

# ALG axioms

**ALG001-0.ax** Abstract algebra axioms, based on Godel set theory

$(\text{associative}(xs, xf) \text{ and } x \in xs \text{ and } y \in xs \text{ and } z \in xs) \Rightarrow \text{apply\_to\_two\_arguments}(xf, \text{apply\_to\_two\_arguments}(xf, x, y), z) = \text{apply\_to\_two\_arguments}(xf, x, \text{apply\_to\_two\_arguments}(xf, y, z))$      $\text{cnf}(\text{associative\_system}_1, \text{axiom})$   
 $\text{associative}(xs, xf) \text{ or } f_{34}(xs, xf) \in xs$      $\text{cnf}(\text{associative\_system}_2, \text{axiom})$   
 $\text{associative}(xs, xf) \text{ or } f_{35}(xs, xf) \in xs$      $\text{cnf}(\text{associative\_system}_3, \text{axiom})$   
 $\text{associative}(xs, xf) \text{ or } f_{36}(xs, xf) \in xs$      $\text{cnf}(\text{associative\_system}_4, \text{axiom})$   
 $\text{apply\_to\_two\_arguments}(xf, \text{apply\_to\_two\_arguments}(xf, f_{34}(xs, xf), f_{35}(xs, xf)), f_{36}(xs, xf)) = \text{apply\_to\_two\_arguments}(xf, f_{34}(xs, xf), f_{35}(xs, xf))$      $\text{cnf}(\text{associative\_system}_5, \text{axiom})$   
 $\text{identity}(xs, xf, xe) \Rightarrow xe \in xs$      $\text{cnf}(\text{identity}_1, \text{axiom})$   
 $(\text{identity}(xs, xf, xe) \text{ and } x \in xs) \Rightarrow \text{apply\_to\_two\_arguments}(xf, xe, x) = x$      $\text{cnf}(\text{identity}_2, \text{axiom})$   
 $(\text{identity}(xs, xf, xe) \text{ and } x \in xs) \Rightarrow \text{apply\_to\_two\_arguments}(xf, x, xe) = x$      $\text{cnf}(\text{identity}_3, \text{axiom})$   
 $xe \in xs \Rightarrow (\text{identity}(xs, xf, xe) \text{ or } f_{37}(xs, xf, xe) \in xs)$      $\text{cnf}(\text{identity}_4, \text{axiom})$   
 $(xe \in xs \text{ and } \text{apply\_to\_two\_arguments}(xf, xe, f_{37}(xs, xf, xe)) = f_{37}(xs, xf, xe) \text{ and } \text{apply\_to\_two\_arguments}(xf, f_{37}(xs, xf, xe), xe) = f_{37}(xs, xf, xe)) \Rightarrow \text{identity}(xs, xf, xe)$      $\text{cnf}(\text{identity}_5, \text{axiom})$   
 $xs' \Rightarrow \text{maps}(xg, xs, xs)$      $\text{cnf}(\text{inverse}_1, \text{axiom})$   
 $(xs' \text{ and } x \in xs) \Rightarrow \text{apply\_to\_two\_arguments}(xf, \text{apply}(xg, x), x) = xe$      $\text{cnf}(\text{inverse}_2, \text{axiom})$   
 $(xs' \text{ and } x \in xs) \Rightarrow \text{apply\_to\_two\_arguments}(xf, x, \text{apply}(xg, x)) = xe$      $\text{cnf}(\text{inverse}_3, \text{axiom})$   
 $\text{maps}(xg, xs, xs) \Rightarrow (xs' \text{ or } f_{38}(xs, xf, xe, xg) \in xs)$      $\text{cnf}(\text{inverse}_4, \text{axiom})$   
 $(\text{maps}(xg, xs, xs) \text{ and } \text{apply\_to\_two\_arguments}(xf, \text{apply}(xg, f_{38}(xs, xf, xe, xg)), f_{38}(xs, xf, xe, xg)) = xe \text{ and } \text{apply\_to\_two\_arguments}(xf, xe, \text{apply}(xg, f_{38}(xs, xf, xe, xg))) = xe) \Rightarrow xs'$      $\text{cnf}(\text{inverse}_5, \text{axiom})$   
 $\text{group}(xs, xf) \Rightarrow \text{closed}(xs, xf)$      $\text{cnf}(\text{group}_1, \text{axiom})$   
 $\text{group}(xs, xf) \Rightarrow \text{associative}(xs, xf)$      $\text{cnf}(\text{group}_2, \text{axiom})$   
 $\text{group}(xs, xf) \Rightarrow \text{identity}(xs, xf, f_{39}(xs, xf))$      $\text{cnf}(\text{group}_3, \text{axiom})$   
 $\text{group}(xs, xf) \Rightarrow xs'$      $\text{cnf}(\text{group}_4, \text{axiom})$   
 $(\text{closed}(xs, xf) \text{ and } \text{associative}(xs, xf) \text{ and } \text{identity}(xs, xf, xe) \text{ and } xs') \Rightarrow \text{group}(xs, xf)$      $\text{cnf}(\text{group}_5, \text{axiom})$   
 $(\text{commutes}(xs, xf) \text{ and } x \in xs \text{ and } y \in xs) \Rightarrow \text{apply\_to\_two\_arguments}(xf, x, y) = \text{apply\_to\_two\_arguments}(xf, y, x)$      $\text{cnf}(\text{commutes}_1, \text{axiom})$   
 $\text{commutes}(xs, xf) \text{ or } f_{41}(xs, xf) \in xs$      $\text{cnf}(\text{commutes}_2, \text{axiom})$   
 $\text{commutes}(xs, xf) \text{ or } f_{42}(xs, xf) \in xs$      $\text{cnf}(\text{commutes}_3, \text{axiom})$   
 $\text{apply\_to\_two\_arguments}(xf, f_{41}(xs, xf), f_{42}(xs, xf)) = \text{apply\_to\_two\_arguments}(xf, f_{42}(xs, xf), f_{41}(xs, xf)) \Rightarrow \text{commutes}(xs, xf)$

**ALG002+0.ax** Median algebra axioms

$\forall x, y: f(x, x, y) = x$      $\text{fof}(\text{majority}, \text{axiom})$   
 $\forall x, y, z: f(x, y, z) = f(z, x, y)$      $\text{fof}(\text{permute}_1, \text{axiom})$   
 $\forall x, y, z: f(x, y, z) = f(x, z, y)$      $\text{fof}(\text{permute}_2, \text{axiom})$   
 $\forall w, x, y, z: f(f(x, w, y), w, z) = f(x, w, f(y, w, z))$      $\text{fof}(\text{associativity}, \text{axiom})$

# ALG problems

**ALG001-1.p** The composition of homomorphisms is a homomorphism

include('Axioms/SET003-0.ax')

$(\text{little\_set}(x) \text{ and } \text{little\_set}(u) \text{ and } \text{ordered\_pair}(x, y) = \text{ordered\_pair}(u, v)) \Rightarrow x = u$      $\text{cnf}(\text{first\_components\_are\_equal}, \text{axiom})$   
 $(\text{little\_set}(x) \text{ and } \text{little\_set}(y) \text{ and } \text{non\_ordered\_pair}(z, x) = \text{non\_ordered\_pair}(z, y)) \Rightarrow x = y$      $\text{cnf}(\text{left\_cancellation}, \text{axiom})$   
 $(\text{little\_set}(x) \text{ and } \text{little\_set}(y) \text{ and } \text{little\_set}(u) \text{ and } \text{little\_set}(v) \text{ and } \text{ordered\_pair}(x, y) = \text{ordered\_pair}(u, v)) \Rightarrow y = v$      $\text{cnf}(\text{second\_components\_are\_equal}, \text{axiom})$   
 $(x \subseteq y \text{ and } y \subseteq x) \Rightarrow x = y$      $\text{cnf}(\text{two\_sets\_equal}, \text{axiom})$   
 $(\text{little\_set}(x) \text{ and } \text{little\_set}(y)) \Rightarrow \text{first}(\text{ordered\_pair}(x, y)) = x$      $\text{cnf}(\text{property\_of\_first}, \text{axiom})$   
 $(\text{little\_set}(x) \text{ and } \text{little\_set}(y)) \Rightarrow \text{second}(\text{ordered\_pair}(x, y)) = y$      $\text{cnf}(\text{property\_of\_second}, \text{axiom})$   
 $\text{ordered\_pair\_predicate}(x) \Rightarrow \text{little\_set}(\text{first}(x))$      $\text{cnf}(\text{first\_component\_is\_small}, \text{axiom})$   
 $\text{ordered\_pair\_predicate}(x) \Rightarrow \text{little\_set}(\text{second}(x))$      $\text{cnf}(\text{second\_component\_is\_small}, \text{axiom})$   
 $\text{little\_set}(x) \Rightarrow x \in \text{singleton\_set}(x)$      $\text{cnf}(\text{property\_of\_singleton\_sets}, \text{axiom})$   
 $\text{little\_set}(\text{ordered\_pair}(x, y))$      $\text{cnf}(\text{ordered\_pairs\_are\_small}_1, \text{axiom})$   
 $\text{ordered\_pair\_predicate}(x) \Rightarrow \text{little\_set}(x)$      $\text{cnf}(\text{ordered\_pairs\_are\_small}_2, \text{axiom})$   
 $(x \subseteq y \text{ and } y \subseteq z) \Rightarrow x \subseteq z$      $\text{cnf}(\text{containment\_is\_transitive}, \text{axiom})$   
 $\text{apply}(xf, y) \subseteq \text{sigma}(\text{image}(\text{singleton\_set}(y), xf))$      $\text{cnf}(\text{image\_and\_apply}_1, \text{axiom})$   
 $\text{image}(\text{singleton\_set}(y), xf) \subseteq \text{apply}(xf, y)$      $\text{cnf}(\text{image\_and\_apply}_2, \text{axiom})$   
 $\text{function}(y) \Rightarrow \text{little\_set}(\text{apply}(y, x))$      $\text{cnf}(\text{function\_values\_are\_small}, \text{axiom})$   
 $\text{relation}(y \circ x)$      $\text{cnf}(\text{composition\_is\_a\_relation}, \text{axiom})$

$\text{range\_of}(y \circ x) \subseteq \text{range\_of}(y)$      $\text{cnf}(\text{range\_of\_composition}, \text{axiom})$   
 $\text{range\_of}(x) \subseteq \text{domain\_of}(y) \Rightarrow \text{domain\_of}(x) = \text{domain\_of}(y \circ x)$      $\text{cnf}(\text{domain\_of\_composition}, \text{axiom})$   
 $(\text{function}(x) \text{ and } \text{function}(y)) \Rightarrow \text{function}(y \circ x)$      $\text{cnf}(\text{composition\_is\_a\_function}, \text{axiom})$   
 $(\text{maps}(x, u, v) \text{ and } \text{maps}(x, v, w)) \Rightarrow \text{maps}(x, u, w)$      $\text{cnf}(\text{maps\_for\_composition}, \text{axiom})$   
 $(\text{little\_set}(x) \text{ and } \text{little\_set}(y) \text{ and } \text{function}(x, y) \in \text{xf}) \Rightarrow \text{apply}(x, y) = y$      $\text{cnf}(\text{apply\_for\_functions}_1, \text{axiom})$   
 $(\text{function}(x) \text{ and } x \in \text{domain\_of}(x) \text{ and } \text{apply}(x, x) = y) \Rightarrow \text{ordered\_pair}(x, y) \in \text{xf}$      $\text{cnf}(\text{apply\_for\_functions}_2, \text{axiom})$   
 $(\text{maps}(x, y, z) \text{ and } x \in \text{domain\_of}(x)) \Rightarrow \text{apply}(x, y) \in z$      $\text{cnf}(\text{apply\_for\_functions}_3, \text{axiom})$   
 $(\text{function}(x) \text{ and } x \in \text{domain\_of}(x)) \Rightarrow \text{apply}(x, \text{apply}(x, y)) \subseteq \text{apply}(x, y)$      $\text{cnf}(\text{apply\_for\_composition}_1, \text{axiom})$   
 $\text{function}(x) \Rightarrow \text{apply}(x \circ x, x) \subseteq \text{apply}(x, \text{apply}(x, x))$      $\text{cnf}(\text{apply\_for\_composition}_2, \text{axiom})$   
 $(\text{function}(x) \text{ and } x \in \text{domain\_of}(x)) \Rightarrow \text{apply}(x, \text{apply}(x, y)) = \text{apply}(x \circ x, y)$      $\text{cnf}(\text{apply\_for\_composition}_3, \text{axiom})$   
 $(x \in \text{xs}_1 \text{ and } y \in \text{xs}_2) \Rightarrow \text{ordered\_pair}(x, y) \in \text{cross\_product}(\text{xs}_1, \text{xs}_2)$      $\text{cnf}(\text{ordered\_pair\_in\_cross\_product}, \text{axiom})$   
 $\text{homomorphism}(f_{60}, f_{62}, f_{63}, f_{64}, f_{65})$      $\text{cnf}(\text{one\_homomorphism}, \text{hypothesis})$   
 $\text{homomorphism}(f_{61}, f_{64}, f_{65}, f_{66}, f_{67})$      $\text{cnf}(\text{another\_homomorphism}, \text{hypothesis})$   
 $\neg \text{homomorphism}(f_{60} \circ f_{61}, f_{62}, f_{63}, f_{66}, f_{67})$      $\text{cnf}(\text{prove\_composition\_is\_a\_homomorphism}, \text{negated\_conjecture})$

**ALG001-2.p** The composition of homomorphisms is a homomorphism

include('Axioms/SET003-0.ax')

$\text{homomorphism}(f_{60}, f_{62}, f_{63}, f_{64}, f_{65})$      $\text{cnf}(\text{one\_homomorphism}, \text{hypothesis})$   
 $\text{homomorphism}(f_{61}, f_{64}, f_{65}, f_{66}, f_{67})$      $\text{cnf}(\text{another\_homomorphism}, \text{hypothesis})$   
 $\neg \text{homomorphism}(f_{60} \circ f_{61}, f_{62}, f_{63}, f_{66}, f_{67})$      $\text{cnf}(\text{prove\_composition\_is\_a\_homomorphism}, \text{negated\_conjecture})$

**ALG001-3.p** The composition of homomorphisms is a homomorphism

include('Axioms/SET004-0.ax')

include('Axioms/SET004-1.ax')

$\text{homomorphism}(x_1, x_2, x_3)$      $\text{cnf}(\text{prove\_composition\_of\_homomorphisms}_1, \text{negated\_conjecture})$   
 $\text{homomorphism}(x_2, x_3, x_4)$      $\text{cnf}(\text{prove\_composition\_of\_homomorphisms}_2, \text{negated\_conjecture})$   
 $\neg \text{homomorphism}(x_2 \circ x_1, x_3, x_4)$      $\text{cnf}(\text{prove\_composition\_of\_homomorphisms}_3, \text{negated\_conjecture})$

**ALG001^5.p** TPS problem THM133

The composition of homomorphisms of binary operators is a homomorphisms. Boyer et al JAR 2 page 284.

$g: \text{\$tType} \quad \text{thf}(g\_type, \text{type})$

$b: \text{\$tType} \quad \text{thf}(b\_type, \text{type})$

$a: \text{\$tType} \quad \text{thf}(a\_type, \text{type})$

$\forall x_1: g \rightarrow b, x_2: b \rightarrow a, x_3: g \rightarrow g \rightarrow g, x_4: b \rightarrow b \rightarrow b, x_5: a \rightarrow a \rightarrow a: ((\forall x: g, y: g: (x_1 @ (x_3 @ x @ y)) =$   
 $(x_2 @ (x_1 @ x)) @ (x_1 @ y)) \text{ and } \forall x: b, y: b: (x_2 @ (x_4 @ x @ y)) = (x_3 @ (x_2 @ x) @ (x_2 @ y))) \Rightarrow \forall x: g, y: g: (x_2 @ (x_5 @$   
 $(x_3 @ (x_2 @ (x_1 @ x)) @ (x_2 @ (x_1 @ y)))) \quad \text{thf}(\text{cTHM133\_pme}, \text{conjecture})$

**ALG002-1.p** In an ordered field, if  $X > 0$  then  $X^{-1} > 0$

$x \cdot 1 = x$      $\text{cnf}(\text{right\_identity}, \text{axiom})$

$\neg 1 \cdot 1 = 0$      $\text{cnf}(\text{not\_abelian}, \text{axiom})$

$x \cdot y = z \Rightarrow (-x) \cdot (-y) = z$      $\text{cnf}(\text{product\_of\_inverses}_1, \text{axiom})$

$(-x) \cdot (-y) = z \Rightarrow x \cdot y = z$      $\text{cnf}(\text{product\_of\_inverses}_2, \text{axiom})$

$x \cdot y = z \Rightarrow x \cdot (-y) = -z$      $\text{cnf}(\text{product\_to\_inverse}, \text{axiom})$

$x \cdot x^{-1} = 1$  or  $x \cdot x = 0$      $\text{cnf}(\text{inverse\_and\_identity}, \text{axiom})$

$\text{greater\_than}_0(x) \Rightarrow \neg \text{greater\_than}_0(-x)$      $\text{cnf}(\text{inverse\_greater\_than}_0, \text{axiom})$

$\text{greater\_than}_0(x) \Rightarrow \neg x \cdot x = 0$      $\text{cnf}(\text{greater\_than}_0\_square, \text{axiom})$

$x \cdot y = z \Rightarrow y \cdot x = z$      $\text{cnf}(\text{commutativity\_of\_product}, \text{axiom})$

$\text{greater\_than}_0(x)$  or  $x \cdot x = 0$  or  $\text{greater\_than}_0(-x)$      $\text{cnf}(\text{product\_and\_inverse}, \text{axiom})$

$(y \cdot z = x \text{ and } y \cdot y = 0) \Rightarrow x \cdot x = 0$      $\text{cnf}(\text{square\_to}_0, \text{axiom})$

$(y \cdot z = x \text{ and } \text{greater\_than}_0(y) \text{ and } \text{greater\_than}_0(z)) \Rightarrow \text{greater\_than}_0(x)$      $\text{cnf}(\text{product\_and\_greater\_than}_0, \text{axiom})$

$\text{greater\_than}_0(a)$      $\text{cnf}(\text{a\_greater\_than}_0, \text{hypothesis})$

$\neg \text{greater\_than}_0(a^{-1})$      $\text{cnf}(\text{prove\_a\_inverse\_greater\_than}_0, \text{negated\_conjecture})$

**ALG003-1.p** Cancellative medial algebras

We prove a property of cancellative medial algebras.

$(x \cdot y = z \text{ and } u \cdot y = z) \Rightarrow x = u$      $\text{cnf}(\text{left\_cancellation}, \text{axiom})$

$(x \cdot y = z \text{ and } x \cdot u = z) \Rightarrow y = u$      $\text{cnf}(\text{right\_cancellation}, \text{axiom})$

$(x \cdot y) \cdot (z \cdot u) = (x \cdot z) \cdot (y \cdot u)$      $\text{cnf}(\text{medial\_law}, \text{axiom})$

$\text{an\_element} \cdot \text{an\_element} = \text{an\_element}$      $\text{cnf}(\text{idempotent\_element}, \text{hypothesis})$

$(a \cdot (d \cdot c)) \cdot ((b \cdot e) \cdot f) \neq (a \cdot (b \cdot c)) \cdot ((d \cdot e) \cdot f)$      $\text{cnf}(\text{prove\_this}, \text{negated\_conjecture})$

**ALG004-1.p** Cancellative medial algebras satisfy the quotient condition.

$(x \cdot y = z \text{ and } u \cdot y = z) \Rightarrow x = u$      $\text{cnf}(\text{left\_cancellation}, \text{axiom})$

$(x \cdot y = z \text{ and } x \cdot u = z) \Rightarrow y = u$     cnf(right\_cancellation, axiom)  
 $(x \cdot y) \cdot (z \cdot u) = (x \cdot z) \cdot (y \cdot u)$     cnf(medial\_law, axiom)  
 $b \cdot b_0 = a \cdot a_0$     cnf(prove\_quotient\_condition\_1, negated\_conjecture)  
 $d \cdot b_0 = c \cdot a_0$     cnf(prove\_quotient\_condition\_2, negated\_conjecture)  
 $b \cdot d_0 = a \cdot c_0$     cnf(prove\_quotient\_condition\_3, negated\_conjecture)  
 $d \cdot d_0 \neq c \cdot c_0$     cnf(prove\_quotient\_condition\_4, negated\_conjecture)

**ALG005-1.p** Associativity of intersection in terms of set difference.

Starting with Kalman's basis for families of sets closed under set difference, we define intersection and show it to be associative.

$x \setminus (y \setminus x) = x$     cnf(set\_difference\_1, axiom)  
 $x \setminus (x \setminus y) = y \setminus (y \setminus x)$     cnf(set\_difference\_2, axiom)  
 $(x \setminus y) \setminus z = (x \setminus z) \setminus (y \setminus z)$     cnf(set\_difference\_3, axiom)  
 $x \cdot y = x \setminus (x \setminus y)$     cnf(intersection, axiom)  
 $(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$     cnf(prove\_associativity\_of\_multiply, negated\_conjecture)

**ALG006-1.p** Simplification of Kalman's set difference basis (part 1)

This is part 1 of a proof that one of the axioms in Kalman's basis for set difference can be simplified.

$x \setminus (y \setminus x) = x$     cnf(set\_difference\_1, axiom)  
 $x \setminus (x \setminus y) = y \setminus (y \setminus x)$     cnf(set\_difference\_2, axiom)  
 $(x \setminus y) \setminus z = (x \setminus z) \setminus (y \setminus z)$     cnf(set\_difference\_3, axiom)  
 $(a \setminus c) \setminus b \neq (a \setminus b) \setminus c$     cnf(prove\_set\_difference\_3\_simplified, negated\_conjecture)

**ALG007-1.p** Simplification of Kalman's set difference basis (part 2)

This is part 2 of a proof that one of the axioms in Kalman's basis for set difference can be simplified.

$x \setminus (y \setminus x) = x$     cnf(set\_difference\_1, axiom)  
 $x \setminus (x \setminus y) = y \setminus (y \setminus x)$     cnf(set\_difference\_2, axiom)  
 $(x \setminus y) \setminus z = (x \setminus z) \setminus y$     cnf(set\_difference\_3\_simplified, axiom)  
 $(a \setminus b) \setminus c \neq (a \setminus c) \setminus (b \setminus c)$     cnf(prove\_set\_difference\_3, negated\_conjecture)

**ALG008-1.p** TC + right identity does not give RC.

An algebra with a right identity satisfying the Thomsen Closure (RC) condition does not necessarily satisfy the Reidemeister Closure (RC) condition.

