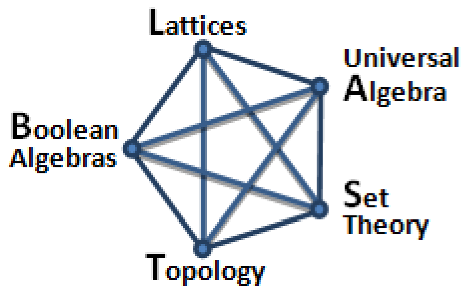
Abstracts

BLAST 2013



NATIONAL SCIENCE FOUNDATION

CHAPMAN UNIVERSITY | SCHMID COLLEGE OF SCIENCE AND TECHNOLOGY

CECAT CENTER OF EXCELLENCE IN COMPUTATION, ALGEBRA AND TOPOLOGY

Invited Speakers

Bernhard Banaschewski McMaster University
William DeMeo University of South Carolina
François Dorais Dartmouth College
Martín Escardó University of Birmingham
Mai Gehrke Université Diderot - Paris 7 & CNRS
Steven Givant Mills College
Heinz-Peter Gumm Universität Marburg
Steve Jackson University of North Texas
Michael Pinsker Université Diderot - Paris 7
Hilary Priestley University of Oxford
Dima Sinapova University of Illinois at Chicago
Sam van Gool Radboud Universiteit Nijmegen

August 5–9, 2013

Contents

Generalized Boolean and Boolean-Like Rings

Hazar Abu-Khuzam
American University of Beirut
hazar@aub.edu.lb
Coauthors: A. Yaqub, University of California, Santa Barbara

A generalized Boolean ring is a ring R such that, for all x, y in $R-(N \cup C)$, $x^n y - xy^n$ is in N and C, where n is a fixed even integer, and N, C are the set of nilpotents and center of R, respectively. A Boolean-like ring is a ring R such that, for all x, y in $R-(N \cup C)$, $x^n y = xy^n$, where again n is a fixed even integer. The structure of certain classes of these rings is considered. In particular, it is proved that a Boolean-like ring with identity is commutative.

Distributive ℓ-pregroups

Richard N. Ball
University of Denver
rball@du.edu
Coauthors: Peter Jipsen and Nikolaos Galatos

A distributive lattice-ordered pregroup, or simply ℓ-pregroup, is a structure of the form $(L, \wedge, \vee, \cdot, 1, {}^l, {}^r)$, where $(L, \wedge, \vee)$ is a distributive lattice, $(L, \cdot, 1)$ is a compatible monoid, i.e., $x(y \vee z) = xy \vee xz$, and x^l and x^r are two additional unary operations which satisfy the inequations

$$x^l x \leq 1 \leq xx^l \quad \text{and} \quad xx^r \leq 1 \leq x^r x.$$

Alternatively, an ℓ-pregroup is a residuated distributive lattice which satisfies the identities $x^{lr} = x = x^{rl}$ and $(xy)^l = y^l x^l$, where $x^l = 1/x$ and $x^r = x \backslash 1$. An ℓ-pregroup which satisfies $x^l = x^r$ is a lattice-ordered group, or ℓ-group, and these objects comprise a well studied subvariety. The outstanding open question in connection with ℓ-pregroups is whether the distributive hypothesis is redundant.

In this talk we analyze the structure of ℓ-pregroups as families of endomorphisms of a chain Ω, since every ℓ-pregroup can be thus represented. The lattice structure is given by pointwise order and the monoid structure by functional composition. The key issue is to determine precisely which endomorphisms have residuals of all orders. We characterize these endomorphisms by means of three simple geometrical properties. This allows us to generate several interesting concrete examples of ℓ-pregroups. We close with some remarks on subdirectly irreducible and simple ℓ-pregroups.

Choice and pointfree topology

Bernhard Banaschewski
McMaster University

Remarks on the interaction between pointfree topology and certain choice principles, specifically the Axiom of Choice (AC), the Axiom of Countable Choice (ACC), and the Boolean Prime Ideal Theorem (PIT), in connection with the following issues:

(1) The equivalence of PIT with certain spatial conditions,

(2) the relation between the classical Stone-Čech compactification and its pointfree counterpart (which does not require any choice principle),

(3) the same regarding the classical Hausdorff Tychonoff Theorem and the pointfree Tychonoff Theorem (which also does not involve any choice principle),

(4) the equivalence of ACC and the Lindelöfness of certain 0-dimensional frames, and

(5) the equivalence of AC and PIT with certain conditions concerning, respectively, the maximal and the prime elements of coherent frames.

On algebras of relations with binary primitive-positive operations

D. A. Bredikhin
Saratov State Technical University (Russsia)
bredikhin@mail.ru

A set of binary relations closed with respect to some collection of operations forms an algebra which is called an algebra of relations. The theory of algebras of relations is an essential part of modern algebraic logic. Tarski was the first to treat algebras of relations from the point of view of universal algebra [1].

An operation on binary relations is called primitive positive [2] if it can be defined by a first order formula in which only existential quantifiers and conjunctions appear.

We shall consider binary primitive positive operations. These operations play a role in logic of predicates similar to a role of binary functions in Boolean algebra. Some results in this direction can be found in [3,4].

For any set Ω of operations on binary relations, let $R\{\Omega, \subset\}$ be the class of partially ordered by the set-theoretical inclusion $\subset$ algebras whose elements are binary relations and whose operations are members of Ω, and let $Var\{\Omega, \subset\}$ be a variety generated by $R\{\Omega \ \subset\}$.

Let's concentrate our attention on the following binary primitive positive operation defined as follows:

$$\rho * \sigma = \{(x, y) \in X \times X : (\exists z)(x, z) \in \rho \wedge (z, x) \in \sigma\}.$$

Theorem. An partially ordered groupoid $(A, *, \leq)$ belongs to the variety $Var\{*, \subset\}$ if and only if it satisfies the identities:

$(xy)y = xy$, $(xy)^2 = xy$, $((xy)z)z = (xy)z$, $x((yz)x) = x(yz)$, $((xy)z)(z(xy)) = (xy)z$, $(x(yz))(yz) = x(yz)$, $((xy)z)u = ((xy)u)z$, $(x((yz)u))(x(yz)) = x((yz)u)$, $(x(yz))(uv) = x((yz))(uv))$, $(xy)z \leq xy$.

REFERENCES:

[1] Tarski A. On the calculus of relations. J. Symbolic Logic. 6(1941), P.73-89.

[2] Börner F., Pöschel R. Clones of operations on binary relations. Contributions to general algebras. - Wien, 7(1991), P.50-70.

[3] Bredikhin D.A. On relation algebras with general superpositions. Colloq. Math. Soc. J. Bolyai. (54) 1994, P.11-124.

[4] Bredikhin D.A. Varietes of groupoids associated with involuted restrictive bisemigroups of binary relations. Semigroup Forum. (44)1992, - P.87-192.

Unification on Subvarieties of Pseudocomplemented Lattices

Leonardo Cabrer

Università degli Studi di Firenze - Dipartimento di Statistica, Informatica, Applicazioni "G. Parenti" - Marie Curie Intra-European Fellowship - FP7

l.cabrer@disia.unifi.it

Given an algebraic language $\mathcal{L}$, *syntactic unification* theory is concerned with the problem of finding a substitution that equalises a finite set of terms simultaneously. In many applications the connectives in $\mathcal{L}$ are assumed to satisfy certain conditions that can be expressed by equations, such as associativity, commutativity, idempotency. Then syntactic unification evolves into *equational unification.* Given an equational theory E in the language $\mathcal{L}$, a unifier for a finite set of pairs of $\mathcal{L}$-terms $U = \{(t_1, s_1), \ldots, (t_m, s_m)\}$ a *unifier* for U is a substitution σ such that $\sigma(t_i) \cong_E \sigma(s_i)$ for each $i \in \{1, \ldots, m\}$.

