

# COM axioms

**COM001+1.ax** Common axioms for progress/preservation proof

$\forall ve: \text{valphaEquivalent}(ve, ve) \quad \text{fof('alpha-equiv-refl', axiom)}$   
 $\forall ve_2, ve_1: (\text{valphaEquivalent}(ve_1, ve_2) \Rightarrow \text{valphaEquivalent}(ve_2, ve_1)) \quad \text{fof('alpha-equiv-sym', axiom)}$   
 $\forall ve_2, ve_1, ve_3: ((\text{valphaEquivalent}(ve_1, ve_2) \text{ and } \text{valphaEquivalent}(ve_2, ve_3)) \Rightarrow \text{valphaEquivalent}(ve_1, ve_3)) \quad \text{fof('alpha-equiv-trans', axiom)}$   
 $\forall vS, vx, vy, ve: (\neg \text{visFreeVar}(vy, ve) \Rightarrow \text{valphaEquivalent}(\text{vabs}(vx, vS, ve), \text{vabs}(vy, vS, \text{vsubst}(vx, \text{vvar}(vy), ve)))) \quad \text{fof('alpha-equiv-abst', axiom)}$   
 $\forall ve, vC, ve_1, vT: ((\text{vtcheck}(vC, ve, vT) \text{ and } \text{valphaEquivalent}(ve, ve_1)) \Rightarrow \text{vtcheck}(vC, ve_1, vT)) \quad \text{fof('alpha-equiv-typing', axiom)}$   
 $\forall ve, vx, ve_1: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{valphaEquivalent}(ve, ve_1)) \Rightarrow \neg \text{visFreeVar}(vx, ve_1)) \quad \text{fof('alpha-equiv-FreeVar', axiom)}$

# COM problems

**COM001-1.p** A program correctness theorem

A simple computing state space, with four states - P3, P4, P5, and P8 (the full version of this state space is in the problem COM002-1). There is a branch at P3 such that the following state is either P4 or P8. P8 has a loop back to P3, while P4 leads to termination. The problem is to show that there is a loop in the computation, passing through P3.

$\text{follows}(\text{goal\_state}, \text{start\_state}) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state}) \quad \text{cnf}(\text{direct\_success}, \text{axiom})$   
 $(\text{succeeds}(\text{goal\_state}, \text{intermediate\_state}) \text{ and } \text{succeeds}(\text{intermediate\_state}, \text{start\_state})) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state})$   
 $(\text{has}(\text{start\_state}, \text{goto}(\text{label})) \text{ and } \text{labels}(\text{label}, \text{goal\_state})) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state}) \quad \text{cnf}(\text{goto\_success}, \text{axiom})$   
 $\text{has}(\text{start\_state}, \text{ifthen}(\text{condition}, \text{goal\_state})) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state}) \quad \text{cnf}(\text{conditional\_success}, \text{axiom})$   
 $\text{labels}(\text{loop}, p_3) \quad \text{cnf}(\text{label\_state}_3, \text{hypothesis})$   
 $\text{has}(p_3, \text{ifthen}(\text{equal\_function}(\text{register\_j}, n), p_4)) \quad \text{cnf}(\text{state}_3, \text{hypothesis})$   
 $\text{has}(p_4, \text{goto}(\text{out})) \quad \text{cnf}(\text{state}_4, \text{hypothesis})$   
 $\text{follows}(p_5, p_4) \quad \text{cnf}(\text{transition\_4\_to}_5, \text{hypothesis})$   
 $\text{follows}(p_8, p_3) \quad \text{cnf}(\text{transition\_3\_to}_8, \text{hypothesis})$   
 $\text{has}(p_8, \text{goto}(\text{loop})) \quad \text{cnf}(\text{state}_8, \text{hypothesis})$   
 $\neg \text{succeeds}(p_3, p_3) \quad \text{cnf}(\text{prove\_there\_is\_a\_loop\_through\_p}_3, \text{negated\_conjecture})$

**COM001.1.p** A program correctness theorem

A simple computing state space, with four states - P3, P4, P5, and P8 (the full version of this state space is in the problem COM002-1). There is a branch at P3 such that the following state is either P4 or P8. P8 has a loop back to P3, while P4 leads to termination. The problem is to show that there is a loop in the computation, passing through P3.

$\text{state}: \$t\text{Type} \quad \text{tff}(\text{state\_type}, \text{type})$   
 $\text{label}: \$t\text{Type} \quad \text{tff}(\text{label\_type}, \text{type})$   
 $\text{statement}: \$t\text{Type} \quad \text{tff}(\text{statement\_type}, \text{type})$   
 $\text{register}: \$t\text{Type} \quad \text{tff}(\text{register\_type}, \text{type})$   
 $\text{number}: \$t\text{Type} \quad \text{tff}(\text{number\_type}, \text{type})$   
 $\text{boolean}: \$t\text{Type} \quad \text{tff}(\text{boolean\_type}, \text{type})$   
 $p_3: \text{state} \quad \text{tff}(p_3\_type, \text{type})$   
 $p_4: \text{state} \quad \text{tff}(p_4\_type, \text{type})$   
 $p_5: \text{state} \quad \text{tff}(p_5\_type, \text{type})$   
 $p_8: \text{state} \quad \text{tff}(p_8\_type, \text{type})$   
 $n: \text{number} \quad \text{tff}(n\_type, \text{type})$   
 $\text{register\_j}: \text{register} \quad \text{tff}(\text{register\_j\_type}, \text{type})$   
 $\text{out}: \text{label} \quad \text{tff}(\text{out\_type}, \text{type})$   
 $\text{loop}: \text{label} \quad \text{tff}(\text{loop\_type}, \text{type})$   
 $\text{equal\_function}: (\text{register} \times \text{number}) \rightarrow \text{boolean} \quad \text{tff}(\text{equal\_function\_type}, \text{type})$   
 $\text{goto}: \text{label} \rightarrow \text{statement} \quad \text{tff}(\text{goto\_type}, \text{type})$   
 $\text{ifthen}: (\text{boolean} \times \text{state}) \rightarrow \text{statement} \quad \text{tff}(\text{ifthen\_type}, \text{type})$   
 $\text{follows}: (\text{state} \times \text{state}) \rightarrow \$o \quad \text{tff}(\text{follows\_type}, \text{type})$   
 $\text{succeeds}: (\text{state} \times \text{state}) \rightarrow \$o \quad \text{tff}(\text{succeeds\_type}, \text{type})$   
 $\text{labels}: (\text{label} \times \text{state}) \rightarrow \$o \quad \text{tff}(\text{labels\_type}, \text{type})$   
 $\text{has}: (\text{state} \times \text{statement}) \rightarrow \$o \quad \text{tff}(\text{has\_type}, \text{type})$   
 $\forall \text{start\_state}: \text{state}, \text{goal\_state}: \text{state}: (\text{follows}(\text{goal\_state}, \text{start\_state}) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state})) \quad \text{tff}(\text{direct\_success}, \text{axiom})$   
 $\forall \text{start\_state}: \text{state}, \text{intermediate\_state}: \text{state}, \text{goal\_state}: \text{state}: ((\text{succeeds}(\text{goal\_state}, \text{intermediate\_state}) \text{ and } \text{succeeds}(\text{intermediate\_state}, \text{start\_state})) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state})) \quad \text{tff}(\text{transitivity\_of\_success}, \text{axiom})$   
 $\forall \text{goal\_state}: \text{state}, \text{label}: \text{label}, \text{start\_state}: \text{state}: ((\text{has}(\text{start\_state}, \text{goto}(\text{label})) \text{ and } \text{labels}(\text{label}, \text{goal\_state})) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state})) \quad \text{tff}(\text{loop\_through\_p}_3, \text{conjecture})$

$\forall \text{goal\_state: state, condition: boolean, start\_state: state: (has(start\_state, ifthen(condition, goal\_state)) \Rightarrow \text{succeeds(goal\_state, start\_state)})$   
 $\text{labels}(\text{loop}, p_3) \quad \text{tff}(\text{label\_state}_3, \text{hypothesis})$   
 $\text{has}(p_3, \text{ifthen}(\text{equal\_function}(\text{register\_j}, n), p_4)) \quad \text{tff}(\text{state}_3, \text{hypothesis})$   
 $\text{has}(p_4, \text{goto}(\text{out})) \quad \text{tff}(\text{state}_4, \text{hypothesis})$   
 $\text{follows}(p_5, p_4) \quad \text{tff}(\text{transition\_4\_to}_5, \text{hypothesis})$   
 $\text{follows}(p_8, p_3) \quad \text{tff}(\text{transition\_3\_to}_8, \text{hypothesis})$   
 $\text{has}(p_8, \text{goto}(\text{loop})) \quad \text{tff}(\text{state}_8, \text{hypothesis})$   
 $\text{succeeds}(p_3, p_3) \quad \text{tff}(\text{prove\_there\_is\_a\_loop\_through\_p}_3, \text{conjecture})$

**COM002-1.p** A program correctness theorem

A computing state space, with eight states - P1 to P8. P1 leads to P3 via P2. There is a branch at P3 such that the following state is either P4 or P6. P6 leads to P8, which has a loop back to P3, while P4 leads to termination. The problem is to show that there is a loop in the computation, passing through P3.

