4th SYSMICS Workshop

Duality in Algebra and Logic

Abstracts

Chapman University

Orange, California, USA

Invited Speakers

Marta Bílková Charles University, Prague
Petr Cintula Czech Academy of Sciences
Antonio Di Nola University of Salerno
Michael Dunn Indiana University Bloomington
Wesley Fussner University of Denver
Nick Galatos University of Denver
Giuseppe Greco Utrecht University
John Harding New Mexico State University
Wesley Holliday University of California at Berkeley
Tadeusz Litak Friedrich-Alexander-University Erlangen-Nürnberg
Pat Morandi New Mexico State University
Daniele Mundici University of Florence
Alessandra Palmigiano Delft University of Technology
Vaughan Pratt Stanford University
Luca Spada University of Salerno
Gavin St. John University of Denver

September 14–17, 2018

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Program

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Understanding infinitary logics via their symmetrizations

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Considering a logic $\vdash$ cast as a Tarskian consequence relation between sets of formulas and formulas, there are several ways how to construct its symmetric version $\Vdash$ with a disjuncture flavour on the right-hand side. The situation becomes even more complex if the logic in question is infinitary. Dunn and Hardegree showed that the proper Cut rule for such symmetric logic is equivalent to Pair Extension Property which plays a crucial rule in the completeness proofs for many modal logics.

As a starting point we show that using the most usual symmetrization ($\Gamma \Vdash \Delta$ if there is finite $\Delta' \subseteq \Delta$ such that $\Gamma \vdash \bigvee \Delta'$) we obtain Pair Extension Property for finitary logics $\vdash$ only and we characterize under which conditions we get at least its restricted form for finite Δs which, perhaps surprisingly, turns out to be equivalent to the Lindenbaum lemma for the starting logic $\vdash$.

In Part 1 we focus on consequences of our general results for the completeness proofs of prominent infinitary modal and algebraic logics.

In Part 2 of our talk, we formulate and prove the result in an abstract setting and consider several alternative symmetrizations of $\vdash$.

The Geometry of Free Algebras in Chang Variety: A Bridge from Semisimplicity to Infinitesimals

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We show that in the wild class of non-archimedean MV-algebras, the algebras in the subvariety generated by the Chang algebra have a quite tractable behaviour. Indeed it seems clear that their behaviour parallels that of archimedean ones, at least in the relationships existing among logic, free algebras and their geometry. We show the existence of a functor that provides a useful mechanism to generate, manipulate and manage infinitesimals, i.e. the algebraic and analytical representatives of perturbations of clear truth values.

Representation and Duality for Distributoids

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Distributoids were introduced in Dunn (1990) as generalizations of Jónsson and Tarski's (1951-1952) *Boolean algebras with operators* (*BAO*). A BAO is a Boolean algebra B together with a set of operations $\{o_i\}_{i \in I}$, where each operation distributes over join in each of its places, i.e.,
(i) $o_i(b_1, \ldots, b_i \vee c_i, \ldots, b_n) = o_i(b_1, \ldots, b_i, \ldots, b_n) \vee o_i(b_1, \ldots, c_i, \ldots, b_n)$, and also (ii) $o_i(b_1, \ldots, 0, \ldots, b_n) = 0$.

A distributoid differs from a BAO in that the Boolean algebra is instead any (for convenience bounded) distributive lattice, and operators need not distribute over join. (i) is generalized so that an o_i can instead distribute over meet, or can "co-distribute" over join changing it to meet, or "co-distribute" over meet changing it to join, and can "mix and match," doing one of these in one of its arguments, and another in another argument. And (ii) is generalized so as to allow for 1 preservation, or inversion of 1 and 0 according to appropriate protocols. Jónsson and Tarski' proved that every BAO is representable as a Boolean algebra of subsets of some set U, where each n-ary operator o_i is represented by a generalized image operator R_i^* defined from an $n+1$-placed relation $R_i$on U. Dunn (1990) contains a similar representation for distributoids showing that each n-ary operator is representable as an n-ary operator on subsets of a set U, where the operator is induced by an $n+1$-placed relation on the set U.