Once a particular unification problem is known to admit E-unifiers, the problem that arises is to find an enumeration of its unifiers. If σ is an E-unifier for U, then $\gamma \circ \sigma$ is also an E-unifier for U whenever γ is a substitution such that $\gamma \circ \sigma$ is well defined. In this case we say that σ is *more general* than $\gamma \circ \sigma$. Therefore, a useful way to determine all the unifiers of a particular problem is to calculate a family of unifiers that are more general than any other unifier of the problem. This set is called a *complete set of unifiers*. It is desirable to obtain a complete set that is not 'redundant' (in the sense that the elements of the set are incomparable). Any such a set is called a *minimal complete set of unifiers*. Depending on the existence and the cardinality of a minimal complete set of unifiers, the *unification type* of a problem is defined (see [1,2]). The unification types have natural ordering. The unification type of the theory E is defined as the smallest common upper bound of the type of its unification problems.

We devote this paper to the study of unification in the implication-free fragment of intuitionistic logic, that is, the equational theory of pseudocomplemented distributive lattices (*p-lattices* for short) and its extensions. It was first observed by Ghilardi in [3] that the equational theory of p-lattices has nullary type (the worst possible case). In this paper we take that result two steps forward. First we prove that Boolean algebras are the only non trivial subvariety of p-lattices that has type one, while the others have nullary type. Secondly, we determine the type of each unification problem in every extension of the equational theory of p-lattices.

The main tools used in this paper are: the algebraic approach to E-unification developed in [3]; the topological duality for p-lattices developed in [5]; the characterisation of subvarieties of p-lattices given in [4] and the description of finite projective p-lattices in these subvarieties given in [6].

References

[1] F. Baader and J.H. Siekmann, Unification theory, in *Handbook of Logic in Artificial Intelligence and Logic Programming* Vol. **2** (D.M. Gabbay, C.J. Hogger and J.A. Robinson. Eds.), Oxford University Press, Oxford, 41–125 (1994).

[2] F. Baader and W. Snyder, Unification theory, in *Handbook of Automated Deduction*, (A. Robinson and A. Voronkov. Eds.) Springer Verlag, Berlin, 445–533 (2001).

[3] S. Ghilardi, Unification through projectivity, *Journal of Logic and Computation* **7**(6) (1997), 733–752.

[4] K.B. Lee, Equational classes of distributive pseudocomplemented lattices, *Canadian Journal of Mathematics* **22** (1970), 881–891.

[5] H.A. Priestley, The construction of spaces dual to pseudocomplemented distributive lattices, *Quarterly Journal of Mathematics. Ox-*

ford Series. **26**(2) (1975), 215–228.

[6] A. Urquhart, Projective distributive p-algebras, *Bulletin of the Australian Mathematical Society* **24** (1981), 269–275.

The finite embeddability property for some noncommutative knotted extensions of FL

Riquelmi Cardona
University of Denver
rcardon3@du.edu

We consider the knotted structural rule $x^m \leq x^n$ for n different than m and m greater or equal than 1. Previously Van Alten proved that commutative residuated lattices that satisfy the knotted rule have the finite embeddability property (FEP). Namely, every finite partial subalgebra of an algebra in the class can be embedded into a finite full algebra in the class. In our work we replace the commutativity property by some slightly weaker conditions. Particularly, we prove the FEP for the variety of residuated lattices that satisfy generalizations of the equation $xyx = x^2y$ and a knotted rule. We also note that the FEP implies the finite model property. Hence the logics modeled by these residuated lattices are decidable.

Variety of orthomodular posets

Ivan Chajda
Palacky University Olomouc, Czech Republic
ivan.chajda@upol.cz
Coauthors: Miroslav Kolařík

Orthomodular posets play an important role in the logic of quantum mechanics. To avoid usual problems with partial algebras, we organize every orthomodular poset onto a total algebra which is presented by several simple identities. This conversion is one-to-one. We show that the variety of orthomodular posts have nice congruence properties (permutability, distributivity, regularity).

An atlas of congruence lattices of finite algebras

William DeMeo
University of South Carolina
williamdemeo@gmail.com
Coauthors: Ralph Freese, Peter Jipsen, Bill Lampe, JB Nation

A longstanding open problem in universal algebra is to characterize those lattices that are isomorphic to congruence lattices of finite algebras. We call such lattices "representable." In this talk we discuss the project of cataloging all small lattices that are known to be representable, and give an overview of the known methods for constructing finite algebras with a given congruence lattice. These include a relatively new method involving "overalgebras," which are built by gluing together smaller algebras, as well as an older approach called "the closure method." The closure method has been extremely useful for identifying many small congruence lattices, and we show pictures of all examples known to us. However, previous implementations of this procedure can only identify congruence lattices of algebras with very small universes (say, 9 or 10 elements). We describe and demonstrate a new fast closure algorithm that allows us to identify congruence lattices on much larger sets than was previously possible. We conclude with some open questions.

General framework for topological Ramsey spaces, canonization theorems, and Tukey types of ultrafilters with weak partition properties

Natasha Dobrinen
University of Denver
natasha.dobrinen@du.edu
Coauthors: Jose Mijares and Timothy Trujillo

We present general methods for constructing new topological Ramsey spaces and prove Ramsey-classification theorems for equivalence relations on barriers on these spaces. Associated to each topological Ramsey space is a notion of ultrafilter selective or Ramsey for that space. Such ultrafilters satisfy weak partition properties. Given any of the ultrafilters associated with our topologicay Ramsey spaces, the Ramsey-classification theorem can be applied to decode the structure of the Tukey types of ultrafilters Tukey reducible to the one under consideration. In particular, we show that each finite Boolean algebra appears as an initial structure in the Tukey types of ultrafilters. This builds on and extends previous work of Dobrinen and Todorcevic.

Prospects for a reverse analysis of topology

François G. Dorais
Dartmouth College
francois.g.dorais@dartmouth.edu

Because of its central role in mathematics, a comprehensive logical analysis of the methods and ideas of topology is very challenging, perhaps even unattainable. A rich array of approaches are needed to analyze all the facets that topology has to offer. In this talk, I will discuss the approach of reverse mathematics: its potential and limitations, as well as prospects to go beyond the current barriers.

I will present and contrast two different ways of presenting general classes of topological spaces that are amenable to analysis in subsystems of second-order arithmetic, the traditional setting for reverse mathematics. The first, inspired by point-set topology, is centered on points and the second, inspired by point-free topology, is centered on the lattice of open sets. Through the reverse analysis of classical theorems in these frameworks, I will illustrate how each point of view allows to isolate the role and strength of certain central ideas of topology.

Since second-order arithmetic has a very limited grasp of the uncountable, I will discuss alternate set-theoretic systems where the reverse analysis of topology can be continued without such size limitations but still admit a smooth transition from subsystems of second-order arithmetic.

Topology in Logic: Some Examples

Patrick Durkin
University of North Dakota
patrick.durkin@my.und.edu

The aim of this talk is to survey three instances of connections between topology and logic. The first is the Compactness Theorem and the origin of its name. Second, an overview of how the interior operator on a space can be used to model propositional modal logic S4 (as shown by McKinsey and Tarski), and the extension of this to a first-order version of S4 via sheaves (due to Awodey and Kishida). Finally, there will be some words on the ideas behind the homotopy interpretation of type theory.

