BOO axioms

BOO001-0.ax Ternary Boolean algebra (equality) axioms
$m(m(v,w,x),y,m(v,w,z)) = m(v,w,m(x,y,z))$ cnf(associativity, axiom)
$m(y,x,x) = x$ cnf(ternary_multiply$_1$, axiom)
$m(x,x,y) = x$ cnf(ternary_multiply$_2$, axiom)
$m(y',y,x) = x$ cnf(left_inverse, axiom)
$m(x,y,y') = x$ cnf(right_inverse, axiom)

BOO002-0.ax Boolean algebra axioms
$x + y{=}x + y$ cnf(closure_of_addition, axiom)
$x \cdot y{=}x \cdot y$ cnf(closure_of_multiplication, axiom)
$x + y{=}z \Rightarrow y + x{=}z$ cnf(commutativity_of_addition, axiom)
$x \cdot y{=}z \Rightarrow y \cdot x{=}z$ cnf(commutativity_of_multiplication, axiom)
$0 + x{=}x$ cnf(additive_identity$_1$, axiom)
$x + 0{=}x$ cnf(additive_identity$_2$, axiom)
$1 \cdot x{=}x$ cnf(multiplicative_identity$_1$, axiom)
$x \cdot 1{=}x$ cnf(multiplicative_identity$_2$, axiom)
$(x \cdot y{=}v_1 \text{ and } x \cdot z{=}v_2 \text{ and } y + z{=}v_3 \text{ and } x \cdot v_3{=}v_4) \Rightarrow v_1 + v_2{=}v_4$ cnf(distributivity$_1$, axiom)
$(x \cdot y{=}v_1 \text{ and } x \cdot z{=}v_2 \text{ and } y + z{=}v_3 \text{ and } v_1 + v_2{=}v_4) \Rightarrow x \cdot v_3{=}v_4$ cnf(distributivity$_2$, axiom)
$(y \cdot x{=}v_1 \text{ and } z \cdot x{=}v_2 \text{ and } y + z{=}v_3 \text{ and } v_3 \cdot x{=}v_4) \Rightarrow v_1 + v_2{=}v_4$ cnf(distributivity$_3$, axiom)
$(y \cdot x{=}v_1 \text{ and } z \cdot x{=}v_2 \text{ and } y + z{=}v_3 \text{ and } v_1 + v_2{=}v_4) \Rightarrow v_3 \cdot x{=}v_4$ cnf(distributivity$_4$, axiom)
$(x + y{=}v_1 \text{ and } x + z{=}v_2 \text{ and } y \cdot z{=}v_3 \text{ and } x + v_3{=}v_4) \Rightarrow v_1 \cdot v_2{=}v_4$ cnf(distributivity$_5$, axiom)
$(x + y{=}v_1 \text{ and } x + z{=}v_2 \text{ and } y \cdot z{=}v_3 \text{ and } v_1 \cdot v_2{=}v_4) \Rightarrow x + v_3{=}v_4$ cnf(distributivity$_6$, axiom)
$(y + x{=}v_1 \text{ and } z + x{=}v_2 \text{ and } y \cdot z{=}v_3 \text{ and } v_3 + x{=}v_4) \Rightarrow v_1 \cdot v_2{=}v_4$ cnf(distributivity$_7$, axiom)
$(y + x{=}v_1 \text{ and } z + x{=}v_2 \text{ and } y \cdot z{=}v_3 \text{ and } v_1 \cdot v_2{=}v_4) \Rightarrow v_3 + x{=}v_4$ cnf(distributivity$_8$, axiom)
$x' + x{=}1$ cnf(additive_inverse$_1$, axiom)
$x + x'{=}1$ cnf(additive_inverse$_2$, axiom)
$x' \cdot x{=}0$ cnf(multiplicative_inverse$_1$, axiom)
$x \cdot x'{=}0$ cnf(multiplicative_inverse$_2$, axiom)
$(x + y{=}u \text{ and } x + y{=}v) \Rightarrow u = v$ cnf(addition_is_well_defined, axiom)
$(x \cdot y{=}u \text{ and } x \cdot y{=}v) \Rightarrow u = v$ cnf(multiplication_is_well_defined, axiom)

BOO003-0.ax Boolean algebra (equality) axioms
$x + y = y + x$ cnf(commutativity_of_add, axiom)
$x \cdot y = y \cdot x$ cnf(commutativity_of_multiply, axiom)
$x \cdot y + z = (x + z) \cdot (y + z)$ cnf(distributivity$_1$, axiom)
$x + y \cdot z = (x + y) \cdot (x + z)$ cnf(distributivity$_2$, axiom)
$(x + y) \cdot z = x \cdot z + y \cdot z$ cnf(distributivity$_3$, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distributivity$_4$, axiom)
$x + x' = 1$ cnf(additive_inverse$_1$, axiom)
$x' + x = 1$ cnf(additive_inverse$_2$, axiom)
$x \cdot x' = 0$ cnf(multiplicative_inverse$_1$, axiom)
$x' \cdot x = 0$ cnf(multiplicative_inverse$_2$, axiom)
$x \cdot 1 = x$ cnf(multiplicative_id$_1$, axiom)
$1 \cdot x = x$ cnf(multiplicative_id$_2$, axiom)
$x + 0 = x$ cnf(additive_id$_1$, axiom)
$0 + x = x$ cnf(additive_id$_2$, axiom)

BOO004-0.ax Boolean algebra (equality) axioms
$x + y = y + x$ cnf(commutativity_of_add, axiom)
$x \cdot y = y \cdot x$ cnf(commutativity_of_multiply, axiom)
$x + y \cdot z = (x + y) \cdot (x + z)$ cnf(distributivity$_1$, axiom)
$x \cdot (y + z) = x \cdot y + x \cdot z$ cnf(distributivity$_2$, axiom)
$x + 0 = x$ cnf(additive_id$_1$, axiom)
$x \cdot 1 = x$ cnf(multiplicative_id$_1$, axiom)
$x + x' = 1$ cnf(additive_inverse$_1$, axiom)
$x \cdot x' = 0$ cnf(multiplicative_inverse$_1$, axiom)

BOO problems

BOO001-1.p In B3 algebra, inverse is an involution
include('Axioms/BOO001-0.ax')
$(a')' \neq a$ cnf(prove_inverse_is_self_cancelling, negated_conjecture)

BOO002-1.p In B3 algebra, X * X∧-1 * Y = Y
$m(m(v,w,x),y,m(v,w,z)) = m(v,w,m(x,y,z))$ cnf(associativity, axiom)
$m(y,x,x) = x$ cnf(ternary_multiply$_1$, axiom)
$m(x,x,y) = x$ cnf(ternary_multiply$_2$, axiom)
$m(y',y,x) = x$ cnf(left_inverse, axiom)
$m(a,a',b) \neq b$ cnf(prove_equation, negated_conjecture)

BOO002-2.p In B3 algebra, X * X∧-1 * Y = Y
$m(m(v,w,x),y,m(v,w,z)) = m(v,w,m(x,y,z))$ cnf(associativity, axiom)
$m(y,x,x) = x$ cnf(ternary_multiply$_1$, axiom)
$m(x,x,y) = x$ cnf(ternary_multiply$_2$, axiom)
$m(y',y,x) = x$ cnf(left_inverse, axiom)
$m(x,y,x) = x$ cnf(extra_lemma, axiom)
$m(a,a',b) \neq b$ cnf(prove_equation, negated_conjecture)