$(x \cdot y = z \text{ and } u \cdot v = z \text{ and } x \cdot w = v_6 \text{ and } v_7 \cdot v = v_6) \Rightarrow u \cdot w = v_7 \cdot y$     cnf(thomsen\_closure, axiom)  
 $x \cdot \text{identity} = x$     cnf(right\_identity, axiom)  
 $c_4 \cdot a = c_3 \cdot b$     cnf(prove\_reidemeister\_1, negated\_conjecture)  
 $c_2 \cdot a = c_1 \cdot b$     cnf(prove\_reidemeister\_2, negated\_conjecture)  
 $c_4 \cdot f = c_3 \cdot \text{identity}$     cnf(prove\_reidemeister\_3, negated\_conjecture)  
 $c_2 \cdot f \neq c_1 \cdot \text{identity}$     cnf(prove\_reidemeister\_4, negated\_conjecture)

**ALG009-1.p** Abstract algebra axioms, based on Godel set theory

include('Axioms/ALG001-0.ax')

include('Axioms/SET003-0.ax')

**ALG010-1.p** Prove J follows from HBACK

Axioms for the quasivariety HBACK are given below. Show that equation J follows.

$a \cdot 1 = 1$     cnf(m\_3, axiom)  
 $1 \cdot a = a$     cnf(m\_4, axiom)  
 $(a \cdot b) \cdot ((c \cdot a) \cdot (c \cdot b)) = 1$     cnf(m\_5, axiom)  
 $(a \cdot b = 1 \text{ and } b \cdot a = 1) \Rightarrow a = b$     cnf(m\_7, axiom)  
 $a \cdot a = 1$     cnf(m\_8, axiom)  
 $a \cdot (b \cdot c) = b \cdot (a \cdot c)$     cnf(m\_9, axiom)  
 $(a \cdot b) \cdot (a \cdot c) = (b \cdot a) \cdot (b \cdot c)$     cnf(h, axiom)  
 $((a \cdot b) \cdot b) \cdot a \cdot a \neq ((b \cdot a) \cdot a) \cdot b \cdot b$     cnf(prove\_j, negated\_conjecture)

**ALG011-1.p** Partition a monoid into 2 partitions

If C,D is a partition of a monoid M, we cannot have  $C * C \subset D$  and  $D * D \subset C$ .

$f(x, f(y, z)) = f(f(x, y), z)$     cnf(f\_is\_associative, axiom)  
 $c(x) \text{ or } d(x)$     cnf(partitions\_union, axiom)  
 $c(x) \Rightarrow \neg d(x)$     cnf(partitions\_exclusive, hypothesis)  
 $c(a_1)$     cnf(partition\_c\_not\_empty, hypothesis)  
 $d(a_2)$     cnf(partition\_d\_not\_empty, hypothesis)  
 $(c(x) \text{ and } c(y)) \Rightarrow d(f(x, y))$     cnf(conjecture\_1, negated\_conjecture)

$(d(x) \text{ and } d(y)) \Rightarrow c(f(x, y))$     cnf(conjecture<sub>2</sub>, negated\_conjecture)

**ALG012-1.p** Partition a monoid into 3 partitions

If C,D1,D2 is a partition of a monoid M, we cannot have  $C * C$  subset  $D1 \cup D2$  and  $Dj * Dj$  subset C.

$f(x, f(y, z)) = f(f(x, y), z)$     cnf(f\_is\_associative, axiom)

$c(x) \text{ or } d_1(x) \text{ or } d_2(x)$     cnf(partitions\_union, axiom)

$c(x) \Rightarrow \neg d_1(x)$     cnf(partitions\_exclusive\_c\_d1, hypothesis)

$c(x) \Rightarrow \neg d_2(x)$     cnf(partitions\_exclusive\_c\_d2, hypothesis)

$d_1(x) \Rightarrow \neg d_2(x)$     cnf(partitions\_exclusive\_d1\_d2, hypothesis)

$c(a_1)$     cnf(partition\_c\_not\_empty, hypothesis)

$d_1(a_2)$     cnf(partition\_d1\_not\_empty, hypothesis)

$d_2(a_3)$     cnf(partition\_d2\_not\_empty, hypothesis)

$(c(x) \text{ and } c(y)) \Rightarrow (d_2(f(x, y)) \text{ or } d_1(f(x, y)))$     cnf(conjecture<sub>1</sub>, negated\_conjecture)

$(d_1(x) \text{ and } d_1(y)) \Rightarrow c(f(x, y))$     cnf(conjecture<sub>2</sub>, negated\_conjecture)

$(d_2(x) \text{ and } d_2(y)) \Rightarrow c(f(x, y))$     cnf(conjecture<sub>3</sub>, negated\_conjecture)

**ALG013-1.p** Partition a monoid into 4 partitions

If C1,C2,D1,D2 is a partition of a monoid M, we cannot have  $Ci * Ci$  subset  $D1 \cup D2$  and  $Dj * Dj$  subset  $C1 \cup C2$ .

$f(x, f(y, z)) = f(f(x, y), z)$     cnf(f\_is\_associative, axiom)

$c_2(x) \text{ or } c_1(x) \text{ or } d_1(x) \text{ or } d_2(x)$     cnf(partitions\_union, axiom)

$c_1(x) \Rightarrow \neg c_2(x)$     cnf(partitions\_exclusive\_c1\_c2, hypothesis)

$c_1(x) \Rightarrow \neg d_1(x)$     cnf(partitions\_exclusive\_c1\_d1, hypothesis)

$c_1(x) \Rightarrow \neg d_2(x)$     cnf(partitions\_exclusive\_c1\_d2, hypothesis)

$c_2(x) \Rightarrow \neg d_1(x)$     cnf(partitions\_exclusive\_c2\_d1, hypothesis)

$c_2(x) \Rightarrow \neg d_2(x)$     cnf(partitions\_exclusive\_c2\_d2, hypothesis)

$d_1(x) \Rightarrow \neg d_2(x)$     cnf(partitions\_exclusive\_d1\_d2, hypothesis)

$c_1(a_1)$     cnf(partition\_c1\_not\_empty, hypothesis)

$c_2(a_2)$     cnf(partition\_c2\_not\_empty, hypothesis)

$d_1(a_3)$     cnf(partition\_d1\_not\_empty, hypothesis)

$d_2(a_4)$     cnf(partition\_d2\_not\_empty, hypothesis)

$(c_1(x) \text{ and } c_1(y)) \Rightarrow (d_2(f(x, y)) \text{ or } d_1(f(x, y)))$     cnf(conjecture<sub>1</sub>, negated\_conjecture)

$(c_2(x) \text{ and } c_2(y)) \Rightarrow (d_2(f(x, y)) \text{ or } d_1(f(x, y)))$     cnf(conjecture<sub>2</sub>, negated\_conjecture)

$(d_1(x) \text{ and } d_1(y)) \Rightarrow (c_2(f(x, y)) \text{ or } c_1(f(x, y)))$     cnf(conjecture<sub>3</sub>, negated\_conjecture)

$(d_2(x) \text{ and } d_2(y)) \Rightarrow (c_2(f(x, y)) \text{ or } c_1(f(x, y)))$     cnf(conjecture<sub>4</sub>, negated\_conjecture)

**ALG018+1.p** Groups 4: CPROPS-SORTED-DISCRIMINANT-PROBLEM-1

$\forall u: (\text{sorti}_1(u) \Rightarrow \forall v: (\text{sorti}_1(v) \Rightarrow \text{sorti}_1(\text{op}_1(u, v))))$     fof(ax<sub>1</sub>, axiom)

$\forall u: (\text{sorti}_2(u) \Rightarrow \forall v: (\text{sorti}_2(v) \Rightarrow \text{sorti}_2(\text{op}_2(u, v))))$     fof(ax<sub>2</sub>, axiom)

$\exists u: (\text{sorti}_1(u) \text{ and } \forall v: (\text{sorti}_1(v) \Rightarrow \text{op}_1(v, v) = u))$     fof(ax<sub>3</sub>, axiom)

$\neg \exists u: (\text{sorti}_2(u) \text{ and } \forall v: (\text{sorti}_2(v) \Rightarrow \text{op}_2(v, v) = u))$     fof(ax<sub>4</sub>, axiom)

$(\forall u: (\text{sorti}_1(u) \Rightarrow \text{sorti}_2(h(u))) \text{ and } \forall v: (\text{sorti}_2(v) \Rightarrow \text{sorti}_1(j(v)))) \Rightarrow \neg \forall w: (\text{sorti}_1(w) \Rightarrow \forall x: (\text{sorti}_1(x) \Rightarrow$

$h(\text{op}_1(w, x) = \text{op}_2(h(w), h(x)))) \text{ and } \forall y: (\text{sorti}_2(y) \Rightarrow \forall z: (\text{sorti}_2(z) \Rightarrow j(\text{op}_2(y, z)) = \text{op}_1(j(y), j(z)))) \text{ and } \forall x_1: (\text{sorti}_2(x_1)$

$h(j(x_1)) = x_1) \text{ and } \forall x_2: (\text{sorti}_1(x_2) \Rightarrow j(h(x_2)) = x_2)$     fof(co<sub>1</sub>, conjecture)

**ALG019+1.p** Groups 4: CPROPS-SORTED-DISCRIMINANT-PROBLEM-2

$\forall u: (\text{sorti}_1(u) \Rightarrow \forall v: (\text{sorti}_1(v) \Rightarrow \text{sorti}_1(\text{op}_1(u, v))))$     fof(ax<sub>1</sub>, axiom)

$\forall u: (\text{sorti}_2(u) \Rightarrow \forall v: (\text{sorti}_2(v) \Rightarrow \text{sorti}_2(\text{op}_2(u, v))))$     fof(ax<sub>2</sub>, axiom)

$\neg \exists u: (\text{sorti}_1(u) \text{ and } \forall v: (\text{sorti}_1(v) \Rightarrow \text{op}_1(v, v) = u))$     fof(ax<sub>3</sub>, axiom)

$\neg \neg \exists u: (\text{sorti}_2(u) \text{ and } \forall v: (\text{sorti}_2(v) \Rightarrow \text{op}_2(v, v) = u))$     fof(ax<sub>4</sub>, axiom)

$(\forall u: (\text{sorti}_1(u) \Rightarrow \text{sorti}_2(h(u))) \text{ and } \forall v: (\text{sorti}_2(v) \Rightarrow \text{sorti}_1(j(v)))) \Rightarrow \neg \forall w: (\text{sorti}_1(w) \Rightarrow \forall x: (\text{sorti}_1(x) \Rightarrow$

$h(\text{op}_1(w, x) = \text{op}_2(h(w), h(x)))) \text{ and } \forall y: (\text{sorti}_2(y) \Rightarrow \forall z: (\text{sorti}_2(z) \Rightarrow j(\text{op}_2(y, z)) = \text{op}_1(j(y), j(z)))) \text{ and } \forall x_1: (\text{sorti}_2(x_1)$

$h(j(x_1)) = x_1) \text{ and } \forall x_2: (\text{sorti}_1(x_2) \Rightarrow j(h(x_2)) = x_2)$     fof(co<sub>1</sub>, conjecture)

**ALG029+1.p** Groups 6: CPROPS-SORTED-DISCRIMINANT-PROBLEM-1

$\forall u: (\text{sorti}_1(u) \Rightarrow \forall v: (\text{sorti}_1(v) \Rightarrow \text{sorti}_1(\text{op}_1(u, v))))$     fof(ax<sub>1</sub>, axiom)

$\forall u: (\text{sorti}_2(u) \Rightarrow \forall v: (\text{sorti}_2(v) \Rightarrow \text{sorti}_2(\text{op}_2(u, v))))$     fof(ax<sub>2</sub>, axiom)

$\forall u: (\text{sorti}_1(u) \Rightarrow \forall v: (\text{sorti}_1(v) \Rightarrow \text{op}_1(u, v) = \text{op}_1(v, u)))$     fof(ax<sub>3</sub>, axiom)

$\neg \forall u: (\text{sorti}_2(u) \Rightarrow \forall v: (\text{sorti}_2(v) \Rightarrow \text{op}_2(u, v) = \text{op}_2(v, u)))$     fof(ax<sub>4</sub>, axiom)

$(\forall u: (\text{sorti}_1(u) \Rightarrow \text{sorti}_2(h(u))) \text{ and } \forall v: (\text{sorti}_2(v) \Rightarrow \text{sorti}_1(j(v)))) \Rightarrow \neg \forall w: (\text{sorti}_1(w) \Rightarrow \forall x: (\text{sorti}_1(x) \Rightarrow$

$h(\text{op}_1(w, x) = \text{op}_2(h(w), h(x)))) \text{ and } \forall y: (\text{sorti}_2(y) \Rightarrow \forall z: (\text{sorti}_2(z) \Rightarrow j(\text{op}_2(y, z)) = \text{op}_1(j(y), j(z)))) \text{ and } \forall x_1: (\text{sorti}_2(x_1)$

$h(j(x_1)) = x_1) \text{ and } \forall x_2: (\text{sorti}_1(x_2) \Rightarrow j(h(x_2)) = x_2)$     fof(co<sub>1</sub>, conjecture)

**ALG030+1.p** Groups 6: CPROPS-SORTED-DISCRIMINANT-PROBLEM-2





$(\forall u: (\text{sort}_1(u) \Rightarrow \text{sort}_2(h(u))) \text{ and } \forall v: (\text{sort}_2(v) \Rightarrow \text{sort}_1(j(v)))) \Rightarrow \neg \forall w: (\text{sort}_1(w) \Rightarrow \forall x: (\text{sort}_1(x) \Rightarrow h(\text{op}_1(w, x) = \text{op}_2(h(w), h(x)))) \text{ and } \forall y: (\text{sort}_2(y) \Rightarrow \forall z: (\text{sort}_2(z) \Rightarrow j(\text{op}_2(y, z) = \text{op}_1(j(y), j(z)))) \text{ and } \forall x_1: (\text{sort}_2(x_1) \Rightarrow h(j(x_1)) = x_1) \text{ and } \forall x_2: (\text{sort}_1(x_2) \Rightarrow j(h(x_2)) = x_2) \quad \text{fof}(\text{co}_1, \text{conjecture})$

**ALG202+1.p** Quasigroups 7 QG5: CPROPS-SORTED-DISCRIMINANT-PROBLEM-2

$\forall u: (\text{sort}_1(u) \Rightarrow \forall v: (\text{sort}_1(v) \Rightarrow \text{sort}_1(\text{op}_1(u, v)))) \quad \text{fof}(\text{ax}_1, \text{axiom})$

$\forall u: (\text{sort}_2(u) \Rightarrow \forall v: (\text{sort}_2(v) \Rightarrow \text{sort}_2(\text{op}_2(u, v)))) \quad \text{fof}(\text{ax}_2, \text{axiom})$

$\forall u: (\text{sort}_1(u) \Rightarrow \text{op}_1(u, u) = u) \quad \text{fof}(\text{ax}_3, \text{axiom})$

$\neg \forall u: (\text{sort}_2(u) \Rightarrow \text{op}_2(u, u) = u) \quad \text{fof}(\text{ax}_4, \text{axiom})$

$(\forall u: (\text{sort}_1(u) \Rightarrow \text{sort}_2(h(u))) \text{ and } \forall v: (\text{sort}_2(v) \Rightarrow \text{sort}_1(j(v)))) \Rightarrow \neg \forall w: (\text{sort}_1(w) \Rightarrow \forall x: (\text{sort}_1(x) \Rightarrow h(\text{op}_1(w, x) = \text{op}_2(h(w), h(x)))) \text{ and } \forall y: (\text{sort}_2(y) \Rightarrow \forall z: (\text{sort}_2(z) \Rightarrow j(\text{op}_2(y, z) = \text{op}_1(j(y), j(z)))) \text{ and } \forall x_1: (\text{sort}_2(x_1) \Rightarrow h(j(x_1)) = x_1) \text{ and } \forall x_2: (\text{sort}_1(x_2) \Rightarrow j(h(x_2)) = x_2) \quad \text{fof}(\text{co}_1, \text{conjecture})$

**ALG203+1.p** Quasigroups 7 QG5: CPROPS-SORTED-DISCRIMINANT-PROBLEM-3

$\forall u: (\text{sort}_1(u) \Rightarrow \forall v: (\text{sort}_1(v) \Rightarrow \text{sort}_1(\text{op}_1(u, v)))) \quad \text{fof}(\text{ax}_1, \text{axiom})$

$\forall u: (\text{sort}_2(u) \Rightarrow \forall v: (\text{sort}_2(v) \Rightarrow \text{sort}_2(\text{op}_2(u, v)))) \quad \text{fof}(\text{ax}_2, \text{axiom})$

$\exists u: (\text{sort}_1(u) \text{ and } \text{op}_1(u, u) = u) \text{ and } \exists v: (\text{sort}_1(v) \text{ and } \text{op}_1(v, v) \neq v) \quad \text{fof}(\text{ax}_3, \text{axiom})$

$\neg \exists u: (\text{sort}_2(u) \text{ and } \text{op}_2(u, u) = u) \text{ and } \exists v: (\text{sort}_2(v) \text{ and } \text{op}_2(v, v) \neq v) \quad \text{fof}(\text{ax}_4, \text{axiom})$

$(\forall u: (\text{sort}_1(u) \Rightarrow \text{sort}_2(h(u))) \text{ and } \forall v: (\text{sort}_2(v) \Rightarrow \text{sort}_1(j(v)))) \Rightarrow \neg \forall w: (\text{sort}_1(w) \Rightarrow \forall x: (\text{sort}_1(x) \Rightarrow h(\text{op}_1(w, x) = \text{op}_2(h(w), h(x)))) \text{ and } \forall y: (\text{sort}_2(y) \Rightarrow \forall z: (\text{sort}_2(z) \Rightarrow j(\text{op}_2(y, z) = \text{op}_1(j(y), j(z)))) \text{ and } \forall x_1: (\text{sort}_2(x_1) \Rightarrow h(j(x_1)) = x_1) \text{ and } \forall x_2: (\text{sort}_1(x_2) \Rightarrow j(h(x_2)) = x_2) \quad \text{fof}(\text{co}_1, \text{conjecture})$

**ALG210+1.p** Star-algebras are closed under multiplication

$\forall a, b, c: \text{times}(\text{times}(a, b), c) = \text{times}(b, \text{times}(c, a)) \quad \text{fof}(\text{axiom}_1, \text{axiom})$

$\forall b: (\text{element}(b) \iff \exists c: (\text{times}(b, c) = b \text{ and } \text{times}(b, b) = c)) \quad \text{fof}(\text{axiom}_2, \text{axiom})$

$\forall a, b: ((\text{element}(a) \text{ and } \text{element}(b)) \Rightarrow \text{element}(\text{times}(a, b))) \quad \text{fof}(\text{conjecture}_1, \text{conjecture})$

**ALG210+2.p** Star-algebras are closed under multiplication

$\forall a, b, c: \text{times}(\text{times}(a, b), c) = \text{times}(b, \text{times}(c, a)) \quad \text{fof}(\text{axiom}_1, \text{axiom})$

$\forall b: (\text{element}(b) \iff \exists c: (\text{times}(b, c) = b \text{ and } \text{times}(b, b) = c)) \quad \text{fof}(\text{axiom}_2, \text{axiom})$

$\forall a, b, c: ((\text{element}(a) \text{ and } \text{element}(b) \text{ and } c = \text{times}(a, b)) \Rightarrow \text{element}(c)) \quad \text{fof}(\text{conjecture}_1, \text{conjecture})$

**ALG211+1.p** Vector spaces and bases

$\forall b, v: (\text{basis\_of}(b, v) \Rightarrow (\text{lin\_ind\_subset}(b, v) \text{ and } \text{a\_subset\_of}(b, \text{vec\_to\_class}(v)))) \quad \text{fof}(\text{basis\_of}, \text{axiom})$

$\forall s, t, v: ((\text{lin\_ind\_subset}(s, v) \text{ and } \text{basis\_of}(t, v)) \Rightarrow \exists u: (\text{a\_subset\_of}(u, t) \text{ and } \text{basis\_of}(\text{union}(s, u), v))) \quad \text{fof}(\text{bg\_2\_2\_5}, \text{axiom})$

$\forall a: (\text{a\_vector\_space}(a) \Rightarrow \exists b: \text{basis\_of}(b, a)) \quad \text{fof}(\text{bg\_remark\_63\_a}, \text{axiom})$

$\forall a, b: (\text{a\_vector\_subspace\_of}(a, b) \Rightarrow \text{a\_vector\_space}(a)) \quad \text{fof}(\text{bg\_2\_4\_a}, \text{axiom})$

$\forall w, v, e: ((\text{a\_vector\_subspace\_of}(w, v) \text{ and } \text{a\_subset\_of}(e, \text{vec\_to\_class}(w))) \Rightarrow (\text{lin\_ind\_subset}(e, w) \iff \text{lin\_ind\_subset}(e, v)))$

$\forall w, v: ((\text{a\_vector\_subspace\_of}(w, v) \text{ and } \text{a\_vector\_space}(v)) \Rightarrow \exists e, f: (\text{basis\_of}(\text{union}(e, f), v) \text{ and } \text{basis\_of}(e, w))) \quad \text{fof}(\text{bg\_2\_}$

**ALG212+1.p** Distributivity, long version

include('Axioms/ALG002+0.ax')

$\forall u, w, x, y, z: f(f(x, y, z), u, w) = f(f(x, u, w), f(y, u, w), f(z, u, w)) \quad \text{fof}(\text{dist\_long}, \text{conjecture})$

**ALG213+1.p** Distributivity, short version

include('Axioms/ALG002+0.ax')

$\forall u, w, x, y, z: f(f(x, y, z), u, w) = f(x, f(y, u, w), f(z, u, w)) \quad \text{fof}(\text{dist\_long}, \text{conjecture})$

**ALG235-1.p** Short equational base for two varieties of groupoids - part 1a

$a \cdot (b \cdot (a \cdot b)) = a \cdot b \quad \text{cnf}(c_{01}, \text{axiom})$

$a \cdot (b \cdot (c \cdot d)) = c \cdot (b \cdot (a \cdot d)) \quad \text{cnf}(c_{02}, \text{axiom})$

$(a \cdot (b \cdot (c \cdot b))) \cdot d = a \cdot (d \cdot ((c \cdot b) \cdot d)) \quad \text{cnf}(c_{03}, \text{axiom})$

$a \cdot (b \cdot (a \cdot (c \cdot (d \cdot c)))) \neq a \cdot (b \cdot (d \cdot c)) \quad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

**ALG236-1.p** Short equational base for two varieties of groupoids - part 1b

$a \cdot (b \cdot (a \cdot b)) = a \cdot b \quad \text{cnf}(c_{01}, \text{axiom})$

$a \cdot (b \cdot (c \cdot d)) = c \cdot (b \cdot (a \cdot d)) \quad \text{cnf}(c_{02}, \text{axiom})$

