Endowing Concurrent Kleene Algebra with Communication Actions

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Outline

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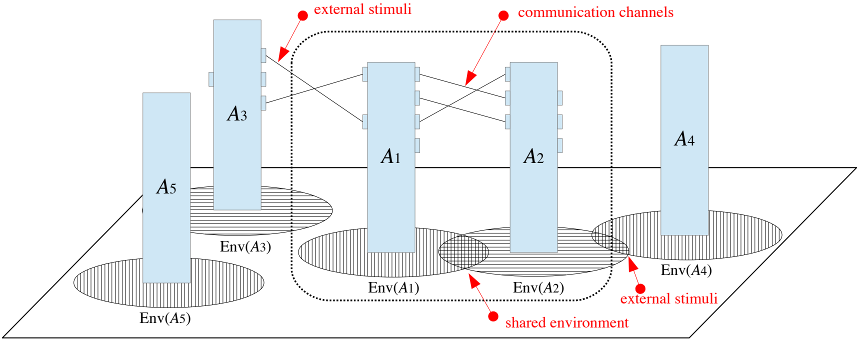
Motivating Question

Question

How can we mathematically formulate the potential for communication condition for covert channel existence in systems of communicating agents?

- We required a formalism that would:
  1. Provide a hybrid model for both communication and concurrency
  2. Lead to a mathematical formulation of the potential for communication

A Hybrid View of Agent Communication



What About Existing Formalisms?

- Looked at existing formalisms for communication and concurrency
  - Temporal Logics
  - Labelled Transition Systems
  - Petri Nets
  - Process Calculi (CCS, CSP, ACP, π-calculus)
- Interested in modelling the behaviour of a system in terms of:
  1. Properties of its states, *or*
  2. Observability of events
- Do not directly, if at all, provide a hybrid model of communication and concurrency that we are interested in

Is Concurrent Kleene Algebra the Answer?

- *Concurrent Kleene Algebra (CKA)* was perhaps the closest formalism to providing a hybrid model
- While CKA can be perceived as a hybrid model for concurrency, the same cannot be said for communication
- Communication in CKA is not directly captured
- CKA does not directly deal with describing how agent behaviours are influenced by external stimuli

Objectives

1. Specify communication in CKA without the need to articulate the state-based system of each action
   - i.e., at a convenient abstract level
2. Express the influence of external stimuli on agent behaviours resulting from the occurrence of external events from
   - Communication among agents
   - Environment of a particular agent

Outline

The Proposed Framework

- Propose a mathematical framework for communication and concurrency called **Communicating Concurrent Kleene Algebra (C^2KA)**
  - Extends the algebraic model of CKA
  - Captures communication and concurrency of agents at the abstract algebraic level
  - Captures the influence of external stimuli on agent behaviour as well as communication through shared environments
  - Presents a different view of communication and concurrency than what was found with existing formalisms

The Proposed Framework

- C^2KA allows for the separation of communicating and concurrent behaviour in a system and its environment
- Can think about concurrent and communicating systems from two different perspectives:
  1. **Behavioural Perspective**: influence of external stimuli as transformations of agent behaviours
  2. **External Event Perspective**: influence of agent behaviours as transformations of external stimuli

Stimuli and Induced Behaviours

Some Terminology

- Every external stimulus *invokes a response* from an agent
- An external stimulus *influences* the behaviour of an agent when the behaviour of then agent changes as a result of the response
- Set of possible influences that any given external stimulus may have on a particular agent are called the *induced behaviours* via external stimuli

A Simple Running Example: One-Place Buffer

- Suppose that a one-place buffer uses two flags to indicate its current status:
  - $flag_1$ denotes the empty/full status
  - $flag_2$ denotes the error status
- Assume that there are two basic system agents:
  - Agent **P** controls $flag_1$
  - Agent **Q** controls $flag_2$

Structure of Agent Behaviours

- Adopt the framework of CKA to describe agent behaviours

Definition (CKA)

A **concurrent Kleene algebra (CKA)** is a structure $(K, +, *, ;, ^{\circledast}, ^{\odot}, 0, 1)$ such that $(K, +, *, ^{\circledast}, 0, 1)$ and $(K, +, ;, ^{\odot}, 0, 1)$ are Kleene algebras linked by the *exchange axiom* given by $(a * b) ; (c * d) \leq_{\mathcal{K}} (b ; c) * (a ; d)$.