BOO003-1.p Multiplication is idempotent (X * X = X)
include('Axioms/BOO002-0.ax')
$\neg\, x \cdot x{=}x$ cnf(prove_both_equalities, negated_conjecture)

BOO003-2.p Multiplication is idempotent (X * X = X)
include('Axioms/BOO003-0.ax')
$a \cdot a \neq a$ cnf(prove_a_times_a_is_a, negated_conjecture)

BOO003-4.p Multiplication is idempotent (X * X = X)
include('Axioms/BOO004-0.ax')
$a \cdot a \neq a$ cnf(prove_a_times_a_is_a, negated_conjecture)

BOO004-1.p Addition is idempotent (X + X = X)
include('Axioms/BOO002-0.ax')
$\neg\, x + x{=}x$ cnf(prove_both_equalities, negated_conjecture)

BOO004-2.p Addition is idempotent (X + X = X)
include('Axioms/BOO003-0.ax')
$a + a \neq a$ cnf(prove_a_plus_a_is_a, negated_conjecture)

BOO004-4.p Addition is idempotent (X + X = X)
include('Axioms/BOO004-0.ax')
$a + a \neq a$ cnf(prove_a_plus_a_is_a, negated_conjecture)

BOO005-1.p Addition is bounded (X + 1 = 1)
include('Axioms/BOO002-0.ax')
$\neg\, x + 1{=}1$ cnf(prove_equations, negated_conjecture)

BOO005-2.p Addition is bounded (X + 1 = 1)
include('Axioms/BOO003-0.ax')
$a + 1 \neq 1$ cnf(prove_a_plus_1_is_a, negated_conjecture)

BOO005-4.p Addition is bounded (X + 1 = 1)
include('Axioms/BOO004-0.ax')
$a + 1 \neq 1$ cnf(prove_a_plus_1_is_a, negated_conjecture)

BOO006-1.p Multiplication is bounded (X * 0 = 0)
include('Axioms/BOO002-0.ax')
$\neg\, x \cdot 0{=}0$ cnf(prove_equations, negated_conjecture)

BOO006-2.p Multiplication is bounded (X * 0 = 0)
include('Axioms/BOO003-0.ax')
$a \cdot 0 \neq 0$ cnf(prove_right_identity, negated_conjecture)

BOO006-4.p Multiplication is bounded (X * 0 = 0)
include('Axioms/BOO004-0.ax')
$a \cdot 0 \neq 0$ cnf(prove_right_identity, negated_conjecture)

BOO007-1.p Product is associative ((X * Y) * Z = X * (Y * Z))
include('Axioms/BOO002-0.ax')

$y \cdot z$=y_times_z cnf(y_times_z, hypothesis)
$x \cdot$ y_times_z=x_times__y_times_z cnf(x_times__y_times_z, hypothesis)
$x \cdot y$=x_times_y cnf(x_times_y, hypothesis)
x_times_y $\cdot\ z$=x_times_y__times_z cnf(x_times_y__times_z, hypothesis)
x_times__y_times_z $\neq$ x_times_y__times_z cnf(prove_equality, negated_conjecture)

BOO007-2.p Product is associative ((X * Y) * Z = X * (Y * Z))
include('Axioms/BOO003-0.ax')
$a \cdot (b \cdot c) \neq (a \cdot b) \cdot c$ cnf(prove_associativity, negated_conjecture)

BOO007-4.p Product is associative ((X * Y) * Z = X * (Y * Z))
include('Axioms/BOO004-0.ax')
$a \cdot (b \cdot c) \neq (a \cdot b) \cdot c$ cnf(prove_associativity, negated_conjecture)

BOO008-1.p Sum is associative ((X + Y) + Z = X + (Y + Z))
include('Axioms/BOO002-0.ax')
$y + z$=y_plus_z cnf(y_plus_z, hypothesis)
$x +$ y_plus_z=x__plus_y_plus_z cnf(x_plus__y_plus_z, hypothesis)
$x + y$=x_plus_y cnf(x_plus_y, hypothesis)
x_plus_y $+\ z$=x_plus_y__plus_z cnf(x_plus_y__plus_z, hypothesis)
x__plus_y_plus_z $\neq$ x_plus_y__plus_z cnf(prove_equality, negated_conjecture)

BOO008-2.p Sum is associative ((X + Y) + Z = X + (Y + Z))
include('Axioms/BOO003-0.ax')
$a + (b + c) \neq (a + b) + c$ cnf(prove_associativity, negated_conjecture)

BOO008-3.p Sum is associative ((X + Y) + Z = X + (Y + Z))
$x + y = x + y$ cnf(closure_of_addition, axiom)
$x \cdot y = x \cdot y$ cnf(closure_of_multiplication, axiom)
$x + y = z \Rightarrow y + x = z$ cnf(commutativity_of_addition, axiom)
$x \cdot y = z \Rightarrow y \cdot x = z$ cnf(commutativity_of_multiplication, axiom)
$0 + x = x$ cnf(additive_identity$_1$, axiom)
$x + 0 = x$ cnf(additive_identity$_2$, axiom)
$1 + x = x$ cnf(multiplicative_identity$_1$, axiom)
$x + 1 = x$ cnf(multiplicative_identity$_2$, axiom)
$(x \cdot y = v_1 \text{ and } x \cdot z = v_2 \text{ and } y + z = v_3 \text{ and } x \cdot v_3 = v_4) \Rightarrow v_1 + v_2 = v_4$ cnf(distributivity$_1$, axiom)
$(x \cdot y = v_1 \text{ and } x \cdot z = v_2 \text{ and } y + z = v_3 \text{ and } v_1 + v_2 = v_4) \Rightarrow x \cdot v_3 = v_4$ cnf(distributivity$_2$, axiom)
$(x + y = v_1 \text{ and } x + z = v_2 \text{ and } y \cdot z = v_3 \text{ and } x + v_3 = v_4) \Rightarrow v_1 \cdot v_2 = v_4$ cnf(distributivity$_5$, axiom)
$(x + y = v_1 \text{ and } x + z = v_2 \text{ and } y \cdot z = v_3 \text{ and } v_1 \cdot v_2 = v_4) \Rightarrow x + v_3 = v_4$ cnf(distributivity$_6$, axiom)
$x' + x = 1$ cnf(additive_inverse$_1$, axiom)
$x + x' = 1$ cnf(additive_inverse$_2$, axiom)
$x' \cdot x = 0$ cnf(multiplicative_inverse$_1$, axiom)
$x \cdot x' = 0$ cnf(multiplicative_inverse$_2$, axiom)
$y + z$=y_plus_z cnf(y_plus_z, hypothesis)
$x +$ y_plus_z=x__plus_y_plus_z cnf(x_plus__y_plus_z, hypothesis)
$x + y$=x_plus_y cnf(x_plus_y, hypothesis)
x_plus_y $+\ z$=x_plus_y__plus_z cnf(x_plus_y__plus_z, hypothesis)
x__plus_y_plus_z $\neq$ x_plus_y__plus_z cnf(prove_equality, negated_conjecture)

BOO008-4.p Sum is associative ((X + Y) + Z = X + (Y + Z))
include('Axioms/BOO004-0.ax')
$a + (b + c) \neq (a + b) + c$ cnf(prove_associativity, negated_conjecture)