$(a \cdot (b \cdot (c \cdot b))) \cdot d = a \cdot (d \cdot ((c \cdot b) \cdot d)) \quad \text{cnf}(c_{03}, \text{axiom})$

$(a \cdot b) \cdot (c \cdot (d \cdot e)) \neq a \cdot (c \cdot ((d \cdot b) \cdot e)) \quad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

**ALG237-1.p** Selfdistributive groupoids are symmetric-by-medial - part 1

$a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c) \quad \text{cnf}(c_{01}, \text{axiom})$

$(a \cdot b) \cdot c = (a \cdot c) \cdot (b \cdot c) \quad \text{cnf}(c_{02}, \text{axiom})$

$((a \cdot b) \cdot (c \cdot d)) \cdot ((a \cdot c) \cdot (b \cdot d)) \neq ((a \cdot c) \cdot (b \cdot d)) \cdot ((a \cdot b) \cdot (c \cdot d)) \quad \text{cnf}(\text{goals}, \text{negated\_conjecture})$

**ALG238-1.p** Selfdistributive groupoids are symmetric-by-medial - part 2





$tptp_2(tptp_2(a)) = a$        $cnf(c_{12}, axiom)$   
 $n(tptp_0(a, b)) = n(tptp_0(tptp_2(b), tptp_2(a)))$        $cnf(c_{13}, axiom)$   
 $v(tptp_1(n(a), n(tptp_0(a, b))), n(b)) \neq n(b)$        $cnf(goals, negated\_conjecture)$

**ALG247^2.p** Push property lemma 0  
include('Axioms/ALG003^0.ax')  
pushprop\_lem0.lthm      thf(thm, conjecture)

**ALG248^1.p** Push property lemma 1  
include('Axioms/ALG003^0.ax')  
pushprop\_lem1.gthm      thf(thm, conjecture)

**ALG248^2.p** Push property lemma 1  
include('Axioms/ALG003^0.ax')  
pushprop\_lem1.lthm      thf(thm, conjecture)

**ALG248^3.p** Push property lemma 1  
include('Axioms/ALG003^0.ax')  
pushprop\_lem1v2.lthm      thf(thm, conjecture)

**ALG249^3.p** Push property lemma 2  
include('Axioms/ALG003^0.ax')  
pushprop\_lem2v2.lthm      thf(thm, conjecture)

**ALG250^3.p** Push property lemma 3  
include('Axioms/ALG003^0.ax')  
pushprop\_lem3v2.lthm      thf(thm, conjecture)

**ALG251^1.p** Push property  
include('Axioms/ALG003^0.ax')  
pushprop\_gthm      thf(thm, conjecture)

**ALG251^2.p** Push property  
include('Axioms/ALG003^0.ax')  
pushprop\_lthm      thf(thm, conjecture)

**ALG251^3.p** Push property  
include('Axioms/ALG003^0.ax')  
pushprop\_lthm\_orig      thf(thm, conjecture)

**ALG252^1.p** Induction lemma  
include('Axioms/ALG003^0.ax')  
induction2lem.gthm      thf(thm, conjecture)

**ALG252^2.p** Induction lemma  
include('Axioms/ALG003^0.ax')  
induction2lem.lthm      thf(thm, conjecture)

**ALG253^1.p** Induction  
include('Axioms/ALG003^0.ax')  
induction2.gthm      thf(thm, conjecture)

**ALG253^2.p** Induction  
include('Axioms/ALG003^0.ax')  
induction2.lthm      thf(thm, conjecture)

**ALG254^1.p** M is a monoid and T is an M-set  
include('Axioms/ALG003^0.ax')  
substmonoid.gthm      thf(thm, conjecture)

**ALG254^2.p** M is a monoid and T is an M-set  
include('Axioms/ALG003^0.ax')  
substmonoid.lthm      thf(thm, conjecture)

**ALG255^1.p** T is an M-set  
include('Axioms/ALG003^0.ax')  
termmset.gthm      thf(thm, conjecture)

**ALG256^1.p** HOASap is injective 1  
include('Axioms/ALG003^0.ax')

hoasapinj1\_gthm thf(thm, conjecture)  
**ALG256^2.p** HOASap is injective 1  
 include('Axioms/ALG003^0.ax')  
 hoasapinj1\_lthm thf(thm, conjecture)  
**ALG257^1.p** HOASap is injective 2  
 include('Axioms/ALG003^0.ax')  
 hoasapinj2\_gthm thf(thm, conjecture)  
**ALG257^2.p** HOASap is injective 2  
 include('Axioms/ALG003^0.ax')  
 hoasapinj2\_lthm thf(thm, conjecture)  
**ALG258^1.p** HOASlam is injective  
 include('Axioms/ALG003^0.ax')  
 hoaslaminj\_gthm thf(thm, conjecture)  
**ALG258^2.p** HOASlam is injective  
 include('Axioms/ALG003^0.ax')  
 hoaslaminj\_lthm thf(thm, conjecture)  
**ALG259^1.p** HOASlam not ap  
 include('Axioms/ALG003^0.ax')  
 hoaslammnotap\_gthm thf(thm, conjecture)  
**ALG259^2.p** HOASlam not ap  
 include('Axioms/ALG003^0.ax')  
 hoaslammnotap\_lthm thf(thm, conjecture)  
**ALG260^1.p** HOASlam not var  
 include('Axioms/ALG003^0.ax')  
 hoaslammnotvar\_gthm thf(thm, conjecture)  
**ALG260^2.p** HOASlam not var  
 include('Axioms/ALG003^0.ax')  
 hoaslammnotvar\_lthm thf(thm, conjecture)  
**ALG261^1.p** HOASap not var  
 include('Axioms/ALG003^0.ax')  
 hoasapnotvar\_gthm thf(thm, conjecture)  
**ALG261^2.p** HOASap not var  
 include('Axioms/ALG003^0.ax')  
 hoasapnotvar\_lthm thf(thm, conjecture)  
**ALG262^2.p** HOAS induction lemma 0  
 include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem0\_lthm thf(thm, conjecture)  
**ALG263^1.p** HOAS induction lemma 1  
 include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem1\_gthm thf(thm, conjecture)  
**ALG263^3.p** HOAS induction lemma 1  
 include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem1v2\_gthm thf(thm, conjecture)  
**ALG264^1.p** HOAS induction lemma 2  
 include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem2\_gthm thf(thm, conjecture)  
**ALG264^3.p** HOAS induction lemma 2  
 include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem2v2\_gthm thf(thm, conjecture)  
**ALG265^2.p** HOAS induction lemma 3aa  
 include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3aa\_lthm thf(thm, conjecture)  
**ALG266^1.p** HOAS induction lemma 3a

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3a\_gthm thf(thm, conjecture)

**ALG266^2.p** HOAS induction lemma 3a

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3a\_lthm thf(thm, conjecture)

**ALG267^1.p** HOAS induction lemma 3b

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3b\_gthm thf(thm, conjecture)

**ALG267^2.p** HOAS induction lemma 3b

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3b\_lthm thf(thm, conjecture)

**ALG268^1.p** HOAS induction lemma 3

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3\_gthm thf(thm, conjecture)

**ALG268^2.p** HOAS induction lemma 3

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3\_lthm thf(thm, conjecture)

**ALG268^3.p** HOAS induction lemma 3

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3v2a\_lthm thf(thm, conjecture)

**ALG268^4.p** HOAS induction lemma 3

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3v2\_f\_lthm thf(thm, conjecture)

**ALG268^5.p** HOAS induction lemma 3

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3v2\_gthm thf(thm, conjecture)

**ALG268^6.p** HOAS induction lemma 3

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lem3v2\_lthm thf(thm, conjecture)

**ALG269^1.p** HOAS induction

include('Axioms/ALG003^0.ax')  
 hoasinduction\_gthm thf(thm, conjecture)

**ALG269^2.p** HOAS induction

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lthm thf(thm, conjecture)

**ALG269^3.p** HOAS induction

include('Axioms/ALG003^0.ax')  
 hoasinduction\_lthm3 thf(thm, conjecture)

**ALG269^4.p** HOAS induction

include('Axioms/ALG003^0.ax')  
 hoasinduction\_no\_psi\_cond\_lthm thf(thm, conjecture)

**ALG270^5.p** TPS problem THM23

$a: \text{\$tType}$  thf(a\_type, type)

$c\_star: a \rightarrow a \rightarrow a$  thf(c\_star, type)

$\forall xx: a, xy: a, xz: a: (c\_star@(c\_star@xx@xy)@xz) = (c\_star@xx@(c\_star@xy@xz)) \Rightarrow \forall w: a, x: a, y: a, z: a: (c\_star@(c\_star@w@(c\_star@x@(c\_star@y@z)))$  thf(cTHM23\_pme, conjecture)

**ALG271^5.p** TPS problem EQUIV-01-03

$g: \text{\$tType}$  thf(g\_type, type)

$cGROUP_1: (g \rightarrow g \rightarrow g) \rightarrow g \rightarrow \text{\$o}$  thf(cGROUP1\_type, type)

$cGROUP_3: (g \rightarrow g \rightarrow g) \rightarrow g \rightarrow \text{\$o}$  thf(cGROUP3\_type, type)

$cGRP\_ASSOC: (g \rightarrow g \rightarrow g) \rightarrow \text{\$o}$  thf(cGRP\_ASSOC\_type, type)

$cGRP\_INVERSE: (g \rightarrow g \rightarrow g) \rightarrow g \rightarrow \text{\$o}$  thf(cGRP\_INVERSE\_type, type)

$cGRP\_RIGHT\_INVERSE: (g \rightarrow g \rightarrow g) \rightarrow g \rightarrow \text{\$o}$  thf(cGRP\_RIGHT\_INVERSE\_type, type)

$cGRP\_RIGHT\_UNIT: (g \rightarrow g \rightarrow g) \rightarrow g \rightarrow \text{\$o}$  thf(cGRP\_RIGHT\_UNIT\_type, type)





$g: \$tType \quad \text{thf}(g\_type, type)$   
 $cGRP\_RIGHT\_INVERSE: (g \rightarrow g \rightarrow g) \rightarrow g \rightarrow \$o \quad \text{thf}(cGRP\_RIGHT\_INVERSE\_type, type)$   
 $cGRP\_RIGHT\_UNIT: (g \rightarrow g \rightarrow g) \rightarrow g \rightarrow \$o \quad \text{thf}(cGRP\_RIGHT\_UNIT\_type, type)$   
 $cGRP\_RIGHT\_INVERSE = (\lambda x f: g \rightarrow g \rightarrow g, x e: g: \forall x a: g: \exists x b: g: (x f @ x a @ x b) = x e) \quad \text{thf}(cGRP\_RIGHT\_INVERSE\_def, definition)$   
 $cGRP\_RIGHT\_UNIT = (\lambda x f: g \rightarrow g \rightarrow g, x e: g: \forall x a: g: (x f @ x a @ x e) = x a) \quad \text{thf}(cGRP\_RIGHT\_UNIT\_def, definition)$   
 $\forall x f: g \rightarrow g \rightarrow g, x e: g: ((\forall x b: g, x c: g, x a: g: (x f @ (x f @ x a @ x b) @ x c) = (x f @ x a @ (x f @ x b @ x c))) \text{ and } cGRP\_RIGHT\_UNIT @ x f @ x e = x e) \text{ and } \forall x a: g: (x f @ x e @ x a) = x a) \quad \text{thf}(cE13A2A, conjecture)$

**ALG280** $\wedge$ **5.p** TPS problem from GRP-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$   
 $cE: a \quad \text{thf}(cE, type)$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, type)$   
 $cJ: a \rightarrow a \quad \text{thf}(cJ, type)$   
 $(\forall x x: a, x y: a, x z: a: (cP @ (cP @ x x @ x y) @ x z) = (cP @ x x @ (cP @ x y @ x z))) \text{ and } \forall x x: a: (cP @ cE @ x x) = x x \text{ and } \forall x y: a: (cP @ (cJ @ x y @ x cE)) \Rightarrow \forall x: a: (cP @ x @ cE) = x \quad \text{thf}(cTHM17\_pme, conjecture)$

**ALG281** $\wedge$ **5.p** TPS problem from GRP-THMS

$cE: \$i \quad \text{thf}(cE, type)$   
 $cJ: \$i \rightarrow \$i \quad \text{thf}(cJ, type)$   
 $cP: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(cP, type)$   
 $(\forall x x: \$i, x y: \$i, x z: \$i: (cP @ (cP @ x x @ x y) @ x z) = (cP @ x x @ (cP @ x y @ x z))) \text{ and } \forall x x: \$i: (cP @ cE @ x x) = x x \text{ and } \forall x y: \$i: (cP @ (cJ @ x y @ cE)) \Rightarrow \forall x: \$i: (cP @ x @ (cJ @ x)) = cE \quad \text{thf}(cTHM16\_pme, conjecture)$

**ALG282** $\wedge$ **5.p** TPS problem from GRP-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, type)$   
 $cE: a \quad \text{thf}(cE, type)$   
 $cJ: a \rightarrow a \quad \text{thf}(cJ, type)$   
 $(\forall x x: a, x y: a, x z: a: (cP @ (cP @ x x @ x y) @ x z) = (cP @ x x @ (cP @ x y @ x z))) \text{ and } \forall x x: a: (cP @ cE @ x x) = x x \text{ and } \forall x y: a: (cP @ (cJ @ x y @ cE)) \Rightarrow \forall x: a, y: a: \exists w: a: (cP @ w @ x) = y \quad \text{thf}(cTHM21\_pme, conjecture)$

**ALG283** $\wedge$ **5.p** TPS problem from GRP-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, type)$   
 $cE: a \quad \text{thf}(cE, type)$   
 $cJ: a \rightarrow a \quad \text{thf}(cJ, type)$   
 $(\forall x x: a, x y: a, x z: a: (cP @ (cP @ x x @ x y) @ x z) = (cP @ x x @ (cP @ x y @ x z))) \text{ and } \forall x x: a: (cP @ cE @ x x) = x x \text{ and } \forall x y: a: (cP @ (cJ @ x y @ cE)) \Rightarrow \forall x: a, y: a: \exists z: a: (cP @ x @ z) = y \quad \text{thf}(cTHM20\_pme, conjecture)$

**ALG284** $\wedge$ **5.p** TPS problem from GRP-THMS

$cJ: \$i \rightarrow \$i \quad \text{thf}(cJ, type)$   
 $cP: \$i \rightarrow \$i \rightarrow \$i \quad \text{thf}(cP, type)$   
 $cE: \$i \quad \text{thf}(cE, type)$   
 $(\forall x x: \$i, x y: \$i, x z: \$i: (cP @ (cP @ x x @ x y) @ x z) = (cP @ x x @ (cP @ x y @ x z))) \text{ and } \forall x x: \$i: (cP @ cE @ x x) = x x \text{ and } \forall x y: \$i: (cP @ (cJ @ x y @ cE)) \Rightarrow \forall x: \$i, y: \$i: (cJ @ (cP @ x @ y)) = (cP @ (cJ @ y) @ (cJ @ x)) \quad \text{thf}(cTHM18\_pme, conjecture)$

**ALG285** $\wedge$ **5.p** TPS problem from GRP-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, type)$   
 $cE: a \quad \text{thf}(cE, type)$   
 $cJ: a \rightarrow a \quad \text{thf}(cJ, type)$   
 $(\forall x x: a, x y: a, x z: a: (cP @ (cP @ x x @ x y) @ x z) = (cP @ x x @ (cP @ x y @ x z))) \text{ and } \forall x x: a: (cP @ cE @ x x) = x x \text{ and } \forall x y: a: (cP @ (cJ @ x y @ cE)) \Rightarrow (\forall x x: a, x y: a, x z: a: (cP @ (cP @ x x @ x y) @ x z) = (cP @ x x @ (cP @ x y @ x z))) \text{ and } \forall x: a, y: a: (\exists u: a: (cP @ x @ u) = y) \text{ and } \exists v: a: (cP @ v @ x) = y) \quad \text{thf}(cTHM22\_pme, conjecture)$

**ALG286** $\wedge$ **5.p** TPS problem from PAIRING-UNPAIRING-ALG-THMS

$a: \$tType \quad \text{thf}(a\_type, type)$   
 $cZ: a \quad \text{thf}(cZ, type)$   
 $u: a \quad \text{thf}(u, type)$   
 $y: a \quad \text{thf}(y, type)$   
 $x: a \quad \text{thf}(x, type)$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, type)$   
 $cR: a \rightarrow a \quad \text{thf}(cR, type)$

$cL: a \rightarrow a \quad \text{thf}(cL, \text{type})$   
 $((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx_0: a, xy_0: a: (cL@(cP@xx_0@xy_0)) = xx_0 \text{ and } \forall xx_0: a, xy_0: a: (cR@(cP@xx_0@xy_0)) = xy_0 \text{ and } \forall xt: a: (xt \neq cZ \iff xt = (cP@(cL@xt)@(cR@xt)))) \Rightarrow (u = (cP@x@y) \Rightarrow u \neq cZ) \quad \text{thf}(cPU\_PAIR\_NOT\_ZEL)$

**ALG287** $\wedge$ **5.p** TPS problem from PAIRING-UNPAIRING-ALG-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$   
 $v: a \quad \text{thf}(v, \text{type})$   
 $u: a \quad \text{thf}(u, \text{type})$   
 $y: a \quad \text{thf}(y, \text{type})$   
 $x: a \quad \text{thf}(x, \text{type})$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, \text{type})$   
 $cR: a \rightarrow a \quad \text{thf}(cR, \text{type})$   
 $cL: a \rightarrow a \quad \text{thf}(cL, \text{type})$   
 $cZ: a \quad \text{thf}(cZ, \text{type})$   
 $((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx_0: a, xy_0: a: (cL@(cP@xx_0@xy_0)) = xx_0 \text{ and } \forall xx_0: a, xy_0: a: (cR@(cP@xx_0@xy_0)) = xy_0 \text{ and } \forall xt: a: (xt \neq cZ \iff xt = (cP@(cL@xt)@(cR@xt)))) \Rightarrow ((cP@x@u) = (cP@y@v) \Rightarrow (x = y \text{ and } u = v)) \quad \text{thf}(cPU\_P\_INJ\_pme, \text{conjecture})$

**ALG288** $\wedge$ **5.p** TPS problem from PU-LAMBDA-MODEL-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, \text{type})$   
 $cR: a \rightarrow a \quad \text{thf}(cR, \text{type})$   
 $cL: a \rightarrow a \quad \text{thf}(cL, \text{type})$   
 $cZ: a \quad \text{thf}(cZ, \text{type})$   
 $((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx: a, xy: a: (cL@(cP@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (cR@(cP@xx@xy)) = xy \text{ and } \forall xt: a: (xt \neq cZ \iff xt = (cP@(cL@xt)@(cR@xt))) \text{ and } \forall x: a \rightarrow \$o: (\exists xt: a: (x@xt \text{ and } \forall xu: a: ((x@xu) \Rightarrow (x@(cL@xu)))) \Rightarrow (x@cZ))) \Rightarrow \forall x: a \rightarrow \$o, xz: a: (\exists xy: a: (x@(cP@xy@xz)) \iff \exists xx: a: (\forall xx_{29}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = xx_{29})) \Rightarrow (x@xx_{29})) \text{ and } \exists xy: a: \forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = (cP@xy@xz)))))) \quad \text{thf}(cPU\_SETR\_CTS\_pme, \text{conjecture})$

**ALG289** $\wedge$ **5.p** TPS problem from PU-LAMBDA-MODEL-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, \text{type})$   
 $cR: a \rightarrow a \quad \text{thf}(cR, \text{type})$   
 $cL: a \rightarrow a \quad \text{thf}(cL, \text{type})$   
 $cZ: a \quad \text{thf}(cZ, \text{type})$   
 $((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx: a, xy: a: (cL@(cP@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (cR@(cP@xx@xy)) = xy \text{ and } \forall xt: a: (xt \neq cZ \iff xt = (cP@(cL@xt)@(cR@xt))) \text{ and } \forall x: a \rightarrow \$o: (\exists xt: a: (x@xt \text{ and } \forall xu: a: ((x@xu) \Rightarrow (x@(cL@xu)))) \Rightarrow (x@cZ))) \Rightarrow \forall x: a \rightarrow \$o, xz: a: (\exists xz_0: a: (x@(cP@xz@xz_0)) \iff \exists xx: a: (\forall xx_{13}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = xx_{13})) \Rightarrow (x@xx_{13})) \text{ and } \exists xz_1: a: \forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = (cP@xz@xz_1)))))) \quad \text{thf}(cPU\_SETL\_CTS\_pme, \text{conjecture})$

**ALG290** $\wedge$ **5.p** TPS problem from PU-LAMBDA-MODEL-THMS

$a: \$tType \quad \text{thf}(a\_type, \text{type})$   
 $cP: a \rightarrow a \rightarrow a \quad \text{thf}(cP, \text{type})$   
 $cG: a \rightarrow \$o \quad \text{thf}(cG, \text{type})$   
 $cX: a \rightarrow \$o \quad \text{thf}(cX, \text{type})$   
 $cR: a \rightarrow a \quad \text{thf}(cR, \text{type})$   
 $cL: a \rightarrow a \quad \text{thf}(cL, \text{type})$   
 $cF: a \rightarrow \$o \quad \text{thf}(cF, \text{type})$   
 $cZ: a \quad \text{thf}(cZ, \text{type})$   
 $((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx: a, xy: a: (cL@(cP@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (cR@(cP@xx@xy)) = xy \text{ and } \forall xt: a: (xt \neq cZ \iff xt = (cP@(cL@xt)@(cR@xt))) \text{ and } \forall x_0: a \rightarrow \$o: (\exists xt: a: (x_0@xt \text{ and } \forall xu: a: ((x_0@xu) \Rightarrow (x_0@(cL@xu)))) \Rightarrow (x_0@cZ))) \Rightarrow (\lambda xy: a: \exists xx: a: (\forall xx_{17}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz: a: ((x_0@xz) \Rightarrow (x_0@(cL@xz)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = xx_{17})) \Rightarrow (cX@xx_{17})) \text{ and } (cF@(cP@xx@xy) \text{ or } cG@(cP@xx@xy)))) \Rightarrow (\lambda xz: a: (\exists xx: a: (\forall xx_{18}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xx_{18})) \Rightarrow (cX@xx_{18})) \text{ and } cF@(cP@xx@xz)) \text{ or } \exists xx: a: (\forall xx_{19}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = xx_{19})) \Rightarrow (cX@xx_{19})) \text{ and } cG@(cP@xx@xz)))) \quad \text{thf}(cPU\_X2310A.p$

