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Parameterised Bisimulations: Some Applications

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- Applications to Compatible Relations
- Conclusions and Future Work

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- Parameterised Bisimulations
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- Parameterised Bisimulations
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Parameterised Bisimulations

- [AK06]A small generalisation of the notion of bisimulations on LTSs.
- Let $\mathcal{L} = \langle \mathbf{P}, \mathcal{O}, \longrightarrow \rangle$, $\longrightarrow \subseteq \mathbf{P} \times \mathcal{O} \times \mathbf{P}$ be any LTS. Let $\rho, \sigma \subseteq \mathcal{O} \times \mathcal{O}$. $R \subseteq \mathbf{P} \times \mathbf{P}$ is a (ρ, σ)**-bisimulation** if pRq implies the following conditions for all $a, b \in \mathcal{O}$.

$$p \xrightarrow{a} p' \Rightarrow \exists b, q'[a\rho b \wedge q \xrightarrow{b} q' \wedge p'Rq']$$

$$q \xrightarrow{b} q' \Rightarrow \exists a, p'[a\sigma b \wedge p \xrightarrow{a} p' \wedge p'Rq']$$

- The largest (ρ, σ)-bisimulation (under set containment) is called (ρ, σ)**-bisimilarity** and denoted $\sqsubseteq_{(\rho,\sigma)}$.

Notation. $R : p \sqsubseteq_{(\rho,\sigma)} q$: "*$R$ is a (ρ, σ)-bisimulation containing (p, q)*"

What do they generalise?

- **Strong bisimilarity**[Mil89]: $\sim\; = \underline{\Box}_{(\equiv,\equiv)}$
- **Efficiency Preorder**[AKH92]: $\precsim\; = \underline{\Box}_{(\preceq,\preceq)}$
- **Elaboration Preorder**[AKN95]: $\precapprox\; = \underline{\Box}_{(\preceq,\hat{\equiv})}$
- **Weak bisimilarity**[Mil89]: $\approx\; = \underline{\Box}_{(\hat{\equiv},\hat{\equiv})}$

where

$$\begin{aligned}
\equiv \;&=\; \text{the identity relation on } Act \text{ or } Act^* \\
\preceq \;&=\; \{(\tau^k a \tau^l, \tau^m a \tau^n) \mid k+l \geq m+n, a \in Act\} \\
\hat{\equiv} \;&=\; \{(s,t) \mid \hat{s} = \hat{t}, s,t \in Act^*\}
\end{aligned}$$

The simulation preorder[Mil89], localities, bisimulation on speed[LV06], amortised bisimilarity[KAK05] etc. may be represented as $\underline{\Box}_{(\rho,\sigma)}$ for appropriately chosen ρ and σ.

Some previous results

Many of the nice algebraic properties of bisimularities are induced by the relations ρ and σ. In fact we have [AK06]

- **Monotonicity.** $(\rho,\sigma) \subseteq (\rho',\sigma') \Rightarrow \underline{\Box}_{(\rho,\sigma)} \subseteq \underline{\Box}_{(\rho',\sigma')}$.
- **Preorders.** $\underline{\Box}_{(\rho,\sigma)}$ is a preorder iff ρ and σ are both preorders.
- **Equivalences.** $\underline{\Box}_{(\rho,\sigma)}$ is an equivalence iff ρ and σ are both preorders and $\sigma = \rho^{-1}$.
- $\langle \mathsf{B}_{(\rho,\sigma)}, \cup, \emptyset \rangle$ is a commutative submonoid of $\langle 2^{\mathbf{P}\times\mathbf{P}}, \cup, \emptyset \rangle$.
- $\underline{\Box}_{(\rho,\sigma)}$ is a preorder if $\langle \mathsf{B}_{(\rho,\sigma)}, \circ, \equiv \rangle$ is a submonoid of $\langle 2^{\mathbf{P}\times\mathbf{P}}, \circ, \equiv \rangle$.

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Compatible Relations

- What about preorders or equivalences that are not necessarily pre-congruences or congruences resp.?
- Is a co-inductive characterisation possible for observational congruence?
- What about other operations such as recursion or substitutions that may not preserve preorders or equivalences?

