DAT axioms

DAT001=0.ax Integer arrays
array: $tType   tff(array_type, type)
read: (array × $int) → $int   tff(read_type, type)
write: (array × $int × $int) → array   tff(write_type, type)
∀u: array, v: $int, w: $int: read(write(u, v, w), v) = w   tff(ax_1, axiom)
∀x: array, y: $int, z: $int, x_1: $int: (y = z or read(write(x, y, x_1), z) = read(x, z))   tff(ax_2, axiom)

DAT002=0.ax Integer collections
collection: $tType   tff(collection_type, type)
empty: collection   tff(empty_type, type)
+: ($int × collection) → collection   tff(add_type, type)
remove: ($int × collection) → collection   tff(remove_type, type)
in: ($int × collection) → $o   tff(in_type, type)
∀u: $int: ¬ in(u, empty)   tff(ax_1, axiom)
∀v: $int, w: collection: in(v, v + w)   tff(ax_2, axiom)
∀x: $int, y: collection: ¬ in(x, remove(x, y))   tff(ax_3, axiom)
∀z: $int, x_1: collection, x_2: $int: ((in(z, x_1) or z = x_2) ⟺ in(z, $x_2 + x_1$))   tff(ax_4, axiom)
∀x_3: $int, x_4: collection, x_5: $int: ((in($x_3$, x_4) and $x_3 \neq x_5$) ⟺ in(x_3, remove(x_5, x_4)))   tff(ax_5, axiom)

DAT002=1.ax Integer collections with counting
count: collection → $int   tff(count_type, type)
∀x_6: collection: $greatereq(count($x_6$), 0)   tff($ax_1$, axiom)
∀x_7: collection: (x_7 = empty ⟺ count(x_7) = 0)   tff(ax_2, axiom)
∀x_8: $int, x_9: collection: (¬ in(x_8, x_9) ⟺ count($x_8 + x_9$) = $sum(count($x_9$), 1))   tff($ax_3$, axiom)
∀x_{10}: $int, x_{11}: collection: (in(x_{10}, x_{11}) ⟺ count($x_{10} + x_{11}$) = count(x_{11}))   tff(ax_4, axiom)
∀x_{12}: $int, x_{13}: collection: (in(x_{12}, x_{13}) ⟺ count(remove(x_{12}, x_{13})) = $difference(count($x_{13}$), 1))   tff($ax_5$, axiom)
∀x_{14}: $int, x_{15}: collection: (¬ in(x_{14}, x_{15}) ⟺ count(remove(x_{14}, x_{15})) = count(x_{15}))   tff(ax_6, axiom)
∀x_{16}: $int, x_{17}: collection: (in(x_{16}, x_{17}) ⇒ $x_{17} = x_{16}$ + remove(x_{16}, x_{17}))   tff(ax_7, axiom)

DAT003=0.ax Pointer data types
record: $tType   tff(record_type, type)
length: record → $int   tff(length_type, type)
next: record → record   tff(next_type, type)
data: record → $int   tff(data_type, type)
$split_1$: record → record   tff(split1_type, type)
$split_2$: record → record   tff(split2_type, type)
isrecord: record → $o   tff(isrecord_type, type)
∀u: record: (¬ isrecord(u) ⇒ length(u) = 0)   tff(ax_1, axiom)
∀u: record: (isrecord(u) ⇒ $greatereq(length(u), 1))   tff(ax_2, axiom)
∀u: record: (isrecord(u) ⇒ length(u) = $sum(length(next(u)), 1))   tff(ax_3, axiom)
∀u: record: (¬ isrecord(u) ⇒ ¬ isrecord($split_1$(u)))   tff(ax_4, axiom)
∀u: record: (isrecord(u) ⇒ isrecord($split_1$(u)))   tff(ax_5, axiom)
∀u: record: (isrecord(u) ⇒ data($split_1$(u)) = data(u))   tff(ax_6, axiom)
∀u: record: ((isrecord(u) and ¬ isrecord(next(u))) ⇒ ¬ isrecord(next($split_1$(u))))   tff(ax_7, axiom)
∀u: record: ((isrecord(u) and isrecord(next(u))) ⇒ next($split_1$(u)) = $split_1$(next(next(u))))   tff(ax_8, axiom)
∀u: record: (¬ isrecord(u) ⇒ ¬ isrecord($split_2$(u)))   tff(ax_9, axiom)
∀u: record: (¬ isrecord(next(u)) ⇒ ¬ isrecord($split_2$(u)))   tff(ax_{10}, axiom)
∀u: record: ((isrecord(u) and isrecord(next(u))) ⇒ isrecord($split_2$(u)))   tff(ax_{11}, axiom)
∀u: record: ((isrecord(u) and isrecord(next(u))) ⇒ data($split_2$(u)) = data(next(u)))   tff(ax_{12}, axiom)
∀u: record: ((isrecord(u) and isrecord(next(u))) ⇒ next($split_2$(u)) = $split_2$(next(next(u))))   tff(ax_{13}, axiom)