**ALG291** $\wedge$ **5.p** TPS problem from PU-LAMBDA-MODEL-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall z: a, p: a \rightarrow a \rightarrow a, l: a \rightarrow a, r: a \rightarrow a, x: a \rightarrow \$o: (((l@z) = z \text{ and } (r@z) = z \text{ and } \forall xx: a, xy: a: (l@(p@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (r@(p@xx@xy)) = xy \text{ and } \forall xt: a: (xt \neq z \iff xt = (p@(l@xt)@(r@xt))) \text{ and } \forall x_0: a \rightarrow \$o: (\exists xt: a: (x_0@xt \text{ and } \forall xu: a: ((x_0@xu) \Rightarrow (x_0@(l@xu)))) \Rightarrow (x_0@z))) \Rightarrow \forall x_0: a \rightarrow \$o, xz: a: (\exists xx: a: (\forall xx_9: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(l@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (r@xv) = xx_9)) \Rightarrow (x@xx_9)) \text{ and } x_0@(p@xx@xy)) \Rightarrow (x@xx_9)) \text{ and } \exists xx_{10}: a: (\forall xx_{10}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(l@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (r@xv) = xx_{10})) \Rightarrow (x@xx_{10})) \text{ and } \exists xx_{12}: a: (\forall xx_{11}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx_{12} \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(l@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (r@xv) = xx_{11})) \Rightarrow (x@xx_{11})) \text{ and } \forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(l@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (r@xv) = (p@xx_{12}@xz)))))) \quad thf(cPU\_X238B\_pme, conjecture)$

**ALG292** $\wedge$ **5.p** TPS problem from PU-LAMBDA-MODEL-THMS

$a: \$tType \quad thf(a\_type, type)$

$\forall z: a, p: a \rightarrow a \rightarrow a, l: a \rightarrow a, r: a \rightarrow a, f: a \rightarrow \$o: (((l@z) = z \text{ and } (r@z) = z \text{ and } \forall xx: a, xy: a: (l@(p@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (r@(p@xx@xy)) = xy \text{ and } \forall xt: a: (xt \neq z \iff xt = (p@(l@xt)@(r@xt))) \text{ and } \forall x: a \rightarrow \$o: (\exists xt: a: (x@xt \text{ and } \forall xu: a: ((x@xu) \Rightarrow (x@(l@xu)))) \Rightarrow (x@z))) \Rightarrow \forall x: a \rightarrow \$o, xz: a: (\exists xx: a: (\forall xx_5: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(l@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (r@xv) = xx_5)) \Rightarrow (x@xx_5)) \text{ and } f@(p@xx@xy)) \Rightarrow (x@xx_5)) \text{ and } \exists xx_6: a: (\forall xx_6: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(l@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (r@xv) = xx_6)) \Rightarrow (x@xx_6)) \text{ and } \exists xx_8: a: (\forall xx_7: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx_8 \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(l@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (r@xv) = xx_7)) \Rightarrow \forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(l@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (r@xv) = xx_7)) \text{ and } f@(p@xx_8@xz)))))) \quad thf(cPU\_X238A\_pme, conjecture)$

**ALG293** $\wedge$ **5.p** TPS problem from PU-LAMBDA-MODEL-THMS

$a: \$tType \quad thf(a\_type, type)$

$cP: a \rightarrow a \rightarrow a \quad thf(cP, type)$

$cR: a \rightarrow a \quad thf(cR, type)$

$cL: a \rightarrow a \quad thf(cL, type)$

$cZ: a \quad thf(cZ, type)$

$((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx: a, xy: a: (cL@(cP@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (cR@(cP@xx@xy)) = xy \text{ and } \forall xt: a: (xt \neq cZ \iff xt = (cP@(cL@xt)@(cR@xt))) \text{ and } \forall x: a \rightarrow \$o: (\exists xt: a: (x@xt \text{ and } \forall xu: a: ((x@xu) \Rightarrow (x@(cL@xu)))) \Rightarrow (x@cZ))) \Rightarrow \forall x: a \rightarrow \$o, xz: a: (\exists xx: a: (\forall xx_{23}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = xx_{23})) \Rightarrow \exists xy: a: (x@(cP@xy@xx_{23})) \text{ and } \exists xz_2: a: (x@(cP@(cP@xx@xy)) \Rightarrow (x@xx_{24})) \text{ and } \exists xx_{26}: a: (\forall xx_{25}: a: (\forall x_0: a \rightarrow \$o: ((x_0@xx_{26} \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = xx_{25})) \Rightarrow \exists xy: a: \forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = (cP@xy@xx_{25})))) \text{ and } \exists xz_3: a: \forall x_0: a \rightarrow \$o: ((x_0@xx \text{ and } \forall xz_0: a: ((x_0@xz_0) \Rightarrow (x_0@(cL@xz_0)))) \Rightarrow \exists xv: a: (x_0@xv \text{ and } (cR@xv) = (cP@(cP@xx_{26}@xz)@xz_3)))))) \quad thf(cPU\_X239\_pme, conjecture)$

**ALG295** $\wedge$ **5.p** TPS problem from SEQUENTIAL-PU-ALG-THMS

$a: \$tType \quad thf(a\_type, type)$

$cZ: a \quad thf(cZ, type)$

$cP: a \rightarrow a \rightarrow a \quad thf(cP, type)$

$cR: a \rightarrow a \quad thf(cR, type)$

$cL: a \rightarrow a \quad thf(cL, type)$

$((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx: a, xy: a: (cL@(cP@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (cR@(cP@xx@xy)) = xy \text{ and } \forall xt: a: (xt \neq cZ \iff xt = (cP@(cL@xt)@(cR@xt))) \Rightarrow (\forall x: a \rightarrow \$o: (\exists xt: a: (x@xt \text{ and } \forall xu: a: ((x@xu) \Rightarrow (x@(cL@xu)))) \Rightarrow (x@cZ)) \iff \forall xt: a: \exists xn: a: (\forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx: a: ((x@xx) \Rightarrow (x@(cP@xx@cZ)))) \Rightarrow (x@xn)) \text{ and } \forall xu: a: (\exists xb: a, xu_{11}: a: ((cP@xn@xu) = (cP@xb@xu_{11})) \text{ and } \forall x: a \rightarrow \$o: ((x@(cP@cZ@xt) \text{ and } \forall xc: a, xv: a: ((x@(cP@(cP@xc@cZ)@(cL@xv)) \text{ and } x@(cP@(cP@xc@cZ)@(cP@cZ@cZ)@(cR@xv)))) \Rightarrow (x@(cP@xb@xu_{11}))) \Rightarrow (xu = cZ))) \quad thf(cPU\_LEM8\_pme, conjecture)$

**ALG296** $\wedge$ **5.p** TPS problem from SEQUENTIAL-PU-ALG-THMS

$a: \$tType \quad thf(a\_type, type)$

$cP: a \rightarrow a \rightarrow a \quad thf(cP, type)$

$cR: a \rightarrow a \quad thf(cR, type)$

$cZ: a \quad thf(cZ, type)$

$cL: a \rightarrow a \quad thf(cL, type)$

$((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx: a, xy: a: (cL@(cP@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (cR@(cP@xx@xy)) = xy \text{ and } \forall xt: a: (xt \neq cZ \iff xt = (cP@(cL@xt)@(cR@xt))) \Rightarrow \forall xt: a, xb: a: (\forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx: a: ((x@xx) \Rightarrow (x@(cP@xx@cZ)) \text{ and } x@(cP@xx@(cP@cZ@cZ)))) \Rightarrow (x@xb)) \Rightarrow \exists xu: a: (\exists xb_0: a, xu_0: a: ((cP@xb@xu) = (cP@xb_0@xu_0)) \text{ and } \forall x: a \rightarrow \$o: ((x@(cP@cZ@xt) \text{ and } \forall xc: a, xv: a: ((x@(cP@xc@xv)) \Rightarrow (x@(cP@(cP@xc@cZ)@(cL@xv)) \text{ and } x@(cP@xb_0@xu_0)))) \text{ and } \forall xv: a: (\exists xb_1: a, xu_0: a: ((cP@xb@xv) = (cP@xb_1@xu_0)) \text{ and } \forall x: a \rightarrow \$o: ((x@(cP@cZ@xt) \text{ and } \forall xc: a, xv: a: ((x@(cP@xc@xv)) \Rightarrow (x@(cP@(cP@xc@cZ)@(cL@xv)) \text{ and } x@(cP@xb_1@xu_0)))))) \quad thf(cPU\_LEM8\_pme, conjecture)$



$(x@(cP@(cP@xc@cZ)@(cL@xv_0)) \text{ and } x@(cP@(cP@xc@(cP@cZ@cZ)@(cR@xv_0)))) \Rightarrow (x@(cP@xb_1@xu_0))) \Rightarrow$   
 $xu = xv))$       thf(cPU\_LEM6\_pme, conjecture)

**ALG297^5.p** TPS problem from S-SEQ-THMS

$a: \$tType$       thf(a\_type, type)

$cP: a \rightarrow a \rightarrow a$       thf(cP, type)

$cZ: a$       thf(cZ, type)

$cR: a \rightarrow a$       thf(cR, type)

$cL: a \rightarrow a$       thf(cL, type)

$t: a$       thf(t, type)

$((cL@cZ) = cZ \text{ and } (cR@cZ) = cZ \text{ and } \forall xx: a, xy: a: (cL@(cP@xx@xy)) = xx \text{ and } \forall xx: a, xy: a: (cR@(cP@xx@xy)) =$

$xy \text{ and } \forall xt_0: a: (xt_0 \neq cZ \iff xt_0 = (cP@(cL@xt_0)@(cR@xt_0)))) \Rightarrow \forall xs: a: (\forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx: a, xy: a: ((x@xx@$

$(x@(cP@xx@xy)))) \Rightarrow (x@xs)) \Rightarrow \forall xb: a: (\forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx: a: ((x@xx \Rightarrow (x@(cP@xx@cZ) \text{ and } x@(cP@xx@$

$(x@xb)) \Rightarrow \forall xu: a: (\exists xb_9: a, xu_{13}: a: ((cP@xb@xu) = (cP@xb_9@xu_{13}) \text{ and } \forall x: a \rightarrow \$o: ((x@(cP@cZ@t) \text{ and } \forall xc: a, xv: a: (($

$(x@(cP@(cP@xc@cZ)@(cL@xv)) \text{ and } x@(cP@(cP@xc@(cP@cZ@cZ)@(cR@xv)))) \Rightarrow (x@(cP@xb_9@xu_{13}))) \Rightarrow$

$\forall x: a \rightarrow \$o: ((x@cZ \text{ and } \forall xx: a, xy: a: ((x@xx \text{ and } x@xy) \Rightarrow (x@(cP@xx@xy)))) \Rightarrow (x@xu))))$       thf(cPU\_LEM7\_pme, co

**ALG298^5.p** TPS problem THM270

$c: \$tType$       thf(c\_type, type)

$b: \$tType$       thf(b\_type, type)

$a: \$tType$       thf(a\_type, type)

$c\_starc: c \rightarrow c \rightarrow c$       thf(c\_starc, type)

$c\_starb: b \rightarrow b \rightarrow b$       thf(c\_starb, type)

$c\_stara: a \rightarrow a \rightarrow a$       thf(c\_stara, type)

$\forall xf: a \rightarrow b, xg: a \rightarrow c, xh: b \rightarrow c: ((\forall xx: a: (xh@(xf@xx)) = (xg@xx) \text{ and } \forall xy: b: \exists xx: a: (xf@xx) = xy \text{ and } \forall xx: a, xy: a: (xf@$

$(c\_starb@(xf@xx)@(xf@xy)) \text{ and } \forall xx: a, xy: a: (xg@(c\_stara@xx@xy)) = (c\_starc@(xg@xx)@(xg@xy))) \Rightarrow \forall xx: b, xy: b: (xh@$

$(c\_starc@(xh@xx)@(xh@xy)))$       thf(cTHM270\_pme, conjecture)

**ALG299-1.p** An equational theory with no nontrivial finite models

A classical example of an equational theory with no nontrivial finite models (found independently by Tarski, Jonsson, Skornyakov and others).

$f(a \cdot b) = a$       cnf(sos<sub>01</sub>, axiom)

$g(a \cdot b) = b$       cnf(sos<sub>02</sub>, axiom)

$tptp_1 \neq tptp_0$       cnf(sos<sub>03</sub>, axiom)

**ALG300-1.p** Identity with no nontrivial finite model

One of the two shortest identities with no nontrivial finite models (in a single binary operation).

$((a \cdot a) \cdot a) \cdot b \cdot (a \cdot c) = b$       cnf(sos<sub>01</sub>, axiom)

$tptp_1 \neq tptp_0$       cnf(sos<sub>02</sub>, axiom)

**ALG301-1.p** Identity with no nontrivial finite model

One of the two shortest identities with no nontrivial finite models (in a single binary operation).

$a \cdot (a \cdot (a \cdot (b \cdot (c \cdot a)))) = b$       cnf(sos<sub>01</sub>, axiom)

$tptp_1 \neq tptp_0$       cnf(sos<sub>02</sub>, axiom)

**ALG302-1.p** Austin's identity

$((a \cdot a) \cdot a) \cdot b \cdot (((a \cdot a) \cdot ((a \cdot a) \cdot a)) \cdot c) = b$       cnf(sos<sub>01</sub>, axiom)

$tptp_1 \neq tptp_0$       cnf(sos<sub>02</sub>, axiom)

**ALG305-1.p** Random graph 3, nu5 polymorphism

$t(y, x, x, x, x) = x$       cnf(polynu5<sub>01</sub>, axiom)

$t(x, y, x, x, x) = x$       cnf(polynu5<sub>02</sub>, axiom)

$t(x, x, y, x, x) = x$       cnf(polynu5<sub>03</sub>, axiom)

$t(x, x, x, y, x) = x$       cnf(polynu5<sub>04</sub>, axiom)

$t(x, x, x, x, y) = x$       cnf(polynu5<sub>05</sub>, axiom)

$(gr(x_0, x_1) \text{ and } gr(x_2, x_3) \text{ and } gr(x_4, x_5) \text{ and } gr(x_6, x_7) \text{ and } gr(x_8, x_9)) \Rightarrow gr(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$       cnf

$\neg gr(n_0, n_0)$       cnf(graph\_n0\_n0, axiom)

$gr(n_0, n_1)$       cnf(graph\_n0\_n1, axiom)

$gr(n_0, n_2)$       cnf(graph\_n0\_n2, axiom)

$gr(n_0, n_3)$       cnf(graph\_n0\_n3, axiom)

$gr(n_0, n_4)$       cnf(graph\_n0\_n4, axiom)

$\neg gr(n_1, n_0)$       cnf(graph\_n1\_n0, axiom)

$\neg gr(n_1, n_1)$       cnf(graph\_n1\_n1, axiom)

$\neg \text{gr}(n_1, n_2)$        $\text{cnf}(\text{graph\_n1\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_1, n_3)$        $\text{cnf}(\text{graph\_n1\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_1, n_4)$        $\text{cnf}(\text{graph\_n1\_n}_4, \text{axiom})$   
 $\text{gr}(n_2, n_0)$        $\text{cnf}(\text{graph\_n2\_n}_0, \text{axiom})$   
 $\text{gr}(n_2, n_1)$        $\text{cnf}(\text{graph\_n2\_n}_1, \text{axiom})$   
 $\neg \text{gr}(n_2, n_2)$        $\text{cnf}(\text{graph\_n2\_n}_2, \text{axiom})$   
 $\text{gr}(n_2, n_3)$        $\text{cnf}(\text{graph\_n2\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_2, n_4)$        $\text{cnf}(\text{graph\_n2\_n}_4, \text{axiom})$   
 $\text{gr}(n_3, n_0)$        $\text{cnf}(\text{graph\_n3\_n}_0, \text{axiom})$   
 $\text{gr}(n_3, n_1)$        $\text{cnf}(\text{graph\_n3\_n}_1, \text{axiom})$   
 $\neg \text{gr}(n_3, n_2)$        $\text{cnf}(\text{graph\_n3\_n}_2, \text{axiom})$   
 $\text{gr}(n_3, n_3)$        $\text{cnf}(\text{graph\_n3\_n}_3, \text{axiom})$   
 $\text{gr}(n_3, n_4)$        $\text{cnf}(\text{graph\_n3\_n}_4, \text{axiom})$   
 $\text{gr}(n_4, n_0)$        $\text{cnf}(\text{graph\_n4\_n}_0, \text{axiom})$   
 $\text{gr}(n_4, n_1)$        $\text{cnf}(\text{graph\_n4\_n}_1, \text{axiom})$   
 $\text{gr}(n_4, n_2)$        $\text{cnf}(\text{graph\_n4\_n}_2, \text{axiom})$   
 $\text{gr}(n_4, n_3)$        $\text{cnf}(\text{graph\_n4\_n}_3, \text{axiom})$   
 $\text{gr}(n_4, n_4)$        $\text{cnf}(\text{graph\_n4\_n}_4, \text{axiom})$   
 $n_0 \neq n_1$        $\text{cnf}(\text{elems\_n0\_n}_1, \text{axiom})$   
 $n_0 \neq n_2$        $\text{cnf}(\text{elems\_n0\_n}_2, \text{axiom})$   
 $n_0 \neq n_3$        $\text{cnf}(\text{elems\_n0\_n}_3, \text{axiom})$   
 $n_0 \neq n_4$        $\text{cnf}(\text{elems\_n0\_n}_4, \text{axiom})$   
 $n_1 \neq n_2$        $\text{cnf}(\text{elems\_n1\_n}_2, \text{axiom})$   
 $n_1 \neq n_3$        $\text{cnf}(\text{elems\_n1\_n}_3, \text{axiom})$   
 $n_1 \neq n_4$        $\text{cnf}(\text{elems\_n1\_n}_4, \text{axiom})$   
 $n_2 \neq n_3$        $\text{cnf}(\text{elems\_n2\_n}_3, \text{axiom})$   
 $n_2 \neq n_4$        $\text{cnf}(\text{elems\_n2\_n}_4, \text{axiom})$   
 $n_3 \neq n_4$        $\text{cnf}(\text{elems\_n3\_n}_4, \text{axiom})$   
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$        $\text{cnf}(\text{elems}, \text{axiom})$

**ALG306-1.p** Random graph 4, edge5 polymorphism

$t(y, y, x, x, x) = x$        $\text{cnf}(\text{polyedge5}_{01}, \text{axiom})$   
 $t(y, x, y, x, x) = x$        $\text{cnf}(\text{polyedge5}_{02}, \text{axiom})$   
 $t(x, x, x, y, x) = x$        $\text{cnf}(\text{polyedge5}_{03}, \text{axiom})$   
 $t(x, x, x, x, y) = x$        $\text{cnf}(\text{polyedge5}_{04}, \text{axiom})$   
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$        $\text{cnf}(\text{graph\_n0\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_0, n_0)$        $\text{cnf}(\text{graph\_n0\_n}_0, \text{axiom})$   
 $\text{gr}(n_0, n_1)$        $\text{cnf}(\text{graph\_n0\_n}_1, \text{axiom})$   
 $\text{gr}(n_0, n_2)$        $\text{cnf}(\text{graph\_n0\_n}_2, \text{axiom})$   
 $\text{gr}(n_0, n_3)$        $\text{cnf}(\text{graph\_n0\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_0, n_4)$        $\text{cnf}(\text{graph\_n0\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_1, n_0)$        $\text{cnf}(\text{graph\_n1\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_1, n_1)$        $\text{cnf}(\text{graph\_n1\_n}_1, \text{axiom})$   
 $\text{gr}(n_1, n_2)$        $\text{cnf}(\text{graph\_n1\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_1, n_3)$        $\text{cnf}(\text{graph\_n1\_n}_3, \text{axiom})$   
 $\text{gr}(n_1, n_4)$        $\text{cnf}(\text{graph\_n1\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_2, n_0)$        $\text{cnf}(\text{graph\_n2\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_2, n_1)$        $\text{cnf}(\text{graph\_n2\_n}_1, \text{axiom})$   
 $\text{gr}(n_2, n_2)$        $\text{cnf}(\text{graph\_n2\_n}_2, \text{axiom})$   
 $\text{gr}(n_2, n_3)$        $\text{cnf}(\text{graph\_n2\_n}_3, \text{axiom})$   
 $\text{gr}(n_2, n_4)$        $\text{cnf}(\text{graph\_n2\_n}_4, \text{axiom})$   
 $\text{gr}(n_3, n_0)$        $\text{cnf}(\text{graph\_n3\_n}_0, \text{axiom})$   
 $\text{gr}(n_3, n_1)$        $\text{cnf}(\text{graph\_n3\_n}_1, \text{axiom})$   
 $\text{gr}(n_3, n_2)$        $\text{cnf}(\text{graph\_n3\_n}_2, \text{axiom})$   
 $\text{gr}(n_3, n_3)$        $\text{cnf}(\text{graph\_n3\_n}_3, \text{axiom})$   
 $\text{gr}(n_3, n_4)$        $\text{cnf}(\text{graph\_n3\_n}_4, \text{axiom})$   
 $\text{gr}(n_4, n_0)$        $\text{cnf}(\text{graph\_n4\_n}_0, \text{axiom})$   
 $\text{gr}(n_4, n_1)$        $\text{cnf}(\text{graph\_n4\_n}_1, \text{axiom})$   
 $\text{gr}(n_4, n_2)$        $\text{cnf}(\text{graph\_n4\_n}_2, \text{axiom})$   
 $\text{gr}(n_4, n_3)$        $\text{cnf}(\text{graph\_n4\_n}_3, \text{axiom})$

$\text{gr}(n_4, n_4)$        $\text{cnf}(\text{graph\_n4\_n}_4, \text{axiom})$   
 $n_0 \neq n_1$        $\text{cnf}(\text{elems\_n0\_n}_1, \text{axiom})$   
 $n_0 \neq n_2$        $\text{cnf}(\text{elems\_n0\_n}_2, \text{axiom})$   
 $n_0 \neq n_3$        $\text{cnf}(\text{elems\_n0\_n}_3, \text{axiom})$   
 $n_0 \neq n_4$        $\text{cnf}(\text{elems\_n0\_n}_4, \text{axiom})$   
 $n_1 \neq n_2$        $\text{cnf}(\text{elems\_n1\_n}_2, \text{axiom})$   
 $n_1 \neq n_3$        $\text{cnf}(\text{elems\_n1\_n}_3, \text{axiom})$   
 $n_1 \neq n_4$        $\text{cnf}(\text{elems\_n1\_n}_4, \text{axiom})$   
 $n_2 \neq n_3$        $\text{cnf}(\text{elems\_n2\_n}_3, \text{axiom})$   
 $n_2 \neq n_4$        $\text{cnf}(\text{elems\_n2\_n}_4, \text{axiom})$   
 $n_3 \neq n_4$        $\text{cnf}(\text{elems\_n3\_n}_4, \text{axiom})$   
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$        $\text{cnf}(\text{elems}, \text{axiom})$