Interactions between Topology and computation

Martin Escardo
University of Birmingham
m.escardo@cs.bham.ac.uk

The interaction between topology and computation goes both ways. I will explore both directions, assuming an audience more familiar with topology than with computation. A dictionary relating topological and computational notions suggests computational readings of topological theorems, and, in the other direction, suggests new theorems in the theory of computation. We will look at Scott domains from programming language semantics, Kleene–Kreisel spaces from computability theory, and at some classical spaces such as compactly generated spaces and their cousins. We will see that some theorems in topology can be seen as types and their proofs as functional programs. I will briefly discuss, towards the end, a new interaction of topology with computation, known as Homotopy Type Theory (HoTT), and some open problems in this new field.

Representations of Complete Boolean Algebras by Ideals

Monroe Eskew
UC Irvine
meskew@math.uci.edu

We present a proof (due to Ashutosh Kumar) that every partial order is forcing-equivalent to forcing with an ideal, i.e. a boolean algebra of the form $P(X)/I$. We improve this to show that for the same partial order, X can take many different cardinalities with I uniform. This stands in contrast to some negative results of Alaoglu-Erdos, Kunen, and Gitik-Shelah concerning countably complete ideals.

Spectral-like Duality for Distributive Hilbert Algebras with Infimum

María Esteban
Universitat de Barcelona
mariaegmp@gmail.com
Coauthors: Sergio Celani (Universidad Nacional del Centro, Argentina)
Ramón Jansana (Universitat de Barcelona)

We present the results of our research on stone-type dualities for certain classes of ordered algebras that do not fall within the scope of extended Priestley-duality. In a forthcoming paper we study a new spectral-like duality for the class of distributive Hilbert algebras with infimum. In the talk we explain the main facts of that duality and we outline how the same strategy could be used for getting a Priestley-style duality for the same class of algebras, as well as dualities for other classes of algebras.

Endomorphisms of Hilbert Algebras

Hernando Gaitan
Universidad Nacional de Colombia
hgaitano@unal.edu.co

In this talk we will describe the congruences of a finite Hilbert algebra in terms of its closure endomorphisms. We use this result to give a necessary and sufficient condition under which two finite Hilbert algebras share the same monoid of endomorphisms.

Developments on higher levels of the substructural hierarchy

Nick Galatos
University of Denver
ngalatos@du.edu

Algebraic methods have been applied to logic at least since the time of Boole, but they have proved very useful in the study of non-classical logics in particular. A recent direction of study, Algebraic Proof Theory (APT), aims to apply algebraic methods directly to Proof Theory, but the interaction turned out to be bidirectional. Traditional Proof Theory studies logical systems based on sequents, which end up being at the second level of a recently introduced formula hierarchy. The systematic algebraic study of APT covers this level and also extends to the third level of hypersequents.

Extensions beyond the third level have proved challenging, but there is hope. I will report on recent results for parts of the third and fourth level of the hierarchy.

On extended and partial real-valued functions in Pointfree Topology

Javier Gutiérrez García
University of the Basque Country, UPV/EHU (Spain)
javier.gutierrezgarcia@ehu.es
Coauthors: Jorge Picado, University of Coimbra (Portugal)

In Pointfree Topology, a continuous real function on a frame L is a map from the frame $\mathfrak{L}(\mathbb{R})$ of reals into L.

The frame of reals can be specified in term of generators and defining relations. By dropping some of the relations we obtain the *extended* and *partial* variants of the frame of reals.

If we replace now the frame of reals with these variants we may speak about extended and partial real functions, the pointfree counterpart of functions on a space X with values in the extended real line and the interval domain, respectively.

In this talk we will present some results and applications from [1] and [2].

References

[1] B.Banaschewski, J. Gutiérrez García and J. Picado, Extended real functions in pointfree topology, J. Pure Appl. Algebra 216 (2012) 905-922.

[2] I. Mozo Carollo, J. Gutiérrez García and J. Picado, The Dedekind order completion of function rings, Preprint DMUC 12-20, Coimbra, 2012 (submitted).

Sheaf representations of MV-algebras via Priestley Duality

Mai Gehrke
LIAFA, CNRS and Paris 7
mgehrke@liafa.univ-paris-diderot.fr

MV-algebras, which are, up to categorical equivalence, the same as unital lattice-ordered abelian groups, were first introduced by Chang in 1958 as semantics for the infinite-valued Lukasiewicz logic. Since then, they have been studied extensively for their own sake and relative to their connections to various other areas of mathematics. Representation theory has played an important role in the theory of MV-algebras and lattice-ordered groups while duality theory has not been used except in some special subclasses such as locally finite or finitely presented MV-algebras. From the point of view of duality theory, the full class of MV-algebras provides an interesting case study because a simple split between topological and discrete components of duality is obstructed for a different reason than the one known from modal logic. This was the topic of several joint papers with Hilary Priestley in which we developed extended dualities, based on Priestley duality, for a type of varieties of distributive lattice ordered algebras including the variety of MV-algebras.

In this talk I will mainly focus on recent work joint with Sam van Gool and Vincenzo Marra. We study representations of MV-algebras via Stone-Priestley duality, using canonical extensions as an essential tool. In particular, the theory of canonical extensions, as applied in the work mentioned above with Hilary Priestley, implies that the dual spaces of MV-algebras carry the structure of topological partial commutative ordered semigroups. We use this structure to obtain two different decompositions of such spaces, one indexed over the prime MV-spectrum, the other over the maximal MV-spectrum. These decompositions each yield sheaf representations of MV-algebras using a new and purely duality-theoretic result that relates certain sheaf representations of distributive lattices to decompositions of their dual spaces.

While the MV-algebraic representation theorems that we obtain in this way already are known, our proofs distinguish themselves by the following features: (1) we use only basic algebraic facts about MV-algebras; (2) we show that the two aforementioned sheaf representations are special cases of a common result, with potential for generalizations; and (3) we show that these results are strongly related to the structure of the Stone-Priestley duals of MV-algebras. In addition, using our analysis of decompositions of dual spaces, we prove that MV-algebras with isomorphic underlying lattices have homeomorphic maximal MV-spectra. This result is an MV-algebraic generalization of a classical theorem by Kaplansky stating that two compact Hausdorff spaces are homeomorphic if, and only if, the lattices of continuous [0,1]-valued functions on the spaces are isomorphic.

Duality Theories for Boolean Algebras with Operators

Steven Givant
Mills College
givant@mills.edu

There are two natural dualities for Boolean algebras. The first is algebraic in nature and concerns the duality between the category of all sets with mappings between sets as the morphisms, and the category of all complete and atomic Boolean algebras with complete homomorphisms between such algebras as the morphisms. The second—the famous Stone duality—is topological in nature and concerns the duality between the category of all Boolean spaces with continuous mappings between such spaces as the morphisms, and the category of all Boolean algebras with homomorphisms between such algebras as the morphisms. Over the last sixty years, these two dualities have gradually been extended to Boolean algebras with normal operators, starting with the work of Jónsson and Tarski, and continuing with the work of Halmos, Hansoul, Goldblatt, and others. There is a third duality—a hybrid between the algebraic and the topological dualities—that seems not to have been noticed before, even in the case of Boolean algebras, and that has important applications.

This talk will survey these three dualities for Boolean algebras with normal operators, and discuss their implications for: (1) dualities between various types of ideals in algebras and subuniverses of relational structures, and the corresponding dualities betweenquotients of algebras and relational substructures; (2) dualities between various types of subuniverses of algebras and congruences on relational structures, and the corresponding dualities between subalgebras and quotients of relational structures; (3) dualities between direct and subdirect products of algebras and unions, or compactifications of unions, of relational structures. Each of the dualities in (1)–(3) involves a lattice isomorphism between a lattice of algebraic structures and a lattice of relational structures.