DAT004=0.ax Array data types
data: $tType   tff(data_type, type)
array: $tType   tff(array_type, type)
mkarray: array   tff(mkarray_type, type)
none: data   tff(none_type, type)
put: (array × $int × data) → array   tff(put_type, type)
get: (array × $int) → data   tff(get_type, type)
∀m: $int: get(mkarray, m) = none   tff(ax_{17}, axiom)

$\forall \text{ar: array}, m\text{: \$int}, d\text{: data: get(put(ar}, m, d), m) = d \qquad \text{tff}(\text{ax}_{18}, \text{axiom})$
$\forall n\text{: \$int}, d\text{: data, ar: array}, m\text{: \$int: } (m \neq n \Rightarrow \text{get(put(ar}, n, d), m) = \text{get(ar}, m)) \qquad \text{tff}(\text{ax}_{19}, \text{axiom})$
$\forall d_2\text{: data, ar: array}, m\text{: \$int}, d_1\text{: data: put(put(ar}, m, d_2), m, d_1) = \text{put(ar}, m, d_1) \qquad \text{tff}(\text{ax}_{20}, \text{axiom})$
$\forall \text{ar: array}, \text{ar}_0\text{: array: } (\text{ar} = \text{ar}_0 \iff \forall n\text{: \$int: get(ar}, n) = \text{get}(\text{ar}_0, n)) \qquad \text{tff}(\text{ax}_{21}, \text{axiom})$

DAT005=0.ax Heap data types
$\text{heap: \$tType} \qquad \text{tff(heap\_type, type)}$
$\text{empty: heap} \qquad \text{tff(empty\_type, type)}$
$\text{get: heap} \to \text{heap} \qquad \text{tff(get\_type, type)}$
$\text{app: (heap} \times \text{\$int)} \to \text{heap} \qquad \text{tff(app\_type, type)}$
$\text{toop: heap} \to \text{\$int} \qquad \text{tff(toop\_type, type)}$
$\text{length: heap} \to \text{\$int} \qquad \text{tff(length\_type, type)}$
$\text{lsls: (heap} \times \text{heap)} \to \text{\$o} \qquad \text{tff(lsls\_type, type)}$
$\forall n\text{: \$int}, h\text{: heap: get(app}(h, n)) = h \qquad \text{tff}(\text{ax}_{17}, \text{axiom})$
$\forall h\text{: heap}, n\text{: \$int: toop(app}(h, n)) = n \qquad \text{tff}(\text{ax}_{18}, \text{axiom})$
$\forall h\text{: heap}, h_0\text{: heap}, n\text{: \$int}, n_0\text{: \$int: (app}(h, n) = \text{app}(h_0, n_0) \iff (h = h_0 \text{ and } n = n_0)) \qquad \text{tff}(\text{ax}_{19}, \text{axiom})$
$\forall h\text{: heap}, n\text{: \$int: empty} \neq \text{app}(h, n) \qquad \text{tff}(\text{ax}_{20}, \text{axiom})$
$\forall h\text{: heap: } (h = \text{empty or } h = \text{app(get}(h), \text{toop}(h))) \qquad \text{tff}(\text{ax}_{21}, \text{axiom})$
$\text{length(empty)} = 0 \qquad \text{tff}(\text{ax}_{22}, \text{axiom})$
$\forall n\text{: \$int}, h\text{: heap: length(app}(h, n)) = \text{\$sum}(1, \text{length}(h)) \qquad \text{tff}(\text{ax}_{23}, \text{axiom})$
$\forall h\text{: heap: } \neg\,\text{lsls}(h, h) \qquad \text{tff}(\text{ax}_{24}, \text{axiom})$
$\forall h_0\text{: heap}, h\text{: heap}, h_1\text{: heap: ((lsls}(h, h_0) \text{ and lsls}(h_0, h_1)) \Rightarrow \text{lsls}(h, h_1)) \qquad \text{tff}(\text{ax}_{25}, \text{axiom})$
$\forall h\text{: heap: } \neg\,\text{lsls}(h, \text{empty}) \qquad \text{tff}(\text{ax}_{26}, \text{axiom})$
$\forall n\text{: \$int}, h_0\text{: heap}, h\text{: heap: (lsls}(h_0, \text{app}(h, n)) \iff (h_0 = h \text{ or lsls}(h_0, h))) \qquad \text{tff}(\text{ax}_{27}, \text{axiom})$