BOO009-1.p Multiplication absorption (X * (X + Y) = X)
include('Axioms/BOO002-0.ax')
$\neg\, x \cdot (x + y) = x$ cnf(prove_equations, negated_conjecture)

BOO009-2.p Multiplication absorption (X * (X + Y) = X)
include('Axioms/BOO003-0.ax')
$a \cdot (a + b) \neq a$ cnf(prove_operation, negated_conjecture)

BOO009-4.p Multiplication absorption (X * (X + Y) = X)
include('Axioms/BOO004-0.ax')
$a \cdot (a + b) \neq a$ cnf(prove_operation, negated_conjecture)

BOO010-1.p Addition absorbtion (X + (X * Y) = X)
include('Axioms/BOO002-0.ax')
$\neg\, x + x \cdot y = x$ cnf(prove_equations, negated_conjecture)

BOO010-2.p Addition absorbtion (X + (X * Y) = X)
include('Axioms/BOO003-0.ax')
$a + a \cdot b \neq a$ cnf(prove_a_plus_ab_is_a, negated_conjecture)

BOO010-4.p Addition absorbtion (X + (X * Y) = X)
include('Axioms/BOO004-0.ax')
$a + a \cdot b \neq a$ cnf(prove_a_plus_ab_is_a, negated_conjecture)

BOO011-1.p Inverse of additive identity = Multiplicative identity
The inverse of the additive identity is the multiplicative identity.
include('Axioms/BOO002-0.ax')
$0' \neq 1$ cnf(prove_equation, negated_conjecture)

BOO011-2.p Inverse of additive identity = Multiplicative identity
The inverse of the additive identity is the multiplicative identity.
include('Axioms/BOO003-0.ax')
$0' \neq 1$ cnf(prove_inverse_of_1_is$_0$, negated_conjecture)

BOO011-4.p Inverse of additive identity = Multiplicative identity
include('Axioms/BOO004-0.ax')
$0' \neq 1$ cnf(prove_inverse_of_1_is$_0$, negated_conjecture)

BOO012-1.p Inverse is an involution
include('Axioms/BOO002-0.ax')
$(x')' \neq x$ cnf(prove_inverse_is_an_involution, negated_conjecture)

BOO012-2.p Inverse is an involution
include('Axioms/BOO003-0.ax')
$(x')' \neq x$ cnf(prove_inverse_is_an_involution, negated_conjecture)

BOO012-3.p Inverse is an involution
include('Axioms/BOO002-0.ax')
$x + x = x$ cnf(x_plus_x_is_x, axiom)
$x \cdot x = x$ cnf(x_times_x_is_x, axiom)
$x + 1 = 1$ cnf(x_plus_multiplicative_identity, axiom)
$x \cdot 0 = 0$ cnf(x_times_additive_identity, axiom)
$x \cdot y = z \;\Rightarrow\; x + z = x$ cnf(sum_product_dual$_1$, axiom)
$x + y = z \;\Rightarrow\; x \cdot z = x$ cnf(sum_product_dual$_2$, axiom)
$x + x \cdot y = x$ cnf(sum_and_multiply, axiom)
$x \cdot (x + y) = x$ cnf(product_and_add, axiom)
(x+y=x_plus_Y and y+z=y_plus_Z and x+y_plus_Z=x_plus_Y_plus_Z) ⇒ x_plus_Y+z=x_plus_Y_plus_Z cnf(associaticity
(x+y=x_plus_Y and y+z=y_plus_Z and x+y_plus_Z=x_plus_Y_plus_Z) ⇒ x_plus_Y+z=x_plus_Y_plus_Z cnf(associaticity
(x·y=x_times_Y and y·z=y_times_Z and x·y_times_Z=x_times_Y_times_Z) ⇒ x_times_Y·z=x_times_Y_times_Z cnf(associa
(x·y=x_times_Y and y·z=y_times_Z and x·y_times_Z=x_times_Y_times_Z) ⇒ x_times_Y·z=x_times_Y_times_Z cnf(associa
$(x')' \neq x$ cnf(prove_inverse_is_an_involution, negated_conjecture)

BOO012-4.p Inverse is an involution
include('Axioms/BOO004-0.ax')
$(x')' \neq x$ cnf(prove_inverse_is_an_involution, negated_conjecture)

BOO013-1.p The inverse of X is unique
include('Axioms/BOO002-0.ax')
$x + y = 1$ cnf(sum_to_multiplicative_identity$_1$, negated_conjecture)
$x + z = 1$ cnf(sum_to_multiplicative_identity$_2$, negated_conjecture)
$x \cdot y = 0$ cnf(product_to_additive_identity$_1$, negated_conjecture)
$x \cdot z = 0$ cnf(product_to_additive_identity$_2$, negated_conjecture)
$y \neq z$ cnf(prove_both_inverse_are_equal, negated_conjecture)

BOO013-2.p The inverse of X is unique
include('Axioms/BOO003-0.ax')
$a + b = 1$ cnf(b_and_multiplicative_identity, hypothesis)
$a + c = 1$ cnf(c_and_multiplicative_identity, hypothesis)

$a \cdot b = 0$ cnf(b_a_additive_identity, hypothesis)
$a \cdot c = 0$ cnf(c_a_additive_identity, hypothesis)
$b \neq c$ cnf(prove_b_is_a, negated_conjecture)

BOO013-3.p The inverse of X is unique
include('Axioms/BOO002-0.ax')
$(x')' = x$ cnf(inverse_is_an_involution, axiom)
$x + y{=}1$ cnf(sum_to_multiplicative_identity$_1$, negated_conjecture)
$x + z{=}1$ cnf(sum_to_multiplicative_identity$_2$, negated_conjecture)
$x \cdot y{=}0$ cnf(product_to_additive_identity$_1$, negated_conjecture)
$x \cdot z{=}0$ cnf(product_to_additive_identity$_2$, negated_conjecture)
$y \neq z$ cnf(prove_both_inverse_are_equal, negated_conjecture)

BOO013-4.p The inverse of X is unique
include('Axioms/BOO004-0.ax')
$a + b = 1$ cnf(b_a_multiplicative_identity, hypothesis)
$a \cdot b = 0$ cnf(b_an_additive_identity, hypothesis)
$b \neq a'$ cnf(prove_a_inverse_is_b, negated_conjecture)

BOO014-1.p DeMorgan for inverse and product (X+Y)∧-1 = (X∧-1) * (Y∧-1)
include('Axioms/BOO002-0.ax')
$x + y{=}$x_plus_y cnf(x_plus_y, negated_conjecture)
$x' \cdot y'{=}$x_inverse_times_y_inverse cnf(x_inverse_times_y_inverse, negated_conjecture)
x_plus_y$' \neq$ x_inverse_times_y_inverse cnf(prove_equation, negated_conjecture)

BOO014-2.p DeMorgan for inverse and product (X+Y)∧-1 = (X∧-1) * (Y∧-1)
include('Axioms/BOO003-0.ax')
$a + b = c$ cnf(a_plus_b_is_c, hypothesis)
$a' \cdot b' = d$ cnf(a_inverse_times_b_inverse_is_d, hypothesis)
$c' \neq d$ cnf(prove_c_inverse_is_d, negated_conjecture)

