COMPUTATIONAL GENERAL ALGEBRA ON TEN DOLLARS A DAY

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ABSTRACT: Computing power has doubled roughly every 18 months for the last 5 decades, but the amount of computing that is used in universal algebraic research has not increased at a similar rate. In this talk I attempt to estimate what can be computed fairly easily for standard varieties of universal algebras assuming we invest a modest amount of resources (such as ten dollars a day).

Ten years ago I started a list [4] of varieties and quasivarieties that collected some basic information for these classes of mathematical structures. While the project underwent several redesigns, its was limited by the storage format (wiki pages) that made it difficult to use and extend the information within a computer algebra system. A version that is currently under development uses a declarative data format that is human- and machine-readable, and integrates well with web browsers (via JavaScript), automated theorem provers such as Prover9/Mace4 [5] (via first-order logic) and computer packages such as Sage [6] and UACalc [3] (via Python/Jython).

Each (quasi)variety is considered as a category and around 100000 smallest members up to isomorphism are computed. But rather than just concentrating on the objects, we also compute generators for the morphisms between them, which requires computing all maximal subalgebras and all minimal congruences of each algebra. Thus we obtain part of the skeleton of each category, from which one can easily calculate (part of) the HS-poset restricted to the subdirectly-irreducible members. For congruence distributive varieties the finite order ideals of this poset give the lattice of finitely generated subvarieties. Often it is more efficient to move to a dual category in which the objects and morphisms are easier to handle. This is greatly aided by the theory of natural dualities developed by Brian Davey and his collaborators (see e.g. [1, 2]).

In this talk I will describe the format of the database and the information that is being collected for each category. Using a cluster of 40 processors, which costs about $10 a day to run, we estimate the rate at which objects and morphisms in these categories can be computed. Details of recent subprojects will also be presented, such as computing all idempotent semirings up to size 8, all reduced contexts up to size 13, all modular lattices up to size 20 (joint work with Nathan Lawless) and constructing finite algebras with prescribed small congruence lattices (joint work with William DeMeo and Ralph Freese).

REFERENCES

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