DAT006=0.ax Tree-heap data types
$\text{heap: \$tType} \qquad \text{tff(heap\_type, type)}$
$\text{empty: heap} \qquad \text{tff(empty\_type, type)}$
$\text{toop: heap} \to \text{\$int} \qquad \text{tff(toop\_type, type)}$
$\text{sel: (heap} \times \text{\$int)} \to \text{\$int} \qquad \text{tff(sel\_type, type)}$
$\text{length: heap} \to \text{\$int} \qquad \text{tff(length\_type, type)}$
$\text{app: (heap} \times \text{\$int)} \to \text{heap} \qquad \text{tff(app\_type, type)}$
$\text{get: heap} \to \text{heap} \qquad \text{tff(get\_type, type)}$
$\text{lsls: (heap} \times \text{heap)} \to \text{\$o} \qquad \text{tff(lsls\_type, type)}$
$\forall m\text{: \$int: sel(empty}, m) = 0 \qquad \text{tff}(\text{ax}_1, \text{axiom})$
$\forall h\text{: heap}, m\text{: \$int}, n\text{: \$int: } (m = \text{\$sum}(1, \text{length}(h)) \Rightarrow \text{sel(app}(h, n), m) = n) \qquad \text{tff}(\text{ax}_2, \text{axiom})$
$\forall n\text{: \$int}, h\text{: heap}, m\text{: \$int: } (m \neq \text{\$sum}(1, \text{length}(h)) \Rightarrow \text{sel(app}(h, n), m) = \text{sel}(h, m)) \qquad \text{tff}(\text{ax}_3, \text{axiom})$
$\forall n\text{: \$int}, h\text{: heap: get(app}(h, n)) = h \qquad \text{tff}(\text{ax}_{20}, \text{axiom})$
$\forall h\text{: heap}, n\text{: \$int: toop(app}(h, n)) = n \qquad \text{tff}(\text{ax}_{21}, \text{axiom})$
$\forall h\text{: heap}, h_0\text{: heap}, n\text{: \$int}, n_0\text{: \$int: (app}(h, n) = \text{app}(h_0, n_0) \iff (h = h_0 \text{ and } n = n_0)) \qquad \text{tff}(\text{ax}_{22}, \text{axiom})$
$\forall h\text{: heap}, n\text{: \$int: empty} \neq \text{app}(h, n) \qquad \text{tff}(\text{ax}_{23}, \text{axiom})$
$\forall h\text{: heap: } (h = \text{empty or } h = \text{app(get}(h), \text{toop}(h))) \qquad \text{tff}(\text{ax}_{24}, \text{axiom})$
$\text{length(empty)} = 0 \qquad \text{tff}(\text{ax}_{25}, \text{axiom})$
$\forall n\text{: \$int}, h\text{: heap: length(app}(h, n)) = \text{\$sum}(1, \text{length}(h)) \qquad \text{tff}(\text{ax}_{26}, \text{axiom})$
$\forall h\text{: heap: } \neg\,\text{lsls}(h, h) \qquad \text{tff}(\text{ax}_{27}, \text{axiom})$
$\forall h_0\text{: heap}, h\text{: heap}, h_1\text{: heap: ((lsls}(h, h_0) \text{ and lsls}(h_0, h_1)) \Rightarrow \text{lsls}(h, h_1)) \qquad \text{tff}(\text{ax}_{28}, \text{axiom})$
$\forall h\text{: heap: } \neg\,\text{lsls}(h, \text{empty}) \qquad \text{tff}(\text{ax}_{29}, \text{axiom})$
$\forall n\text{: \$int}, h_0\text{: heap}, h\text{: heap: (lsls}(h_0, \text{app}(h, n)) \iff (h_0 = h \text{ or lsls}(h_0, h))) \qquad \text{tff}(\text{ax}_{30}, \text{axiom})$

DAT problems

DAT001=1.p Recursive list sort
$\text{list: \$tType} \qquad \text{tff(list\_type, type)}$
$\text{nil: list} \qquad \text{tff(nil\_type, type)}$
$\text{mycons: (\$int} \times \text{list)} \to \text{list} \qquad \text{tff(mycons\_type, type)}$
$\text{sorted: list} \to \text{\$o} \qquad \text{tff(sorted\_type, type)}$
$\text{sorted(nil)} \qquad \text{tff(empty\_is\_sorted, axiom)}$
$\forall x\text{: \$int: sorted(mycons}(x, \text{nil})) \qquad \text{tff(single\_is\_sorted, axiom)}$
$\forall x\text{: \$int}, y\text{: \$int}, r\text{: list: ((\$less}(x, y) \text{ and sorted(mycons}(y, r))) \Rightarrow \text{sorted(mycons}(x, \text{mycons}(y, r)))) \qquad \text{tff(recursive\_sort, axio}$
$\text{sorted(mycons(1, mycons(2, mycons(4, mycons(7, mycons(100, nil))))))} \qquad \text{tff(check\_list, conjecture)}$

DAT002=1.p Recursive list Fibonacci sort
A list is Fibonacci sorted if it is sorted, and every element is greater of equal to the sum of its two predecessors (from the third element onwards).
$\text{list: \$tType} \quad \text{tff(list\_type, type)}$
$\text{nil: list} \quad \text{tff(nil\_type, type)}$
$\text{mycons: (\$int} \times \text{list)} \to \text{list} \quad \text{tff(mycons\_type, type)}$
$\text{fib\_sorted: list} \to \text{\$o} \quad \text{tff(sorted\_type, type)}$
$\text{fib\_sorted(nil)} \quad \text{tff(empty\_fib\_sorted, axiom)}$
$\forall x\text{: \$int: fib\_sorted(mycons(}x\text{, nil))} \quad \text{tff(single\_is\_fib\_sorted, axiom)}$
$\forall x\text{: \$int, }y\text{: \$int: (\$less(}x, y\text{)} \Rightarrow \text{fib\_sorted(mycons(}x\text{, mycons(}y\text{, nil))))} \quad \text{tff(double\_is\_fib\_sorted\_if\_ordered, axiom)}$
$\forall x\text{: \$int, }y\text{: \$int, }z\text{: \$int, }r\text{: list: ((\$less(}x, y\text{) and \$greatereq(}z\text{, \$sum(}x, y\text{)) and fib\_sorted(mycons(}y\text{, mycons(}z, r\text{)))))} \Rightarrow$
$\text{fib\_sorted(mycons(}x\text{, mycons(}y\text{, mycons(}z, r\text{)))))} \quad \text{tff(recursive\_fib\_sort, axiom)}$
$\text{fib\_sorted(mycons(1, mycons(2, mycons(4, mycons(7, mycons(100, nil))))))} \quad \text{tff(check\_list, conjecture)}$