BOO014-3.p DeMorgan for inverse and product (X+Y)∧-1 = (X∧-1) * (Y∧-1)
include('Axioms/BOO002-0.ax')
$(x')' = x$ cnf(inverse_is_self_cancelling, axiom)
$(x + y{=}1 \text{ and } x + z{=}1 \text{ and } x \cdot y{=}0 \text{ and } x \cdot z{=}0) \Rightarrow y = z$ cnf(inverse_is_unique, axiom)
$x + y{=}$x_plus_y cnf(x_plus_y, hypothesis)
$x' \cdot y'{=}$x_inverse_times_y_inverse cnf(x_inverse_times_y_inverse, hypothesis)
x_plus_y$' \neq$ x_inverse_times_y_inverse cnf(prove_equation, negated_conjecture)

BOO014-4.p DeMorgan for inverse and product (X+Y)∧-1 = (X∧-1) * (Y∧-1)
include('Axioms/BOO004-0.ax')
$(a + b)' \neq a' \cdot b'$ cnf(prove_c_inverse_is_d, negated_conjecture)

BOO015-1.p DeMorgan for inverse and sum (X∧-1 + Y∧-1) = (X * Y)∧-1
include('Axioms/BOO002-0.ax')
$x \cdot y{=}$x_times_y cnf(x_times_y, negated_conjecture)
$x' + y'{=}$x_inverse_plus_y_inverse cnf(x_inverse_plus_y_inverse, negated_conjecture)
x_times_y$' \neq$ x_inverse_plus_y_inverse cnf(prove_equation, negated_conjecture)

BOO015-2.p DeMorgan for inverse and sum (X∧-1 + Y∧-1) = (X * Y)∧-1
include('Axioms/BOO003-0.ax')
$a \cdot b = c$ cnf(a_times_b_is_c, hypothesis)
$a' + b' = d$ cnf(a_inverse_plus_b_inverse_is_d, hypothesis)
$c' \neq d$ cnf(prove_c_inverse_is_d, negated_conjecture)

BOO015-4.p DeMorgan for inverse and sum (X∧-1 + Y∧-1) = (X * Y)∧-1
include('Axioms/BOO004-0.ax')
$(a \cdot b)' \neq a' + b'$ cnf(prove_c_inverse_is_d, negated_conjecture)

BOO016-1.p Relating product and sum (X * Y = Z → X + Z = X)
include('Axioms/BOO002-0.ax')
$x \cdot y{=}z$ cnf(x_times_y, hypothesis)
$\neg\, x + z{=}x$ cnf(prove_sum, negated_conjecture)

BOO016-2.p Relating product and sum (X * Y = Z → X + Z = X)
include('Axioms/BOO003-0.ax')

$x \cdot y = z$ cnf(x_times_y, hypothesis)
$x + z \neq x$ cnf(prove_sum, negated_conjecture)

BOO017-1.p Relating sum and product (X + Y = Z → X * Z = X)
include('Axioms/BOO002-0.ax')
$x + y{=}z$ cnf(x_plus_y, hypothesis)
$\neg\, x \cdot z{=}x$ cnf(prove_product, negated_conjecture)

BOO017-2.p Relating sum and product (X + Y = Z → X * Z = X)
include('Axioms/BOO003-0.ax')
$x + y = z$ cnf(x_times_y, hypothesis)
$x \cdot z \neq x$ cnf(prove_sum, negated_conjecture)

BOO018-4.p Inverse of multiplicative identity = Additive identity
include('Axioms/BOO004-0.ax')
$1' \neq 0$ cnf(prove_inverse_of_1_is$_0$, negated_conjecture)

BOO019-1.p Prove the independance of Ternary Boolean algebra axiom
$m(m(v, w, x), y, m(v, w, z)) = m(v, w, m(x, y, z))$ cnf(associativity, axiom)
$m(x, x, y) = x$ cnf(ternary_multiply$_2$, axiom)
$m(y', y, x) = x$ cnf(left_inverse, axiom)
$m(x, y, y') = x$ cnf(right_inverse, axiom)
$m(y, x, x) \neq x$ cnf(prove_ternary_multiply_1_independant, negated_conjecture)

BOO020-1.p Frink's Theorem
Prove that Frink's implicational basis for Boolean algebra implies Huntington's equational basis for Boolean algebra.
$x + x = x$ cnf(frink$_1$, axiom)
$((x + y) + z) + u = (y + z) + x \;\Rightarrow\; ((x + y) + z) + u' = n_0$ cnf(frink$_2$, axiom)
$((x + y) + z) + u' = n_0 \;\Rightarrow\; ((x + y) + z) + u = (y + z) + x$ cnf(frink$_3$, axiom)
$((a + b')' + (a' + b')' = b$ and $(a + b) + c = a + (b + c)) \;\Rightarrow\; b + a \neq a + b$ cnf(prove_huntington, negated_conjecture)

BOO021-1.p A Basis for Boolean Algebra
This theorem starts with a (self-dual independent) basis_ for Boolean algebra and derives commutativity of product.
$(x + y) \cdot y = y$ cnf(multiply_add, axiom)
$x \cdot (y + z) = y \cdot x + z \cdot x$ cnf(multiply_add_property, axiom)
$x + x' = n_1$ cnf(additive_inverse, axiom)
$x \cdot y + y = y$ cnf(add_multiply, axiom)
$x + y \cdot z = (y + x) \cdot (z + x)$ cnf(add_multiply_property, axiom)
$x \cdot x' = n_0$ cnf(multiplicative_inverse, axiom)
$b \cdot a \neq a \cdot b$ cnf(prove_commutativity_of_multiply, negated_conjecture)

BOO022-1.p A Basis for Boolean Algebra
This theorem starts with a (self-dual independent) 6-basis for Boolean algebra and derives associativity of product.
$(x + y) \cdot y = y$ cnf(multiply_add, axiom)
$x \cdot (y + z) = y \cdot x + z \cdot x$ cnf(multiply_add_property, axiom)
$x + x' = n_1$ cnf(additive_inverse, axiom)
$x \cdot y + y = y$ cnf(add_multiply, axiom)
$x + y \cdot z = (y + x) \cdot (z + x)$ cnf(add_multiply_property, axiom)
$x \cdot x' = n_0$ cnf(multiplicative_inverse, axiom)
$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$ cnf(prove_associativity_of_multiply, negated_conjecture)

BOO023-1.p Half of Padmanabhan's 6-basis with Pixley, part 1.
Part 1 (of 3) of the proof that half of Padmanaban's self-dual independent 6-basis for Boolean Algebra, together with a Pixley polynomial, is a basis for Boolean algebra.
$(x + y) \cdot y = y$ cnf(multiply_add, axiom)
$x \cdot (y + z) = y \cdot x + z \cdot x$ cnf(multiply_add_property, axiom)
$x + x' = n_1$ cnf(additive_inverse, axiom)
$\text{pixley}(x, y, z) = x \cdot y' + (x \cdot z + y' \cdot z)$ cnf(pixley_defn, axiom)
$\text{pixley}(x, x, y) = y$ cnf(pixley$_1$, axiom)
$\text{pixley}(x, y, y) = x$ cnf(pixley$_2$, axiom)
$\text{pixley}(x, y, x) = x$ cnf(pixley$_3$, axiom)
$a + b \cdot c \neq (a + b) \cdot (a + c)$ cnf(prove_add_multiply_property, negated_conjecture)

BOO024-1.p Half of Padmanabhan's 6-basis with Pixley, part 2.