Our Contribution

- Observational Congruence is a parameterised bisimilarity
- Amortised bisimilarity [KAK05] is preserved under recursion
- Hyperbisimilarity in the Fusion calculus [Vic98] is a parameterised hyper-bisimilarity

Unifying Feature

They are all different!

- Observational Congruence : requires abstracting the transitions in the LTS generated by the operational semantics
- Amortised bisimilarity: requires transforming the LTS generated by the operational semantics
- Hyperbisimilarity: requires changing the operational semantics and generalising hyper-bisimilarity to a parameterised hyper-bisimilarity that preserves substitution effects.

All the proofs are very technical and involved!

Observational Congruence

- $\tau.\mathbf{0} \approx \mathbf{0}$. But $a.\mathbf{0} + \tau.\mathbf{0} \not\approx a.\mathbf{0} + \mathbf{0}$ for any $a \neq \tau$.
- Observational congruence is the largest symmetric relation ($\approx^+$) on $\mathbf{P}$ such that $p \approx^+ q$ implies for all $\mu \in Act$, if $p \xrightarrow{\mu} p'$ then for some q', $m, n \geq 0$, $q \xrightarrow{\tau^m \mu \tau^n} q'$ and $p' \approx q'$.
- $\tau.\mathbf{0} \approx \mathbf{0}$ but $\tau.\mathbf{0} \not\approx^+ \mathbf{0}$. However $\tau.\tau.\mathbf{0} \approx^+ \tau.\mathbf{0}$
- $\approx^+$ is always expressed in terms of $\approx$. There has been no co-inductive characterisation of $\approx^+$.

A Derived LTS

Let

- $\mathcal{L} = \langle \mathbf{P}, Act, \longrightarrow \rangle$ be an LTS,
- $q \approx\stackrel{\tau}{\longrightarrow} q'$ denote $q \stackrel{\tau}{\longrightarrow} q'$ and $q \approx q'$.

Define $\mathcal{L}_\dagger = \langle \mathbf{P}, Act.\tau^*, \longrightarrow_\dagger \rangle$ such that $p \stackrel{\mu\tau^n}{\longrightarrow}_\dagger p'$ iff

- $p \stackrel{\mu}{\longrightarrow} p_0 \approx\stackrel{\tau}{\longrightarrow} p_1 \approx\stackrel{\tau}{\longrightarrow} \cdots \approx\stackrel{\tau}{\longrightarrow} p_n \equiv p'$ and
- there does not exist any p_{n+1} such that $p' \approx\stackrel{\tau}{\longrightarrow} p_{n+1}$.

No μ-derivative of any process in $\mathcal{L}_\dagger$ has any τ-derivative that is weakly bisimilar to itself. Hence both $\tau.\mathbf{0}$ and $\tau.\tau.\mathbf{0}$ have $\mathbf{0}$ as their only τ-derivative in $\mathcal{L}_\dagger$.

An equivalent LTS

Since $Act^+ = (Act.\tau^*)^+$ we may identify

$$\mathcal{L}_\dagger = \langle \mathbf{P}, Act.\tau^*, \longrightarrow_\dagger \rangle \text{ with } \mathcal{L}_\dagger^+ = \langle \mathbf{P}, Act^+, \longrightarrow_\dagger^+ \rangle$$

where $\longrightarrow_\dagger^+$ denotes one or more transitions via $\longrightarrow_\dagger$.

Theorem 1 *For divergence-free finite-state agents p, q,*

1. *For all $s \in Act^+$, if $p \stackrel{s}{\longrightarrow}_\dagger^+ p'$ then $p \stackrel{s}{\longrightarrow} p'$.*
2. *For all $s \in Act^+$, if $p \stackrel{s}{\longrightarrow} p'$ then there exists $s' \widehat{=} s$ such that $p \stackrel{s'}{\longrightarrow}_\dagger^+ p''$ and $p' \approx p''$.*
3. *$p \sqsubseteq_{(\widehat{=}_+, \widehat{=}_+)} q$ in $\mathcal{L}_\dagger^+$ iff $p \approx^+ q$, where $\widehat{=}_+$ is the restriction of $\widehat{=}$ to Act^+.*

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Amortised Bisimilarity

Introduced in [KAK05] in CCS – generalises and extends the "faster than" preorder defined by Lüttgen and Vogler in [LV06].