About the same time Goldblatt (1989) came up with a similar generalization of BAOs, but it did not allow for co-distribution nor did it allow "mix and match" within a single operator (though it did allow this for different operators – one operator might preserve join and another might preserve meet). Goldblatt went on to provide a representation theorem and a full duality theorem.

Dunn (1990) introduced *gaggles* (*generalized Galois logics*) along with distributoids, and representations for both. Gaggles are distributoids where families of operators are linked together by a common abstraction of Galois connections and residuation. Bimbó and Dunn (2008) proved a full duality theorem for *gaggles*, but neither Dunn (1990) nor Bimbó and Dunn (2008) provided a duality theorem for distributoids. I will take the opportunity to fill this gap.

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Functional duals for residuation algebras

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In residuated algebraic structures based on distributive lattices, the residuated operations typically manifest on duals as ternary relations. Perhaps due to the conceptual difficulty in working with such ternary relations, duality-theoretic methods have enjoyed fewer applications to residuated algebras generally than they have in the special case of Heyting algebras (where the residuated operations may be captured by a binary, rather than ternary, relation). However, in some cases the ternary relation dualizing the residuated operations may be interpreted as functional, i.e., as a (possibly partially-defined) binary operation. This provides a different perspective from which to bring duality-theoretic tools to bear on residuated algebras, opening new avenues of inquiry. Here we employ the theory of canonical extensions to characterize the circumstances in which the dual ternary relation is functional in the aforementioned sense, putting special emphasis on the case when the dual partial operation is total. Moreover, we provide a partial answer to a question of Gehrke, asking whether there is any first-order condition that characterizes functionality of duals.

Is there a useful duality for residuated frames?

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Abstract: Residuated frames are relational semantics for substructural logic, reminiscent of Kripke frames, but recast in a two-sorted language (to account for the lack of distributivity) and expanded (to account for the extra multiplicative connectives). Substructural logics can, of course, also be studied proof-theoretically and algebraically. An interesting feature of residuated frames is that they relate surprisingly well with both of these alternative approaches and, to an extent, link them. We will give a detailed overview of the various applications that residuated frames have enjoyed in the short period of time after their definition. Their use so far has been guided by these applications and in that sense they have more than justified their existence. However, these structures beg for a more categorical and duality-theoretic approach that goes beyond the ones already provided: one that is in line with the applications and demonstrates the needed flexibility for them. In other words, the whole talk is just the introduction to an open question.

Focusing via display

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The focusing discipline in logic was introduced to analyse proof search, where the most natural and successful applications are in verifications, logical and functional programming [LM09]. Focusing has became an important tool in formal linguistics, where it is used, for instance, to account for different readings or disambiguate sentences [MV18, Bas12]. I will present an analysis of the focused Lambek-Grishin calculus fLG [MM13] where, via a suitable translation, the (de-)focusing rules are simulated thanks to (heterogeneous) modalities in a proper multi-type display calculus (mD-fLG). The modalities have a standard interpretation in a simple semantics built in accordance with the syntax.

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Boolean subalgebras of orthomodular structures

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We develop a direct method to recover an orthomodular poset from its poset of Boolean subalgebras. For this a new notion of direction is introduced. Directions are also used to characterize in purely order-theoretic terms those posets that are isomorphic to the poset of Boolean subalgebras of an orthomodular poset. These posets are characterized by simple conditions defining orthodomains and the additional requirement of having enough directions. Excepting pathologies involving maximal Boolean subalgebras of four elements, it is shown that there is an equivalence between the category of orthomodular posets and a suitable category of orthodomains with enough directions with morphisms suitably defined. Furthermore, we develop a representation of orthodomains with enough directions, and hence of orthomodular posets, as certain hypergraphs. This hypergraph approach extends the technique of Greechie diagrams and resembles projective geometry. Using such hypergraphs, every orthomodular poset can be represented by a set of points and lines where each line contains exactly three points.