Part 2 (of 3) of the proof that half of Padmanaban's self-dual independent 6-basis for Boolean Algebra, together with a Pixley polynomial, is a basis for Boolean algebra.
$(x + y) \cdot y = y$ cnf(multiply_add, axiom)
$x \cdot (y + z) = y \cdot x + z \cdot x$ cnf(multiply_add_property, axiom)
$x + x' = n_1$ cnf(additive_inverse, axiom)
$\text{pixley}(x, y, z) = x \cdot y' + (x \cdot z + y' \cdot z)$ cnf(pixley_defn, axiom)
$\text{pixley}(x, x, y) = y$ cnf(pixley_1, axiom)
$\text{pixley}(x, y, y) = x$ cnf(pixley_2, axiom)
$\text{pixley}(x, y, x) = x$ cnf(pixley_3, axiom)
$a \cdot b + b \neq b$ cnf(prove_add_multiply, negated_conjecture)

BOO025-1.p Half of Padmanabhan's 6-basis with Pixley, part 3.
Part 3 (of 3) of the proof that half of Padmanaban's self-dual independent 6-basis for Boolean Algebra, together with a Pixley polynomial, is a basis for Boolean algebra.
$(x + y) \cdot y = y$ cnf(multiply_add, axiom)
$x \cdot (y + z) = y \cdot x + z \cdot x$ cnf(multiply_add_property, axiom)
$x + x' = n_1$ cnf(additive_inverse, axiom)
$\text{pixley}(x, y, z) = x \cdot y' + (x \cdot z + y' \cdot z)$ cnf(pixley_defn, axiom)
$\text{pixley}(x, x, y) = y$ cnf(pixley_1, axiom)
$\text{pixley}(x, y, y) = x$ cnf(pixley_2, axiom)
$\text{pixley}(x, y, x) = x$ cnf(pixley_3, axiom)
$b \cdot b' \neq a \cdot a'$ cnf(prove_equal_identity, negated_conjecture)

BOO026-1.p Absorption from self-dual independent 2-basis
This is part of a proof that there exists an independent self-dual 2-basis for Boolean Algebra. You may note that the basis below has more than 2 equations; but don't worry, it can be reduced to 2 (large) equations by Pixley reduction.
$x \cdot (y + z) = y \cdot x + z \cdot x$ cnf(multiply_add_property, axiom)
$x + x' = n_1$ cnf(additive_inverse, axiom)
$x + y \cdot z = (y + x) \cdot (z + x)$ cnf(add_multiply_property, axiom)
$x \cdot x' = n_0$ cnf(multiplicative_inverse, axiom)
$x \cdot x' + (x \cdot y + x' \cdot y) = y$ cnf(pixley_1, axiom)
$x \cdot y' + (x \cdot y + y' \cdot y) = x$ cnf(pixley_2, axiom)
$x \cdot y' + (x \cdot x + y' \cdot x) = x$ cnf(pixley_3, axiom)
$(x + x') \cdot ((x + y) \cdot (x' + y)) = y$ cnf(pixley1_dual, axiom)
$(x + y') \cdot ((x + y) \cdot (y' + y)) = x$ cnf(pixley2_dual, axiom)
$(x + y') \cdot ((x + x) \cdot (y' + x)) = x$ cnf(pixley3_dual, axiom)
$(a + b) \cdot b \neq b$ cnf(prove_multiply_add, negated_conjecture)

BOO027-1.p Independence of self-dual 2-basis.
Show that half of the self-dual 2-basis in DUAL-BA-3 is not a basis for Boolean Algebra.
$x \cdot (y + z) = y \cdot x + z \cdot x$ cnf(multiply_add_property, axiom)
$x + x' = 1$ cnf(additive_inverse, axiom)
$x \cdot x' + (x \cdot y + x' \cdot y) = y$ cnf(pixley_1, axiom)
$x \cdot y' + (x \cdot y + y' \cdot y) = x$ cnf(pixley_2, axiom)
$x \cdot y' + (x \cdot x + y' \cdot x) = x$ cnf(pixley_3, axiom)
$a + a \neq a$ cnf(prove_idempotence, negated_conjecture)

BOO028-1.p Self-dual 2-basis from majority reduction, part 1.
This is part of a proof that there exists an independent self-dual-2-basis for Boolean algebra by majority reduction.
$x + y \cdot (x \cdot z) = x$ cnf(l_1, axiom)
$(x \cdot y + y \cdot z) + y = y$ cnf(l_3, axiom)
$(x + y) \cdot (x + y') = x$ cnf(b_1, axiom)
$x \cdot (y + (x + z)) = x$ cnf(l_2, axiom)
$((x + y) \cdot (y + z)) \cdot y = y$ cnf(l_4, axiom)
$x \cdot y + x \cdot y' = x$ cnf(b_2, axiom)
$x + y = y + x$ cnf(commutativity_of_add, axiom)
$x \cdot y = y \cdot x$ cnf(commutativity_of_multiply, axiom)
$(x + y) + z = x + (y + z)$ cnf(associativity_of_add, axiom)
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associativity_of_multiply, axiom)
$a \cdot (b + c) \neq b \cdot a + c \cdot a$ cnf(prove_multiply_add_property, negated_conjecture)

BOO029-1.p Self-dual 2-basis from majority reduction, part 3.

This is part of a proof that there exists an independent self-dual-2-basis for Boolean algebra by majority reduction.
$x + y \cdot (x \cdot z) = x$ cnf(l_1, axiom)
$(x \cdot y + y \cdot z) + y = y$ cnf(l_3, axiom)
$(x + y) \cdot (x + y') = x$ cnf(b_1, axiom)
$x \cdot (y + (x + z)) = x$ cnf(l_2, axiom)
$((x + y) \cdot (y + z)) \cdot y = y$ cnf(l_4, axiom)
$x \cdot y + x \cdot y' = x$ cnf(b_2, axiom)
$x + y = y + x$ cnf(commutativity_of_add, axiom)
$x \cdot y = y \cdot x$ cnf(commutativity_of_multiply, axiom)
$(x + y) + z = x + (y + z)$ cnf(associativity_of_add, axiom)
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associativity_of_multiply, axiom)
$b + b' \neq a + a'$ cnf(prove_equal_inverse, negated_conjecture)

BOO030-1.p Independence of a BA 2-basis by majority reduction.
This shows that the self-dual 2-basis for Boolean algebra (majority reduction) of problem DUAL-BA-5 is independent, in particular, that half of the 2-basis is not a basis.
$x + y \cdot (x \cdot z) = x$ cnf(l_1, axiom)
$(x \cdot y + y \cdot z) + y = y$ cnf(l_3, axiom)
$(x + y) \cdot (x + y') = x$ cnf(b_1, axiom)
$(x \cdot y + x) \cdot (x + y) = x$ cnf(majority_1, axiom)
$(x \cdot x + y) \cdot (x + x) = x$ cnf(majority_2, axiom)
$(x \cdot y + y) \cdot (x + y) = y$ cnf(majority_3, axiom)
$(a')' \neq a$ cnf(prove_inverse_involution, negated_conjecture)