DAT002^1.p Recursive list Fibonacci sort
A list is Fibonacci sorted if it is sorted, and every element is greater of equal to the sum of its two predecessors (from the third element onwards).
$\text{list: \$tType} \quad \text{thf(list\_type, type)}$
$\text{nil: list} \quad \text{thf(nil\_type, type)}$
$\text{mycons: \$int} \to \text{list} \to \text{list} \quad \text{thf(mycons\_type, type)}$
$\text{fib\_sorted: list} \to \text{\$o} \quad \text{thf(sorted\_type, type)}$
$\text{fib\_sorted@nil} \quad \text{thf(empty\_fib\_sorted, axiom)}$
$\forall x\text{: \$int: (fib\_sorted@(mycons@}x\text{@nil))} \quad \text{thf(single\_is\_fib\_sorted, axiom)}$
$\forall x\text{: \$int, }y\text{: \$int: ((\$less@}x\text{@}y\text{)} \Rightarrow \text{(fib\_sorted@(mycons@}x\text{@(mycons@}y\text{@nil))))} \quad \text{thf(double\_is\_fib\_sorted\_if\_ordered, axiom}$
$\forall x\text{: \$int, }y\text{: \$int, }z\text{: \$int, }r\text{: list: ((\$less@}x\text{@}y\text{ and \$greatereq@}z\text{@(\$sum@}x\text{@}y\text{) and fib\_sorted@(mycons@}y\text{@(mycons@}z\text{@}r\text{)))}$
$\text{(fib\_sorted@(mycons@}x\text{@(mycons@}y\text{@(mycons@}z\text{@}r\text{)))))} \quad \text{thf(recursive\_fib\_sort, axiom)}$
$\text{fib\_sorted@(mycons@1@(mycons@2@(mycons@4@(mycons@7@(mycons@100@nil)))))} \quad \text{thf(check\_list, conjecture)}$

DAT003=1.p Element 3 is 33
include('Axioms/DAT001=0.ax')
$\forall u\text{: array, }v\text{: array: (}u = \text{write(write(write(write(}v\text{, 3, 33), 4, 444), 5, 55), 4, 44)} \Rightarrow \text{read(}u\text{, 3)} = 33\text{)} \quad \text{tff(co}_1\text{, conjecture)}$

DAT004=1.p Element 4 is 44 or 66
include('Axioms/DAT001=0.ax')
$\forall u\text{: array, }v\text{: array, }w\text{: \$int: (}u = \text{write(write(write(write(}v\text{, 3, 33), 4, 44), 5, 55), }w\text{, 66)} \Rightarrow \text{(read(}u\text{, 4)} = 44 \text{ or read(}u\text{, 4)} = 66\text{))} \quad \text{tff(co}_1\text{, conjecture)}$

DAT005=1.p Element between 33 and 44
include('Axioms/DAT001=0.ax')
$\forall u\text{: array, }v\text{: array, }w\text{: \$int: ((}u = \text{write(write(write(write(}v\text{, 3, 33), 4, 444), 5, 55), 4, 44) and \$lesseq(3, }w\text{) and \$lesseq(}w\text{, 4))} \Rightarrow$
$\text{(\$lesseq(33, read(}u, w\text{)) and \$lesseq(read(}u, w\text{), 44)))} \quad \text{tff(co}_1\text{, conjecture)}$

DAT006=1.p Some element is 33
include('Axioms/DAT001=0.ax')
$\forall u\text{: array, }v\text{: array: (}u = \text{write(write(write(write(}v\text{, 3, 33), 4, 444), 5, 55), 4, 44)} \Rightarrow \exists w\text{: \$int: read(}u, w\text{)} = 33\text{)} \quad \text{tff(co}_1\text{, conjec}$

DAT007=1.p Element between 30 and 40
include('Axioms/DAT001=0.ax')
$\forall u\text{: array, }v\text{: array: (}u = \text{write(write(write(write(}v\text{, 3, 33), 4, 444), 5, 55), 4, 44)} \Rightarrow \exists w\text{: \$int: (\$less(read(}u, w\text{), 40) and \$less(30,}$

DAT008=1.p An element greater than its index
include('Axioms/DAT001=0.ax')
$\forall u\text{: array, }v\text{: array: ((}\forall w\text{: \$int: \$greater(read(}v, w\text{), }w\text{) and }u = \text{write(write(}v\text{, 3, 5), 7, 9))} \Rightarrow \forall x\text{: \$int: \$greater(read(}u, x\text{), }x\text{))}$