Possibilities for Boolean, Heyting, and modal algebras

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I will give an overview of an alternative to the standard duality theory for Boolean, Heyting, and modal algebras descending from Stone (1934, 1937) and Jónsson and Tarski (1951). While the standard theory forms the foundation of the well-known "possible world semantics" in logic, the alternative theory forms the foundation of the recently investigated "possibility semantics" in logic.

The talk will be based on the following papers:

W. H. Holliday, "Possibility Frames and Forcing for Modal Logic," 2015.

G. Bezhanishvili and W. H. Holliday, "Locales, Nuclei, and Dragalin Frames," 2016.

G. Bezhanishvili and W. H. Holliday, "A Semantic Hierarchy for Intuitionistic Logic," 2018.

N. Bezhanishvili and W. H. Holliday, "Choice-Free Stone Duality," 2018.

De Vries duality for completely regular spaces

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De Vries duality yields a dual equivalence between the category of compact Hausdorff spaces and a category of complete Boolean algebras with a proximity relation on them, known as de Vries algebras. This duality sends a compact Hausdorff space X to the pair $(B, \prec)$, where B is the set of regular open subsets of X, and $U \prec V$ iff $\overline{U} \subseteq V$. We extend de Vries duality to completely regular spaces by replacing the category of de Vries algebras with certain extensions of de Vries algebras which we call de Vries extensions. This is done by first formulating a duality between a category of compactifications of completely regular spaces and de Vries extensions, and then specializing to the extensions that correspond to Stone-Čech compactifications.

Semantic and syntactic consequence

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According to the Tarskian paradigm, a formula ϕ is a syntactic consequence of a set Θ of formulas in a logic L if every model that satisfies all formulas of Θ also satisfies ϕ. While the notion of a "model" is clear in first-order logic, when L is a propositional logic, one usually rephrases the Tarskian definition by saying that ϕ is a semantic consequence of Θ if every "valuation" that assigns "true" to every formula of Θ also assigns the same value to ϕ. Now in many cases L also has a syntactic-algorithmic notion of consequence that does not coincide with the semantic one. We discuss this incompleteness phenomenon in algebraic logic, with particular reference to many-valued logics.

Logical Foundations of Categorization Theory

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I will start by discussing the idea that meaning, evaluation, and decision-making can be understood in terms of one and the same cognitive mechanism, that of categorization. This is why categorization is key to the theories and methodologies of a wide range of fields: from linguistics, where categories are central to the mechanisms of grammar generation and compositional meaning, to cognitive science, where categorical perception has been identified as the most basic cognitive activity, and used to analyze higher-order cognitive activities; AI, where classification techniques are core to data mining, text mining, and database theory; sociology, where categorization is used to explain the construction of social identity and the organization of experience; business science, where categories are used to predict how products and producers are perceived and evaluated by consumers and investors. This talk reports on the initial results and future challenges of a research program aimed at building an overarching logical theory, technically rooted in duality, generalized Sahlqvist correspondence and structural proof theory, in which meaning-attribution, value-attribution and decision-making can be formally analyzed in their dynamic aspects and in relationship to one another. The initial results I will present include the novel framework of conceptual approximation spaces and their associated rough concepts (this framework unifies rough set theory and FCA), and an approach to reconcile the classical and the prototype views on categorization, based on an epistemic logic of concepts in which the core notion of typicality is captured via a 'common knowledge operator' construction.

The Duality of Time and Information in Concurrency and Branching Time

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I reprise my CONCUR'92 talk of this title in the light of subsequent developments in the theory of Chu spaces, which greatly broaden the class of schedules and their dual automata so representable. Whereas Chu spaces over 2 are essentially limited to interleaving concurrency, Chu spaces over 3 accommodate true concurrency including its representation by higher dimensional automata. Chu spaces over 4 further extend this to a notion of branching time which is sufficiently sensitive to the timing of branches as to be able to distinguish $a(b + c)$ and $ab + ac$.

Kakutani duality, for groups

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Based on joint work with V. Marra (University of Milan).