BOO031-1.p Dual BA 3-basis, proof of distributivity.
This is part of a proof of the existence of a self-dual 3-basis for Boolean algebra by majority reduction.
$x \cdot y + (y \cdot z + z \cdot x) = (x + y) \cdot ((y + z) \cdot (z + x))$ cnf(distributivity, axiom)
$x + y \cdot (x \cdot z) = x$ cnf(l_1, axiom)
$(x \cdot y + y \cdot z) + y = y$ cnf(l_3, axiom)
$(x + x') \cdot y = y$ cnf(property_3, axiom)
$x \cdot (y + (x + z)) = x$ cnf(l_2, axiom)
$((x + y) \cdot (y + z)) \cdot y = y$ cnf(l_4, axiom)
$x \cdot x' + y = y$ cnf(property3_dual, axiom)
$x + x' = n_1$ cnf(additive_inverse, axiom)
$x \cdot x' = n_0$ cnf(multiplicative_inverse, axiom)
$(x + y) + z = x + (y + z)$ cnf(associativity_of_add, axiom)
$(x \cdot y) \cdot z = x \cdot (y \cdot z)$ cnf(associativity_of_multiply, axiom)
$a \cdot (b + c) \neq b \cdot a + c \cdot a$ cnf(prove_multiply_add_property, negated_conjecture)

BOO032-1.p Independence of a system of Boolean algebra
This is part of a proof that a self-dual 3-basis for Boolean algebra is independent.
$x + y \cdot (x \cdot z) = x$ cnf(l_1, axiom)
$(x \cdot y + y \cdot z) + y = y$ cnf(l_3, axiom)
$(x + x') \cdot y = y$ cnf(property_3, axiom)
$x \cdot (y + (x + z)) = x$ cnf(l_2, axiom)
$((x + y) \cdot (y + z)) \cdot y = y$ cnf(l_4, axiom)
$x \cdot x' + y = y$ cnf(property3_dual, axiom)
$(x + y) \cdot x + x \cdot y = x$ cnf(majority_1, axiom)
$(x + x) \cdot y + x \cdot x = x$ cnf(majority_2, axiom)
$(x + y) \cdot y + x \cdot y = y$ cnf(majority_3, axiom)
$(x \cdot y + x) \cdot (x + y) = x$ cnf(majority1_dual, axiom)
$(x \cdot x + y) \cdot (x + x) = x$ cnf(majority2_dual, axiom)
$(x \cdot y + y) \cdot (x + y) = y$ cnf(majority3_dual, axiom)
$(a')' \neq a$ cnf(prove_inverse_involution, negated_conjecture)

BOO033-1.p Independence of a system of Boolean algebra.
This is part of a proof that a self-dual 3-basis for Boolean algebra is independent.
$x \cdot y + (y \cdot z + z \cdot x) = (x + y) \cdot ((y + z) \cdot (z + x))$ cnf(distributivity, axiom)
$x + y \cdot (x \cdot z) = x$ cnf(l_1, axiom)
$(x \cdot y + y \cdot z) + y = y$ cnf(l_3, axiom)
$(x + x') \cdot y = y$ cnf(property_3, axiom)

$(x \cdot y + x) \cdot (x + y) = x \quad \mathrm{cnf}(\mathrm{majority}_1, \mathrm{axiom})$
$(x \cdot x + y) \cdot (x + x) = x \quad \mathrm{cnf}(\mathrm{majority}_2, \mathrm{axiom})$
$(x \cdot y + y) \cdot (x + y) = y \quad \mathrm{cnf}(\mathrm{majority}_3, \mathrm{axiom})$
$(a')' \neq a \quad \mathrm{cnf}(\mathrm{prove\_inverse\_involution}, \mathrm{negated\_conjecture})$

BOO034-1.p Ternary Boolean Algebra Single axiom is sound.
We show that that an equation (which turns out to be a single axiom for TBA) can be derived from the axioms of TBA.
include('Axioms/BOO001-0.ax')
$m(m(a, a', b), (m(m(c, d, e), f, m(c, d, g)))', m(d, m(g, f, e), c)) \neq b \quad \mathrm{cnf}(\mathrm{prove\_single\_axiom}, \mathrm{negated\_conjecture})$

BOO035-1.p Ternary Boolean Algebra Single axiom is complete
We show that that the standard axioms for TAB can be derived from an equation that turns out to be a single axiom for TBA.
$m(m(x, x', y), (m(m(z, u, v), w, m(z, u, v_6)))', m(u, m(v_6, w, v), z)) = y \quad \mathrm{cnf}(\mathrm{single\_axiom}, \mathrm{axiom})$
$(m(m(d, e, a), b, m(d, e, c)) = m(d, e, m(a, b, c))$ and $m(b, a, a) = a$ and $m(a, b, b') = a$ and $m(a, a, b) = a) \Rightarrow m(b', b, a) \neq a \quad \mathrm{cnf}(\mathrm{prove\_tba\_axioms}, \mathrm{negated\_conjecture})$

BOO036-1.p Ternary Boolean algebra (equality) axioms
include('Axioms/BOO001-0.ax')

BOO037-1.p Boolean algebra axioms
include('Axioms/BOO002-0.ax')

BOO037-2.p Boolean algebra (equality) axioms
include('Axioms/BOO003-0.ax')

BOO037-3.p Boolean algebra (equality) axioms
include('Axioms/BOO004-0.ax')

BOO038-1.p DN-1 is a single axiom for Boolean algebra
Show that equation DN-1 is a single axiom for Boolean algebra in terms of disjunction and negation by deriving the Huntington 3-basis.
$(((a + b)' + c)' + (a + (c' + (c + d)')')')' = c \quad \mathrm{cnf}(\mathrm{dn}_1, \mathrm{axiom})$
$(b + a = a + b$ and $(a + b) + c = a + (b + c)) \Rightarrow (a' + b)' + (a' + b')' \neq a \quad \mathrm{cnf}(\mathrm{huntinton}, \mathrm{negated\_conjecture})$

BOO039-1.p Sh-1 is a single axiom for Boolean algebra
Show that equation Sh-1 is a single axiom for Boolean algebra in terms of the Sheffer stroke by deriving the Meredith 2-basis.
$(a \uparrow ((b \uparrow a) \uparrow a)) \uparrow (b \uparrow (c \uparrow a)) = b \quad \mathrm{cnf}(\mathrm{sh}_1, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \quad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}, \mathrm{negated\_conjecture})$

BOO040-1.p Single axiom C1 for Boolean algebra in the Sheffer stroke
$(a \uparrow ((b \uparrow a) \uparrow a)) \uparrow (b \uparrow (a \uparrow c)) = b \quad \mathrm{cnf}(c_1, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \quad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}, \mathrm{negated\_conjecture})$

BOO041-1.p Single axiom C2 for Boolean algebra in the Sheffer stroke
$(a \uparrow (a \uparrow (b \uparrow a))) \uparrow (b \uparrow (c \uparrow a)) = b \quad \mathrm{cnf}(c_2, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \quad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}, \mathrm{negated\_conjecture})$

BOO042-1.p Single axiom C3 for Boolean algebra in the Sheffer stroke
$(a \uparrow (a \uparrow (a \uparrow b))) \uparrow (b \uparrow (c \uparrow a)) = b \quad \mathrm{cnf}(c_3, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \quad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}, \mathrm{negated\_conjecture})$

BOO043-1.p Single axiom C4 for Boolean algebra in the Sheffer stroke
$(a \uparrow (a \uparrow (a \uparrow b))) \uparrow (b \uparrow (a \uparrow c)) = b \quad \mathrm{cnf}(c_4, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \quad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}, \mathrm{negated\_conjecture})$

