Solving the general quadratic equation

Where does the quadratic formula come from?

We want to find all values x such that $ax^2+bx+c=0$.

If $a=0$ then we can easily solve $bx+c=0$
(unless $b=0\neq c$, in which case there are no solutions).

So assume $a\neq 0$, and divide by a on both sides.

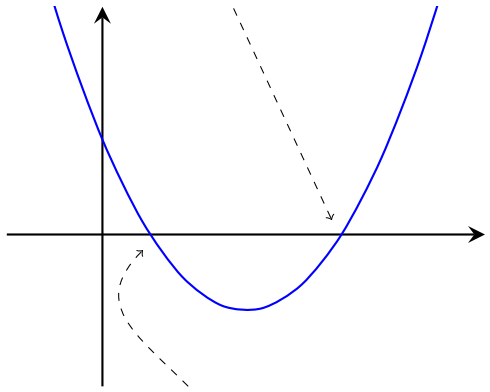
$$\Leftrightarrow \quad x^2+\frac{b}{a}x+\frac{c}{a}=0. \quad \text{Now complete the square}$$

$$\Leftrightarrow \quad x^2+2\frac{b}{2a}x+\square-\square+\frac{c}{a}=0$$

$$\Leftrightarrow \quad (x+\frac{b}{2a})^2=\square-\frac{c}{a}=\frac{\square-4ac}{4a^2}$$

$$\Leftrightarrow \quad x+\frac{b}{2a}=\pm\sqrt{\frac{\square-4ac}{4a^2}}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$



$$x=\frac{-b-\sqrt{b^2-4ac}}{2a}$$