Kakutani (-Yoshida-Stone) duality asserts that an arbitrary unital real vector lattice (V, u) is isomorphic to $(\mathbf{C}(X), 1)$ (the algebra of all continuous functions from X into $\mathbb{R}$) for some compact Hausdorff space X, if, and only if, V is Archimedean and norm-complete (with respect to the norm induced by the unit). Here we are interested in extending Kakutani duality to the category of unital Abelian ℓ-groups. The main obstacle is represented by the fact that, while in the former case the definable maps are characterised by Stone-Weierstrass theorem as all continuous functions, in the latter the definable maps are exactly (Goodearl-Handelman 1980) the *denominator respecting maps*. If I is any set, one can define a *concrete denominator map* on the Tychonoff cube $[0, 1]^I$, by sending each point in $[0, 1]^I \setminus \mathbb{Q}^I$ to

0 and otherwise to the least common multiple of the denominators of the coordinates written in reduced form (the `lcm` being 0, in case the set of denominators is unbounded). Similarly, one can endow any compact Hausdorff X space with an *abstract denominator map* which is simply a map from X into $\mathbb{N}$. The crux of the matter is then to characterise those compact Hausdorff spaces, endowed with such an abstract denominator map, that are homeomorphic to some closed subspace of some Tychonoff cube, in a way that corresponding points have equal denominators (abstract equal to concrete).

In this talk a separation property that characterises when such a homeomorphism exists will be discussed.

Undecidability of $\mathsf{FL_e}$ in the presence of structural rules

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The decidability of the equational and quasi-equational theories for the variety of commutative residuated lattices ($\mathcal{CRL}$) axiomatized by $\{\cdot, 1, \leq\}$-inequalities has been fully classified. It was shown by van Alten (2005) that quasi-equational theories axiomatized by *knotted inequations* (k_n^m) (i.e. universally quantified inequations of the form $x^n \leq x^m$ for $n \neq m$) are not only decidable, but enjoy the finite embeddability property (FEP). In fact, Galatos and Jipsen (2013) showed that $\mathcal{CRL} + (\mathrm{k}_n^m) + \Gamma$ has the FEP for any set Γ of $\{\cdot, 1, \vee\}$-equations. Viewed proof-theoretically, these results show that deducibility in the Full Lambek calculus with exchange ($\mathsf{FL_e}$) axiomatized by knotted inference rules are decidable.

Dropping commutativity, Horčík (2015) showed that $\mathcal{RL} + (\mathrm{k}_n^m)$ has an undecidable word problem for any $1 \leq n < m$ or $2 \leq m < n$, which implies deducibility is undecidable in the corresponding logic. By utilizing the theory of residuated frames, as developed by Galatos and Jipsen (2013), and an encoding based on counter machines with an undecidable halting problem, Horčík constructs a residuated lattice $\mathbf{R}$ in $\mathcal{RL} + (\mathrm{k}_1^2) + (\mathrm{k}_3^2)$ such that for any variety $\mathcal{V} \subseteq \mathcal{RL}$, $\mathbf{R} \in \mathcal{V}$ implies $\mathcal{V}$ has undecidable word problem. Bootstrapping this result via a deduction theorem, Chvalovský and Horčík (2016) showed that the equational theory of $\mathcal{RL} + (\mathrm{k}_n^m)$ is undecidable for any $1 \leq n < m$, and thus provability in the corresponding knotted extension of FL is undecidable. As a consequence, these results capture many non-commutative varieties satisfying equations in the signature $\{\cdot, 1, \vee\}$. For example, $\mathcal{RL} + (x \leq x^2 \vee x^3)$ has an undecidable word problem since it contains $\mathbf{R}$.

However, in general these results do not hold in the commutative case, as it would contradict van Alten's result. With the exception of knotted rules, little is known about the decidability questions for varieties of commutative residuated lattices axiomatized by equations in the signature $\{\cdot, 1, \vee\}$, e.g., are the (quasi)equational theories for $\mathcal{CRL} + (x \leq x^2 \vee x^3)$ or $\mathcal{CRL} + (xy \leq x^2y \vee x^3y^2 \vee 1)$ decidable?