DAT009=1.p Every element greater than its index
include('Axioms/DAT001=0.ax')
$\forall u\text{: array, }v\text{: array, }w\text{: \$int: ((}\forall x\text{: \$int: \$greater(read(}v, x\text{), }x\text{) and }u = \text{write(}v, w\text{, \$sum(}w\text{, 3)))} \Rightarrow \forall y\text{: \$int: \$greater(read(}u, y$

DAT010=1.p All elements are less than 100
include('Axioms/DAT001=0.ax')
$\forall u\text{: array, }v\text{: array: ((}u = \text{write(write(write(write(}v\text{, 3, 33), 4, 444), 5, 55), 4, 44) and }\forall w\text{: \$int: \$less(read(}v, w\text{), 100))} \Rightarrow$
$\forall x\text{: \$int: \$less(read(}u, x\text{), 100))} \quad \text{tff(co}_1\text{, conjecture)}$

DAT011=1.p Compare elements 1
include('Axioms/DAT001=0.ax')

$\forall u\colon \text{array}, v\colon \text{array}, w\colon \text{\$int}, x\colon \text{\$int}\colon ((\forall y\colon \text{\$int}\colon ((\text{\$lesseq}(w, y) \text{ and } \text{\$lesseq}(y, x)) \Rightarrow \text{\$greater}(\text{read}(v, y), 0)) \text{ and } u = \text{write}(v, \text{\$sum}(x, 1), 3)) \Rightarrow \forall z\colon \text{\$int}\colon ((\text{\$lesseq}(w, z) \text{ and } \text{\$lesseq}(z, \text{\$sum}(x, 1))) \Rightarrow \text{\$greater}(\text{read}(u, z), 0)))$ $\text{tff}(\text{co}_1, \text{conject}$

DAT012=1.p Compare elements 2
include('Axioms/DAT001=0.ax')
$\forall u\colon \text{array}, v\colon \text{array}, w\colon \text{\$int}, x\colon \text{\$int}\colon ((\forall y\colon \text{\$int}\colon ((\text{\$lesseq}(w, y) \text{ and } \text{\$lesseq}(y, x)) \Rightarrow \text{\$greater}(\text{read}(u, y), 0)) \text{ and } v = \text{write}(u, \text{\$sum}(w, 2), \text{\$sum}(\text{read}(u, \text{\$sum}(w, 1)), 1))) \Rightarrow \forall z\colon \text{\$int}\colon ((\text{\$lesseq}(w, z) \text{ and } \text{\$lesseq}(z, x)) \Rightarrow \text{\$greater}(\text{read}(v, z), 0)))$

DAT013=1.p Compare elements 3
include('Axioms/DAT001=0.ax')
$\forall u\colon \text{array}, v\colon \text{\$int}, w\colon \text{\$int}\colon (\forall x\colon \text{\$int}\colon ((\text{\$lesseq}(v, x) \text{ and } \text{\$lesseq}(x, w)) \Rightarrow \text{\$greater}(\text{read}(u, x), 0)) \Rightarrow \forall y\colon \text{\$int}\colon ((\text{\$lesseq}(\text{\$sum}($
$\text{\$greater}(\text{read}(u, y), 0)))$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT014=1.p Compare elements 4
include('Axioms/DAT001=0.ax')
$\forall u\colon \text{array}\colon ((\forall v\colon \text{\$int}\colon ((\text{\$lesseq}(1, v) \text{ and } \text{\$lesseq}(v, 10)) \Rightarrow \text{\$greater}(\text{read}(u, v), v)) \text{ and } \forall w\colon \text{\$int}\colon ((\text{\$lesseq}(11, w) \text{ and } \text{\$lesseq}($
$\text{\$greater}(\text{read}(u, w), \text{\$difference}(20, w)))) \Rightarrow \forall x\colon \text{\$int}\colon ((\text{\$lesseq}(6, x) \text{ and } \text{\$lesseq}(x, 15)) \Rightarrow \text{\$greater}(\text{read}(u, x), 5)))$ $\text{tff}(\text{c}$

