3))$ thf(cSET80-pme, conjecture)

SYO121^5.p TPS problem from BASIC-FO-THMS
cQ: \$i → \$i → \$o thf(cQ, type)
 $\exists e: \$i: \forall k: \$i: (cQ@k@e@k)$ thf(cTHM₂₇, conjecture)

SYO122^5.p TPS problem from BASIC-FO-THMS
cEQ: \$i → \$i → \$o thf(cEQ, type)
 $\forall xu: \$i, xx: \$i, xy: \$i: (cEQ@xu@xy)$ thf(cA_4A₃, conjecture)

SYO123^5.p TPS problem from BASIC-FO-THMS
b: \$i thf(b, type)

cP: $\$i \rightarrow \o thf(cP, type)
 a: $\$i$ thf(a, type)
 $\forall xx: \$i: (cP@xx) \Rightarrow (cP@a \text{ and } cP@b)$ thf(cALLCONJ₂, conjecture)

SYO124^5.p TPS problem from BASIC-FO-THMS

cQ: $\$o$ thf(cQ, type)
 cR: $\$o$ thf(cR, type)
 cP: $\$o$ thf(cP, type)
 $(cP \text{ and } (cR \text{ or } cQ)) \Rightarrow ((cP \text{ and } cR) \text{ or } cQ)$ thf(cDISJ_BUG, conjecture)

SYO125^5.p TPS problem from BASIC-FO-THMS

cR: $\$o$ thf(cR, type)
 cP: $\$o$ thf(cP, type)
 cQ: $\$o$ thf(cQ, type)
 $((cP \iff cQ) \text{ and } (cQ \iff cR)) \Rightarrow (cP \iff cR)$ thf(cTRIV₄, conjecture)

SYO126^5.p TPS problem from BASIC-FO-THMS

cS: $((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \o thf(cS, type)
 cP: $((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \o thf(cP, type)
 $c_0: (\$i \rightarrow \$o) \rightarrow \$o$ thf(c_0 , type)
 $cP@c_0 \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o: ((cP@x) \Rightarrow (cP@(cS@x)))$ thf(cSUPERSET_NAT, conjecture)

SYO128^5.p TPS problem from BASIC-FO-THMS

r: $\$o$ thf(r, type)
 p: $\$o$ thf(p, type)
 q: $\$o$ thf(q, type)
 $(p \text{ and } (q \text{ or } r)) \Rightarrow ((p \text{ and } q) \text{ or } (p \text{ and } r))$ thf(cDISJ_BUG₂, conjecture)

SYO129^5.p TPS problem from BASIC-FO-THMS

c: $\$i$ thf(c, type)
 cP: $\$i \rightarrow \o thf(cP, type)
 b: $\$i$ thf(b, type)
 a: $\$i$ thf(a, type)
 $\forall xx: \$i: (cP@xx) \Rightarrow (cP@a \text{ and } cP@b \text{ and } cP@c)$ thf(cALLCONJ₃, conjecture)

SYO130^5.p TPS problem from BASIC-FO-THMS

cP: $\$i \rightarrow \o thf(cP, type)
 $\exists xx: \$i: (cP@xx) \Rightarrow \exists xx: \$i: (cP@xx) \Rightarrow \forall xx: \$i: \neg cP@xx$ thf(cSIMPLER_BUG, conjecture)

SYO131^5.p TPS problem from BASIC-FO-THMS

f: $\$i \rightarrow \i thf(f, type)
 cR: $\$i \rightarrow \$i \rightarrow \$o$ thf(cR, type)
 a: $\$i$ thf(a, type)
 g: $\$i \rightarrow \i thf(g, type)
 $(cR@(g@(f@a)) @ (f@(g@(f@a)))) \Rightarrow \exists xx: \$i: (cR@xx@(f@xx))$ thf(cEXAMPLE₁, conjecture)

SYO132^5.p TPS problem from BASIC-FO-THMS

cX: $\$i$ thf(cX, type)
 cS: $\$i \rightarrow \o thf(cS, type)
 cT: $\$i \rightarrow \o thf(cT, type)
 cP: $(\$i \rightarrow \$o) \rightarrow \$o$ thf(cP, type)
 $(cP@cS \text{ and } cT@cX \text{ and } cS = cT) \Rightarrow (cP@cT \text{ and } cS@cX)$ thf(cTHM₅₀₃, conjecture)

SYO133^5.p TPS problem from BASIC-FO-THMS

cP₂: $\$i \rightarrow \o thf(cP₂, type)
 cP₁: $\$i \rightarrow \o thf(cP₁, type)
 $\exists xy: \$i: \forall xx: \$i: ((cP₁@xx \text{ and } ((cP₁@xy) \Rightarrow (cP₂@xx))) \Rightarrow (cP₂@xy))$ thf(cBAFFLER₂, conjecture)

SYO134^5.p TPS problem from BASIC-FO-THMS

a: $\$i$ thf(a, type)
 cQ: $\$i \rightarrow \o thf(cQ, type)
 cP: $\$i \rightarrow \o thf(cP, type)
 $((cP@a) \Rightarrow \forall xx: \$i: (cQ@xx)) \Rightarrow (\forall xx: \$i: (cP@xx) \Rightarrow (cQ@a))$ thf(cADDHYP₁, conjecture)

SYO135^5.p TPS problem from BASIC-FO-THMS

x: $\$i$ thf(x, type)

cQ: \$i → \$o thf(cQ, type)
 b: \$i thf(b, type)
 cP: \$i → \$o thf(cP, type)
 a: \$i thf(a, type)
 $(\forall \text{xx}_0: \text{i: (cP@xx}_0\text{) or cQ@x}) \Rightarrow ((\text{cP}@a \text{ and cP}@b) \text{ or cQ}@x) \quad \text{thf(cDUP_BUG, conjecture)}$

SYO136^5.p TPS problem from BASIC-FO-THMS
 cQ: \$i → \$o thf(cQ, type)
 x: \$i thf(x, type)
 cB: \$o thf(cB, type)
 cP: \$i → \$o thf(cP, type)
 $((\exists \text{xx}_0: \text{i: (cP@xx}_0\text{)} \Rightarrow \text{cB}) \Rightarrow \text{cB}) \Rightarrow ((\text{cQ}@x) \Rightarrow \exists \text{xx}_0: \text{i: (cQ@xx}_0\text{)}) \quad \text{thf(cADDHYP}_3\text{, conjecture)}$

SYO137^5.p TPS problem from BASIC-FO-THMS
 cN: \$i → \$o thf(cN, type)
 cM: \$i → \$o thf(cM, type)
 $(\forall \text{xx}: \text{i: ((cM@xx) \iff (cN@xx))} \iff \forall \text{xx}: \text{i: (cM@xx)} \iff \forall \text{xx}: \text{i: (cN@xx)}) \quad \text{thf(cX}_{2304}\text{, conjecture)}$

SYO138^5.p TPS problem from BASIC-FO-THMS
 cS: \$i → \$o thf(cS, type)
 cP: \$i → \$o thf(cP, type)
 $(\exists \text{xx}: \text{i: (cP@xx)} \Rightarrow \exists \text{xx}: \text{i: (cP@xx)}) \Rightarrow \forall \text{xx}: \text{i: ((cP@xx) \Rightarrow (\text{cS}@xx))} \quad \text{thf(cSIMPLE_BUG, conjecture)}$

SYO139^5.p TPS problem from BASIC-FO-THMS
 cQ: \$i → \$o thf(cQ, type)
 cP: \$i → \$o thf(cP, type)
 $\exists \text{xy}: \text{i: ((\exists \text{xx}: \text{i: (cP@xx} \text{ or } \neg \text{cP}@xx\text{)} \text{ and cQ}@xy) \text{ or } \neg \text{cQ}@xy)} \quad \text{thf(cCOUNTER}_1\text{, conjecture)}$

SYO140^5.p TPS problem from BASIC-FO-THMS
 cQ: \$i → \$o thf(cQ, type)
 cP: \$i → \$o thf(cP, type)
 $\exists \text{xx}: \text{i: (cP@xx)} \Rightarrow (\forall \text{xx}: \text{i: ((cP@xx) \Rightarrow (\text{cQ}@xx))} \Rightarrow \exists \text{xx}: \text{i: (cQ@xx)}) \quad \text{thf(cADDHYP}_6\text{, conjecture)}$

SYO141^5.p TPS problem from BASIC-FO-THMS
 a: \$i thf(a, type)
 cQ: \$i → \$o thf(cQ, type)
 x: \$i thf(x, type)
 cR: \$i → \$o thf(cR, type)
 cP: \$i → \$o thf(cP, type)
 $((\text{cP}@a) \Rightarrow \forall \text{xx}_0: \text{i: (cQ@xx}_0\text{)}) \Rightarrow (\forall \text{xx}_0: \text{i: (cP@xx}_0\text{)} \Rightarrow ((\text{cR}@x) \Rightarrow (\text{cQ}@a))) \quad \text{thf(cADDHYP}_2\text{, conjecture)}$

SYO142^5.p TPS problem from BASIC-FO-THMS
 a: \$i thf(a, type)
 f: \$i → \$i thf(f, type)
 p: \$i → \$o thf(p, type)
 $\exists \text{xx}: \text{i, xy}: \text{i: (((p@xx) \Rightarrow (p@(f@(f@xy)))) \text{ and } ((p@xy) \Rightarrow (p@(f@(f@a)))))} \quad \text{thf(cTEST}_4\text{, conjecture)}$

SYO143^5.p TPS problem from BASIC-FO-THMS
 cQ: \$i → \$o thf(cQ, type)
 cP: \$i → \$o thf(cP, type)
 $(\neg \exists \text{xy}: \text{i: } \forall \text{xx}: \text{i: (cP@xy)} \Rightarrow (\text{cP}@xx)) \text{ or } \exists \text{xz}: \text{i: (cQ@xz)} \Rightarrow \exists \text{xz}: \text{i: (cQ@xz)} \quad \text{thf(cDUP_BUG}_1\text{, conjecture)}$

SYO144^5.p TPS problem from BASIC-FO-THMS
 cC: \$o thf(cC, type)
 x: \$i thf(x, type)
 cR₃: \$i → \$o thf(cR₃, type)
 cR₂: \$i → \$o thf(cR₂, type)
 cR₁: \$i → \$o thf(cR₁, type)
 cQ: \$i → \$o thf(cQ, type)
 cP: \$i → \$o thf(cP, type)
 $(\forall \text{xx}_0: \text{i: (cP@xx}_0\text{)} \Rightarrow \forall \text{xx}_0: \text{i: (cQ@xx}_0\text{)} \Rightarrow ((\text{cR}_1@x \text{ or cR}_2@x \text{ or cR}_3@x) \Rightarrow \text{cC}) \quad \text{thf(cADDHYP}_7\text{, conjecture)}$

SYO145^5.p TPS problem from BASIC-FO-THMS
 cP: \$i → \$i → \$o thf(cP, type)
 $\forall \text{xx}: \text{i: } \exists \text{xy}: \text{i: (cP@xx@xy)} \Rightarrow \forall \text{xx}: \text{i: } \exists \text{xy}: \text{i, xz}: \text{i: (cP@xy@xz \text{ and cP}@xx@xy)} \quad \text{thf(cEXPVAR_BUG, conjecture)}$

SYO146^5.p TPS problem from BASIC-FO-THMS

$a: \$i \text{ thf}(a, \text{type})$
 $cQ: \$i \rightarrow \$o \text{ thf}(cQ, \text{type})$
 $c: \$i \text{ thf}(c, \text{type})$
 $cP: \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cP, \text{type})$
 $b: \$i \text{ thf}(b, \text{type})$
 $\forall xx: \$i: (\forall xy: \$i: (cP@xx@xy) \text{ or } cQ@xx) \Rightarrow ((cP@a@b \text{ and } cP@a@c) \text{ or } cQ@a) \text{ thf(cDUP_EXPL}_1\text{, conjecture)}$

SYO147^5.p TPS problem from BASIC-FO-THMS

$cP_3: \$i \rightarrow \$o \text{ thf}(cP_3, \text{type})$
 $cP_2: \$i \rightarrow \$o \text{ thf}(cP_2, \text{type})$
 $cP_1: \$i \rightarrow \$o \text{ thf}(cP_1, \text{type})$
 $\exists xy: \$i: \forall xx: \$i: ((cP_1@xx \text{ and } (cP_1@xy) \Rightarrow (cP_2@xx)) \text{ and } ((cP_2@xy) \Rightarrow (cP_3@xx))) \Rightarrow (cP_3@xy) \text{ thf(cBAFFLER}_3\text{)}$

SYO148^5.p TPS problem from BASIC-FO-THMS

$a: \$tType \text{ thf}(a_type, \text{type})$
 $cZ: a \rightarrow \$o \text{ thf}(cZ, \text{type})$
 $cY: a \rightarrow \$o \text{ thf}(cY, \text{type})$
 $cX: a \rightarrow \$o \text{ thf}(cX, \text{type})$
 $\forall u: a: (\neg cX@u \iff ((cY@u) \iff (cZ@u))) \Rightarrow cX = (\lambda u: a: \neg(cY@u) \iff (cZ@u)) \text{ thf(cBOOL}_{25}\text{, conjecture)}$

SYO149^5.p TPS problem from BASIC-FO-THMS

$a: \$i \text{ thf}(a, \text{type})$
 $f: \$i \rightarrow \$i \text{ thf}(f, \text{type})$
 $cP: \$i \rightarrow \$o \text{ thf}(cP, \text{type})$
 $cQ: \$i \rightarrow \$o \text{ thf}(cQ, \text{type})$
 $(\forall xx: \$i: ((cP@xx) \Rightarrow (cQ@(f@xx))) \text{ and } \forall xy: \$i: ((cQ@xy) \Rightarrow (cP@xy)) \text{ and } cP@a \Rightarrow (cP@(f@a)) \text{ thf(cSIMPLEPQ)}$

SYO150^5.p TPS problem from BASIC-FO-THMS

$cC: \$o \text{ thf}(cC, \text{type})$
 $x: \$i \text{ thf}(x, \text{type})$
 $cR_4: \$i \rightarrow \$o \text{ thf}(cR_4, \text{type})$
 $cR_3: \$i \rightarrow \$o \text{ thf}(cR_3, \text{type})$
 $cR_2: \$i \rightarrow \$o \text{ thf}(cR_2, \text{type})$
 $cR_1: \$i \rightarrow \$o \text{ thf}(cR_1, \text{type})$
 $cQ: \$i \rightarrow \$o \text{ thf}(cQ, \text{type})$
 $cP: \$i \rightarrow \$o \text{ thf}(cP, \text{type})$
 $(\forall xx_0: \$i: (cP@xx_0) \Rightarrow \forall xx_0: \$i: (cQ@xx_0)) \Rightarrow ((cR_1@x \text{ or } cR_2@x \text{ or } cR_3@x \text{ or } cR_4@x) \Rightarrow cC) \text{ thf(cADDHYP}_8\text{, conjecture)}$

SYO151^5.p TPS problem from BASIC-FO-THMS

$cQ: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cQ, \text{type})$
 $\forall x: \$i, y: \$i: (\exists g: \$i: (cQ@g@x@y) \text{ and } \exists j: \$i: (cQ@x@j@y) \text{ and } \exists f: \$i: (cQ@x@y@f)) \text{ thf(cHYP}_2\text{, conjecture)}$

SYO152^5.p TPS problem from BASIC-FO-THMS

$cR: \$o \text{ thf}(cR, \text{type})$
 $cS: \$o \text{ thf}(cS, \text{type})$
 $cP: \$o \text{ thf}(cP, \text{type})$
 $cQ: \$o \text{ thf}(cQ, \text{type})$
 $\exists xx: \$i, xy: \$i: (((\forall xu: \$i, xv: \$i: cP \text{ or } cQ) \text{ and } \forall xz: \$i: cR) \Rightarrow \neg(\neg cP \text{ and } \neg cS) \text{ or } \neg cR) \text{ thf(cQUANTIFIER_BUG, conjecture)}$

SYO153^5.p TPS problem from BASIC-FO-THMS

$cAPP: \$i \rightarrow \$i \rightarrow \$i \text{ thf}(cAPP, \text{type})$
 $cReduct: \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cReduct, \text{type})$
 $cApp: \$i \rightarrow \$i \rightarrow \$i \text{ thf}(cApp, \text{type})$
 $\forall xx: \$i, xy: \$i, xz: \$i: ((cReduct@xx@xy) \Rightarrow (cReduct@(cAPP@xx@xz)@(cApp@xy@xz) \text{ and } cReduct@(cAPP@xz@xx)@(cApp@xy@xz))) \text{ thf(cAPP_REDUCE)}$

SYO154^5.p TPS problem from BASIC-FO-THMS

$cR: \$i \rightarrow \$o \text{ thf}(cR, \text{type})$
 $g: \$i \rightarrow \$i \text{ thf}(g, \text{type})$
 $cQ: \$i \rightarrow \$o \text{ thf}(cQ, \text{type})$
 $f: \$i \rightarrow \$i \text{ thf}(f, \text{type})$
 $cP: \$i \rightarrow \$o \text{ thf}(cP, \text{type})$
 $a: \$i \text{ thf}(a, \text{type})$
 $(cP@a \text{ and } \forall xx: \$i: ((cP@xx) \Rightarrow (cQ@(f@xx)))) \text{ and } \forall xy: \$i: ((cQ@xy) \Rightarrow (cR@(g@xy)))) \Rightarrow \exists xw: \$i: (cR@xw) \text{ thf(cDUP_REDUCE)}$

SYO155^5.p TPS problem from BASIC-FO-THMS

cS: \$i → \$o thf(cS, type)
 cP: \$i → \$o thf(cP, type)
 $((\exists_{xx}: \$i: (cP@{xx}) \Rightarrow \exists_{xx}: \$i: (cP@{xx})) \text{ and } (\exists_{xx}: \$i: (cP@{xx}) \Rightarrow \exists_{xx}: \$i: (cP@{xx})) \Rightarrow \forall_{xx}: \$i: ((cP@{xx}) \Rightarrow (cS@{xx}))$ thf(cPELL264, conjecture)

SYO156^5.p TPS problem from BASIC-FO-THMS

cR: \$o thf(cR, type)
 cP: \$o thf(cP, type)
 cQ: \$o thf(cQ, type)
 $(\neg cP \text{ or } cQ) \text{ and } (cP \text{ or } \neg cQ) \text{ and } (\neg cQ \text{ or } cR) \text{ and } (cQ \text{ or } \neg cR) \text{ and } (\neg cP \text{ or } \neg cR) \text{ and } (cP \text{ or } cR)$ thf(cCNF_NTRIV)

SYO157^5.p TPS problem from BASIC-FO-THMS

cR: \$i → \$o thf(cR, type)
 cP: \$i → \$o thf(cP, type)
 cS: \$i → \$o thf(cS, type)
 cQ: \$i → \$o thf(cQ, type)
 $(\forall_{xx}: \$i: ((cP@{xx}) \Rightarrow (cR@{xx})) \Rightarrow \forall_{xx}: \$i: ((cQ@{xx}) \Rightarrow (cS@{xx}))) \text{ and } (\forall_{xx}: \$i: ((cQ@{xx}) \Rightarrow (cS@{xx})) \Rightarrow \forall_{xx}: \$i: ((cP@{xx}) \Rightarrow (cR@{xx})))$ thf(cPELL261, conjecture)

SYO158^5.p TPS problem from BASIC-FO-THMS

cQ: \$i → \$i → \$i → \$o thf(cQ, type)
 $\forall x: \$i, y: \$i, z: \$i, u: \$i, v: \$i, vV: \$i: ((cQ@{x}{y}{z}{u}) \text{ and } cQ@{y}{z}{v}{vV}) \Rightarrow ((cQ@{x}{v}{vV}) \iff (cQ@{u}{z}{vV}))$ thf(cHY)

SYO159^5.p TPS problem from BASIC-FO-THMS

cC: \$i → \$o thf(cC, type)
 cB: \$i → \$i → \$o thf(cB, type)
 cA: \$i → \$o thf(cA, type)
 $\neg \forall_{xx}: \$i: ((cA@{xx}) \Rightarrow \exists_{xy}: \$i: (cB@{xx}{xy}) \text{ and } cC@{xy})) \Rightarrow \exists_{xu}: \$i: (cA@{xu}) \text{ and } \neg \exists_{xv}: \$i: \neg (cB@{xu}{xv}) \Rightarrow \neg cC@{xv})$ thf(cNNF_EXAMPLE, conjecture)

SYO160^5.p TPS problem from BASIC-FO-THMS

c: \$i thf(c, type)
 cR: \$i → \$o thf(cR, type)
 b: \$i thf(b, type)
 a: \$i thf(a, type)
 cQ: \$i → \$o thf(cQ, type)
 cP: \$i → \$o thf(cP, type)
 $((\forall_{xx}: \$i: (cP@{xx}) \text{ or } \forall_{xx}: \$i: (cQ@{xx})) \text{ and } \forall_{xx}: \$i: ((cP@{xx}) \Rightarrow (cR@{xx})) \text{ and } \forall_{xx}: \$i: ((cQ@{xx}) \Rightarrow (cR@{xx}))) \Rightarrow (cR@a \text{ and } cR@b \text{ and } cR@c)$ thf(cDISJ_THIRD, conjecture)

SYO161^5.p TPS problem from BASIC-FO-THMS

cP: \$i → \$o thf(cP, type)
 $\exists_{xx}: \$i: \forall_{xy}: \$i: (((cP@{xx}) \Rightarrow (cP@{xy})) \text{ and } ((cP@{xy}) \Rightarrow (cP@{xx}))) \Rightarrow ((\exists_{xx}: \$i: (cP@{xx}) \Rightarrow \forall_{xy}: \$i: (cP@{xy})) \text{ and } (\forall_{xy}: \$i: (cP@{xy}) \Rightarrow \exists_{xx}: \$i: (cP@{xx})))$ thf(cX2125_HALFB, conjecture)

SYO163^5.p TPS problem from BASIC-FO-THMS

cT: \$i → \$o thf(cT, type)
 i: \$i → \$i → \$i thf(i, type)
 n: \$i → \$i thf(n, type)
 z: \$i thf(z, type)
 $\forall_{xx}: \$i, xy: \$i: (cT@{(i@{xx}{i@{xy}{xx}})} \text{ and } cT@{(i@{(i@{xx}{i@{xy}{z}})}@{(i@{(i@{xx}{xy}})}@{(i@{xx}{z}})}}) \text{ and } cT@{(i@{(i@{(n@{xx}}}{cT@{xy}}))}}$ thf(cMORGAN_AXIOMS, conjecture)

SYO164^5.p TPS problem from BASIC-FO-THMS

cR: \$i → \$o thf(cR, type)
 cP: \$i → \$o thf(cP, type)
 cS: \$i → \$o thf(cS, type)
 cQ: \$i → \$o thf(cQ, type)
 $((\exists_{xx}: \$i: (cP@{xx}) \Rightarrow \exists_{xx}: \$i: (cQ@{xx})) \text{ and } (\exists_{xx}: \$i: (cQ@{xx}) \Rightarrow \exists_{xx}: \$i: (cP@{xx}))) \Rightarrow ((\forall_{xx}: \$i: ((cP@{xx}) \Rightarrow (cR@{xx}))) \Rightarrow \forall_{xx}: \$i: ((cQ@{xx}) \Rightarrow (cS@{xx}))) \text{ and } (\forall_{xx}: \$i: ((cQ@{xx}) \Rightarrow (cS@{xx}))) \Rightarrow \forall_{xx}: \$i: ((cP@{xx}) \Rightarrow (cR@{xx})))$ thf(cPELL262, conjecture)

SYO165^5.p TPS problem from BASIC-FO-THMS

cP: \$i → \$o thf(cP, type)

$s: \$i \text{ thf}(s, \text{type})$
 $cE: \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cE, \text{type})$
 $cR: \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cR, \text{type})$
 $(\forall xx: \$i, xz: \$i: ((cE@xz@xx) \Rightarrow \exists xy: \$i: (cE@xy@xx \text{ and } \forall xw: \$i: ((cR@xy@xw) \Rightarrow \neg cE@xw@xx))) \text{ and } \forall xs_0: \$i, xx: \$i: (\forall (cP@xy) \Rightarrow (cP@xx))) \Rightarrow \forall xx: \$i: ((cE@xx@s) \Rightarrow (cP@xx)) \text{ thf(cTHM117A, conjecture)}$

SYO166^5.p TPS problem from BASIC-FO-THMS

$n: \$i \rightarrow \$i \text{ thf}(n, \text{type})$
 $i: \$i \rightarrow \$i \rightarrow \$i \text{ thf}(i, \text{type})$
 $cT: \$i \rightarrow \$o \text{ thf}(cT, \text{type})$
 $z: \$i \text{ thf}(z, \text{type})$
 $\forall xx: \$i, xy: \$i: (cT@(i@xx@(i@xy@xx)) \text{ and } cT@(i@(i@xx@(i@xy@z))@((i@(i@xx@xy)@i@xx@z))) \text{ and } cT@(i@(i@(n@xx@(cT@xy)))) \Rightarrow \forall xx: \$i: (cT@(i@(n@(n@xx)))@xx)) \text{ thf(cPELL}_{67}\text{, conjecture)}$

SYO167^5.p TPS problem from BASIC-FO-THMS

$n: \$i \rightarrow \$i \text{ thf}(n, \text{type})$
 $i: \$i \rightarrow \$i \rightarrow \$i \text{ thf}(i, \text{type})$
 $cT: \$i \rightarrow \$o \text{ thf}(cT, \text{type})$
 $z: \$i \text{ thf}(z, \text{type})$
 $\forall xx: \$i, xy: \$i: (cT@(i@xx@(i@xy@xx)) \text{ and } cT@(i@(i@xx@(i@xy@z))@((i@(i@xx@xy)@i@xx@z))) \text{ and } cT@(i@(i@(n@xx@(cT@xy)))) \Rightarrow \forall xx: \$i: (cT@(i@xx@(n@(n@xx)))) \text{ thf(cPELL}_{66}\text{, conjecture)}$

SYO168^5.p TPS problem from BASIC-FO-THMS

$n: \$i \rightarrow \$i \text{ thf}(n, \text{type})$
 $i: \$i \rightarrow \$i \rightarrow \$i \text{ thf}(i, \text{type})$
 $cT: \$i \rightarrow \$o \text{ thf}(cT, \text{type})$
 $z: \$i \text{ thf}(z, \text{type})$
 $\forall xx: \$i, xy: \$i: (cT@(i@xx@(i@xy@xx)) \text{ and } cT@(i@(i@xx@(i@xy@z))@((i@(i@xx@xy)@i@xx@z))) \text{ and } \forall xx_0: \$i, xy_0: \$i: (cT@(i@xx@(i@xy@z))@((i@(i@xx@xy)@i@xx@z))) \Rightarrow \forall xx: \$i: (cT@(i@xx@(n@(n@xx)))) \text{ thf(cPELL}_{68}\text{, conjecture)}$

SYO169^5.p TPS problem from BASIC-FO-THMS

$ab: \$i \text{ thf}(ab, \text{type})$
 $a: \$i \text{ thf}(a, \text{type})$
 $b: \$i \text{ thf}(b, \text{type})$
 $cP: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cP, \text{type})$
 $cPx: \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cPx, \text{type})$
 $e: \$i \text{ thf}(e, \text{type})$
 $(\forall xx: \$i: (cP@e@xx@xx) \text{ and } \forall xy: \$i: (cP@xy@e@xy) \text{ and } \forall xz: \$i: (cP@xz@xz@e) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i, xyz: \$i, xxy: \$i, xyy: \$i, xxyy: \$i, xxyyz: \$i, xxyz: \$i, xxyyz) \Leftrightarrow ((cP@a@b@ab) \Rightarrow (cP@b@a@ab)) \text{ thf(cGRP_COMM, conjecture)}$

SYO170^5.p TPS problem from BASIC-FO-THMS

$ab: \$i \text{ thf}(ab, \text{type})$
 $a: \$i \text{ thf}(a, \text{type})$
 $b: \$i \text{ thf}(b, \text{type})$
 $cP: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o \text{ thf}(cP, \text{type})$
 $e: \$i \text{ thf}(e, \text{type})$
 $(\forall xx: \$i: (cP@e@xx@xx) \text{ and } \forall xy: \$i: (cP@xy@e@xy) \text{ and } \forall xz: \$i: (cP@xz@xz@e) \text{ and } \forall xx: \$i, xy: \$i, xz: \$i, xxy: \$i, xyz: \$i, xxyy: \$i, xxyyz: \$i, xxyz: \$i, xxyyz) \Leftrightarrow ((cP@a@b@ab) \Rightarrow (cP@b@a@ab)) \text{ thf(cTHM}_{105}\text{, conjecture)}$

SYO171^5.p TPS problem from BASIC-FO-THMS

$a: \$i \text{ thf}(a, \text{type})$
 $g: \$i \rightarrow \$i \rightarrow \$o \text{ thf}(g, \text{type})$
 $f: \$i \rightarrow \$i \text{ thf}(f, \text{type})$
 $\neg \forall a: \$i: (g@a@a \text{ or } g@(f@a)@a) \text{ and } \forall a: \$i: (g@a@a \text{ or } g@a@(f@a)) \text{ and } \forall a: \$i, b: \$i: (\neg g@a@b \text{ or } g@(f@b)@b) \text{ and } \forall a: \$i,$

SYO173^5.p TPS problem from BASIC-FO-THMS

$nt: \$i \rightarrow \$i \text{ thf}(nt, \text{type})$
 $imp: \$i \rightarrow \$i \rightarrow \$i \text{ thf}(imp, \text{type})$
 $cT: \$i \rightarrow \$o \text{ thf}(cT, \text{type})$
 $\neg \forall xp: \$i, xq: \$i: (\neg cT@(imp@xp@xq) \text{ or } \neg cT@xp \text{ or } cT@xq) \text{ and } \forall xp: \$i, xq: \$i: (cT@(imp@xp@(imp@xq@xp))) \text{ and } \forall xp: \$i, xq: \$i: (cT@(imp@xp@xq) \Rightarrow (cT@xp @ (imp@xq@xp)))$

SYO174^5.p TPS problem from BASIC-FO-THMS

$cQ: \$i \rightarrow \$o \text{ thf}(cQ, \text{type})$
 $cP: \$i \rightarrow \$o \text{ thf}(cP, \text{type})$

$cS: \$i \rightarrow \$o \quad \text{thf}(cS, \text{type})$
 $cR: \$i \rightarrow \$o \quad \text{thf}(cR, \text{type})$
 $((\exists xx: \$i. \forall xy: \$i. ((cP@xx) \iff (cP@xy))) \iff (\exists xx: \$i. (cQ@xx) \iff \forall xy: \$i. (cR@xy))) \iff$
 $(\exists xx: \$i. \forall xy: \$i. ((cQ@xx) \iff (cQ@xy))) \iff (\exists xx: \$i. (cR@xx) \iff \forall xy: \$i. (cS@xy))) \iff$
 $((\exists xx: \$i. \forall xy: \$i. ((cR@xx) \iff (cR@xy))) \iff (\exists xx: \$i. (cS@xx) \iff \forall xy: \$i. (cP@xy))) \iff$
 $(\exists xx: \$i. \forall xy: \$i. ((cS@xx) \iff (cS@xy)) \iff (\exists xx: \$i. (cP@xx) \iff \forall xy: \$i. (cQ@xy)))) \quad \text{thf}(c\text{THM}_{138}, \text{conjecture})$

SYO175^5.p TPS problem from BASIC-FO-THMS

```

p: $tType      thf(p_type, type)
a: $tType      thf(a_type, type)
cGRAIN: p      thf(cGRAIN, type)
cVEG_EATS: a → p → $o      thf(cVEG_EATS, type)
cMEAT_EATS: a → a → $o      thf(cMEAT_EATS, type)
cSNAIL: a      thf(cSNAIL, type)
cCATERPILLAR: a      thf(cCATERPILLAR, type)
cBIRD: a       thf(cBIRD, type)
cWOLF: a       thf(cWOLF, type)
cFOX: a        thf(cFOX, type)
cSMALLER: a → a → $o      thf(cSMALLER, type)
(∀xx: a: (∀xy: p: (cVEG_EATS@xx@xy) or ∀xz: a: ((cSMALLER@xz@xx and ∃xw: p: (cVEG_EATS@xz@xw)) ⇒
(cMEAT_EATS@xx@xz))) and cSMALLER@cCATERPILLAR@cBIRD and cSMALLER@cSNAIL@cBIRD and cSMALLER
∃xx: a, xy: a: (cMEAT_EATS@xx@xy and cVEG_EATS@xy@cGRAIN)      thf(cPUZ031_1_HO, conjecture)

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SYO176^5.p TPS problem from BASIC-FO-THMS

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cG: $o      thf(cG, type)
cM: $o      thf(cM, type)
cK: $o      thf(cK, type)
cR: $o      thf(cR, type)
cC: $o      thf(cC, type)
cB: $o      thf(cB, type)
cF: $o      thf(cF, type)
cP: $o      thf(cP, type)
cN: $o      thf(cN, type)
cE: $o      thf(cE, type)
cL: $o      thf(cL, type)

((cL and cP) ⇒ cM) and ((cG and ¬cR) ⇒ cM) and ((¬cK and cN and cM) ⇒ cF) and ((¬cG and ¬cP) ⇒
cR) and ((cK and cB) ⇒ cC) and ((cR and ¬cN and ¬cF) ⇒ cP) and ((cL and cM) ⇒ cC) and ((cE and ¬cK and cG and
¬cM) and ((¬cG and ¬cR) ⇒ cK) and ((cK and cL and cE) ⇒ ¬cM) and ((cR and cE) ⇒ ¬cC) and ((cG and ¬cK and cB and
¬cB) and ((cN and ¬cP and ¬cF) ⇒ cC) and ((cG and cB and ¬cR) ⇒ ¬cC) and ((cR and ¬cK and ¬cM) ⇒
cG)      thf(cPORKCHOP, conjecture)

```

SYQ178\5.p TPS problem from BASIC-FO-THMS

cM: \$o thf(cM, type)
cN: \$o thf(cN, type)
cG: \$o thf(cG, type)
cK: \$o thf(cK, type)
cE: \$o thf(cE, type)
cR: \$o thf(cR, type)
cF: \$o thf(cF, type)
cC: \$o thf(cC, type)
cB: \$o thf(cB, type)
cP: \$o thf(cP, type)
cL: \$o thf(cL, type)

$\neg cL$ and cE and $(\neg cF$ or $cB)$ and $(\neg cL$ or $\neg cP$ or cI)

SYO179^5.p TPS problem from
 cG: \$i → \$i → \$o thf(cG, type)
 cR: \$i → \$i → \$o thf(cR, type)
 cF: \$i thf(cF, type)
 cE: \$i thf(cE, type)
 cD: \$i thf(cD, type)

cC: \$i thf(cC, type)

cB: \$i thf(cB, type)

cA: \$i thf(cA, type)

((cR@cA@cB or cG@cA@cB) and (cR@cA@cC or cG@cA@cC) and (cR@cA@cD or cG@cA@cD) and (cR@cA@cE or cG@cA@cE) and (cR@cA@cF or cG@cA@cF))
 $\exists x_a: \$i, x_b: \$i, x_c: \$i: (cR@xa@xb \text{ and } cR@xa@xc \text{ and } cR@xb@xc) \text{ or } \exists x_a: \$i, x_b: \$i, x_c: \$i: (cG@xa@xb \text{ and } cG@xa@xc \text{ and } cG@xb@xc)$

SYO180^5.p TPS problem from BASIC-FO-THMS

$\forall l: \$o, p: \$o, m: \$o, g: \$o, r: \$o, e: \$o, n: \$o, f: \$o, b: \$o, c: \$o: (((l \text{ and } p) \Rightarrow m) \text{ and } ((g \text{ and } \neg r) \Rightarrow m) \text{ and } ((\neg k \text{ and } n \text{ and } m) \Rightarrow f) \text{ and } ((\neg g \text{ and } \neg p) \Rightarrow r) \text{ and } ((k \text{ and } b) \Rightarrow c) \text{ and } ((r \text{ and } \neg n \text{ and } \neg f) \Rightarrow p) \text{ and } ((l \text{ and } m) \Rightarrow c) \text{ and } ((e \text{ and } \neg k \text{ and } g \text{ and } \neg n) \Rightarrow \neg m) \text{ and } ((\neg g \text{ and } \neg r) \Rightarrow k) \text{ and } ((k \text{ and } l \text{ and } e) \Rightarrow \neg m) \text{ and } ((r \text{ and } e) \Rightarrow \neg c) \text{ and } ((g \text{ and } \neg k \text{ and } \neg m) \Rightarrow \neg b) \text{ and } ((n \text{ and } \neg p \text{ and } \neg f) \Rightarrow c) \text{ and } ((g \text{ and } b \text{ and } \neg r) \Rightarrow \neg c) \text{ and } ((r \text{ and } \neg k \text{ and } \neg m) \Rightarrow g) \Rightarrow ((e \text{ and } l) \Rightarrow (f \text{ and } \neg b))) \quad \text{thf(cPORKCHOP}_2\text{, conjecture)}$

SYO183^5.p TPS problem CT2

a: \$o thf(a, type)

$\exists x_p: \$o \rightarrow \$o: (xp@a) \quad \text{thf(cCT}_2\text{, conjecture)}$

SYO183^6.p TPS problem THM123

Trivial theorem to test TPS.