With a method similar in spirit to the above, the present work defines an infinite set D of $\{\cdot, 1, \vee\}$-equations where for any finite $\Gamma \subset D$ there exists $\mathbf{R}_\Gamma \in \mathcal{CRL} + \Gamma$ such that for any variety $\mathcal{V} \subseteq \mathcal{RL}$, $\mathbf{R}_\Gamma \in \mathcal{V}$ implies $\mathcal{V}$ has an undecidable word problem. This is achieved by encoding the computation of a counter machine with an undecidable halting problem in the language of commutative semirings (where the role of $\vee$ is essential), and the soundness of the encoding is proved using the theory of residuated frames. Furthermore, there is an infinite proper subset $D_\epsilon \subset D$ such that the above can be extended to undecidability of the equational theory via a deduction theorem if $\mathcal{V} \subseteq \mathcal{CRL} + (\mathsf{d})$ for some $(\mathsf{d}) \in D_\epsilon$. Membership of D corresponds to whether there exists special positive solutions to certain systems of linear equations and, in a sense, captures most rules in the signature. For instance, $(x \leq x^2 \vee x^3) \in D_\epsilon$ and $(xy \leq x^2y \vee x^3y^2 \vee 1) \in D \setminus D_\epsilon$.

These results imply undecidability for deducibility and provability in the corresponding structural extensions of $\mathsf{FL_e}$, e.g., we show $\mathcal{CRL} + (x \leq x^2 \vee x^3)$ has an undecidable equational theory, and thus provability in $\mathsf{FL_e}$ + (d) is undecidable, where

$$\frac{\Gamma, \Delta, \Delta, \Pi \Rightarrow \phi \quad \Gamma, \Delta, \Delta, \Delta, \Pi \Rightarrow \phi}{\Gamma, \Delta, \Pi \Rightarrow \phi}\ (\mathsf{d}).$$

Goldblatt-Thomason for LE-logics

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We extend the Goldblatt-Thomason theorem from classical modal logic to arbitrary signatures of normal LE-logics. This result builds on Drew

Moshier's category theoretic perspective on polarities and uses extended Birkhoff's duality/adjunction for lattices and polarities.

MTL-algebras via rotations of basic hoops

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The investigation on the algebraic properties of BL-algebras and basic hoops has been very successful in the past twenty years. One of the reasons is a result proved in [3] by Aglianò and Montagna, that characterizes completely the totally ordered BL-algebras and basic hoops as ordinal sums of Wajsberg hoops. This allows to obtain information on the lattice of subvarieties of BL-algebras and basic hoops (and, dually, on the lattice of axiomatic extensions of the corresponding logics). All these results rely heavily on the fact that BL-algebras are *divisible*, but if we move beyond divisibility, into MTL-algebras, analogous results are lacking. Thus it is interesting to focus on relevant subvarieties of MTL-algebras in which the structure can be well-understood.

In particular, in [4, 5] generalized notions of *rotation* are studied, which generate interesting varieties of MTL-algebras having a retraction into a particular MV-subalgebra (or, as a special case, into their Boolean skeleton). Such classes are proven to be categorically equivalent to categories whose objects are quadruples made of an MV-algebra (or a Boolean algebra), a GMTL-algebra, an operator named *external join* that codifies how they interact, and a nucleus on the GMTL-algebra.

We use such constructions to lift information from the lattice of subvarieties of basic hoops (that is largely known) to some parts of the lattice of subvarieties of MTL-algebras. In particular, we will use the characterization of splitting algebras in the lattice of subvarieties of basic hoops (and interesting subvarieties such as Wajsberg hoops, cancellative hoops and Gödel hoops) in [1, 2] to obtain splitting algebras in the lattice of subvarieties of varieties generated by their rotations.

Moreover, we show that the rotation construction also preserves the amalgamation property, enjoyed by the varieties of basic hoops, Wajsberg hoops, cancellative hoops and Gödel hoops ([6]). Thus we obtain that the varieties generated by rotations of such algebras have the amalgamation property, or equivalently, that the corresponding logics enjoy the deductive interpolation property.

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