$\text{follows}(\text{goal\_state}, \text{start\_state}) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state}) \quad \text{cnf}(\text{direct\_success}, \text{axiom})$   
 $(\text{succeeds}(\text{goal\_state}, \text{intermediate\_state}) \text{ and } \text{succeeds}(\text{intermediate\_state}, \text{start\_state})) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state})$   
 $(\text{has}(\text{start\_state}, \text{goto}(\text{label})) \text{ and } \text{labels}(\text{label}, \text{goal\_state})) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state}) \quad \text{cnf}(\text{goto\_success}, \text{axiom})$   
 $\text{has}(\text{start\_state}, \text{ifthen}(\text{condition}, \text{goal\_state})) \Rightarrow \text{succeeds}(\text{goal\_state}, \text{start\_state}) \quad \text{cnf}(\text{conditional\_success}, \text{axiom})$   
 $\text{has}(p_1, \text{assign}(\text{register\_j}, n_0)) \quad \text{cnf}(\text{state}_1, \text{hypothesis})$   
 $\text{follows}(p_2, p_1) \quad \text{cnf}(\text{transition\_1\_to}_2, \text{hypothesis})$   
 $\text{has}(p_2, \text{assign}(\text{register\_k}, n_1)) \quad \text{cnf}(\text{state}_2, \text{hypothesis})$   
 $\text{labels}(\text{loop}, p_3) \quad \text{cnf}(\text{label\_state}_3, \text{hypothesis})$   
 $\text{follows}(p_3, p_2) \quad \text{cnf}(\text{transition\_2\_to}_3, \text{hypothesis})$   
 $\text{has}(p_3, \text{ifthen}(\text{equal\_function}(\text{register\_j}, n), p_4)) \quad \text{cnf}(\text{state}_3, \text{hypothesis})$   
 $\text{has}(p_4, \text{goto}(\text{out})) \quad \text{cnf}(\text{state}_4, \text{hypothesis})$   
 $\text{follows}(p_5, p_4) \quad \text{cnf}(\text{transition\_4\_to}_5, \text{hypothesis})$   
 $\text{follows}(p_6, p_3) \quad \text{cnf}(\text{transition\_3\_to}_6, \text{hypothesis})$   
 $\text{has}(p_6, \text{assign}(\text{register\_k}, \text{times}(n_2, \text{register\_k}))) \quad \text{cnf}(\text{state}_6, \text{hypothesis})$   
 $\text{follows}(p_7, p_6) \quad \text{cnf}(\text{transition\_6\_to}_7, \text{hypothesis})$   
 $\text{has}(p_7, \text{assign}(\text{register\_j}, \text{register\_j} + n_1)) \quad \text{cnf}(\text{state}_7, \text{hypothesis})$   
 $\text{follows}(p_8, p_7) \quad \text{cnf}(\text{transition\_7\_to}_8, \text{hypothesis})$   
 $\text{has}(p_8, \text{goto}(\text{loop})) \quad \text{cnf}(\text{state}_8, \text{hypothesis})$   
 $\neg \text{succeeds}(p_3, p_3) \quad \text{cnf}(\text{prove\_there\_is\_a\_loop\_through\_p}_3, \text{negated\_conjecture})$

**COM002-2.p** A program correctness theorem.

A computing state space, with eight states - P1 to P8. P1 leads to P3 via P2. There is a branch at P3 such that the following state is either P4 or P6. P6 leads to P8, which has a loop back to P3, while P4 leads to termination. The problem is to show that there is a loop in the computation, passing through P3.

$\text{fails}(\text{goal\_state}, \text{start\_state}) \Rightarrow \neg \text{follows}(\text{goal\_state}, \text{start\_state}) \quad \text{cnf}(\text{direct\_success}, \text{axiom})$   
 $\text{fails}(\text{goal\_state}, \text{start\_state}) \Rightarrow (\text{fails}(\text{goal\_state}, \text{intermediate\_state}) \text{ or } \text{fails}(\text{intermediate\_state}, \text{start\_state})) \quad \text{cnf}(\text{transitivity}, \text{axiom})$   
 $(\text{fails}(\text{goal\_state}, \text{start\_state}) \text{ and } \text{has}(\text{start\_state}, \text{goto}(\text{label}))) \Rightarrow \neg \text{labels}(\text{label}, \text{goal\_state}) \quad \text{cnf}(\text{goto\_success}, \text{axiom})$   
 $\text{fails}(\text{goal\_state}, \text{start\_state}) \Rightarrow \neg \text{has}(\text{start\_state}, \text{ifthen}(\text{condition}, \text{goal\_state})) \quad \text{cnf}(\text{conditional\_success}, \text{axiom})$   
 $\text{has}(p_1, \text{assign}(\text{register\_j}, n_0)) \quad \text{cnf}(\text{state}_1, \text{hypothesis})$   
 $\text{follows}(p_2, p_1) \quad \text{cnf}(\text{transition\_1\_to}_2, \text{hypothesis})$   
 $\text{has}(p_2, \text{assign}(\text{register\_k}, n_1)) \quad \text{cnf}(\text{state}_2, \text{hypothesis})$   
 $\text{labels}(\text{loop}, p_3) \quad \text{cnf}(\text{label\_state}_3, \text{hypothesis})$   
 $\text{follows}(p_3, p_2) \quad \text{cnf}(\text{transition\_2\_to}_3, \text{hypothesis})$   
 $\text{has}(p_3, \text{ifthen}(\text{equal\_function}(\text{register\_j}, n), p_4)) \quad \text{cnf}(\text{state}_3, \text{hypothesis})$   
 $\text{has}(p_4, \text{goto}(\text{out})) \quad \text{cnf}(\text{state}_4, \text{hypothesis})$   
 $\text{follows}(p_5, p_4) \quad \text{cnf}(\text{transition\_4\_to}_5, \text{hypothesis})$   
 $\text{follows}(p_6, p_3) \quad \text{cnf}(\text{transition\_3\_to}_6, \text{hypothesis})$   
 $\text{has}(p_6, \text{assign}(\text{register\_k}, \text{times}(n_2, \text{register\_k}))) \quad \text{cnf}(\text{state}_6, \text{hypothesis})$   
 $\text{follows}(p_7, p_6) \quad \text{cnf}(\text{transition\_6\_to}_7, \text{hypothesis})$   
 $\text{has}(p_7, \text{assign}(\text{register\_j}, \text{register\_j} + n_1)) \quad \text{cnf}(\text{state}_7, \text{hypothesis})$   
 $\text{follows}(p_8, p_7) \quad \text{cnf}(\text{transition\_7\_to}_8, \text{hypothesis})$   
 $\text{has}(p_8, \text{goto}(\text{loop})) \quad \text{cnf}(\text{state}_8, \text{hypothesis})$   
 $\text{fails}(p_3, p_3) \quad \text{cnf}(\text{prove\_there\_is\_a\_loop\_through\_p}_3, \text{negated\_conjecture})$

**COM003+1.p** The halting problem is undecidable

$\exists x: (\text{algorithm}(x) \text{ and } \forall y: (\text{program}(y) \Rightarrow \forall z: \text{decides}(x, y, z))) \Rightarrow \exists w: (\text{program}(w) \text{ and } \forall y: (\text{program}(y) \Rightarrow \forall z: \text{decides}(w, y, z))) \quad \text{fof}(p_1, \text{axiom})$

$\forall w: ((\text{program}(w) \text{ and } \forall y: (\text{program}(y) \Rightarrow \forall z: \text{decides}(w, y, z))) \Rightarrow \forall y, z: (((\text{program}(y) \text{ and } \text{halts}_2(y, z)) \Rightarrow (\text{halts}_3(w, y, z) \text{ and } \text{outputs}(w, \text{good}))) \text{ and } ((\text{program}(y) \text{ and } \neg \text{halts}_2(y, z)) \Rightarrow (\text{halts}_3(w, y, z) \text{ and } \text{outputs}(w, \text{bad}))))))$   
 $\exists w: (\text{program}(w) \text{ and } \forall y: (((\text{program}(y) \text{ and } \text{halts}_2(y, y)) \Rightarrow (\text{halts}_3(w, y, y) \text{ and } \text{outputs}(w, \text{good}))) \text{ and } ((\text{program}(y) \text{ and } (\text{halts}_3(w, y, y) \text{ and } \text{outputs}(w, \text{bad})))))) \Rightarrow \exists v: (\text{program}(v) \text{ and } \forall y: (((\text{program}(y) \text{ and } \text{halts}_2(y, y)) \Rightarrow (\text{halts}_2(v, y) \text{ and } \text{outputs}(v, \text{good}))) \text{ and } ((\text{program}(y) \text{ and } \neg \text{halts}_2(v, y) \text{ and } \text{outputs}(v, \text{bad})))))) \quad \text{fof}(p_3, \text{axiom})$   
 $\exists v: (\text{program}(v) \text{ and } \forall y: (((\text{program}(y) \text{ and } \text{halts}_2(y, y)) \Rightarrow (\text{halts}_2(v, y) \text{ and } \text{outputs}(v, \text{good}))) \text{ and } ((\text{program}(y) \text{ and } \neg \text{halts}_2(v, y) \text{ and } \text{outputs}(v, \text{bad})))))) \Rightarrow \exists u: (\text{program}(u) \text{ and } \forall y: (((\text{program}(y) \text{ and } \text{halts}_2(y, y)) \Rightarrow \neg \text{halts}_2(u, y) \text{ and } ((\text{program}(y) \text{ and } \text{halts}_2(u, y) \text{ and } \text{outputs}(u, \text{bad})))))) \quad \text{fof}(p_4, \text{axiom})$   
 $\neg \exists x_1: (\text{algorithm}(x_1) \text{ and } \forall y_1: (\text{program}(y_1) \Rightarrow \forall z_1: \text{decides}(x_1, y_1, z_1))) \quad \text{fof}(\text{prove\_this}, \text{conjecture})$