- **Priced actions**: $CA \cap Act = \emptyset$
- Priced actions cannot be restricted or relabelled and do not have complements. Hence cannot take part in synchronization.
- $\mathcal{A} = Act \cup CA$
- **Cost function**: $c : \mathcal{A} \to \mathbb{N}$, $c(\mu) = 0$ for all $\mu \in Act$
- $\rho \subseteq \mathcal{A} \times \mathcal{A}$ such that
  - $\rho \restriction Act = \equiv_{Act}$ and
  - $\rho \setminus \equiv_{Act} \subseteq CA \times CA$

Strong Amortised Bisimilarity

- A **strong amortised bisimulation** is a family of relations $\mathscr{R} = \{R_i \mid i \in \mathbb{N}\}$ such that whenever $(p, q) \in R_i$ for some $i \in \mathbb{N}$,

$$p \xrightarrow{a} p' \Rightarrow \exists b, q'[a\rho b \wedge q \xrightarrow{b} q' \wedge p' R_j q']$$

$$q \xrightarrow{b} q' \Rightarrow \exists a, p'[a\rho b \wedge p \xrightarrow{a} p' \wedge p' R_j q']$$

  where $j = i + c(b) - c(a)$.

- p is **amortised cheaper (more cost efficient)** than q ($p \prec_0^\rho q$ or simply $p \prec^\rho q$) if pR_0q for some strong amortised ρ-bisimulation $\mathscr{R}$.

- p is **amortised cheaper** than q **up to credit** i ($p \prec_i^\rho q$) if pR_iq. The index i gives the maximum credit which p requires to bisimulate q.

Compatibility with Recursion

- Milner's techniques not directly applicable to bisimulation defined as a family of relations.
- Necessary to "massage" the LTS generated by the operational semantics
- – Remember the cost of reaching a state.
  – $\mathcal{L}_C = \langle \mathbb{N} \times \mathbf{P}, \mathcal{A} \times \mathbb{N}, \longrightarrow_C \rangle$ is such that

$$p \xrightarrow{a} p' \Rightarrow m : p \xrightarrow{(a,n)}_C n : p', \text{ where } n = m + c(a)$$

- Express amortised bisimulation as a parameterised bisimilarity.

Theorem 2 *Let* $\gamma_\rho = \{((a,m),(b,n)) \mid a\rho b, m \leq n\}$. *Then* $m : p \sqsubseteq_{(\gamma_\rho,\gamma_\rho)} m : q$ *iff* $p \prec_0^\rho q$, *for all* $m \in \mathbb{N}$.

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Name-passing Calculi

- Substitutions are an integral part of name-passing calculi
- Names may be identified by some synchronization
- Under CCS-like synchronization *dynamic conflation* of names (passed as parameters) does not propagate without the meta-operation of substitution

$$\overline{u}x.\overline{x}.p \mid u(y).y.q \xrightarrow{\tau} \overline{x}.p \mid x.\{x/y\}.q$$

- Further bisimilarity is not preserved under parallel composition in name-passing calculi, because of the possibilities of synchronization.

Fusion Calculus

Why not the π-calculus?

- Symmetry between input and output.
- Properly contains the π-calculus.
- Explicit identification of names (fusions) as a global equivalence relation (restricted only by scope of application).
- There is only one meaningful notion of bisimilarity in the Fusion calculus (*c.f.* early and late bisimulations in the π-calculus).
- Substitutions are not symmetric, but fusions are. Makes it possible to give a generalised theory of parameterised bisimulations that respects the dynamic equivalence generated by fusions.

Man, It is still not that easy!

- In the presence of a relation ρ if $x\rho y$ and y is "fused" with z then we require $x\rho z$ to hold too!
- We need to dynamically remember what names are "equivalent", which the original semantics of the fusion calculus fails to provide.
- Change the operational semantics to have an environment ψ of name identifications.