$\forall x_p: \$o: \exists x_f: \$o \rightarrow \$o: (xf@xp) \quad \text{thf(cTHM}_{123}\text{, conjecture})$

SYO184^5.p TPS problem CT9

$\exists x_x: \$o, x_y: \$o: xx \quad \text{thf(cCT}_9\text{, conjecture)}$

SYO185^5.p TPS problem CT17

y: \$o thf(y, type)

$\exists x_x: \$o: (y \Rightarrow y) \quad \text{thf(cCT}_{17}\text{, conjecture})$

SYO186^5.p TPS problem CT11

$\forall x_y: \$o: \exists x_{y_0}: \$o: xy_0 \quad \text{thf(cCT}_{11}\text{, conjecture})$

SYO187^5.p TPS problem CT10

$\exists x_x: \$o: \forall x_y: \$o: xx \quad \text{thf(cCT}_{10}\text{, conjecture})$

SYO188^5.p TPS problem CT19

$\exists x_q: (\$o \rightarrow \$o) \rightarrow \$o: \forall x_p: \$o \rightarrow \$o: (xq@xp) \quad \text{thf(cCT}_{19}\text{, conjecture})$

SYO189^5.p TPS problem CT5

$\exists x_x: \$o: \forall x_y: \$o: (xx \Rightarrow xy) \quad \text{thf(cCT}_5\text{, conjecture})$

SYO190^5.p TPS problem CT15

$\exists x_x: \$o, x_y: \$o: (xx \text{ or } xy) \quad \text{thf(cCT}_{15}\text{, conjecture})$

SYO191^5.p TPS problem CT14

$\exists x_z: \$o, x_x: \$o: xx = xx \quad \text{thf(cCT}_{14}\text{, conjecture})$

SYO192^5.p TPS problem CT12

$\exists x_x: \$o, x_y: \$o: xx = xy \quad \text{thf(cCT}_{12}\text{, conjecture})$

SYO193^5.p TPS problem CT20

$\neg \forall x_q: (\$o \rightarrow \$o) \rightarrow \$o: \neg \exists x_p: \$o \rightarrow \$o: (xq@xp) \quad \text{thf(cCT}_{20}\text{, conjecture})$

SYO194^5.p TPS problem CT23

$\forall x_x: \$i: \exists x_f: \$i \rightarrow \$i: (xf@xx) = xx \quad \text{thf(cCT}_{23}\text{, conjecture})$

SYO195^5.p TPS problem CT25

$\neg \forall x_x: \$o, x_y: \$o: xx = xy \quad \text{thf(cCT}_{25}\text{, conjecture})$

SYO196^5.p TPS problem CT21

$\exists x_p: \$o \rightarrow \$o: (xp@\forall x_y: \$o: (xy \text{ and } \neg xy)) \quad \text{thf(cCT}_{21}\text{, conjecture})$

SYO197^5.p TPS problem CT18

$\exists x_y: \$o: \forall x_z: \$o: \exists x_x: \$o: (xz \Rightarrow xx) \quad \text{thf(cCT}_{18}\text{, conjecture})$

SYO198^5.p TPS problem CT8

$\forall x_x: \$o, x_y: \$o: \exists x_p: \$o \rightarrow \$o: (xp@xx \text{ and } xp@xy) \quad \text{thf(cCT}_8\text{, conjecture})$

SYO199^5.p TPS problem CT7

$\exists x_p: \$o \rightarrow \$o: \forall x_x: \$o, x_y: \$o: (xp@xx \text{ and } xp@xy) \quad \text{thf(cCT}_7\text{, conjecture})$

SYO200^5.p TPS problem CT4

$\forall x_x: \$o, x_y: \$o: \exists x_p: \$o \rightarrow \$o: ((xp@xx) \Rightarrow (xp@xy)) \quad \text{thf(cCT}_4\text{, conjecture})$

SYO202^5.p TPS problem CT22
$$\neg \exists x p: \$o \rightarrow \$o: (xp@xp = xp \text{ and } \neg xp@xp = xp) \quad \text{thf(cCT}_{22}, \text{conjecture})$$
SYO203^5.p TPS problem PROP-2003-3-13
$$r: \$o \quad \text{thf}(r, \text{type})$$

$$p: \$o \quad \text{thf}(p, \text{type})$$

$$q: \$o \quad \text{thf}(q, \text{type})$$

$$(q \Rightarrow r) \Rightarrow ((\neg q \Rightarrow \neg p) \Rightarrow (p \Rightarrow r)) \quad \text{thf(cPROP_2003_3}_{13}, \text{conjecture})$$
SYO204^5.p TPS problem CT313
$$\exists x z: \$o, xx: \$o: \forall xy: \$o: (xx \Rightarrow xy) \quad \text{thf(cCT}_{313}, \text{conjecture})$$
SYO205^5.p TPS problem CT27
$$\exists xx: \$o, xy: \$o: xx = xy = xy = xx \quad \text{thf(cCT}_{27}, \text{conjecture})$$
SYO206^5.p TPS problem CT265
$$\exists xx: \$o: \forall xy: \$o: xx = xy = xy = xx \quad \text{thf(cCT}_{265}, \text{conjecture})$$
SYO207^5.p TPS problem CT26
$$\forall xx: \$o, xy: \$o: xx = xy = xy = xx \quad \text{thf(cCT}_{26}, \text{conjecture})$$
SYO208^5.p TPS problem CT31
$$a: \$tType \quad \text{thf(a_type, type)}$$

$$\neg \exists x g: a \rightarrow a \rightarrow \$o: \forall x f: a \rightarrow \$o: \exists x j: a: (xg@xj) = xf = xf = xf \quad \text{thf(cCT}_{31}, \text{conjecture})$$
SYO209^5.p TPS problem CT29
$$a: \$tType \quad \text{thf(a_type, type)}$$

$$\neg \exists x g: a \rightarrow a \rightarrow \$o: \forall x f: a \rightarrow \$o: \exists x j: a: \forall x p: (a \rightarrow \$o) \rightarrow \$o: ((xp@(xg@xj)) \Rightarrow (xp@xf)) \quad \text{thf(cCT}_{29}, \text{conjecture})$$
SYO210^5.p TPS problem from BASIC-HO-PROP-THMS
$$\exists xx: \$o: xx = \neg xx \quad \text{thf(cCT}_6, \text{conjecture})$$
SYO211^5.p TPS problem from BASIC-HO-PROP-THMS
$$x: \$i \quad \text{thf}(x, \text{type})$$

$$\exists xy: \$i: \forall x f: \$i \rightarrow \$i: (xf@x) = x \quad \text{thf(cCT}_{24}, \text{conjecture})$$
SYO212^5.p TPS problem from BASIC-HO-PROP-THMS
$$b: \$o \quad \text{thf}(b, \text{type})$$

$$a: \$o \quad \text{thf}(a, \text{type})$$

$$\forall x p: \$o \rightarrow \$o: ((xp@a \text{ and } xp@b) \Rightarrow (xp@(a \text{ and } b))) \quad \text{thf(cEMB}_2, \text{conjecture})$$
SYO213^5.p TPS problem from BASIC-HO-PROP-THMS
$$a: \$tType \quad \text{thf(a_type, type)}$$

$$\neg \exists x g: a \rightarrow a \rightarrow \$o: \forall x f: a \rightarrow \$o: \exists x j: a: \forall x p: \$o \rightarrow \$o: ((xp@(xg@xj)) = xf) \Rightarrow (xp@xf = xf) \quad \text{thf(cCT}_{30}, \text{conjecture})$$
SYO214^5.p TPS problem THM12
$$\forall r: \$i \rightarrow \$o, s: \$i \rightarrow \$o: (r = s \Rightarrow \forall x: \$i: ((s@x) \Rightarrow (r@x))) \quad \text{thf(cTHM}_{12}, \text{conjecture})$$
SYO215^5.p TPS problem THM26
$$\exists x: \$i, y: \$i: x \neq y \Rightarrow \forall u: \$i: \exists z: \$i: z \neq u \quad \text{thf(cTHM}_{26}, \text{conjecture})$$
SYO216^5.p TPS problem THM107
$$b: \$tType \quad \text{thf}(b_type, type)$$

$$\forall x: b \rightarrow b, y: b \rightarrow b, z: b \rightarrow b: (\lambda w: b: (x@(y@(z@w)))) = (\lambda w: b: (x@(y@(z@w)))) \quad \text{thf(cTHM107_pme, conjecture})$$
SYO217^5.p TPS problem THM174

Principle of extensionality for binary relations.

$$b: \$tType \quad \text{thf}(b_type, type)$$

$$a: \$tType \quad \text{thf}(a_type, type)$$

$$\forall x r: b \rightarrow a \rightarrow \$o, xs: b \rightarrow a \rightarrow \$o: (\forall xx: b, xy: a: ((xr@xx@xy) \iff (xs@xx@xy)) \Rightarrow xr = xs) \quad \text{thf(cTHM}_{174}, \text{conjecture})$$
SYO218^5.p TPS problem THM7B

Half-proved version of thm7 for test purposes.

$$\forall xx: \$i, xy: \$i: (\forall x q: (\$i \rightarrow \$o) \rightarrow \$i: (xx = (xq@\lambda x z: \$i: xx = xz) \Rightarrow xy = (xq@\lambda x z: \$i: xy = xz)) \Rightarrow xx = xy) \quad \text{thf(cTHM7B_pme, conjecture})$$
SYO219^5.p TPS problem THM6
$$cS: \$i \rightarrow \$i \quad \text{thf}(cS, \text{type})$$

$$\forall m: \$i: (cS@m) \neq m \Rightarrow \neg \exists g: \$i \rightarrow \$i \rightarrow \$i: \forall f: \$i \rightarrow \$i: \exists j: \$i: (g@j) = f \quad \text{thf(cTHM}_6, \text{conjecture})$$
SYO220^5.p TPS problem THM47A

$\forall x: \$i, y: \$i: (x = y \Rightarrow \forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall z: \$i: (r@z@z) \Rightarrow (r@x@y))) \quad \text{thf(cTHM47A, conjecture)}$

SYO221^5.p TPS problem BLEDSOE6

$b: \$i \quad \text{thf}(b, \text{type})$

$cP: \$i \rightarrow \$o \quad \text{thf}(cP, \text{type})$

$a: \$i \quad \text{thf}(a, \text{type})$

$\exists a: \$i \rightarrow \$o: ((cP@a \text{ and } a \neq b) \Rightarrow (\forall xx: \$i: ((a@xx) \Rightarrow (cP@xx)) \text{ and } \exists xy: \$i: (a@xy) \text{ and } \neg a@b)) \quad \text{thf(cBLEDSOE}_6, \text{conjecture})$

SYO222^5.p TPS problem THM115A

Sunil's example.

$f: \$i \rightarrow \$i \quad \text{thf}(f, \text{type})$

$a: \$i \quad \text{thf}(a, \text{type})$

$cP: \$i \rightarrow \$o \quad \text{thf}(cP, \text{type})$

$\exists a: \$i \rightarrow \$o: (\forall xx: \$i: ((a@(f@xx)) \Rightarrow (cP@xx)) \text{ and } ((cP@a \text{ and } \forall xx: \$i, xy: \$i: ((f@xx) = (f@xy) \Rightarrow xx = xy)) \Rightarrow \exists xz: \$i: (a@xz))) \quad \text{thf(cTHM115A, conjecture)}$

SYO223^5.p TPS problem LING2

$cJ: \$i \quad \text{thf}(cJ, \text{type})$

$c\text{LIKE}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(c\text{LIKE}, \text{type})$

$c\text{UNIQUE}: \$i \rightarrow \$o \quad \text{thf}(c\text{UNIQUE}, \text{type})$

$cS: \$i \quad \text{thf}(cS, \text{type})$

$cP: \$i \quad \text{thf}(cP, \text{type})$

$(\forall x: \$i: ((c\text{UNIQUE}@x) \Rightarrow \forall z: \$i: x = z) \text{ and } c\text{UNIQUE}@cS) \Rightarrow \exists xan: \$i \rightarrow \$o: ((xan@cP) = (c\text{LIKE}@cP@cS) \text{ and } (xan@cJ) = (c\text{UNIQUE}@x \text{ and } c\text{LIKE}@cJ@x))) \quad \text{thf(cLING}_2, \text{conjecture})$

SYO224^5.p TPS problem LING1

$cJ: \$i \quad \text{thf}(cJ, \text{type})$

$c\text{LIKE}: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(c\text{LIKE}, \text{type})$

$c\text{WRH}: \$i \rightarrow \$o \quad \text{thf}(c\text{WRH}, \text{type})$

$cW: \$i \rightarrow \$o \quad \text{thf}(cW, \text{type})$

$c\text{UNIQUE}: \$i \rightarrow \$o \quad \text{thf}(c\text{UNIQUE}, \text{type})$

$cS: \$i \quad \text{thf}(cS, \text{type})$

$cP: \$i \quad \text{thf}(cP, \text{type})$

$(\forall x: \$i: ((c\text{UNIQUE}@x) \Rightarrow \forall z: \$i: ((c\text{WRH}@z \text{ and } cW@z) \Rightarrow x = z)) \text{ and } c\text{UNIQUE}@cS \text{ and } cW@cS \text{ and } c\text{WRH}@cS) \Rightarrow \exists xan: \$i \rightarrow \$o: ((xan@cP) = (c\text{LIKE}@cP@cS) \text{ and } (xan@cJ) = (c\text{UNIQUE}@x \text{ and } cW@x \text{ and } c\text{WRH}@x \text{ and } c\text{LIKE}@cJ@x)))$

SYO225^5.p TPS problem THM126-CORRECTED

$g: \$t\text{Type} \quad \text{thf}(g_type, \text{type})$

$b: \$t\text{Type} \quad \text{thf}(b_type, \text{type})$

$a: \$t\text{Type} \quad \text{thf}(a_type, \text{type})$

$\forall xh_1: g \rightarrow b, xh_2: b \rightarrow a, xs_1: g \rightarrow \$o, xf_1: g \rightarrow g \rightarrow g, xs_2: b \rightarrow \$o, xf_2: b \rightarrow b \rightarrow b, xh_{10}: g \rightarrow b, xh_{20}: b \rightarrow a, xs_{10}: g \rightarrow \$o, xf_{10}: g \rightarrow g \rightarrow g, xs_{20}: b \rightarrow \$o, xf_{20}: b \rightarrow b \rightarrow b, xs_3: a \rightarrow \$o, xf_3: a \rightarrow a \rightarrow a: ((\forall xx: g, xy: g: ((xs_{10}@xx \text{ and } xs_{10}@xy) \Rightarrow (xs_{10}@xf_{10}@xx@xy))) \text{ and } \forall xx: g: ((xs_{10}@xx) \Rightarrow (xs_{20}@xh_{10}@xx))) \text{ and } \forall xx: g: ((xs_{10}@xx) \Rightarrow (xs_{20}@xh_{10}@xx)) \text{ and } \forall xx: g, xy: g: ((xs_{10}@xx \text{ and } xs_{10}@xy) \Rightarrow (xh_{10}@xf_{10}@xx@xy)) = (xf_{20}@xh_{10}@xx@(xh_{10}@xy))) \text{ and } \forall xx: g, xy: g: ((xs_{10}@xx \text{ and } xs_{10}@xy) \Rightarrow (xf_3@xx@xy)) \text{ and } \forall xx: g, xy: g: ((xs_{20}@xx) \Rightarrow (xs_3@xx@xy)) \text{ and } \forall xx: g, xy: g: ((xs_{20}@xx) \Rightarrow (xh_{20}@xf_{20}@xx@xy)) = (xf_3@xh_{20}@xx@(xh_{20}@xy))) \Rightarrow (\forall xx: g, xy: g: ((xs_{10}@xx) \Rightarrow (xs_{10}@xf_{10}@xx@xy)) \Rightarrow (xs_{10}@xf_{10}@xx@xy)) \text{ and } \forall xx: g, xy: g: ((xs_{10}@xx) \Rightarrow (xs_3@xx@xy)) \text{ and } \forall xx: g, xy: g: ((xs_{10}@xx) \Rightarrow (xh_{20}@xf_{20}@xx@xy)) = (xf_3@xh_{20}@xx@(xh_{20}@xy)))) \quad \text{thf(cTHM126_CORRECTED_pme, conjecture)}$

SYO226^5.p TPS problem THM47B

$\forall x: \$i, y: \$i: (\forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall z: \$i: (r@z@z) \Rightarrow (r@x@y)) \Rightarrow x = y) \quad \text{thf(cTHM47B, conjecture)}$

SYO227^5.p TPS problem BLEDSOE4-W-AX

$c: \$i \quad \text{thf}(c, \text{type})$

$b: \$i \quad \text{thf}(b, \text{type})$

$a: \$i \quad \text{thf}(a, \text{type})$

$c_less: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(c_less, \text{type})$

$\forall xx: \$i, xy: \$i: ((c_less@xx@xy) \Rightarrow xx \neq xy) \Rightarrow ((c_less@a@b \text{ and } c_less@b@c) \Rightarrow \exists a: \$i \rightarrow \$o: (\neg a@a \text{ and } a@b \text{ and } \neg a@c))$

SYO228^5.p TPS problem THM126-EXPANDED

$g: \$t\text{Type} \quad \text{thf}(g_type, \text{type})$

$b: \$t\text{Type} \quad \text{thf}(b_type, \text{type})$

$a: \$t\text{Type} \quad \text{thf}(a_type, \text{type})$

$\forall xh_1: g \rightarrow b, xh_2: b \rightarrow a, xs_1: g \rightarrow \$o, xf_1: g \rightarrow g \rightarrow g, xs_2: b \rightarrow \$o, xf_2: b \rightarrow b \rightarrow b, xs_3: a \rightarrow \$o, xf_3: a \rightarrow a \rightarrow a: ((\forall xx: g, xy: g: ((xs_1@xx \text{ and } xs_1@xy) \Rightarrow (xs_1@(xf_1@xx@xy)))) \text{ and } \forall xx: b, xy: b: ((xs_2@xx \text{ and } xs_2@xy) \Rightarrow (xs_2@(xf_2@xx@xy))) \text{ and } \forall xx: g: ((xs_1@xx) \Rightarrow (xs_2@(xh_1@xx))) \text{ and } \forall xx: g, xy: g: ((xs_1@xx \text{ and } xs_1@xy) \Rightarrow (xh_1@(xf_1@xx@xy)) = (xf_2@(xh_1@xx)@(xh_1@xy))) \text{ and } \forall xx: b, xy: b: ((xs_2@xx \text{ and } xs_2@xy) \Rightarrow (xs_2@(xf_2@xx@xy))) \text{ and } \forall xx: xs_3: a \rightarrow \$o, xf_3: a \rightarrow a \rightarrow a: ((xs_3@xx@xy) = (xf_3@xx@xy)) \text{ and } \forall xx: b: ((xs_2@xx) \Rightarrow (xs_3@(xh_2@xx))) \text{ and } \forall xx: b, xy: b: ((xs_2@xx \text{ and } xs_2@xy) \Rightarrow (xh_2@(xf_2@xx@xy)) = (xf_3@(xh_2@xx)@(xh_2@xy)))) \Rightarrow (\forall xx: g, xy: g: ((xs_1@xx \text{ and } xs_1@xy) \Rightarrow (xs_1@(xf_1@xx@xy))) \text{ and } (xs_3@xx@xy) = (xf_3@xx@xy)) \text{ and } \forall xx: g: ((xs_1@xx) \Rightarrow (xs_3@(xh_2@xx))) \text{ and } \forall xx: g, xy: g: ((xs_1@xx \text{ and } xs_1@xy) \text{ and } xs_1@xy \text{ and } xh_2@xx) = (xf_3@(xh_2@xx)@(xh_1@xy)))) \quad \text{thf(cTHM126_EXPANDED_pme, conjecture)}$

SYO229^5.p TPS problem from BASIC-HO-EQ-THMS

cB: \$i → \$o thf(cB, type)

cA: \$i → \$o thf(cA, type)

$\exists xu: \$i \rightarrow \$o: xu = (\lambda xx: \$i: (cA@xx \text{ and } cB@xx)) \quad \text{thf(cSV}_3\text{, conjecture)}$

SYO230^5.p TPS problem from BASIC-HO-EQ-THMS

cB: \$i → \$o thf(cB, type)

cA: \$i → \$o thf(cA, type)

$\exists xu: \$i \rightarrow \$o: xu = (\lambda xx: \$i: (cA@xx \text{ or } cB@xx)) \quad \text{thf(cSV}_2\text{, conjecture)}$

SYO231^5.p TPS problem from BASIC-HO-EQ-THMS

cB: \$i → \$o thf(cB, type)

cA: \$i → \$o thf(cA, type)

$\exists xu: \$i \rightarrow \$o: xu = (\lambda xx: \$i: (\neg cA@xx \text{ and } cB@xx)) \quad \text{thf(cSV}_6\text{, conjecture)}$

SYO232^5.p TPS problem from BASIC-HO-EQ-THMS

a: \$tType thf(a_type, type)

cP: a → \$o thf(cP, type)

x: a thf(x, type)

cB: \$o thf(cB, type)

y: \$i thf(y, type)

$(y = y \Rightarrow cB) \Rightarrow ((cP@x) \Rightarrow \exists xx_0: a: (cP@xx_0)) \quad \text{thf(cADDHYP}_4\text{, conjecture)}$

SYO233^5.p TPS problem from BASIC-HO-EQ-THMS

cN: \$i → \$i thf(cN, type)

cM: \$i → \$i thf(cM, type)

$\forall xx: \$i, xp: \$i \rightarrow \$o: ((xp@(cM@xx)) \Rightarrow (xp@(cN@xx))) \Rightarrow cM = cN \quad \text{thf(cLEIBNIZ, conjecture)}$

SYO234^5.p TPS problem from BASIC-HO-EQ-THMS

d: \$tType thf(d_type, type)

c: \$tType thf(c_type, type)

cB: d → c → \$o thf(cB, type)

cA: c → d → \$o thf(cA, type)

$\exists xu: c \rightarrow d \rightarrow \$o: xu = (\lambda xx: c, xy: d: (cA@xx@xy \text{ or } cB@xy@xx)) \quad \text{thf(cSV}_4\text{, conjecture)}$

SYO235^5.p TPS problem from BASIC-HO-EQ-THMS

cB: \$o thf(cB, type)

cA: \$o thf(cA, type)

$(\forall q: \$o \rightarrow \$o: ((q@cA) \Rightarrow (q@cB)) \text{ or } (cA \Leftrightarrow cB)) \Rightarrow cA = cB \quad \text{thf(cTHM}_{505}\text{, conjecture)}$

SYO236^5.p TPS problem from BASIC-HO-EQ-THMS

b: \$tType thf(b_type, type)

a: \$tType thf(a_type, type)

g: b → a thf(g, type)

f: b → a thf(f, type)

$(\forall q: (b \rightarrow a) \rightarrow \$o: ((q@f) \Rightarrow (q@g)) \text{ or } \forall xx: b: (f@xx) = (g@xx)) \Rightarrow f = g \quad \text{thf(cTHM}_{504}\text{, conjecture)}$

SYO237^5.p TPS problem from BASIC-HO-EQ-THMS

g: \$i → \$i thf(g, type)

p: (\$i → \$i) → \$o thf(p, type)

x: \$i thf(x, type)

q: \$i → \$o thf(q, type)

f: \$i → \$i thf(f, type)

$\forall xx_0: \$i: (f@xx_0) = (g@xx_0) \Rightarrow ((p@\lambda xx_0: \$i: (f@xx_0)) \Rightarrow ((q@x) \Rightarrow (p@\lambda xx_0: \$i: (g@xx_0)))) \quad \text{thf(cTHM}_{508}\text{, conjecture)}$

SYO238^5.p TPS problem from BASIC-HO-EQ-THMS

cB: \$o thf(cB, type)
 x: \$i thf(x, type)
 cP: \$i → \$o thf(cP, type)
 f: \$i → \$i thf(f, type)
 $(\text{true} \Rightarrow \forall xx_0: \$i: (f@xx_0) = xx_0) \Rightarrow (((\lambda xx_0: \$i: (f@(f@xx_0))) = (\lambda xx_0: \$i: xx_0)) \Rightarrow cB) \Rightarrow ((cP@x) \Rightarrow cB))$ thf(cADDHYP₅, conjecture)

SYO239^5.p TPS problem from BASIC-HO-EQ-THMS

s: \$i → \$i thf(s, type)
 c₂: \$i thf(c₂, type)
 c_star: \$i → \$i → \$i thf(c_star, type)
 $\forall xx: \$i: (\exists xu: \$i: xx = (c_star@c_2@xu) \iff \neg \exists xv: \$i: (s@xx) = (c_star@c_2@xv)) \Rightarrow \exists a: \$i \rightarrow \$o: \forall xx: \$i: ((a@xx) \iff \neg a@(s@xx))$ thf(cBLEDSOE_FENG_SV_EO1_W_LEM, conjecture)

SYO240^5.p TPS problem from BASIC-HO-EQ-THMS

a: \$tType thf(a_type, type)
 b: \$tType thf(b_type, type)
 cK: (a → b → \$o) → a → b → \$o thf(cK, type)
 $\forall xu: a \rightarrow b \rightarrow \$o, xv: a \rightarrow b \rightarrow \$o: (\forall xx: a, xy: b: ((xu@xx@xy) \Rightarrow (xv@xx@xy)) \Rightarrow \forall xx: a, xy: b: ((cK@xu@xx@xy) \Rightarrow (cK@xv@xx@xy))) \Rightarrow \exists xu: a \rightarrow b \rightarrow \$o: (cK@xu) = xu$ thf(cTHM2B, conjecture)

SYO241^5.p TPS problem from BASIC-HO-EQ-THMS

$\forall xh: (\$i \rightarrow \$o) \rightarrow \$i: (\forall xp: \$i \rightarrow \$o, xq: \$i \rightarrow \$o: ((xh@xp) = (xh@xq) \Rightarrow xp = xq) \Rightarrow \neg \exists xt: \$i \rightarrow \$o: (\neg xt@(xh@xt) \text{ and } (xh@xz: \$i: \exists xt_0: \$i \rightarrow \$o: (\neg xt_0@(xh@xt_0) \text{ and } xz = (xh@xt_0))) = (xh@xt)))$ thf(cTHM143_EX)

SYO242^5.p TPS problem from BASIC-HO-EQ-THMS

a: \$tType thf(a_type, type)
 $\exists xf: (a \rightarrow \$o) \rightarrow a: \forall x: a \rightarrow \$o: (\exists xt: a: (x@xt) \Rightarrow (x@(xf@x))) \Rightarrow \exists r: a \rightarrow a \rightarrow \$o: (\forall xx: a, xy: a: ((r@xx@xy) \text{ and } r@xy@xx) = xy) \text{ and } \forall s: a \rightarrow \$o: (\exists xu: a: (s@xu) \Rightarrow \exists xv: a: (s@xv) \text{ and } \forall xz: a: (r@xv@xz)))$ thf(cTHM₅₃₉, conjecture)

SYO243^5.p TPS problem from BASIC-HO-EQ-THMS

f: \$i → \$i thf(f, type)
 g: \$i → \$i thf(g, type)
 $\forall xr: \$i \rightarrow \$i \rightarrow \$o: (\forall xx: \$i: \exists xy: \$i: (xr@xx@xy) \Rightarrow \exists xh: \$i \rightarrow \$i: \forall xx: \$i: (xr@xx@(xh@xx)) \Rightarrow (\forall xx: \$i, xy: \$i: ((g@xx) = (g@xy)) \Rightarrow (f@xx) = (f@xy)) \Rightarrow \exists xh: \$i \rightarrow \$i: \forall xx: \$i, xy: \$i: ((g@xx) = xy \Rightarrow (xh@xy) = (f@xx)))$ thf(cTHM588LEM)

SYO244^5.p TPS problem from BASIC-HO-EQ-THMS

a: \$tType thf(a_type, type)
 $\exists xf: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow a \rightarrow \$o: \forall x: (a \rightarrow \$o) \rightarrow \$o: (\exists xt: a \rightarrow \$o: (x@xt) \Rightarrow (x@(xf@x))) \Rightarrow \forall a: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: (\exists x: (a \rightarrow \$o) \rightarrow \$o: (a@x) \text{ and } \forall x: (a \rightarrow \$o) \rightarrow \$o: ((a@x) \Rightarrow \exists xu: a \rightarrow \$o: (x@xu))) \Rightarrow (\lambda xx: a: \forall xa: (a \rightarrow \$o) \rightarrow \$o: ((a@xa) \Rightarrow \exists xb: a \rightarrow \$o: (xa@xb) \text{ and } xb@xx))) = (\lambda xx: a: \exists xf: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow a \rightarrow \$o: \forall xa: (a \rightarrow \$o) \rightarrow \$o: ((a@xa) \Rightarrow (xa@(xf@xa) \text{ and } xf@xa@xx))))$ thf(cTHM535B, conjecture)

SYO245^5.p TPS problem from BASIC-HO-EQ-THMS

$\forall xh: (\$i \rightarrow \$o) \rightarrow \$i: (\exists xt: \$i \rightarrow \$o: (\neg xt@(xh@xt) \text{ and } (xh@xz: \$i: \exists xt_0: \$i \rightarrow \$o: (\neg xt_0@(xh@xt_0) \text{ and } xz = (xh@xt_0))) = (xh@xt)) \Rightarrow \exists xt: \$i \rightarrow \$o: (\neg xt@(xh@xt) \text{ and } (xh@xz: \$i: \exists xt_0: \$i \rightarrow \$o: (\neg xt_0@(xh@xt_0) \text{ and } xz = (xh@xt_0))) = (xh@xt))) \Rightarrow \neg \exists xh: (\$i \rightarrow \$o) \rightarrow \$i: \forall xx: \$i \rightarrow \$o, xy: \$i \rightarrow \$o: ((xh@xx) = (xh@xy) \Rightarrow xx = xy)$ thf(cTHM193A, conjecture)

SYO246^5.p TPS problem from BASIC-HO-EQ-THMS

a: \$tType thf(a_type, type)
 $\forall xs: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: (\forall x: (a \rightarrow \$o) \rightarrow \$o: ((xs@x) \Rightarrow \exists xt: a \rightarrow \$o: (x@xt)) \Rightarrow \exists xf: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow a \rightarrow \$o: \forall x: (a \rightarrow \$o) \rightarrow \$o: ((xs@x) \Rightarrow (x@(xf@x)))) \Rightarrow \forall a: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: (\exists x: (a \rightarrow \$o) \rightarrow \$o: (a@x) \text{ and } \forall x: (a \rightarrow \$o) \rightarrow \$o: ((a@x) \Rightarrow \exists xu: a \rightarrow \$o: (x@xu))) \Rightarrow (\lambda xx: a: \forall xa: (a \rightarrow \$o) \rightarrow \$o: ((a@xa) \Rightarrow \exists xb: a \rightarrow \$o: (xa@xb) \text{ and } xb@xx))) = (\lambda xx: a: \exists xf: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow a \rightarrow \$o: \forall xa: (a \rightarrow \$o) \rightarrow \$o: ((a@xa) \Rightarrow (xa@(xf@xa) \text{ and } xf@xa@xx))))$ thf(cTHM₅₃₅, conjecture)

SYO247^5.p TPS problem from BASIC-HO-EQ-THMS

cP₆: \$i thf(cP₆, type)
 cP₅: \$i thf(cP₅, type)
 cP₄: \$i thf(cP₄, type)
 cP₃: \$i thf(cP₃, type)
 cP₂: \$i thf(cP₂, type)
 cP₁: \$i thf(cP₁, type)

Test theorem for nested primitive substitutions.

a: \$tType thf(a_type, type)
 $\forall x: \$o \rightarrow \$o, xp: a \rightarrow \$o, xq: a \rightarrow \$o: \exists xh: \$o: ((xr@xh) \iff (xr@\forall xx: a: (xp@xx \text{ or } xq@xx)))$

thf(cTHM₁₂₁, conjecture)

SYO257^5.p TPS problem THM84

h: \$i → \$i thf(h, type)
g: \$i → \$i thf(g, type)
cP: \$i → \$i → \$o thf(cP, type)
 $\exists xx: \$i: \forall xy: \$i: (\forall xf: \$i \rightarrow \$i, xz: \$i: (cP@xz@(xf@xx) \text{ and } cP@xx@xy) \Rightarrow (cP@xy@g(h@xy)))$

thf(cTHM₈₄, conjecture)

SYO258^5.p TPS problem BLEDSOE-FENG-6

a: \$i thf(a, type)
cQ: \$i → (\$i → \$o) → \$o thf(cQ, type)
cP: \$i → \$i → \$o thf(cP, type)
b: \$i thf(b, type)
 $(cP@a@b \text{ and } \forall e: \$i \rightarrow \$o: (cQ@b@e)) \Rightarrow \exists a: \$i \rightarrow \$o: \forall xg: \$i: ((a@xg) \Rightarrow (\exists xx: \$i: (cP@xg@xx \text{ and } cQ@xx@a) \text{ and } a@a))$

SYO259^5.p TPS problem THM125B

Trivial theorem to test flexible-flexible pairs.