**COM003+2.p** The halting problem is undecidable

$\forall x: (\text{program\_decides}(x) \iff \forall y: (\text{program}(y) \Rightarrow \forall z: \text{decides}(x, y, z))) \quad \text{fof}(\text{program\_decides\_def}, \text{axiom})$   
 $\forall x: (\text{program\_program\_decides}(x) \iff (\text{program}(x) \text{ and } \text{program\_decides}(x))) \quad \text{fof}(\text{program\_program\_decides\_def}, \text{axiom})$   
 $\forall x: (\text{algorithm\_program\_decides}(x) \iff (\text{algorithm}(x) \text{ and } \text{program\_decides}(x))) \quad \text{fof}(\text{algorithm\_program\_decides\_def}, \text{axiom})$   
 $\forall x, y: (\text{program\_halts}_2(x, y) \iff (\text{program}(x) \text{ and } \text{halts}_2(x, y))) \quad \text{fof}(\text{program\_halts}_2\_def, \text{axiom})$   
 $\forall x, y, z, w: (\text{halts}_3\_outputs(x, y, z, w) \iff (\text{halts}_3(x, y, z) \text{ and } \text{outputs}(x, w))) \quad \text{fof}(\text{halts}_3\_outputs\_def, \text{axiom})$   
 $\forall x, y: (\text{program\_not\_halts}_2(x, y) \iff (\text{program}(x) \text{ and } \neg \text{halts}_2(x, y))) \quad \text{fof}(\text{program\_not\_halts}_2\_def, \text{axiom})$   
 $\forall x, y, w: (\text{halts}_2\_outputs(x, y, w) \iff (\text{halts}_2(x, y) \text{ and } \text{outputs}(x, w))) \quad \text{fof}(\text{halts}_2\_outputs\_def, \text{axiom})$   
 $\forall x, y, z, w: (\text{program\_halts}_2\_halts}_3\_outputs(x, y, z, w) \iff (\text{program\_halts}_2(y, z) \Rightarrow \text{halts}_3\_outputs(x, y, z, w))) \quad \text{fof}(\text{program\_halts}_2\_halts}_3\_outputs\_def, \text{axiom})$   
 $\forall x, y, z, w: (\text{program\_not\_halts}_2\_halts}_3\_outputs(x, y, z, w) \iff (\text{program\_not\_halts}_2(y, z) \Rightarrow \text{halts}_3\_outputs(x, y, z, w))) \quad \text{fof}(\text{program\_not\_halts}_2\_halts}_3\_outputs\_def, \text{axiom})$   
 $\forall x, y, w: (\text{program\_halts}_2\_halts}_2\_outputs(x, y, w) \iff (\text{program\_halts}_2(y, y) \Rightarrow \text{halts}_2\_outputs(x, y, w))) \quad \text{fof}(\text{program\_halts}_2\_halts}_2\_outputs\_def, \text{axiom})$   
 $\forall x, y, w: (\text{program\_not\_halts}_2\_halts}_2\_outputs(x, y, w) \iff (\text{program\_not\_halts}_2(y, y) \Rightarrow \text{halts}_2\_outputs(x, y, w))) \quad \text{fof}(\text{program\_not\_halts}_2\_halts}_2\_outputs\_def, \text{axiom})$   
 $\exists x: \text{algorithm\_program\_decides}(x) \Rightarrow \exists w: \text{program\_program\_decides}(w) \quad \text{fof}(p_1, \text{axiom})$   
 $\forall w: (\text{program\_program\_decides}(w) \Rightarrow \forall y, z: (\text{program\_halts}_2\_halts}_3\_outputs(w, y, z, \text{good}) \text{ and } \text{program\_not\_halts}_2\_halts}_3\_outputs(w, y, z, \text{bad}))) \quad \text{fof}(\text{program\_program\_decides\_def}, \text{axiom})$   
 $\exists w: (\text{program}(w) \text{ and } \forall y: (\text{program\_halts}_2\_halts}_3\_outputs(w, y, y, \text{good}) \text{ and } \text{program\_not\_halts}_2\_halts}_3\_outputs(w, y, y, \text{bad}))) \quad \text{fof}(\text{program\_program\_decides\_def}, \text{axiom})$   
 $\exists v: (\text{program}(v) \text{ and } \forall y: (\text{program\_halts}_2\_halts}_2\_outputs(v, y, \text{good}) \text{ and } \text{program\_not\_halts}_2\_halts}_2\_outputs(v, y, \text{bad}))) \quad \text{fof}(\text{program\_program\_decides\_def}, \text{axiom})$   
 $\exists v: (\text{program}(v) \text{ and } \forall y: (\text{program\_halts}_2\_halts}_2\_outputs(v, y, \text{good}) \text{ and } \text{program\_not\_halts}_2\_halts}_2\_outputs(v, y, \text{bad}))) \Rightarrow \exists u: (\text{program}(u) \text{ and } \forall y: ((\text{program\_halts}_2(y, y) \Rightarrow \neg \text{halts}_2(u, y)) \text{ and } \text{program\_not\_halts}_2\_halts}_2\_outputs(u, y, \text{good}))) \quad \text{fof}(\text{prove\_this}, \text{conjecture})$   
 $\neg \exists x: \text{algorithm\_program\_decides}(x) \quad \text{fof}(\text{prove\_this}, \text{conjecture})$

**COM003+3.p** The halting problem is undecidable

$\exists x: (\text{algorithm}(x) \text{ and } \forall y: (\text{program}(y) \Rightarrow \forall z: \text{decides}(x, y, z))) \Rightarrow \exists w: (\text{program}(w) \text{ and } \forall y: (\text{program}(y) \Rightarrow \forall z: \text{decides}(w, y, z))) \quad \text{fof}(p_1, \text{axiom})$   
 $\forall w: ((\text{program}(w) \text{ and } \forall y: (\text{program}(y) \Rightarrow \forall z: \text{decides}(w, y, z))) \Rightarrow \forall y, z: (((\text{program}(y) \text{ and } \text{halts}_2(y, z)) \Rightarrow (\text{halts}_3(w, y, z) \text{ and } \text{outputs}(w, \text{good}))) \text{ and } ((\text{program}(y) \text{ and } \neg \text{halts}_2(y, z)) \Rightarrow (\text{halts}_3(w, y, z) \text{ and } \text{outputs}(w, \text{bad}))))))$   
 $\forall w: ((\text{program}(w) \text{ and } \forall y, z: (((\text{program}(y) \text{ and } \text{halts}_2(y, z)) \Rightarrow (\text{halts}_3(w, y, z) \text{ and } \text{outputs}(w, \text{good}))) \text{ and } ((\text{program}(y) \text{ and } (\text{halts}_3(w, y, z) \text{ and } \text{outputs}(w, \text{bad})))))) \Rightarrow \exists v: (\text{program}(v) \text{ and } \forall y: (((\text{program}(y) \text{ and } \text{halts}_3(w, y, y) \text{ and } \text{outputs}(w, \text{good})) \text{ and } \neg \text{halts}_2(v, y) \text{ and } ((\text{program}(y) \text{ and } \text{halts}_3(w, y, y) \text{ and } \text{outputs}(w, \text{bad})) \Rightarrow (\text{halts}_2(v, y) \text{ and } \text{outputs}(v, \text{bad})))))) \quad \text{fof}(p_2, \text{axiom})$   
 $\neg \exists x_1: (\text{algorithm}(x_1) \text{ and } \forall y_1: (\text{program}(y_1) \Rightarrow \forall z_1: \text{decides}(x_1, y_1, z_1))) \quad \text{fof}(\text{prove\_this}, \text{conjecture})$

**COM003.1.p** The halting problem is undecidable

`program: $tType    tff(program_type, type)`  
`algorithm: $tType    tff(algorithm_type, type)`  
`input: $tType    tff(input_type, type)`  
`output: $tType    tff(output_type, type)`  
`bad: output    tff(bad_type, type)`  
`good: output    tff(good_type, type)`  
`decides: (algorithm × program × input) → $o    tff(decides_type, type)`  
`halts2: (program × input) → $o    tff(halts2_type, type)`  
`halts3: (program × program × input) → $o    tff(halts3_type, type)`  
`outputs: (program × output) → $o    tff(outputs_type, type)`  
`algorithm_of: program → algorithm    tff(algorithm_of_type, type)`  
`as_input: program → input    tff(as_input_type, type)`  
 $\exists x: \text{algorithm}: \forall y: \text{program}, z: \text{input}: \text{decides}(x, y, z) \Rightarrow \exists w: \text{program}: \forall y: \text{program}, z: \text{input}: \text{decides}(\text{algorithm\_of}(w), y, z)$   
 $\forall w: \text{program}, y: \text{program}, z: \text{input}: (\text{decides}(\text{algorithm\_of}(w), y, z) \Rightarrow \forall y: \text{program}, z: \text{input}: ((\text{halts}_2(y, z) \Rightarrow (\text{halts}_3(w, y, z) \text{ and } \text{outputs}(w, \text{good}))) \text{ and } (\neg \text{halts}_2(y, z) \Rightarrow (\text{halts}_3(w, y, z) \text{ and } \text{outputs}(w, \text{bad})))))) \quad \text{tff}(p_2, \text{axiom})$   
 $\exists w: \text{program}: \forall y: \text{program}: ((\text{halts}_2(y, \text{as\_input}(y)) \Rightarrow (\text{halts}_3(w, y, \text{as\_input}(y)) \text{ and } \text{outputs}(w, \text{good}))) \text{ and } (\neg \text{halts}_2(y, \text{as\_input}(y)) \Rightarrow (\text{halts}_3(w, y, \text{as\_input}(y)) \text{ and } \text{outputs}(w, \text{bad})))))) \Rightarrow \exists v: \text{program}: \forall y: \text{program}: ((\text{halts}_2(y, \text{as\_input}(y)) \Rightarrow (\text{halts}_2(v, \text{as\_input}(y)) \text{ and } \text{outputs}(v, \text{bad})))) \quad \text{tff}(p_3, \text{axiom})$