Duality for sheaves of distributive-lattice-ordered algebras over stably compact spaces

Sam van Gool
Radboud University Nijmegen and Université Diderot - Paris 7
samvangool@me.com
Coauthors: Mai Gehrke (Université Diderot - Paris 7 & CNRS)

A sheaf representation of a universal algebra A over a topological space Y can be viewed as a special subdirect product decomposition of A indexed by Y. Indeed, in case Y is a Boolean space, sheaf representations of A correspond exactly to weak Boolean product decompositions of A. Moreover, if A is a distributive lattice, then Boolean sheaf representations of A correspond to decompositions of the (Stone-Priestley) dual space of A into a Boolean sum, i.e., a disjoint sum indexed over the underlying Boolean space satisfying a certain patching property for the topology on the sum.

We study sheaf representations of distributive lattices over *stably compact spaces*, which are the natural T_0 spaces associated to compact ordered spaces. In particular, the class of stably compact spaces contains both the class of compact Hausdorff spaces and the class of spectral spaces. We introduce an appropriate condition on sheaves, that we call *fitted*, which allows us to prove the following:

Theorem. Fitted sheaf representations of a distributive lattice A over a stably compact space Y are in one-to-one correspondence with patching decompositions of the Stone-Priestley dual space of A over the space Y.

Here, a *patching decomposition* of a topological space X over a space Y is most conveniently described by a continuous map from X to Y which satisfies a certain patching property, reflecting the patching property of the sheaf. If the indexing space Y is Boolean, then such a patching decomposition precisely corresponds to a Boolean sum. However, since stably compact spaces may have a non-trivial specialization order, the correct notion of *patching decomposition* is no longer a disjoint sum of spaces, but rather corresponds to an ordered sum with an appropriate topological property. The interest of the theorem thus lies in the fact that it opens the way for studying sheaf representations of varieties of distributive-lattice-ordered algebras via decompositions of their dual spaces, which may be indexed over any stably compact space.

The Admissible Rules of BD_2

Jeroen Goudsmit
Utrecht University
J.P.Goudsmit@uu.nl

The admissible rules of a logic are those rules that can be added without making new theorems derivable. Algebraically, these rules correspond to quasi-equations that hold in free algebras. Many axiomatic extensions of intuitionistic propositional logic (IPC) have non-trivial admissible rules. An example of such an admissible rule is $\neg C \to A \vee B/(\neg C \to A) \vee (\neg C \to B)$, which in IPC is not derivable, yet admissible (Harrop, 1960). In fact, Prucnal (1979) showed it to be admissible for all intermediate logics.

Some intermediate logics enjoy a nice characterisation of their admissible rules. Iemhoff (2001) and Rozière (1992) independently proved that all admissible rules of IPC derive from the so-called Visser rules. The Visser rules are useful in describing the admissible rules of many an intermediate logic (Iemhoff, 2005). The intermediate logics BD_2, the weakest intermediate logic of the second finite slice (Chagrov and Zakharyaschev, 1997), is not amendable to this approach, for Citkin (2012) showed that it does not admit the Visser rules.

The logic BD_2 does enjoy many nice properties. It is decidable (Jankov, 1963), it is one of the three pre-tabular intermediate logics (Maksimova, 1972) and one of the seven intermediate logics with interpolation (Maksimova, 1979). We present a scheme of rules similar to but distinct from the Visser rules, inspired by Skura (1992), and characterise its admissibility among intermediate logics. We then show that this scheme suffices to derive all admissible rules of BD_2. The main technical tools are Jankov-de Jongh characteristic formulae and the characterisation of finitely generated projective Heyting algebras by Ghilardi (1999).

References

- A. Chagrov and M. Zakharyaschev (1997). Modal Logic. Vol. 77. Oxford Logic Guides. Oxford University Press.
- A. Citkin (2012). A note on admissible rules and the disjunction property in intermediate logics. In: Archive for Mathematical Logic 51 (1), pp. 1–14.
- S. Ghilardi (1999). Unification in Intuitionistic Logic. In: The Journal of Symbolic Logic 64.2, pp. 859–880.
- R. Harrop (1960). Concerning Formulas of the Types $A \to B \vee C$, $A \to (\exists x)B(x)$ in Intuitionistic Formal Systems. In: The Journal of Symbolic Logic 25.1, pp. 27–32.
- R. Iemhoff (2001). A(nother) characterization of intuitionistic propositional logic. In: Annals of Pure and Applied Logic 113.1-3. pp. 161–173.
- R. Iemhoff (2005). Intermediate Logics and Visser's Rules. In: Notre Dame Journal of Formal Logic 46.1, pp. 65–81.
- V.A. Jankov (1963). Some superconstructive propositional calculi. In: Soviet Mathematics Doklady 4, pp. 1103–1105.
- L.L. Maksimova (1972). Pretabular superintuitionist logic. In: Algebra and Logic 11.5, pp. 308–314.

- L.L. Maksimova (1979). Interpolation properties of superintuitionistic logics. In: Studia Logica 38.4, pp. 419–428.
- T. Prucnal (1979). On Two Problems of Harvey Friedman. In: Studia Logica 38 (3), pp. 247–262.
- P. Rozière (1992). Règles admissibles en calcul propositionnel intuitionniste. PhD thesis. Université de Paris VII.
- T. Skura (1992). Refutation Calculi for Certain Intermediate Propositional Logics. In: Notre Dame Journal of Formal Logic 33.4, pp. 552–560.

Coalgebras - an introductory tutorial

H. Peter Gumm
Universität Marburg
gumm@Mathematatik.Uni-Marburg.de

State based systems are typical for Computer Science: Automata, Transition systems, Objects in Programming, Nondeterministic, Probabilistic and Fuzzy Systems, all are based on notions of state and of state change.

Typically, states are internal and not directly visible, but only their effects are observable by tests. Depending on the system type, observations may be structured as lists, streams, languages, trees, etc.. Thus questions of observational equivalence, minimization, and logical descriptions arise.

Just as Universal Algebra serves to provide a unified theory for all kinds of algebraic structures, leading to genuine results of its own, and yielding blueprints for algebraic investigations in the individual fields, so does Universal Coalgebra provide a unified theory for all the above mentioned state based system, with a rich theory of its own and interesting mathematical properties. From the general viewpoint of Universal Coalgebra it turns out then that further mathematical objects fit into the general framework, amongst them neighbourhood systems and, in particular, topological spaces.

Universal Coalgebra develops a structure theory and both a universal logic - coequations together with a co-Birkhoff theorem - and local logics - e.g. coalgebraic modal logic - together with correctness and completeness theorems.

In our first lecture we plan to start with some examples, leading to the notion of types, coalgebras of these types, and their basic (co)algebraic structure theory. In the second lecture we shall introduce the logical notions such as coequations and modalities and show, respectively, their appropriateness and completeness.

We need to make mild use of category theoretical language, but we will always strive to give a very intuitive and graphical account of all notions, results and proofs.

If time permits we finish with the investigation of special types - given by functor properties - and finally connect them to some classical universal algebraic notions.

Boolean Spaces of Finite Type and Modal Logic

Georges Hansoul
University of Liege, Belgium
G.hansoul@ulg.ac.be

A (metric boolean) space is said of finite type if it has finitely many orbits under the action of its automorphism group. These spaces have been introduced by Pierce (in 1972). He gives a decomposition result of a space of finite type in terms of its pseudo indecomposable subspaces. He also obtains a complete description of the semigroup $U(\infty)$ of isomorphic types of pseudo indecomposable spaces of finite types, naturally endowed with a transitive relation $\triangleleft$ linked to his decomposition theorem.

At the same time (1971), Segerberg was studying the canonical model for the modal logic $K4$. The points of finite depth in the free variable canonical model form a frame $(W_{K4}(0), R)$ and it can be observed that this frame is isomorphic to $(U(\infty), \triangleleft)$.

We show that this pleasant similarity is by no means a mere luck. There is a strong link between the two subjects, which can be found in studying the derivational logic of the spaces of finite type.