DAT015=1.p Some element is 50
include('Axioms/DAT001=0.ax')
$\forall u\colon \text{array}\colon (\forall v\colon \text{\$int}\colon ((\text{\$lesseq}(20, v) \text{ and } \text{\$lesseq}(v, 30)) \Rightarrow \text{read}(u, v) = \text{\$sum}(v, 25)) \Rightarrow \exists w\colon \text{\$int}\colon \text{read}(u, w) = 50)$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT016=1.p Some element is 53
include('Axioms/DAT001=0.ax')
$\forall u\colon \text{array}\colon (\forall v\colon \text{\$int}\colon ((\text{\$lesseq}(20, v) \text{ and } \text{\$lesseq}(v, 30)) \Rightarrow \text{read}(u, v) = \text{\$sum}(\text{\$product}(2, v), 3)) \Rightarrow \exists w\colon \text{\$int}\colon \text{read}(u, w) = 53)$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT017=1.p Arrays with different elements
include('Axioms/DAT001=0.ax')
$\forall u\colon \text{array}, v\colon \text{array}, w\colon \text{\$int}, x\colon \text{\$int}, y\colon \text{\$int}\colon ((\text{read}(v, w) \neq \text{read}(v, x) \text{ and } u = \text{write}(\text{write}(\text{write}(v, x, 0), y, \text{\$sum}(\text{read}(v, y), 1)),$
$\exists z\colon \text{\$int}\colon \text{read}(u, z) \neq \text{read}(v, z))$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT018=1.p Compare elements 5
include('Axioms/DAT001=0.ax')
$\forall u\colon \text{array}, v\colon \text{array}, w\colon \text{\$int}, x\colon \text{\$int}\colon ((u = \text{write}(\text{write}(\text{write}(v, w, 3), \text{\$sum}(w, 2), 2), \text{\$sum}(w, 4), 1) \text{ and } \text{\$lesseq}(w, x) \text{ and } \text{\$lesse}$
$\exists y\colon \text{\$int}\colon (\text{\$lesseq}(x, y) \text{ and } \text{\$lesseq}(y, \text{\$sum}(x, 3)) \text{ and } \text{\$lesseq}(\text{read}(u, y), 3)))$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT019=1.p 3 is in the colection
include('Axioms/DAT002=0.ax')
$\text{in}(3, 1 + (3 + (5 + \text{empty})))$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT020=1.p 4 is not in the collection
include('Axioms/DAT002=0.ax')
$\neg\,\text{in}(4, 1 + (3 + (5 + \text{empty})))$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT021=1.p Sum of two elements is less than 9
include('Axioms/DAT002=0.ax')
$\forall u\colon \text{collection}, v\colon \text{\$int}, w\colon \text{\$int}\colon ((u = 5{+}(3{+}(1{+}\text{empty})) \text{ and } \text{in}(v, u) \text{ and } \text{in}(w, u) \text{ and } v \neq w) \Rightarrow \text{\$less}(\text{\$sum}(v, w), 9))$ $\text{tff}($

DAT022=1.p Elements stay positive
include('Axioms/DAT002=0.ax')
$\forall u\colon \text{collection}, v\colon \text{collection}\colon ((\forall w\colon \text{\$int}\colon (\text{in}(w, v) \Rightarrow \text{\$greater}(w, 0)) \text{ and } u = 2{+}\text{remove}(7, v)) \Rightarrow \forall x\colon \text{\$int}\colon (\text{in}(x, u) \Rightarrow \text{\$greater}(x, 0)))$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT023=1.p Removing 1 and 2 ensures elements are greater than 2
include('Axioms/DAT002=0.ax')
$\forall u\colon \text{collection}, v\colon \text{collection}\colon ((\forall w\colon \text{\$int}\colon (\text{in}(w, v) \Rightarrow \text{\$greater}(w, 0)) \text{ and } u = \text{remove}(4, \text{remove}(1, \text{remove}(2, v)))) \Rightarrow \forall x\colon \text{\$int}\colon (\text{in}(x, u) \Rightarrow \text{\$greater}(x, 2)))$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT024=1.p Without 0 or 1 all elements are greater or equal to 2
include('Axioms/DAT002=0.ax')
$\forall u\colon \text{collection}\colon ((\forall v\colon \text{\$int}\colon (\text{in}(v, u) \Rightarrow \text{\$greatereq}(v, 0)) \text{ and } \neg\,\text{in}(0, u) \text{ and } \neg\,\text{in}(1, u)) \Rightarrow \forall w\colon \text{\$int}\colon (\text{in}(w, u) \Rightarrow \text{\$greatereq}(w, 2)))$ $\text{tff}(\text{co}_1, \text{conjecture})$

DAT025=1.p With 0 and 1 removed all elements are greater or equal to 2
include('Axioms/DAT002=0.ax')

$\forall u$: collection: $((\forall v\colon \$int\colon (in(v,u) \Rightarrow \$greatereq(v,0))$ and $\forall w\colon \$int\colon (in(w,u) \iff in(w, remove(0, remove(1,u))))) \Rightarrow \forall x\colon \$int\colon (in(x,u) \Rightarrow \$greatereq(x,2)))$ tff(co_1, conjecture)

DAT026=1.p Replacing 2 by something larger keeps elements positive
include('Axioms/DAT002=0.ax')
$\forall u$: collection, v: collection, w: \$int: $((\forall x\colon \$int\colon (in(x,v) \Rightarrow \$greater(x,0))$ and $in(w,v)$ and $u = \$sum(w,2) + remove(w,v)) \Rightarrow \forall y\colon \$int\colon (in(y,u) \Rightarrow \$greater(y,0)))$ tff(co_1, conjecture)

DAT027=1.p Replacing 2 by something positive keeps elements positive
include('Axioms/DAT002=0.ax')
$\forall u$: collection, v: collection, w: \$int, x: \$int: $((\forall y\colon \$int\colon (in(y,v) \Rightarrow \$greater(y,0))$ and $in(w,v)$ and $\$greatereq(x,0)$ and $u = \$sum(w,x) + remove(w,v)) \Rightarrow \forall z\colon \$int\colon (in(z,u) \Rightarrow \$greater(z,0)))$ tff(co_1, conjecture)