$\exists v: \text{program}: \forall y: \text{program}: ((\text{halts}_2(y, \text{as\_input}(y)) \Rightarrow (\text{halts}_2(v, \text{as\_input}(y)) \text{ and } \text{outputs}(v, \text{good}))) \text{ and } (\neg \text{halts}_2(y, \text{as\_input}(y)) \text{ and } (\text{halts}_2(v, \text{as\_input}(y)) \text{ and } \text{outputs}(v, \text{bad})))) \Rightarrow \exists u: \text{program}: \forall y: \text{program}: ((\text{halts}_2(y, \text{as\_input}(y)) \Rightarrow \neg \text{halts}_2(u, \text{as\_input}(y)) \text{ and } (\text{halts}_2(u, \text{as\_input}(y)) \text{ and } \text{outputs}(u, \text{bad})))) \quad \text{tff}(p_4, \text{axiom})$   
 $\neg \exists x_1: \text{algorithm}: \forall y_1: \text{program}, z_1: \text{input}: \text{decides}(x_1, y_1, z_1) \quad \text{tff}(\text{prove\_this, conjecture})$

#### COM004-1.p Part of completeness of resolution

Part of [Bun83]'s proof of the completeness of resolution uses the notion of failure nodes. This proves a special case when a parent is the empty failure node.

$(\text{failure\_node}(x, \text{or}(c, p)) \text{ and } \text{failure\_node}(y, \text{or}(d, q)) \text{ and } \text{contradictory}(p, q) \text{ and } \text{siblings}(x, y)) \Rightarrow \text{failure\_node}(\text{parent\_of}(x, y))$   
 $\text{contradictory}(\neg x, x) \quad \text{cnf}(\text{not\_x\_contradicts\_x, axiom})$   
 $\text{contradictory}(x, \neg x) \quad \text{cnf}(\text{x\_contradicts\_not\_x, axiom})$   
 $\text{siblings}(\text{left\_child\_of}(x), \text{right\_child\_of}(x)) \quad \text{cnf}(\text{n\_left\_and\_n\_right\_are\_siblings, axiom})$   
 $\text{failure\_node}(\text{n\_left}, \text{or}(\text{empty}, \text{atom})) \quad \text{cnf}(\text{n\_left\_is\_atom, hypothesis})$   
 $\text{failure\_node}(\text{n\_right}, \text{or}(\text{empty}, \neg \text{atom})) \quad \text{cnf}(\text{n\_right\_is\_not\_atom, hypothesis})$   
 $\text{n\_left} = \text{left\_child\_of}(n) \quad \text{cnf}(\text{n\_left\_equals\_left\_child\_of\_n, hypothesis})$   
 $\text{n\_right} = \text{right\_child\_of}(n) \quad \text{cnf}(\text{n\_right\_equals\_right\_child\_of\_n, hypothesis})$   
 $\neg \text{failure\_node}(z, \text{or}(\text{empty}, \text{empty})) \quad \text{cnf}(\text{goal\_is\_there\_an\_empty\_node, negated\_conjecture})$

#### COM007+1.p Preservation of the Diamond Property under reflexive closure

$\text{reflexive\_rewrite}(a, b) \text{ and } \text{reflexive\_rewrite}(a, c) \quad \text{fof}(\text{assumption, axiom})$   
 $\forall a: ((\text{reflexive\_rewrite}(b, a) \text{ and } \text{reflexive\_rewrite}(c, a)) \Rightarrow \text{goal}) \quad \text{fof}(\text{goal\_ax, axiom})$   
 $\forall a: a = a \quad \text{fof}(\text{reflexivity, axiom})$   
 $\forall a, b: (a = b \Rightarrow b = a) \quad \text{fof}(\text{symmetry, axiom})$   
 $\forall a, b, c: ((a = b \text{ and } \text{reflexive\_rewrite}(b, c)) \Rightarrow \text{reflexive\_rewrite}(a, c)) \quad \text{fof}(\text{substitution, axiom})$   
 $\forall a, b: (a = b \Rightarrow \text{reflexive\_rewrite}(a, b)) \quad \text{fof}(\text{equalish\_in\_reflexive\_rewrite, axiom})$   
 $\forall a, b: (\text{rewrite}(a, b) \Rightarrow \text{reflexive\_rewrite}(a, b)) \quad \text{fof}(\text{rewrite\_in\_reflexive\_rewrite, axiom})$   
 $\forall a, b: (\text{reflexive\_rewrite}(a, b) \Rightarrow (a = b \text{ or } \text{rewrite}(a, b))) \quad \text{fof}(\text{equalish\_or\_rewrite, axiom})$   
 $\forall a, b, c: ((\text{rewrite}(a, b) \text{ and } \text{rewrite}(a, c)) \Rightarrow \exists d: (\text{rewrite}(b, d) \text{ and } \text{rewrite}(c, d))) \quad \text{fof}(\text{rewrite\_diamond, axiom})$   
 $\text{goal} \quad \text{fof}(\text{goal\_to\_be\_proved, conjecture})$

#### COM007+2.p Preservation of the Diamond Property under reflexive closure

$\text{reflexive\_rewrite}(a, b) \text{ and } \text{reflexive\_rewrite}(a, c) \quad \text{fof}(\text{assumption, axiom})$   
 $\forall a: ((\text{reflexive\_rewrite}(b, a) \text{ and } \text{reflexive\_rewrite}(c, a)) \Rightarrow \text{goal}) \quad \text{fof}(\text{goal\_ax, axiom})$   
 $\forall a, b: (a = b \Rightarrow \text{reflexive\_rewrite}(a, b)) \quad \text{fof}(\text{equal\_in\_reflexive\_rewrite, axiom})$   
 $\forall a, b: (\text{rewrite}(a, b) \Rightarrow \text{reflexive\_rewrite}(a, b)) \quad \text{fof}(\text{rewrite\_in\_reflexive\_rewrite, axiom})$   
 $\forall a, b: (\text{reflexive\_rewrite}(a, b) \Rightarrow (a = b \text{ or } \text{rewrite}(a, b))) \quad \text{fof}(\text{equal\_or\_rewrite, axiom})$   
 $\forall a, b, c: ((\text{rewrite}(a, b) \text{ and } \text{rewrite}(a, c)) \Rightarrow \exists d: (\text{rewrite}(b, d) \text{ and } \text{rewrite}(c, d))) \quad \text{fof}(\text{rewrite\_diamond, axiom})$   
 $\text{goal} \quad \text{fof}(\text{goal\_to\_be\_proved, conjecture})$

#### COM008+1.p Induction step in Newman's Lemma

$\forall a: ((\text{transitive\_reflexive\_rewrite}(b, a) \text{ and } \text{transitive\_reflexive\_rewrite}(c, a)) \Rightarrow \text{goal}) \quad \text{fof}(\text{found, axiom})$   
 $\text{transitive\_reflexive\_rewrite}(a, b) \text{ and } \text{transitive\_reflexive\_rewrite}(a, c) \quad \text{fof}(\text{assumption, axiom})$   
 $\forall a: a = a \quad \text{fof}(\text{reflexivity, axiom})$   
 $\forall a, b: (a = b \Rightarrow b = a) \quad \text{fof}(\text{symmetry, axiom})$   
 $\forall a, b: (a = b \Rightarrow \text{transitive\_reflexive\_rewrite}(a, b)) \quad \text{fof}(\text{equality\_in\_transitive\_reflexive\_rewrite, axiom})$   
 $\forall a, b: (\text{rewrite}(a, b) \Rightarrow \text{transitive\_reflexive\_rewrite}(a, b)) \quad \text{fof}(\text{rewrite\_in\_transitive\_reflexive\_rewrite, axiom})$   
 $\forall a, b, c: ((\text{transitive\_reflexive\_rewrite}(a, b) \text{ and } \text{transitive\_reflexive\_rewrite}(b, c)) \Rightarrow \text{transitive\_reflexive\_rewrite}(a, c)) \quad \text{fof}(\text{transitive\_reflexive\_rewrite, axiom})$   
 $\forall a, b, c: ((\text{rewrite}(a, b) \text{ and } \text{rewrite}(a, c)) \Rightarrow \exists d: (\text{transitive\_reflexive\_rewrite}(b, d) \text{ and } \text{transitive\_reflexive\_rewrite}(c, d))) \quad \text{fof}(\text{rewrite\_diamond, axiom})$   
 $\forall a, b, c: ((\text{rewrite}(a, a) \text{ and } \text{transitive\_reflexive\_rewrite}(a, b) \text{ and } \text{transitive\_reflexive\_rewrite}(a, c)) \Rightarrow \exists d: (\text{transitive\_reflexive\_rewrite}(a, d) \text{ and } \text{transitive\_reflexive\_rewrite}(b, d) \text{ and } \text{transitive\_reflexive\_rewrite}(c, d))) \quad \text{fof}(\text{transitive\_reflexive\_rewrite\_diamond, axiom})$   
 $\forall a, b: (\text{transitive\_reflexive\_rewrite}(a, b) \Rightarrow (a = b \text{ or } \exists c: (\text{rewrite}(a, c) \text{ and } \text{transitive\_reflexive\_rewrite}(c, b)))) \quad \text{fof}(\text{equalish\_or\_rewrite, axiom})$   
 $\text{goal} \quad \text{fof}(\text{goal\_to\_be\_proved, conjecture})$