$\forall xa: \$i, xb: \$i, xc: \$i, p: \$i \rightarrow \$o: \exists xm: \$i \rightarrow \$o, xn: \$i \rightarrow \$o: ((xn@xa \text{ or } xm@xa) \text{ and } (p@xb \text{ or } xn@xb) \text{ and } (xm@xc \text{ or } \neg p@xc))$

SYO260^5.p TPS problem THM125A

Trivial theorem to test flexible-flexible pairs.

$\forall xa: \$i, xb: \$i, xc: \$i, p: \$i \rightarrow \$o: \exists xm: \$i \rightarrow \$o, xn: \$i \rightarrow \$o: ((xm@xa \text{ or } xn@xa) \text{ and } (p@xb \text{ or } xn@xb) \text{ and } (xm@xc \text{ or } \neg p@xc))$

SYO261^5.p TPS problem BLEDSOE-FENG-SV-I1

n: \$i thf(n, type)
cP: \$i → \$o thf(cP, type)
c1_plus: \$i → \$i thf(c1_plus, type)
cO: \$i thf(cO, type)
 $(\forall a: \$i \rightarrow \$o: ((a@cO \text{ and } \forall xx: \$i: ((a@xx) \Rightarrow (a@(c1_plus@xx)))) \Rightarrow (a@n)) \text{ and } cP@cO \text{ and } \forall xx: \$i: ((cP@xx) \Rightarrow (cP@(c1_plus@xx))) \Rightarrow (cP@n) \text{ thf(cBLEDSOE_FENG_SV_I1, conjecture)}$

SYO262^5.p TPS problem THM19SK1

cP: \$i → \$i → \$o thf(cP, type)
cE: (\$i → \$o) → \$i thf(cE, type)
 $\neg \forall xx: \$i: (cP@xx@(cE@\lambda xy: \$i: (cP@xx@xy))) \text{ and } \forall xf: \$i \rightarrow \$i: \neg cP@(\lambda xx: \$i: \neg cP@xx@(xf@xx)) @ (\lambda xf: \$i: (cE@\lambda xx: \$i: \neg cP@xx@(xf@xx)))$

SYO263^5.p TPS problem THM125D

Trivial theorem to test flexible-flexible pairs.

$\forall xa: \$i, xb: \$i, xc: \$i, p: \$i \rightarrow \$o: \exists xm: \$i \rightarrow \$o, xn: \$i \rightarrow \$o: ((xn@xa \text{ or } xm@xa) \text{ and } (p@xb \text{ or } xn@xb) \text{ and } (xm@xc \text{ or } \neg p@xc)) \text{ thf(cTHM125D, conjecture)}$

SYO264^5.p TPS problem THM125C

Trivial theorem to test flexible-flexible pairs.

$\forall xa: \$i, xb: \$i, xc: \$i, p: \$i \rightarrow \$o: \exists xm: \$i \rightarrow \$o, xn: \$i \rightarrow \$o: ((xm@xa \text{ or } xn@xa) \text{ and } (p@xb \text{ or } xn@xb) \text{ and } (xm@xc \text{ or } \neg p@xc)) \text{ thf(cTHM125C, conjecture)}$

SYO265^5.p TPS problem X5210

a: \$tType thf(a_type, type)
x: \$i thf(x, type)
 $(\lambda xx: a, xy: a: xx = xy@x) = (\lambda xz: a: \exists xy: a: (xy = x \text{ and } xz = xy)) \text{ thf(cX}_{5210}, \text{conjecture})$

SYO266^5.p TPS problem THM44

cQ: \$i → \$o thf(cQ, type)
cP: \$i → \$o thf(cP, type)
 $\exists s: \$i \rightarrow \$o: \forall x: \$i: ((s@x \text{ or } cP@x) \text{ and } (\neg s@x \text{ or } cQ@x)) \iff \forall y: \$i: (cP@y \text{ or } cQ@y) \text{ thf(cTHM}_{44}, \text{conjecture})$

SYO267^5.p TPS problem THM111

$\forall p: \$i \rightarrow \$o: \exists m: (\$i \rightarrow \$i) \rightarrow \$o: \forall g: \$i \rightarrow \$i, h: \$i \rightarrow \$i: ((m@g \text{ and } m@h) \Rightarrow (m@\lambda z: \$i: (g@(h@z))) \text{ and } \forall y: \$i: ((p@y) \Rightarrow (p@(g@y)))) \text{ thf(cTHM}_{111}, \text{conjecture})$

SYO268^5.p TPS problem X5308

a: \$tType thf(a_type, type)
b: \$tType thf(b_type, type)
r: \$i → \$i → \$o thf(r, type)

$\exists xj: (b \rightarrow \$o) \rightarrow b: \forall xp: b \rightarrow \$o: (\exists xx: b: (xp@xx) \Rightarrow (xp@(xj@xp))) \Rightarrow (\forall xx: a: \exists xy: b: (r@xx@xy) \iff \exists xf: a \rightarrow b: \forall xx: a: (r@xx@(xf@xx)))$ thf(cX₅₃₀₈, conjecture)

SYO269^5.p TPS problem THM112D

$\forall p: \$i \rightarrow \$o: \exists xm_9: (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$o, xm_{10}: (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$o: (\forall xw_1: \$i: (xm_9@\lambda xx: \$i: xx@xw_1 \text{ or } xm_{10}@g@xx: \$i: xx@g@xx))$
 $\$i, h: \$i \rightarrow \$i: ((\forall xw_1: \$i: (xm_9@g@xw_1 \text{ or } xm_{10}@g@xw_1) \text{ and } \forall xw_1: \$i: (xm_9@g@xw_1 \text{ or } xm_{10}@g@xw_1) \text{ and } \forall xw_1: \$i: (xm_9@g@xx: \$i: (g@(h@xx))@xw_1 \text{ or } xm_{10}@g@xx: \$i: (g@(h@xx))@xw_1) \text{ and } \forall y: \$i: ((p@y) \Rightarrow (p@(g@y))))))$ thf(cX₅₃₀₈, conjecture)

SYO270^5.p TPS problem THM85

$h: \$i \rightarrow \i thf(h, type)

$g: \$i \rightarrow \i thf(g, type)

$cP: \$i \rightarrow \$i \rightarrow \$o$ thf(cP, type)

$\exists xx: \$i, xf: \$i \rightarrow \$i: \forall xy: \$i: ((\forall xz: \$i: (cP@xz@(xf@xx)) \text{ and } cP@xx@xy) \Rightarrow (cP@xy@(g@(h@xy))))$ thf(cTHM₈₅, conjecture)

SYO271^5.p TPS problem X5500

$b: \$tType$ thf(b_type, type)

$a: \$tType$ thf(a_type, type)

$cJ: (b \rightarrow \$o) \rightarrow b$ thf(cJ, type)

$\forall p: b \rightarrow \$o: (\exists xx: b: (p@xx) \Rightarrow (p@(cJ@p))) \Rightarrow \forall xf: b \rightarrow a, xg: b \rightarrow a: ((xf@(\lambda xx: b: (xf@xx) \neq (xg@xx))) = (xg@(\lambda xx: b: (xf@xx) \neq (xg@xx)))) \Rightarrow xf = xg$ thf(cX₅₅₀₀, conjecture)

SYO272^5.p TPS problem THM301A

$cHALF: \$i \rightarrow \$i \rightarrow \$o$ thf(cHALF, type)

$cDOUBLE: \$i \rightarrow \$i \rightarrow \$o$ thf(cDOUBLE, type)

$cS: \$i \rightarrow \i thf(cS, type)

$c_0: \$i$ thf(c₀, type)

$(\forall xu: \$i, xv: \$i: ((cDOUBLE@xu@xv) \iff \forall q: \$i \rightarrow \$i \rightarrow \$o: ((q@c_0@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((q@xx@xy) \Rightarrow (q@(cS@xx)@(cS@(cS@xy)))) \Rightarrow (q@xu@xv))) \text{ and } cHALF@c_0@c_0 \text{ and } cHALF@(cS@c_0)@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((cHALF@c_0@c_0 \text{ and } cHALF@(cS@xx)@(cS@xy)))) \Rightarrow \forall xu: \$i, xv: \$i: ((cDOUBLE@xu@xv) \Rightarrow (cHALF@xv@xu)))$ thf(cTHM_{301A}, conjecture)

SYO274^5.p TPS problem THM48-EXPD

$c: \$tType$ thf(c_type, type)

$b: \$tType$ thf(b_type, type)

$a: \$tType$ thf(a_type, type)

$cG: c \rightarrow b$ thf(cG, type)

$cF: b \rightarrow a$ thf(cF, type)

$(\forall xx: b, xy: b: (\forall xq: a \rightarrow \$o: ((xq@(cF@xx)) \Rightarrow (xq@(cF@xy))) \Rightarrow \forall xq: b \rightarrow \$o: ((xq@xx) \Rightarrow (xq@xy))) \text{ and } \forall xx: c, xy: c: (\forall xq: a \rightarrow \$o: ((xq@(cG@xx)) \Rightarrow (xq@(cG@xy))) \Rightarrow \forall xq: c \rightarrow \$o: ((xq@xx) \Rightarrow (xq@xy))) \Rightarrow \forall xx: c, xy: c: (\forall xq: a \rightarrow \$o: ((xq@(cF@(cG@xx))) \Rightarrow (xq@(cF@(cG@xy)))) \Rightarrow \forall xq: c \rightarrow \$o: ((xq@xx) \Rightarrow (xq@xy)))$ thf(cTHM_{48_EXPD}, conjecture)

SYO275^5.p TPS problem THM300A

$cS: \$i \rightarrow \i thf(cS, type)

$cDOUBLE: \$i \rightarrow \$i \rightarrow \$o$ thf(cDOUBLE, type)

$cHALF: \$i \rightarrow \$i \rightarrow \$o$ thf(cHALF, type)

$c_0: \$i$ thf(c₀, type)

$(\forall xu: \$i, xv: \$i: ((cHALF@xu@xv) \iff \forall q: \$i \rightarrow \$i \rightarrow \$o: ((q@c_0@c_0 \text{ and } q@(cS@c_0)@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((q@xx@xy) \Rightarrow (q@(cS@xx)@(cS@xy)))) \Rightarrow (q@xu@xv))) \text{ and } cDOUBLE@c_0@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((cDOUBLE@xx@xy) \Rightarrow (cDOUBLE@(cS@xx)@(cS@xy)))) \Rightarrow \forall xu: \$i, xv: \$i: ((cHALF@xu@xv) \Rightarrow (cDOUBLE@xv@xu \text{ or } cDOUBLE@(cS@xx)@(cS@xy))))$ thf(cTHM_{300A}, conjecture)

SYO276^5.p TPS problem BLEDSOE-FENG-SV-I2

$m: \$i$ thf(m, type)

$cP: \$i \rightarrow \o thf(cP, type)

$n: \$i$ thf(n, type)

$s: \$i \rightarrow \i thf(s, type)

$cO: \$i$ thf(cO, type)

$(\forall a: \$i \rightarrow \$i \rightarrow \$o: ((a@cO@cO \text{ and } \forall xx: \$i, xy: \$i: ((a@xx@xy) \Rightarrow (a@(s@xx)@(s@xy)))) \Rightarrow (a@n@m)) \text{ and } cP@n) \Rightarrow (cP@m)$ thf(cBLEDSOE_FENG_SV_I₂, conjecture)

SYO277^5.p TPS problem THM47D

$\forall x: \$i, y: \$i: (\forall xq: \$i \rightarrow \$o: ((xq@x) \Rightarrow (xq@y)) \iff \forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall z: \$i: (r@z@z) \Rightarrow (r@x@y)))$ thf(cTHM_{47D}, conjecture)

SYO278^5.p TPS problem from BASIC-HO-THMS

$a: \$tType$ thf(a_type, type)

$q: \$o$ thf(q, type)

$r: ((a \rightarrow \$o) \rightarrow \$o) \rightarrow \$o$ thf(r, type)

$\forall xp: (a \rightarrow \$o) \rightarrow \$o: (r@xp) \Rightarrow q \quad \text{thf(cPRIMQWFF, conjecture)}$

SYO279 \wedge 5.p TPS problem from BASIC-HO-THMS

cLAM: $(\$i \rightarrow \$i) \rightarrow \$i \quad \text{thf(cLAM, type)}$

cAPP: $\$i \rightarrow \$i \rightarrow \$i \quad \text{thf(cAPP, type)}$

cReduct: $\$i \rightarrow \$i \rightarrow \$o \quad \text{thf(cReduct, type)}$

$\forall xx: \$i \rightarrow \$i, xy: \$i: (\text{cReduct} @ (\text{cAPP} @ (\text{cLAM} @ xx) @ xy) @ (xx @ xy)) \quad \text{thf(cBETA_CONVERSION, conjecture)}$

SYO280 \wedge 5.p TPS problem from BASIC-HO-THMS

s: $\$i \rightarrow \$i \quad \text{thf(s, type)}$

$\exists a: \$i \rightarrow \$o: \forall xx: \$i: ((a@xx) \iff \neg a@(s@xx)) \quad \text{thf(cBLEDSOE_FENG_SV_EO}_1\text{, conjecture)}$

SYO281 \wedge 5.p TPS problem from BASIC-HO-THMS

$\forall xp: \$i \rightarrow \$o: \exists xq: \$i \rightarrow \$o: \forall xx: \$i: ((xp@xx) \Rightarrow (xq@xx)) \quad \text{thf(cTRIV}_3\text{, conjecture)}$

SYO282 \wedge 5.p TPS problem from BASIC-HO-THMS

b: $\$tType \quad \text{thf(b_type, type)}$

$\forall xx: b \rightarrow \$o: \exists xy: b: (\exists xx_0: b: (xx@xx_0) \Rightarrow (xx@xy)) \quad \text{thf(cL}_51\text{, conjecture)}$

SYO284 \wedge 5.p TPS problem from BASIC-HO-THMS

r: $\$i \rightarrow \$i \rightarrow \$o \quad \text{thf(r, type)}$

$\forall xp: \$i \rightarrow \$o, xy: \$i: ((xp@xy) \Rightarrow \exists xx: \$i: (r@xx@xy)) \quad \text{thf(cTRIV}_1\text{, conjecture)}$

SYO285 \wedge 5.p TPS problem from BASIC-HO-THMS

y: $\$i \quad \text{thf(y, type)}$

cQ: $\$i \rightarrow \$o \quad \text{thf(cQ, type)}$

cR: $\$i \rightarrow \$o \quad \text{thf(cR, type)}$

$\forall xp: \$i \rightarrow \$o: (xp@y) \Rightarrow (\forall xx: \$i: (cR@xx) \text{ and } cQ@y) \quad \text{thf(cTEST}_2\text{, conjecture)}$

SYO286 \wedge 5.p TPS problem from BASIC-HO-THMS

y: $\$i \quad \text{thf(y, type)}$

cQ: $\$i \rightarrow \$o \quad \text{thf(cQ, type)}$

cR: $\$i \rightarrow \$o \quad \text{thf(cR, type)}$

$\forall xp: \$i \rightarrow \$o: (xp@y) \Rightarrow (\exists xx: \$i: (cR@xx) \Rightarrow (cQ@y)) \quad \text{thf(cTEST}_1\text{, conjecture)}$

SYO287 \wedge 5.p TPS problem from BASIC-HO-THMS

cB: $\$i \rightarrow \$i \rightarrow \$o \quad \text{thf(cB, type)}$

cA: $\$i \rightarrow \$o \quad \text{thf(cA, type)}$

c₀: $\$i \quad \text{thf(c}_0\text{, type)}$

$\exists xv: \$i \rightarrow \$i \rightarrow \$o: \forall xx: \$i: ((xv@xx@c_0) \iff (cA@xx \text{ or } cB@xx@xx)) \quad \text{thf(cSV}_9\text{, conjecture)}$

SYO288 \wedge 5.p TPS problem from BASIC-HO-THMS

cB: $\$i \rightarrow \$i \rightarrow \$o \quad \text{thf(cB, type)}$

cA: $\$i \rightarrow \$o \quad \text{thf(cA, type)}$

c₀: $\$i \quad \text{thf(c}_0\text{, type)}$

$\exists xv: \$i \rightarrow \$i \rightarrow \$o: \forall xx: \$i: ((xv@xx@c_0) \iff (cA@xx \text{ and } cB@xx@xx)) \quad \text{thf(cSV}_{10}\text{, conjecture)}$

SYO289 \wedge 5.p TPS problem from BASIC-HO-THMS

$\forall xr: \$i \rightarrow \$i \rightarrow \$o, xy: \$i: (\forall xp: \$i \rightarrow \$o: (xp@xy) \Rightarrow \exists xx: \$i: (xr@xx@xy)) \quad \text{thf(cTRIV}_2\text{, conjecture)}$

SYO290 \wedge 5.p TPS problem from BASIC-HO-THMS

a: $\$tType \quad \text{thf(a_type, type)}$

g₂: $a \rightarrow a \quad \text{thf(g}_2\text{, type)}$

g₁: $a \rightarrow a \quad \text{thf(g}_1\text{, type)}$

p: $(a \rightarrow a) \rightarrow \$o \quad \text{thf(p, type)}$

$\neg p@\lambda xw: a: (g_1@(g_2@xw)) \text{ or } p@\lambda xx: a: (g_1@(g_2@xx)) \quad \text{thf(cTRANS_BUG_EX}_2\text{, conjecture)}$

SYO291 \wedge 5.p TPS problem from BASIC-HO-THMS

y: $\$i \quad \text{thf(y, type)}$

cQ: $\$i \rightarrow \$o \quad \text{thf(cQ, type)}$

x: $\$i \quad \text{thf(x, type)}$

f: $\$i \rightarrow \$i \quad \text{thf(f, type)}$

cR: $\$i \rightarrow \$o \quad \text{thf(cR, type)}$

$\forall xp: \$i \rightarrow \$o: (xp@y) \Rightarrow (cR@x \text{ and } cR@(f@x) \text{ and } cQ@y) \quad \text{thf(cTEST}_3\text{, conjecture)}$

SYO292 \wedge 5.p TPS problem from BASIC-HO-THMS

cP: $\$i \rightarrow \$i \rightarrow \$o \quad \text{thf(cP, type)}$

$\exists a: \$i \rightarrow \$o: \forall xx: \$i, xy: \$i: ((a@xx \text{ or } a@xy) \Rightarrow (cP@xx@xy))$	thf(cBLEDSOE5E, conjecture)
SYO293^5.p TPS problem from BASIC-HO-THMS	
cP: $\$i \rightarrow \$i \rightarrow \$o$ thf(cP, type)	
$\exists a: \$i \rightarrow \$o: \forall xx: \$i, xy: \$i: ((a@xx \text{ and } a@xy) \Rightarrow (cP@xx@xy))$	thf(cBLEDSOE5A, conjecture)
SYO294^5.p TPS problem from BASIC-HO-THMS	
cN: $(\$i \rightarrow \$o) \rightarrow \$o$ thf(cN, type)	
cS: $((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow (\$i \rightarrow \$o) \rightarrow \o thf(cS, type)	
c ₀ : $(\$i \rightarrow \$o) \rightarrow \$o$ thf(c ₀ , type)	
$\forall p: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o: ((p@c_0 \text{ and } \forall x: (\$i \rightarrow \$o) \rightarrow \$o: ((p@x) \Rightarrow (p@(cS@x)))) \Rightarrow (p@cN))$	thf(cNATN ₁ , conjecture)
SYO295^5.p TPS problem from BASIC-HO-THMS	
b: $\$i$ thf(b, type)	
y: $\$i$ thf(y, type)	
a: $\$i$ thf(a, type)	
x: $\$i$ thf(x, type)	
$x \neq y \Rightarrow \exists xf: \$i \rightarrow \$i: ((xf@x) = a \text{ and } (xf@y) = b)$	thf(cX ₅₃₁₁ , conjecture)
SYO296^5.p TPS problem from BASIC-HO-THMS	
q: $\$i \rightarrow \o thf(q, type)	
p: $\$i \rightarrow \o thf(p, type)	
$\exists a: \$i \rightarrow \$o: \forall xx: \$i, xy: \$i: ((a@xx \text{ or } a@xy) \Rightarrow (p@xx \text{ and } q@xy))$	thf(cBLEDSOE5D, conjecture)
SYO297^5.p TPS problem from BASIC-HO-THMS	
n: $\$tType$ thf(n_type, type)	
cS: $n \rightarrow n$ thf(cS, type)	
c ₀ : n thf(c ₀ , type)	
$\forall xp: n \rightarrow \$o: ((xp@c_0 \text{ and } \forall xx: n: ((xp@xx) \Rightarrow (xp@(cS@xx)))) \Rightarrow \forall xx: n: (xp@xx))$	thf(cPA_IND, conjecture)
SYO298^5.p TPS problem from BASIC-HO-THMS	
q: $\$i \rightarrow \$i \rightarrow \$o$ thf(q, type)	
p: $\$i \rightarrow \$i \rightarrow \$o$ thf(p, type)	
$\exists a: \$i \rightarrow \$o: \forall l: \$i: \exists xy: \$i: \forall xz: \$i: ((a@xz) \Rightarrow (p@xz@xy \text{ and } q@l@xy))$	thf(cBLEDSOE5B, conjecture)
SYO299^5.p TPS problem from BASIC-HO-THMS	
$\forall p: \$o, q: \$o, r: \$o: \exists s: \$o: (((p \Leftarrow q) \Leftarrow r) \Rightarrow (p \Leftarrow (q \Leftarrow s)))$	thf(cTHM ₅₁ , conjecture)
SYO300^5.p TPS problem from BASIC-HO-THMS	
$\forall p: ((\$i \rightarrow \$i) \rightarrow \$i) \rightarrow ((\$i \rightarrow \$i) \rightarrow \$i) \rightarrow \$o: \exists x: (\$i \rightarrow \$i) \rightarrow \$i: (\forall xz: (\$i \rightarrow \$i) \rightarrow \$i: (p@xz@xz) \Rightarrow (p@\lambda u: \$i \rightarrow \$i: (u@(x@\lambda v: \$i: v))@x))$	thf(cUNIFTHM ₁ , conjecture)
SYO301^5.p TPS problem from BASIC-HO-THMS	
f: $\$i \rightarrow \i thf(f, type)	
cP: $\$i \rightarrow \o thf(cP, type)	
b: $\$i$ thf(b, type)	
$(cP@(f@b)) \Rightarrow \exists xs: \$i, a: \$i \rightarrow \$o: (\forall xx: \$i: ((a@xx) \Rightarrow (cP@xx)) \text{ and } a@(f@xs))$	thf(cBLEDSOE ₅ , conjecture)
SYO302^5.p TPS problem from BASIC-HO-THMS	
c: $\$i$ thf(c, type)	
b: $\$i$ thf(b, type)	
a: $\$i$ thf(a, type)	
c_less_: $\$i \rightarrow \$i \rightarrow \$o$ thf(c_less_, type)	
$(c_less_\text{@}a\text{@}b \text{ and } c_less_\text{@}b\text{@}c) \Rightarrow \exists a: \$i \rightarrow \$o: (\neg a@a \text{ and } a@b \text{ and } \neg a@c)$	thf(cBLEDSOE ₄ , conjecture)
SYO303^5.p TPS problem from BASIC-HO-THMS	
b: $\$i$ thf(b, type)	
cS: $\$i \rightarrow \o thf(cS, type)	
a: $\$i$ thf(a, type)	
cT: $\$i \rightarrow \o thf(cT, type)	
c ₀ : $\$i$ thf(c ₀ , type)	
$\forall p: \$i \rightarrow \$o: (\neg p@c_0 \text{ or } \forall xx: \$i: (p@xx)) \Rightarrow (\neg cS@c_0 \text{ and } cT@c_0 \text{ or } (cT@a \text{ and } cS@b))$	thf(cMIN_QUAN_BUG, conjecture)
SYO304^5.p TPS problem from UNKNOWN	
a: $\$tType$ thf(a_type, type)	
$\forall xq: (a \rightarrow a \rightarrow \$o) \rightarrow \$o: ((xq@\lambda x: a, y: a: \forall xq_0: a \rightarrow \$o: ((xq_0@x) \Rightarrow (xq_0@y))) \Rightarrow (xq@\lambda x: a, y: a: x = y))$	thf(cE2_eq_pme, conjecture)

SYO305^5.p TPS problem from BASIC-HO-THMS

cB: \$i → \$o thf(cB, type)
cA: \$i → \$o thf(cA, type)
 $\forall z_1: \$i: ((cA@z_1) \Rightarrow (cB@z_1)) \Rightarrow \forall z_1: \$i \rightarrow \$o: (\forall z_3: \$i: ((z_1@z_3) \Rightarrow (cA@z_3)) \Rightarrow \forall z_3: \$i: ((z_1@z_3) \Rightarrow (cB@z_3)))$ thf(cSET79_pme, conjecture)

SYO306^5.p TPS problem from BASIC-HO-THMS

cT: \$i → \$o thf(cT, type)
cS: \$i → \$o thf(cS, type)
 $\forall p: \$i \rightarrow \$o: (\forall xx: \$i: (p@xx) \Rightarrow \forall xx: \$i: (p@xx)) \Rightarrow (\forall xx: \$i: (cS@xx \text{ and } cT@xx) \Rightarrow \forall xx: \$i: (cS@xx \text{ and } cT@xx))$ thf(cBL1, conjecture)

SYO307^5.p TPS problem from BASIC-HO-THMS

cT: \$i → \$o thf(cT, type)
q: \$i → \$o thf(q, type)
cR: \$i → \$o thf(cR, type)
p: \$i → \$o thf(p, type)
 $\exists a: \$i \rightarrow \$o: \forall xx: \$i, xy: \$i, xz: \$i: ((a@xx \text{ or } (a@xy \text{ and } a@xz)) \Rightarrow ((p@xx \text{ and } cR@xy) \text{ or } (q@xx \text{ and } cT@xz)))$ thf(cBL1, conjecture)

SYO308^5.p TPS problem from BASIC-HO-THMS

cB: \$i → \$o thf(cB, type)
cA: \$i → \$o thf(cA, type)
 $\forall z_1: \$i \rightarrow \$o: (\forall z_3: \$i: ((z_1@z_3) \Rightarrow (cA@z_3)) \text{ or } \forall z_4: \$i: ((z_1@z_4) \Rightarrow (cB@z_4))) \Rightarrow \forall z_5: \$i: ((z_1@z_5) \Rightarrow (cA@z_5 \text{ or } cB@z_5))$ thf(cSET81_pme, conjecture)

SYO309^5.p TPS problem from BASIC-HO-THMS

cS: \$i → \$i thf(cS, type)
cQ: \$i → \$i thf(cQ, type)
cR: \$i → \$i → \$o thf(cR, type)
 $\forall xx: \$i: (\exists xu: \$i: (cR@xx@(cQ@xu)) \iff \neg \exists xv: \$i: (cR@(cS@xx)@(cQ@xv))) \Rightarrow \exists a: \$i \rightarrow \$o: \forall xx: \$i: ((a@xx) \iff \neg a@(cS@xx))$ thf(cEO1, conjecture)

SYO310^5.p TPS problem from BASIC-HO-THMS

$\exists xop: \$o \rightarrow \$o \rightarrow \$o: (xop@$true@$true \text{ and } xop@$false@$false \text{ and } \neg xop@$false@$true \text{ and } \neg xop@$true@$false) \text{ and } \forall xm: \$o: ((xm@$true) \iff (xm@($true \Rightarrow $true)))$ thf(cOMEGA_EXAMPLE, conjecture)

SYO311^5.p TPS problem from BASIC-HO-THMS

$\exists xop: \$o \rightarrow \$o \rightarrow \$o: (xop@$true@$true \text{ and } \neg xop@$false@$false \text{ and } \neg xop@$false@$true \text{ and } \neg xop@$true@$false) \text{ and } \forall xm: \$o: ((xm@$true) \iff (xm@($true \Rightarrow $true)))$ thf(cOMEGA_EXAMPLE2, conjecture)

SYO312^5.p TPS problem from BASIC-HO-THMS

cE2: (\$i → \$o) → (\$i → \$o) → \$o thf(cE2, type)
cE1: \$i → \$i → \$o thf(cE1, type)
 $\neg \exists xh: ($i \rightarrow $o) \rightarrow $i: (\forall xx: $i: (cE1@xx@xx) \text{ and } \forall xp: $i \rightarrow $o, xq: $i \rightarrow $o: ((cE2@xp@xq) \Rightarrow ((xp@(xh@xp)) \iff (xq@(xh@xq)))) \text{ and } \forall xp: $i \rightarrow $o, xq: $i \rightarrow $o: ((cE1@(xh@xp)@(xh@xq)) \Rightarrow (cE2@xp@xq)))$ thf(cTHM118, conjecture)

SYO313^5.p TPS problem from BASIC-HO-THMS

cELESS: \$i → \$i → \$o thf(cELESS, type)
 $\forall a: \$i \rightarrow \$o: ((\exists xr: \$i: (a@xr) \text{ and } \exists xu: \$i: \forall xx: \$i: ((a@xx) \Rightarrow (cELESS@xx@xu))) \Rightarrow \exists xl: \$i: (\forall xx: \$i: ((a@xx) \Rightarrow (cELESS@xx@xl)) \text{ and } \forall xy: \$i: (\forall xx: \$i: ((a@xx) \Rightarrow (cELESS@xx@xy)) \Rightarrow (cELESS@xl@xy))))$ thf(cELUB, conjecture)