Operational Semantics

$$\text{PrefL}\ \frac{-}{\alpha.p \xrightarrow{\alpha} p}$$

$$\text{SumL}\ \frac{p \xrightarrow{\alpha} p'}{p + q \xrightarrow{\alpha} p'}$$

$$\text{ParL}\ \frac{p \xrightarrow{\alpha} p'}{p|q \xrightarrow{\alpha} p'|q}$$

$$\text{PasL}\ \frac{p \xrightarrow{\alpha} p'}{(z)p \xrightarrow{\alpha} (z)p'}, z \notin n(\alpha)$$

$$\text{ScL}\ \frac{p \xrightarrow{\varphi} p', z\varphi x, x \neq z}{(z)p \xrightarrow{\varphi\backslash z} \{x/z\}p'}$$

$$\text{PrefR}\ \frac{-}{(\psi, \alpha.p) \xrightarrow{\alpha} (\psi \oplus \psi(\alpha), p)}$$

$$\text{SumR}\ \frac{(\psi, p) \xrightarrow{\alpha} (\psi', p')}{(\psi, p + q) \xrightarrow{\alpha} (\psi', p')}$$

$$\text{ParR}\ \frac{(\psi, p) \xrightarrow{\alpha} (\psi', p')}{(\psi, p|q) \xrightarrow{\alpha} (\psi', p'|q)}$$

$$\text{PasR}\ \frac{(\psi, p) \xrightarrow{\alpha} (\psi', p')}{(\psi, (z)p) \xrightarrow{\alpha} (\psi', (z)p')}, z \notin n(\alpha)$$

$$\text{ScR}\ \frac{(\psi, p) \xrightarrow{\varphi} (\psi \oplus \varphi, p'), z\varphi x, x \neq z}{(\psi, (z)p) \xrightarrow{\varphi\backslash z} (\psi \oplus (\varphi\backslash z), \{x/z\}p')}$$

Operational Semantics (contd.)

$$\textsc{OpL}\frac{p \xrightarrow{(\tilde{y})a\tilde{x}} p', z \in \tilde{x} - \tilde{y}, a \notin \{z, \bar{z}\}}{(z)p \xrightarrow{(z\tilde{y})a\tilde{x}} p'}$$

$$\textsc{OpR}\frac{(\psi, p) \xrightarrow{(\tilde{y})a\tilde{x}} (\psi, p'), z \in \tilde{x} - \tilde{y}, a \notin \{z, \bar{z}\}}{(\psi, (z)p) \xrightarrow{(z\tilde{y})a\tilde{x}} (\psi, p')}$$

$$\textsc{ComL}\frac{p \xrightarrow{u\tilde{x}} p', q \xrightarrow{\bar{u}\tilde{y}} q', |\tilde{x}| = |\tilde{y}|}{p|q \xrightarrow{\{\tilde{x}=\tilde{y}\}} p'|q'}$$

$$\textsc{ComR}\frac{(\psi, p) \xrightarrow{u\tilde{x}} (\psi, p'), (\psi, q) \xrightarrow{\bar{v}\tilde{y}} (\psi, q'), |\tilde{x}| = |\tilde{y}|, u\ \psi\ v}{(\psi, p|q) \xrightarrow{\{\tilde{x}=\tilde{y}\}} (\psi \oplus \{\tilde{x} = \tilde{y}\}, p'|q')}$$

Substitution Effects

- Every action has a "side-effect" viz. the creation of an equivalence on names. On sequences $\boldsymbol{\alpha}$ of actions this effect is

$$E(\boldsymbol{\alpha}) = \begin{cases} \varphi \oplus E(\boldsymbol{\alpha}') & \text{if } \boldsymbol{\alpha} = \varphi.\boldsymbol{\alpha}' \\ E(\boldsymbol{\alpha}') & \text{if } \boldsymbol{\alpha} = \alpha.\boldsymbol{\alpha}' \\ \mathbf{1} & \text{if } \boldsymbol{\alpha} = \epsilon \end{cases}$$

  where $\varphi \oplus \varphi'$ is the smallest equivalence containing both φ and φ'.