#### COM008+2.p Induction step in Newman's Lemma

$\forall a: ((\text{transitive\_reflexive\_rewrite}(b, a) \text{ and } \text{transitive\_reflexive\_rewrite}(c, a)) \Rightarrow \text{goal}) \quad \text{fof}(\text{found, axiom})$   
 $\text{transitive\_reflexive\_rewrite}(a, b) \text{ and } \text{transitive\_reflexive\_rewrite}(a, c) \quad \text{fof}(\text{assumption, axiom})$   
 $\forall a, b: (a = b \Rightarrow \text{transitive\_reflexive\_rewrite}(a, b)) \quad \text{fof}(\text{equality\_in\_transitive\_reflexive\_rewrite, axiom})$   
 $\forall a, b: (\text{rewrite}(a, b) \Rightarrow \text{transitive\_reflexive\_rewrite}(a, b)) \quad \text{fof}(\text{rewrite\_in\_transitive\_reflexive\_rewrite, axiom})$   
 $\forall a, b, c: ((\text{transitive\_reflexive\_rewrite}(a, b) \text{ and } \text{transitive\_reflexive\_rewrite}(b, c)) \Rightarrow \text{transitive\_reflexive\_rewrite}(a, c)) \quad \text{fof}(\text{transitive\_reflexive\_rewrite, axiom})$   
 $\forall a, b, c: ((\text{rewrite}(a, b) \text{ and } \text{rewrite}(a, c)) \Rightarrow \exists d: (\text{transitive\_reflexive\_rewrite}(b, d) \text{ and } \text{transitive\_reflexive\_rewrite}(c, d))) \quad \text{fof}(\text{rewrite\_diamond, axiom})$   
 $\forall a, b, c: ((\text{rewrite}(a, a) \text{ and } \text{transitive\_reflexive\_rewrite}(a, b) \text{ and } \text{transitive\_reflexive\_rewrite}(a, c)) \Rightarrow \exists d: (\text{transitive\_reflexive\_rewrite}(a, d) \text{ and } \text{transitive\_reflexive\_rewrite}(b, d) \text{ and } \text{transitive\_reflexive\_rewrite}(c, d))) \quad \text{fof}(\text{transitive\_reflexive\_rewrite\_diamond, axiom})$   
 $\forall a, b: (\text{transitive\_reflexive\_rewrite}(a, b) \Rightarrow (a = b \text{ or } \exists c: (\text{rewrite}(a, c) \text{ and } \text{transitive\_reflexive\_rewrite}(c, b)))) \quad \text{fof}(\text{equal\_or\_rewrite, axiom})$   
 $\text{goal} \quad \text{fof}(\text{goal\_to\_be\_proved, conjecture})$

**COM009-1.p** Problem about UNITY theory

```
include('Axioms/MS001-2.ax')
```

```
include('Axioms/MS001-0.ax')
```

```
c.in(c.Relation_OId, c.UNITY_OActs(v_F, t_a), tc_set(tc_prod(t_a, t_a)))    cnf(cls.UNITY_OId_in_Acts0, axiom)
c.in(c.Relation_OId, c.UNITY_OAllowedActs(v_F, t_a), tc_set(tc_prod(t_a, t_a)))    cnf(cls.UNITY_OId_in_AllowedActs0, a
c.UNITY_Oall_total(v_F, t_a) => c.UNITY_Ototalize(v_F, t_a) = v_F    cnf(cls.UNITY_Oall_total_imp_totalize0, axiom)
c.UNITY_Oall_total(c.UNITY_Ototalize(v_F, t_a), t_a)    cnf(cls.UNITY_Oall_total_totalize0, axiom)
c.in(v_F, c.UNITY_Oconstrains(v_A, c.UNIV, t_a), tc.UNITY_Oprogram(t_a))    cnf(cls.UNITY_Oconstrains_UNIV20, axio
c.in(v_F, c.UNITY_Oconstrains(c.UNIV, v_B, t_a), tc.UNITY_Oprogram(t_a)) => v_B = c.UNIV    cnf(cls.UNITY_Oconstr
c.in(v_F, c.UNITY_Oconstrains(c.UNIV, c.UNIV, t_a), tc.UNITY_Oprogram(t_a))    cnf(cls.UNITY_Oconstrains_UNIV_if
c.in(v_F, c.UNITY_Oconstrains(v_A, c.emptyset, t_a), tc.UNITY_Oprogram(t_a)) => v_A = c.emptyset    cnf(cls.UNITY_O
c.in(v_F, c.UNITY_Oconstrains(c.emptyset, c.emptyset, t_a), tc.UNITY_Oprogram(t_a))    cnf(cls.UNITY_Oconstrains_em
c.in(v_F, c.UNITY_Oconstrains(c.emptyset, v_B, t_a), tc.UNITY_Oprogram(t_a))    cnf(cls.UNITY_Oconstrains_empty0, ax
c.insert(c.Relation_OId, c.UNITY_OActs(v_F, t_a), tc_set(tc_prod(t_a, t_a))) = c.UNITY_OActs(v_F, t_a)    cnf(cls.UNITY_
c.insert(c.Relation_OId, c.UNITY_OAllowedActs(v_F, t_a), tc_set(tc_prod(t_a, t_a))) = c.UNITY_OAllowedActs(v_F, t_a)
c.UNITY_Ototalize(v_F, t_a) = v_F => ~c.UNITY_Oall_total(v_F, t_a)    cnf(cls.conjecture0, negated_conjecture)
c.UNITY_Oall_total(v_F, t_a) or c.UNITY_Ototalize(v_F, t_a) = v_F    cnf(cls.conjecture1, negated_conjecture)
```

**COM009-2.p** Problem about UNITY theory

```
c.UNITY_Ototalize(v_F, t_a) = v_F => ~c.UNITY_Oall_total(v_F, t_a)    cnf(cls.conjecture0, negated_conjecture)
```

```
c.UNITY_Oall_total(v_F, t_a) or c.UNITY_Ototalize(v_F, t_a) = v_F    cnf(cls.conjecture1, negated_conjecture)
```

```
c.UNITY_Oall_total(v_F, t_a) => c.UNITY_Ototalize(v_F, t_a) = v_F    cnf(cls.UNITY_Oall_total_imp_totalize0, axiom)
```

```
c.UNITY_Oall_total(c.UNITY_Ototalize(v_F, t_a), t_a)    cnf(cls.UNITY_Oall_total_totalize0, axiom)
```

**COM010-1.p** Problem about UNITY theory

```
include('Axioms/MS001-1.ax')
```

```
include('Axioms/MS001-0.ax')
```

```
c.UNITY_OActs(c.UNITY_Omk_program(c.Pair(v_init, c.Pair(v_acts, v_allowed, tc_set(tc_set(tc_prod(t_a, t_a))), tc_set(tc_set(tc_
c.insert(c.Relation_OId, v_acts, tc_set(tc_prod(t_a, t_a)))    cnf(cls.UNITY_OActs_eq0, axiom)
```

```
c.UNITY_OActs(v_F, t_a) ≠ c.emptyset    cnf(cls.UNITY_OActs_nonempty0, axiom)
```

```
c.UNITY_OAllowedActs(c.UNITY_Omk_program(c.Pair(v_init, c.Pair(v_acts, v_allowed, tc_set(tc_set(tc_prod(t_a, t_a))), tc_s
c.insert(c.Relation_OId, v_allowed, tc_set(tc_prod(t_a, t_a)))    cnf(cls.UNITY_OAllowedActs_eq0, axiom)
```

```
c.in(c.Relation_OId, c.UNITY_OActs(v_F, t_a), tc_set(tc_prod(t_a, t_a)))    cnf(cls.UNITY_OId_in_Acts0, axiom)
```

```
c.in(c.Relation_OId, c.UNITY_OAllowedActs(v_F, t_a), tc_set(tc_prod(t_a, t_a)))    cnf(cls.UNITY_OId_in_AllowedActs0, a
```

```
c.UNITY_OInit(c.UNITY_Omk_program(c.Pair(v_y, c.Pair(v_acts, v_allowed, tc_set(tc_set(tc_prod(t_a, t_a))), tc_set(tc_set(tc_
v_y    cnf(cls.UNITY_OInit_eq0, axiom)
```

```
c.insert(c.Relation_OId, c.UNITY_OActs(v_F, t_a), tc_set(tc_prod(t_a, t_a))) = c.UNITY_OActs(v_F, t_a)    cnf(cls.UNITY_
```

```
c.insert(c.Relation_OId, c.UNITY_OAllowedActs(v_F, t_a), tc_set(tc_prod(t_a, t_a))) = c.UNITY_OAllowedActs(v_F, t_a)
```

```
c.UNITY_Omk_program(c.Pair(c.UNITY_OInit(v_y, t_a), c.Pair(c.UNITY_OActs(v_y, t_a), c.UNITY_OAllowedActs(v_y, t_a)
v_y    cnf(cls.UNITY_Osurjective_mk_program0, axiom)
```

```
c.UNITY_OInit(v_F, t_a) = c.UNITY_OInit(v_G, t_a)    cnf(cls.conjecture0, negated_conjecture)
```

```
c.UNITY_OActs(v_F, t_a) = c.UNITY_OActs(v_G, t_a)    cnf(cls.conjecture1, negated_conjecture)
```

```
c.UNITY_OAllowedActs(v_F, t_a) = c.UNITY_OAllowedActs(v_G, t_a)    cnf(cls.conjecture2, negated_conjecture)
```

```
v_F ≠ v_G    cnf(cls.conjecture3, negated_conjecture)
```