SYO314^5.p TPS problem from BASIC-HO-THMS

z: \$i thf(z, type)
 $\exists xp: \$i \rightarrow \$o, xq: \$i \rightarrow \$o, xx: \$i, xy: \$i: (xp@xx \text{ and } \neg xp@xy \text{ and } xq@xy \text{ and } \neg xq@z) \iff \exists xp: \$i \rightarrow \$o, xq: \$i \rightarrow \$o, xx: \$i, xy: \$i: (\neg xq@z \text{ and } xq@xy \text{ and } xp@xx \text{ and } \neg xp@xy)$ thf(cSILLYWFF2, conjecture)

SYO315^5.p TPS problem from BASIC-HO-THMS

z: \$i thf(z, type)
 $\exists xp: \$i \rightarrow \$o, xq: \$i \rightarrow \$o, xx: \$i, xy: \$i: (xp@xx \text{ and } \neg xp@xy \text{ and } xq@xy \text{ and } \neg xq@z) \Rightarrow \exists xp: \$i \rightarrow \$o, xq: \$i \rightarrow \$o, xx: \$i, xy: \$i: (\neg xq@z \text{ and } xq@xy \text{ and } xp@xx \text{ and } \neg xp@xy)$ thf(cSILLYWFF, conjecture)

SYO316^5.p TPS problem from BASIC-HO-THMS

c0: \$i thf(c0, type)
cDOUBLE: \$i → \$i → \$o thf(cDOUBLE, type)
cHALF: \$i → \$i → \$o thf(cHALF, type)
cS: \$i → \$i thf(cS, type)

$\forall_{xx}: \$i, xy: \$i: ((cDOUBLE@{xx}@{xy}) \Rightarrow (cDOUBLE@({cS}@{xx})@({cS}@({cS}@{xy})))) \Rightarrow ((\forall_{xu}: \$i, xv: \$i: (cHALF@{xu}@{xv}) \text{ and } \$i \rightarrow \$i \rightarrow \$o: (\forall_{xz}: \$i: (xp@{xu}@{c0}@{xz}) \Rightarrow \forall_{xz}: \$i: (xp@{xu}@{xv}@{xz}))) \Rightarrow \neg cDOUBLE@{c0}@{c0}) \quad \text{thf}(cTHM300_BUG, conjecture)$

SYO317^5.p TPS problem from BASIC-HO-THMS

$cP: \$i \rightarrow \$o \quad \text{thf}(cP, type)$
 $f: \$i \rightarrow \$i \quad \text{thf}(f, type)$
 $cEQ: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cEQ, type)$
 $a: \$i \quad \text{thf}(a, type)$

$(\forall_{xx}: \$i: (cEQ@{xx}@{xx}) \text{ and } \forall_{xx}: \$i, xy: \$i: ((cEQ@{xx}@{xy} \text{ and } cP@{xx}) \Rightarrow (cP@{xy})) \text{ and } cP@a \text{ and } \forall_{xy}: \$i, xz: \$i: ((cEQ@({cEQ}@{xy}@{xz}))) \Rightarrow \exists a: \$i \rightarrow \$o: (\forall_{xx}: \$i: ((a@({f}@{xx}))) \Rightarrow (cP@{xx})) \text{ and } \exists_{xz}: \$i: (a@{xz})) \quad \text{thf}(cTHM_{304}, conjecture)$

SYO318^5.p TPS problem from BASIC-HO-THMS

$c_0: \$i \quad \text{thf}(c_0, type)$
 $c_2: \$i \quad \text{thf}(c_2, type)$
 $c_star: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(c_star, type)$
 $c_less_l: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(c_less_l, type)$
 $div: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(div, type)$

$(\neg c_less_l@c_0@c_0 \text{ and } \neg c_less_l@(div@c_0@c_2)@c_0 \text{ and } \neg c_less_l(c_star@c_2@c_0)@c_0) \Rightarrow \exists a: \$i \rightarrow \$o: (\forall_{xx}: \$i, xy: \$i: ((a@{xx} \text{ and } a@{yy}) \Rightarrow (a@{xy})) \text{ and } a@a) \quad \text{thf}(cBLEDS, conjecture)$

SYO323^5.p TPS problem from BASIC-HO-THMS

$cS: \$i \rightarrow \$i \quad \text{thf}(cS, type)$
 $cHALF: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cHALF, type)$
 $c_0: \$i \quad \text{thf}(c_0, type)$
 $cDOUBLE: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cDOUBLE, type)$

$\forall q: \$i \rightarrow \$i \rightarrow \$o, xu: \$i, xv: \$i: ((cDOUBLE@{xu}@{xv} \text{ and } q@c_0@c_0) \text{ and } \forall_{xx}: \$i, xy: \$i: ((q@{xx}@{xy}) \Rightarrow (q@({cS}@{xx})@({cS}@({cS}@{xy})))) \text{ and } cHALF@c_0@c_0 \text{ and } cHALF@c_0@({cS}@{c_0}) \text{ and } \forall_{xx}: \$i, xy: \$i: ((cHALF@{xx}@{xy}) \Rightarrow (cHALF@({cS}@({cS}@{xx})))) \Rightarrow \exists a: \$i \rightarrow \$o: (\forall_{xx}: \$i, xy: \$i: ((a@{xx} \text{ and } a@{yy}) \Rightarrow (a@{xy}))) \quad \text{thf}(cTHM143_EXPANDB, conjecture)$

SYO324^5.p TPS problem from BASIC-HO-THMS

$a: \$tType \quad \text{thf}(a_type, type)$
 $cK: (a \rightarrow \$o) \rightarrow a \rightarrow \$o \quad \text{thf}(cK, type)$

$\forall x: a \rightarrow \$o, y: a \rightarrow \$o: (\forall_{xx}: a: ((x@{xx}) \Rightarrow (y@{xx})) \Rightarrow \forall_{xx}: a: ((cK@{x}@{xx}) \Rightarrow (cK@{y}@{xx}))) \Rightarrow \forall_{xx}: a: ((cK@{\lambda xx_0}: a: \exists s: a \rightarrow \$o: (\forall_{xx_1}: a: ((s@{xx_1}) \Rightarrow (cK@s@{xx_1}))) \text{ and } s@s@{xx_0})@{xx}) \Rightarrow (cK@({cK}@{\lambda xx_0}: a: \exists s: a \rightarrow \$o: (\forall_{xx_1}: a: ((s@{xx_1}) \Rightarrow (cK@s@{xx_1}))) \text{ and } s@s@{xx_0}))@{xx}) \quad \text{thf}(cTHM116_1SS, conjecture)$

SYO326^5.p TPS problem from BASIC-HO-THMS

$cC: \$i \rightarrow \$o \quad \text{thf}(cC, type)$
 $f: \$i \rightarrow \$i \quad \text{thf}(f, type)$
 $cB: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cB, type)$
 $cA: \$i \rightarrow \$o \quad \text{thf}(cA, type)$

$\exists_{xu}: \$i \rightarrow \$i \rightarrow \$o: (\forall_{xw}: \$i, xz: \$i: ((cA@{xw} \text{ and } cB@{xz}@{xw}) \Rightarrow (xu@({f}@{xw})@{xz})) \text{ and } \forall_{xz}: \$i: ((cC@{xz}) \Rightarrow (xu@{xz}@{xz})) \text{ and } \forall_{xv}: \$i \rightarrow \$i \rightarrow \$o: ((\forall_{xw}: \$i, xz: \$i: ((cA@{xw} \text{ and } cB@{xz}@{xw}) \Rightarrow (xv@({f}@{xw})@{xz})) \text{ and } \forall_{xz}: \$i: ((cC@{xz}) \Rightarrow (xv@{xz}@{xz}))) \Rightarrow \forall_{xx}: \$i, xy: \$i: ((xu@{xx}@{xy}) \Rightarrow (xv@{xx}@{xy}))) \quad \text{thf}(cSV_8, conjecture)$

SYO327^5.p TPS problem from BASIC-HO-THMS

$cHALF: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cHALF, type)$
 $cDOUBLE: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cDOUBLE, type)$
 $cS: \$i \rightarrow \$i \quad \text{thf}(cS, type)$
 $c_0: \$i \quad \text{thf}(c_0, type)$

$(\forall q: \$i \rightarrow \$i \rightarrow \$o, xu: \$i, xv: \$i: ((cDOUBLE@{xu}@{xv} \text{ and } q@c_0@c_0) \text{ and } \forall_{xx}: \$i, xy: \$i: ((q@{xx}@{xy}) \Rightarrow (q@({cS}@{xx})@({cS}@({cS}@{xy})))) \text{ and } cHALF@c_0@c_0 \text{ and } cHALF@c_0@({cS}@{c_0}) \text{ and } \forall_{xx}: \$i, xy: \$i: ((cHALF@{xx}@{xy}) \Rightarrow (cHALF@({cS}@({cS}@{xx})))) \Rightarrow \forall_{xu}: \$i, xv: \$i: ((cDOUBLE@{xu}@{xv}) \Rightarrow (cHALF@{xv}@{xu}))) \quad \text{thf}(cDOUBLE_TO_HALF_6, conjecture)$

SYO328^5.p TPS problem from BASIC-HO-THMS

$a: \$tType \quad \text{thf}(a_type, type)$
 $cG2_0: a \rightarrow a \quad \text{thf}(cG2_0, type)$
 $cG1_0: a \rightarrow a \quad \text{thf}(cG1_0, type)$
 $cP_0: (a \rightarrow a) \rightarrow \$o \quad \text{thf}(cP_0, type)$
 $j_7: a \rightarrow a \quad \text{thf}(j_7, type)$

$cF_0: a \rightarrow a \quad \text{thf}(cF_0, \text{type})$
 $j_6: a \rightarrow a \quad \text{thf}(j_6, \text{type})$
 $p_6: (a \rightarrow a) \rightarrow \$o \quad \text{thf}(p_6, \text{type})$
 $cJ_1: a \rightarrow a \quad \text{thf}(cJ_1, \text{type})$
 $p_4: (a \rightarrow a) \rightarrow \$o \quad \text{thf}(p_4, \text{type})$
 $cJ_0: a \rightarrow a \quad \text{thf}(cJ_0, \text{type})$
 $((p_4 @ \lambda x u_3: a: x u_3 \text{ and } ((p_4 @ c J_0) \Rightarrow (p_4 @ \lambda x x_4: a: (c F_0 @ (c J_0 @ x x_4)))))) \Rightarrow (p_4 @ c G_1_0) \text{ and } ((p_6 @ \lambda x u_4: a: x u_4 \text{ and } ((p_6 @ c J_0 @ x x_5: a: (c F_0 @ (c J_1 @ x x_5)))))) \Rightarrow (p_6 @ \lambda x x: a: (c G_2_0 @ x x)) \Rightarrow ((c P_0 @ \lambda x u_5: a: x u_5 \text{ and } ((c P_0 @ j_6) \Rightarrow (c P_0 @ \lambda x x_7: a: (c F_0 @ (j_6 @ x x_7)))))) \text{ and } ((c P_0 @ j_7) \Rightarrow (c P_0 @ \lambda x x_7: a: (c F_0 @ (j_7 @ x x_7)))) \Rightarrow (c P_0 @ \lambda x x_6: a: (c G_1_0 @ (c G_2_0 @ x x_6)))$

SYO329^5.p TPS problem from BASIC-HO-THMS

$cS: \$i \rightarrow \$i \quad \text{thf}(cS, \text{type})$
 $cDOUBLE: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cDOUBLE, \text{type})$
 $cHALF: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cHALF, \text{type})$
 $c_0: \$i \quad \text{thf}(c_0, \text{type})$
 $(\forall x u: \$i, x v: \$i: ((cHALF @ x u @ x v) \Rightarrow \forall q: \$i \rightarrow \$i \rightarrow \$o: ((q @ c_0 @ c_0 \text{ and } q @ (cS @ c_0)) @ c_0 \text{ and } \forall x x: \$i, x y: \$i: ((q @ x x @ x y) \Rightarrow (q @ (cS @ (cS @ x x)) @ (cS @ x y)))) \Rightarrow (q @ x u @ x v)) \text{ and } cDOUBLE @ c_0 @ c_0 \text{ and } \forall x x: \$i, x y: \$i: ((cDOUBLE @ x x @ x y) \Rightarrow (cDOUBLE @ (cS @ x x) @ (cS @ (cS @ x y)))) \Rightarrow \forall x u: \$i, x v: \$i: ((cHALF @ x u @ x v) \Rightarrow (cDOUBLE @ x v @ x u \text{ or } cDOUBLE @ (cS @ (cS @ x u) @ x v))))$

SYO330^5.p TPS problem from BASIC-HO-THMS

$cNUMBER: \$i \rightarrow \$o \quad \text{thf}(cNUMBER, \text{type})$
 $cODD: \$i \rightarrow \$o \quad \text{thf}(cODD, \text{type})$
 $cEVEN: \$i \rightarrow \$o \quad \text{thf}(cEVEN, \text{type})$
 $cS: \$i \rightarrow \$i \quad \text{thf}(cS, \text{type})$
 $c_0: \$i \quad \text{thf}(c_0, \text{type})$
 $(cEVEN @ c_0 \text{ and } \forall x n: \$i: ((cEVEN @ x n) \Rightarrow (cEVEN @ (cS @ (cS @ x n)))) \text{ and } cODD @ (cS @ c_0) \text{ and } \forall x n: \$i: ((cODD @ x n) \Rightarrow (cODD @ (cS @ (cS @ x n)))) \text{ and } \forall p: \$i \rightarrow \$o, q: \$i \rightarrow \$o: ((p @ c_0 \text{ and } q @ c_0 \text{ and } \forall x x: \$i: ((p @ x x \text{ and } q @ x x) \Rightarrow (p @ (cS @ x x) \text{ and } q @ (cS @ x x)))) \text{ and } \forall x n: \$i: ((cNUMBER @ x n) \iff (cEVEN @ x n \text{ or } cODD @ x n)) \Rightarrow \forall x n: \$i: (cNUMBER @ x n))$

SYO331^5.p TPS problem from BASIC-HO-THMS

$a: \$tType \quad \text{thf}(a_type, \text{type})$
 $cK: (a \rightarrow \$o) \rightarrow a \rightarrow \$o \quad \text{thf}(cK, \text{type})$
 $\forall x: a \rightarrow \$o: (\forall x x: a: ((x @ x x) \Rightarrow \exists s: a \rightarrow \$o: (\forall x x_0: a: ((s @ x x_0) \Rightarrow (cK @ s @ x x_0)) \text{ and } s @ x x)) \Rightarrow \forall x x: a: ((cK @ x @ x x) \Rightarrow (cK @ \lambda x x_0: a: \exists s: a \rightarrow \$o: (\forall x x_1: a: ((s @ x x_1) \Rightarrow (cK @ s @ x x_1)) \text{ and } s @ x x_0 @ x x)) \Rightarrow \forall x x: a: (\exists s: a \rightarrow \$o: (\forall x x_0: a: ((s @ x x_0) \Rightarrow (cK @ s @ x x_0)) \text{ and } s @ x x) \Rightarrow (cK @ \lambda x x_0: a: \exists s: a \rightarrow \$o: (\forall x x_1: a: ((s @ x x_1) \Rightarrow (cK @ s @ x x_1)) \text{ and } s @ x x_0 @ x x))) \quad \text{thf}(cTHM116_2SS_pme, conjecture)$

SYO332^5.p TPS problem from BASIC-HO-THMS

$b: \$tType \quad \text{thf}(b_type, \text{type})$
 $a: \$tType \quad \text{thf}(a_type, \text{type})$
 $(\forall x s: (b \rightarrow \$o) \rightarrow \$o: (\forall x: b \rightarrow \$o: ((x s @ x) \Rightarrow \exists x y: b: (x @ x y)) \Rightarrow \exists x f: (b \rightarrow \$o) \rightarrow b: \forall x: b \rightarrow \$o: ((x s @ x) \Rightarrow (x @ (x f @ x))) \Rightarrow \forall x r: a \rightarrow b \rightarrow \$o: \exists x g: a \rightarrow b: \forall x x: a: (\exists x y: b: (x r @ x x @ x y) \Rightarrow (x r @ x x @ (x g @ x x))) \text{ and } (\forall x r: (a \rightarrow \$o) \rightarrow a \rightarrow \$o: \exists x g: (a \rightarrow \$o) \rightarrow a: \forall x x: a \rightarrow \$o: (\exists x y: a: (x r @ x x @ x y) \Rightarrow (x r @ x x @ (x g @ x x))) \Rightarrow \forall x s: (a \rightarrow \$o) \rightarrow \$o: (\forall x: a \rightarrow \$o: ((x s @ x) \Rightarrow \exists x t: a: (x @ x t)) \Rightarrow \exists x f: (a \rightarrow \$o) \rightarrow a: \forall x: a \rightarrow \$o: ((x s @ x) \Rightarrow (x @ (x f @ x)))) \quad \text{thf}(cTHM561, conjecture)$

SYO333^5.p TPS problem from BASIC-HO-THMS

$u: \$i \quad \text{thf}(u, \text{type})$
 $v: \$i \quad \text{thf}(v, \text{type})$
 $cDOUBLE: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cDOUBLE, \text{type})$
 $cHALF: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cHALF, \text{type})$
 $cS: \$i \rightarrow \$i \quad \text{thf}(cS, \text{type})$
 $c_0: \$i \quad \text{thf}(c_0, \text{type})$
 $cSx: \$i \quad \text{thf}(cSx, \text{type})$
 $(cDOUBLE @ c_0 @ c_0 \text{ and } \forall x x: \$i, x y: \$i: ((cDOUBLE @ x x @ x y) \Rightarrow (cDOUBLE @ cSx @ (cS @ (cS @ x y)))) \text{ and } cHALF @ c_0 @ c_0 \text{ and } (cHALF @ (cS @ x x) @ (cS @ (cS @ x y))) \text{ and } \forall q: \$i \rightarrow \$i \rightarrow \$o, x u_0: \$i, x v_0: \$i: ((cHALF @ x u_0 @ x v_0) \text{ and } q @ c_0 @ c_0 \text{ and } q @ c_0 @ (cS @ (q @ (cS @ x x) @ (cS @ (cS @ x y)))) \Rightarrow (q @ x u_0 @ x v_0)) \Rightarrow ((cHALF @ u @ v) \Rightarrow (cDOUBLE @ v @ u)) \quad \text{thf}(cHALF_TO_DOUBLE, conjecture)$

SYO334^5.p TPS problem from BASIC-HO-THMS

$c: \$tType \quad \text{thf}(c_type, \text{type})$
 $b: \$tType \quad \text{thf}(b_type, \text{type})$
 $a: \$tType \quad \text{thf}(a_type, \text{type})$
 $c_starc: c \rightarrow c \rightarrow c \quad \text{thf}(c_starc, \text{type})$

c_starb: $b \rightarrow b \rightarrow b$ thf(c_starb, type)
c_stara: $a \rightarrow a \rightarrow a$ thf(c_stara, type)
 $\forall xf: a \rightarrow b, xg: a \rightarrow c, xh: b \rightarrow c: ((\forall xx: a, xq: c \rightarrow \$o: ((xq@(xh@{xf@xx})) \Rightarrow (xq@(xg@xx))) \text{ and } \forall xy: b: \exists xx: a: \forall xq: b \rightarrow \$o: ((xq@(xf@xx)) \Rightarrow (xq@xy)) \text{ and } \forall xx: a, xy: a, xq: b \rightarrow \$o: ((xq@(xf@{c_stara@xx@xy})) \Rightarrow (xq@(c_starb@{xf@xx}@{xf@xx}))) \Rightarrow (\forall xx: b, xy: b, xq: c \rightarrow \$o: ((xq@(xh@{c_starb@{xh@xx}}) \Rightarrow (xq@(c_stara@{xh@xx}@{xh@xy})))))) \Rightarrow \forall xx: b, xy: b, xq: c \rightarrow \$o: ((xq@(xh@{c_starb@{xh@xx}}) \Rightarrow (xq@(c_stara@{xh@xx}@{xh@xy}))))))$ thf(cTHM270_INST, conjecture)

SYO335^5.p TPS problem from BASIC-HO-THMS

cS: $\$i \rightarrow \i thf(cS, type)
cDOUBLE: $\$i \rightarrow \$i \rightarrow \$o$ thf(cDOUBLE, type)
cHALF: $\$i \rightarrow \$i \rightarrow \$o$ thf(cHALF, type)
 $c_0: \$i$ thf(c_0 , type)
 $(\forall xu: \$i, xv: \$i: ((cHALF@{xu@xv}) \Rightarrow \forall q: \$i \rightarrow \$i \rightarrow \$o: ((q@c_0@c_0) \text{ and } q@(cS@c_0)@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((q@xx@xy) \Rightarrow (q@(cS@(cS@xx))@{cS@xy}))) \Rightarrow (q@{xu@xv})) \text{ and } cDOUBLE@c_0@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((cDOUBLE@xx@xy) \Rightarrow (cDOUBLE@{cS@xx}@{cS@{cS@xy}}))) \text{ and } \forall xx: \$i, xy: \$i: ((cDOUBLE@xx@xy) \Rightarrow (cDOUBLE@{cS@xx}@{cS@{cS@xy}}))) \text{ and } \forall xx: \$i, xy: \$i: ((cDOUBLE@xx@xy) \Rightarrow (cDOUBLE@{cS@xx}@{cS@{cS@xy}}))) \Rightarrow \forall xu: \$i, xv: \$i: ((cHALF@{xu@xv}) \Rightarrow (cDOUBLE@{xv@xu} \text{ or } cDOUBLE@{cS@{xv@xu}})))$

SYO336^5.p TPS problem from BASIC-HO-THMS

cS: $\$i \rightarrow \i thf(cS, type)
cDOUBLE: $\$i \rightarrow \$i \rightarrow \$o$ thf(cDOUBLE, type)
cHALF: $\$i \rightarrow \$i \rightarrow \$o$ thf(cHALF, type)
 $c_0: \$i$ thf(c_0 , type)
 $(cDOUBLE@c_0@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((cDOUBLE@xx@xy) \Rightarrow (cDOUBLE@{cS@xx}@{cS@{cS@xy}}))) \text{ and } \forall xx: \$i, xy: \$i: ((cDOUBLE@{cS@xx}@{cS@{cS@xy}}))) \text{ and } \forall xx: \$i, xy: \$i: ((cDOUBLE@xx@xy) \Rightarrow (cDOUBLE@{cS@xx}@{cS@{cS@xy}}))) \text{ and } \forall xx: \$i, xy: \$i: ((cHALF@{xu@xv} \text{ and } xp@c_0@c_0 \text{ and } xq@c_0@c_0 \text{ and } xp@(cS@c_0)@c_0 \text{ and } xq@(cS@c_0)@c_0 \text{ and } xq@(cS@{cS@xx})@{cS@{cS@xy}}) \Rightarrow (xp@{xu@xv} \text{ and } xq@{xu@xv})) \Rightarrow \forall xu: \$i, xv: \$i: ((cHALF@{xu@xv}) \Rightarrow (cDOUBLE@{xv@xu} \text{ or } cDOUBLE@{cS@{xv@xu}})))$ thf(cTRY₄, conjecture)

SYO337^5.p TPS problem from BASIC-HO-THMS

cS: $\$i \rightarrow \i thf(cS, type)
cDOUBLE: $\$i \rightarrow \$i \rightarrow \$o$ thf(cDOUBLE, type)
cHALF: $\$i \rightarrow \$i \rightarrow \$o$ thf(cHALF, type)
 $c_0: \$i$ thf(c_0 , type)
 $(\forall xu: \$i, xv: \$i: ((cHALF@{xu@xv} \text{ and } \forall xp: \$i \rightarrow \$i \rightarrow \$o, xq: \$i \rightarrow \$i \rightarrow \$o: (((xp@c_0@c_0) \text{ or } xq@c_0@c_0) \text{ and } (xp@(cS@c_0)@c_0 \text{ or } xq@(cS@c_0)@c_0) \text{ or } (xp@{cS@{cS@xx}}@{cS@{cS@xy}}) \Rightarrow (xp@{xu@xv} \text{ or } xq@{xu@xv})) \text{ or } (\neg cHALF@{xu@xv} \text{ and } \$i \rightarrow \$o: ((q@c_0@c_0) \text{ and } q@(cS@c_0)@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((q@xx@xy) \Rightarrow (q@(cS@{cS@xx})@{cS@{cS@xy}}))) \Rightarrow (q@{xu@xv}) \text{ and } cDOUBLE@c_0@c_0 \text{ and } \forall xx: \$i, xy: \$i: ((cDOUBLE@xx@xy) \Rightarrow (cDOUBLE@{cS@xx}@{cS@{cS@xy}}))) \text{ and } \forall xu: \$i, xv: \$i: ((cHALF@{xu@xv}) \Rightarrow (cDOUBLE@{xv@xu} \text{ or } cDOUBLE@{cS@{xv@xu}})))$ thf(cTHM300A_EXPAND, conjecture)

SYO338^5.p TPS problem from BASIC-HO-THMS

a: \$tType thf(a_type, type)
 $\forall r: a \rightarrow a \rightarrow \$o, u: (a \rightarrow \$o) \rightarrow a: (\exists x: a, y: a, z: a: ((r@{x@y} \text{ or } r@{x@y}) \text{ and } (r@{y@z} \text{ or } r@{y@z}) \text{ and } (\neg r@{x@z} \text{ or } \neg r@{x@z})) \text{ and } \$o: (\exists xz: a: ((xs@{xz} \text{ or } xs@{xz}) \text{ and } (\neg r@{xz@{u@xs}} \text{ or } \neg r@{xz@{u@xs}})) \text{ or } \exists xj: a: (\forall xk: a: (\neg xs@{xk} \text{ or } \neg xs@{xk} \text{ or } r@{xk@{xj}})) \text{ or } a: (\exists xx: a, xy: a: ((r@{xx@{xy}} \text{ or } r@{xx@{xy}}) \text{ and } (\neg r@{xf@{xx}}@{xf@{xy}} \text{ or } \neg r@{xf@{xx}}@{xf@{xy}})) \text{ or } \exists xw: a: ((r@{xw@{xf@{xw}}}) \text{ or } (r@{xw@{xf@{xw}}}))$

SYO339^5.p TPS problem from BASIC-HO-THMS

b: \$i thf(b, type)
a: \$i thf(a, type)
 $\forall xr: \$i \rightarrow \$i \rightarrow \$o, xf: \$i \rightarrow \$i, x0: \$i: ((\forall a: \$i \rightarrow \$o: ((\exists xl: \$i: (a@{xl}) \text{ and } \exists xu: \$i: ((a@{xx}) \Rightarrow (xr@{xx@{xu}}))) \Rightarrow \exists xl: \$i: (\forall xx: \$i: ((a@{xx}) \Rightarrow (xr@{xx@{xl}}))) \text{ and } \forall xy: \$i: ((\forall xx: \$i: ((a@{xx}) \Rightarrow (xr@{xx@{xy}}))) \Rightarrow (xr@{xl@{xy}}))) \text{ and } \forall xx: \$i: ((xr@{x0@{xf@{xx}}}) \Rightarrow \exists xt: \$i: (xr@{xt@{xx}} \text{ and } \forall xs: \$i: ((xr@{xt@{xs}} \text{ and } xr@{xs@{xx}}) \Rightarrow (xr@{x0@{xf@{xs}}})) \text{ and } \forall xx: \$i: ((xr@{xf@{xx}}@{x0}) \Rightarrow \exists xt: \$i: (xr@{xx@{xt}} \text{ and } \forall xs: \$i: ((xr@{xs@{xt}} \text{ and } xr@{xx@{xs}}) \Rightarrow (xr@{xf@{xs}}@{x0}))) \text{ and } xr@a@b \text{ and } xr@{xf@{a}}@{x0} \text{ and } xr@{x0@{xf@{b}}}) \Rightarrow \exists xx: \$i: (xr@{a@{xx}} \text{ and } xr@{xx@{b}} \text{ and } xr@{xf@{a}}@{x0}))$

SYO340^5.p TPS problem from BASIC-HO-THMS

cG: $\$i \rightarrow \i thf(cG, type)
cK: $\$i \rightarrow \i thf(cK, type)
cR: $\$i \rightarrow \$i \rightarrow \$o$ thf(cR, type)
cF: $\$i \rightarrow \$i \rightarrow \$i$ thf(cF, type)
cP: $\$i \rightarrow \o thf(cP, type)
 $\forall x: \$i, y: \$i, q: \$i \rightarrow \$i \rightarrow \$o: (\exists q1: \$i \rightarrow \$i \rightarrow \$o, q2: \$i \rightarrow \$i \rightarrow \$o: (q1@{(cF@{x@{(cG@x)}})@{y}} \text{ and } \forall y0: \$i: ((\neg q1@{x@{y0}} \text{ or } cP@{x@{y0}}) \text{ and } \exists q1: \$i \rightarrow \$i \rightarrow \$o: ((cP@{(cF@{x@{(cG@x)}})@{y}}) \text{ or } q@{(cF@{x@{(cG@x)}})@{y}}) \text{ and } (\neg cP@{(cF@{x@{(cG@x)}})@{y}}) \text{ or } q1@{(cF@{x@{(cG@x)}})@{y}}))$

SYO341^5.p TPS problem from BASIC-HO-THMS

cG: \$i → \$i thf(cG, type)
 cK: \$i thf(cK, type)
 cR: \$i → \$i → \$o thf(cR, type)
 cF: \$i → \$i → \$i thf(cF, type)
 cP: \$i → \$o thf(cP, type)

$\forall x: \$i, y: \$i, q: \$i \rightarrow \$i \rightarrow \$o: (\exists q_1: \$i \rightarrow \$i \rightarrow \$o, q_2: \$i \rightarrow \$i \rightarrow \$o: (q_1 @ (cF @ x @ (cG @ x)) @ y \text{ and } \forall y_0: \$i: ((\neg q_1 @ x @ y_0 \text{ or } cP @ x @ (cF @ x @ (cG @ x)) @ y) \text{ and } \exists q_1: \$i \rightarrow \$i \rightarrow \$o: ((cP @ (cF @ x @ (cG @ y)) \text{ or } q @ (cF @ x @ (cG @ y)) @ (cG @ y)) \text{ and } (\neg cP @ (cF @ x @ (cG @ y)) \text{ or } q_1 @ (cF @ x @ (cG @ y)) @ (cG @ y)))$

SYO344^5.p TPS problem THM618

Simple E-unification example.

cD: \$i thf(cD, type)
 cQ: \$i → \$o thf(cQ, type)
 cC: \$i thf(cC, type)
 $cC \neq cD \text{ or } \neg cQ@cC \text{ or } cQ@cD \quad \text{thf}(c\text{THM}_{618}, \text{conjecture})$

SYO345^5.p TPS problem THM615

cH: \$o → \$i thf(cH, type)
 $(cH @ (cH @ \$\text{true})) = (cH @ \$\text{false}) = (cH @ \$\text{false}) \quad \text{thf}(c\text{THM}_{615}, \text{conjecture})$

SYO346^5.p TPS problem THM621

Simple extensionality example.

cB: \$o thf(cB, type)
 cP: \$o → \$o thf(cP, type)
 cA: \$o thf(cA, type)
 $cA \neq cB \text{ or } \neg cP@cA \text{ or } cP@cB \quad \text{thf}(c\text{THM}_{621}, \text{conjecture})$

SYO347^5.p TPS problem THM620

Simple extensionality example.

cD: \$i → \$o thf(cD, type)
 cR: \$(i → \$o) → \$o thf(cR, type)
 cC: \$i → \$o thf(cC, type)
 $cC \neq cD \text{ or } \neg cR@cC \text{ or } cR@cD \quad \text{thf}(c\text{THM}_{620}, \text{conjecture})$

SYO348^5.p TPS problem E1EXT

Example from [Ben99] p.115.

b: \$o thf(b, type)
 a: \$o thf(a, type)
 $(a \iff b) \Rightarrow \forall p: \$o \rightarrow \$o: ((p @ a) \Rightarrow (p @ b)) \quad \text{thf}(c\text{E1EXT}, \text{conjecture})$

SYO349^5.p TPS problem THM617

Simple extensionality example.

cB: \$o thf(cB, type)
 cP: \$o → \$o thf(cP, type)
 cA: \$o thf(cA, type)
 $\neg cA \text{ or } \neg cB \text{ or } \neg cP@cA \text{ or } cP@cB \quad \text{thf}(c\text{THM}_{617}, \text{conjecture})$

SYO350^5.p TPS problem E1FUNC

Example from [Ben99] p.115.

g: \$i → \$i thf(g, type)
 p: \$(i → \$i) → \$o thf(p, type)
 f: \$i → \$i thf(f, type)
 $\forall x: \$i: (f @ x) = (g @ x) \Rightarrow ((p @ f) \Rightarrow (p @ g)) \quad \text{thf}(c\text{E1FUNC}, \text{conjecture})$

SYO351^5.p TPS problem E6EXT

Example from [Ben99] p.116.

a: \$tType thf(a_type, type)
 $(\lambda r: a \rightarrow \$o: \forall xx: a: ((r @ xx) \Rightarrow \$\text{false})) = (\lambda xy: a \rightarrow \$o: (\lambda xx: a: \$\text{false}) = xy) \quad \text{thf}(c\text{E6EXT_pme}, \text{conjecture})$

SYO352^5.p TPS problem E5EXT

Nontrivial direction of functional extensionality using Leibniz equality.

n: \$i → \$i thf(n, type)
 m: \$i → \$i thf(m, type)
 $\forall x: \$i, p: \$i \rightarrow \$o: ((p @ (m @ x)) \Rightarrow (p @ (n @ x))) \Rightarrow \forall q: (\$i \rightarrow \$i) \rightarrow \$o: ((q @ m) \Rightarrow (q @ n)) \quad \text{thf}(c\text{E5EXT}, \text{conjecture})$

SYO353^5.p TPS problem E1LEIBEQ1

Example about alternative defns of equality.

a: \$tType thf(a_type, type)
v: *a* thf(v, type)
u: *a* thf(u, type)
 $\forall xq: a \rightarrow \$o: ((xq@u) \Rightarrow (xq@v)) \Rightarrow \forall q: a \rightarrow a \rightarrow \$o: (\forall z: a: (q@z@z) \Rightarrow (q@u@v))$ thf(cE1LEIBEQ1_pme, conjecture)

SYO354^5.p TPS problem E4EXT

n: \$i → \$i thf(n, type)
m: \$i → \$i thf(m, type)
 $\forall x: \$i, p: \$i \rightarrow \$o: ((p@(m@x)) \Rightarrow (p@(n@x))) \Rightarrow \forall q: (\$i \rightarrow \$i) \rightarrow \$o: ((q@\lambda x: \$i: (m@x)) \Rightarrow (q@\lambda x: \$i: (n@x)))$ thf(cE4EXT_pme, conjecture)

SYO355^5.p TPS problem THM613

If there's one individual, then there's only one function.