BOO044-1.p Single axiom C5 for Boolean algebra in the Sheffer stroke
$(a \uparrow (a \uparrow (b \uparrow c))) \uparrow (b \uparrow (c \uparrow a)) = b \quad \mathrm{cnf}(c_5, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \quad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}, \mathrm{negated\_conjecture})$

BOO045-1.p Single axiom C6 for Boolean algebra in the Sheffer stroke
$(a \uparrow (a \uparrow (b \uparrow c))) \uparrow (c \uparrow (a \uparrow b)) = c \quad \mathrm{cnf}(c_6, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \quad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}, \mathrm{negated\_conjecture})$

BOO046-1.p Single axiom C7 for Boolean algebra in the Sheffer stroke
$(a \uparrow (a \uparrow (b \uparrow b))) \uparrow (b \uparrow (c \uparrow a)) = b \quad \mathrm{cnf}(c_7, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \quad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}, \mathrm{negated\_conjecture})$

BOO047-1.p Single axiom C8 for Boolean algebra in the Sheffer stroke
$((a \uparrow (a \uparrow b)) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_8, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO048-1.p Single axiom C9 for Boolean algebra in the Sheffer stroke
$((a \uparrow (b \uparrow b)) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_9, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO049-1.p Single axiom C10 for Boolean algebra in the Sheffer stroke
$((a \uparrow (b \uparrow c)) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{10}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO050-1.p Single axiom C11 for Boolean algebra in the Sheffer stroke
$((a \uparrow (b \uparrow c)) \uparrow a) \uparrow (c \uparrow (a \uparrow b)) = c$ cnf(c_{11}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO051-1.p Single axiom C12 for Boolean algebra in the Sheffer stroke
$((a \uparrow (a \uparrow b)) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{12}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO052-1.p Single axiom C13 for Boolean algebra in the Sheffer stroke
$(((a \uparrow b) \uparrow a) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_{13}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO053-1.p Single axiom C14 for Boolean algebra in the Sheffer stroke
$(((a \uparrow b) \uparrow a) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{14}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO054-1.p Single axiom C15 for Boolean algebra in the Sheffer stroke
$(((a \uparrow b) \uparrow c) \uparrow c) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{15}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO055-1.p Single axiom C16 for Boolean algebra in the Sheffer stroke
$(((a \uparrow b) \uparrow c) \uparrow c) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_{16}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO056-1.p Single non-axiom M5A for Boolean algebra in the Sheffer stroke
$(a \uparrow ((b \uparrow c) \uparrow a)) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(m5A, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO057-1.p Single non-axiom M5B for Boolean algebra in the Sheffer stroke
$((a \uparrow (b \uparrow c)) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(m5B, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO058-1.p Single non-axiom M6A for Boolean algebra in the Sheffer stroke
$((a \uparrow (b \uparrow a)) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(m6A, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO059-1.p Single non-axiom M6B for Boolean algebra in the Sheffer stroke
$(a \uparrow (a \uparrow (b \uparrow a))) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(m6B, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO060-1.p Single non-axiom M6C for Boolean algebra in the Sheffer stroke
$((a \uparrow (b \uparrow b)) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(m6C, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO061-1.p Single non-axiom M6D for Boolean algebra in the Sheffer stroke
$(a \uparrow (a \uparrow (b \uparrow b))) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(m6D, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO066-1.p Single non-axiom M8A for Boolean algebra in the Sheffer stroke
$((a \uparrow (b \uparrow c)) \uparrow a) \uparrow (c \uparrow (b \uparrow a)) = c$ cnf(m8A, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) = a \Rightarrow a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis, negated_conjecture)

BOO067-1.p Ternary Boolean Algebra Single axiom is complete, part 1
$m(m(a, a', b), (m(m(c, d, e), f, m(c, d, g)))', m(d, m(g, f, e), c)) = b$ cnf(single_axiom, axiom)
$m(m(d, e, a), b, m(d, e, c)) \neq m(d, e, m(a, b, c))$ cnf(prove_tba_axioms$_1$, negated_conjecture)

BOO068-1.p Ternary Boolean Algebra Single axiom is complete, part 2
$m(m(a, a', b), (m(m(c, d, e), f, m(c, d, g)))', m(d, m(g, f, e), c)) = b$ cnf(single_axiom, axiom)

$m(b, a, a) \neq a$ cnf(prove_tba_axioms$_2$, negated_conjecture)

BOO069-1.p Ternary Boolean Algebra Single axiom is complete, part 3
$m(m(a, a', b), (m(m(c, d, e), f, m(c, d, g)))', m(d, m(g, f, e), c)) = b$ cnf(single_axiom, axiom)
$m(a, b, b') \neq a$ cnf(prove_tba_axioms$_3$, negated_conjecture)

BOO070-1.p Ternary Boolean Algebra Single axiom is complete, part 4
$m(m(a, a', b), (m(m(c, d, e), f, m(c, d, g)))', m(d, m(g, f, e), c)) = b$ cnf(single_axiom, axiom)
$m(a, a, b) \neq a$ cnf(prove_tba_axioms$_4$, negated_conjecture)

BOO071-1.p Ternary Boolean Algebra Single axiom is complete, part 5
$m(m(a, a', b), (m(m(c, d, e), f, m(c, d, g)))', m(d, m(g, f, e), c)) = b$ cnf(single_axiom, axiom)
$m(b', b, a) \neq a$ cnf(prove_tba_axioms$_5$, negated_conjecture)

BOO072-1.p DN-1 is a single axiom for Boolean algebra, part 1
$(((a + b)' + c)' + (a + (c' + (c + d)')')')' = c$ cnf(dn$_1$, axiom)
$b + a \neq a + b$ cnf(huntinton$_1$, negated_conjecture)

BOO073-1.p DN-1 is a single axiom for Boolean algebra, part 2
$(((a + b)' + c)' + (a + (c' + (c + d)')')')' = c$ cnf(dn$_1$, axiom)
$(a + b) + c \neq a + (b + c)$ cnf(huntinton$_2$, negated_conjecture)

BOO074-1.p DN-1 is a single axiom for Boolean algebra, part 3
$(((a + b)' + c)' + (a + (c' + (c + d)')')')' = c$ cnf(dn$_1$, axiom)
$(a' + b)' + (a' + b')' \neq a$ cnf(huntinton$_3$, negated_conjecture)

BOO075-1.p Sh-1 is a single axiom for Boolean algebra, part 1
$(a \uparrow ((b \uparrow a) \uparrow a)) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(sh$_1$, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO076-1.p Sh-1 is a single axiom for Boolean algebra, part 2
$(a \uparrow ((b \uparrow a) \uparrow a)) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(sh$_1$, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO077-1.p Axiom C1 for Boolean algebra in the Sheffer stroke, part 1
$(a \uparrow ((b \uparrow a) \uparrow a)) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_1, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO078-1.p Axiom C1 for Boolean algebra in the Sheffer stroke, part 2
$(a \uparrow ((b \uparrow a) \uparrow a)) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_1, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO079-1.p Axiom C2 for Boolean algebra in the Sheffer stroke, part 1
$(a \uparrow (a \uparrow (b \uparrow a))) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_2, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO080-1.p Axiom C2 for Boolean algebra in the Sheffer stroke, part 2
$(a \uparrow (a \uparrow (b \uparrow a))) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_2, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO081-1.p Axiom C3 for Boolean algebra in the Sheffer stroke, part 1
$(a \uparrow (a \uparrow (a \uparrow b))) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_3, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO082-1.p Axiom C3 for Boolean algebra in the Sheffer stroke, part 2
$(a \uparrow (a \uparrow (a \uparrow b))) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_3, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO083-1.p Axiom C4 for Boolean algebra in the Sheffer stroke, part 1
$(a \uparrow (a \uparrow (a \uparrow b))) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_4, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO084-1.p Axiom C4 for Boolean algebra in the Sheffer stroke, part 2
$(a \uparrow (a \uparrow (a \uparrow b))) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_4, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO085-1.p Axiom C5 for Boolean algebra in the Sheffer stroke, part 1
$(a \uparrow (a \uparrow (b \uparrow c))) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_5, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO086-1.p Axiom C5 for Boolean algebra in the Sheffer stroke, part 2