Projective bichains

John Harding
New Mexico State University
jharding@nmsu.edu
Coauthors: Carol and Elbert Walker

A bisemilattice is an algebra with two binary operations $\cdot$ and $+$ that are commutative, associative, and idempotent. A Birkhoff system is a bisemilattice where the two operations are linked by a weakened version of the law of absorption: $x \cdot (x+y) = x + (x \cdot y)$. A bisemilattice carries two partial orderings, and we call a bisemilattice a bichain if each of these partial orderings is linear. Conversely, any two linear orderings of a set yields a bichain. A simple argument shows each bichain is a Birkhoff system.

Our purpose here is to describe the finite bichains that are (weakly) projective in the variety of Birkhoff systems. This study is related to problems in the axiomatics of type-2 fuzzy logic.

Sequential and countability properties in frames

David Holgate
University of the Western Cape, South Africa
dholgate@uwc.ac.za
Coauthors: Jacques Masuret (University of Stellenbosch) Mark Sioen (Free University, Brussels)

We propose and investigate a notion of sequence (of "points") in a frame. Allied topological properties of sequential closedness and sequential compactness follow. We also consider the interaction of sequential properties with countability related properties such as countable compactness and countable nearly closedness.

A Topological / Modal Logic Theory of Everything

Norman Howes
5430 Havenwoods Drive, Houston, TX 77066
normhowes@comcast.net
Coauthors: John Howes

The topological part of the theory is from the author's book "Modern Analysis and Topology." The modal logic part is from the author's series of 6 distributed system theory books for the IEEE Computer Society. The author presented a paper at the 2006 World Congress of Computer Scientists, Engineers and Applications, titled "A Theory of Distributed Systems." Since then the theory has been used to model many types of systems in physical reality. These models have relativistic and uncertainty behaviors. The modal projection operator replaces the projection operator from the standard model and the behavior function replaces the wave function. Recently we were able to model human consciousness systems with the theory and are finishing a book on that subject titled "The Collective Consciousness."

A new forcing and some applications in the theory of countable equivalence relations

Steve Jackson
University of North Texas
jackson@unt.edu

We introduce a new forcing which generically adds minimal 2-colorings. The forcing, and variations of it, can be used to obtain new results about countable Borel equivalence relations. We will explain the forcing and present several applications.

On Categorical Relationship among various Fuzzy Topological Systems, Fuzzy Topological Spaces and related Algebraic Structures

Purbita Jana
Department of Pure Mathematics, University of Calcutta
purbita_presi@yahoo.co.in

Interconnection among the categories of fuzzy topological systems, fuzzy topological spaces and algebraic structures is established [2,6]. The origin goes back to similar interconnection among topological systems, topological spaces and locales studied by Vickers [7] in the context of geometric logic. Furthermore we establish the interconnection among the categories of fuzzy topological systems whose underlying sets are fuzzy sets, fuzzy topological spaces on fuzzy sets and locales: it is a significant generalization of previous interconnections. It therefore allows us to conveniently analyze the relative merits of previous connections, give coherence to known results, and indicate appropriate directions for future development. Furthermore duality between Lukasiewicz n valued algebra with constants and n-fuzzy Boolean space is established via n-fuzzy Boolean system [2] which was done directly in [4].

[1] George E. Strecker, Adamek Jiri, Horst Herrlich, Abstract and Concrete Categories, John Wiley & Sons. ISBN 0-471-60922-6, 1990.

[2] M.K. Chakraborty and P. Jana, Some Topological Systems: their Categorical Relationship with Fuzzy Topological Spaces and related Algebras, Fuzzy Sets and Systems(submitted (2012)).

[3] P.T. Johnstone, Stone Spaces, Cambridge University Press, 1982.

[4] Yoshihiro Maruyama, Fuzzy Topology and Lukasiewicz Logics from the Viewpoint of Duality Theory, Studia Logica, 94, 2010, pp. 245-269.

[5] James R. Munkres, Topology, Prentic-Hall of India Private Limited, 2000.

[6] Apostolos Syropoulos and Valeria de Paiva, Fuzzy topological systems, 8th Panhellenic Logic Symposium, Ioannina, Greece, July, 2011.

[7] Steven J. Vickers, Topology Via Logic, volume 5, Cambridge Tracts in Theoretical Computer Science University Press, 1989.

[8] L.A. Zadeh, Fuzzy sets, Information and Control, 8, 1965, pp. 338-353.

Maximal subsets of pairwise summable elements in generalized effect algebras

Jiri Janda

Department of Mathematics and Statistics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37 Brno, Czech Republic

98599@mail.muni.cz

Coauthors: Zdenka Riecanova, Department of Mathematics, Slovak University of Technology, Slovak Republic.

Effect algebras were presented as algebraic structures suitable for modeling unsharp and non-compatible events in quantum mechanics (D. Foulis, 1994). Mutually equivalent generalizations (unbounded version) called generalized effect algebra were introduced by several authors (D. Foulis and M.K. Bennett, G. Kalmbach and Z. Riecanova, J. Hedlikova and S. Pulmannova and F. Kopka and F. Chovanec).

In Z. Riecanova: "Generalization of Blocks for D-Lattices and Lattice-Ordered Effect Algebras" was shown, that every lattice effect algebra is a set theoretical union of its maximal compatible sub-sets called compatibility blocks. Our work is an analogy of this result for the case of generalized effect algebras and their maximal pairwise summable sub-sets.

We show that in every generalized effect algebra G, a maximal pairwise summable subset is a sub-generalized effect algebra, called a summability block. If G is lattice ordered, then its every summability block is a generalized MV-effect algebra. Moreover, if every element of G has an infinite isotropic index, then G is covered by its summability blocks, which are generalized MV-effect algebras in the case when G is lattice ordered. We also present relations between summability blocks and compatibility blocks of G.

The patch topology, Smyth's stable compactification and the assembly of a frame

Olaf Klinke
German Cancer Research Center (DKFZ)
olf@aatal-apotheke.de

We showcase a method that produces various known and novel constructions in point-free topology, such as the assembly of a frame and a patch frame for arbitrary continuous frames.

The patch construction for stably continuous frames is well-studied and represents the point-free analogue of the patch topology of a stably compact space. Various attempts have been made to extend the patch construction beyond the stably compact case.

Here we show how to obtain a patch of any continuous frame as the pushout of a diagram involving the original frame, its smallest stable compactification according to Smyth, and the well-known patch of the latter (a.k.a. Fell compactification). We show that the well-known assembly of a frame can be obtained using the same method, by replacing the smallest stable compactification with the largest. As a byproduct, our constructions each possess a certain universal property involving quasi-proximities.

We give a description of the pushout via generators and relations. This description exhibits bitopological structure in both the patch and the assembly while being simpler than previous bitopological descriptions.

Extending set-functors to topological categories

Alexander Kurz
University of Leicester
kurz@mcs.le.ac.uk
Coauthors: Adriana Balan, Jiri Velebil

Many constructions familiar from sets can be extended in a canonical way to categories C with richer structure (our examples will be of a topological nature, but the techniques and results are more general). In many cases the extension is based on an an obvious functor Set $\to$ C that is dense (in a certain technical sense).

For example, if the construction in question is taking powersets and C is posets, then one obtains the convex subsets with the Egli-Milner order; similarly, if C is metric spaces, one obtains closed subspaces with the Hausdorff distance.

In this talk, we show how these and many other extensions of set-functors can be uniformly accounted for: (1) Represent a given object of C as a weighted colimit of 'discrete' objects; (2) Apply the set-functor to the discrete objects; (3) Compute the colimit in C. Thus, the essence of the construction is encoding the structure of a C-object in a diagram of discrete objects.