$\forall xa: \$i, xb: \$i: xa = xb \Rightarrow \forall xf: \$i \rightarrow \$i, xg: \$i \rightarrow \$i: xf = xg$ thf(cTHM613, conjecture)

SYO356^5.p TPS problem E1LEIBEQ2

Example from [Ben9] about alternative defns of equality.

a: \$tType thf(a_type, type)
v: *a* thf(v, type)
u: *a* thf(u, type)
 $\forall q: a \rightarrow a \rightarrow \$o: (\forall z: a: (q@z@z) \Rightarrow (q@u@v)) \Rightarrow \forall xq: a \rightarrow \$o: ((xq@u) \Rightarrow (xq@v))$ thf(cE1LEIBEQ2_pme, conjecture)

SYO357^5.p TPS problem E2LEIBEQ2

Example from [Ben99] about alternative defns of equality.

atype: \$tType thf(a_type, type)
a: \$o thf(a, type)
v: atype thf(v, type)
u: atype thf(u, type)
b: \$o thf(b, type)
 $\forall p: atype \rightarrow \$o: (((a \text{ or } \neg a) \text{ and } p@u) \Rightarrow ((b \text{ or } \neg b) \text{ and } p@v)) \Rightarrow \forall xq: atype \rightarrow \$o: ((xq@u) \Rightarrow (xq@v))$ thf(cE2LEIBEQ2_pme, conjecture)

SYO358^5.p TPS problem E2LEIBEQ1

Example from [Ben99] about alternative defns of equality.

atype: \$tType thf(a_type, type)
a: \$o thf(a, type)
v: atype thf(v, type)
b: \$o thf(b, type)
u: atype thf(u, type)
 $\forall xq: atype \rightarrow \$o: ((xq@u) \Rightarrow (xq@v)) \Rightarrow \forall p: atype \rightarrow \$o: (((a \text{ or } \neg a) \text{ and } p@u) \Rightarrow ((b \text{ or } \neg b) \text{ and } p@v))$ thf(cE2LEIBEQ1_pme, conjecture)

SYO359^5.p TPS problem EXT1

Theorem about extensionality.

b: \$tType thf(b_type, type)
 g : \$tType thf(g_type, type)
g: $b \rightarrow \$o$ thf(g, type)
h: $(b \rightarrow \$o) \rightarrow g$ thf(h, type)
f: $b \rightarrow \$o$ thf(f, type)
 $(\forall xx: b: ((f@xx) \iff (g@xx)) \Rightarrow \forall xq: (b \rightarrow \$o) \rightarrow \$o: ((xq@f) \Rightarrow (xq@g))) \Rightarrow (\forall xx: b: ((f@xx) \iff (g@xx)) \Rightarrow (h@f) = (h@g))$ thf(cEXT1, conjecture)

SYO360^5.p TPS problem EDEC1

A version of EDEC from [Ben99] using = instead of LeibEq.

a: \$tType thf(a_type, type)
j: $a \rightarrow a$ thf(j, type)
g: $(a \rightarrow a) \rightarrow a \rightarrow a$ thf(g, type)
h: $a \rightarrow a$ thf(h, type)
f: $(a \rightarrow a) \rightarrow a \rightarrow a$ thf(f, type)
 $(\forall x: a \rightarrow a, y: a: (f@x@y) = (g@x@y) \text{ and } \forall z: a: (h@z) = (j@z)) \Rightarrow (f@h) = (g@j)$ thf(cEDEC1, conjecture)

SYO361^5.p TPS problem THM47

Shows equivalence of two definitions of =.

$\forall x: \$i, y: \$i: (x = y \iff \forall r: \$i \rightarrow \$i \rightarrow \$o: (\forall z: \$i: (r@z@z) \Rightarrow (r@x@y)))$ thf(cTHM47, conjecture)

SYO362^5.p TPS problem THM631A

If a set function preserves unions, then it is monotone.

cK: $(\$i \rightarrow \$o) \rightarrow \$i \rightarrow \o thf(cK, type)

$\forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (cK@\lambda xz: \$i: (x@xz \text{ or } y@xz)) = (\lambda xw: \$i: (cK@x@xw \text{ or } cK@y@xw)) \Rightarrow \forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (\forall xx: \$i: ((x@xx) \Rightarrow (y@xx)) \Rightarrow \forall xx: \$i: ((cK@x@xx) \Rightarrow (cK@y@xx)))$ thf(cTHM631A_pme, conjecture)

SYO363⁵.p TPS problem EDEC2

EDEC from [Ben99], using Primeq in the antecedent instead of Leibeq.

a: \$tType thf(a_type, type)

j: $a \rightarrow a$ thf(j, type)

g: $(a \rightarrow a) \rightarrow a \rightarrow a$ thf(g, type)

h: $a \rightarrow a$ thf(h, type)

f: $(a \rightarrow a) \rightarrow a \rightarrow a$ thf(f, type)

$(\forall x: a \rightarrow a, y: a: (f@x@y) = (g@x@y) \text{ and } \forall z: a: (h@z) = (j@z)) \Rightarrow \forall xq: (a \rightarrow a) \rightarrow \$o: ((xq@(f@h)) \Rightarrow (xq@(g@j)))$ thf(cEDEC2_pme, conjecture)

SYO364⁵.p TPS problem EDEC

Example from [Ben99] about decomposition (using Leibniz equality).

a: \$tType thf(a_type, type)

j: $a \rightarrow a$ thf(j, type)

g: $(a \rightarrow a) \rightarrow a \rightarrow a$ thf(g, type)

h: $a \rightarrow a$ thf(h, type)

f: $(a \rightarrow a) \rightarrow a \rightarrow a$ thf(f, type)

$(\forall x: a \rightarrow a, y: a, xq: a \rightarrow \$o: ((xq@(f@x@y)) \Rightarrow (xq@(g@x@y))) \text{ and } \forall z: a, xq: a \rightarrow \$o: ((xq@(h@z)) \Rightarrow (xq@(j@z))) \Rightarrow \forall xq: (a \rightarrow a) \rightarrow \$o: ((xq@(f@h)) \Rightarrow (xq@(g@j)))$ thf(cEDEC_pme, conjecture)

SYO365⁵.p TPS problem from EXTENSIONALITY

cA: \$o thf(cA_type, type)

cC: $\$o \rightarrow \o thf(cC_type, type)

cEXT₂: \$o thf(cEXT2_type, type)

cEXT_eq₀: $\$o \rightarrow \text{thf}(\text{cEXT_eq_0_type}, \text{type})$

cEXT₂ = $((\text{cC}@cA) \Rightarrow (\text{cC}@\neg\neg cA))$ thf(cEXT2_def, definition)

cEXT_eq₀ = $(\forall xp: \$o, xq: \$o: ((xp \iff xq) \Rightarrow xp = xq))$ thf(cEXT_eq_0_def, definition)

cEXT_eq₀ \Rightarrow cEXT₂ thf(cEXT2A, conjecture)

SYO366⁵.p TPS problem from EXTENSIONALITY

cP: $\$o \rightarrow \o thf(cP, type)

cA: \$o thf(cA, type)

$(cP@(cA \Rightarrow cA)) \Rightarrow (cP@\$true)$ thf(cTRIVEXT₄, conjecture)

SYO367⁵.p TPS problem from EXTENSIONALITY

cA: \$o thf(cA, type)

cC: $\$o \rightarrow \o thf(cC, type)

$(cC@cA) \Rightarrow (cC@\neg\neg cA)$ thf(cEXT₂, conjecture)

SYO368⁵.p TPS problem from EXTENSIONALITY

b: \$o thf(b, type)

cP: $\$o \rightarrow \o thf(cP, type)

a: \$o thf(a, type)

$((a \iff b) \text{ and } cP@a) \Rightarrow (cP@b)$ thf(cTRIVEXT₁, conjecture)

SYO369⁵.p TPS problem from EXTENSIONALITY

cB: \$o thf(cB, type)

cA: \$o thf(cA, type)

cP: $\$o \rightarrow \o thf(cP, type)

$(cP@(cA \Rightarrow cB)) \Rightarrow (cP@(\neg cA \text{ or } cB))$ thf(cTRIVEXT₅, conjecture)

SYO370⁵.p TPS problem from EXTENSIONALITY

cA: $\$i \rightarrow \o thf(cA, type)

cP: $(\$i \rightarrow \$o) \rightarrow \$o$ thf(cP, type)

$(cP@cA) \Rightarrow (cP@\lambda xx: \$i: \neg\neg cA@xx)$ thf(cTRIVEXT₃, conjecture)

SYO371⁵.p TPS problem from EXTENSIONALITY

cQ: \$o thf(cQ, type)

cP: \$o thf(cP, type)

$\forall r: \$o \rightarrow \$o: ((r@cP) \Rightarrow (r@cQ)) \Rightarrow (cP \iff cQ)$ thf(cEXT_O_LEIB, conjecture)

SYO372^5.p TPS problem from EXTENSIONALITY

$b: \$tType \quad \text{thf}(b_type, type)$
 $gtype: \$tType \quad \text{thf}(g_type, type)$
 $g: b \rightarrow \$o \quad \text{thf}(g, type)$
 $h: (b \rightarrow \$o) \rightarrow gtype \quad \text{thf}(h, type)$
 $f: b \rightarrow \$o \quad \text{thf}(f, type)$
 $\forall xx: b: ((f@xx) \iff (g@xx)) \Rightarrow (h@f) = (h@g) \quad \text{thf(cEXT_SET}_2\text{, conjecture)}$

SYO373^5.p TPS problem from EXTENSIONALITY

$b: \$tType \quad \text{thf}(b_type, type)$
 $g: b \rightarrow \$o \quad \text{thf}(g, type)$
 $f: b \rightarrow \$o \quad \text{thf}(f, type)$
 $\forall xx: b: ((f@xx) \iff (g@xx)) \Rightarrow \forall xq: (b \rightarrow \$o) \rightarrow \$o: ((xq@f) \Rightarrow (xq@g)) \quad \text{thf(cEXT_SET, conjecture)}$

SYO374^5.p TPS problem from EXTENSIONALITY

$cN: \$i \rightarrow \$i \quad \text{thf}(cN, type)$
 $cM: \$i \rightarrow \$i \quad \text{thf}(cM, type)$
 $\forall xx: \$i, xp: \$i \rightarrow \$o: ((xp@(cM@xx)) \iff (xp@(cN@xx))) \Rightarrow \forall xq: (\$i \rightarrow \$i) \rightarrow \$o: ((xq@cM) \iff (xq@cN)) \quad \text{thf(cEXT_A_LEIB}_2\text{, conjecture)}$

SYO375^5.p TPS problem from EXTENSIONALITY

$\forall x: \$i \rightarrow \$o, y: \$i \rightarrow \$o: (\forall w: \$i: ((x@w) \iff (y@w)) \Rightarrow \forall xx: \$i: ((x@xx) \iff (y@xx))) \quad \text{thf(cEXT_SET}_1\text{, conjecture)}$

SYO376^5.p TPS problem from EXTENSIONALITY

$cG: \$o \rightarrow \$o \quad \text{thf}(cG, type)$
 $cF: \$o \rightarrow \$o \quad \text{thf}(cF, type)$
 $cP: (\$o \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cP, type)$
 $(cP@cF \text{ and } cP@cG) \Rightarrow (cP@\lambda xx: \$o: (cF@xx \text{ or } cG@xx) \text{ or } cP@\lambda xx: \$o: (\neg cF@xx \text{ or } cG@xx)) \quad \text{thf(cTHM}_{627}\text{, conjecture)}$

SYO377^5.p TPS problem from EXTENSIONALITY

$b: \$o \quad \text{thf}(b, type)$
 $a: \$o \quad \text{thf}(a, type)$
 $\forall xx: \$o, xy: \$o: ((xx \iff xy) \Rightarrow \forall xq: \$o \rightarrow \$o: ((xq@xx) \Rightarrow (xq@xy))) \Rightarrow \forall xp: \$o \rightarrow \$o: ((xp@a \text{ and } xp@b) \Rightarrow (xp@(a \text{ and } b))) \quad \text{thf(cEXT_IMP_EMB}_2\text{, conjecture)}$

SYO378^5.p TPS problem from QUANTDEPTH-THMS

$c: \$i \quad \text{thf}(c_type, type)$
 $cQDP_0: \$i \rightarrow \$o \quad \text{thf}(cQDP0_type, type)$
 $cQDP_1: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cQDP1_type, type)$
 $cQDP_2: ((\$i \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cQDP2_type, type)$
 $cQDP_0 = (\lambda xz: \$i: xz = c) \quad \text{thf}(cQDP0_def, definition)$
 $cQDP_1 = (\lambda xz: \$i \rightarrow \$o: (xz = cQDP_0 \text{ and } \exists xt: \$i: (xz@xt))) \quad \text{thf}(cQDP1_def, definition)$
 $cQDP_2 = (\lambda xz: (\$i \rightarrow \$o) \rightarrow \$o: (xz = cQDP_1 \text{ and } \exists xt: \$i \rightarrow \$o: (xz@xt))) \quad \text{thf}(cQDP2_def, definition)$
 $\exists xs: (\$i \rightarrow \$o) \rightarrow \$o: (cQDP_2@xs) \quad \text{thf}(cQDTHM}_2\text{, conjecture})$

SYO379^5.p TPS problem from QUANTDEPTH-THMS

$c: \$i \quad \text{thf}(c_type, type)$
 $cQDP_0: \$i \rightarrow \$o \quad \text{thf}(cQDP0_type, type)$
 $cQDP_1: (\$i \rightarrow \$o) \rightarrow \$o \quad \text{thf}(cQDP1_type, type)$
 $cQDP_0 = (\lambda xz: \$i: xz = c) \quad \text{thf}(cQDP0_def, definition)$
 $cQDP_1 = (\lambda xz: \$i \rightarrow \$o: (xz = cQDP_0 \text{ and } \exists xt: \$i: (xz@xt))) \quad \text{thf}(cQDP1_def, definition)$
 $\exists xs: \$i \rightarrow \$o: (cQDP_1@xs) \quad \text{thf}(cQDTHM}_1\text{, conjecture})$

SYO380^5.p TPS problem X-2002-12-17-B

Suggested by a student's answer on a test.

$cR: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cR, type)$
 $x: \$i \quad \text{thf}(x, type)$
 $\exists xy: \$i: ((cR@x@xy) \Rightarrow \exists xx_0: \$i: (cR@xy@xx_0)) \quad \text{thf}(cX_2002_12_17_B, conjecture)$

SYO381^5.p TPS problem X-2002-12-17

Suggested by a student's answer on a test.

$cR: \$i \rightarrow \$i \rightarrow \$o \quad \text{thf}(cR, type)$
 $\exists xx: \$i: (\exists xz: \$i: \forall xy: \$i: (cR@xz@xy) \iff \forall xw: \$i: (cR@xx@xw)) \quad \text{thf}(cX_2002_12_{17}, conjecture)$

SYO382^5.p TPS problem THM407


```

d: $i      thf(d, type)
c: $i      thf(c, type)
b: $i      thf(b, type)
a: $i      thf(a, type)
cP3: $i → $i → $o      thf(cP3, type)
cQ3: $i → $i → $i → $o      thf(cQ3, type)
cP2: $i → $i → $o      thf(cP2, type)
cQ2: $i → $i → $i → $o      thf(cQ2, type)
cP1: $i → $i → $o      thf(cP1, type)
cQ1: $i → $i → $i → $o      thf(cQ1, type)
¬cQ1@a@b@c and cP1@a@a and cP1@b@b and cP1@c@c and ∀xx: $i: (cP1@d@xx) and ∀xx: $i, xy: $i: (¬cP1@xx@xy or cP1@y@x)

```

SYO389^5.p TPS problem from MISC

Reflexivity of constants.

```

b: $tType      thf(b_type, type)
cA: b → $o      thf(cA, type)
cA = cA      thf(cEQ_THM, conjecture)

```

SYO390^5.p TPS problem UNKNOWN

```
∀xp: $o, xq: $o: ((xp ⇐⇒ xq) ⇒ xp = xq)      thf(cEXT_eq0, conjecture)
```

SYO392^1.p Ted Sider's modal proposition logic theorem 01

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mnot@(mbox_k@(mdia_k@(mbox_k@p))))@((mdia_k@(mbox_k@(mdia_k@(mnot@p))))))      thf(prove, conj)

```

SYO393^1.p Ted Sider's modal proposition logic theorem 02

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mnot@(mdia_k@(mor@p@q)))@((mand@(mnot@(mdia_k@p))@((mnot@(mdia_k@q))))))      thf(prove, conj)

```

SYO394^1.p Ted Sider's modal proposition logic theorem 03

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@((mdia_k@(mand@p@q))@((mand@((mdia_k@p)@((mdia_k@q))))))      thf(prove, conjecture)

```

SYO395^1.p Ted Sider's modal proposition logic theorem 04

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mbox_k@(mimplies@(mnot@p)@p))@((mbox_k@p)))      thf(prove, conjecture)

```

SYO396^1.p Ted Sider's modal proposition logic theorem 05

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mbox_k@(mimplies@p@(mnot@p)))@((mnot@(mdia_k@p))))      thf(prove, conjecture)

```

SYO397^1.p Ted Sider's modal proposition logic theorem 06

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_k@p)@((mbox_k@(mimplies@q@p))))      thf(prove, conjecture)

```

SYO398^1.p Ted Sider's modal proposition logic theorem 07

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)

```

mvalid(@(mimplies(@(mbox_k@(mnot@p))@(@mbox_k@mimplies@p@q))) thf(prove, conjecture)

SYO399¹.p Ted Sider's modal proposition logic theorem 08
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 $r: \$i \rightarrow \o thf(r_type, type)
 mvalid(@(mequiv(@(mnot@(mdia_k@(mand@q@r))))@(@mbox_k@mimplies@q@mnot@r))) thf(prove, conjecture)

SYO400¹.p Ted Sider's modal proposition logic theorem 09
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 mvalid(@(mimplies(@(mand@mbox_k@mimplies@p@q))@(@mbox_k@mimplies@p@mnot@q)))@(@mbox_k@mimplies@p@mnot@q) thf(prove, conjecture)

SYO401¹.p Ted Sider's modal proposition logic theorem 10
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 mvalid(@(mimplies(@(mand@mbox_k@p)@(@mbox_k@q))@(@mbox_k@(mand@p@q))) thf(prove, conjecture)

SYO402¹.p Ted Sider's modal proposition logic theorem 11
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 mvalid(@(mimplies(@(mbox_k@(mequiv@p@q))@(@mequiv@mbox_k@p)@(@mbox_k@q))) thf(prove, conjecture)

SYO403¹.p Ted Sider's modal proposition logic theorem 12
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 mvalid(@(mimplies(@(mor@mbox_k@p)@(@mbox_k@q))@(@mbox_k@(mor@p@q))) thf(prove, conjecture)

SYO404¹.p Ted Sider's modal proposition logic theorem 13
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 mvalid(@(mequiv(@(mand@mbox_k@mimplies@q@p))@(@mbox_k@mimplies@mnot@q@p))@(@mbox_k@p)) thf(prove, conjecture)

SYO405¹.p Ted Sider's modal proposition logic theorem 14
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 mvalid(@(mimplies(@(mand@mbox_k@mimplies@p@q))@(@mbox_k@mimplies@p@mnot@q))@(@mnot@mdia_k@p)) thf(prove, conjecture)

SYO406¹.p Ted Sider's modal proposition logic theorem 15
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 mvalid(@(mimplies(@(mbox_k@(mor@p@q))@(@mor@mbox_k@p)@(@mdia_k@q))) thf(prove, conjecture)

SYO407¹.p Ted Sider's modal proposition logic theorem 16
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 $q: \$i \rightarrow \o thf(q_type, type)
 $r: \$i \rightarrow \o thf(r_type, type)

mvalid@($\text{mimplies} @ (\text{mand} @ (\text{mbox_k} @ (\text{mimplies} @ p @ q)) @ (\text{mbox_k} @ (\text{mimplies} @ q @ r))) @ (\text{mbox_k} @ (\text{mimplies} @ p @ r))$) thf(

SYO408 \wedge 1.p Ted Sider's modal proposition logic theorem 17
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 $q: \$i \rightarrow \$o \quad \text{thf}(q_type, type)$
 mvalid@($\text{mimplies} @ (\text{mand} @ (\text{mbox_k} @ p) @ (\text{mbox_k} @ q)) @ (\text{mbox_k} @ (\text{mequiv} @ p @ q))$) thf(prove, conjecture)

SYO409 \wedge 1.p Ted Sider's modal proposition logic theorem 18
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 $q: \$i \rightarrow \$o \quad \text{thf}(q_type, type)$
 mvalid@($\text{mequiv} @ (\text{mdia_k} @ (\text{mor} @ p @ q)) @ (\text{mor} @ (\text{mdia_k} @ p) @ (\text{mdia_k} @ q))$) thf(prove, conjecture)

SYO410 \wedge 1.p Ted Sider's modal proposition logic theorem 19
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 $q: \$i \rightarrow \$o \quad \text{thf}(q_type, type)$
 mvalid@($\text{mimplies} @ (\text{mand} @ (\text{mdia_k} @ p) @ (\text{mbox_k} @ q)) @ (\text{mdia_k} @ (\text{mand} @ p @ q))$) thf(prove, conjecture)

SYO411 \wedge 1.p Ted Sider's modal proposition logic theorem 20
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 $q: \$i \rightarrow \$o \quad \text{thf}(q_type, type)$
 $r: \$i \rightarrow \$o \quad \text{thf}(r_type, type)$
 mvalid@($\text{mimplies} @ (\text{mand} @ (\text{mbox_k} @ (\text{mimplies} @ p @ q)) @ (\text{mdia_k} @ (\text{mand} @ p @ r))) @ (\text{mdia_k} @ (\text{mand} @ q @ r))$) thf(prove, c

SYO412 \wedge 1.p Ted Sider's modal proposition logic theorem 21
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 $q: \$i \rightarrow \$o \quad \text{thf}(q_type, type)$
 mvalid@($\text{mequiv} @ (\text{mdia_k} @ (\text{mimplies} @ p @ q)) @ (\text{mimplies} @ (\text{mbox_k} @ p) @ (\text{mdia_k} @ q))$) thf(prove, conjecture)

SYO413 \wedge 1.p Ted Sider's modal proposition logic theorem 22
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 $q: \$i \rightarrow \$o \quad \text{thf}(q_type, type)$
 mvalid@($\text{mimplies} @ (\text{mdia_k} @ p) @ (\text{mimplies} @ (\text{mbox_k} @ q) @ (\text{mdia_k} @ q))$) thf(prove, conjecture)

SYO414 \wedge 1.p Ted Sider's modal proposition logic theorem 23
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 $q: \$i \rightarrow \$o \quad \text{thf}(q_type, type)$
 $r: \$i \rightarrow \$o \quad \text{thf}(r_type, type)$
 mvalid@($\text{mimplies} @ (\text{mdia_k} @ (\text{mimplies} @ p @ (\text{mand} @ q @ r))) @ (\text{mand} @ (\text{mimplies} @ (\text{mbox_k} @ p) @ (\text{mdia_k} @ q))) @ (\text{mimplies} @ (\text{mbox_k} @ (\text{mbox_k} @ (\text{mimplies} @ p @ q)) @ (\text{mdia_k} @ (\text{mbox_k} @ r))))$) thf(

SYO415 \wedge 1.p Ted Sider's modal proposition logic theorem 24
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 $q: \$i \rightarrow \$o \quad \text{thf}(q_type, type)$
 mvalid@($\text{mimplies} @ (\text{mand} @ (\text{mbox_k} @ (\text{mdia_k} @ p)) @ (\text{mdia_k} @ (\text{mbox_k} @ (\text{mimplies} @ p @ q)))) @ (\text{mdia_k} @ (\text{mdia_k} @ q))$) thf(

SYO416 \wedge 1.p Ted Sider's modal proposition logic theorem 25
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^2.ax')
 $p: \$i \rightarrow \$o \quad \text{thf}(p_type, type)$
 mvalid@($\text{mimplies} @ (\text{mbox_d} @ (\text{mbox_d} @ p)) @ (\text{mbox_d} @ (\text{mdia_d} @ p))$) thf(prove, conjecture)

SYO417¹.p Ted Sider's modal proposition logic theorem 26

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_d@(mbox_d@p))@(mdia_d@(mdia_d@p)))      thf(prove, conjecture)
```

SYO418¹.p Ted Sider's modal proposition logic theorem 27

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_d@p)@(mdia_d@mimplies@q@p)))      thf(prove, conjecture)
```

SYO419¹.p Ted Sider's modal proposition logic theorem 28

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mnot@(mbox_d@(mand@p@mnot@p)))      thf(prove, conjecture)
```

SYO420¹.p Ted Sider's modal proposition logic theorem 29

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_d@p)@(mbox_d@mimplies@p@q))@(mdia_d@q))      thf(prove, conjecture)
```

SYO421¹.p Ted Sider's modal proposition logic theorem 30

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mnot@(mand@(mbox_d@p)@(mbox_d@mnot@p)))      thf(prove, conjecture)
```

SYO422¹.p Ted Sider's modal proposition logic theorem 31

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mdia_d@mimplies@mimplies@p@q@p))      thf(prove, conjecture)
```

SYO423¹.p Ted Sider's modal proposition logic theorem 32

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mnot@mbox_d@(mand@mbox_d@(mand@p@q))@(mbox_d@mimplies@p@mnot@q))))      thf(prove, conjecture)
```

SYO424¹.p Ted Sider's modal proposition logic theorem 33

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@mor@mdia_d@mnot@p)@(mdia_d@mnot@q))@mdia_d@mand@p@q))      thf(prove, conjecture)
```

SYO425¹.p Ted Sider's modal proposition logic theorem 34

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
mvalid@mimplies@mbox_m@p@mbox_m@mdia_m@p))      thf(prove, conjecture)
```

SYO426¹.p Ted Sider's modal proposition logic theorem 35

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@mimplies@mdia_m@mbox_m@p@mbox_m@mor@p@q))      thf(prove, conjecture)
```

SYO427¹.p Ted Sider's modal proposition logic theorem 36

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@((mbox_m@p)@(mdia_m@((mbox_m@mimplies@p@q))))@(mdia_m@q)))      thf(prove, conjecture)
```

SYO428¹.p Ted Sider's modal proposition logic theorem 37

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mdia_m@mimplies@p@(mbox_m@q))@(mimplies@mbox_m@p@(mdia_m@q)))      thf(prove, conjecture)
```

SYO429¹.p Ted Sider's modal proposition logic theorem 38

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mdia_b@mbox_b@p)@(mbox_b@mdia_b@p))      thf(prove, conjecture)
```

SYO430¹.p Ted Sider's modal proposition logic theorem 39

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mdia_b@mbox_b@p)@(mdia_b@mbox_b@(mdia_b@mbox_b@p)))      thf(prove, conjecture)
```

SYO431¹.p Ted Sider's modal proposition logic theorem 40

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mnot@(mdia_b@((mand@((mdia_b@mbox_b@(mdia_b@p))@((mbox_b@mnot@p)))))))      thf(prove, conjecture)
```

SYO432¹.p Ted Sider's modal proposition logic theorem 41

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@mbox_b@p)@(mbox_b@mdia_b@mbox_b@mimplies@p@q))@((mbox_b@q))      thf(prove, conjecture)
```

SYO433¹.p Ted Sider's modal proposition logic theorem 42

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@((mdia_s4@p)@(mbox_s4@q))@(mdia_s4@((mand@p@mbox_s4@q))))))      thf(prove, conjecture)
```

SYO434¹.p Ted Sider's modal proposition logic theorem 43

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_s4@p)@(mbox_s4@((mdia_s4@mbox_s4@p))))      thf(prove, conjecture)
```

SYO435¹.p Ted Sider's modal proposition logic theorem 44

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_s4@((mdia_s4@p)@(mbox_s4@((mdia_s4@mbox_s4@p))))))      thf(prove, conjecture)
```

SYO436¹.p Ted Sider's modal proposition logic theorem 45

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mor@mbox_s4@p)@(mbox_s4@q))@((mbox_s4@((mor@mbox_s4@p)@(mbox_s4@q))))      thf(prove, conjecture)
```

SYO437¹.p Ted Sider's modal proposition logic theorem 46

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@(mimplies@(mbox_s4@(mimplies@(mbox_s4@(mequiv@p@q))@r))@ (mbox_s4@(mimplies@(mbox_s4@(mequiv@p@q))))
```

SYO438¹.p Ted Sider's modal proposition logic theorem 47

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_s4@(mdia_s4@(mbox_s4@(mdia_s4@p))))@ (mbox_s4@(mdia_s4@p)))      thf(prove, conjecture)
```

SYO439¹.p Ted Sider's modal proposition logic theorem 48

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mdia_s4@(mbox_s4@p))@ (mdia_s4@(mbox_s4@(mdia_s4@mbox_s4@p))))      thf(prove, conjecture)
```

SYO440¹.p Ted Sider's modal proposition logic theorem 49

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mdia_s4@(mbox_s4@(mdia_s4@mbox_s4@p))))@ (mdia_s4@(mbox_s4@p))      thf(prove, conjecture)
```

SYO441¹.p Ted Sider's modal proposition logic theorem 50

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mdia_s5@(mdia_s5@(mdia_s5@mbox_s5@p))))@ (mbox_s5@p)      thf(prove, conjecture)
```

SYO442¹.p Ted Sider's modal proposition logic theorem 51

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mbox_s5@(mdia_s5@(mdia_s5@mbox_s5@p))))@ (mbox_s5@mbox_s5@p)      thf(prove, conjecture)
```

SYO443¹.p Ted Sider's modal proposition logic theorem 52

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mequiv@(mbox_s5@(mor@(mnot@p)@ (mbox_s5@q)))@ (mor@(mbox_s5@mnot@p)@ (mbox_s5@q)))      thf(prove, conjecture)
```

SYO444¹.p Ted Sider's modal proposition logic theorem 53

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mequiv@(mbox_s5@(mor@(mnot@p)@ (mdia_s5@q)))@ (mor@(mnot@(mdia_s5@p))@ (mdia_s5@q)))      thf(prove, conjecture)
```

SYO445¹.p Ted Sider's modal proposition logic theorem 54

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mequiv@(mor@(mbox_s5@p)@ (mdia_s5@q))@ (mbox_s5@(mor@p@ (mdia_s5@q))))      thf(prove, conjecture)
```

SYO446¹.p Ted Sider's modal proposition logic theorem 55

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mequiv@(mdia_s5@(mand@p@ (mdia_s5@q)))@ (mand@(mdia_s5@p)@ (mdia_s5@q)))      thf(prove, conjecture)
```

SYO447^1.p Ted Sider's modal proposition logic theorem 56

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mequiv@(mand@((mdia_s5@p)@(mbox_s5@q))@((mdia_s5@(mand@p@mbox_s5@q))))      thf(prove, conjecture)
```

SYO448^1.p Ted Sider's modal proposition logic theorem 57

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@mbox_s5@mimplies@mbox_s5@p)@((mbox_s5@mimplies@mbox_s5@q@mbox_s5@p)))      thf(prove, conjecture)
```

SYO449^1.p Ted Sider's modal proposition logic theorem 58

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@mbox_s5@(mequiv@mbox_s5@mimplies@((mdia_s5@p)@q))@((mbox_s5@mimplies@p@mbox_s5@q)))      thf(prove, conjecture)
```