- $a \leq_{\mathcal{K}} b$ indicates that *a is a sub-behaviour of b* if and only if $a + b = b$

Structure of Agent Behaviours

Running Example: One-Place Buffer

- Consider the following set of events:

$$P_1 \stackrel{\text{def}}{=} (\mathit{flag}_1 := \mathit{off}) \qquad Q_1 \stackrel{\text{def}}{=} (\mathit{flag}_2 := \mathit{off})$$
$$P_2 \stackrel{\text{def}}{=} (\mathit{flag}_1 := \mathit{on}) \qquad Q_2 \stackrel{\text{def}}{=} (\mathit{flag}_2 := \mathit{on})$$

- K is generated by the set of basic behaviours $\{P_1, P_2, Q_1, Q_2, 0, 1\}$
  - *Inactive agent* 0 is interpreted as abort
  - *Idle agent* 1 is interpreted as skip

Structure of External Stimuli

- Each discrete, observable event introduced to a system is considered to be an *external stimulus* which invokes a response from each system agent

Definition (Stimulus Structure)

Let $\mathcal{S} \stackrel{\text{def}}{=} (S, \oplus, \odot, \mathfrak{d}, \mathfrak{n})$ be an idempotent semiring with a multiplicatively absorbing $\mathfrak{d}$ and identity $\mathfrak{n}$. We call $\mathcal{S}$ a **stimulus structure**.

- $s \leq_{\mathcal{S}} t$ indicates that *s is sub-stimulus of t* if and only if $s \oplus t = t$

Structure of External Stimuli

Running Example: One-Place Buffer

- Behaviour of each agent in the one-place buffer system is influenced by a number of external stimuli:
  - *in* places an item in the buffer
  - *out* removes an item from the buffer
  - *error* generates an error
- S is generated by the set of basic external stimuli $\{in, out, error, \mathfrak{d}, \mathfrak{n}\}$
  - *Deactivation stimulus* $\mathfrak{d}$ is interpreted as a kill signal
  - *Neutral stimulus* $\mathfrak{n}$ is interpreted as any stimulus with no influence that belongs to the complement of the set of external stimuli which may be introduced to a system

Communicating Concurrent Kleene Algebra (C^{2}KA)

Definition (C^{2}KA)

A **Communicating Concurrent Kleene Algebra (C^{2}KA)** is a system $(\mathcal{S}, \mathcal{K})$, where $\mathcal{S} = (S, \oplus, \odot, \mathfrak{d}, \mathfrak{n})$ is a stimulus structure and $\mathcal{K} = (K, +, *, ;, ^{\circledast}, ^{\odot}, 0, 1)$ is a CKA such that $({}_{\mathcal{S}}K, +)$ is a unitary and zero-preserving *left $\mathcal{S}$-semimodule* with mapping $\circ : S \times K \to K$ and $(S_{\mathcal{K}}, \oplus)$ is a unitary and zero-preserving *right $\mathcal{K}$-semimodule* with mapping $\lambda : S \times K \to S$, and where the following axioms are satisfied for all $a, b, c \in K$ and $s, t \in S$:

1. $s \circ (a \,;\, b) = (s \circ a) \,;\, (\lambda(s, a) \circ b)$
2. $c \leq_{\mathcal{K}} a \ \vee\ (s \circ a) \,;\, (\lambda(s, c) \circ b) = 0$
3. $\lambda(s \odot t, a) = \lambda(s, (t \circ a)) \odot \lambda(t, a)$

Communicating Concurrent Kleene Algebra (C^2KA)

- A C^2KA consists of two semimodules
  - $({}_{\mathcal{S}}K, +)$ describes how the stimulus structure $\mathcal{S}$ acts upon the CKA $\mathcal{K}$ via the *next behaviour mapping* $\circ$
  - $(S_{\mathcal{K}}, \oplus)$ describes how the CKA $\mathcal{K}$ acts upon the stimulus structure $\mathcal{S}$ via the *next stimulus mapping* λ
- Together $({}_{\mathcal{S}}K, +)$ and $(S_{\mathcal{K}}, \oplus)$ characterise the response invoked by an external stimulus on the behaviour of an agent as a next behaviour and a next stimulus

Initiating Agent Behaviours

- Agent behaviour can be initiated in two ways:
  1. **Reactivation**: A C^2KA is *with reactivation* if $s \circ 1 \neq 1$ for some $s \in S \backslash \{\mathfrak{d}\}$
     - Passive idle agent may be influenced to behave as any active agent
  2. **Stimulus Initiation**: $a \in K \backslash \{0, 1\}$ is a *stimulus initiator* if and only if $\lambda(\mathfrak{n}, a) \neq \mathfrak{n}$
     - May generate a new stimulus without outside influence

