

# The axioms of Zermelo-Fraenkel set theory with choice **ZFC**

In principle all of mathematics can be derived from these axioms

- Extensionality**  $\forall X \forall Y [X = Y \Leftrightarrow \forall z(z \in X \Leftrightarrow z \in Y)]$
- Pairing**  $\forall x \forall y \exists Z \forall z [z \in Z \Leftrightarrow z = x \text{ or } z = y]$
- Union**  $\forall X \exists Y \forall y [y \in Y \Leftrightarrow \exists Z(Z \in X \text{ and } y \in Z)]$
- Empty set**  $\exists X \forall y [y \notin X]$  (this set  $X$  is denoted by  $\emptyset$ )
- Infinity**  $\exists X [\emptyset \in X \text{ and } \forall x(x \in X \Rightarrow x \cup \{x\} \in X)]$
- Power set**  $\forall X \exists Y \forall Z [Z \in Y \Leftrightarrow \forall z(z \in Z \Rightarrow z \in X)]$
- Replacement**  $\forall x \in X \exists! y P(x, y) \Rightarrow [\exists Y \forall y (y \in Y \Leftrightarrow \exists x \in X (P(x, y)))]$
- Regularity**  $\forall X [X \neq \emptyset \Rightarrow \exists Y \in X (X \cap Y = \emptyset)]$
- Axiom of choice**  $\forall X [\emptyset \notin X \text{ and } \forall Y, Z \in X (Y \neq Z \Rightarrow Y \cap Z = \emptyset) \Rightarrow \exists Y \forall Z \in X \exists! z \in Z (z \in Y)]$

$\forall$  = for all      $\exists!$  = there exists a unique      $P$  is any formula that does not contain  $Y$

$$z \in X \cup Y \Leftrightarrow z \in X \text{ or } z \in Y \qquad z \in X \cap Y \Leftrightarrow z \in X \text{ and } z \in Y$$