(The relevant notions of density and diagram are those of enriched category theory, but no previous knowledge of enriched category theory will be assumed for the presentation.)

Generating all finite modular lattices of a given size

Nathan Lawless
Chapman University
lawle108@mail.chapman.edu

Modular lattices, introduced by R. Dedekind, are an important subvariety of lattices that includes all distributive lattices. In 2002, Heitzig and Reinhold developed an efficient algorithm to enumerate all finite lattices up to isomorphism and used it to count the number of lattices up to size 18. Here we present an improvement and adaptation of this algorithm which is used to enumerate all modular lattices up to size 22. Additionally, we use this approach to enumerate other types of lattices such as semimodular, semidistributive, two-distributive and selfdual lattices.

Lattice-ordered matrix algebras - A brief survey

Jingjing Ma
University of Houston-Clear Lake
ma@uhcl.edu

Let R be a lattice-ordered ring and $M_n(R)$ ($n \geq 2$) be the $n \times n$ matrix ring over R. A natural lattice order on $M_n(R)$ to make it into a lattice-ordered ring is the entrywise lattice order in which a matrix $A = (a_{ij}) \in M_n(R)$ is positive if each a_{ij} is positive in R. If R has the positive identity element, then clearly the identity matrix of $M_n(R)$ is positive with respect to the entrywise lattice order.

In 1966, E. Weinberg initiated study of lattice orders on matrix algebras. Let $\mathbb{Q}$ be the field of rational numbers. He proved that if $M_2(\mathbb{Q})$ is a lattice-ordered ring in which the identity matrix is positive, then it is isomorphic to $M_2(\mathbb{Q})$ with the entrywise lattice order. He also conjectured that this fact is true for any matrix ring $M_n(\mathbb{Q})$ with $n > 2$.

In 2002, Weinberg's conjecture was proved for any matrix algebra $M_n(F)$ over a totally ordered subfield F of the field $\mathbb{R}$ of real numbers.

In this talk, I will briefly survey research activities on lattice-ordered matrix algebras since 2002.

Nilpotence and dualizability

Peter Mayr
Johannes Kepler University Linz
peter.mayr@jku.at
Coauthors: Wolfram Bentz (Lisbon)

An algebraic structure is dualizable in the sense of Clark and Davey if there exists a certain natural duality between the quasi-variety it generates and some category of topological-relational structures. As a classical example, Boolean algebras are dualized by Boolean spaces via Stone's representation theorem.

Dualizable algebras in congruence distributive varieties have been fully characterized by Davey, Heindorf and McKenzie (1995). In contrast, it is wide open which algebras in congruence permutable varieties can be dualized. In the work of Quackenbush and Szabó on groups (2002) and Szabó on rings (1999) nilpotence appears as an obstacle to dualizability. We extend these results and show that actually a stronger version of nilpotence, called supernilpotence by Aichinger and Mudrinski (2010), is the real culprit. In particular, we will present a non-abelian nilpotent expansion of a group that is dualizable.

This is a joint work with Wolfram Bentz (Lisbon).

Graph theory and modal logic

Yutaka Miyazaki
Osaka University of Economics and Law, Osaka, JAPAN
y-miya@keiho-u.ac.jp

Kripke frame is a graph. We stand on this fundamental observation, and investigate the properties of graphs by means of propositional modal logics. In this talk, we use the framework of modal (hybrid) languages and techniques to formulate and analyze a few kinds of properties of undirected graphs, i.e. irreflexive and symmetric Kripke frames.

Tense MV-algebras and related functions

Jan Paseka
Department of Mathematics and Statistics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37 Brno, Czech Republic
paseka@math.muni.cz
Coauthors: Michal Botur

The main aim of this lecture is to study functions between MV-algebras that are related to the so called tense MV-algebras. Tense MV-algebras introduced by D. Diagonescu and G. Georgescu are just MV-algebras with new unary operations G and H which express universal time quantifiers.

We present a general representation theorem for these functions between semisimple MV-algebras ,i.e., we show that any such fuction is induced by a relation analogously to classical works in this field of logic. This result yields both a solution of Open problem by D. Diagonescu and G. Georgescu about representation for some classes of tense MV-algebras solved by the coauthor and strenghtens also a result on a broader class of operators on semisimple MV-algebras by the speaker.

As a byproduct of our study we describe the meets of extremal states on MV-algebras and we give a new characterization of extremal states.

Point-free generalized nearness and subfitness

Jorge Picado
Department of Mathematics, University of Coimbra, Portugal
picado@mat.uc.pt
Coauthors: Ales Pultr (Charles University, Prague, Czech Republic)

The standardly used concept of nearness in the point-free context [1] is that of a filter of covers $\mathcal{N}$ of a frame L, admissible in the sense that each element a in L is the join of all the b uniformly below it. This makes sense in regular frames only. If we replace elements with sublocales in the admissibility condition, then, unlike with uniformity, it becomes possible to speak of a nearness in more general frames (precisely, the subfit ones [2]). Further, one is sometimes interested in the non-symmetric variant which (even in the regular case) cannot be dealt simply with covers that make everything naturally symmetric.

In this talk we will discuss the nearness extended in both the mentioned direction: it is generalized in the sense the cover nearness was generalized in [2], and it allows for non-symmetry as well [3]. In this extension one encounters inherent bitopological concepts of fitness and subfitness. It may be of interest that although one gets the subfitness as a necessary and sufficient condition for the existence of nearness in the quite general context again (and fitness as the hereditary variant), it is not quite a smooth extension (while the fitness is).

References

[1] B. Banaschewski and A. Pultr, Cauchy points of uniform and nearness frames, *Quaest. Math.* 19 (1996) 101–127.

[2] H. Herrlich and A. Pultr, Nearness, subfitness and sequential regularity, *Appl. Categ. Structures* 8 (2000) 67–80.

[3] J. Picado and A. Pultr, Subfit biframes and non-symmetric nearness, *preprint DMUC 13-12*, April 2013 (submitted).

The 42 reducts of the random ordered graph

Michael Pinsker
Université Diderot - Paris 7
marula@gmx.at

The random ordered graph is the up to isomorphism unique countable homogeneous structure that embeds all finite linearly ordered graphs. I will present a complete classification of all structures which have a first order definition in the random ordered graph. This is equivalent to a classification of the closed permutation groups which contain the automorphism group of the random ordered graph.

Our classification includes in particular all structures definable in the order of the rationals (previously classified by Cameron '76), the random graph (Thomas '91) and the random tournament (Bennett '97). We obtained the result by the method of canonical functions, which is based on structural Ramsey theory and which I will outline.

(Joint work with M. Bodirsky and A. Pongrácz)

A homogeneous algebraic definition of Euclidean space

Vaughan Pratt
Stanford University
pratt@cs.stanford.edu

Euclidean spaces have been defined both algebraically and logically, respectively as a suitable inner product space, and via Tarski's first-order axioms. Only the latter however is homogeneous; for the former, inner product maps a pair of points (suitably construed as vectors) to a real, entailing two sorts. Can there be a definition that is both algebraic and homogeneous?

Our previous work, some presented at earlier BLASTs, focused on affine geometry, easier because the affine spaces over any fixed field form a variety. Euclidean spaces cannot (at least naively) form even a quasivariety: for one thing the real line carries too little structure for its direct square to be the real Euclidean plane. Some modifications are therefore in order.