SYO450^1.p Ted Sider's propositional modal logic wff 01

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@mimplies@mbox_k@(mimplies@p@((mdia_k@mimplies@q@r)))@((mdia_k@mimplies@q@mimplies@mbox_k@p)))      thf(prove, conjecture)
```

SYO450^2.p Ted Sider's propositional modal logic wff 01

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@mimplies@mbox_d@(mimplies@p@((mdia_d@mimplies@q@r)))@((mdia_d@mimplies@q@mimplies@mbox_d@p)))      thf(prove, conjecture)
```

SYO450^3.p Ted Sider's propositional modal logic wff 01

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@mimplies@mbox_m@(mimplies@p@((mdia_m@mimplies@q@r)))@((mdia_m@mimplies@q@mimplies@mbox_m@p)))      thf(prove, conjecture)
```

SYO450^4.p Ted Sider's propositional modal logic wff 01

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@mimplies@mbox_b@(mimplies@p@((mdia_b@mimplies@q@r)))@((mdia_b@mimplies@q@mimplies@mbox_b@p)))      thf(prove, conjecture)
```

SYO450^5.p Ted Sider's propositional modal logic wff 01

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@mimplies@mbox_s4@(mimplies@p@((mdia_s4@mimplies@q@r)))@((mdia_s4@mimplies@q@mimplies@mbox_s4@p)))      thf(prove, conjecture)
```

SYO450^6.p Ted Sider's propositional modal logic wff 01

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
```

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

r: \$i → \$o thf(r_type, type)

mvalid@(mimplies@(mbox_s5@(mimplies@p@(mdia_s5@(mimplies@q@r))))@((mdia_s5@(mimplies@q@(mimplies@mbox_s5@)))

SYO451 \wedge 1.p Ted Sider's propositional modal logic wff 02

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^1.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@(mimplies@(mbox_k@(mor@p@(mdia_k@q))))@((mor@mbox_k@p)@((mdia_k@q))) thf(prove, conjecture)

SYO451 \wedge 2.p Ted Sider's propositional modal logic wff 02

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^2.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@(mimplies@(mbox_d@(mor@p@(mdia_d@q))))@((mor@mbox_d@p)@((mdia_d@q))) thf(prove, conjecture)

SYO451 \wedge 3.p Ted Sider's propositional modal logic wff 02

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^3.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@(mimplies@(mbox_m@(mor@p@(mdia_m@q))))@((mor@mbox_m@p)@((mdia_m@q))) thf(prove, conjecture)

SYO451 \wedge 4.p Ted Sider's propositional modal logic wff 02

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^4.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@(mimplies@(mbox_b@(mor@p@(mdia_b@q))))@((mor@mbox_b@p)@((mdia_b@q))) thf(prove, conjecture)

SYO451 \wedge 5.p Ted Sider's propositional modal logic wff 02

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^5.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@(mimplies@(mbox_s4@(mor@p@(mdia_s4@q))))@((mor@mbox_s4@p)@((mdia_s4@q))) thf(prove, conjecture)

SYO451 \wedge 6.p Ted Sider's propositional modal logic wff 02

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^6.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@(mimplies@(mbox_s5@(mor@p@(mdia_s5@q))))@((mor@mbox_s5@p)@((mdia_s5@q))) thf(prove, conjecture)

SYO452 \wedge 1.p Ted Sider's propositional modal logic wff 03

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^1.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@(mimplies@(mdia_k@(mand@p@(mdia_k@q))))@((mimplies@mbox_k@((mdia_k@p))@((mdia_k@mbox_k@p)))) thf

SYO452 \wedge 2.p Ted Sider's propositional modal logic wff 03

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^2.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@(mimplies@(mdia_d@(mand@p@(mdia_d@q))))@((mimplies@mbox_d@((mdia_d@p))@((mdia_d@mbox_d@p)))) thf

SYO452 \wedge 3.p Ted Sider's propositional modal logic wff 03

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^3.ax')

p: \$i → \$o thf(p_type, type)

$q: \$i \rightarrow \$o \quad \text{thf(q_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mdia_m} @ (\text{mand} @ p @ (\text{mdia_m} @ q))) @ (\text{mimplies} @ (\text{mbox_m} @ (\text{mdia_m} @ p)) @ (\text{mdia_m} @ (\text{mbox_m} @ p))))$

SYO452^4.p Ted Sider's propositional modal logic wff 03

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mdia\_b} @ (\text{mand} @ p @ (\text{mdia\_b} @ q))) @ (\text{mimplies} @ (\text{mbox\_b} @ (\text{mdia\_b} @ p)) @ (\text{mdia\_b} @ (\text{mbox\_b} @ p))))$


```

SYO452^5.p Ted Sider's propositional modal logic wff 03

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mdia\_s}_4 @ (\text{mand} @ p @ (\text{mdia\_s}_4 @ q))) @ (\text{mimplies} @ (\text{mbox\_s}_4 @ (\text{mdia\_s}_4 @ p)) @ (\text{mdia\_s}_4 @ (\text{mbox\_s}_4 @ p))))$


```

SYO452^6.p Ted Sider's propositional modal logic wff 03

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mdia\_s}_5 @ (\text{mand} @ p @ (\text{mdia\_s}_5 @ q))) @ (\text{mimplies} @ (\text{mbox\_s}_5 @ (\text{mdia\_s}_5 @ p)) @ (\text{mdia\_s}_5 @ (\text{mbox\_s}_5 @ p))))$


```

SYO453^1.p Ted Sider's propositional modal logic wff 04

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mbox\_k} @ (\text{mequiv} @ (\text{mdia\_k} @ p) @ (\text{mdia\_k} @ q))) @ (\text{mbox\_k} @ (\text{mequiv} @ p @ (\text{mbox\_k} @ q))))$


```

SYO453^2.p Ted Sider's propositional modal logic wff 04

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mbox\_d} @ (\text{mequiv} @ (\text{mdia\_d} @ p) @ (\text{mdia\_d} @ q))) @ (\text{mbox\_d} @ (\text{mequiv} @ p @ (\text{mbox\_d} @ q))))$


```

SYO453^3.p Ted Sider's propositional modal logic wff 04

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mbox\_m} @ (\text{mequiv} @ (\text{mdia\_m} @ p) @ (\text{mdia\_m} @ q))) @ (\text{mbox\_m} @ (\text{mequiv} @ p @ (\text{mbox\_m} @ q))))$


```

SYO453^4.p Ted Sider's propositional modal logic wff 04

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mbox\_b} @ (\text{mequiv} @ (\text{mdia\_b} @ p) @ (\text{mdia\_b} @ q))) @ (\text{mbox\_b} @ (\text{mequiv} @ p @ (\text{mbox\_b} @ q))))$


```

SYO453^5.p Ted Sider's propositional modal logic wff 04

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mbox\_s}_4 @ (\text{mequiv} @ (\text{mdia\_s}_4 @ p) @ (\text{mdia\_s}_4 @ q))) @ (\text{mbox\_s}_4 @ (\text{mequiv} @ p @ (\text{mbox\_s}_4 @ q))))$


```

SYO453^6.p Ted Sider's propositional modal logic wff 04

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')


$p: \$i \rightarrow \$o \quad \text{thf(p\_type, type)}$



$q: \$i \rightarrow \$o \quad \text{thf(q\_type, type)}$



$\text{mvalid} @ (\text{mimplies} @ (\text{mbox\_s}_5 @ (\text{mequiv} @ (\text{mdia\_s}_5 @ p) @ (\text{mdia\_s}_5 @ q))) @ (\text{mbox\_s}_5 @ (\text{mequiv} @ p @ (\text{mbox\_s}_5 @ q))))$


```

SYO454^1.p Ted Sider's propositional modal logic wff 05

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@((mdia_k@(mand@((mdia_k@p)@(mnot@q)))@((mbox_k@(mimplies@p@(mbox_k@q))))))      thf(prove, conjecture)
```

SYO454^2.p Ted Sider's propositional modal logic wff 05

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@((mdia_d@(mand@((mdia_d@p)@(mnot@q)))@((mbox_d@(mimplies@p@(mbox_d@q))))))      thf(prove, conjecture)
```

SYO454^3.p Ted Sider's propositional modal logic wff 05

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@((mdia_m@(mand@((mdia_m@p)@(mnot@q)))@((mbox_m@(mimplies@p@(mbox_m@q))))))      thf(prove, conjecture)
```

SYO454^4.p Ted Sider's propositional modal logic wff 05

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@((mdia_b@(mand@((mdia_b@p)@(mnot@q)))@((mbox_b@(mimplies@p@(mbox_b@q))))))      thf(prove, conjecture)
```

SYO454^5.p Ted Sider's propositional modal logic wff 05

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@((mdia_s4@(mand@((mdia_s4@p)@(mnot@q)))@((mbox_s4@(mimplies@p@(mbox_s4@q))))))      thf(prove, conjecture)
```

SYO454^6.p Ted Sider's propositional modal logic wff 05

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@((mdia_s5@(mand@((mdia_s5@p)@(mnot@q)))@((mbox_s5@(mimpires@p@(mbox_s5@q))))))      thf(prove, conjecture)
```

SYO455^1.p Ted Sider's propositional modal logic wff 06

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@((mimpires@((mand@((mbox_k@p)@((mbox_k@(mor@((mnot@p)@q))))@((mdia_k@q))))))      thf(prove, conjecture)
```

SYO455^2.p Ted Sider's propositional modal logic wff 06

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@((mimpires@((mand@((mbox_d@p)@((mbox_d@(mor@((mnot@p)@q))))@((mdia_d@q))))))      thf(prove, conjecture)
```

SYO455^3.p Ted Sider's propositional modal logic wff 06

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@((mimpires@((mand@((mbox_m@p)@((mbox_m@(mor@((mnot@p)@q))))@((mdia_m@q))))))      thf(prove, conjecture)
```

SYO455^4.p Ted Sider's propositional modal logic wff 06

```
include('Axioms/LCL013^0.ax')
```

```

include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_b@p)@(mbox_b@(mor@(mnot@p)@q)))@(mdia_b@q))      thf(prove, conjecture)

SYO455^5.p Ted Sider's propositional modal logic wff 06
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_s4@p)@(mbox_s4@(mor@(mnot@p)@q)))@(mdia_s4@q))      thf(prove, conjecture)

SYO455^6.p Ted Sider's propositional modal logic wff 06
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_s5@p)@(mbox_s5@(mor@(mnot@p)@q)))@(mdia_s5@q))      thf(prove, conjecture)

SYO456^1.p Ted Sider's propositional modal logic wff 07
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mdia_k@(mbox_k@(mdia_k@(mbox_k@p))))@(mdia_k@(mbox_k@p)))      thf(prove, conjecture)

SYO456^2.p Ted Sider's propositional modal logic wff 07
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mdia_d@(mbox_d@(mdia_d@(mbox_d@p))))@(mdia_d@(mbox_d@p)))      thf(prove, conjecture)

SYO456^3.p Ted Sider's propositional modal logic wff 07
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mdia_m@(mbox_m@(mdia_m@(mbox_m@p))))@(mdia_m@(mbox_m@p)))      thf(prove, conjecture)

SYO456^4.p Ted Sider's propositional modal logic wff 07
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mdia_b@(mbox_b@(mdia_b@(mbox_b@p))))@(mdia_b@(mbox_b@p)))      thf(prove, conjecture)

SYO456^6.p Ted Sider's propositional modal logic wff 07
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mdia_s5@(mbox_s5@(mdia_s5@(mbox_s5@p))))@(mdia_s5@(mbox_s5@p)))      thf(prove, conjecture)

SYO457^1.p Ted Sider's propositional modal logic wff 08
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mdia_k@(mbox_k@p))@(mdia_k@(mimplies@q@p)))      thf(prove, conjecture)

SYO457^2.p Ted Sider's propositional modal logic wff 08
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mdia_d@(mbox_d@p))@(mdia_d@(mimplies@q@p)))      thf(prove, conjecture)

SYO457^3.p Ted Sider's propositional modal logic wff 08

```

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mdia_m@(mbox_m@p))@(mdia_m@mimplies@q@p)))    thf(prove, conjecture)

```

SYO457^4.p Ted Sider's propositional modal logic wff 08

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mdia_b@(mbox_b@p))@(mdia_b@mimplies@q@p)))    thf(prove, conjecture)

```

SYO457^5.p Ted Sider's propositional modal logic wff 08

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mdia_s4@(mbox_s4@p))@(mdia_s4@mimplies@q@p)))    thf(prove, conjecture)

```

SYO457^6.p Ted Sider's propositional modal logic wff 08

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mdia_s5@(mbox_s5@p))@(mdia_s5@mimplies@q@p)))    thf(prove, conjecture)

```

SYO458^2.p Ted Sider's propositional modal logic wff 09

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_d@p)@(mbox_d@q))@(mbox_d@(mequiv@p@q)))    thf(prove, conjecture)

```

SYO458^3.p Ted Sider's propositional modal logic wff 09

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_m@p)@(mbox_m@q))@(mbox_m@(mequiv@p@q)))    thf(prove, conjecture)

```

SYO458^4.p Ted Sider's propositional modal logic wff 09

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_b@p)@(mbox_b@q))@(mbox_b@(mequiv@p@q)))    thf(prove, conjecture)

```

SYO458^5.p Ted Sider's propositional modal logic wff 09

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_s4@p)@(mbox_s4@q))@(mbox_s4@(mequiv@p@q)))    thf(prove, conjecture)

```

SYO458^6.p Ted Sider's propositional modal logic wff 09

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mand@(mbox_s5@p)@(mbox_s5@q))@(mbox_s5@(mequiv@p@q)))    thf(prove, conjecture)

```

SYO459^1.p Ted Sider's propositional modal logic wff 10

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')

```

$p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mdia_k} @ (\text{mbox_k} @ p)) @ (\text{mdia_k} @ (\text{mbox_k} @ (\text{mdia_k} @ (\text{mbox_k} @ p))))) \quad \text{thf(prove, conjecture)}$

SYO459^{^2}.p Ted Sider's propositional modal logic wff 10
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^2.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mdia_d} @ (\text{mbox_d} @ p)) @ (\text{mdia_d} @ (\text{mbox_d} @ (\text{mdia_d} @ p)))) \quad \text{thf(prove, conjecture)}$

SYO459^{^3}.p Ted Sider's propositional modal logic wff 10
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^3.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mdia_m} @ (\text{mbox_m} @ p)) @ (\text{mdia_m} @ (\text{mbox_m} @ (\text{mdia_m} @ p)))) \quad \text{thf(prove, conjecture)}$

SYO459^{^4}.p Ted Sider's propositional modal logic wff 10
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^4.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mdia_b} @ (\text{mbox_b} @ p)) @ (\text{mdia_b} @ (\text{mbox_b} @ (\text{mdia_b} @ p)))) \quad \text{thf(prove, conjecture)}$

SYO459^{^6}.p Ted Sider's propositional modal logic wff 10
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mdia_s5} @ (\text{mbox_s5} @ p)) @ (\text{mdia_s5} @ (\text{mbox_s5} @ (\text{mdia_s5} @ p)))) \quad \text{thf(prove, conjecture)}$

SYO460^{^1}.p Ted Sider's propositional modal logic wff 11
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $q: \$i \rightarrow \$o \quad \text{thf(q_type, type)}$
 $r: \$i \rightarrow \$o \quad \text{thf(r_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mor} @ (\text{mdia_k} @ (\text{mand} @ p @ q)) @ (\text{mdia_k} @ (\text{mand} @ p @ r))) @ (\text{mdia_k} @ p)) \quad \text{thf(prove, conjecture)}$

SYO460^{^2}.p Ted Sider's propositional modal logic wff 11
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^2.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $q: \$i \rightarrow \$o \quad \text{thf(q_type, type)}$
 $r: \$i \rightarrow \$o \quad \text{thf(r_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mor} @ (\text{mdia_d} @ (\text{mand} @ p @ q)) @ (\text{mdia_d} @ (\text{mand} @ p @ r))) @ (\text{mdia_d} @ p)) \quad \text{thf(prove, conjecture)}$

SYO460^{^3}.p Ted Sider's propositional modal logic wff 11
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^3.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $q: \$i \rightarrow \$o \quad \text{thf(q_type, type)}$
 $r: \$i \rightarrow \$o \quad \text{thf(r_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mor} @ (\text{mdia_m} @ (\text{mand} @ p @ q)) @ (\text{mdia_m} @ (\text{mand} @ p @ r))) @ (\text{mdia_m} @ p)) \quad \text{thf(prove, conjecture)}$

SYO460^{^4}.p Ted Sider's propositional modal logic wff 11
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^4.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $q: \$i \rightarrow \$o \quad \text{thf(q_type, type)}$
 $r: \$i \rightarrow \$o \quad \text{thf(r_type, type)}$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mor} @ (\text{mdia_b} @ (\text{mand} @ p @ q)) @ (\text{mdia_b} @ (\text{mand} @ p @ r))) @ (\text{mdia_b} @ p)) \quad \text{thf(prove, conjecture)}$

SYO460^{^5}.p Ted Sider's propositional modal logic wff 11
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^5.ax')
 $p: \$i \rightarrow \$o \quad \text{thf(p_type, type)}$
 $q: \$i \rightarrow \$o \quad \text{thf(q_type, type)}$
 $r: \$i \rightarrow \$o \quad \text{thf(r_type, type)}$

mvalid@($\text{mimplies} @ (\text{mor} @ (\text{mdia_s}_4 @ (\text{mand} @ p @ q)) @ (\text{mdia_s}_4 @ (\text{mand} @ p @ r))) @ (\text{mdia_s}_4 @ p)$) thf(prove, conjecture)

SYO460 \wedge 6.p Ted Sider's propositional modal logic wff 11

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^6.ax')

$p: \$i \rightarrow \o thf(p_type, type)

$q: \$i \rightarrow \o thf(q_type, type)

$r: \$i \rightarrow \o thf(r_type, type)

mvalid@($\text{mimplies} @ (\text{mor} @ (\text{mdia_s}_5 @ (\text{mand} @ p @ q)) @ (\text{mdia_s}_5 @ (\text{mand} @ p @ r))) @ (\text{mdia_s}_5 @ p)$) thf(prove, conjecture)

SYO461 \wedge 1.p Ted Sider's propositional modal logic wff 12

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^1.ax')

$p: \$i \rightarrow \o thf(p_type, type)

mvalid@($\text{mimplies} @ (\text{mdia_k} @ (\text{mor} @ p @ (\text{mnot} @ p))) @ (\text{mand} @ p @ (\text{mnot} @ p))$) thf(prove, conjecture)

SYO461 \wedge 2.p Ted Sider's propositional modal logic wff 12

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^2.ax')

$p: \$i \rightarrow \o thf(p_type, type)

mvalid@($\text{mimplies} @ (\text{mdia_d} @ (\text{mor} @ p @ (\text{mnot} @ p))) @ (\text{mand} @ p @ (\text{mnot} @ p))$) thf(prove, conjecture)

SYO461 \wedge 3.p Ted Sider's propositional modal logic wff 12

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^3.ax')

$p: \$i \rightarrow \o thf(p_type, type)

mvalid@($\text{mimplies} @ (\text{mdia_m} @ (\text{mor} @ p @ (\text{mnot} @ p))) @ (\text{mand} @ p @ (\text{mnot} @ p))$) thf(prove, conjecture)

SYO461 \wedge 4.p Ted Sider's propositional modal logic wff 12

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^4.ax')

$p: \$i \rightarrow \o thf(p_type, type)

mvalid@($\text{mimplies} @ (\text{mdia_b} @ (\text{mor} @ p @ (\text{mnot} @ p))) @ (\text{mand} @ p @ (\text{mnot} @ p))$) thf(prove, conjecture)

SYO461 \wedge 5.p Ted Sider's propositional modal logic wff 12

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^5.ax')

$p: \$i \rightarrow \o thf(p_type, type)

mvalid@($\text{mimplies} @ (\text{mdia_s}_4 @ (\text{mor} @ p @ (\text{mnot} @ p))) @ (\text{mand} @ p @ (\text{mnot} @ p))$) thf(prove, conjecture)

SYO461 \wedge 6.p Ted Sider's propositional modal logic wff 12

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^6.ax')

$p: \$i \rightarrow \o thf(p_type, type)

mvalid@($\text{mimplies} @ (\text{mdia_s}_5 @ (\text{mor} @ p @ (\text{mnot} @ p))) @ (\text{mand} @ p @ (\text{mnot} @ p))$) thf(prove, conjecture)

SYO462 \wedge 2.p Ted Sider's propositional modal logic wff 13

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^2.ax')

$p: \$i \rightarrow \o thf(p_type, type)

$q: \$i \rightarrow \o thf(q_type, type)

mvalid@($\text{mimplies} @ (\text{mbox_d} @ (\text{mequiv} @ p @ q)) @ (\text{mequiv} @ (\text{mbox_d} @ p) @ (\text{mbox_d} @ q))$) thf(prove, conjecture)

SYO462 \wedge 3.p Ted Sider's propositional modal logic wff 13

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^3.ax')

$p: \$i \rightarrow \o thf(p_type, type)

$q: \$i \rightarrow \o thf(q_type, type)

mvalid@($\text{mimplies} @ (\text{mbox_m} @ (\text{mequiv} @ p @ q)) @ (\text{mequiv} @ (\text{mbox_m} @ p) @ (\text{mbox_m} @ q))$) thf(prove, conjecture)

SYO462 \wedge 4.p Ted Sider's propositional modal logic wff 13

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^4.ax')

$p: \$i \rightarrow \o thf(p_type, type)

$q: \$i \rightarrow \o thf(q_type, type)

mvalid@((mimplies@mbox_b@(mequiv@p@q))@((mequiv@mbox_b@p)@(mbox_b@q))) thf(prove, conjecture)

SYO462^5.p Ted Sider's propositional modal logic wff 13

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^5.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_s4@(mequiv@p@q))@((mequiv@mbox_s4@p)@(mbox_s4@q))) thf(prove, conjecture)

SYO462^6.p Ted Sider's propositional modal logic wff 13

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^6.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_s5@(mequiv@p@q))@((mequiv@mbox_s5@p)@(mbox_s5@q))) thf(prove, conjecture)

SYO463^1.p Ted Sider's propositional modal logic wff 14

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^1.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_k@(mequiv@p@q))@((mbox_k@(mequiv@mbox_k@p)@(mbox_k@q)))) thf(prove, conjecture)

SYO463^2.p Ted Sider's propositional modal logic wff 14

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^2.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_d@(mequiv@p@q))@((mbox_d@(mequiv@mbox_d@p)@(mbox_d@q)))) thf(prove, conjecture)

SYO463^3.p Ted Sider's propositional modal logic wff 14

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^3.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_m@(mequiv@p@q))@((mbox_m@(mequiv@mbox_m@p)@(mbox_m@q)))) thf(prove, conjecture)

SYO463^4.p Ted Sider's propositional modal logic wff 14

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^4.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_b@(mequiv@p@q))@((mbox_b@(mequiv@mbox_b@p)@(mbox_b@q)))) thf(prove, conjecture)

SYO463^5.p Ted Sider's propositional modal logic wff 14

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^5.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_s4@(mequiv@p@q))@((mbox_s4@(mequiv@mbox_s4@p)@(mbox_s4@q)))) thf(prove, conjecture)

SYO463^6.p Ted Sider's propositional modal logic wff 14

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^6.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_s5@(mequiv@p@q))@((mbox_s5@(mequiv@mbox_s5@p)@(mbox_s5@q)))) thf(prove, conjecture)

SYO464^1.p Ted Sider's propositional modal logic wff 15

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^1.ax')

p: \$i → \$o thf(p_type, type)

q: \$i → \$o thf(q_type, type)

mvalid@((mimplies@mbox_k@(mand@p@q))@((mbox_k@(mbox_k@(mimplies@((mdia_k@p)@(mdia_k@q)))))) thf(prove, conjecture)

SYO464^2.p Ted Sider's propositional modal logic wff 15

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_d@(mand@p@q))@(mbox_d@(mbox_d@(mimplies@(mdia_d@p)@(mdia_d@q))))))  thf(prove, conjecture)
```

SYO464^3.p Ted Sider's propositional modal logic wff 15

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_m@(mand@p@q))@(mbox_m@(mbox_m@(mimplies@(mdia_m@p)@(mdia_m@q))))))  thf(prove, conjecture)
```

SYO464^4.p Ted Sider's propositional modal logic wff 15

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_b@(mand@p@q))@(mbox_b@(mbox_b@(mimplies@(mdia_b@p)@(mdia_b@q))))))  thf(prove, conjecture)
```

SYO464^5.p Ted Sider's propositional modal logic wff 15

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_s4@(mand@p@q))@(mbox_s4@(mbox_s4@(mimplies@(mdia_s4@p)@(mdia_s4@q))))))  thf(prove, conjecture)
```

SYO464^6.p Ted Sider's propositional modal logic wff 15

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_s5@(mand@p@q))@(mbox_s5@(mbox_s5@(mimplies@(mdia_s5@p)@(mdia_s5@q))))))  thf(prove, conjecture)
```

SYO465^1.p Ted Sider's propositional modal logic wff 16

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_k@(mdia_k@p))@(mdia_k@(mbox_k@p))))  thf(prove, conjecture)
```

SYO465^2.p Ted Sider's propositional modal logic wff 16

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_d@(mdia_d@p))@(mdia_d@(mbox_d@p))))  thf(prove, conjecture)
```

SYO465^3.p Ted Sider's propositional modal logic wff 16

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_m@(mdia_m@p))@(mdia_m@(mbox_m@p))))  thf(prove, conjecture)
```

SYO465^4.p Ted Sider's propositional modal logic wff 16

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_b@(mdia_b@p))@(mdia_b@(mbox_b@p))))  thf(prove, conjecture)
```

SYO465^5.p Ted Sider's propositional modal logic wff 16

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_s4@(mdia_s4@p))@(mdia_s4@(mbox_s4@p))))  thf(prove, conjecture)
```

SYO465⁶.p Ted Sider's propositional modal logic wff 16

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_s5@(mdia_s5@p))@(mdia_s5@(mbox_s5@p)))      thf(prove, conjecture)
```

SYO467¹.p Ted Sider's propositional modal logic wff 18

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mdia_k@(mbox_k@p))@(mbox_k@(mdia_k@p)))      thf(prove, conjecture)
```

SYO467².p Ted Sider's propositional modal logic wff 18

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mdia_d@(mbox_d@p))@(mbox_d@(mdia_d@p)))      thf(prove, conjecture)
```

SYO467³.p Ted Sider's propositional modal logic wff 18

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mdia_m@(mbox_m@p))@(mbox_m@(mdia_m@p)))      thf(prove, conjecture)
```

SYO467⁴.p Ted Sider's propositional modal logic wff 18

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mdia_b@(mbox_b@p))@(mbox_b@(mdia_b@p)))      thf(prove, conjecture)
```

SYO467⁵.p Ted Sider's propositional modal logic wff 18

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mdia_s4@(mbox_s4@p))@(mbox_s4@(mdia_s4@p)))      thf(prove, conjecture)
```

SYO467⁶.p Ted Sider's propositional modal logic wff 18

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
mvalid@(mequiv@(mdia_s5@(mbox_s5@p))@(mbox_s5@(mdia_s5@p)))      thf(prove, conjecture)
```

SYO468¹.p Ted Sider's propositional modal logic wff 19

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@(mbox_k@(mimplies@(mbox_k@p)@q))@(mbox_k@(mimplies@(mbox_k@q)@p)))      thf(prove, conjecture)
```

SYO468².p Ted Sider's propositional modal logic wff 19

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@(mbox_d@(mimplies@(mbox_d@p)@q))@(mbox_d@(mimplies@(mbox_d@q)@p)))      thf(prove, conjecture)
```

SYO468³.p Ted Sider's propositional modal logic wff 19

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@(mbox_m@(mimplies@(mbox_m@p)@q))@(mbox_m@(mimplies@(mbox_m@q)@p)))      thf(prove, conjecture)
```

SYO468⁴.p Ted Sider's propositional modal logic wff 19

```
include('Axioms/LCL013^0.ax')
```

```

include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@(mbox_b@(mimplies@(mbox_b@p)@q))@(mbox_b@(mimplies@(mbox_b@q)@p)))      thf(prove, conjecture)

SYO468^5.p Ted Sider's propositional modal logic wff 19
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@(mbox_s4@(mimplies@(mbox_s4@p)@q))@(mbox_s4@(mimplies@(mbox_s4@q)@p)))      thf(prove, conjecture)

SYO468^6.p Ted Sider's propositional modal logic wff 19
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mor@(mbox_s5@(mimplies@(mbox_s5@p)@q))@(mbox_s5@(mimplies@(mbox_s5@q)@p)))      thf(prove, conjecture)

SYO469^1.p Ted Sider's propositional modal logic wff 20
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_k@(mimplies@(mbox_k@p)@q))@(mbox_k@(mimplies@(mbox_k@p)@(mbox_k@q))))      thf(prove, conjecture)

SYO469^2.p Ted Sider's propositional modal logic wff 20
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_d@(mimplies@(mbox_d@p)@q))@(mbox_d@(mimplies@(mbox_d@p)@(mbox_d@q))))      thf(prove, conjecture)

SYO469^3.p Ted Sider's propositional modal logic wff 20
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies@(mbox_m@(mimplies@(mbox_m@p)@q))@(mbox_m@(mimplies@(mbox_m@p)@(mbox_m@q))))      thf(prove, conjecture)

SYO469^4.p Ted Sider's propositional modal logic wff 20
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mbox_b@(mimplies(@(mbox_b@p)@q))@(mbox_b@(mimplies(@(mbox_b@p)@(mbox_b@q))))))      thf(prove, conjecture)

SYO469^5.p Ted Sider's propositional modal logic wff 20
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mbox_s4@(mimplies(@(mbox_s4@p)@q))@(mbox_s4@(mimplies(@(mbox_s4@p)@(mbox_s4@q))))))      thf(prove, conjecture)

SYO469^6.p Ted Sider's propositional modal logic wff 20
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mbox_s5@(mimplies(@(mbox_s5@p)@q))@(mbox_s5@(mimplies(@(mbox_s5@p)@(mbox_s5@q))))))      thf(prove, conjecture)

SYO470^1.p Ted Sider's propositional modal logic wff 21
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)

```

mvalid@(mequiv@(mdia_k@(mdia_k@(mbox_k@p)))@(mbox_k@p)) thf(prove, conjecture)

SYO470 \wedge 2.p Ted Sider's propositional modal logic wff 21
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^2.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mequiv@(mdia_d@(mdia_d@(mbox_d@p)))@(mbox_d@p)) thf(prove, conjecture)

SYO470 \wedge 3.p Ted Sider's propositional modal logic wff 21
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^3.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mequiv@(mdia_m@(mdia_m@(mbox_m@p)))@(mbox_m@p)) thf(prove, conjecture)

SYO470 \wedge 4.p Ted Sider's propositional modal logic wff 21
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^4.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mequiv@(mdia_b@(mdia_b@(mbox_b@p)))@(mbox_b@p)) thf(prove, conjecture)

SYO470 \wedge 5.p Ted Sider's propositional modal logic wff 21
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^5.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mequiv@(mdia_s4@(mdia_s4@(mbox_s4@p)))@(mbox_s4@p)) thf(prove, conjecture)

SYO470 \wedge 6.p Ted Sider's propositional modal logic wff 21
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mequiv@(mdia_s5@(mdia_s5@(mbox_s5@p)))@(mbox_s5@p)) thf(prove, conjecture)

SYO471 \wedge 1.p Ted Sider's propositional modal logic wff 22
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^1.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mimplies(@(mdia_k@(mdia_k@p))@(mbox_k@(mdia_k@p))) thf(prove, conjecture)

SYO471 \wedge 2.p Ted Sider's propositional modal logic wff 22
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^2.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mimplies(@(mdia_d@(mdia_d@p))@(mbox_d@(mdia_d@p))) thf(prove, conjecture)

SYO471 \wedge 3.p Ted Sider's propositional modal logic wff 22
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^3.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mimplies(@(mdia_m@(mdia_m@p))@(mbox_m@(mdia_m@p))) thf(prove, conjecture)

SYO471 \wedge 4.p Ted Sider's propositional modal logic wff 22
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^4.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mimplies(@(mdia_b@(mdia_b@p))@(mbox_b@(mdia_b@p))) thf(prove, conjecture)

SYO471 \wedge 5.p Ted Sider's propositional modal logic wff 22
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^5.ax')
 $p: \$i \rightarrow \o thf(p_type, type)
 mvalid@(mimplies(@(mdia_s4@(mdia_s4@p))@(mbox_s4@(mdia_s4@p))) thf(prove, conjecture)

SYO471 \wedge 6.p Ted Sider's propositional modal logic wff 22
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $p: \$i \rightarrow \o thf(p_type, type)

mvalid@(mimplies(@(mdia_s5@(mdia_s5@p))@((mbox_s5@(mdia_s5@p)))) thf(prove, conjecture)

SYO472¹.p Ted Sider's propositional modal logic wff 23

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@(mimplies(@(mbox_k@(mimplies@p@(mbox_k@(mimplies@q@r))))@((mimplies@q@(mbox_k@(mimplies@p@r))))
```

SYO472².p Ted Sider's propositional modal logic wff 23

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@(mimplies(@(mbox_d@(mimplies@p@(mbox_d@(mimplies@q@r))))@((mimplies@q@(mbox_d@(mimplies@p@r))))
```