**ALG307-1.p** Random graph 5, nu5 polymorphism

$t(y, x, x, x, x) = x$        $\text{cnf}(\text{polynu5}_{01}, \text{axiom})$   
 $t(x, y, x, x, x) = x$        $\text{cnf}(\text{polynu5}_{02}, \text{axiom})$   
 $t(x, x, y, x, x) = x$        $\text{cnf}(\text{polynu5}_{03}, \text{axiom})$   
 $t(x, x, x, y, x) = x$        $\text{cnf}(\text{polynu5}_{04}, \text{axiom})$   
 $t(x, x, x, x, y) = x$        $\text{cnf}(\text{polynu5}_{05}, \text{axiom})$   
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$        $\text{cnf}(\text{graph\_5\_nu5\_polymorphism}, \text{axiom})$   
 $\neg \text{gr}(n_0, n_0)$        $\text{cnf}(\text{graph\_n0\_n}_0, \text{axiom})$   
 $\text{gr}(n_0, n_1)$        $\text{cnf}(\text{graph\_n0\_n}_1, \text{axiom})$   
 $\text{gr}(n_0, n_2)$        $\text{cnf}(\text{graph\_n0\_n}_2, \text{axiom})$   
 $\text{gr}(n_0, n_3)$        $\text{cnf}(\text{graph\_n0\_n}_3, \text{axiom})$   
 $\text{gr}(n_0, n_4)$        $\text{cnf}(\text{graph\_n0\_n}_4, \text{axiom})$   
 $\text{gr}(n_1, n_0)$        $\text{cnf}(\text{graph\_n1\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_1, n_1)$        $\text{cnf}(\text{graph\_n1\_n}_1, \text{axiom})$   
 $\text{gr}(n_1, n_2)$        $\text{cnf}(\text{graph\_n1\_n}_2, \text{axiom})$   
 $\text{gr}(n_1, n_3)$        $\text{cnf}(\text{graph\_n1\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_1, n_4)$        $\text{cnf}(\text{graph\_n1\_n}_4, \text{axiom})$   
 $\text{gr}(n_2, n_0)$        $\text{cnf}(\text{graph\_n2\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_2, n_1)$        $\text{cnf}(\text{graph\_n2\_n}_1, \text{axiom})$   
 $\text{gr}(n_2, n_2)$        $\text{cnf}(\text{graph\_n2\_n}_2, \text{axiom})$   
 $\text{gr}(n_2, n_3)$        $\text{cnf}(\text{graph\_n2\_n}_3, \text{axiom})$   
 $\text{gr}(n_2, n_4)$        $\text{cnf}(\text{graph\_n2\_n}_4, \text{axiom})$   
 $\text{gr}(n_3, n_0)$        $\text{cnf}(\text{graph\_n3\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_3, n_1)$        $\text{cnf}(\text{graph\_n3\_n}_1, \text{axiom})$   
 $\text{gr}(n_3, n_2)$        $\text{cnf}(\text{graph\_n3\_n}_2, \text{axiom})$   
 $\text{gr}(n_3, n_3)$        $\text{cnf}(\text{graph\_n3\_n}_3, \text{axiom})$   
 $\text{gr}(n_3, n_4)$        $\text{cnf}(\text{graph\_n3\_n}_4, \text{axiom})$   
 $\text{gr}(n_4, n_0)$        $\text{cnf}(\text{graph\_n4\_n}_0, \text{axiom})$   
 $\text{gr}(n_4, n_1)$        $\text{cnf}(\text{graph\_n4\_n}_1, \text{axiom})$   
 $\text{gr}(n_4, n_2)$        $\text{cnf}(\text{graph\_n4\_n}_2, \text{axiom})$   
 $\text{gr}(n_4, n_3)$        $\text{cnf}(\text{graph\_n4\_n}_3, \text{axiom})$   
 $\text{gr}(n_4, n_4)$        $\text{cnf}(\text{graph\_n4\_n}_4, \text{axiom})$   
 $n_0 \neq n_1$        $\text{cnf}(\text{elems\_n0\_n}_1, \text{axiom})$   
 $n_0 \neq n_2$        $\text{cnf}(\text{elems\_n0\_n}_2, \text{axiom})$   
 $n_0 \neq n_3$        $\text{cnf}(\text{elems\_n0\_n}_3, \text{axiom})$   
 $n_0 \neq n_4$        $\text{cnf}(\text{elems\_n0\_n}_4, \text{axiom})$   
 $n_1 \neq n_2$        $\text{cnf}(\text{elems\_n1\_n}_2, \text{axiom})$   
 $n_1 \neq n_3$        $\text{cnf}(\text{elems\_n1\_n}_3, \text{axiom})$   
 $n_1 \neq n_4$        $\text{cnf}(\text{elems\_n1\_n}_4, \text{axiom})$   
 $n_2 \neq n_3$        $\text{cnf}(\text{elems\_n2\_n}_3, \text{axiom})$   
 $n_2 \neq n_4$        $\text{cnf}(\text{elems\_n2\_n}_4, \text{axiom})$   
 $n_3 \neq n_4$        $\text{cnf}(\text{elems\_n3\_n}_4, \text{axiom})$   
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$        $\text{cnf}(\text{elems}, \text{axiom})$

**ALG308-1.p** Random graph 6, nu5 polymorphism

$t(y, x, x, x, x) = x$        $\text{cnf}(\text{polynu5}_{01}, \text{axiom})$   
 $t(x, y, x, x, x) = x$        $\text{cnf}(\text{polynu5}_{02}, \text{axiom})$

$t(x, x, y, x, x) = x$     cnf(polynu5<sub>03</sub>, axiom)  
 $t(x, x, x, y, x) = x$     cnf(polynu5<sub>04</sub>, axiom)  
 $t(x, x, x, x, y) = x$     cnf(polynu5<sub>05</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$     cnf  
 $\text{gr}(n_0, n_0)$     cnf(graph\_n0\_n0, axiom)  
 $\text{gr}(n_0, n_1)$     cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$     cnf(graph\_n0\_n2, axiom)  
 $\neg \text{gr}(n_0, n_3)$     cnf(graph\_n0\_n3, axiom)  
 $\neg \text{gr}(n_0, n_4)$     cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$     cnf(graph\_n0\_n5, axiom)  
 $\text{gr}(n_1, n_0)$     cnf(graph\_n1\_n0, axiom)  
 $\text{gr}(n_1, n_1)$     cnf(graph\_n1\_n1, axiom)  
 $\text{gr}(n_1, n_2)$     cnf(graph\_n1\_n2, axiom)  
 $\text{gr}(n_1, n_3)$     cnf(graph\_n1\_n3, axiom)  
 $\neg \text{gr}(n_1, n_4)$     cnf(graph\_n1\_n4, axiom)  
 $\text{gr}(n_1, n_5)$     cnf(graph\_n1\_n5, axiom)  
 $\neg \text{gr}(n_2, n_0)$     cnf(graph\_n2\_n0, axiom)  
 $\text{gr}(n_2, n_1)$     cnf(graph\_n2\_n1, axiom)  
 $\neg \text{gr}(n_2, n_2)$     cnf(graph\_n2\_n2, axiom)  
 $\text{gr}(n_2, n_3)$     cnf(graph\_n2\_n3, axiom)  
 $\neg \text{gr}(n_2, n_4)$     cnf(graph\_n2\_n4, axiom)  
 $\neg \text{gr}(n_2, n_5)$     cnf(graph\_n2\_n5, axiom)  
 $\text{gr}(n_3, n_0)$     cnf(graph\_n3\_n0, axiom)  
 $\text{gr}(n_3, n_1)$     cnf(graph\_n3\_n1, axiom)  
 $\neg \text{gr}(n_3, n_2)$     cnf(graph\_n3\_n2, axiom)  
 $\text{gr}(n_3, n_3)$     cnf(graph\_n3\_n3, axiom)  
 $\text{gr}(n_3, n_4)$     cnf(graph\_n3\_n4, axiom)  
 $\text{gr}(n_3, n_5)$     cnf(graph\_n3\_n5, axiom)  
 $\neg \text{gr}(n_4, n_0)$     cnf(graph\_n4\_n0, axiom)  
 $\text{gr}(n_4, n_1)$     cnf(graph\_n4\_n1, axiom)  
 $\neg \text{gr}(n_4, n_2)$     cnf(graph\_n4\_n2, axiom)  
 $\text{gr}(n_4, n_3)$     cnf(graph\_n4\_n3, axiom)  
 $\neg \text{gr}(n_4, n_4)$     cnf(graph\_n4\_n4, axiom)  
 $\text{gr}(n_4, n_5)$     cnf(graph\_n4\_n5, axiom)  
 $\text{gr}(n_5, n_0)$     cnf(graph\_n5\_n0, axiom)  
 $\text{gr}(n_5, n_1)$     cnf(graph\_n5\_n1, axiom)  
 $\text{gr}(n_5, n_2)$     cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$     cnf(graph\_n5\_n3, axiom)  
 $\text{gr}(n_5, n_4)$     cnf(graph\_n5\_n4, axiom)  
 $\neg \text{gr}(n_5, n_5)$     cnf(graph\_n5\_n5, axiom)  
 $n_0 \neq n_1$     cnf(elems\_n0\_n1, axiom)  
 $n_0 \neq n_2$     cnf(elems\_n0\_n2, axiom)  
 $n_0 \neq n_3$     cnf(elems\_n0\_n3, axiom)  
 $n_0 \neq n_4$     cnf(elems\_n0\_n4, axiom)  
 $n_0 \neq n_5$     cnf(elems\_n0\_n5, axiom)  
 $n_1 \neq n_2$     cnf(elems\_n1\_n2, axiom)  
 $n_1 \neq n_3$     cnf(elems\_n1\_n3, axiom)  
 $n_1 \neq n_4$     cnf(elems\_n1\_n4, axiom)  
 $n_1 \neq n_5$     cnf(elems\_n1\_n5, axiom)  
 $n_2 \neq n_3$     cnf(elems\_n2\_n3, axiom)  
 $n_2 \neq n_4$     cnf(elems\_n2\_n4, axiom)  
 $n_2 \neq n_5$     cnf(elems\_n2\_n5, axiom)  
 $n_3 \neq n_4$     cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$     cnf(elems\_n3\_n5, axiom)  
 $n_4 \neq n_5$     cnf(elems\_n4\_n5, axiom)  
 $x = n_0 \text{ or } x = n_1 \text{ or } x = n_2 \text{ or } x = n_3 \text{ or } x = n_4 \text{ or } x = n_5$     cnf(elems, axiom)

**ALG309-1.p** Random graph 7, nu5 polymorphism

$t(y, x, x, x, x) = x$     cnf(polynu5<sub>01</sub>, axiom)  
 $t(x, y, x, x, x) = x$     cnf(polynu5<sub>02</sub>, axiom)

$t(x, x, y, x, x) = x$     cnf(polynu5<sub>03</sub>, axiom)  
 $t(x, x, x, y, x) = x$     cnf(polynu5<sub>04</sub>, axiom)  
 $t(x, x, x, x, y) = x$     cnf(polynu5<sub>05</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$     cnf  
 $\text{gr}(n_0, n_0)$     cnf(graph\_n0\_n0, axiom)  
 $\neg \text{gr}(n_0, n_1)$     cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$     cnf(graph\_n0\_n2, axiom)  
 $\neg \text{gr}(n_0, n_3)$     cnf(graph\_n0\_n3, axiom)  
 $\neg \text{gr}(n_0, n_4)$     cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$     cnf(graph\_n0\_n5, axiom)  
 $\text{gr}(n_1, n_0)$     cnf(graph\_n1\_n0, axiom)  
 $\text{gr}(n_1, n_1)$     cnf(graph\_n1\_n1, axiom)  
 $\text{gr}(n_1, n_2)$     cnf(graph\_n1\_n2, axiom)  
 $\neg \text{gr}(n_1, n_3)$     cnf(graph\_n1\_n3, axiom)  
 $\neg \text{gr}(n_1, n_4)$     cnf(graph\_n1\_n4, axiom)  
 $\text{gr}(n_1, n_5)$     cnf(graph\_n1\_n5, axiom)  
 $\text{gr}(n_2, n_0)$     cnf(graph\_n2\_n0, axiom)  
 $\neg \text{gr}(n_2, n_1)$     cnf(graph\_n2\_n1, axiom)  
 $\neg \text{gr}(n_2, n_2)$     cnf(graph\_n2\_n2, axiom)  
 $\neg \text{gr}(n_2, n_3)$     cnf(graph\_n2\_n3, axiom)  
 $\neg \text{gr}(n_2, n_4)$     cnf(graph\_n2\_n4, axiom)  
 $\text{gr}(n_2, n_5)$     cnf(graph\_n2\_n5, axiom)  
 $\neg \text{gr}(n_3, n_0)$     cnf(graph\_n3\_n0, axiom)  
 $\text{gr}(n_3, n_1)$     cnf(graph\_n3\_n1, axiom)  
 $\text{gr}(n_3, n_2)$     cnf(graph\_n3\_n2, axiom)  
 $\neg \text{gr}(n_3, n_3)$     cnf(graph\_n3\_n3, axiom)  
 $\text{gr}(n_3, n_4)$     cnf(graph\_n3\_n4, axiom)  
 $\neg \text{gr}(n_3, n_5)$     cnf(graph\_n3\_n5, axiom)  
 $\neg \text{gr}(n_4, n_0)$     cnf(graph\_n4\_n0, axiom)  
 $\text{gr}(n_4, n_1)$     cnf(graph\_n4\_n1, axiom)  
 $\neg \text{gr}(n_4, n_2)$     cnf(graph\_n4\_n2, axiom)  
 $\neg \text{gr}(n_4, n_3)$     cnf(graph\_n4\_n3, axiom)  
 $\neg \text{gr}(n_4, n_4)$     cnf(graph\_n4\_n4, axiom)  
 $\neg \text{gr}(n_4, n_5)$     cnf(graph\_n4\_n5, axiom)  
 $\text{gr}(n_5, n_0)$     cnf(graph\_n5\_n0, axiom)  
 $\text{gr}(n_5, n_1)$     cnf(graph\_n5\_n1, axiom)  
 $\neg \text{gr}(n_5, n_2)$     cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$     cnf(graph\_n5\_n3, axiom)  
 $\text{gr}(n_5, n_4)$     cnf(graph\_n5\_n4, axiom)  
 $\text{gr}(n_5, n_5)$     cnf(graph\_n5\_n5, axiom)  
 $n_0 \neq n_1$     cnf(elems\_n0\_n1, axiom)  
 $n_0 \neq n_2$     cnf(elems\_n0\_n2, axiom)  
 $n_0 \neq n_3$     cnf(elems\_n0\_n3, axiom)  
 $n_0 \neq n_4$     cnf(elems\_n0\_n4, axiom)  
 $n_0 \neq n_5$     cnf(elems\_n0\_n5, axiom)  
 $n_1 \neq n_2$     cnf(elems\_n1\_n2, axiom)  
 $n_1 \neq n_3$     cnf(elems\_n1\_n3, axiom)  
 $n_1 \neq n_4$     cnf(elems\_n1\_n4, axiom)  
 $n_1 \neq n_5$     cnf(elems\_n1\_n5, axiom)  
 $n_2 \neq n_3$     cnf(elems\_n2\_n3, axiom)  
 $n_2 \neq n_4$     cnf(elems\_n2\_n4, axiom)  
 $n_2 \neq n_5$     cnf(elems\_n2\_n5, axiom)  
 $n_3 \neq n_4$     cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$     cnf(elems\_n3\_n5, axiom)  
 $n_4 \neq n_5$     cnf(elems\_n4\_n5, axiom)  
 $x = n_0 \text{ or } x = n_1 \text{ or } x = n_2 \text{ or } x = n_3 \text{ or } x = n_4 \text{ or } x = n_5$     cnf(elems, axiom)

**ALG310-1.p** Random graph 8, nu5 polymorphism

$t(y, x, x, x, x) = x$     cnf(polynu5<sub>01</sub>, axiom)  
 $t(x, y, x, x, x) = x$     cnf(polynu5<sub>02</sub>, axiom)

$t(x, x, y, x, x) = x$     cnf(polynu5<sub>03</sub>, axiom)  
 $t(x, x, x, y, x) = x$     cnf(polynu5<sub>04</sub>, axiom)  
 $t(x, x, x, x, y) = x$     cnf(polynu5<sub>05</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$     cnf  
 $\neg \text{gr}(n_0, n_0)$     cnf(graph\_n0\_n0, axiom)  
 $\neg \text{gr}(n_0, n_1)$     cnf(graph\_n0\_n1, axiom)  
 $\text{gr}(n_0, n_2)$     cnf(graph\_n0\_n2, axiom)  
 $\text{gr}(n_0, n_3)$     cnf(graph\_n0\_n3, axiom)  
 $\text{gr}(n_0, n_4)$     cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$     cnf(graph\_n0\_n5, axiom)  
 $\neg \text{gr}(n_1, n_0)$     cnf(graph\_n1\_n0, axiom)  
 $\neg \text{gr}(n_1, n_1)$     cnf(graph\_n1\_n1, axiom)  
 $\text{gr}(n_1, n_2)$     cnf(graph\_n1\_n2, axiom)  
 $\text{gr}(n_1, n_3)$     cnf(graph\_n1\_n3, axiom)  
 $\text{gr}(n_1, n_4)$     cnf(graph\_n1\_n4, axiom)  
 $\neg \text{gr}(n_1, n_5)$     cnf(graph\_n1\_n5, axiom)  
 $\neg \text{gr}(n_2, n_0)$     cnf(graph\_n2\_n0, axiom)  
 $\text{gr}(n_2, n_1)$     cnf(graph\_n2\_n1, axiom)  
 $\text{gr}(n_2, n_2)$     cnf(graph\_n2\_n2, axiom)  
 $\text{gr}(n_2, n_3)$     cnf(graph\_n2\_n3, axiom)  
 $\text{gr}(n_2, n_4)$     cnf(graph\_n2\_n4, axiom)  
 $\text{gr}(n_2, n_5)$     cnf(graph\_n2\_n5, axiom)  
 $\text{gr}(n_3, n_0)$     cnf(graph\_n3\_n0, axiom)  
 $\text{gr}(n_3, n_1)$     cnf(graph\_n3\_n1, axiom)  
 $\neg \text{gr}(n_3, n_2)$     cnf(graph\_n3\_n2, axiom)  
 $\neg \text{gr}(n_3, n_3)$     cnf(graph\_n3\_n3, axiom)  
 $\neg \text{gr}(n_3, n_4)$     cnf(graph\_n3\_n4, axiom)  
 $\text{gr}(n_3, n_5)$     cnf(graph\_n3\_n5, axiom)  
 $\text{gr}(n_4, n_0)$     cnf(graph\_n4\_n0, axiom)  
 $\text{gr}(n_4, n_1)$     cnf(graph\_n4\_n1, axiom)  
 $\text{gr}(n_4, n_2)$     cnf(graph\_n4\_n2, axiom)  
 $\text{gr}(n_4, n_3)$     cnf(graph\_n4\_n3, axiom)  
 $\neg \text{gr}(n_4, n_4)$     cnf(graph\_n4\_n4, axiom)  
 $\text{gr}(n_4, n_5)$     cnf(graph\_n4\_n5, axiom)  
 $\neg \text{gr}(n_5, n_0)$     cnf(graph\_n5\_n0, axiom)  
 $\neg \text{gr}(n_5, n_1)$     cnf(graph\_n5\_n1, axiom)  
 $\text{gr}(n_5, n_2)$     cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$     cnf(graph\_n5\_n3, axiom)  
 $\text{gr}(n_5, n_4)$     cnf(graph\_n5\_n4, axiom)  
 $\neg \text{gr}(n_5, n_5)$     cnf(graph\_n5\_n5, axiom)  
 $n_0 \neq n_1$     cnf(elems\_n0\_n1, axiom)  
 $n_0 \neq n_2$     cnf(elems\_n0\_n2, axiom)  
 $n_0 \neq n_3$     cnf(elems\_n0\_n3, axiom)  
 $n_0 \neq n_4$     cnf(elems\_n0\_n4, axiom)  
 $n_0 \neq n_5$     cnf(elems\_n0\_n5, axiom)  
 $n_1 \neq n_2$     cnf(elems\_n1\_n2, axiom)  
 $n_1 \neq n_3$     cnf(elems\_n1\_n3, axiom)  
 $n_1 \neq n_4$     cnf(elems\_n1\_n4, axiom)  
 $n_1 \neq n_5$     cnf(elems\_n1\_n5, axiom)  
 $n_2 \neq n_3$     cnf(elems\_n2\_n3, axiom)  
 $n_2 \neq n_4$     cnf(elems\_n2\_n4, axiom)  
 $n_2 \neq n_5$     cnf(elems\_n2\_n5, axiom)  
 $n_3 \neq n_4$     cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$     cnf(elems\_n3\_n5, axiom)  
 $n_4 \neq n_5$     cnf(elems\_n4\_n5, axiom)  
 $x = n_0 \text{ or } x = n_1 \text{ or } x = n_2 \text{ or } x = n_3 \text{ or } x = n_4 \text{ or } x = n_5$     cnf(elems, axiom)

**ALG311-1.p** Random graph 9, nu5 polymorphism

$t(y, x, x, x, x) = x$     cnf(polynu5<sub>01</sub>, axiom)  
 $t(x, y, x, x, x) = x$     cnf(polynu5<sub>02</sub>, axiom)