In this talk we define a Euclidean space to be an algebraic structure (E, c, a, b, t) with similarity type 3-3-3-3 (no constants so no origin O) satisfying certain pure equations (no Horn or other clauses). The modifications preventing these spaces from forming a variety are (i) we allow the four operations to be partial, and (ii) we introduce a fifth non-associative "domain-independent" operation $x + y$ of *compatible sum* of points, which is commutative, idempotent, satisfies $x + y = x$ when y is undefined, and otherwise (namely when both arguments are defined and distinct) is undefined. Sum serves to combine partial operations, enlarging their domains where compatibly defined. Binary sum generalizes to Sum: $2^E \to E$ analogously to the generalization of binary join to Sup: $2^L \to L$ on lattices, with Sum mapping each singleton $\{x\}$ to x and being undefined on all other sets.

The operation $c(A, B, C)$ returns the circumcenter of triangle ABC, catering for both flatness and line-line intersection. The operations a, b, t each applied to (A, B, C) intersect the line CX with the sphere AB (center A, radius AB) where X is respectively A, B, or T (the tangent point on sphere AB from C that is nearest B). Complementing the Mohr-Mascheroni compass-only theorem, we obtain a specified intersection of two coplanar circles in 14 steps.

Our main theorem is that in every Euclidean space so defined having two or more (even uncountably many) dimensions, all lines defined by two distinct points are isomorphic, and (taking the two points to be respectively 0 and 1) bear the structure of an ordered extension of the field of constructible real numbers. Furthermore every such field extension arises in this way for some Euclidean space.

Duality theory and B L A S T : selected themes

Hilary Priestley

Mathematical Institute, University of Oxford

hap@maths.ox.ac.uk

These two tutorial talks will focus on the framework of natural duality theory, in particular as this applies to quasivarieties and varieties of lattice-based and distributive lattice-based algebras, and to unital semilattices. Overall we shall seek to connect, in different ways and in varying degrees, with the topics from which BLAST acquires its acronym.

The first talk will take as its starting point Stone duality for Boolean algebras and Priestley duality for distributive lattices, and will present further dual equivalences which follow the same pattern. The examples, some old and some new, will be chosen to illustrate the underlying theory at various levels of generality.

In the second talk we shall adopt a broader perspective and discuss links between ideas from duality theory on the one hand and canonical extensions, profinite completions and zero-dimensional Bohr compactifications on the other.

On the filter theory of residuated lattices

Jiri Rachunek
Palacky University in Olomouc, Czech Republic
jiri.rachunek@upol.cz
Coauthors: Dana Salounova (VSB-Technical University of Ostrava, Czech Republic)

Bounded integral commutative residuated lattices (= residuated lattices, in short) form a large class of algebras which contains e.g. algebras that are algebraic counterparts of some important many-valued and fuzzy propositional logics (MV-algebras, BL-algebras, MV-algebras), as well as of the intuitionistic logic (Heyting algebras). Filters of such algebras correspond to deductive systems of appropriate logics.

In the talk we will deal with the so-called extended filters of filters associated with subsets of residuated lattices. We have shown that all such extended filters of any residuated lattice can be interpreted as the relative pseudocomplements of the complete Heyting algebra of filters. This fact enable us to investigate extended filters using the technique of Heyting algebras. Especially we have described the sets of filters which are in connection with the relative pseudocomplements that are associated with a given filter and characterize filters which are stable with respect to a fixed filter.

Ideal extension of semigroups and their applications

Hamidreza Rahimi
Islamic Azad University, Central Tehran Branch
rahimi@iauctb.ac.ir

Let S and T be disjoint semigroups, S having an identity 1_S and T having a zero element 0. A semigroup Ω is called an [ideal] extension of S by T if it contains S as an ideal and if the Rees factor semigroup $\frac{\Omega}{S}$ is isomorphic to T, i.e. $\frac{\Omega}{G} \simeq S$ [1].Ideal extension for topological semigroup as subdirect product of $S \times T$ was studied by Christoph in 1970 [2].

In this talk we intoduce ideal extension for topological semigroups using a new method, then we investigate the compactification spaces of these structures. As a consequence, we use this result to characterize compactification spaces for Brant λ-extension of topological semigroups.

[1] A. H. Clifford and J. B. Preston, The Algebraic Theory of Semigroups I, American Mathematical Society, Surveys 7, (1961).

[2] Francis T. Christoph Jr, Ideal extensions of topological semigroups, *Can. J. Math*, **Vol. XXII, No. 6**, (1970), pp. 1168-1175.

[3] O. Gutik and D. Repovs, On Brandt λ^0-extensions of monoids with zero,*Semigroup Forum*, **vol. 80, no. 1**, (2010) pp. 8.32.

[4] H. Rahimi, Function spaces of Rees matrix semigroups,*Bulletin of the Iranian Mathematical Society* **Vol. 38 No. 1** (2012), pp 27-38.

[5] H. Rahimi, M. Asgari, Function Spaces of Topological Inverse Semigroups,*J. Basic. Appl. Sci. Res***2(10)** (2012), 10652-10655.

[6] H. Rahimi, Function spaces on tensor product of semigroups,*Iranian Journal of Science and Technology* **Vol. A3** (2011), 223-228.

A Stochastic Lambda-Calculus

Dana S. Scott
Visiting Scholar, UC Berkeley
dana.scott@cs.cmu.edu

Many authors have suggested ways of adding random elements and probability assessments to versions of Church's Lambda-Calculus. When asked recently about models, the speaker realized that the so-called Graph Model (based on using enumeration operators acting on the powerset of the integers) could easily be expanded to a Boolean-valued powerset interpretation using standard measure algebras. It is also possible to use a powerset topology to define random variables taking values in the lambda-calculus model. The talk will report on how a continuation-passing semantics can be used for modeling a branching combinator using random coin tossing. The idea can also be employed for introducing many other random combinators.

Combinatorial properties of singular cardinals

Dima Sinapova
University of Illinois at Chicago
`sinapova@math.uic.edu`

Consistency results about singular cardinals involve forcing in the presence of large cardinal axioms. We will discuss some old and new Prikry type forcings and their impact on combinatorial principles such as scales and squares.

On some topological properties of point-free function rings

Mark Sioen
VUB, Free University of Brussels (Belgium)
`msioen@vub.ac.be`

For a topological space X, it is customary to equip the f-ring $C(X)$ or its bounded part $C^*(X)$ with well-known topologies or uniformities (like e.g. those of uniform convergence, pointwise convergence, convergence on preferred subsets), which thank their importance to several fundamental theorems like a.o. the Stone-Weierstrass Theorem or Dini's Theorem. These theorems are classically proved subject to possible supplementary (often compactness) conditions on X. Alternatively, one can also in many cases characterize exactly those X for which the conclusion of such a theorem holds, i.e. those X that have the Stone-Weierstrass or Dini property. In point-free topology, for a frame L, one encounters as a counterpart to $C(X)$ (resp. $C^*(X)$), the well-studied f-ring $\mathcal{R}L$ of real-valued continuous functions on L and its bounded part $\mathcal{R}^*L$. A point-free Stone-Weierstrass Theorem has been proved by B. Banaschewski. It is the aim of this talk to discuss some topological properties of the f-ring $\mathcal{R}L$ or its bounded part which are point-free counterparts of e.g. the Stone-Weierstrass and Dini properties.

On a co-universal arrow in the construct of n-ary hyperalgebras

Josef Slapal
Brno University of Technology
slapal@fme.vutbr.cz

We define and discuss a special product of n-ary hyperalgebras which lies between their cartesian product and the cartesian product of their idempotent hulls. We study the endofunctor of the construct of n-ary hyperalgebras given by the product introduced and a fixed n-ary hyperalgebra. The n-ary hyperalgebras are characterized for which the point-wise defined function spaces together with the corresponding evaluation maps constitute co-universal arrows with respect to the endofunctor. Some further properties of the product of n-ary hyperalgebras are dealt with, too.