**COM010-2.p** Problem about UNITY theory

```
c.UNITY_OInit(v_F, t_a) = c.UNITY_OInit(v_G, t_a)    cnf(cls.conjecture0, negated_conjecture)
```

```
c.UNITY_OActs(v_F, t_a) = c.UNITY_OActs(v_G, t_a)    cnf(cls.conjecture1, negated_conjecture)
```

```
c.UNITY_OAllowedActs(v_F, t_a) = c.UNITY_OAllowedActs(v_G, t_a)    cnf(cls.conjecture2, negated_conjecture)
```

```
v_F ≠ v_G    cnf(cls.conjecture3, negated_conjecture)
```

```
c.UNITY_Omk_program(c.Pair(c.UNITY_OInit(v_y, t_a), c.Pair(c.UNITY_OActs(v_y, t_a), c.UNITY_OAllowedActs(v_y, t_a)
v_y    cnf(cls.UNITY_Osurjective_mk_program0, axiom)
```

**COM011-1.p** Problem about UNITY theory

```
include('Axioms/MS001-2.ax')
```

```
include('Axioms/MS001-0.ax')
```

```
c.in(c.Relation_OId, c.UNITY_OActs(v_F, t_a), tc_set(tc_prod(t_a, t_a)))    cnf(cls.UNITY_OId_in_Acts0, axiom)
```

```
c.in(c.Relation_OId, c.UNITY_OAllowedActs(v_F, t_a), tc_set(tc_prod(t_a, t_a)))    cnf(cls.UNITY_OId_in_AllowedActs0, a
```

```
c.in(v_F, c.UNITY_Oconstrains(v_A, c.UNIV, t_a), tc.UNITY_Oprogram(t_a))    cnf(cls.UNITY_Oconstrains_UNIV20, axio
```

```
c.in(v_F, c.UNITY_Oconstrains(c.UNIV, v_B, t_a), tc.UNITY_Oprogram(t_a)) => v_B = c.UNIV    cnf(cls.UNITY_Oconstr
```

```
c.in(v_F, c.UNITY_Oconstrains(c.UNIV, c.UNIV, t_a), tc.UNITY_Oprogram(t_a))    cnf(cls.UNITY_Oconstrains_UNIV_if
```

$c.in(v\_F, c\_UNITY\_Oconstrains(v\_A, c\_emptyset, t\_a), tc\_UNITY\_Oprogram(t\_a)) \Rightarrow v\_A = c\_emptyset \quad cnf(cls\_UNITY\_Oconstrains\_empty0, axiom)$   
 $c.in(v\_F, c\_UNITY\_Oconstrains(c\_emptyset, c\_emptyset, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_UNITY\_Oconstrains\_empty0, axiom)$   
 $c.in(v\_F, c\_UNITY\_Oconstrains(c\_emptyset, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_UNITY\_Oconstrains\_empty0, axiom)$   
 $(c.in(v\_F, c\_UNITY\_Oconstrains(v\_A, v\_A\_H, t\_a), tc\_UNITY\_Oprogram(t\_a)) \text{ and } c.lessequals(v\_B, v\_A, tc\_set(t\_a))) \Rightarrow$   
 $c.in(v\_F, c\_UNITY\_Oconstrains(v\_B, v\_A\_H, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_UNITY\_Oconstrains\_weaken\_L0, axiom)$   
 $c.insert(c\_Relation\_OId, c\_UNITY\_OActs(v\_F, t\_a), tc\_set(tc\_prod(t\_a, t\_a))) = c\_UNITY\_OActs(v\_F, t\_a) \quad cnf(cls\_UNITY\_OActs\_id, axiom)$   
 $c.insert(c\_Relation\_OId, c\_UNITY\_OAllowedActs(v\_F, t\_a), tc\_set(tc\_prod(t\_a, t\_a))) = c\_UNITY\_OAllowedActs(v\_F, t\_a)$   
 $(c.in(v\_F, c\_UNITY\_Oconstrains(c\_minus(v\_A, v\_B, tc\_set(t\_a)), c\_union(v\_A, v\_B, t\_a), t\_a), tc\_UNITY\_Oprogram(t\_a)) \text{ and } c.$   
 $c.in(v\_F, c\_WFair\_Oensures(v\_A, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_WFair\_OensuresI0, axiom)$   
 $c.in(v\_F, c\_WFair\_Oensures(v\_A, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a)) \Rightarrow c.in(v\_F, c\_WFair\_OleadsTo(v\_A, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a))$   
 $(c.in(v\_F, c\_WFair\_Otransient(v\_A, t\_a), tc\_UNITY\_Oprogram(t\_a)) \text{ and } c.lessequals(v\_B, v\_A, tc\_set(t\_a))) \Rightarrow c.in(v\_F, c\_WFair\_Otransient(v\_A, t\_a), tc\_UNITY\_Oprogram(t\_a))$   
 $c.in(v\_F, c\_UNITY\_Oconstrains(v\_A, c\_union(v\_A, v\_B, t\_a), t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_conjecture0, negated\_conjecture)$   
 $c.in(v\_F, c\_WFair\_Otransient(v\_A, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_conjecture1, negated\_conjecture)$   
 $\neg c.in(v\_F, c\_WFair\_OleadsTo(v\_A, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_conjecture2, negated\_conjecture)$

### COM011-2.p Problem about UNITY theory

$c.in(v\_F, c\_UNITY\_Oconstrains(v\_A, c\_union(v\_A, v\_B, t\_a), t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_conjecture0, negated\_conjecture)$   
 $c.in(v\_F, c\_WFair\_Otransient(v\_A, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_conjecture1, negated\_conjecture)$   
 $\neg c.in(v\_F, c\_WFair\_OleadsTo(v\_A, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_conjecture2, negated\_conjecture)$   
 $c.in(v\_c, c\_minus(v\_A, v\_B, tc\_set(t\_a)), t\_a) \Rightarrow c.in(v\_c, v\_A, t\_a) \quad cnf(cls\_Set\_ODiffE1, axiom)$   
 $c.in(c\_Main\_OsubsetI\_1(v\_A, v\_B, t\_a), v\_A, t\_a) \text{ or } c.lessequals(v\_A, v\_B, tc\_set(t\_a)) \quad cnf(cls\_Set\_OsubsetI0, axiom)$   
 $c.in(c\_Main\_OsubsetI\_1(v\_A, v\_B, t\_a), v\_B, t\_a) \Rightarrow c.lessequals(v\_A, v\_B, tc\_set(t\_a)) \quad cnf(cls\_Set\_OsubsetI1, axiom)$   
 $(c.in(v\_F, c\_UNITY\_Oconstrains(v\_A, v\_A\_H, t\_a), tc\_UNITY\_Oprogram(t\_a)) \text{ and } c.lessequals(v\_B, v\_A, tc\_set(t\_a))) \Rightarrow$   
 $c.in(v\_F, c\_UNITY\_Oconstrains(v\_B, v\_A\_H, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_UNITY\_Oconstrains\_weaken\_L0, axiom)$   
 $(c.in(v\_F, c\_UNITY\_Oconstrains(c\_minus(v\_A, v\_B, tc\_set(t\_a)), c\_union(v\_A, v\_B, t\_a), t\_a), tc\_UNITY\_Oprogram(t\_a)) \text{ and } c.$   
 $c.in(v\_F, c\_WFair\_Oensures(v\_A, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a)) \quad cnf(cls\_WFair\_OensuresI0, axiom)$   
 $c.in(v\_F, c\_WFair\_Oensures(v\_A, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a)) \Rightarrow c.in(v\_F, c\_WFair\_OleadsTo(v\_A, v\_B, t\_a), tc\_UNITY\_Oprogram(t\_a))$   
 $(c.in(v\_F, c\_WFair\_Otransient(v\_A, t\_a), tc\_UNITY\_Oprogram(t\_a)) \text{ and } c.lessequals(v\_B, v\_A, tc\_set(t\_a))) \Rightarrow c.in(v\_F, c\_WFair\_Otransient(v\_A, t\_a), tc\_UNITY\_Oprogram(t\_a))$