$t(x, x, y, x, x) = x$     cnf(polynu5<sub>03</sub>, axiom)  
 $t(x, x, x, y, x) = x$     cnf(polynu5<sub>04</sub>, axiom)  
 $t(x, x, x, x, y) = x$     cnf(polynu5<sub>05</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$     cnf  
 $\neg \text{gr}(n_0, n_0)$     cnf(graph\_n0\_n0, axiom)  
 $\neg \text{gr}(n_0, n_1)$     cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$     cnf(graph\_n0\_n2, axiom)  
 $\neg \text{gr}(n_0, n_3)$     cnf(graph\_n0\_n3, axiom)  
 $\neg \text{gr}(n_0, n_4)$     cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$     cnf(graph\_n0\_n5, axiom)  
 $\neg \text{gr}(n_1, n_0)$     cnf(graph\_n1\_n0, axiom)  
 $\neg \text{gr}(n_1, n_1)$     cnf(graph\_n1\_n1, axiom)  
 $\neg \text{gr}(n_1, n_2)$     cnf(graph\_n1\_n2, axiom)  
 $\text{gr}(n_1, n_3)$     cnf(graph\_n1\_n3, axiom)  
 $\text{gr}(n_1, n_4)$     cnf(graph\_n1\_n4, axiom)  
 $\text{gr}(n_1, n_5)$     cnf(graph\_n1\_n5, axiom)  
 $\text{gr}(n_2, n_0)$     cnf(graph\_n2\_n0, axiom)  
 $\neg \text{gr}(n_2, n_1)$     cnf(graph\_n2\_n1, axiom)  
 $\text{gr}(n_2, n_2)$     cnf(graph\_n2\_n2, axiom)  
 $\neg \text{gr}(n_2, n_3)$     cnf(graph\_n2\_n3, axiom)  
 $\neg \text{gr}(n_2, n_4)$     cnf(graph\_n2\_n4, axiom)  
 $\neg \text{gr}(n_2, n_5)$     cnf(graph\_n2\_n5, axiom)  
 $\neg \text{gr}(n_3, n_0)$     cnf(graph\_n3\_n0, axiom)  
 $\text{gr}(n_3, n_1)$     cnf(graph\_n3\_n1, axiom)  
 $\neg \text{gr}(n_3, n_2)$     cnf(graph\_n3\_n2, axiom)  
 $\text{gr}(n_3, n_3)$     cnf(graph\_n3\_n3, axiom)  
 $\neg \text{gr}(n_3, n_4)$     cnf(graph\_n3\_n4, axiom)  
 $\text{gr}(n_3, n_5)$     cnf(graph\_n3\_n5, axiom)  
 $\neg \text{gr}(n_4, n_0)$     cnf(graph\_n4\_n0, axiom)  
 $\text{gr}(n_4, n_1)$     cnf(graph\_n4\_n1, axiom)  
 $\neg \text{gr}(n_4, n_2)$     cnf(graph\_n4\_n2, axiom)  
 $\text{gr}(n_4, n_3)$     cnf(graph\_n4\_n3, axiom)  
 $\neg \text{gr}(n_4, n_4)$     cnf(graph\_n4\_n4, axiom)  
 $\neg \text{gr}(n_4, n_5)$     cnf(graph\_n4\_n5, axiom)  
 $\text{gr}(n_5, n_0)$     cnf(graph\_n5\_n0, axiom)  
 $\text{gr}(n_5, n_1)$     cnf(graph\_n5\_n1, axiom)  
 $\neg \text{gr}(n_5, n_2)$     cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$     cnf(graph\_n5\_n3, axiom)  
 $\text{gr}(n_5, n_4)$     cnf(graph\_n5\_n4, axiom)  
 $\text{gr}(n_5, n_5)$     cnf(graph\_n5\_n5, axiom)  
 $n_0 \neq n_1$     cnf(elems\_n0\_n1, axiom)  
 $n_0 \neq n_2$     cnf(elems\_n0\_n2, axiom)  
 $n_0 \neq n_3$     cnf(elems\_n0\_n3, axiom)  
 $n_0 \neq n_4$     cnf(elems\_n0\_n4, axiom)  
 $n_0 \neq n_5$     cnf(elems\_n0\_n5, axiom)  
 $n_1 \neq n_2$     cnf(elems\_n1\_n2, axiom)  
 $n_1 \neq n_3$     cnf(elems\_n1\_n3, axiom)  
 $n_1 \neq n_4$     cnf(elems\_n1\_n4, axiom)  
 $n_1 \neq n_5$     cnf(elems\_n1\_n5, axiom)  
 $n_2 \neq n_3$     cnf(elems\_n2\_n3, axiom)  
 $n_2 \neq n_4$     cnf(elems\_n2\_n4, axiom)  
 $n_2 \neq n_5$     cnf(elems\_n2\_n5, axiom)  
 $n_3 \neq n_4$     cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$     cnf(elems\_n3\_n5, axiom)  
 $n_4 \neq n_5$     cnf(elems\_n4\_n5, axiom)  
 $x = n_0 \text{ or } x = n_1 \text{ or } x = n_2 \text{ or } x = n_3 \text{ or } x = n_4 \text{ or } x = n_5$     cnf(elems, axiom)

**ALG312-1.p** Random graph 10, nu5 polymorphism

$t(y, x, x, x, x) = x$     cnf(polynu5<sub>01</sub>, axiom)  
 $t(x, y, x, x, x) = x$     cnf(polynu5<sub>02</sub>, axiom)

$t(x, x, y, x, x) = x$     cnf(polynu5<sub>03</sub>, axiom)  
 $t(x, x, x, y, x) = x$     cnf(polynu5<sub>04</sub>, axiom)  
 $t(x, x, x, x, y) = x$     cnf(polynu5<sub>05</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$     cnf  
 $\neg \text{gr}(n_0, n_0)$     cnf(graph\_n0\_n0, axiom)  
 $\neg \text{gr}(n_0, n_1)$     cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$     cnf(graph\_n0\_n2, axiom)  
 $\neg \text{gr}(n_0, n_3)$     cnf(graph\_n0\_n3, axiom)  
 $\text{gr}(n_0, n_4)$     cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$     cnf(graph\_n0\_n5, axiom)  
 $\neg \text{gr}(n_1, n_0)$     cnf(graph\_n1\_n0, axiom)  
 $\neg \text{gr}(n_1, n_1)$     cnf(graph\_n1\_n1, axiom)  
 $\text{gr}(n_1, n_2)$     cnf(graph\_n1\_n2, axiom)  
 $\neg \text{gr}(n_1, n_3)$     cnf(graph\_n1\_n3, axiom)  
 $\neg \text{gr}(n_1, n_4)$     cnf(graph\_n1\_n4, axiom)  
 $\neg \text{gr}(n_1, n_5)$     cnf(graph\_n1\_n5, axiom)  
 $\neg \text{gr}(n_2, n_0)$     cnf(graph\_n2\_n0, axiom)  
 $\text{gr}(n_2, n_1)$     cnf(graph\_n2\_n1, axiom)  
 $\neg \text{gr}(n_2, n_2)$     cnf(graph\_n2\_n2, axiom)  
 $\neg \text{gr}(n_2, n_3)$     cnf(graph\_n2\_n3, axiom)  
 $\neg \text{gr}(n_2, n_4)$     cnf(graph\_n2\_n4, axiom)  
 $\neg \text{gr}(n_2, n_5)$     cnf(graph\_n2\_n5, axiom)  
 $\neg \text{gr}(n_3, n_0)$     cnf(graph\_n3\_n0, axiom)  
 $\text{gr}(n_3, n_1)$     cnf(graph\_n3\_n1, axiom)  
 $\neg \text{gr}(n_3, n_2)$     cnf(graph\_n3\_n2, axiom)  
 $\text{gr}(n_3, n_3)$     cnf(graph\_n3\_n3, axiom)  
 $\neg \text{gr}(n_3, n_4)$     cnf(graph\_n3\_n4, axiom)  
 $\text{gr}(n_3, n_5)$     cnf(graph\_n3\_n5, axiom)  
 $\neg \text{gr}(n_4, n_0)$     cnf(graph\_n4\_n0, axiom)  
 $\text{gr}(n_4, n_1)$     cnf(graph\_n4\_n1, axiom)  
 $\neg \text{gr}(n_4, n_2)$     cnf(graph\_n4\_n2, axiom)  
 $\neg \text{gr}(n_4, n_3)$     cnf(graph\_n4\_n3, axiom)  
 $\neg \text{gr}(n_4, n_4)$     cnf(graph\_n4\_n4, axiom)  
 $\text{gr}(n_4, n_5)$     cnf(graph\_n4\_n5, axiom)  
 $\neg \text{gr}(n_5, n_0)$     cnf(graph\_n5\_n0, axiom)  
 $\text{gr}(n_5, n_1)$     cnf(graph\_n5\_n1, axiom)  
 $\neg \text{gr}(n_5, n_2)$     cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$     cnf(graph\_n5\_n3, axiom)  
 $\neg \text{gr}(n_5, n_4)$     cnf(graph\_n5\_n4, axiom)  
 $\neg \text{gr}(n_5, n_5)$     cnf(graph\_n5\_n5, axiom)  
 $n_0 \neq n_1$     cnf(elems\_n0\_n1, axiom)  
 $n_0 \neq n_2$     cnf(elems\_n0\_n2, axiom)  
 $n_0 \neq n_3$     cnf(elems\_n0\_n3, axiom)  
 $n_0 \neq n_4$     cnf(elems\_n0\_n4, axiom)  
 $n_0 \neq n_5$     cnf(elems\_n0\_n5, axiom)  
 $n_1 \neq n_2$     cnf(elems\_n1\_n2, axiom)  
 $n_1 \neq n_3$     cnf(elems\_n1\_n3, axiom)  
 $n_1 \neq n_4$     cnf(elems\_n1\_n4, axiom)  
 $n_1 \neq n_5$     cnf(elems\_n1\_n5, axiom)  
 $n_2 \neq n_3$     cnf(elems\_n2\_n3, axiom)  
 $n_2 \neq n_4$     cnf(elems\_n2\_n4, axiom)  
 $n_2 \neq n_5$     cnf(elems\_n2\_n5, axiom)  
 $n_3 \neq n_4$     cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$     cnf(elems\_n3\_n5, axiom)  
 $n_4 \neq n_5$     cnf(elems\_n4\_n5, axiom)  
 $x = n_0 \text{ or } x = n_1 \text{ or } x = n_2 \text{ or } x = n_3 \text{ or } x = n_4 \text{ or } x = n_5$     cnf(elems, axiom)

**ALG313-1.p** Random graph 11, nu5 polymorphism

$t(y, x, x, x, x) = x$     cnf(polynu5<sub>01</sub>, axiom)  
 $t(x, y, x, x, x) = x$     cnf(polynu5<sub>02</sub>, axiom)



$t(x, x, y, x, x) = x$     cnf(polynu5<sub>03</sub>, axiom)  
 $t(x, x, x, y, x) = x$     cnf(polynu5<sub>04</sub>, axiom)  
 $t(x, x, x, x, y) = x$     cnf(polynu5<sub>05</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$     cnf  
 $\neg \text{gr}(n_0, n_0)$     cnf(graph\_n0\_n0, axiom)  
 $\text{gr}(n_0, n_1)$     cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$     cnf(graph\_n0\_n2, axiom)  
 $\text{gr}(n_0, n_3)$     cnf(graph\_n0\_n3, axiom)  
 $\neg \text{gr}(n_0, n_4)$     cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$     cnf(graph\_n0\_n5, axiom)  
 $\text{gr}(n_1, n_0)$     cnf(graph\_n1\_n0, axiom)  
 $\neg \text{gr}(n_1, n_1)$     cnf(graph\_n1\_n1, axiom)  
 $\text{gr}(n_1, n_2)$     cnf(graph\_n1\_n2, axiom)  
 $\text{gr}(n_1, n_3)$     cnf(graph\_n1\_n3, axiom)  
 $\text{gr}(n_1, n_4)$     cnf(graph\_n1\_n4, axiom)  
 $\neg \text{gr}(n_1, n_5)$     cnf(graph\_n1\_n5, axiom)  
 $\text{gr}(n_2, n_0)$     cnf(graph\_n2\_n0, axiom)  
 $\text{gr}(n_2, n_1)$     cnf(graph\_n2\_n1, axiom)  
 $\text{gr}(n_2, n_2)$     cnf(graph\_n2\_n2, axiom)  
 $\text{gr}(n_2, n_3)$     cnf(graph\_n2\_n3, axiom)  
 $\text{gr}(n_2, n_4)$     cnf(graph\_n2\_n4, axiom)  
 $\text{gr}(n_2, n_5)$     cnf(graph\_n2\_n5, axiom)  
 $\neg \text{gr}(n_3, n_0)$     cnf(graph\_n3\_n0, axiom)  
 $\text{gr}(n_3, n_1)$     cnf(graph\_n3\_n1, axiom)  
 $\text{gr}(n_3, n_2)$     cnf(graph\_n3\_n2, axiom)  
 $\text{gr}(n_3, n_3)$     cnf(graph\_n3\_n3, axiom)  
 $\text{gr}(n_3, n_4)$     cnf(graph\_n3\_n4, axiom)  
 $\text{gr}(n_3, n_5)$     cnf(graph\_n3\_n5, axiom)  
 $\text{gr}(n_4, n_0)$     cnf(graph\_n4\_n0, axiom)  
 $\text{gr}(n_4, n_1)$     cnf(graph\_n4\_n1, axiom)  
 $\text{gr}(n_4, n_2)$     cnf(graph\_n4\_n2, axiom)  
 $\text{gr}(n_4, n_3)$     cnf(graph\_n4\_n3, axiom)  
 $\neg \text{gr}(n_4, n_4)$     cnf(graph\_n4\_n4, axiom)  
 $\neg \text{gr}(n_4, n_5)$     cnf(graph\_n4\_n5, axiom)  
 $\text{gr}(n_5, n_0)$     cnf(graph\_n5\_n0, axiom)  
 $\text{gr}(n_5, n_1)$     cnf(graph\_n5\_n1, axiom)  
 $\text{gr}(n_5, n_2)$     cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$     cnf(graph\_n5\_n3, axiom)  
 $\neg \text{gr}(n_5, n_4)$     cnf(graph\_n5\_n4, axiom)  
 $\neg \text{gr}(n_5, n_5)$     cnf(graph\_n5\_n5, axiom)  
 $n_0 \neq n_1$     cnf(elems\_n0\_n1, axiom)  
 $n_0 \neq n_2$     cnf(elems\_n0\_n2, axiom)  
 $n_0 \neq n_3$     cnf(elems\_n0\_n3, axiom)  
 $n_0 \neq n_4$     cnf(elems\_n0\_n4, axiom)  
 $n_0 \neq n_5$     cnf(elems\_n0\_n5, axiom)  
 $n_1 \neq n_2$     cnf(elems\_n1\_n2, axiom)  
 $n_1 \neq n_3$     cnf(elems\_n1\_n3, axiom)  
 $n_1 \neq n_4$     cnf(elems\_n1\_n4, axiom)  
 $n_1 \neq n_5$     cnf(elems\_n1\_n5, axiom)  
 $n_2 \neq n_3$     cnf(elems\_n2\_n3, axiom)  
 $n_2 \neq n_4$     cnf(elems\_n2\_n4, axiom)  
 $n_2 \neq n_5$     cnf(elems\_n2\_n5, axiom)  
 $n_3 \neq n_4$     cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$     cnf(elems\_n3\_n5, axiom)  
 $n_4 \neq n_5$     cnf(elems\_n4\_n5, axiom)  
 $x = n_0 \text{ or } x = n_1 \text{ or } x = n_2 \text{ or } x = n_3 \text{ or } x = n_4 \text{ or } x = n_5$     cnf(elems, axiom)

**ALG314-1.p** Random graph 12, nu5 polymorphism

$t(y, x, x, x, x) = x$     cnf(polynu5<sub>01</sub>, axiom)  
 $t(x, y, x, x, x) = x$     cnf(polynu5<sub>02</sub>, axiom)

$t(x, x, y, x, x) = x$     cnf(polynu5<sub>03</sub>, axiom)  
 $t(x, x, x, y, x) = x$     cnf(polynu5<sub>04</sub>, axiom)  
 $t(x, x, x, x, y) = x$     cnf(polynu5<sub>05</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$     cnf  
 $\neg \text{gr}(n_0, n_0)$     cnf(graph\_n0\_n0, axiom)  
 $\text{gr}(n_0, n_1)$     cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$     cnf(graph\_n0\_n2, axiom)  
 $\neg \text{gr}(n_0, n_3)$     cnf(graph\_n0\_n3, axiom)  
 $\text{gr}(n_0, n_4)$     cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$     cnf(graph\_n0\_n5, axiom)  
 $\text{gr}(n_1, n_0)$     cnf(graph\_n1\_n0, axiom)  
 $\text{gr}(n_1, n_1)$     cnf(graph\_n1\_n1, axiom)  
 $\text{gr}(n_1, n_2)$     cnf(graph\_n1\_n2, axiom)  
 $\neg \text{gr}(n_1, n_3)$     cnf(graph\_n1\_n3, axiom)  
 $\text{gr}(n_1, n_4)$     cnf(graph\_n1\_n4, axiom)  
 $\text{gr}(n_1, n_5)$     cnf(graph\_n1\_n5, axiom)  
 $\neg \text{gr}(n_2, n_0)$     cnf(graph\_n2\_n0, axiom)  
 $\neg \text{gr}(n_2, n_1)$     cnf(graph\_n2\_n1, axiom)  
 $\text{gr}(n_2, n_2)$     cnf(graph\_n2\_n2, axiom)  
 $\neg \text{gr}(n_2, n_3)$     cnf(graph\_n2\_n3, axiom)  
 $\neg \text{gr}(n_2, n_4)$     cnf(graph\_n2\_n4, axiom)  
 $\neg \text{gr}(n_2, n_5)$     cnf(graph\_n2\_n5, axiom)  
 $\text{gr}(n_3, n_0)$     cnf(graph\_n3\_n0, axiom)  
 $\text{gr}(n_3, n_1)$     cnf(graph\_n3\_n1, axiom)  
 $\neg \text{gr}(n_3, n_2)$     cnf(graph\_n3\_n2, axiom)  
 $\text{gr}(n_3, n_3)$     cnf(graph\_n3\_n3, axiom)  
 $\text{gr}(n_3, n_4)$     cnf(graph\_n3\_n4, axiom)  
 $\text{gr}(n_3, n_5)$     cnf(graph\_n3\_n5, axiom)  
 $\neg \text{gr}(n_4, n_0)$     cnf(graph\_n4\_n0, axiom)  
 $\text{gr}(n_4, n_1)$     cnf(graph\_n4\_n1, axiom)  
 $\text{gr}(n_4, n_2)$     cnf(graph\_n4\_n2, axiom)  
 $\text{gr}(n_4, n_3)$     cnf(graph\_n4\_n3, axiom)  
 $\neg \text{gr}(n_4, n_4)$     cnf(graph\_n4\_n4, axiom)  
 $\text{gr}(n_4, n_5)$     cnf(graph\_n4\_n5, axiom)  
 $\text{gr}(n_5, n_0)$     cnf(graph\_n5\_n0, axiom)  
 $\text{gr}(n_5, n_1)$     cnf(graph\_n5\_n1, axiom)  
 $\neg \text{gr}(n_5, n_2)$     cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$     cnf(graph\_n5\_n3, axiom)  
 $\text{gr}(n_5, n_4)$     cnf(graph\_n5\_n4, axiom)  
 $\text{gr}(n_5, n_5)$     cnf(graph\_n5\_n5, axiom)  
 $n_0 \neq n_1$     cnf(elems\_n0\_n1, axiom)  
 $n_0 \neq n_2$     cnf(elems\_n0\_n2, axiom)  
 $n_0 \neq n_3$     cnf(elems\_n0\_n3, axiom)  
 $n_0 \neq n_4$     cnf(elems\_n0\_n4, axiom)  
 $n_0 \neq n_5$     cnf(elems\_n0\_n5, axiom)  
 $n_1 \neq n_2$     cnf(elems\_n1\_n2, axiom)  
 $n_1 \neq n_3$     cnf(elems\_n1\_n3, axiom)  
 $n_1 \neq n_4$     cnf(elems\_n1\_n4, axiom)  
 $n_1 \neq n_5$     cnf(elems\_n1\_n5, axiom)  
 $n_2 \neq n_3$     cnf(elems\_n2\_n3, axiom)  
 $n_2 \neq n_4$     cnf(elems\_n2\_n4, axiom)  
 $n_2 \neq n_5$     cnf(elems\_n2\_n5, axiom)  
 $n_3 \neq n_4$     cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$     cnf(elems\_n3\_n5, axiom)  
 $n_4 \neq n_5$     cnf(elems\_n4\_n5, axiom)  
 $x = n_0 \text{ or } x = n_1 \text{ or } x = n_2 \text{ or } x = n_3 \text{ or } x = n_4 \text{ or } x = n_5$     cnf(elems, axiom)

**ALG315-1.p** Random graph 13, nu5 polymorphism

$t(y, x, x, x, x) = x$     cnf(polynu5<sub>01</sub>, axiom)  
 $t(x, y, x, x, x) = x$     cnf(polynu5<sub>02</sub>, axiom)

$t(x, x, y, x, x) = x$     cnf(polynu5<sub>03</sub>, axiom)  
 $t(x, x, x, y, x) = x$     cnf(polynu5<sub>04</sub>, axiom)  
 $t(x, x, x, x, y) = x$     cnf(polynu5<sub>05</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$     cnf  
 $\neg \text{gr}(n_0, n_0)$     cnf(graph\_n0\_n0, axiom)  
 $\neg \text{gr}(n_0, n_1)$     cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$     cnf(graph\_n0\_n2, axiom)  
 $\text{gr}(n_0, n_3)$     cnf(graph\_n0\_n3, axiom)  
 $\text{gr}(n_0, n_4)$     cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$     cnf(graph\_n0\_n5, axiom)  
 $\neg \text{gr}(n_0, n_6)$     cnf(graph\_n0\_n6, axiom)  
 $\text{gr}(n_1, n_0)$     cnf(graph\_n1\_n0, axiom)  
 $\text{gr}(n_1, n_1)$     cnf(graph\_n1\_n1, axiom)  
 $\text{gr}(n_1, n_2)$     cnf(graph\_n1\_n2, axiom)  
 $\neg \text{gr}(n_1, n_3)$     cnf(graph\_n1\_n3, axiom)  
 $\text{gr}(n_1, n_4)$     cnf(graph\_n1\_n4, axiom)  
 $\neg \text{gr}(n_1, n_5)$     cnf(graph\_n1\_n5, axiom)  
 $\neg \text{gr}(n_1, n_6)$     cnf(graph\_n1\_n6, axiom)  
 $\text{gr}(n_2, n_0)$     cnf(graph\_n2\_n0, axiom)  
 $\text{gr}(n_2, n_1)$     cnf(graph\_n2\_n1, axiom)  
 $\text{gr}(n_2, n_2)$     cnf(graph\_n2\_n2, axiom)  
 $\text{gr}(n_2, n_3)$     cnf(graph\_n2\_n3, axiom)  
 $\text{gr}(n_2, n_4)$     cnf(graph\_n2\_n4, axiom)  
 $\text{gr}(n_2, n_5)$     cnf(graph\_n2\_n5, axiom)  
 $\text{gr}(n_2, n_6)$     cnf(graph\_n2\_n6, axiom)  
 $\text{gr}(n_3, n_0)$     cnf(graph\_n3\_n0, axiom)  
 $\neg \text{gr}(n_3, n_1)$     cnf(graph\_n3\_n1, axiom)  
 $\text{gr}(n_3, n_2)$     cnf(graph\_n3\_n2, axiom)  
 $\text{gr}(n_3, n_3)$     cnf(graph\_n3\_n3, axiom)  
 $\text{gr}(n_3, n_4)$     cnf(graph\_n3\_n4, axiom)  
 $\neg \text{gr}(n_3, n_5)$     cnf(graph\_n3\_n5, axiom)  
 $\text{gr}(n_3, n_6)$     cnf(graph\_n3\_n6, axiom)  
 $\neg \text{gr}(n_4, n_0)$     cnf(graph\_n4\_n0, axiom)  
 $\text{gr}(n_4, n_1)$     cnf(graph\_n4\_n1, axiom)  
 $\text{gr}(n_4, n_2)$     cnf(graph\_n4\_n2, axiom)  
 $\text{gr}(n_4, n_3)$     cnf(graph\_n4\_n3, axiom)  
 $\neg \text{gr}(n_4, n_4)$     cnf(graph\_n4\_n4, axiom)  
 $\text{gr}(n_4, n_5)$     cnf(graph\_n4\_n5, axiom)  
 $\text{gr}(n_4, n_6)$     cnf(graph\_n4\_n6, axiom)  
 $\text{gr}(n_5, n_0)$     cnf(graph\_n5\_n0, axiom)  
 $\text{gr}(n_5, n_1)$     cnf(graph\_n5\_n1, axiom)  
 $\text{gr}(n_5, n_2)$     cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$     cnf(graph\_n5\_n3, axiom)  
 $\text{gr}(n_5, n_4)$     cnf(graph\_n5\_n4, axiom)  
 $\text{gr}(n_5, n_5)$     cnf(graph\_n5\_n5, axiom)  
 $\text{gr}(n_5, n_6)$     cnf(graph\_n5\_n6, axiom)  
 $\text{gr}(n_6, n_0)$     cnf(graph\_n6\_n0, axiom)  
 $\text{gr}(n_6, n_1)$     cnf(graph\_n6\_n1, axiom)  
 $\text{gr}(n_6, n_2)$     cnf(graph\_n6\_n2, axiom)  
 $\text{gr}(n_6, n_3)$     cnf(graph\_n6\_n3, axiom)  
 $\neg \text{gr}(n_6, n_4)$     cnf(graph\_n6\_n4, axiom)  
 $\text{gr}(n_6, n_5)$     cnf(graph\_n6\_n5, axiom)  
 $\text{gr}(n_6, n_6)$     cnf(graph\_n6\_n6, axiom)  
 $n_0 \neq n_1$     cnf(elems\_n0\_n1, axiom)  
 $n_0 \neq n_2$     cnf(elems\_n0\_n2, axiom)  
 $n_0 \neq n_3$     cnf(elems\_n0\_n3, axiom)  
 $n_0 \neq n_4$     cnf(elems\_n0\_n4, axiom)  
 $n_0 \neq n_5$     cnf(elems\_n0\_n5, axiom)  
 $n_0 \neq n_6$     cnf(elems\_n0\_n6, axiom)