Boolean topological graphs of semigroups

Michał Stronkowski
Warsaw University of Technology
m.stronkowski@mini.pw.edu.pl
Coauthors: Belinda Trotta

The Boolean core H_{BC} of a universal Horn class H is the topological prevariety generated by the finite structures of H (treated as topological structures with the discrete topology). Specifically, H_{BC} is the class of topologically closed substructures of products, with nonempty indexing sets, of finite members of H (up to isomorphism). Also H_{BC} is the class of profinite structures built, as inverse limits, from finite members of H (up to isomorphism) [2].

If V is a variety of semigroups, V_{BC} consist of all Boolean (compact, totally disconnected) topological semigroups satisfying identities true in V. In particular, V_{BC} is first-order definable (relative to the class of all Boolean topological groupoids) [1].

However the situation changes radically when we shift to relational structures. The graph of a semigroup $(S, \circ)$ is the relational structure (A, R), where R is the graph of the multiplication $\circ$. This means that R is the relation consisting of those triples $(a, b, c) \in A^3$ which satisfy $a \circ b = c$. We proved the following fact.

THEOREM. Let C be a class of semigroups possessing a nontrivial member with a neutral element and H be the universal Horn class generated by the graphs of semigroups from C. Then H_{BC} is not first-order axiomatizable.

REFERENCES
[1] David M. Clark, Brian A. Davey, Ralph S. Freese, and Marcel Jackson. Standard topological algebras: syntactic and principal congruences and profiniteness. Algebra Univers., 52:343-376, 2004.
[2] David. M. Clark, Brian A. Davey, Marcel G. Jackson, and Jane G. Pitkethly. The axiomatizability of topological prevarieties. Adv. Math., 218(5):1604-1653, 2008.

Natural extension of median algebras

Bruno Teheux
University of Luxembourg
`bruno.teheux@uni.lu`
Coauthors: Georges Hansoul

The concept of natural extension for algebras in an internally residually finite prevariety $\mathcal{A}$ has been introduced as a generalization of canonical extension of lattice-based algebras. An algebra A embeds as a topological dense subalgebra of its natural extension, which is a topological algebra isomorphic to the profinite completion of A. If $\mathcal{A} = \mathbb{ISP}(M)$ is a dualisable (in the sense of natural duality) finitely generated prevariety then the natural extension of A can be concretely obtained as the topological algebra of the structure preserving maps from the dual of A to the structure $M_\sim$ that generates the duality.

This tool has already been applied as an alternative way to study canonical extension of finitely generated varieties of lattice-based algebras. Actually, it is clearly well designed for any dualisable finitely generated variety, in which algebras do not necessarily have a lattice reduct.

In this talk we propose to illustrate these constructions for the variety $\mathcal{A}$ of median algebras, *i.e.*, for $\mathcal{A} = \mathbb{ISP}(M)$ where $M = \langle \{0,1\}, (\cdot,\cdot,\cdot)\rangle$ where $(\cdot,\cdot,\cdot)$ is the ternary majority function on $\{0,1\}$. In the perspective of illustrating the techniques of natural extension, this variety is interesting for at least three reasons: (1) it admits a strong duality, (2) it is not lattice-based but (3) it has nevertheless some interesting order-theoretic properties. It is indeed known that any median algebra can be regarded as a median semilattice (that is, a meet-semilattice in which every principal ideal is a distributive lattice and in which any three elements have an upper bound whenever each pair of them is bounded above) and conversely.

Hence, we investigate and illustrate how natural extension behaves with regard to median algebras and their correspondent median semilattices. We also show how to define a new topology that is used to extend maps between median algebras to maps between their natural extension. This allows us to consider natural extension of expansions of median algebras and the problem of preservation of equations through natural extension.

A selective for $\mathcal{R}$ but not Ramsey for $\mathcal{R}$ ultrafilter.

Timothy Trujillo
University of Denver
du.timothy.trujillo@gmail.com

In [1], J. Mijares introduced the relation of almost-reduction $\leq^*$ in an arbitrary topological Ramsey space $\mathcal{R}$ as a generalization of almost-inclusion $\subseteq^*$ on $[\omega]^\omega$. This leads naturally to the definition of Selective for $\mathcal{R}$ and Ramsey for $\mathcal{R}$ ultrafilters. If $\mathcal{R}$ is taken to be the familiar Ellentuck space then the definitions of selective for $\mathcal{R}$ and Ramsey for $\mathcal{R}$ ultrafilter reduce to the well-known concepts of selective ultrafilter and Ramsey ultrafilter, respectively. Therefore an ultrafilter $\mathcal{U}$ is selective for the Ellentuck space if and only if it Ramsey for the Ellentuck space.

In [1], J. Mijares shows that in any topological Ramsey space $\mathcal{R}$, if $\mathcal{U}$ is Ramsey for $\mathcal{R}$ then $\mathcal{U}$ is selective for $\mathcal{R}$. In this talk, we give examples of topological Ramsey spaces constructed from trees such that, assuming CH, MA or forcing with $(\mathcal{R}, \leq^*)$, there exists an ultrafilter that is selective for $\mathcal{R}$ but not Ramsey for $\mathcal{R}$.

[1] J. Mijares, *A notion of selective ultrafilter corresponding to topological Ramsey spaces, Math. Log. Quart.* **53**, No. 3, 255-267 (2007).

Aronszajn trees and the successors of a singular cardinal

Spencer Unger
UCLA
sunger@cmu.edu

The tree property at an uncountable cardinal κ is a generalization of the classical König Infinity Lemma. The construction of an Aronszajn tree witnesses that the tree property fails at $\aleph_1$. A result of Mitchell shows that the tree property at $\aleph_2$ is equiconsistent with the existence of a weakly compact cardinal. Mitchell's result uses the modern set theoretic tools of forcing, large cardinals and inner models.

An old question about extending Mitchell's result asks whether it is consistent relative to large cardinals that every regular cardinal greater than $\aleph_1$ has the tree property. Motivated by this question we prove the following theorem: If there are a supercompact cardinal and a weakly compact cardinal above it, then it is relatively consistent that there is a singular strong limit cardinal κ of cofinality ω such that there are no special κ^+-trees and κ^{++} has the tree property.

Axiomatizing Set Bands

Lawrence Valby
UC Berkeley
`valby@berkeley.edu`

There is a certain class of idempotent semigroups (I've been calling them set bands) which arises naturally from thinking about conversational update systems (abstractly these are just actions, or functions of the form $A \times B \to A$). Yalcin and Rothschild pointed out recently that actions expressible using set intersection are exactly the commutative and idempotent actions. However, what if we allow both intersection and union? Set bands are a single-sorted translation of this problem. They form a quasivariety, but not a variety. I consider some partial progress toward concretely specifying an axiomatization for these set bands.

Gelfand semirings , m-semirings and the Zariski topology

Jorge Vielma and Luz Ruza
Universidad de los Andes- Mérida- Venezuela
`vielma@ula.ve`

In this work by a semiring R we understand a commutative semiring with identity and we consider in its prime spectrum the Zariski topology tz. We denote by Tz the smallest Alexandroff topology containing tz. Also Tz* denotes its corresponding cotopology. We say that R is a Gelfand semiring if every prime ideal is contained in a unique maximal ideal. We say that R is an m-semiring if each prime ideal contains only one minimal prime ideal. We give a characterization of such semirings in terms of the smallest Alexandroff topology Tz in Spec(R) containing the Zariski topology tz and the Zariski topology tz itself. Other results involving the topology Tz* are also given.

Notes on orthoalgebras in categories

Taewon Yang
New Mexico State University
yangjong@nmsu.edu
Coauthors: John Harding

Orthoalgebras were introduced in the early 1980s by Foulis and Randall as extended models of the propositional logic of quantum mechanics with tensor products. Using the canonical orthoalgebra of all decompositions of an object in a suitable category, we show that every interval in the orthoalgebra also arises from decompositions.