Isotonicity Laws

Proposition

Let $(\mathcal{S}, \mathcal{K})$ be a C^2KA. For all $a, b \in K$ and $s, t \in S$:

1. $a \leq_{\mathcal{K}} b \ \wedge \ s \leq_{\mathcal{S}} t \implies s \circ a \leq_{\mathcal{K}} t \circ b$
2. $a \leq_{\mathcal{K}} b \ \wedge \ s \leq_{\mathcal{S}} t \implies \lambda(s, a) \leq_{\mathcal{S}} \lambda(t, b)$

Corollary

In a C^2KA where the underlying CKA and stimulus structure are built up from quantales, the following laws hold:

1. $a \leq_{\mathcal{K}} b \implies s \circ a \leq_{\mathcal{K}} s \circ b$
2. $s \leq_{\mathcal{S}} t \implies s \circ a \leq_{\mathcal{K}} t \circ a$
3. $s \circ (a \,;\, b + b \,;\, a) \leq_{\mathcal{K}} s \circ (a * b)$
4. $s \circ a^{\circledS} \leq_{\mathcal{K}} s \circ a^{\circledast}$
5. $s \circ a^{\circledS} = +(n \mid n \geq 0 : s \circ a^n)$
6. $s \leq_{\mathcal{S}} t \implies \lambda(s, a) \leq_{\mathcal{S}} \lambda(t, a)$
7. $a \leq_{\mathcal{K}} b \implies \lambda(s, a) \leq_{\mathcal{S}} \lambda(s, b)$
8. $\lambda(s, (a \,;\, b + b \,;\, a)) \leq_{\mathcal{S}} \lambda(s, (a * b))$
9. $\lambda(s, a^{\circledS}) \leq_{\mathcal{S}} \lambda(s, a^{\circledast})$
10. $\lambda(s, a^{\circledS}) = \oplus(n \mid n \geq 0 : \lambda(s, a^n))$

A Comment on a Model for C^2KA

Structure of Agent Behaviours

$\big(PR(EV), \cup, *, ;, ^{\circledast}, ^{\circled{;}}, \emptyset, \{\emptyset\}\big)$ is a CKA.

- A CKA can be modelled as sets of programs and traces
- EV is a set of event occurrences
- A *trace* is a set of events and a *program* is a set of traces
- $TR(EV) \stackrel{\text{def}}{=} \mathcal{P}(EV)$ denotes the set of all traces over EV
- $PR(EV) \stackrel{\text{def}}{=} \mathcal{P}(TR(EV))$ denotes the set of all programs

A Comment on a Model for C^{2}KA
Structure of External Stimuli

$(\mathcal{P}(\Lambda), \cup, \bullet, \emptyset, \{\epsilon\})$ is a stimulus structure.

- A stimulus structure can be modelled by sets of strings
- Λ is a set of alphabet symbols
- $\bullet$ denotes set concatenation
- ϵ is the empty string

A Comment on a Model for C^{2}KA

$\big(Q, \Sigma, \Theta, F, G\big)$ is a C^{2}KA.

- A C^{2}KA can be modelled as a Mealy automaton
- The set of states Q is a subset of $PR(EV)$ (i.e., the set K)
- The input alphabet Σ and output alphabet Θ are given by the stimulus structure such that $\Sigma = \Theta = S$
- The transition function $F : \Sigma \times Q \to Q$ corresponds to the next behaviour mapping $\circ : S \times K \to K$
- The output function $G : \Sigma \times Q \to \Theta$ corresponds to the next stimulus mapping $\lambda : S \times K \to S$

A Comment on a Model for C^2KA

- Proposed model is also equipped with two operations:
  - Operation $;$ is associative
    - *Cascading Product* of Mealy automata
  - Operation $+$ is associative, idempotent, and commutative
    - *Full Direct Product* of Mealy automata

Specifying Systems of Communicating Agents with C^2KA

- Three levels of specification:
  1. Stimulus-Response Specification of Agents
  2. Abstract Behaviour Specification
  3. Concrete Behaviour Specification
- Context of the given problem helps to dictate at which level we need to work

Stimulus-Response Specification

Running Example: One-Place Buffer

$$\mathbf{P} \overset{\text{def}}{=} P_1 + P_2 \qquad\qquad \mathbf{Q} \overset{\text{def}}{=} Q_1 + Q_2$$

$\circ_{\mathbf{P}}$	$\mathfrak{n}$	*in*	*out*	*error*
P_1	P_1	P_2	P_1	P_1
P_2	P_2	P_2	P_1	P_2