DAT028=1.p Comparing elements in two collections 1
include('Axioms/DAT002=0.ax')
$\forall u$: collection, v: collection: $((\forall w\colon \$int\colon (in(w,v) \Rightarrow \$greater(w,0))$ and $\forall x\colon \$int\colon (in(x,u) \Rightarrow \exists y\colon \$int\colon (in(y,v)$ and \$greater(
$\forall z\colon \$int\colon (in(z,u) \Rightarrow \$greater(z,1)))$ tff(co_1, conjecture)

DAT029=1.p Comparing elements in two collections 2
include('Axioms/DAT002=0.ax')
$\forall u$: collection, v: collection: $((\forall w\colon \$int\colon (in(w,v) \Rightarrow \$greater(w,0))$ and $\forall x\colon \$int\colon (in(x,u) \Rightarrow \exists y\colon \$int\colon (in(y,v)$ and \$greater(
$\forall z\colon \$int\colon (in(z,u) \Rightarrow \$greater(z,2)))$ tff(co_1, conjecture)

DAT030=1.p Comparing elements in two collections 3
include('Axioms/DAT002=0.ax')
$\forall u$: collection, v: collection: $((\forall w\colon \$int\colon (in(w,v) \Rightarrow \$greater(w,0))$ and $\forall x\colon \$int\colon (in(x,u) \Rightarrow \exists y\colon \$int, z\colon \$int\colon (in(y,v)$ and in
$\$sum(y,z)))) \Rightarrow \forall x_1\colon \$int\colon (in(x_1,u) \Rightarrow \$greater(x_1,1)))$ tff(co_1, conjecture)

DAT031=1.p Some element is betweeb 20 and 40
include('Axioms/DAT002=0.ax')
$\forall u$: collection: $(u = 10{+}(30{+}(50{+}empty)) \Rightarrow \exists v\colon \$int\colon (\$lesseq(20,v)$ and $\$lesseq(v,40)$ and $in(v,u)))$ tff(co_1, conjecture

DAT032=1.p Removing one elements changes count by one
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection: $(\$greatereq(count(remove(5,u)),7) \Rightarrow \$greatereq(count(remove(4,u)),6))$ tff(co_1, conjecture)

DAT033=1.p Count changes are consistent with adding and removal
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection: $(count(5+u) = count(3+u) \Rightarrow count(remove(5,u)) = count(remove(3,u)))$ tff(co_1, conjecture)

DAT034=1.p Adding an element increases the count by at least one
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int: $\$greatereq(\$sum(count(u),1), count(v+u))$ tff(co_1, conjecture)

DAT035=1.p Adding an element greater than 0 - 1
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int, w: \$int: $(\$greater(w,0) \Rightarrow \$greatereq(\$sum(count(u),w), count(v{+}u)))$ tff(co_1, conjecture)

DAT036=1.p Adding an element greater than 0 - 2
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int, w: \$int: $(\$greater(w,0) \Rightarrow \$greatereq(\$sum(count(u),w), count(v{+}(v{+}u))))$ tff(co_1, conjecture)

DAT037=1.p If 2 is the only element, there are not 5 elements
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection: $((in(2,u)$ and $count(u) = 1) \Rightarrow \neg in(5,u))$ tff(co_1, conjecture)

DAT038=1.p If 2 and 3 are the only elements, there are not 5 elements
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection: $((in(2,u)$ and $in(3,u)$ and $count(u) = 2) \Rightarrow \neg in(5,u))$ tff(co_1, conjecture)

DAT039=1.p If 2 and 3 are the only elements, then no elements are larger

include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int: ((in(2, u) and in(3, u) and count(u) = 2 and \$greater(v, 3)) $\Rightarrow$ $\neg$ in(v, u)) tff(co_1, conjecture)

DAT040=1.p Only elements less than 3 or greater than 6
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int, w: \$int: ((\$less(v, 3) and \$less(6, w) and in(v, u) and in(w, u) and count(u) = 2) $\Rightarrow$ $\neg$ in(5, u)) tff(c

DAT041=1.p Adding an element makes list longer
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\exists u$: collection, v: \$int: \$greater(\$sum(count(u), 1), count($v + u$)) tff(co_1, conjecture)

DAT042=1.p Some collection has 3 as an element
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\exists u$: collection: count(u) = 3 tff(co_1, conjecture)

DAT043=1.p Three different elements
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int, w: \$int, x: \$int: ((\$greater(v, w) and \$greater(w, x) and in(v, u) and in(w, u) and in(x, u)) $\Rightarrow$ \$greater(count(u), 2)) tff(co_1, conjecture)

DAT044=1.p Adding a larger element to the collection 1
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int: ($\forall w$: \$int: (in(w, u) $\Rightarrow$ \$greater(v, w)) $\Rightarrow$ \$greater(count(v+u), count(u))) tff(co_1, conjecture)