$n_1 \neq n_2$       cnf(elems\_n1\_n2, axiom)  
 $n_1 \neq n_3$       cnf(elems\_n1\_n3, axiom)  
 $n_1 \neq n_4$       cnf(elems\_n1\_n4, axiom)  
 $n_1 \neq n_5$       cnf(elems\_n1\_n5, axiom)  
 $n_1 \neq n_6$       cnf(elems\_n1\_n6, axiom)  
 $n_2 \neq n_3$       cnf(elems\_n2\_n3, axiom)  
 $n_2 \neq n_4$       cnf(elems\_n2\_n4, axiom)  
 $n_2 \neq n_5$       cnf(elems\_n2\_n5, axiom)  
 $n_2 \neq n_6$       cnf(elems\_n2\_n6, axiom)  
 $n_3 \neq n_4$       cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$       cnf(elems\_n3\_n5, axiom)  
 $n_3 \neq n_6$       cnf(elems\_n3\_n6, axiom)  
 $n_4 \neq n_5$       cnf(elems\_n4\_n5, axiom)  
 $n_4 \neq n_6$       cnf(elems\_n4\_n6, axiom)  
 $n_5 \neq n_6$       cnf(elems\_n5\_n6, axiom)  
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$  or  $x = n_5$  or  $x = n_6$       cnf(elems, axiom)

**ALG316-1.p** Random graph 14, siggers polymorphism

$t(x, x, x, x) = x$       cnf(polysiggers<sub>01</sub>, axiom)  
 $t(x, y, x, z) = t(y, x, z, y)$       cnf(polysiggers<sub>02</sub>, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6), t(x_1, x_3, x_5, x_7))$       cnf(preserves<sub>03</sub>, axiom)  
 $\neg \text{gr}(n_0, n_0)$       cnf(graph\_n0\_n0, axiom)  
 $\neg \text{gr}(n_0, n_1)$       cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$       cnf(graph\_n0\_n2, axiom)  
 $\neg \text{gr}(n_0, n_3)$       cnf(graph\_n0\_n3, axiom)  
 $\neg \text{gr}(n_0, n_4)$       cnf(graph\_n0\_n4, axiom)  
 $\neg \text{gr}(n_0, n_5)$       cnf(graph\_n0\_n5, axiom)  
 $\text{gr}(n_0, n_6)$       cnf(graph\_n0\_n6, axiom)  
 $\neg \text{gr}(n_1, n_0)$       cnf(graph\_n1\_n0, axiom)  
 $\neg \text{gr}(n_1, n_1)$       cnf(graph\_n1\_n1, axiom)  
 $\neg \text{gr}(n_1, n_2)$       cnf(graph\_n1\_n2, axiom)  
 $\neg \text{gr}(n_1, n_3)$       cnf(graph\_n1\_n3, axiom)  
 $\neg \text{gr}(n_1, n_4)$       cnf(graph\_n1\_n4, axiom)  
 $\text{gr}(n_1, n_5)$       cnf(graph\_n1\_n5, axiom)  
 $\neg \text{gr}(n_1, n_6)$       cnf(graph\_n1\_n6, axiom)  
 $\neg \text{gr}(n_2, n_0)$       cnf(graph\_n2\_n0, axiom)  
 $\text{gr}(n_2, n_1)$       cnf(graph\_n2\_n1, axiom)  
 $\text{gr}(n_2, n_2)$       cnf(graph\_n2\_n2, axiom)  
 $\neg \text{gr}(n_2, n_3)$       cnf(graph\_n2\_n3, axiom)  
 $\neg \text{gr}(n_2, n_4)$       cnf(graph\_n2\_n4, axiom)  
 $\text{gr}(n_2, n_5)$       cnf(graph\_n2\_n5, axiom)  
 $\neg \text{gr}(n_2, n_6)$       cnf(graph\_n2\_n6, axiom)  
 $\neg \text{gr}(n_3, n_0)$       cnf(graph\_n3\_n0, axiom)  
 $\neg \text{gr}(n_3, n_1)$       cnf(graph\_n3\_n1, axiom)  
 $\neg \text{gr}(n_3, n_2)$       cnf(graph\_n3\_n2, axiom)  
 $\neg \text{gr}(n_3, n_3)$       cnf(graph\_n3\_n3, axiom)  
 $\neg \text{gr}(n_3, n_4)$       cnf(graph\_n3\_n4, axiom)  
 $\neg \text{gr}(n_3, n_5)$       cnf(graph\_n3\_n5, axiom)  
 $\text{gr}(n_3, n_6)$       cnf(graph\_n3\_n6, axiom)  
 $\neg \text{gr}(n_4, n_0)$       cnf(graph\_n4\_n0, axiom)  
 $\neg \text{gr}(n_4, n_1)$       cnf(graph\_n4\_n1, axiom)  
 $\neg \text{gr}(n_4, n_2)$       cnf(graph\_n4\_n2, axiom)  
 $\neg \text{gr}(n_4, n_3)$       cnf(graph\_n4\_n3, axiom)  
 $\text{gr}(n_4, n_4)$       cnf(graph\_n4\_n4, axiom)  
 $\neg \text{gr}(n_4, n_5)$       cnf(graph\_n4\_n5, axiom)  
 $\neg \text{gr}(n_4, n_6)$       cnf(graph\_n4\_n6, axiom)  
 $\neg \text{gr}(n_5, n_0)$       cnf(graph\_n5\_n0, axiom)  
 $\neg \text{gr}(n_5, n_1)$       cnf(graph\_n5\_n1, axiom)  
 $\neg \text{gr}(n_5, n_2)$       cnf(graph\_n5\_n2, axiom)  
 $\neg \text{gr}(n_5, n_3)$       cnf(graph\_n5\_n3, axiom)

$\neg \text{gr}(n_5, n_4)$        $\text{cnf}(\text{graph\_n5\_n}_4, \text{axiom})$   
 $\text{gr}(n_5, n_5)$        $\text{cnf}(\text{graph\_n5\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_5, n_6)$        $\text{cnf}(\text{graph\_n5\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_6, n_0)$        $\text{cnf}(\text{graph\_n6\_n}_0, \text{axiom})$   
 $\text{gr}(n_6, n_1)$        $\text{cnf}(\text{graph\_n6\_n}_1, \text{axiom})$   
 $\text{gr}(n_6, n_2)$        $\text{cnf}(\text{graph\_n6\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_6, n_3)$        $\text{cnf}(\text{graph\_n6\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_6, n_4)$        $\text{cnf}(\text{graph\_n6\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_6, n_5)$        $\text{cnf}(\text{graph\_n6\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_6, n_6)$        $\text{cnf}(\text{graph\_n6\_n}_6, \text{axiom})$   
 $n_0 \neq n_1$        $\text{cnf}(\text{elems\_n0\_n}_1, \text{axiom})$   
 $n_0 \neq n_2$        $\text{cnf}(\text{elems\_n0\_n}_2, \text{axiom})$   
 $n_0 \neq n_3$        $\text{cnf}(\text{elems\_n0\_n}_3, \text{axiom})$   
 $n_0 \neq n_4$        $\text{cnf}(\text{elems\_n0\_n}_4, \text{axiom})$   
 $n_0 \neq n_5$        $\text{cnf}(\text{elems\_n0\_n}_5, \text{axiom})$   
 $n_0 \neq n_6$        $\text{cnf}(\text{elems\_n0\_n}_6, \text{axiom})$   
 $n_1 \neq n_2$        $\text{cnf}(\text{elems\_n1\_n}_2, \text{axiom})$   
 $n_1 \neq n_3$        $\text{cnf}(\text{elems\_n1\_n}_3, \text{axiom})$   
 $n_1 \neq n_4$        $\text{cnf}(\text{elems\_n1\_n}_4, \text{axiom})$   
 $n_1 \neq n_5$        $\text{cnf}(\text{elems\_n1\_n}_5, \text{axiom})$   
 $n_1 \neq n_6$        $\text{cnf}(\text{elems\_n1\_n}_6, \text{axiom})$   
 $n_2 \neq n_3$        $\text{cnf}(\text{elems\_n2\_n}_3, \text{axiom})$   
 $n_2 \neq n_4$        $\text{cnf}(\text{elems\_n2\_n}_4, \text{axiom})$   
 $n_2 \neq n_5$        $\text{cnf}(\text{elems\_n2\_n}_5, \text{axiom})$   
 $n_2 \neq n_6$        $\text{cnf}(\text{elems\_n2\_n}_6, \text{axiom})$   
 $n_3 \neq n_4$        $\text{cnf}(\text{elems\_n3\_n}_4, \text{axiom})$   
 $n_3 \neq n_5$        $\text{cnf}(\text{elems\_n3\_n}_5, \text{axiom})$   
 $n_3 \neq n_6$        $\text{cnf}(\text{elems\_n3\_n}_6, \text{axiom})$   
 $n_4 \neq n_5$        $\text{cnf}(\text{elems\_n4\_n}_5, \text{axiom})$   
 $n_4 \neq n_6$        $\text{cnf}(\text{elems\_n4\_n}_6, \text{axiom})$   
 $n_5 \neq n_6$        $\text{cnf}(\text{elems\_n5\_n}_6, \text{axiom})$   
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$  or  $x = n_5$  or  $x = n_6$        $\text{cnf}(\text{elems}, \text{axiom})$

**ALG317-1.p** Random graph 15, nu5 polymorphism

$t(y, x, x, x, x) = x$        $\text{cnf}(\text{polynu5}_{01}, \text{axiom})$   
 $t(x, y, x, x, x) = x$        $\text{cnf}(\text{polynu5}_{02}, \text{axiom})$   
 $t(x, x, y, x, x) = x$        $\text{cnf}(\text{polynu5}_{03}, \text{axiom})$   
 $t(x, x, x, y, x) = x$        $\text{cnf}(\text{polynu5}_{04}, \text{axiom})$   
 $t(x, x, x, x, y) = x$        $\text{cnf}(\text{polynu5}_{05}, \text{axiom})$   
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$        $\text{cnf}(\text{graph\_15\_nu5\_polymorphism}, \text{axiom})$   
 $\neg \text{gr}(n_0, n_0)$        $\text{cnf}(\text{graph\_n0\_n}_0, \text{axiom})$   
 $\text{gr}(n_0, n_1)$        $\text{cnf}(\text{graph\_n0\_n}_1, \text{axiom})$   
 $\neg \text{gr}(n_0, n_2)$        $\text{cnf}(\text{graph\_n0\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_0, n_3)$        $\text{cnf}(\text{graph\_n0\_n}_3, \text{axiom})$   
 $\text{gr}(n_0, n_4)$        $\text{cnf}(\text{graph\_n0\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_0, n_5)$        $\text{cnf}(\text{graph\_n0\_n}_5, \text{axiom})$   
 $\text{gr}(n_0, n_6)$        $\text{cnf}(\text{graph\_n0\_n}_6, \text{axiom})$   
 $\text{gr}(n_1, n_0)$        $\text{cnf}(\text{graph\_n1\_n}_0, \text{axiom})$   
 $\text{gr}(n_1, n_1)$        $\text{cnf}(\text{graph\_n1\_n}_1, \text{axiom})$   
 $\text{gr}(n_1, n_2)$        $\text{cnf}(\text{graph\_n1\_n}_2, \text{axiom})$   
 $\text{gr}(n_1, n_3)$        $\text{cnf}(\text{graph\_n1\_n}_3, \text{axiom})$   
 $\text{gr}(n_1, n_4)$        $\text{cnf}(\text{graph\_n1\_n}_4, \text{axiom})$   
 $\text{gr}(n_1, n_5)$        $\text{cnf}(\text{graph\_n1\_n}_5, \text{axiom})$   
 $\text{gr}(n_1, n_6)$        $\text{cnf}(\text{graph\_n1\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_2, n_0)$        $\text{cnf}(\text{graph\_n2\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_2, n_1)$        $\text{cnf}(\text{graph\_n2\_n}_1, \text{axiom})$   
 $\text{gr}(n_2, n_2)$        $\text{cnf}(\text{graph\_n2\_n}_2, \text{axiom})$   
 $\text{gr}(n_2, n_3)$        $\text{cnf}(\text{graph\_n2\_n}_3, \text{axiom})$   
 $\text{gr}(n_2, n_4)$        $\text{cnf}(\text{graph\_n2\_n}_4, \text{axiom})$   
 $\text{gr}(n_2, n_5)$        $\text{cnf}(\text{graph\_n2\_n}_5, \text{axiom})$

$\neg \text{gr}(n_2, n_6)$        $\text{cnf}(\text{graph\_n2\_n}_6, \text{axiom})$   
 $\text{gr}(n_3, n_0)$        $\text{cnf}(\text{graph\_n3\_n}_0, \text{axiom})$   
 $\text{gr}(n_3, n_1)$        $\text{cnf}(\text{graph\_n3\_n}_1, \text{axiom})$   
 $\text{gr}(n_3, n_2)$        $\text{cnf}(\text{graph\_n3\_n}_2, \text{axiom})$   
 $\text{gr}(n_3, n_3)$        $\text{cnf}(\text{graph\_n3\_n}_3, \text{axiom})$   
 $\text{gr}(n_3, n_4)$        $\text{cnf}(\text{graph\_n3\_n}_4, \text{axiom})$   
 $\text{gr}(n_3, n_5)$        $\text{cnf}(\text{graph\_n3\_n}_5, \text{axiom})$   
 $\text{gr}(n_3, n_6)$        $\text{cnf}(\text{graph\_n3\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_4, n_0)$        $\text{cnf}(\text{graph\_n4\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_4, n_1)$        $\text{cnf}(\text{graph\_n4\_n}_1, \text{axiom})$   
 $\text{gr}(n_4, n_2)$        $\text{cnf}(\text{graph\_n4\_n}_2, \text{axiom})$   
 $\text{gr}(n_4, n_3)$        $\text{cnf}(\text{graph\_n4\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_4, n_4)$        $\text{cnf}(\text{graph\_n4\_n}_4, \text{axiom})$   
 $\text{gr}(n_4, n_5)$        $\text{cnf}(\text{graph\_n4\_n}_5, \text{axiom})$   
 $\text{gr}(n_4, n_6)$        $\text{cnf}(\text{graph\_n4\_n}_6, \text{axiom})$   
 $\text{gr}(n_5, n_0)$        $\text{cnf}(\text{graph\_n5\_n}_0, \text{axiom})$   
 $\text{gr}(n_5, n_1)$        $\text{cnf}(\text{graph\_n5\_n}_1, \text{axiom})$   
 $\text{gr}(n_5, n_2)$        $\text{cnf}(\text{graph\_n5\_n}_2, \text{axiom})$   
 $\text{gr}(n_5, n_3)$        $\text{cnf}(\text{graph\_n5\_n}_3, \text{axiom})$   
 $\text{gr}(n_5, n_4)$        $\text{cnf}(\text{graph\_n5\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_5, n_5)$        $\text{cnf}(\text{graph\_n5\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_5, n_6)$        $\text{cnf}(\text{graph\_n5\_n}_6, \text{axiom})$   
 $\text{gr}(n_6, n_0)$        $\text{cnf}(\text{graph\_n6\_n}_0, \text{axiom})$   
 $\text{gr}(n_6, n_1)$        $\text{cnf}(\text{graph\_n6\_n}_1, \text{axiom})$   
 $\text{gr}(n_6, n_2)$        $\text{cnf}(\text{graph\_n6\_n}_2, \text{axiom})$   
 $\text{gr}(n_6, n_3)$        $\text{cnf}(\text{graph\_n6\_n}_3, \text{axiom})$   
 $\text{gr}(n_6, n_4)$        $\text{cnf}(\text{graph\_n6\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_6, n_5)$        $\text{cnf}(\text{graph\_n6\_n}_5, \text{axiom})$   
 $\text{gr}(n_6, n_6)$        $\text{cnf}(\text{graph\_n6\_n}_6, \text{axiom})$   
 $n_0 \neq n_1$        $\text{cnf}(\text{elems\_n0\_n}_1, \text{axiom})$   
 $n_0 \neq n_2$        $\text{cnf}(\text{elems\_n0\_n}_2, \text{axiom})$   
 $n_0 \neq n_3$        $\text{cnf}(\text{elems\_n0\_n}_3, \text{axiom})$   
 $n_0 \neq n_4$        $\text{cnf}(\text{elems\_n0\_n}_4, \text{axiom})$   
 $n_0 \neq n_5$        $\text{cnf}(\text{elems\_n0\_n}_5, \text{axiom})$   
 $n_0 \neq n_6$        $\text{cnf}(\text{elems\_n0\_n}_6, \text{axiom})$   
 $n_1 \neq n_2$        $\text{cnf}(\text{elems\_n1\_n}_2, \text{axiom})$   
 $n_1 \neq n_3$        $\text{cnf}(\text{elems\_n1\_n}_3, \text{axiom})$   
 $n_1 \neq n_4$        $\text{cnf}(\text{elems\_n1\_n}_4, \text{axiom})$   
 $n_1 \neq n_5$        $\text{cnf}(\text{elems\_n1\_n}_5, \text{axiom})$   
 $n_1 \neq n_6$        $\text{cnf}(\text{elems\_n1\_n}_6, \text{axiom})$   
 $n_2 \neq n_3$        $\text{cnf}(\text{elems\_n2\_n}_3, \text{axiom})$   
 $n_2 \neq n_4$        $\text{cnf}(\text{elems\_n2\_n}_4, \text{axiom})$   
 $n_2 \neq n_5$        $\text{cnf}(\text{elems\_n2\_n}_5, \text{axiom})$   
 $n_2 \neq n_6$        $\text{cnf}(\text{elems\_n2\_n}_6, \text{axiom})$   
 $n_3 \neq n_4$        $\text{cnf}(\text{elems\_n3\_n}_4, \text{axiom})$   
 $n_3 \neq n_5$        $\text{cnf}(\text{elems\_n3\_n}_5, \text{axiom})$   
 $n_3 \neq n_6$        $\text{cnf}(\text{elems\_n3\_n}_6, \text{axiom})$   
 $n_4 \neq n_5$        $\text{cnf}(\text{elems\_n4\_n}_5, \text{axiom})$   
 $n_4 \neq n_6$        $\text{cnf}(\text{elems\_n4\_n}_6, \text{axiom})$   
 $n_5 \neq n_6$        $\text{cnf}(\text{elems\_n5\_n}_6, \text{axiom})$   
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$  or  $x = n_5$  or  $x = n_6$        $\text{cnf}(\text{elems}, \text{axiom})$

**ALG319-1.p** Random graph 17, siggers polymorphism

$t(x, x, x, x) = x$        $\text{cnf}(\text{polysiggers}_{01}, \text{axiom})$

$t(x, y, x, z) = t(y, x, z, y)$        $\text{cnf}(\text{polysiggers}_{02}, \text{axiom})$

$(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6), t(x_1, x_3, x_5, x_7))$        $\text{cnf}(\text{preserves}, \text{axiom})$

$\neg \text{gr}(n_0, n_0)$        $\text{cnf}(\text{graph\_n0\_n}_0, \text{axiom})$

$\neg \text{gr}(n_0, n_1)$        $\text{cnf}(\text{graph\_n0\_n}_1, \text{axiom})$

$\text{gr}(n_0, n_2)$        $\text{cnf}(\text{graph\_n0\_n}_2, \text{axiom})$

$\neg \text{gr}(n_0, n_3)$        $\text{cnf}(\text{graph\_n0\_n}_3, \text{axiom})$



$n_2 \neq n_6$       cnf(elems\_n2\_n6, axiom)  
 $n_3 \neq n_4$       cnf(elems\_n3\_n4, axiom)  
 $n_3 \neq n_5$       cnf(elems\_n3\_n5, axiom)  
 $n_3 \neq n_6$       cnf(elems\_n3\_n6, axiom)  
 $n_4 \neq n_5$       cnf(elems\_n4\_n5, axiom)  
 $n_4 \neq n_6$       cnf(elems\_n4\_n6, axiom)  
 $n_5 \neq n_6$       cnf(elems\_n5\_n6, axiom)  
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$  or  $x = n_5$  or  $x = n_6$       cnf(elems, axiom)