### COM012+1.p Newman's lemma on rewriting systems 01, 00 expansion

$\forall w_0: (aElement_0(w_0) \Rightarrow \$true) \quad fof(mElmSort, axiom)$   
 $\forall w_0: (aRewritingSystem_0(w_0) \Rightarrow \$true) \quad fof(mRelSort, axiom)$   
 $\forall w_0, w_1: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1)) \Rightarrow \forall w_2: (aReductOfIn_0(w_2, w_0, w_1) \Rightarrow aElement_0(w_2))) \quad fof(mRReduct, axiom)$   
 $\forall w_0, w_1: ((aElement_0(w_0) \text{ and } aElement_0(w_1)) \Rightarrow (iLess_0(w_0, w_1) \Rightarrow \$true)) \quad fof(mWFOrd, axiom)$   
 $\forall w_0, w_1, w_2: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1) \text{ and } aElement_0(w_2)) \Rightarrow (sdtmndtplgtdt_0(w_0, w_1, w_2) \Rightarrow \$true)) \quad fof(mTCbr, axiom)$   
 $\forall w_0, w_1, w_2: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1) \text{ and } aElement_0(w_2)) \Rightarrow (sdtmndtplgtdt_0(w_0, w_1, w_2) \iff (aReductOfIn_0(w_2, w_0, w_1) \text{ or } \exists w_3: (aElement_0(w_3) \text{ and } aReductOfIn_0(w_3, w_0, w_1) \text{ and } sdtmndtplgtdt_0(w_3, w_1, w_2)))))) \quad fof(mTCDef, definition)$   
 $\forall w_0, w_1, w_2, w_3: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1) \text{ and } aElement_0(w_2) \text{ and } aElement_0(w_3)) \Rightarrow ((sdtmndtplgtdt_0(w_0, w_1, w_2) \text{ and } sdtmndtplgtdt_0(w_0, w_1, w_3))) \quad fof(mTCTrans, axiom)$   
 $\forall w_0, w_1, w_2: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1) \text{ and } aElement_0(w_2)) \Rightarrow (sdtmndtasgtdt_0(w_0, w_1, w_2) \iff (w_0 = w_2 \text{ or } sdtmndtplgtdt_0(w_0, w_1, w_2)))) \quad fof(mTCRDef, definition)$   
 $aElement_0(xx) \text{ and } aRewritingSystem_0(xR) \text{ and } aElement_0(xy) \text{ and } aElement_0(xz) \quad fof(m_{-349}, hypothesis)$   
 $(sdtmndtasgtdt_0(xx, xR, xy) \text{ and } sdtmndtasgtdt_0(xy, xR, xz)) \Rightarrow sdtmndtasgtdt_0(xx, xR, xz) \quad fof(m_{-}, conjecture)$

### COM012+3.p Newman's lemma on rewriting systems 01, 02 expansion

$\forall w_0: (aElement_0(w_0) \Rightarrow \$true) \quad fof(mElmSort, axiom)$   
 $\forall w_0: (aRewritingSystem_0(w_0) \Rightarrow \$true) \quad fof(mRelSort, axiom)$   
 $\forall w_0, w_1: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1)) \Rightarrow \forall w_2: (aReductOfIn_0(w_2, w_0, w_1) \Rightarrow aElement_0(w_2))) \quad fof(mRReduct, axiom)$   
 $\forall w_0, w_1: ((aElement_0(w_0) \text{ and } aElement_0(w_1)) \Rightarrow (iLess_0(w_0, w_1) \Rightarrow \$true)) \quad fof(mWFOrd, axiom)$   
 $\forall w_0, w_1, w_2: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1) \text{ and } aElement_0(w_2)) \Rightarrow (sdtmndtplgtdt_0(w_0, w_1, w_2) \Rightarrow \$true)) \quad fof(mTCbr, axiom)$   
 $\forall w_0, w_1, w_2: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1) \text{ and } aElement_0(w_2)) \Rightarrow (sdtmndtplgtdt_0(w_0, w_1, w_2) \iff (aReductOfIn_0(w_2, w_0, w_1) \text{ or } \exists w_3: (aElement_0(w_3) \text{ and } aReductOfIn_0(w_3, w_0, w_1) \text{ and } sdtmndtplgtdt_0(w_3, w_1, w_2)))))) \quad fof(mTCDef, definition)$   
 $\forall w_0, w_1, w_2, w_3: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1) \text{ and } aElement_0(w_2) \text{ and } aElement_0(w_3)) \Rightarrow ((sdtmndtplgtdt_0(w_0, w_1, w_2) \text{ and } sdtmndtplgtdt_0(w_0, w_1, w_3))) \quad fof(mTCTrans, axiom)$   
 $\forall w_0, w_1, w_2: ((aElement_0(w_0) \text{ and } aRewritingSystem_0(w_1) \text{ and } aElement_0(w_2)) \Rightarrow (sdtmndtasgtdt_0(w_0, w_1, w_2) \iff (w_0 = w_2 \text{ or } sdtmndtplgtdt_0(w_0, w_1, w_2)))) \quad fof(mTCRDef, definition)$   
 $aElement_0(xx) \text{ and } aRewritingSystem_0(xR) \text{ and } aElement_0(xy) \text{ and } aElement_0(xz) \quad fof(m_{-349}, hypothesis)$

$((xx = xy \text{ or } ((\text{aReductOfIn}_0(xy, xx, xR) \text{ or } \exists w_0: (\text{aElement}_0(w_0) \text{ and } \text{aReductOfIn}_0(w_0, xx, xR) \text{ and } \text{sdtmndtplgtdt}_0(w_0, xR, xx) \text{ or } ((\text{aReductOfIn}_0(xz, xy, xR) \text{ or } \exists w_0: (\text{aElement}_0(w_0) \text{ and } \text{aReductOfIn}_0(w_0, xy, xR) \text{ and } \text{sdtmndtplgtdt}_0(w_0, xR, xz)))) \text{ and } (xx = xz \text{ or } \text{aReductOfIn}_0(xz, xx, xR) \text{ or } \exists w_0: (\text{aElement}_0(w_0) \text{ and } \text{aReductOfIn}_0(w_0, xx, xR) \text{ and } \text{sdtmndtplgtdt}_0(w_0, xR, xz))))$

### COM024+5.p TPS problem THM9

A very naive version of the recursion theorem. TM X Y is the output of Turing machine X on input Y, TH F is the number of a Turing machine that computes function F.

cTM:  $\$i \rightarrow \$i \rightarrow \$i$  thf(cTM, type)

cTH:  $(\$i \rightarrow \$i) \rightarrow \$i$  thf(cTH, type)

$\forall g: \$i \rightarrow \$i: (\text{cTM}@\text{cTH}@g) = g \Rightarrow \forall f: \$i \rightarrow \$i: \exists n: \$i: (\text{cTM}@(f@n)) = (\text{cTM}@n)$  thf(cTHM9, conjecture)

### COM123+1.p T-Weak-FreeVar-abs-1 step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vT: ((\neg \text{visFreeVar}(vx, veabs) \text{ and } \text{vtcheck}(vC, veabs, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), veabs, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vy, vS_1, vT: ((vx \neq vy \text{ and } \neg \text{visFreeVar}(vx, \text{vabs}(vy, vS_1, veabs)) \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, veabs), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, veabs), vT))$  fof('T-Weak-FreeVar-abs-1', conjecture)

### COM124+1.p T-Weak-FreeVar-abs-2 step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

include('Axioms/COM001+1.ax')

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vy, vS_1, ve_1, vT: ((vx \neq vy \text{ and } \neg \text{visFreeVar}(vx, \text{vabs}(vy, vS_1, ve_1)) \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, ve_1), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, ve_1), vT))$  fof('T-Weak-FreeVar-abs-1-gen', axiom)

$\forall vx, vS, vC, vy, vS_1, vT: ((vx = vy \text{ and } \neg \text{visFreeVar}(vx, \text{vabs}(vy, vS_1, veabs)) \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, veabs), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, veabs), vT))$  fof('T-Weak-FreeVar-abs-2', conjecture)

### COM125+1.p T-Weak-FreeVar-abs step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vT: ((\neg \text{visFreeVar}(vx, veabs) \text{ and } \text{vtcheck}(vC, veabs, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), veabs, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vy, vS_1, vT: ((vx \neq vy \text{ and } \neg \text{visFreeVar}(vx, \text{vabs}(vy, vS_1, veabs)) \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, veabs), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, veabs), vT))$  fof('T-Weak-FreeVar-abs-1', axiom)

$\forall vx, vS, vC, vy, vS_1, vT: ((vx = vy \text{ and } \neg \text{visFreeVar}(vx, \text{vabs}(vy, vS_1, veabs)) \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, veabs), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, veabs), vT))$  fof('T-Weak-FreeVar-abs-2', axiom)

$\forall vx, vS, vC, vy, vS_1, vT: ((\neg \text{visFreeVar}(vx, \text{vabs}(vy, vS_1, veabs)) \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, veabs), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, veabs), vT))$  fof('T-Weak-FreeVar-abs-2', axiom)

### COM126+1.p T-Weak-FreeVar-app step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vT: ((\neg \text{visFreeVar}(vx, ve1app) \text{ and } \text{vtcheck}(vC, ve1app, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve1app, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vT: ((\neg \text{visFreeVar}(vx, ve2app) \text{ and } \text{vtcheck}(vC, ve2app, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve2app, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vT: ((\neg \text{visFreeVar}(vx, vapp(ve1app, ve2app)) \text{ and } \text{vtcheck}(vC, vapp(ve1app, ve2app), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), vapp(ve1app, ve2app), vT))$  fof('T-Strong',

