

Database of Finite Residuated Lattices

<http://vychodil.inf.upol.cz/order/>

Structure of the Input Files

Finite residuated lattices are encoded in ASCII text files (using Unix-style newlines). Each line defines either a lattice order \leq or a multiplication \otimes defined on the last lattice order. A line beginning with colon “:” and continuing with a sequence of 0s and 1s encodes a lattice order. A line beginning with 0 or a capital letter (e.g., A,B,…) encodes a multiplication. All n -element residuated lattices are encoded in the file `RES n .txt.bz2`. For instance, an excerpt of `RES06.txt.bz2` encoding all 6-element residuated lattices is

```
:001011
0000000000
:011010
00A0B0BCAB
AOAAB0BCAD
:011011
0000000000
0000B0B00B
⋮
```

Encoding of Lattice Orders and Multiplications

In the encoding, we assume that the support set of L contains 0 and 1 denoting the least and the greatest element of the lattices, respectively. Further elements of lattices are encoded by capital letters A,B,…. A lattice ordering \leq on $L = \{0, A, \dots, 1\}$ is encoded by a vector of 0s and 1s representing values in the inner triangle of the upper-triangular adjacency matrix in which rows and columns corresponding to elements of L are listed in the following order: 0, A, …, 1. The first $n - 3$ values in the encoding determine whether $A \leq B$, $A \leq C$, $A \leq D, \dots$; next $n - 4$ values in the encoding determine whether $B \leq C$, $C \leq D, \dots$, and so on. A multiplication is encoded by a vector of the results $a \otimes b$ in which we skip results that are trivial, e.g. $a \otimes 0$, $a \otimes 1$, etc. Moreover, due to the commutativity of \otimes , the vector contains results only for $a \otimes b$ where a precedes (or, is equal to) b in the table.

Example

Let $L = \{0, A, B, C, D, 1\}$. Consider the third and fourth lines in `RES06.txt.bz2`. The third line `:011010` says that $A \not\leq B$, $A \leq C$, $A \leq D$; $B \not\leq C$, $B \leq D$; and $C \not\leq D$. The fourth line `00A0B0BCAB` says that $A \otimes A = 0$, $A \otimes B = 0$, $A \otimes C = A$, $A \otimes D = 0$; $B \otimes B = B$, $B \otimes C = A$, $B \otimes D = B$; $C \otimes C = C$, $C \otimes D = A$; and $D \otimes D = B$. Both the vectors can be seen as encodings of the red regions in the following tables of \leq and \otimes :

\leq	0	A	B	C	D	1
0	1	1	1	1	1	1
A	0	1	0	1	1	1
B	0	0	1	0	1	1
C	0	0	0	1	0	1
D	0	0	0	0	1	1
1	0	0	0	0	0	1

\otimes	0	A	B	C	D	1
0	0	0	0	0	0	0
A	0	0	0	A	0	A
B	0	0	B	0	B	B
C	0	A	0	C	A	C
D	0	0	B	A	B	D
1	0	A	B	C	D	1

Bibliography

More information can be found in the following paper:

Belohlavek R., Vychodil V.: Residuated Lattices of Size ≤ 12 .
Order, <http://dx.doi.org/10.1007/s11083-010-9143-7>.