- A relation ρ is **conservative** iff $\forall \boldsymbol{\alpha}, \boldsymbol{\beta}$ if $\boldsymbol{\alpha} \rho \boldsymbol{\beta}$ then $E(\boldsymbol{\alpha}) = E(\boldsymbol{\beta})$. It is **substitution-closed** iff $\forall x, y, \boldsymbol{\alpha}, \boldsymbol{\beta}$ if $\boldsymbol{\alpha} \rho \boldsymbol{\beta}$ then $(\{x/y\}\boldsymbol{\alpha})\rho(\{x/y\}\boldsymbol{\beta})$.

Generalised (ρ, σ)-bisimulation

- For any $\rho, \sigma \subseteq Act^2$, a relation $\mathcal{G} \subseteq (Env \times \mathbf{P})^2$ is a **generalised (ρ, σ)-bisimulation** if $(\psi, p)\mathcal{G}(\omega, q)$ implies for all $\boldsymbol{\alpha_1}, \boldsymbol{\beta_2} \in Act^*$,

$$(\psi, p) \xrightarrow{\boldsymbol{\alpha_1}} (\psi', p') \Rightarrow \exists \boldsymbol{\alpha_2} : \boldsymbol{\alpha_1} \rho' \boldsymbol{\alpha_2}, (\omega', q') : (\omega, q) \xrightarrow{\boldsymbol{\alpha_2}} (\omega', q') \wedge (\psi', p') \; \mathcal{G} \; (\omega', q')$$

$$(\omega, q) \xrightarrow{\boldsymbol{\beta_2}} (\omega', q') \Rightarrow \exists \boldsymbol{\beta_1} : \boldsymbol{\beta_1} \sigma' \boldsymbol{\beta_2}, (\psi', p') : (\psi, p) \xrightarrow{\boldsymbol{\beta_1}} (\psi', p') \wedge (\psi', p') \; \mathcal{G} \; (\omega', q')$$

  where $\rho' = \psi \circ \rho \circ \omega$ and $\sigma' = \psi \circ \sigma \circ \omega$.

- A generalised (ρ, σ)-bisimulation $\mathcal{G}$ is a **(ρ, σ)-bisimulation** if for all $\psi, \omega \in Env$ and $p, q \in \mathbf{P}$, $(\psi, p) \; \mathcal{G} \; (\omega, q)$ implies $\psi = \omega$.

- $\square_{(\rho,\sigma)}$ is the largest (ρ, σ)-bisimulation.

(ρ,σ)-hyperbisimulation

- $\mathscr{H} \subseteq \mathbf{P}^2$ is a (ρ,σ)-hyperbisimulation if $p\mathscr{H}q$ implies for every $\psi \in Env$, there exists $\mathcal{G} : (\psi, p)\Box_{(\rho,\sigma)}(\psi, q)$
- $\underline{\Box}_{(\rho,\sigma)}$ is (ρ,σ)-hyperbisimilarity.
- If ρ and σ are both conservative and substitution closed relations on actions then for all $\psi \in Env$ and θ such that θ is a substitutive effect of ψ we have $(\psi, p)\Box_{(\rho,\sigma)}(\psi, q)$ if and only if $(\mathbf{1}, \theta p)\Box_{(\rho,\sigma)}(\mathbf{1}, \theta q)$.
- Furthermore if $p\underline{\Box}_{(\rho,\sigma)}q$ then for all substitutions θ we have $\theta p\underline{\Box}_{(\rho,\sigma)}\theta q$.
- $\underline{\Box}_{(\equiv,\equiv)}$ coincides with the strong hyper-bisimilarity of the Fusion Calculus.

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Conclusions and Open Questions

Contributions:

- A generalisation of the notion of bisimulation from which certain other bisimulation theories may be derived.
- Coinductive characterisations of some (pre-)congruence relations which were not defined coinductively before.
- The inheritance properties may also be generalised to hyper-bisimulations and similar “hyper” versions of the “vanilla”-flavoured relations may be obtained.

Open Questions

- How do these parameterisations fit within a categorical framework?
- What kind of algorithms may be designed for checking (ρ, σ)-hyper-bisimilarity?

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Thank You!

Any Questions?