### COM127+1.p T-Weak-FreeVar-var step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$  fof('T-Strong',

$\forall vx, vS, vC, vy, vT: ((\neg \text{visFreeVar}(vx, \text{vvar}(vy)) \text{ and } \text{vtcheck}(vC, \text{vvar}(vy), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vvar}(vy), vT))$  fof('T-Strong',







$\forall vx, vS, vC, vy, vT: ((\neg \text{visFreeVar}(vx, \text{vvar}(vy))) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vvar}(vy), vT)) \Rightarrow \text{vtcheck}(vC, \text{vvar}(vy), vT))$

**COM139+1.p** T-Weak-abs-1 step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{veabs}, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{veabs}, vT))$  fof

$\forall vx, vS, vC, vy, vS_1, vT: ((vx \neq vy \text{ and } \text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, \text{veabs}), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, \text{veabs}), vT))$  fof('T-Weak-abs-1', conjecture)

**COM140+1.p** T-Weak-abs-2 step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

include('Axioms/COM001+1.ax')

$\forall vx, vS, vC, vy, vS_1, ve_1, vT: ((vx \neq vy \text{ and } \text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, ve_1), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, ve_1), vT))$  fof('T-Weak-abs-1-gen', axiom)

$\forall vx, vS, vC, vy, vS_1, vT: ((vx = vy \text{ and } \text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, \text{veabs}), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, \text{veabs}), vT))$  fof('T-Weak-abs-2', conjecture)

**COM141+1.p** T-Weak-abs step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{veabs}, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{veabs}, vT))$  fof

$\forall vx, vS, vC, vy, vS_1, vT: ((vx \neq vy \text{ and } \text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, \text{veabs}), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, \text{veabs}), vT))$  fof('T-Weak-abs-1', axiom)

$\forall vx, vS, vC, vy, vS_1, vT: ((vx = vy \text{ and } \text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, \text{veabs}), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, \text{veabs}), vT))$  fof('T-Weak-abs-2', axiom)

$\forall vx, vS, vC, vy, vS_1, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{vabs}(vy, vS_1, \text{veabs}), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vabs}(vy, vS_1, \text{veabs}), vT))$

**COM142+1.p** T-Weak-app step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{ve1app}, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{ve1app}, vT))$

$\forall vx, vS, vC, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{ve2app}, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{ve2app}, vT))$

$\forall vx, vS, vC, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{vapp}(\text{ve1app}, \text{ve2app}), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vapp}(\text{ve1app}, \text{ve2app}), vT))$

**COM143+1.p** T-Weak-var step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, vy, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, \text{vvar}(vy), vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), \text{vvar}(vy), vT))$

**COM144+1.p** T-Preservation-T-abs step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$  fof('T-Weak-F

$\forall vT, vC, vx, ve, ve_2, vT_2: ((\text{vtcheck}(vC, ve, vT) \text{ and } \text{vtcheck}(\text{vbind}(vx, vT, vC), ve_2, vT_2)) \Rightarrow \text{vtcheck}(vC, \text{vsubst}(vx, ve, ve_2), vT_2))$

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$  fof('T-

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$  fof('T-Strong',

$\forall vC, veout, vT: ((\text{vreduce}(ve_1) = \text{vsomeExp}(veout) \text{ and } \text{vtcheck}(vC, ve_1, vT)) \Rightarrow \text{vtcheck}(vC, veout, vT))$  fof('T-Preserva

$\forall vx, vS, vC, veout, vT: ((\text{vreduce}(\text{vabs}(vx, vS, ve_1)) = \text{vsomeExp}(veout) \text{ and } \text{vtcheck}(vC, \text{vabs}(vx, vS, ve_1), vT)) \Rightarrow \text{vtcheck}(vC, veout, vT))$  fof('T-Preservation-T-abs', conjecture)

**COM145+1.p** T-Preservation-T-app step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$  fof('T-Weak-F

$\forall vT, vC, vx, ve, ve_2, vT_2: ((\text{vtcheck}(vC, ve, vT) \text{ and } \text{vtcheck}(\text{vbind}(vx, vT, vC), ve_2, vT_2)) \Rightarrow \text{vtcheck}(vC, \text{vsubst}(vx, ve, ve_2), vT_2))$

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$     fof('T-Weak-F')  
 $\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$     fof('T-Strong')  
 $\forall vC, veout, vT: ((\text{vreduce}(ve_1) = \text{vsomeExp}(veout) \text{ and } \text{vtcheck}(vC, ve_1, vT)) \Rightarrow \text{vtcheck}(vC, veout, vT))$     fof('T-Preserva')  
 $\forall vC, veout, vT: ((\text{vreduce}(ve_2) = \text{vsomeExp}(veout) \text{ and } \text{vtcheck}(vC, ve_2, vT)) \Rightarrow \text{vtcheck}(vC, veout, vT))$     fof('T-Preserva')  
 $\forall vC, veout, vT: ((\text{vreduce}(\text{vapp}(ve_1, ve_2)) = \text{vsomeExp}(veout) \text{ and } \text{vtcheck}(vC, \text{vapp}(ve_1, ve_2), vT)) \Rightarrow \text{vtcheck}(vC, veout, vT))$     fof('T-Preserva')

**COM146+1.p** T-Preservation-T-var step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$     fof('T-Weak-F')  
 $\forall vT, vC, vx, ve, ve_2, vT_2: ((\text{vtcheck}(vC, ve, vT) \text{ and } \text{vtcheck}(\text{vbind}(vx, vT, vC), ve_2, vT_2)) \Rightarrow \text{vtcheck}(vC, \text{vsubst}(vx, ve, ve_2), vT_2))$     fof('T-Weak-F')  
 $\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$     fof('T-Weak-F')  
 $\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$     fof('T-Strong')  
 $\forall vx, vC, veout, vT: ((\text{vreduce}(\text{vvar}(vx)) = \text{vsomeExp}(veout) \text{ and } \text{vtcheck}(vC, \text{vvar}(vx), vT)) \Rightarrow \text{vtcheck}(vC, veout, vT))$     fof('T-Strong')

**COM147+1.p** T-Progress-T-abs step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$     fof('T-Weak-F')  
 $\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$     fof('T-Strong')  
 $\forall vT: ((\text{vtcheck}(\text{vempty}, ve_1, vT) \text{ and } \neg \text{visValue}(ve_1)) \Rightarrow \exists veout: \text{vreduce}(ve_1) = \text{vsomeExp}(veout))$     fof('T-Progress-T-abs')  
 $\forall vTin, vx, vS: ((\text{vtcheck}(\text{vempty}, \text{vabs}(vx, vS, ve_1), vTin) \text{ and } \neg \text{visValue}(\text{vabs}(vx, vS, ve_1))) \Rightarrow \exists veout: \text{vreduce}(\text{vabs}(vx, vS, ve_1)) = \text{vsomeExp}(veout))$     fof('T-Progress-T-abs', conjecture)

**COM148+1.p** T-Progress-T-app step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$     fof('T-Weak-F')  
 $\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$     fof('T-Strong')  
 $\forall vT: ((\text{vtcheck}(\text{vempty}, ve_1, vT) \text{ and } \neg \text{visValue}(ve_1)) \Rightarrow \exists veout: \text{vreduce}(ve_1) = \text{vsomeExp}(veout))$     fof('T-Progress-T-app')  
 $\forall vT: ((\text{vtcheck}(\text{vempty}, ve_2, vT) \text{ and } \neg \text{visValue}(ve_2)) \Rightarrow \exists veout: \text{vreduce}(ve_2) = \text{vsomeExp}(veout))$     fof('T-Progress-T-app')  
 $\forall vT: ((\text{vtcheck}(\text{vempty}, \text{vapp}(ve_1, ve_2), vT) \text{ and } \neg \text{visValue}(\text{vapp}(ve_1, ve_2))) \Rightarrow \exists veout: \text{vreduce}(\text{vapp}(ve_1, ve_2)) = \text{vsomeExp}(veout))$     fof('T-Progress-T-app', conjecture)

**COM149+1.p** T-Progress-T-var step in progress/preservation proof

This problem is a step within the proof of progress and preservation for the standard type system for the simply-typed lambda calculus.

include('Axioms/COM001+0.ax')

$\forall vx, vS, vC, ve, vT: ((\text{vlookup}(vx, vC) = \text{vnoType} \text{ and } \text{vtcheck}(vC, ve, vT)) \Rightarrow \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT))$     fof('T-Weak-F')  
 $\forall vx, vS, vC, ve, vT: ((\neg \text{visFreeVar}(vx, ve) \text{ and } \text{vtcheck}(\text{vbind}(vx, vS, vC), ve, vT)) \Rightarrow \text{vtcheck}(vC, ve, vT))$     fof('T-Strong')  
 $\forall vT, vx: ((\text{vtcheck}(\text{vempty}, \text{vvar}(vx), vT) \text{ and } \neg \text{visValue}(\text{vvar}(vx))) \Rightarrow \exists veout: \text{vreduce}(\text{vvar}(vx)) = \text{vsomeExp}(veout))$     fof('T-Strong')

**COM150+1.p** Axioms for progress/preservation proof

include('Axioms/COM001+0.ax')

include('Axioms/COM001+1.ax')

**COM151+1.p** Common axioms for progress/preservation proof

include('Axioms/COM001+0.ax')

include('Axioms/COM001+1.ax')