$(a \uparrow (a \uparrow (b \uparrow c))) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_5, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO087-1.p Axiom C6 for Boolean algebra in the Sheffer stroke, part 1
$(a \uparrow (a \uparrow (b \uparrow c))) \uparrow (c \uparrow (a \uparrow b)) = c$ cnf(c_6, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO088-1.p Axiom C6 for Boolean algebra in the Sheffer stroke, part 2
$(a \uparrow (a \uparrow (b \uparrow c))) \uparrow (c \uparrow (a \uparrow b)) = c$ cnf(c_6, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO089-1.p Axiom C7 for Boolean algebra in the Sheffer stroke, part 1
$(a \uparrow (a \uparrow (b \uparrow b))) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_7, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO090-1.p Axiom C7 for Boolean algebra in the Sheffer stroke, part 2
$(a \uparrow (a \uparrow (b \uparrow b))) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_7, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO091-1.p Axiom C8 for Boolean algebra in the Sheffer stroke, part 1
$((a \uparrow (a \uparrow b)) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_8, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO092-1.p Axiom C8 for Boolean algebra in the Sheffer stroke, part 2
$((a \uparrow (a \uparrow b)) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_8, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO093-1.p Axiom C9 for Boolean algebra in the Sheffer stroke, part 1
$((a \uparrow (b \uparrow b)) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_9, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO094-1.p Axiom C9 for Boolean algebra in the Sheffer stroke, part 2
$((a \uparrow (b \uparrow b)) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_9, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO095-1.p Axiom C10 for Boolean algebra in the Sheffer stroke, part 1
$((a \uparrow (b \uparrow c)) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{10}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO096-1.p Axiom C10 for Boolean algebra in the Sheffer stroke, part 2
$((a \uparrow (b \uparrow c)) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{10}, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO097-1.p Axiom C11 for Boolean algebra in the Sheffer stroke, part 1
$((a \uparrow (b \uparrow c)) \uparrow a) \uparrow (c \uparrow (a \uparrow b)) = c$ cnf(c_{11}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO098-1.p Axiom C11 for Boolean algebra in the Sheffer stroke, part 2
$((a \uparrow (b \uparrow c)) \uparrow a) \uparrow (c \uparrow (a \uparrow b)) = c$ cnf(c_{11}, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO099-1.p Axiom C12 for Boolean algebra in the Sheffer stroke, part 1
$((a \uparrow (a \uparrow b)) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{12}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO100-1.p Axiom C12 for Boolean algebra in the Sheffer stroke, part 2
$((a \uparrow (a \uparrow b)) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{12}, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO101-1.p Axiom C13 for Boolean algebra in the Sheffer stroke, part 1
$(((a \uparrow b) \uparrow a) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_{13}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO102-1.p Axiom C13 for Boolean algebra in the Sheffer stroke, part 2
$(((a \uparrow b) \uparrow a) \uparrow a) \uparrow (b \uparrow (c \uparrow a)) = b$ cnf(c_{13}, axiom)
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a$ cnf(prove_meredith_2_basis$_2$, negated_conjecture)

BOO103-1.p Axiom C14 for Boolean algebra in the Sheffer stroke, part 1
$(((a \uparrow b) \uparrow a) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b$ cnf(c_{14}, axiom)
$(a \uparrow a) \uparrow (b \uparrow a) \neq a$ cnf(prove_meredith_2_basis$_1$, negated_conjecture)

BOO104-1.p Axiom C14 for Boolean algebra in the Sheffer stroke, part 2
$(((a \uparrow b) \uparrow a) \uparrow a) \uparrow (b \uparrow (a \uparrow c)) = b \qquad \mathrm{cnf}(c_{14}, \mathrm{axiom})$
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \qquad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}_2, \mathrm{negated\_conjecture})$

BOO105-1.p Axiom C15 for Boolean algebra in the Sheffer stroke, part 1
$(((a \uparrow b) \uparrow c) \uparrow c) \uparrow (b \uparrow (a \uparrow c)) = b \qquad \mathrm{cnf}(c_{15}, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) \neq a \qquad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}_1, \mathrm{negated\_conjecture})$

BOO106-1.p Axiom C15 for Boolean algebra in the Sheffer stroke, part 2
$(((a \uparrow b) \uparrow c) \uparrow c) \uparrow (b \uparrow (a \uparrow c)) = b \qquad \mathrm{cnf}(c_{15}, \mathrm{axiom})$
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \qquad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}_2, \mathrm{negated\_conjecture})$

BOO107-1.p Axiom C16 for Boolean algebra in the Sheffer stroke, part 1
$(((a \uparrow b) \uparrow c) \uparrow c) \uparrow (b \uparrow (c \uparrow a)) = b \qquad \mathrm{cnf}(c_{16}, \mathrm{axiom})$
$(a \uparrow a) \uparrow (b \uparrow a) \neq a \qquad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}_1, \mathrm{negated\_conjecture})$

BOO108-1.p Axiom C16 for Boolean algebra in the Sheffer stroke, part 2
$(((a \uparrow b) \uparrow c) \uparrow c) \uparrow (b \uparrow (c \uparrow a)) = b \qquad \mathrm{cnf}(c_{16}, \mathrm{axiom})$
$a \uparrow (b \uparrow (a \uparrow c)) \neq ((c \uparrow b) \uparrow b) \uparrow a \qquad \mathrm{cnf}(\mathrm{prove\_meredith\_2\_basis}_2, \mathrm{negated\_conjecture})$

BOO109+1.p Josef Urban's problem using the Wajsberg axiom
$\forall a, b, c, d\colon p((a \uparrow (b \uparrow c)) \uparrow (((d \uparrow c) \uparrow ((a \uparrow d) \uparrow (a \uparrow d))) \uparrow (a \uparrow (a \uparrow b)))) \qquad \mathrm{fof}(\mathrm{wajsbergs\_axiom}, \mathrm{axiom})$
$\forall p, q, r\colon ((p(p \uparrow (q \uparrow r)) \text{ and } p(p)) \Rightarrow p(r)) \qquad \mathrm{fof}(\mathrm{modus\_ponens\_for\_nand}, \mathrm{axiom})$
$\forall a, b\colon p((a \uparrow (b \uparrow b)) \uparrow (((b \uparrow b) \uparrow ((a \uparrow a) \uparrow (a \uparrow a))) \uparrow ((b \uparrow b) \uparrow ((a \uparrow a) \uparrow (a \uparrow a))))) \qquad \mathrm{fof}(\mathrm{tautology}, \mathrm{conjecture})$