$\circ_{\mathbf{Q}}$	$\mathfrak{n}$	*in*	*out*	*error*
Q_1	Q_1	Q_1	Q_1	Q_2
Q_2	Q_2	Q_2	Q_2	Q_2

$\lambda_{\mathbf{P}}$	$\mathfrak{n}$	*in*	*out*	*error*
P_1	$\mathfrak{n}$	$\mathfrak{n}$	*error*	$\mathfrak{n}$
P_2	$\mathfrak{n}$	*error*	$\mathfrak{n}$	$\mathfrak{n}$

$\lambda_{\mathbf{Q}}$	$\mathfrak{n}$	*in*	*out*	*error*
Q_1	$\mathfrak{n}$	$\mathfrak{n}$	$\mathfrak{n}$	$\mathfrak{n}$
Q_2	$\mathfrak{n}$	$\mathfrak{n}$	$\mathfrak{n}$	$\mathfrak{n}$

$$\forall(P_i, Q_i \mid 1 \leq i \leq 2 \;:\; \mathfrak{d} \circ P_i = 0 \;\wedge\; \mathfrak{d} \circ Q_i = 0 \;\wedge\; \lambda(\mathfrak{d}, P_i) = \mathfrak{d} \;\wedge\; \lambda(\mathfrak{d}, Q_i) = \mathfrak{d}\,)$$

$$\textbf{Buffer} \overset{\text{def}}{=} \mathbf{P} \,;\, \mathbf{Q} = (P_1 + P_2) \,;\, (Q_1 + Q_2)$$

Abstract Behaviour Specification

Running Example: One-Place Buffer

- Consider the following context:
  - Buffer can only behave as empty or full
  - Buffer may only be influenced by *in* and *out* stimuli
    - *error* is an uncontrollable stimulus

$$
\begin{aligned}
& (in \oplus out) \circ (P_1 \,;\, Q_1 + P_2 \,;\, Q_1) \\
= \;& in \circ (P_1 \,;\, Q_1) + out \circ (P_1 \,;\, Q_1) + in \circ (P_2 \,;\, Q_1) + out \circ (P_2 \,;\, Q_1) \\
= \;& (in \circ P_1) \,;\, \big(\lambda(in, P_1) \circ Q_1\big) + (out \circ P_1) \,;\, \big(\lambda(out, P_1) \circ Q_1\big) + \\
& (in \circ P_2) \,;\, \big(\lambda(in, P_2) \circ Q_1\big) + (out \circ P_2) \,;\, \big(\lambda(out, P_2) \circ Q_1\big) \\
= \;& P_2 \,;\, Q_1 + P_1 \,;\, Q_2 + P_2 \,;\, Q_2 + P_1 \,;\, Q_1
\end{aligned}
$$

Concrete Behaviour Specification

Running Example: One-Place Buffer

$$
\begin{array}{llllll}
\text{EMPTY} & \stackrel{\text{def}}{=} & P_1 \mathbin{;} Q_1 & = & (\mathit{flag}_1 := \mathit{off} \quad ; \quad \mathit{flag}_2 := \mathit{off}) \\
\text{FULL} & \stackrel{\text{def}}{=} & P_2 \mathbin{;} Q_1 & = & (\mathit{flag}_1 := \mathit{on} \quad ; \quad \mathit{flag}_2 := \mathit{off}) \\
\text{UNDERFLOW} & \stackrel{\text{def}}{=} & P_1 \mathbin{;} Q_2 & = & (\mathit{flag}_1 := \mathit{off} \quad ; \quad \mathit{flag}_2 := \mathit{on}) \\
\text{OVERFLOW} & \stackrel{\text{def}}{=} & P_2 \mathbin{;} Q_2 & = & (\mathit{flag}_1 := \mathit{on} \quad ; \quad \mathit{flag}_2 := \mathit{on})
\end{array}
$$

C^{2}KA and Orbits, Stabilisers, and Fixed Points

- Orbits, stabilisers, and fixed points allow us to:
  1. Perceive a kind of topology of a system
  2. Gain some insight into the communication channels that can be established
  3. Model the possible reactions of a system to a stimulus
  4. Alleviate the state explosion problem in model checking

C^2KA and Orbits, Stabilisers, and Fixed Points

- Two complementary notions of orbits, stabilisers, and fixed points
- Can think about concurrent and communicating systems from two different perspectives:
  1. **Behavioural Perspective**: action of external stimuli on agent behaviours described by $({}_{\mathcal{S}}K, +)$
  2. **External Event Perspective**: action of agent behaviours on external stimuli described by $(S_{\mathcal{K}}, \oplus)$

Orbits

Definition (Orbit)

The **orbit** of a in $\mathcal{S}$ is the set $\mathrm{Orb}(a) = \{s \circ a \mid s \in S\}$.