SYO472³.p Ted Sider's propositional modal logic wff 23

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@(mimplies(@(mbox_m@(mimplies@p@(mbox_m@(mimplies@q@r))))@((mimplies@q@(mbox_m@(mimplies@p@r))))
```

SYO472⁴.p Ted Sider's propositional modal logic wff 23

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@(mimplies(@(mbox_b@(mimplies@p@(mbox_b@(mimplies@q@r))))@((mimplies@q@(mbox_b@(mimplies@p@r))))
```

SYO472⁵.p Ted Sider's propositional modal logic wff 23

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@(mimplies(@(mbox_s4@(mimplies@p@(mbox_s4@(mimplies@q@r))))@((mimplies@q@(mbox_s4@(mimplies@p@r))))
```

SYO472⁶.p Ted Sider's propositional modal logic wff 23

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
r: $i → $o      thf(r_type, type)
mvalid@(mimplies(@(mbox_s5@(mimplies@p@(mbox_s5@(mimplies@q@r))))@((mimplies@q@(mbox_s5@(mimplies@p@r))))
```

SYO473¹.p Ted Sider's propositional modal logic wff 24

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mbox_k@(mimplies@(mbox_k@(mequiv@p@q))@((mdia_k@q))))@((mbox_k@(mimplies@(mbox_k@(mequiv
```

SYO473².p Ted Sider's propositional modal logic wff 24

```
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(mimplies(@(mbox_d@(mimplies@(mbox_d@(mequiv@p@q))@((mdia_d@q))))@((mbox_d@(mimplies@(mbox_d@(mequiv
```

SYO473³.p Ted Sider's propositional modal logic wff 24

SYO473^5.p Ted Sider's propositional modal logic wff 24

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)

```

```

SYO473^6.p Ted Sider's prop
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
myvalid@mimplyies@mbox s-@(i

```

SYQ474\1.p Ted Sider's propositional modal logic wff 25

```

SMTLIB1p Tcf Slicer's properties
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@({implies@({mbox_k@({m

```

SYO474^2.p Ted Sider's propositional modal logic wff 25

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^2.ax')
p: $i → $o      thf(p_type, type)
mvalid@((mimplies@((mbox_d@(n

```

SYO474\3.p Ted Sider's propositional modal logic wff 25
 incl. 1. (iA, iB, iC, iD, iE, iF, iG) 0123456789

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^3.ax')
p: $i → $o      thf(p_type, type)
mvalid@({implies@({mbox_m@({n

```

SYO474\4.p Ted Sider's propositional modal logic wff 25

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^4.ax')
p: $i → $o      thf(p_type, type)
q: $i → $o      thf(q_type, type)
mvalid@(implies@(mbox_b@(n

```

SYO474^5.p Ted Sider's propositional modal logic wff 25

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^5.ax')
p: $! → $o      thf(p_type, type)
mvalid@(mimplies@(mbox_s4@(n

```

SYU474\6.p Ted Sider's propositional modal logic wff 25
 $\vdash \perp \wedge (\Diamond \perp \rightarrow \Box \perp)$

```

include('Axioms/LCL013_0.ax')
include('Axioms/LCL013^6.ax')
p: $i → $o      thf(p_type, type)
mvalid@({implies@({mbox_s5@(n

```

STO475\0.p Ted Sider's SS quantified modal logic wif 01

```

include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mforall_ind@λx: mu: (mbox_s5@(f@x)))@(mbox_s5@(mforall_ind@λx: mu: (f@x))))      thf(prove, conj)
SYO476^6.p Ted Sider's S5 quantified modal logic wff 02
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mforall_ind@λx: mu: (f@x)))@(mforall_ind@λx: mu: (mbox_s5@(f@x))))      thf(prove, conj)
SYO477^6.p Ted Sider's S5 quantified modal logic wff 03
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mbox_s5@(mforall_ind@λx: mu: (f@x)))@(mexists_ind@λx: mu: (mbox_s5@(f@x))))      thf(prove, conj)
SYO478^6.p Ted Sider's S5 quantified modal logic wff 04
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mexists_ind@λx: mu: (mbox_s5@(f@x)))@(mexists_ind@λx: mu: (f@x))))      thf(prove, conj)
SYO479^6.p Ted Sider's S5 quantified modal logic wff 05
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mforall_ind@λx: mu: (mdia_s5@(f@x)))@(mdia_s5@(mforall_ind@λx: mu: (f@x))))      thf(prove, conj)
SYO480^6.p Ted Sider's S5 quantified modal logic wff 06
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mdia_s5@(mforall_ind@λx: mu: (f@x)))@(mforall_ind@λx: mu: (mdia_s5@(f@x))))      thf(prove, conj)
SYO481^6.p Ted Sider's S5 quantified modal logic wff 07
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
mvalid@(mforall_ind@λx: mu: (mforall_ind@λy: mu: (mimplies@(mnot@(meq_ind@x@y))@(mbox_s5@(mnot@(meq_ind@x@y))
SYO482^6.p Ted Sider's S5 quantified modal logic wff 08
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
mvalid@(mforall_ind@λx: mu: (mbox_s5@(mexists_ind@λy: mu: (meq_ind@x@y))))      thf(prove, conjecture)
SYO483^6.p Ted Sider's S5 quantified modal logic wff 09
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
a: mu      thf(a_type, type)
mvalid@(mexists_ind@λx: mu: (mbox_s5@(meq_ind@x@a)))      thf(prove, conjecture)
SYO484^6.p Ted Sider's S5 quantified modal logic wff 10
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
mvalid@(mforall_ind@λx: mu: (mbox_s5@(mforall_ind@λy: mu: (meq_ind@x@y))))      thf(prove, conjecture)
SYO485^6.p Ted Sider's S5 quantified modal logic wff 11
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
mvalid@(mforall_ind@λx: mu: (mdia_s5@(mexists_ind@λy: mu: (meq_ind@x@y))))      thf(prove, conjecture)
SYO486^6.p Ted Sider's S5 quantified modal logic wff 12
include('Axioms/LCL013^0.ax')
include('Axioms/LCL013^6.ax')
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mbox_s5@(mexists_ind@λx: mu: (f@x)))@(mdia_s5@(mforall_ind@λx: mu: (f@x))))      thf(prove, conj)

```

SYO487^6.p Ted Sider's S5 quantified modal logic wff 13
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mdia_s5} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (f @ x))) @ (\text{mexists_ind} @ \lambda x: \text{mu}: (\text{mdia_s5} @ (f @ x)))) \quad \text{thf}(\text{prove}, \text{conjecture})$

SYO488^6.p Ted Sider's S5 quantified modal logic wff 14
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mdia_s5} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (f @ x))) @ (\text{mnot} @ (\text{mexists_ind} @ \lambda x: \text{mu}: (\text{mbox_s5} @ (\text{mnot} @ (f @ x)))))$

SYO489^6.p Ted Sider's S5 quantified modal logic wff 15
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $a: \text{mu} \quad \text{thf}(a_\text{type}, \text{type})$
 $f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$
 $g: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(g_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mand} @ (\text{mdia_s5} @ (f @ a)) @ (\text{mdia_s5} @ (g @ a))) @ (\text{mdia_s5} @ (\text{mand} @ (f @ a) @ (g @ a)))) \quad \text{thf}(\text{prove}, \text{conjecture})$

SYO490^6.p Ted Sider's S5 quantified modal logic wff 16
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $a: \text{mu} \quad \text{thf}(a_\text{type}, \text{type})$
 $r: \text{mu} \rightarrow \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(r_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mexists_ind} @ \lambda x: \text{mu}: (\text{mdia_s5} @ (r @ a @ x))) @ (\text{mdia_s5} @ (\text{mbox_s5} @ (\text{mexists_ind} @ \lambda x: \text{mu}: (\text{mexists_ind} @ \lambda y: \text{mu}: (y @ r @ a @ x))))))$

SYO491^6.p Ted Sider's S5 quantified modal logic wff 17
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$
 $g: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(g_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mbox_s5} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mimplies} @ (f @ x) @ (g @ x)))) @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mbox_s5} @ (\text{mimplies} @ (f @ x) @ (g @ x))))))$

SYO492^6.p Ted Sider's S5 quantified modal logic wff 18
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$
 $g: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(g_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mbox_s5} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mor} @ (f @ x) @ (g @ x)))) @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mor} @ (\text{mbox_s5} @ (f @ x) @ (g @ x))))))$

SYO493^6.p Ted Sider's S5 quantified modal logic wff 19
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$
 $g: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(g_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mexists_ind} @ \lambda x: \text{mu}: (\text{mbox_s5} @ (\text{mor} @ (f @ x) @ (g @ x)))) @ (\text{mbox_s5} @ (\text{mor} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (f @ x) @ (g @ x))))))$

SYO494^6.p Ted Sider's S5 quantified modal logic wff 20
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$
 $g: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(g_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mimplies} @ (f @ x) @ (\text{mdia_s5} @ (g @ x)))) @ (\text{mdia_s5} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mimplies} @ (f @ x) @ (g @ x))))))$

SYO495^6.p Ted Sider's S5 quantified modal logic wff 21
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')
 $f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$
 $g: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(g_\text{type}, \text{type})$
 $\text{mvalid} @ (\text{mimplies} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mor} @ (\text{mbox_s5} @ (f @ x)) @ (\text{mbox_s5} @ (g @ x)))) @ (\text{mbox_s5} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mimplies} @ (f @ x) @ (g @ x))))))$

SYO496^6.p Ted Sider's S5 quantified modal logic wff 22
 include('Axioms/LCL013^0.ax')
 include('Axioms/LCL013^6.ax')

$f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$

$g: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(g_\text{type}, \text{type})$

$\text{mvalid} @ (\text{mimplies} @ (\text{mbox_s5} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mimplies} @ (f @ x) @ (g @ x)))) @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mimplies} @ (f @ x) @ (g @ x))))$

SYO497^6.p Ted Sider's S5 quantified modal logic wff 23

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^6.ax')

$f: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(f_\text{type}, \text{type})$

$\text{mvalid} @ (\text{mimplies} @ (\text{mand} @ (\text{mbox_s5} @ (\text{mforall_ind} @ \lambda x: \text{mu}: (\text{mimplies} @ (f @ x) @ (\text{mbox_s5} @ (f @ x)))))) @ (\text{mdia_s5} @ (\text{mexists_ind} @ \lambda x: \text{mu}: (\text{mimplies} @ (f @ x) @ (g @ x))))))$

SYO498^6.p Ted Sider's S5 quantified modal logic wff 24

include('Axioms/LCL013^0.ax')

include('Axioms/LCL013^6.ax')

$n: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(n_\text{type}, \text{type})$

$o: \text{mu} \rightarrow \$i \rightarrow \$o \quad \text{thf}(o_\text{type}, \text{type})$

$\text{mvalid} @ (\text{mimplies} @ (\text{mexists_ind} @ \lambda x: \text{mu}: (\text{mand} @ (n @ x) @ (\text{mand} @ (\text{mforall_ind} @ \lambda y: \text{mu}: (\text{mimplies} @ (n @ y) @ (\text{meq_ind} @ y @ x)))))))$

SYO499^1.p Explosive confrontation

The Mensa Example: There are not 3 distinct values of type \$o.

$a: \$o \quad \text{thf}(a, \text{type})$

$b: \$o \quad \text{thf}(b, \text{type})$

$c: \$o \quad \text{thf}(c, \text{type})$

$f: \$o \rightarrow \$i \quad \text{thf}(f, \text{type})$

$f_1: \$o \rightarrow \$i \quad \text{thf}(f_1, \text{type})$

$f_2: \$o \rightarrow \$i \quad \text{thf}(f_2, \text{type})$

$g: \$o \rightarrow \$i \quad \text{thf}(g, \text{type})$

$g_1: \$o \rightarrow \$i \quad \text{thf}(g_1, \text{type})$

$g_2: \$o \rightarrow \$i \quad \text{thf}(g_2, \text{type})$

$(f @ a) = (g @ b) \text{ or } (f @ b) \neq (g @ a) \text{ or } (f_1 @ a) = (g_1 @ c) \text{ or } (f_1 @ c) \neq (g_1 @ a) \text{ or } (f_2 @ b) = (g_2 @ c) \text{ or } (f_2 @ c) \neq (g_2 @ b) \quad \text{thf}(co)$

SYO500^1.002.p Two function variant of the Kaminski equation

$x: \$o \quad \text{thf}(x, \text{type})$

$f_0: \$o \rightarrow \$o \quad \text{thf}(f_0, \text{type})$

$f_1: \$o \rightarrow \$o \quad \text{thf}(f_1, \text{type})$

$(f_0 @ (f_0 @ (f_0 @ (f_1 @ x)))) = (f_0 @ (f_1 @ (f_1 @ (f_1 @ x)))) \quad \text{thf(kaminski}_2, \text{conjecture)}$

SYO500^1.003.p Three function variant of the Kaminski equation

$x: \$o \quad \text{thf}(x, \text{type})$

$f_0: \$o \rightarrow \$o \quad \text{thf}(f_0, \text{type})$

$f_1: \$o \rightarrow \$o \quad \text{thf}(f_1, \text{type})$

$f_2: \$o \rightarrow \$o \quad \text{thf}(f_2, \text{type})$

$(f_0 @ (f_1 @ (f_1 @ (f_2 @ (f_2 @ x))))) = (f_0 @ (f_0 @ (f_1 @ (f_2 @ (f_2 @ (f_2 @ x))))) \quad \text{thf(kaminski}_3, \text{conjecture})$

SYO500^1.004.p Four function variant of the Kaminski equation

$x: \$o \quad \text{thf}(x, \text{type})$

$f_0: \$o \rightarrow \$o \quad \text{thf}(f_0, \text{type})$

$f_1: \$o \rightarrow \$o \quad \text{thf}(f_1, \text{type})$

$f_2: \$o \rightarrow \$o \quad \text{thf}(f_2, \text{type})$

$f_3: \$o \rightarrow \$o \quad \text{thf}(f_3, \text{type})$

$(f_0 @ (f_1 @ (f_1 @ (f_2 @ (f_2 @ (f_2 @ (f_3 @ x))))))) = (f_0 @ (f_1 @ (f_1 @ (f_2 @ (f_3 @ (f_3 @ (f_3 @ x))))))) \quad \text{thf(kaminski}_4, \text{conjecture})$

SYO500^1.005.p Five function variant of the Kaminski equation

$x: \$o \quad \text{thf}(x, \text{type})$

$f_0: \$o \rightarrow \$o \quad \text{thf}(f_0, \text{type})$

$f_1: \$o \rightarrow \$o \quad \text{thf}(f_1, \text{type})$

$f_2: \$o \rightarrow \$o \quad \text{thf}(f_2, \text{type})$

$f_3: \$o \rightarrow \$o \quad \text{thf}(f_3, \text{type})$

$f_4: \$o \rightarrow \$o \quad \text{thf}(f_4, \text{type})$

$(f_0 @ (f_1 @ (f_1 @ (f_2 @ (f_2 @ (f_2 @ (f_3 @ (f_3 @ (f_3 @ x)))))))) = (f_0 @ (f_0 @ (f_1 @ (f_1 @ (f_2 @ (f_2 @ (f_3 @ (f_3 @ (f_4 @ (f_4 @ x)))))))))))$

SYO500^1.006.p Six function variant of the Kaminski equation

$x: \$o \quad \text{thf}(x, \text{type})$

$f_0: \$o \rightarrow \$o \quad \text{thf}(f_0, \text{type})$

$f_1: \$o \rightarrow \$o \quad \text{thf}(f_1, \text{type})$

SYO500\1.007.p Seven function variant of the Kaminski equation

SYO500\1.008.p Eight function variant of the Kaminski equation

```

x: $o      thf(x,type)
f0: $o → $o      thf(f0,type)
f1: $o → $o      thf(f1,type)
f2: $o → $o      thf(f2,type)
f3: $o → $o      thf(f3,type)
f4: $o → $o      thf(f4,type)
f5: $o → $o      thf(f5,type)
f6: $o → $o      thf(f6,type)
f7: $o → $o      thf(f7,type)
(f0@(f0@(f0@(f1@(f1@(f2@(f2@(f3@(f3@(f4@(f4@(f4@(f5@(f6@(f6@(f6@(f7@x))))))))))))))) = (f0@(f1@(f1@(f1@(f2@(f3@(f3@(f4@(f4@(f5@(f6@(f6@(f6@(f7@x)))))))))))))))

```

SYO500\^1.p The Kaminski equation

f: \$o → \$o thf(*f*, type)
x: \$o thf(*x*, type)
 $(f @ (f @ (f @ x))) = (f @ x)$ thf(con, conjecture)

SYO501\1.p An unsatisfiable normal set with embedded formulas

$x: \$i \quad \text{thf}(x, \text{type})$
 $y: \$o \quad \text{thf}(y, \text{type})$
 $f: \$i \rightarrow \$o \rightarrow \$i \quad \text{thf}(f, \text{type})$
 $p: \$i \rightarrow \$o \quad \text{thf}(p, \text{type})$
 $\neg p @ (f @ x @ \neg \neg y) \text{ or } p @ (f @ x @ y) \quad \text{thf(claim, conjecture)}$

SYO502\1.p Rules sym and con handle positive equations at i

$a: \$i \quad \text{thf}(a, \text{type})$
 $b: \$i \quad \text{thf}(b, \text{type})$
 $f: \$i \rightarrow \$i \quad \text{thf}(f, \text{type})$
 $g: \$i \rightarrow \$i \quad \text{thf}(g, \text{type})$
 $a \neq b \text{ or } (f @ a) \neq (g @ b) \text{ or } (f @ b) = (g @ a) \quad \text{thf(claim, conjecture)}$

SYO503\1.p Tableau with two branches

```

a: $o      thf(a,type)
b: $o      thf(b,type)
c: $o      thf(c,type)
f: $o → $o      thf(f,type)
g: $o → $o      thf(g,type)
p: ($o → $o) → $o      thf(p,type)
a = b or ¬ f@a or ¬ f@b or ¬ g@a or ¬ g@b or ¬ p@f or p@g      thf(claim,conjecture)

```

SYO504^1.p Hoeschele p.21

$a: \$o \quad \text{thf}(a, \text{type})$
 $b: \$o \quad \text{thf}(b, \text{type})$
 $h: \$o \rightarrow \$o \rightarrow \$o \quad \text{thf}(h, \text{type})$
 $i: \$o \rightarrow \$o \rightarrow \$o \quad \text{thf}(i, \text{type})$
 $g: (\$o \rightarrow \$o \rightarrow \$o) \rightarrow \$o \quad \text{thf}(g, \text{type})$

$f: ((\$o \rightarrow \$o \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \quad \text{thf}(f, \text{type})$
 $((h@a@b) = (a \text{ and } b) \text{ and } (i@a@b) = (a \text{ or } b) \text{ and } (f@g) = (i@a@b) \text{ and } h@a@b) \Rightarrow (f@g) \quad \text{thf}(\text{claim}, \text{conjecture})$

SYO505^1.p Explosive confrontation

The Mensa Example at type oo: There are not 5 distinct values of type oo.

```

a: $o → $o      thf(a, type)
b: $o → $o      thf(b, type)
c: $o → $o      thf(c, type)
d: $o → $o      thf(d, type)
e: $o → $o      thf(e, type)
f0: ($o → $o) → $i      thf(f0, type)
g0: ($o → $o) → $i      thf(g0, type)
f1: ($o → $o) → $i      thf(f1, type)
g1: ($o → $o) → $i      thf(g1, type)
f2: ($o → $o) → $i      thf(f2, type)
g2: ($o → $o) → $i      thf(g2, type)
f3: ($o → $o) → $i      thf(f3, type)
g3: ($o → $o) → $i      thf(g3, type)
f4: ($o → $o) → $i      thf(f4, type)
g4: ($o → $o) → $i      thf(g4, type)
f5: ($o → $o) → $i      thf(f5, type)
g5: ($o → $o) → $i      thf(g5, type)
f6: ($o → $o) → $i      thf(f6, type)
g6: ($o → $o) → $i      thf(g6, type)
f7: ($o → $o) → $i      thf(f7, type)
g7: ($o → $o) → $i      thf(g7, type)
f8: ($o → $o) → $i      thf(f8, type)
g8: ($o → $o) → $i      thf(g8, type)
f9: ($o → $o) → $i      thf(f9, type)
g9: ($o → $o) → $i      thf(g9, type)
(f0@a) = (g0@b)      thf(anotb1, axiom)
(f0@b) ≠ (g0@a)      thf(anotb2, axiom)
(f1@a) = (g1@c)      thf(anotc1, axiom)
(f1@c) ≠ (g1@a)      thf(anotc2, axiom)
(f2@a) = (g2@d)      thf(anotd1, axiom)
(f2@d) ≠ (g2@a)      thf(anotd2, axiom)
(f3@a) = (g3@e)      thf(anote1, axiom)
(f3@e) ≠ (g3@a)      thf(anote2, axiom)
(f4@b) = (g4@c)      thf(bnotc1, axiom)
(f4@c) ≠ (g4@b)      thf(bnotc2, axiom)
(f5@b) = (g5@d)      thf(bnotd1, axiom)
(f5@d) ≠ (g5@b)      thf(bnotd2, axiom)
(f6@b) = (g6@e)      thf(bnote1, axiom)
(f6@e) ≠ (g6@b)      thf(bnote2, axiom)
(f7@c) = (g7@d)      thf(cnotd1, axiom)
(f7@d) ≠ (g7@c)      thf(cnotd2, axiom)
(f8@c) = (g8@e)      thf(cnote1, axiom)
(f8@e) ≠ (g8@c)      thf(cnote2, axiom)
(f9@d) = (g9@e)      thf(dnote1, axiom)
(f9@e) ≠ (g9@d)      thf(dnote2, axiom)
$false      thf(mensaoo, conjecture)

```

SYO506^1.p (if (X = Y) then X else Y) = Y

c: \$o → \$i → \$i → \$i thf(c, type)

∀x: \$i, y: \$i: (c@x = y@x@y) = y or ¬∀x: \$i, y: \$i: (c@\$true@x@y) = x or ¬∀x: \$i, y: \$i: (c@\$false@x@y) = y thf(claim, conjecture)

SYO507^1.p Example 4.1

f: \$i → \$o thf(f, type)

p: (\$i → \$o) → \$o thf(p, type)

¬p@f and ¬p@λx: \$i: ¬¬f@x thf(claim, conjecture)

SYO509 \wedge 1.p Existence of choice functions for binary relations

a: \$tType thf(a, type)
b: \$tType thf(b, type)
 $\exists c_1: (a \rightarrow b \rightarrow \$o) \rightarrow a, c_2: (a \rightarrow b \rightarrow \$o) \rightarrow b: \forall r: a \rightarrow b \rightarrow \$o: (\exists x: a, y: b: (r @ x @ y) \Rightarrow (r @ (c_1 @ r) @ (c_2 @ r)))$ thf(claim, c)

SYO511 \wedge 1.p Two different choice operators at type i

eps: (\$i → \$o) → \$i thf(eps, type)
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p)))$ thf(epschoice, axiom)
eps₂: (\$i → \$o) → \$i thf(eps₂, type)
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps}_2 @ p)))$ thf(eps2choice, axiom)
eps = eps₂ thf(claim, conjecture)

SYO512 \wedge 1.p Choice operator used to obtain functions from total relations

A choice operator can be used to obtain functions from total relations.

eps: (\$i → \$o) → \$i thf(eps, type)
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p)))$ thf(epschoice, axiom)
r: \$i → \$i → \$o thf(r, type)
 $\forall x: \$i: \exists y: \$i: (r @ x @ y)$ thf(rttotal, axiom)
 $\forall x: \$i: (r @ x @ (\text{eps} @ (r @ x)))$ thf(claim, conjecture)

SYO513 \wedge 1.p There is a choice operator at type o

$\exists c: (\$o \rightarrow \$o) \rightarrow \$o: \forall p: \$o \rightarrow \$o: (\exists x: \$o: (p @ x) \Rightarrow (p @ (c @ p)))$ thf(choiceo, conjecture)

SYO514 \wedge 1.p A choice operator at type oo

$\exists c: ((\$o \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \rightarrow \$o: \forall p: (\$o \rightarrow \$o) \rightarrow \$o: (\exists x: \$o \rightarrow \$o: (p @ x) \Rightarrow (p @ (c @ p)))$ thf(choiceoo, conjecture)

SYO515 \wedge 1.p A choice operator at type oo

t: ((\\$o → \\$o) → \\$o) → \\$o → \\$o thf(t_type, type)
 $t = (\lambda p: (\$o \rightarrow \$o) \rightarrow \$o, x: \$o: (\neg p @ \lambda x: \$o: \$false \text{ and } (p @ \lambda x: \$o: \$true \text{ or } (p @ \lambda x: \$o: \neg x) = \neg x)))$ thf(t, definition)
 $\forall p: (\$o \rightarrow \$o) \rightarrow \$o: (\exists y: \$o \rightarrow \$o: (p @ y) \Rightarrow (p @ (t @ p)))$ thf(choiceoo₁, conjecture)

SYO516 \wedge 1.p Every functional relation corresponds to a function

r: \$i → \$i → \$o thf(r_type, type)
 $\forall x: \$i: \exists y: \$i: (r @ x) = (\lambda z: \$i: y = z) \Rightarrow \exists f: \$i \rightarrow \$i: \forall x: \$i: (r @ x @ (f @ x))$ thf(descr₂, conjecture)

SYO517 \wedge 1.p A description operator at type i

$\exists d: (\$i \rightarrow \$o) \rightarrow \$i: \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \text{ and } \forall y: \$i: ((p @ y) \Rightarrow x = y)) \Rightarrow (p @ (d @ p))$ thf(descri, conjecture)

SYO518 \wedge 1.p There is an if-then-else operator at type i

$\exists i: \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i: \forall x: \$i, y: \$i: ((i @ \$true @ x @ y) = x \text{ and } (i @ \$false @ x @ y) = y)$ thf(ifi, conjecture)

SYO519 \wedge 1.p For any X,Y:i, there is a function swapping X and Y

$\forall x: \$i, y: \$i: \exists f: \$i \rightarrow \$i: ((f @ x) = y \text{ and } (f @ y) = x)$ thf(ifi, conjecture)

SYO520 \wedge 1.p A simple problem with a choice operator

eps: (\$i → \$o) → \$i thf(eps, type)
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p)))$ thf(epschoice, axiom)
p: \$i → \$o thf(p, type)
 $p @ (\text{eps} @ \lambda x: \$i: \neg p @ x)$ thf(ax₁, axiom)
 $\neg p @ (\text{eps} @ p)$ thf(ax₂, axiom)

SYO521=1.p There are more than two integers

a: \$int tff(a_type, type)
b: \$int tff(b_type, type)
 $\exists x: \$int: (\$sum(2, 2) = x \text{ and } \forall y: \$int: (y = a \text{ or } y = b))$ tff(a, conjecture)

SYO522=1.p Functions are either odd or even

f: (\$int × \$int × \$int) → \$int tff(f_type, type)
 $\exists x: \$int, y: \$int, z: \$int: f(x, x, y) = \$product(2, z) \text{ or } \exists x: \$int, y: \$int, z: \$int: f(x, y, y) = \$sum(\$product(2, z), 1)$ tff(fxxx, conjecture)

SYO523=1.p Injective pigeon hole function

f: \$int → \$int tff(f_type, type)
 $(\forall x: \$int, y: \$int: (f(x) = f(y) \Rightarrow x = y) \text{ and } \$less(6, f(3)) \text{ and } \$less(f(3), 9) \text{ and } \$less(6, f(4)) \text{ and } \$less(f(4), 9)) \Rightarrow (\$lesseq(f(5), 6) \text{ or } \$lesseq(9, f(5)))$ tff(injective_f_pigeonhole, conjecture)

SYO524=1.p Monotone function

f: \$int → \$int tff(f_type, type)
 $(\forall u: \$int: \$lesseq(f(\$sum(u, 1)), f(\$sum(u, 2))) \text{ and } \$lesseq(f(7), 3)) \Rightarrow \$lesseq(f(4), 3)$ tff(co₁, conjecture)

SYO526^1.p The BQFQFE problem

```

a: $o      thf(a, type)
b: $o      thf(b, type)
a ⇔ b      thf(ab, axiom)
f: $i → $o  thf(f, type)
g: $i → $o  thf(g, type)
f = g      thf(fg, axiom)
f = (λx: $i: a)  thf(fa, axiom)
g = (λx: $i: b)  thf(gb, conjecture)

```

SYO527^1.p Skolem Property on two types

For every total relation r on a * b, there is a corresponding function from a to b.

```

a: $tType   thf(a, type)
b: $tType   thf(b, type)
r: a → b → $o  thf(r, type)
∀x: a: ∃y: b: (r@x@y)  thf(rttotal, axiom)
∃f: a → b: ∀x: a: (r@x@(f@x))  thf(skolem, conjecture)

```

SYO528^1.p There can be 4 distinct choice operators on type \$o

```

eps1: ($o → $o) → $o  thf(eps1, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps1@p)))  thf(choiceax1, axiom)
eps2: ($o → $o) → $o  thf(eps2, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps2@p)))  thf(choiceax2, axiom)
eps3: ($o → $o) → $o  thf(eps3, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps3@p)))  thf(choiceax3, axiom)
eps4: ($o → $o) → $o  thf(eps4, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps4@p)))  thf(choiceax4, axiom)
eps1 ≠ eps2  thf(choiceax12, axiom)
eps1 ≠ eps3  thf(choiceax13, axiom)
eps1 ≠ eps4  thf(choiceax14, axiom)
eps2 ≠ eps3  thf(choiceax23, axiom)
eps2 ≠ eps4  thf(choiceax24, axiom)
eps3 ≠ eps4  thf(choiceax34, axiom)

```

SYO529^1.p There cannot be 5 distinct choice operators on type \$o

```

eps1: ($o → $o) → $o  thf(eps1, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps1@p)))  thf(choiceax1, axiom)
eps2: ($o → $o) → $o  thf(eps2, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps2@p)))  thf(choiceax2, axiom)
eps3: ($o → $o) → $o  thf(eps3, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps3@p)))  thf(choiceax3, axiom)
eps4: ($o → $o) → $o  thf(eps4, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps4@p)))  thf(choiceax4, axiom)
eps5: ($o → $o) → $o  thf(eps5, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(eps5@p)))  thf(choiceax5, axiom)
eps1 ≠ eps2  thf(choiceax12, axiom)
eps1 ≠ eps3  thf(choiceax13, axiom)
eps1 ≠ eps4  thf(choiceax14, axiom)
eps1 ≠ eps5  thf(choiceax15, axiom)
eps2 ≠ eps3  thf(choiceax23, axiom)
eps2 ≠ eps4  thf(choiceax24, axiom)
eps2 ≠ eps5  thf(choiceax25, axiom)
eps3 ≠ eps4  thf(choiceax34, axiom)
eps3 ≠ eps5  thf(choiceax35, axiom)
eps4 ≠ eps5  thf(choiceax45, axiom)

```

SYO530^1.p Binary choice on individuals

epsa and epsb work together to give an a and b such that R a b holds, if such an a and b exist for a binary relation R on \$i. A choice operator on i can be used to define a choice operator on i*i (Curried). In this version, the solution is given and the goal is to check that it works.

```
eps: ($i → $o) → $i  thf(eps, type)
```

$$\begin{aligned}
 & \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf(choiceax, axiom)} \\
 & \text{epsa: } (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(epsa, type)} \\
 & \text{epsa} = (\lambda r: \$i \rightarrow \$i \rightarrow \$o: (\text{eps} @ \lambda x: \$i: \exists y: \$i: (r @ x @ y))) \quad \text{thf(epсад, definition)} \\
 & \text{epsb: } (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(epsb, type)} \\
 & \text{epsb} = (\lambda r: \$i \rightarrow \$i \rightarrow \$o: (\text{eps} @ \lambda y: \$i: (r @ (\text{epsa} @ r) @ y))) \quad \text{thf(epсад, definition)} \\
 & \forall r: \$i \rightarrow \$i \rightarrow \$o: (\exists x: \$i, y: \$i: (r @ x @ y) \Rightarrow (r @ (\text{epsa} @ r) @ (\text{epsb} @ r))) \quad \text{thf(conj, conjecture)}
 \end{aligned}$$

SYO531^1.p Binary choice on individuals 2

There is an Epsb such that epsa and Epsb work together to give an a and b such that R a b holds, if such an a and b exist for a binary relation R on \$i. A choice operator on i can be used to define a choice operator on i*i (Curried). In this version, the first half of the solution is given.

$$\begin{aligned}
 & \text{eps: } (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)} \\
 & \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf(choiceax, axiom)} \\
 & \text{epsa: } (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(epsa, type)} \\
 & \text{epsa} = (\lambda r: \$i \rightarrow \$i \rightarrow \$o: (\text{eps} @ \lambda x: \$i: \exists y: \$i: (r @ x @ y))) \quad \text{thf(epсад, definition)} \\
 & \exists \text{epsb: } (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i: \forall r: \$i \rightarrow \$i \rightarrow \$o: (\exists x: \$i, y: \$i: (r @ x @ y) \Rightarrow (r @ (\text{epsa} @ r) @ (\text{epsb} @ r))) \quad \text{thf(conj, conjecture)}
 \end{aligned}$$