**ALG320-1.p** Random graph 18, nu5 polymorphism

$t(y, x, x, x, x) = x$       cnf(polynu5\_01, axiom)  
 $t(x, y, x, x, x) = x$       cnf(polynu5\_02, axiom)  
 $t(x, x, y, x, x) = x$       cnf(polynu5\_03, axiom)  
 $t(x, x, x, y, x) = x$       cnf(polynu5\_04, axiom)  
 $t(x, x, x, x, y) = x$       cnf(polynu5\_05, axiom)  
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$       cnf  
 $\neg \text{gr}(n_0, n_0)$       cnf(graph\_n0\_n0, axiom)  
 $\neg \text{gr}(n_0, n_1)$       cnf(graph\_n0\_n1, axiom)  
 $\neg \text{gr}(n_0, n_2)$       cnf(graph\_n0\_n2, axiom)  
 $\neg \text{gr}(n_0, n_3)$       cnf(graph\_n0\_n3, axiom)  
 $\text{gr}(n_0, n_4)$       cnf(graph\_n0\_n4, axiom)  
 $\text{gr}(n_0, n_5)$       cnf(graph\_n0\_n5, axiom)  
 $\text{gr}(n_0, n_6)$       cnf(graph\_n0\_n6, axiom)  
 $\neg \text{gr}(n_1, n_0)$       cnf(graph\_n1\_n0, axiom)  
 $\text{gr}(n_1, n_1)$       cnf(graph\_n1\_n1, axiom)  
 $\text{gr}(n_1, n_2)$       cnf(graph\_n1\_n2, axiom)  
 $\text{gr}(n_1, n_3)$       cnf(graph\_n1\_n3, axiom)  
 $\neg \text{gr}(n_1, n_4)$       cnf(graph\_n1\_n4, axiom)  
 $\neg \text{gr}(n_1, n_5)$       cnf(graph\_n1\_n5, axiom)  
 $\text{gr}(n_1, n_6)$       cnf(graph\_n1\_n6, axiom)  
 $\neg \text{gr}(n_2, n_0)$       cnf(graph\_n2\_n0, axiom)  
 $\neg \text{gr}(n_2, n_1)$       cnf(graph\_n2\_n1, axiom)  
 $\text{gr}(n_2, n_2)$       cnf(graph\_n2\_n2, axiom)  
 $\text{gr}(n_2, n_3)$       cnf(graph\_n2\_n3, axiom)  
 $\text{gr}(n_2, n_4)$       cnf(graph\_n2\_n4, axiom)  
 $\text{gr}(n_2, n_5)$       cnf(graph\_n2\_n5, axiom)  
 $\text{gr}(n_2, n_6)$       cnf(graph\_n2\_n6, axiom)  
 $\neg \text{gr}(n_3, n_0)$       cnf(graph\_n3\_n0, axiom)  
 $\neg \text{gr}(n_3, n_1)$       cnf(graph\_n3\_n1, axiom)  
 $\text{gr}(n_3, n_2)$       cnf(graph\_n3\_n2, axiom)  
 $\text{gr}(n_3, n_3)$       cnf(graph\_n3\_n3, axiom)  
 $\text{gr}(n_3, n_4)$       cnf(graph\_n3\_n4, axiom)  
 $\text{gr}(n_3, n_5)$       cnf(graph\_n3\_n5, axiom)  
 $\neg \text{gr}(n_3, n_6)$       cnf(graph\_n3\_n6, axiom)  
 $\text{gr}(n_4, n_0)$       cnf(graph\_n4\_n0, axiom)  
 $\neg \text{gr}(n_4, n_1)$       cnf(graph\_n4\_n1, axiom)  
 $\neg \text{gr}(n_4, n_2)$       cnf(graph\_n4\_n2, axiom)  
 $\text{gr}(n_4, n_3)$       cnf(graph\_n4\_n3, axiom)  
 $\text{gr}(n_4, n_4)$       cnf(graph\_n4\_n4, axiom)  
 $\text{gr}(n_4, n_5)$       cnf(graph\_n4\_n5, axiom)  
 $\neg \text{gr}(n_4, n_6)$       cnf(graph\_n4\_n6, axiom)  
 $\text{gr}(n_5, n_0)$       cnf(graph\_n5\_n0, axiom)  
 $\neg \text{gr}(n_5, n_1)$       cnf(graph\_n5\_n1, axiom)  
 $\text{gr}(n_5, n_2)$       cnf(graph\_n5\_n2, axiom)  
 $\text{gr}(n_5, n_3)$       cnf(graph\_n5\_n3, axiom)  
 $\text{gr}(n_5, n_4)$       cnf(graph\_n5\_n4, axiom)  
 $\text{gr}(n_5, n_5)$       cnf(graph\_n5\_n5, axiom)  
 $\text{gr}(n_5, n_6)$       cnf(graph\_n5\_n6, axiom)  
 $\neg \text{gr}(n_6, n_0)$       cnf(graph\_n6\_n0, axiom)  
 $\neg \text{gr}(n_6, n_1)$       cnf(graph\_n6\_n1, axiom)



$\neg \text{gr}(n_6, n_2)$        $\text{cnf}(\text{graph\_n6\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_6, n_3)$        $\text{cnf}(\text{graph\_n6\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_6, n_4)$        $\text{cnf}(\text{graph\_n6\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_6, n_5)$        $\text{cnf}(\text{graph\_n6\_n}_5, \text{axiom})$   
 $\text{gr}(n_6, n_6)$        $\text{cnf}(\text{graph\_n6\_n}_6, \text{axiom})$   
 $n_0 \neq n_1$        $\text{cnf}(\text{elems\_n0\_n}_1, \text{axiom})$   
 $n_0 \neq n_2$        $\text{cnf}(\text{elems\_n0\_n}_2, \text{axiom})$   
 $n_0 \neq n_3$        $\text{cnf}(\text{elems\_n0\_n}_3, \text{axiom})$   
 $n_0 \neq n_4$        $\text{cnf}(\text{elems\_n0\_n}_4, \text{axiom})$   
 $n_0 \neq n_5$        $\text{cnf}(\text{elems\_n0\_n}_5, \text{axiom})$   
 $n_0 \neq n_6$        $\text{cnf}(\text{elems\_n0\_n}_6, \text{axiom})$   
 $n_1 \neq n_2$        $\text{cnf}(\text{elems\_n1\_n}_2, \text{axiom})$   
 $n_1 \neq n_3$        $\text{cnf}(\text{elems\_n1\_n}_3, \text{axiom})$   
 $n_1 \neq n_4$        $\text{cnf}(\text{elems\_n1\_n}_4, \text{axiom})$   
 $n_1 \neq n_5$        $\text{cnf}(\text{elems\_n1\_n}_5, \text{axiom})$   
 $n_1 \neq n_6$        $\text{cnf}(\text{elems\_n1\_n}_6, \text{axiom})$   
 $n_2 \neq n_3$        $\text{cnf}(\text{elems\_n2\_n}_3, \text{axiom})$   
 $n_2 \neq n_4$        $\text{cnf}(\text{elems\_n2\_n}_4, \text{axiom})$   
 $n_2 \neq n_5$        $\text{cnf}(\text{elems\_n2\_n}_5, \text{axiom})$   
 $n_2 \neq n_6$        $\text{cnf}(\text{elems\_n2\_n}_6, \text{axiom})$   
 $n_3 \neq n_4$        $\text{cnf}(\text{elems\_n3\_n}_4, \text{axiom})$   
 $n_3 \neq n_5$        $\text{cnf}(\text{elems\_n3\_n}_5, \text{axiom})$   
 $n_3 \neq n_6$        $\text{cnf}(\text{elems\_n3\_n}_6, \text{axiom})$   
 $n_4 \neq n_5$        $\text{cnf}(\text{elems\_n4\_n}_5, \text{axiom})$   
 $n_4 \neq n_6$        $\text{cnf}(\text{elems\_n4\_n}_6, \text{axiom})$   
 $n_5 \neq n_6$        $\text{cnf}(\text{elems\_n5\_n}_6, \text{axiom})$   
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$  or  $x = n_5$  or  $x = n_6$        $\text{cnf}(\text{elems}, \text{axiom})$

**ALG321-1.p** Random graph 19, nu5 polymorphism

$t(y, x, x, x, x) = x$        $\text{cnf}(\text{polynu5}_{01}, \text{axiom})$   
 $t(x, y, x, x, x) = x$        $\text{cnf}(\text{polynu5}_{02}, \text{axiom})$   
 $t(x, x, y, x, x) = x$        $\text{cnf}(\text{polynu5}_{03}, \text{axiom})$   
 $t(x, x, x, y, x) = x$        $\text{cnf}(\text{polynu5}_{04}, \text{axiom})$   
 $t(x, x, x, x, y) = x$        $\text{cnf}(\text{polynu5}_{05}, \text{axiom})$   
 $(\text{gr}(x_0, x_1) \text{ and } \text{gr}(x_2, x_3) \text{ and } \text{gr}(x_4, x_5) \text{ and } \text{gr}(x_6, x_7) \text{ and } \text{gr}(x_8, x_9)) \Rightarrow \text{gr}(t(x_0, x_2, x_4, x_6, x_8), t(x_1, x_3, x_5, x_7, x_9))$        $\text{cnf}$   
 $\neg \text{gr}(n_0, n_0)$        $\text{cnf}(\text{graph\_n0\_n}_0, \text{axiom})$   
 $\text{gr}(n_0, n_1)$        $\text{cnf}(\text{graph\_n0\_n}_1, \text{axiom})$   
 $\text{gr}(n_0, n_2)$        $\text{cnf}(\text{graph\_n0\_n}_2, \text{axiom})$   
 $\text{gr}(n_0, n_3)$        $\text{cnf}(\text{graph\_n0\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_0, n_4)$        $\text{cnf}(\text{graph\_n0\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_0, n_5)$        $\text{cnf}(\text{graph\_n0\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_0, n_6)$        $\text{cnf}(\text{graph\_n0\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_1, n_0)$        $\text{cnf}(\text{graph\_n1\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_1, n_1)$        $\text{cnf}(\text{graph\_n1\_n}_1, \text{axiom})$   
 $\text{gr}(n_1, n_2)$        $\text{cnf}(\text{graph\_n1\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_1, n_3)$        $\text{cnf}(\text{graph\_n1\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_1, n_4)$        $\text{cnf}(\text{graph\_n1\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_1, n_5)$        $\text{cnf}(\text{graph\_n1\_n}_5, \text{axiom})$   
 $\text{gr}(n_1, n_6)$        $\text{cnf}(\text{graph\_n1\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_2, n_0)$        $\text{cnf}(\text{graph\_n2\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_2, n_1)$        $\text{cnf}(\text{graph\_n2\_n}_1, \text{axiom})$   
 $\neg \text{gr}(n_2, n_2)$        $\text{cnf}(\text{graph\_n2\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_2, n_3)$        $\text{cnf}(\text{graph\_n2\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_2, n_4)$        $\text{cnf}(\text{graph\_n2\_n}_4, \text{axiom})$   
 $\text{gr}(n_2, n_5)$        $\text{cnf}(\text{graph\_n2\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_2, n_6)$        $\text{cnf}(\text{graph\_n2\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_3, n_0)$        $\text{cnf}(\text{graph\_n3\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_3, n_1)$        $\text{cnf}(\text{graph\_n3\_n}_1, \text{axiom})$   
 $\text{gr}(n_3, n_2)$        $\text{cnf}(\text{graph\_n3\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_3, n_3)$        $\text{cnf}(\text{graph\_n3\_n}_3, \text{axiom})$

$\text{gr}(n_3, n_4)$        $\text{cnf}(\text{graph\_n3\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_3, n_5)$        $\text{cnf}(\text{graph\_n3\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_3, n_6)$        $\text{cnf}(\text{graph\_n3\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_4, n_0)$        $\text{cnf}(\text{graph\_n4\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_4, n_1)$        $\text{cnf}(\text{graph\_n4\_n}_1, \text{axiom})$   
 $\text{gr}(n_4, n_2)$        $\text{cnf}(\text{graph\_n4\_n}_2, \text{axiom})$   
 $\text{gr}(n_4, n_3)$        $\text{cnf}(\text{graph\_n4\_n}_3, \text{axiom})$   
 $\text{gr}(n_4, n_4)$        $\text{cnf}(\text{graph\_n4\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_4, n_5)$        $\text{cnf}(\text{graph\_n4\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_4, n_6)$        $\text{cnf}(\text{graph\_n4\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_5, n_0)$        $\text{cnf}(\text{graph\_n5\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_5, n_1)$        $\text{cnf}(\text{graph\_n5\_n}_1, \text{axiom})$   
 $\neg \text{gr}(n_5, n_2)$        $\text{cnf}(\text{graph\_n5\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_5, n_3)$        $\text{cnf}(\text{graph\_n5\_n}_3, \text{axiom})$   
 $\text{gr}(n_5, n_4)$        $\text{cnf}(\text{graph\_n5\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_5, n_5)$        $\text{cnf}(\text{graph\_n5\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_5, n_6)$        $\text{cnf}(\text{graph\_n5\_n}_6, \text{axiom})$   
 $\neg \text{gr}(n_6, n_0)$        $\text{cnf}(\text{graph\_n6\_n}_0, \text{axiom})$   
 $\neg \text{gr}(n_6, n_1)$        $\text{cnf}(\text{graph\_n6\_n}_1, \text{axiom})$   
 $\neg \text{gr}(n_6, n_2)$        $\text{cnf}(\text{graph\_n6\_n}_2, \text{axiom})$   
 $\neg \text{gr}(n_6, n_3)$        $\text{cnf}(\text{graph\_n6\_n}_3, \text{axiom})$   
 $\neg \text{gr}(n_6, n_4)$        $\text{cnf}(\text{graph\_n6\_n}_4, \text{axiom})$   
 $\neg \text{gr}(n_6, n_5)$        $\text{cnf}(\text{graph\_n6\_n}_5, \text{axiom})$   
 $\neg \text{gr}(n_6, n_6)$        $\text{cnf}(\text{graph\_n6\_n}_6, \text{axiom})$   
 $n_0 \neq n_1$        $\text{cnf}(\text{elems\_n0\_n}_1, \text{axiom})$   
 $n_0 \neq n_2$        $\text{cnf}(\text{elems\_n0\_n}_2, \text{axiom})$   
 $n_0 \neq n_3$        $\text{cnf}(\text{elems\_n0\_n}_3, \text{axiom})$   
 $n_0 \neq n_4$        $\text{cnf}(\text{elems\_n0\_n}_4, \text{axiom})$   
 $n_0 \neq n_5$        $\text{cnf}(\text{elems\_n0\_n}_5, \text{axiom})$   
 $n_0 \neq n_6$        $\text{cnf}(\text{elems\_n0\_n}_6, \text{axiom})$   
 $n_1 \neq n_2$        $\text{cnf}(\text{elems\_n1\_n}_2, \text{axiom})$   
 $n_1 \neq n_3$        $\text{cnf}(\text{elems\_n1\_n}_3, \text{axiom})$   
 $n_1 \neq n_4$        $\text{cnf}(\text{elems\_n1\_n}_4, \text{axiom})$   
 $n_1 \neq n_5$        $\text{cnf}(\text{elems\_n1\_n}_5, \text{axiom})$   
 $n_1 \neq n_6$        $\text{cnf}(\text{elems\_n1\_n}_6, \text{axiom})$   
 $n_2 \neq n_3$        $\text{cnf}(\text{elems\_n2\_n}_3, \text{axiom})$   
 $n_2 \neq n_4$        $\text{cnf}(\text{elems\_n2\_n}_4, \text{axiom})$   
 $n_2 \neq n_5$        $\text{cnf}(\text{elems\_n2\_n}_5, \text{axiom})$   
 $n_2 \neq n_6$        $\text{cnf}(\text{elems\_n2\_n}_6, \text{axiom})$   
 $n_3 \neq n_4$        $\text{cnf}(\text{elems\_n3\_n}_4, \text{axiom})$   
 $n_3 \neq n_5$        $\text{cnf}(\text{elems\_n3\_n}_5, \text{axiom})$   
 $n_3 \neq n_6$        $\text{cnf}(\text{elems\_n3\_n}_6, \text{axiom})$   
 $n_4 \neq n_5$        $\text{cnf}(\text{elems\_n4\_n}_5, \text{axiom})$   
 $n_4 \neq n_6$        $\text{cnf}(\text{elems\_n4\_n}_6, \text{axiom})$   
 $n_5 \neq n_6$        $\text{cnf}(\text{elems\_n5\_n}_6, \text{axiom})$   
 $x = n_0$  or  $x = n_1$  or  $x = n_2$  or  $x = n_3$  or  $x = n_4$  or  $x = n_5$  or  $x = n_6$        $\text{cnf}(\text{elems}, \text{axiom})$

**ALG440-1.p** Malcev, wnu2, wnu3 implies majority

$m(a, a, b) = b$        $\text{cnf}(\text{sos}, \text{axiom})$   
 $m(a, b, b) = a$        $\text{cnf}(\text{sos}_{001}, \text{axiom})$   
 $u(a, a, a) = a$        $\text{cnf}(\text{sos}_{002}, \text{axiom})$   
 $v(a, a) = a$        $\text{cnf}(\text{sos}_{003}, \text{axiom})$   
 $u(a, a, b) = u(a, b, a)$        $\text{cnf}(\text{sos}_{004}, \text{axiom})$   
 $u(a, a, b) = u(b, a, a)$        $\text{cnf}(\text{sos}_{005}, \text{axiom})$   
 $v(a, b) = v(b, a)$        $\text{cnf}(\text{sos}_{006}, \text{axiom})$   
 $u(a, a, b) = v(a, b)$        $\text{cnf}(\text{sos}_{007}, \text{axiom})$   
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5) \text{ and } r(x_6, x_7, x_8)) \Rightarrow r(m(x_0, x_3, x_6), m(x_1, x_4, x_7), m(x_2, x_5, x_8))$        $\text{cnf}(\text{sos}_{008}, \text{axiom})$   
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5) \text{ and } r(x_6, x_7, x_8)) \Rightarrow r(u(x_0, x_3, x_6), u(x_1, x_4, x_7), u(x_2, x_5, x_8))$        $\text{cnf}(\text{sos}_{009}, \text{axiom})$   
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5)) \Rightarrow r(v(x_0, x_3), v(x_1, x_4), v(x_2, x_5))$        $\text{cnf}(\text{sos}_{010}, \text{axiom})$   
 $r(a, a, b)$        $\text{cnf}(\text{sos}_{011}, \text{axiom})$

$r(a, b, a)$     cnf(sos<sub>012</sub>, axiom)  
 $r(b, a, a)$     cnf(sos<sub>013</sub>, axiom)  
 $\neg r(a, a, a)$     cnf(goals, negated\_conjecture)

**ALG441-1.p** Malcev, wnu2, wnu4 implies majority

$m(a, a, b) = b$     cnf(sos, axiom)  
 $m(a, b, b) = a$     cnf(sos<sub>001</sub>, axiom)  
 $u(a, a) = a$     cnf(sos<sub>002</sub>, axiom)  
 $v(a, a, a, a) = a$     cnf(sos<sub>003</sub>, axiom)  
 $u(a, b) = u(b, a)$     cnf(sos<sub>004</sub>, axiom)  
 $v(a, a, a, b) = v(a, a, b, a)$     cnf(sos<sub>005</sub>, axiom)  
 $v(a, a, b, a) = v(a, b, a, a)$     cnf(sos<sub>006</sub>, axiom)  
 $v(a, b, a, a) = v(b, a, a, a)$     cnf(sos<sub>007</sub>, axiom)  
 $u(a, b) = v(a, a, a, b)$     cnf(sos<sub>008</sub>, axiom)  
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5) \text{ and } r(x_6, x_7, x_8)) \Rightarrow r(m(x_0, x_3, x_6), m(x_1, x_4, x_7), m(x_2, x_5, x_8))$     cnf(sos<sub>009</sub>, axiom)  
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5)) \Rightarrow r(u(x_0, x_3), u(x_1, x_4), u(x_2, x_5))$     cnf(sos<sub>010</sub>, axiom)  
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5) \text{ and } r(x_6, x_7, x_8) \text{ and } r(x_9, x_{10}, x_{11})) \Rightarrow r(v(x_0, x_3, x_6, x_9), v(x_1, x_4, x_7, x_{10}), v(x_2, x_5, x_8, x_{11}))$   
 $r(a, a, b)$     cnf(sos<sub>012</sub>, axiom)  
 $r(a, b, a)$     cnf(sos<sub>013</sub>, axiom)  
 $r(b, a, a)$     cnf(sos<sub>014</sub>, axiom)  
 $\neg r(a, a, a)$     cnf(goals, negated\_conjecture)

**ALG442-1.p** Malcev, wnu3, wnu4 implies majority

$m(a, a, b) = b$     cnf(sos, axiom)  
 $m(a, b, b) = a$     cnf(sos<sub>001</sub>, axiom)  
 $u(a, a, a) = a$     cnf(sos<sub>002</sub>, axiom)  
 $v(a, a, a, a) = a$     cnf(sos<sub>003</sub>, axiom)  
 $u(a, a, b) = u(a, b, a)$     cnf(sos<sub>004</sub>, axiom)  
 $u(a, a, b) = u(b, a, a)$     cnf(sos<sub>005</sub>, axiom)  
 $v(a, a, a, b) = v(a, a, b, a)$     cnf(sos<sub>006</sub>, axiom)  
 $v(a, a, b, a) = v(a, b, a, a)$     cnf(sos<sub>007</sub>, axiom)  
 $v(a, b, a, a) = v(b, a, a, a)$     cnf(sos<sub>008</sub>, axiom)  
 $u(a, a, b) = v(a, a, a, b)$     cnf(sos<sub>009</sub>, axiom)  
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5) \text{ and } r(x_6, x_7, x_8)) \Rightarrow r(m(x_0, x_3, x_6), m(x_1, x_4, x_7), m(x_2, x_5, x_8))$     cnf(sos<sub>010</sub>, axiom)  
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5) \text{ and } r(x_6, x_7, x_8)) \Rightarrow r(u(x_0, x_3, x_6), u(x_1, x_4, x_7), u(x_2, x_5, x_8))$     cnf(sos<sub>011</sub>, axiom)  
 $(r(x_0, x_1, x_2) \text{ and } r(x_3, x_4, x_5) \text{ and } r(x_6, x_7, x_8) \text{ and } r(x_9, x_{10}, x_{11})) \Rightarrow r(v(x_0, x_3, x_6, x_9), v(x_1, x_4, x_7, x_{10}), v(x_2, x_5, x_8, x_{11}))$   
 $r(a, a, b)$     cnf(sos<sub>013</sub>, axiom)  
 $r(a, b, a)$     cnf(sos<sub>014</sub>, axiom)  
 $r(b, a, a)$     cnf(sos<sub>015</sub>, axiom)  
 $\neg r(a, a, a)$     cnf(goals, negated\_conjecture)

**ALG443+1.p** Axioms for Median algebra

include('Axioms/ALG002+0.ax')

**ALG444^1.p** Axioms for Untyped lambda sigma calculus

include('Axioms/ALG003^0.ax')