- Set of all possible behavioural responses from an agent to any external stimulus from $\mathcal{S}$
  - Set of all possible future behaviours
- **Running Example**:

$$\begin{aligned} \mathrm{Orb}(\textsc{empty}) &= \{\textsc{empty}, \textsc{full}, \textsc{underflow}, \textsc{overflow}\} \\ \mathrm{Orb}(\textsc{overflow}) &= \{\textsc{underflow}, \textsc{overflow}\} \end{aligned}$$

Another Interpretation of Orbits

Definition (Induced Behaviour)

Let $a, b \in K$ be agent behaviours such that $a \neq b$. We say that b **is induced by a via external stimuli** (denoted by $a \lhd b$) if and only if $\exists(s \mid s \in S : s \circ a = b)$.

- Equivalently, $a \lhd b \iff b \in \mathrm{Orb}(a)$ for $a \neq b$
- **Running Example**:
  - $\mathrm{EMPTY} \lhd \mathrm{UNDERFLOW}$ via the external stimulus *out*
  - $\mathrm{EMPTY} \lhd \mathrm{OVERFLOW}$ via the external stimulus $in \odot in$

Strong Orbits

Definition (Strong Orbit)

The **strong orbit** of a in $\mathcal{S}$ is the set
$\mathrm{Orb}_{\mathrm{S}}(a) = \{b \in K \mid \mathrm{Orb}(b) = \mathrm{Orb}(a)\}$.

- Two agents are in the same strong orbit ($a \sim_{\mathcal{K}} b$) if and only if their orbits are identical
- If $a \sim_{\mathcal{K}} b$, then $\exists(s, t \mid s, t \in S : s \circ a = b \land t \circ b = a)$
  - s and t can be perceived as *inverses* of one another
- **Running Example**: We have two strong orbits: $\{\mathrm{EMPTY}, \mathrm{FULL}\}$ and $\{\mathrm{UNDERFLOW}, \mathrm{OVERFLOW}\}$

Stabilisers

Definition (Stabiliser)

The **stabiliser** of a in $\mathcal{S}$ is the set $\mathrm{Stab}(a) = \{s \in S \mid s \circ a = a\}$.

- Set of external stimuli which have no observable influence (or act as neutral stimuli) on an agent
- **Running Example**: $\mathrm{Stab}(\textsc{empty})$ is generated by $\{error, in \odot out\}$

Fixed Point Behaviours

Definition (Fixed Point)

An element $a \in K$ is a **fixed point** if $\forall(s \mid s \in S\backslash\{\mathfrak{d}\} : s \circ a = a)$.

- Not influenced by any external stimulus other than the deactivation stimulus $\mathfrak{d}$
- May be any number of fixed points with respect to $\circ$
- When $a \in K$ is a fixed point, $\mathrm{Orb}(a) = \{0, a\}$ and $\mathrm{Stab}(a) = S\backslash\{\mathfrak{d}\}$
- **Running Example**: With regard to the specification, the behaviour Q_2 is a fixed point

Topological Insights and Induced Behaviours

Proposition

Let $a, b, c \in K$ be agent behaviours.

1. a is a fixed point $\implies \forall(b \mid b \in K \land b \neq 0 \land b \neq a : \neg(a \lhd b))$
2. $a \sim_{\mathcal{K}} b \implies a \lhd b \land b \lhd a$
3. $a \sim_{\mathcal{K}} b \implies (a \lhd c \iff b \lhd c)$

Outline

Conclusion

- C^2KA extends the algebraic setting of CKA to capture the influence of external stimuli on the behaviour of system agents
- C^2KA supports the ability to work in either a state-based or event-based model for both the specification of communicating and concurrent behaviour
- To the best of our knowledge, such a formalism does not currently exist in the literature
  - Required for studying the necessary conditions for covert channel existence

Current and Future Work

- Developed a formulation of the potential for communication condition for covert channels using C^2KA
- Prototype tool to support the automated computation and specification of systems of communicating agents using C^2KA
- Adapt C^2KA for solving interface equations
- Use C^2KA to capture and explain the influence of external stimuli on agent behaviour in social networking environments

Outline

Questions

Questions?