SYO532^1.p Binary choice on individuals 3

Epsa and (epsb Epsa) work together to give an a and b such that R a b holds, if such an a and b exist for a binary relation R on \$i. A choice operator on i can be used to define a choice operator on i*i (Curried). In this version, the second half of the solution is given.

$$\begin{aligned}
 & \text{eps: } (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)} \\
 & \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf(choiceax, axiom)} \\
 & \text{epsb: } ((\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i) \rightarrow (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(epsb, type)} \\
 & \text{epsb} = (\lambda \text{epsa: } (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i, r: \$i \rightarrow \$i \rightarrow \$o: (\text{eps} @ \lambda y: \$i: (r @ (\text{epsa} @ r) @ y))) \quad \text{thf(epсад, definition)} \\
 & \exists \text{epsa: } (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i: \forall r: \$i \rightarrow \$i \rightarrow \$o: (\exists x: \$i, y: \$i: (r @ x @ y) \Rightarrow (r @ (\text{epsa} @ r) @ (\text{epsb} @ \text{epsa} @ r))) \quad \text{thf(conj, conjecture)}
 \end{aligned}$$

SYO533^1.p Binary choice on individuals 4

There is an Epsa such that Epsa and (epsb Epsa) work together to give an a and b such that R a b holds, if such an a and b exist for a binary relation R on \$i. A choice operator on i can be used to define a choice operator on i*i (Curried). In this version the prover must synthesize both parts of the solution.

$$\exists \text{epsa: } (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i, \text{epsb: } (\$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i: \forall r: \$i \rightarrow \$i \rightarrow \$o: (\exists x: \$i, y: \$i: (r @ x @ y) \Rightarrow (r @ (\text{epsa} @ r) @ (\text{epsb} @ r))) \quad \text{thf(conj, conjecture)}$$

SYO534^1.p 3-ary choice on individuals

epsa, epsb and epse work together to give an a, b and c such that R a b c holds, if such an a, b and c exist for a 3-ary relation R on \$i. A choice operator on i can be used to define a choice operator on i*i*i (Curried). In this version, the solution is given and the goal is to check that it works.

$$\begin{aligned}
 & \text{eps: } (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)} \\
 & \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf(choiceax, axiom)} \\
 & \text{epsa: } (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(epса, type)} \\
 & \text{epsa} = (\lambda r: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: (\text{eps} @ \lambda x: \$i: \exists y: \$i, z: \$i: (r @ x @ y @ z))) \quad \text{thf(epсад, definition)} \\
 & \text{epsb: } (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(epsb, type)} \\
 & \text{epsb} = (\lambda r: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: (\text{eps} @ \lambda y: \$i: \exists z: \$i: (r @ (\text{epsа} @ r) @ y @ z))) \quad \text{thf(epсад, definition)} \\
 & \text{epsc: } (\$i \rightarrow \$i \rightarrow \$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(epsc, type)} \\
 & \text{epsc} = (\lambda r: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: (\text{eps} @ \lambda z: \$i: (r @ (\text{epsа} @ r) @ (\text{epsb} @ r) @ z))) \quad \text{thf(epscd, definition)} \\
 & \forall r: \$i \rightarrow \$i \rightarrow \$i \rightarrow \$o: (\exists x: \$i, y: \$i, z: \$i: (r @ x @ y @ z) \Rightarrow (r @ (\text{epsа} @ r) @ (\text{epsb} @ r) @ (\text{epsc} @ r))) \quad \text{thf(conj, conjecture)}
 \end{aligned}$$

SYO535^1.p Choice on relations between individuals and functions

$$\begin{aligned}
 & \text{eps: } (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)} \\
 & \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf(choiceax, axiom)} \\
 & \text{epsii: } ((\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i \quad \text{thf(epси, type)} \\
 & \forall p: (\$i \rightarrow \$i) \rightarrow \$o: (\exists x: \$i \rightarrow \$i: (p @ x) \Rightarrow (p @ (\text{epsii} @ p))) \quad \text{thf(choiceaxii, axiom)} \\
 & \text{epsa: } (\$i \rightarrow (\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \quad \text{thf(epса, type)} \\
 & \text{epsa} = (\lambda r: \$i \rightarrow (\$i \rightarrow \$i) \rightarrow \$o: (\text{eps} @ \lambda x: \$i: \exists y: \$i \rightarrow \$i: (r @ x @ y))) \quad \text{thf(epсад, definition)} \\
 & \text{epsb: } (\$i \rightarrow (\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i \quad \text{thf(epsb, type)} \\
 & \text{epsb} = (\lambda r: \$i \rightarrow (\$i \rightarrow \$i) \rightarrow \$o: (\text{epsii} @ \lambda y: \$i \rightarrow \$i: (r @ (\text{epsа} @ r) @ y))) \quad \text{thf(epсад, definition)} \\
 & \forall r: \$i \rightarrow (\$i \rightarrow \$i) \rightarrow \$o: (\exists x: \$i, y: \$i \rightarrow \$i: (r @ x @ y) \Rightarrow (r @ (\text{epsа} @ r) @ (\text{epsb} @ r))) \quad \text{thf(conj, conjecture)}
 \end{aligned}$$

SYO536^1.p Choice on relations between functions and individuals

$$\begin{aligned}
 & \text{eps: } (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)} \\
 & \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf(choiceax, axiom)} \\
 & \text{epsii: } ((\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i \quad \text{thf(epси, type)}
 \end{aligned}$$

$$\begin{aligned} \forall p: (\$i \rightarrow \$i) \rightarrow \$o: (\exists x: \$i \rightarrow \$i: (p @ x) \Rightarrow (p @ (\text{epsii} @ p))) & \quad \text{thf(choiceaxii, axiom)} \\ \text{epsa: } ((\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$o) \rightarrow \$i \rightarrow \$i & \quad \text{thf(epsa, type)} \\ \text{epsa} = (\lambda r: (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$o: (\text{epsii} @ \lambda x: \$i \rightarrow \$i: \exists y: \$i: (r @ x @ y))) & \quad \text{thf(epsad, definition)} \\ \text{epsb: } ((\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$o) \rightarrow \$i & \quad \text{thf(epsb, type)} \\ \text{epsb} = (\lambda r: (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$o: (\text{eps} @ \lambda y: \$i: (r @ (\text{epsa} @ r) @ y))) & \quad \text{thf(epsbd, definition)} \\ \forall r: (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$o: (\exists x: \$i \rightarrow \$i, y: \$i: (r @ x @ y) \Rightarrow (r @ (\text{epsa} @ r) @ (\text{epsb} @ r))) & \quad \text{thf(conj, conjecture)} \end{aligned}$$

SYO537^1.p Choice on binary relations between functions

$$\begin{aligned} \text{epsii: } ((\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i & \quad \text{thf(epsii, type)} \\ \forall p: (\$i \rightarrow \$i) \rightarrow \$o: (\exists x: \$i \rightarrow \$i: (p @ x) \Rightarrow (p @ (\text{epsii} @ p))) & \quad \text{thf(choiceaxii, axiom)} \\ \text{epsa: } ((\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i & \quad \text{thf(epsa, type)} \\ \text{epsa} = (\lambda r: (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$i) \rightarrow \$o: (\text{epsii} @ \lambda x: \$i \rightarrow \$i: \exists y: \$i \rightarrow \$i: (r @ x @ y))) & \quad \text{thf(epsad, definition)} \\ \text{epsb: } ((\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i & \quad \text{thf(epsb, type)} \\ \text{epsb} = (\lambda r: (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$i) \rightarrow \$o: (\text{epsii} @ \lambda y: \$i \rightarrow \$i: (r @ (\text{epsa} @ r) @ y))) & \quad \text{thf(epsbd, definition)} \\ \forall r: (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$i) \rightarrow \$o: (\exists x: \$i \rightarrow \$i, y: \$i \rightarrow \$i: (r @ x @ y) \Rightarrow (r @ (\text{epsa} @ r) @ (\text{epsb} @ r))) & \quad \text{thf(conj, conjecture)} \end{aligned}$$

SYO538^1.p If-then-else on \$i defined from choice on \$i

A choice operator on \$i is used to define an if-then-else operator at \$i. Check that it works.

$$\begin{aligned} \text{eps: } (\$i \rightarrow \$o) \rightarrow \$i & \quad \text{thf(eps, type)} \\ \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) & \quad \text{thf(choiceax, axiom)} \\ \text{if: } \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i & \quad \text{thf(if, type)} \\ \text{if} = (\lambda b: \$o, x: \$i, y: \$i: (\text{eps} @ \lambda z: \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) & \quad \text{thf(ifd, definition)} \\ \forall x: \$i, y: \$i: ((\text{if}@{\$true}@x@y) = x \text{ and } (\text{if}@{\$false}@x@y) = y) & \quad \text{thf(conj, conjecture)} \end{aligned}$$

SYO539^1.p Range of if-then-else on \$i defined from choice on \$i

A choice operator on \$i is used to define an if-then-else operator at \$i. Check that it always returns the then-part or the else-part.

$$\begin{aligned} \text{eps: } (\$i \rightarrow \$o) \rightarrow \$i & \quad \text{thf(eps, type)} \\ \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) & \quad \text{thf(choiceax, axiom)} \\ \text{if: } \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i & \quad \text{thf(if, type)} \\ \text{if} = (\lambda b: \$o, x: \$i, y: \$i: (\text{eps} @ \lambda z: \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) & \quad \text{thf(ifd, definition)} \\ \forall b: \$o, x: \$i, y: \$i: ((\text{if}@b@x@y) = x \text{ or } (\text{if}@b@x@y) = y) & \quad \text{thf(conj, conjecture)} \end{aligned}$$

SYO540^1.p Property of if-then-else on \$i defined from choice on \$i

A choice operator on \$i is used to define an if-then-else operator at \$i. Check that if the then-part and the else-part are both X, it returns X.

$$\begin{aligned} \text{eps: } (\$i \rightarrow \$o) \rightarrow \$i & \quad \text{thf(eps, type)} \\ \forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) & \quad \text{thf(choiceax, axiom)} \\ \text{if: } \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i & \quad \text{thf(if, type)} \\ \text{if} = (\lambda b: \$o, x: \$i, y: \$i: (\text{eps} @ \lambda z: \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) & \quad \text{thf(ifd, definition)} \\ \forall b: \$o, x: \$i: (\text{if}@b@x@x) = x & \quad \text{thf(conj, conjecture)} \end{aligned}$$

SYO541^1.p If-then-else on \$i>\$i defined from choice on \$i>\$i

A choice operator on (\$i>\$i) is used to define an if-then-else operator at (\$i>\$i). Check that it works.

$$\begin{aligned} \text{epsii: } ((\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i & \quad \text{thf(epsii, type)} \\ \forall p: (\$i \rightarrow \$i) \rightarrow \$o: (\exists x: \$i \rightarrow \$i: (p @ x) \Rightarrow (p @ (\text{epsii} @ p))) & \quad \text{thf(choiceaxii, axiom)} \\ \text{if: } \$o \rightarrow (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$i & \quad \text{thf(if, type)} \\ \text{if} = (\lambda b: \$o, x: \$i \rightarrow \$i, y: \$i \rightarrow \$i: (\text{epsii} @ \lambda z: \$i \rightarrow \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) & \quad \text{thf(ifd, definition)} \\ \forall x: \$i \rightarrow \$i, y: \$i \rightarrow \$i: ((\text{if}@{\$true}@x@y) = x \text{ and } (\text{if}@{\$false}@x@y) = y) & \quad \text{thf(conj, conjecture)} \end{aligned}$$

SYO542^1.p If-then-else on \$i>\$i defined from choice on \$i>\$i

A choice operator on (\$i>\$i) is used to define an if-then-else operator at (\$i>\$i). Check that it always returns the then-part or the else-part.

$$\begin{aligned} \text{epsii: } ((\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i & \quad \text{thf(epsii, type)} \\ \forall p: (\$i \rightarrow \$i) \rightarrow \$o: (\exists x: \$i \rightarrow \$i: (p @ x) \Rightarrow (p @ (\text{epsii} @ p))) & \quad \text{thf(choiceaxii, axiom)} \\ \text{if: } \$o \rightarrow (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$i & \quad \text{thf(if, type)} \\ \text{if} = (\lambda b: \$o, x: \$i \rightarrow \$i, y: \$i \rightarrow \$i: (\text{epsii} @ \lambda z: \$i \rightarrow \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) & \quad \text{thf(ifd, definition)} \\ \forall b: \$o, x: \$i \rightarrow \$i, y: \$i \rightarrow \$i: ((\text{if}@b@x@y) = x \text{ or } (\text{if}@b@x@y) = y) & \quad \text{thf(conj, conjecture)} \end{aligned}$$

SYO543^1.p If-then-else on \$i>\$i defined from choice on \$i>\$i

A choice operator on (\$i>\$i) is used to define an if-then-else operator at (\$i>\$i). Check that if the then-part and else-part are both X, then it returns X.

$$\text{epsii: } ((\$i \rightarrow \$i) \rightarrow \$o) \rightarrow \$i \rightarrow \$i & \quad \text{thf(epsii, type)}$$

$\forall p: (\$i \rightarrow \$i) \rightarrow \$o: (\exists x: \$i \rightarrow \$i: (p@x) \Rightarrow (p@(epsii@p))) \quad \text{thf(choiceaxii, axiom)}$
 if: $\$o \rightarrow (\$i \rightarrow \$i) \rightarrow (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$i \quad \text{thf(if, type)}$
 $\text{if} = (\lambda b: \$o, x: \$i \rightarrow \$i, y: \$i \rightarrow \$i: (\text{epsii}@z: \$i \rightarrow \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) \quad \text{thf(ifd, definition)}$
 $\forall b: \$o, x: \$i \rightarrow \$i: (\text{if}@b@x@x) = x \quad \text{thf(conj, conjecture)}$

SYO544^1.p Case operator from ($\$o > \o) to $\$i$ defined from choice on $\$i$

A case operator from ($\$o > \o) (with 4 elements) to $\$i$ is defined using a choice operator on $\$i$. Check all 4 equations.

eps: $(\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)}$
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p@x) \Rightarrow (p@(eps@p))) \quad \text{thf(choiceax, axiom)}$
 case: $(\$o \rightarrow \$o) \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad \text{thf(caseoo, type)}$
 $\text{case} = (\lambda b: \$o \rightarrow \$o, x: \$i, y: \$i, u: \$i, v: \$i: (\text{eps}@z: \$i: ((b = (\lambda a: \$o: \$false) \text{ and } z = x) \text{ or } (b = \neg \text{ and } z = y) \text{ or } (b = (\lambda a: \$o: a) \text{ and } z = u) \text{ or } (b = (\lambda a: \$o: \$true) \text{ and } z = v)))) \quad \text{thf(caseood, definition)}$
 $f_0: \$o \rightarrow \$o \quad \text{thf}(f_0, type)$
 $(f_0@\$false) = \$false \quad \text{thf}(f0f, axiom)$
 $(f_0@\$true) = \$false \quad \text{thf}(f0t, axiom)$
 $f_1: \$o \rightarrow \$o \quad \text{thf}(f_1, type)$
 $(f_1@\$false) = \$true \quad \text{thf}(f1f, axiom)$
 $(f_1@\$true) = \$false \quad \text{thf}(f1t, axiom)$
 $f_2: \$o \rightarrow \$o \quad \text{thf}(f_2, type)$
 $(f_2@\$false) = \$false \quad \text{thf}(f2f, axiom)$
 $(f_2@\$true) = \$true \quad \text{thf}(f2t, axiom)$
 $f_3: \$o \rightarrow \$o \quad \text{thf}(f_3, type)$
 $(f_3@\$false) = \$true \quad \text{thf}(f3f, axiom)$
 $(f_3@\$true) = \$true \quad \text{thf}(f3t, axiom)$
 $\forall x: \$i, y: \$i, u: \$i, v: \$i: ((\text{case}@f_0@x@y@u@v) = x \text{ and } (\text{case}@f_1@x@y@u@v) = y \text{ and } (\text{case}@f_2@x@y@u@v) = u \text{ and } (\text{case}@f_3@x@y@u@v) = v) \quad \text{thf(conj, conjecture)}$

SYO545^1.p Property of case from ($\$o > \o) to $\$i$ defined from choice on $\$i$

A choice operator on $\$i$ is used to define an if-then-else operator at $\$i$. Check that case always returns one of the four given results.

eps: $(\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)}$
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p@x) \Rightarrow (p@(eps@p))) \quad \text{thf(choiceax, axiom)}$
 case: $(\$o \rightarrow \$o) \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad \text{thf(caseoo, type)}$
 $\text{case} = (\lambda b: \$o \rightarrow \$o, x: \$i, y: \$i, u: \$i, v: \$i: (\text{eps}@z: \$i: ((b = (\lambda a: \$o: \$false) \text{ and } z = x) \text{ or } (b = \neg \text{ and } z = y) \text{ or } (b = (\lambda a: \$o: a) \text{ and } z = u) \text{ or } (b = (\lambda a: \$o: \$true) \text{ and } z = v)))) \quad \text{thf(caseood, definition)}$
 $f: \$o \rightarrow \$o \quad \text{thf}(f, type)$
 $\forall x: \$i, y: \$i, u: \$i, v: \$i: ((\text{case}@f@x@y@u@v) = x \text{ or } (\text{case}@f@x@y@u@v) = y \text{ or } (\text{case}@f@x@y@u@v) = u \text{ or } (\text{case}@f@x@y@u@v) = v) \quad \text{thf(conj, conjecture)}$

SYO546^1.p Property of case from ($\$o > \o) to $\$i$ defined from choice on $\$i$

A choice operator on $\$i$ is used to define an if-then-else operator at $\$i$. Check that case always returns one of the four given results.

eps: $(\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)}$
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p@x) \Rightarrow (p@(eps@p))) \quad \text{thf(choiceax, axiom)}$
 case: $(\$o \rightarrow \$o) \rightarrow \$i \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad \text{thf(caseoo, type)}$
 $\text{case} = (\lambda b: \$o \rightarrow \$o, x: \$i, y: \$i, u: \$i, v: \$i: (\text{eps}@z: \$i: ((b = (\lambda a: \$o: \$false) \text{ and } z = x) \text{ or } (b = \neg \text{ and } z = y) \text{ or } (b = (\lambda a: \$o: a) \text{ and } z = u) \text{ or } (b = (\lambda a: \$o: \$true) \text{ and } z = v)))) \quad \text{thf(caseood, definition)}$
 $f: \$o \rightarrow \$o \quad \text{thf}(f, type)$
 $\forall x: \$i: (\text{case}@f@x@x@x@x) = x \quad \text{thf(conj, conjecture)}$

SYO547^1.p Choice Complement

The choice operator applied to complements of predicates chooses an element not in the predicate, if there is one.

eps: $(\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(eps, type)}$
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p@x) \Rightarrow (p@(eps@p))) \quad \text{thf(choiceax, axiom)}$
 epscomp: $(\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf(epscomp, type)}$
 $\text{epscomp} = (\lambda p: \$i \rightarrow \$o: (\text{eps}@x: \$i: \neg p@x)) \quad \text{thf(epscompd, definition)}$
 $\forall p: \$i \rightarrow \$o: (\exists x: \$i: \neg p@x \Rightarrow \neg p@(epscomp@p)) \quad \text{thf(choicem, conjecture)}$

SYO548^1.p Choice complement

There is an operator that chooses an element not in the predicate, if there is one.

$\exists e: (\$i \rightarrow \$o) \rightarrow \$i: \forall p: \$i \rightarrow \$o: (\exists x: \$i: \neg p@x \Rightarrow \neg p@(e@p)) \quad \text{thf(choicem, conjecture)}$

SYO549^1.p The eta double negation problem

$p: (\$o \rightarrow \$o) \rightarrow (\$o \rightarrow \$o) \rightarrow \$o \quad \text{thf}(p, \text{type})$
 $f: \$o \rightarrow \$o \quad \text{thf}(f, \text{type})$
 $g: \$o \rightarrow \$o \quad \text{thf}(g, \text{type})$
 $p @ \lambda x: \$o: (f @ \neg \neg x) @ g \quad \text{thf}(\text{pfg}, \text{axiom})$
 $p @ f @ \lambda x: \$o: (g @ \neg \neg x) \quad \text{thf}(\text{pfgc}, \text{conjecture})$

SYO550^1.p The identity function on individuals exists

$\exists f: \$i \rightarrow \$i: \forall x: \$i: (f @ x) = x \quad \text{thf}(\text{claim}, \text{conjecture})$

SYO551^1.p The identity function on functions from $\$i$ to $\$i$ exists

$\exists f: (\$i \rightarrow \$i) \rightarrow \$i \rightarrow \$i: \forall x: \$i \rightarrow \$i: (f @ x) = x \quad \text{thf}(\text{claim}, \text{conjecture})$

SYO552^1.p The first projection exists

There is a binary function that returns its first argument

$\exists f: \$i \rightarrow \$i \rightarrow \$i: \forall x: \$i, y: \$i: (f @ x @ y) = x \quad \text{thf}(\text{claim}, \text{conjecture})$

SYO553^1.p The second projection exists.

There is a binary function that returns its second argument.

$\exists f: \$i \rightarrow \$i \rightarrow \$i: \forall x: \$i, y: \$i: (f @ x @ y) = y \quad \text{thf}(\text{claim}, \text{conjecture})$

SYO554^1.p Teucke's example

$p: \$i \rightarrow \$o \quad \text{thf}(p, \text{type})$
 $s: \$i \quad \text{thf}(s, \text{type})$
 $t: \$i \quad \text{thf}(t, \text{type})$
 $u: \$i \quad \text{thf}(u, \text{type})$
 $p @ s \text{ or } p @ t \quad \text{thf}(\text{pst}, \text{axiom})$
 $\neg p @ u \text{ or } \neg p @ t \quad \text{thf}(\text{puv}, \text{axiom})$
 $s = t \quad \text{thf}(\text{st}, \text{axiom})$
 $t = u \quad \text{thf}(\text{tu}, \text{axiom})$

SYO555^1.p If-then-else defined from choice is independent of choice

$\text{eps}_1: (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf}(\text{eps}_1, \text{type})$

$\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps}_1 @ p))) \quad \text{thf}(\text{choiceax}_1, \text{axiom})$

$\text{if}_1: \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{if}_1, \text{type})$

$\text{if}_1 = (\lambda b: \$o, x: \$i, y: \$i: (\text{eps}_1 @ \lambda z: \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) \quad \text{thf}(\text{if1d}, \text{definition})$

$\text{eps}_2: (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf}(\text{eps}_2, \text{type})$

$\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps}_2 @ p))) \quad \text{thf}(\text{choiceax}_2, \text{axiom})$

$\text{if}_2: \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{if}_2, \text{type})$

$\text{if}_2 = (\lambda b: \$o, x: \$i, y: \$i: (\text{eps}_2 @ \lambda z: \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) \quad \text{thf}(\text{if2d}, \text{definition})$

$\text{if}_1 = \text{if}_2 \quad \text{thf}(\text{conj}, \text{conjecture})$

SYO556^1.p Relationship between if-then-else and choice on $\$$

$\text{eps}: (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf}(\text{eps}, \text{type})$

$\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf}(\text{choiceax}, \text{axiom})$

$\text{if}: \$o \rightarrow \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(\text{if}, \text{type})$

$\text{if} = (\lambda b: \$o, x: \$i, y: \$i: (\text{eps} @ \lambda z: \$i: ((b \text{ and } z = x) \text{ or } (\neg b \text{ and } z = y)))) \quad \text{thf}(\text{ifd}, \text{definition})$

$\forall p: \$i \rightarrow \$o: (\text{eps} @ p) = (\text{if} @ \exists x: \$i: (p @ x) @ (\text{eps} @ p) @ (\text{eps} @ \lambda x: \$i: \$\text{false})) \quad \text{thf}(\text{conj}, \text{conjecture})$

SYO557^1.p Exists on $\$i$ can be expressed in terms of choice on $\$$

$\text{eps}: (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf}(\text{eps}, \text{type})$

$\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf}(\text{choiceax}, \text{axiom})$

$(\lambda p: \$i \rightarrow \$o: (p @ (\text{eps} @ p))) = (\lambda p: \$i \rightarrow \$o: ?:?(p)) \quad \text{thf}(\text{conj}, \text{conjecture})$

SYO558^1.p Forall on $\$i$ can be expressed in terms of choice on $\$$

$\text{eps}: (\$i \rightarrow \$o) \rightarrow \$i \quad \text{thf}(\text{eps}, \text{type})$

$\forall p: \$i \rightarrow \$o: (\exists x: \$i: (p @ x) \Rightarrow (p @ (\text{eps} @ p))) \quad \text{thf}(\text{choiceax}, \text{axiom})$

$(\lambda p: \$i \rightarrow \$o: (p @ (\text{eps} @ \lambda x: \$i: \neg p @ x))) = (\lambda p: \$i \rightarrow \$o: !!(p)) \quad \text{thf}(\text{conj}, \text{conjecture})$

SYO559^1.p Choice on $\$o > \o applied to choice on $\$o$ cannot be negated

$\text{epso}: (\$o \rightarrow \$o) \rightarrow \$o \quad \text{thf}(\text{epso}, \text{type})$

$\forall p: \$o \rightarrow \$o: (\exists x: \$o: (p @ x) \Rightarrow (p @ (\text{epso} @ p))) \quad \text{thf}(\text{choiceaxo}, \text{axiom})$

$\text{epsoo}: ((\$o \rightarrow \$o) \rightarrow \$o) \rightarrow \$o \rightarrow \$o \quad \text{thf}(\text{epsoo}, \text{type})$

$\forall p: (\$o \rightarrow \$o) \rightarrow \$o: (\exists x: \$o \rightarrow \$o: (p @ x) \Rightarrow (p @ (\text{epsoo} @ p))) \quad \text{thf}(\text{choiceaxoo}, \text{axiom})$

$(\text{epsoo} @ \text{epso} @ \$\text{false}) \Rightarrow (\text{epsoo} @ \text{epso} @ \$\text{true}) \quad \text{thf}(c, \text{conjecture})$

SYO560^1.p Choice on $\$o > \o applied to choice on $\$o$ is identity or constant

```

epso: ($o → $o) → $o      thf(epso, type)
∀p: $o → $o: (∃x: $o: (p@x) ⇒ (p@(epso@p)))      thf(choiceaxo, axiom)
epsoo: ((\$o → \$o) → \$o) → \$o → \$o      thf(epsoo, type)
∀p: (\$o → \$o) → \$o: (∃x: \$o → \$o: (p@x) ⇒ (p@(epsoo@p)))      thf(choiceaxoo, axiom)
q: (\$o → \$o) → \$o      thf(q, type)
q@λx: $o: $true      thf(qkt, axiom)
q@λx: $o: $false      thf(qkf, axiom)
q@λx: $o: x      thf(qid, axiom)
q@(epsoo@epso)      thf(c, conjecture)

```

SYO561+1.p Distinct objects

"Apple" ≠ "Microsoft" fof(apple_not_microsoft, conjecture)

SYO561_1.p Distinct objects

```

company: $tType      tff(company_type, type)
apple: company      tff(apple_company, type)
microsoft: company      tff(microsoft_company, type)
$distinct(apple, microsoft)      tff(distinct_companies, axiom)
apple ≠ microsoft      tff(apple_not_microsoft, conjecture)

```

SYO561_2.p Distinct objects

"Apple" ≠ "Microsoft" tff(apple_not_microsoft, conjecture)

SYO562_1.p If-then-else

```

a: $i      tff(a_type, type)
f: $i → $i      tff(f_type, type)
p: $i → $o      tff(p_type, type)
q: ($i × $i) → $o      tff(q_type, type)
∀z: $i: $ite_f(∃x: p(f(x)), ∀x: q(x, x), q(z, a))      tff(ite_f, axiom)
p($ite_t(q(a, f(a)), a, f(a)))      tff(ite_t, axiom)
q(a, a)      tff(fact, axiom)
q(f(a), f(a))      tff(prove, conjecture)

```

SYO563+1.p Unequal numbers - reals

1.0 ≠ 2.0 fof(one_not_equal_to_2, conjecture)

SYO563+2.p Unequal numbers - rationals

1/1 ≠ 2/1 fof(one_not_equal_to_2, conjecture)

SYO563+3.p Unequal numbers - integers

1 ≠ 2 fof(one_not_equal_to_2, conjecture)

SYO564^7.p Barcan scheme instance. (Ted Sider's qml wwf 1)

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mforall_ind@λx: mu: (mbox_s4@(f@x)))@(mbox_s4@(mforall_ind@λx: mu: (f@x))))      thf(con, conjecture)

```

SYO565^7.p Fitting and Mendelsohn problem

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
a: mu → $i → $o      thf(a_type, type)
mvalid@(mequiv@(mbox_s4@(mforall_ind@λx: mu: (a@x)))@(mdia_s4@(mforall_ind@λx: mu: (mbox_s4@(a@x))))      thf(co)

```

SYO566^7.p Girle problem

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
g: mu → $i → $o      thf(g_type, type)
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mbox_s4@(mforall_ind@λx: mu: (mimplies@(f@x)@(g@x))))@(mimplies@(mforall_ind@λx: mu: (f@x))@

```

SYO567^7.p Girle problem

```

include('Axioms/LCL015^0.ax')

```

```

include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
g: mu → $i → $o      thf(g_type, type)
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mforall_ind@λx: mu: (mimplies@(f@x)@(mbox_s4@(g@x))))))@((mimplies@((mforall_ind@λx: mu: (f@x)))@((mforall_ind@λy: mu: (g@y))))))@((mforall_ind@λz: mu: (mbox_s4@((f@z)@((g@z))))))@((mforall_ind@λw: mu: (mbox_s4@((g@w)@((f@w)))))))

```

SYO568^7.p Girle problem

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
f: mu → $i → $o      thf(f_type, type)
a: mu      thf(a_type, type)
∀v: $i: (exists_in_world@a@v)      thf(existence_of_a_ax, axiom)
minvalid@(mimplies@(mforall_ind@λx: mu: (mor@((mbox_s4@(f@x))@(mbox_s4@(mnot@(f@x))))))@((mbox_s4@(mequiv@((mbo

```

SYO569^7.p Fitting and Mendelsohn problem

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
g: mu → $i → $o      thf(g_type, type)
f: mu → $i → $o      thf(f_type, type)
mvalid@(mimplies@(mand@((mforall_ind@(\lambda x: mu: (mbox_s4@(mbox_s4@(f@x))))))@
  (mdia_s4@(mexists_ind@(\lambda x: mu: (g@x))))))

```

SYO570^7.p Forbes problem

SYO571\7.p Quantified modal logics wffs. problem 9.

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
a: mu      thf(a_type, type)
∀v: $i: (exists_in_world@a@v)      thf(existence_of_a_ax, axiom)
mvalid@(mexists_ind@λx: mu: (mbox_s4@(qmltpeq@x@a)))      thf(con, conjecture)

```

SYO572\7.p Quantified modal logics wffs. problem 13.

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
f: mu → $i → $o      thf(f_type, type)
minvalid@(mimplies@((mdia_s4@(mforall_ind@λx: mu: (f@x)))@((mexists_ind@λx: mu: (mdia_s4@(f@x)))))      thf(con, conjecture)

```

SYO573\7.p Quantified modal logics wffs. problem 15.

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
g: mu → $i → $o      thf(g_type, type)
f: mu → $i → $o      thf(f_type, type)
a: mu      thf(a_type, type)
∀v: $i: (exists_in_world@a@v)      thf(existence_of_a_ax, axiom)
mvalid@(mimplies@(mand@((mdia_s4@(f@a))@(mdia_s4@(g@a))))@((mdia_s4@(mand@(f@a)@(g@a)))))      thf(con, conjecture)

```

SYO574\7.p Modal Propositional Logic Theorems. problem 37

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
q: $i → $o      thf(q_type, type)
p: $i → $o      thf(p_type, type)
mvalid@((mimplies@((mdia_s4@(mimplies@p@mbox_s4@q)))@((mimplies@mbox_s4@p)@((mdia_s4@q))))      thf(con, conjecture)

```

SYO575\7.p Modal Propositional Logic Theorems. problem 50

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@(#equiv@(EDIA_S4@(#EDIA_S4@(#MBOX_S4@#p))))@(#MBOX_S4@#p))      thf(con, conjecture)

```

SYO576^7.p Mixed Modal Propositional Logic WFFs. problem 7

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
p: $i → $o      thf(p_type, type)
mvalid@(#implies@(#EDIA_S4@(#MBOX_S4@(#EDIA_S4@(#MBOX_S4@#p))))@(#EDIA_S4@(#MBOX_S4@#p)))      thf(con, conjecture)

```

SYO577^7.p Mixed Modal Propositional Logic WFFs. problem 19

```

include('Axioms/LCL015^0.ax')
include('Axioms/LCL013^5.ax')
include('Axioms/LCL015^1.ax')
q: $i → $o      thf(q_type, type)
p: $i → $o      thf(p_type, type)
mvalid@(#or@(#MBOX_S4@(#implies@(#MBOX_S4@#p)@#q))@(#MBOX_S4@(#implies@(#MBOX_S4@#q)@#p)))      thf(con, conjecture)

```