DAT045=1.p Adding a larger element to the collection 2
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int: ($\forall w$: \$int: (in(w, u) $\Rightarrow$ \$greater(v, w)) $\Rightarrow$ \$greater(count(v+u), count(u))) tff(co_1, conjecture)

DAT046=1.p The collection of 1 and 2 has size 2
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
count(1 + (3 + empty)) = 2 tff(co_1, conjecture)

DAT047=1.p Adding and removing 3 leaves size 1
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
count(5 + remove(3, 3 + empty)) = 1 tff(co_1, conjecture)

DAT048=1.p Removing an non-existent element from collection of size 3
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
count(1 + (5 + remove(3, 2 + empty))) = 3 tff(co_1, conjecture)

DAT049=1.p Removing what you've added does change the size
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int: count(remove(v, $v + u$)) = count(remove(v, u)) tff(co_1, conjecture)

DAT050=1.p Adding 0 to equal size results
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')
$\forall u$: collection, v: \$int: count(0 + remove(v, $v + u$)) = count(0 + remove(v, u)) tff(co_1, conjecture)

DAT051=1.p Even and odd numbered elements 1
include('Axioms/DAT003=0.ax')
$\forall u$: record, v: record: ((isrecord(u) and isrecord(next(u)) and v = next(next(u)) and (\$product(2, length($split_2$(v))) = \$difference(length(v), 1) or \$product(2, length($split_2$(v))) = length(v))) $\Rightarrow$ (\$product(2, length($split_2$(u))) = \$difference(leng length(u))) tff(co_1, conjecture)

DAT052=1.p Even and odd numbered elements 2

include('Axioms/DAT003=0.ax')
∀u: record, v: record: ((isrecord(u) and isrecord(next(u)) and v = next(next(u)) and $sum(length(split$_1$($v$)), length(split$_2$($v$))
length(v)) ⇒ $sum(length(split$_1$($u$)), length(split$_2$($u$))) = length($u$)) tff(co$_1$, conjecture)

DAT053=1.p Sublist of odd numbered elements
include('Axioms/DAT003=0.ax')
a: record tff(a_type, type)
b: record tff(b_type, type)
(isrecord(b) and isrecord(next(b)) and next(next(b)) = a and ($product(2, length(split$_1$($a$))) = $sum(length($a$), 1) or $produc
length(a))) ⇒ ($product(2, length(split$_1$($b$))) = $sum(length($b$), 1) or $product(2, length(split$_1$($b$))) = length($b$)) tff(co$_1$,

DAT054=1.p Decreasing pointer list
element: $tType tff(element_type, type)
data: element → $int tff(data_type, type)
next: element → element tff(next_type, type)
iselem: element → $o tff(iselem_type, type)
a: element tff(a_type, type)
(∀x: element, z: $int: (¬ iselem($x$) or data($x$) ≠ z or $less(0, z)) and ∀x: element, y: element: (¬ iselem(x) or ¬ iselem(next(y))
y or $less(data($y$), data($x$))) and iselem($a$) and iselem(next($a$)) and iselem(next(next($a$)))) ⇒ $lesseq(3, data($a$)) tff(dec

DAT055=1.p Boyer-Moore min-max problem
list: $tType tff(list_type, type)
a: list tff(a_type, type)
l: $int tff(l_type, type)
k: $int tff(k_type, type)
min: list → $int tff(min_type, type)
max: list → $int tff(max_type, type)
(∀x: list: $lesseq(min($x$), max($x$)) and $lesseq($l$, min($a$)) and $less(0, k)) ⇒ $less($l$, $sum(max($a$), k)) tff(boyer_moore_ma

DAT056^1.p List operation requiring induction
lst: $tType thf(ty_n_tc__Foo__Olst_It__J, type)
a: $tType thf(ty_n_t_, type)
ap: lst → lst → lst thf(sy_c_Foo_Oap_001t_, type)
cns: a → lst → lst thf(sy_c_Foo_Olst_OCns_001t_, type)
nl: lst thf(sy_c_Foo_Olst_ONl_001t_, type)
xs: lst thf(sy_v_xs, type)
∀lst: lst: (∀ys: lst, zs: lst: (ap@nl@(ap@ys@zs)) = (ap@(ap@nl@ys)@zs) ⇒ (∀a: a, lst$_2$: lst: (∀ys$_3$: lst, zs$_2$: lst: (ap@lst$_2$@(ap@
(ap@(ap@lst$_2$@ys$_3$)@zs$_2$) ⇒ ∀ys: lst, zs: lst: (ap@(cns@a@lst$_2$)@(ap@ys@zs)) = (ap@(ap@(cns@a@lst$_2$)@ys)@zs)) ⇒
∀ys$_3$: lst, zs$_2$: lst: (ap@lst@(ap@ys$_3$@zs$_2$)) = (ap@(ap@lst@ys$_3$)@zs$_2$))) thf(fact_0_lst_Oinduct_091where_AP_A_061_A_C_F
∀ys$_2$: lst, xs: lst, x: a: (ap@(cns@x@xs)@ys$_2$) = (cns@x@(ap@xs@ys$_2$)) thf(fact_1p_Osimps_I2_J, axiom)
∀ys$_2$: lst: (ap@nl@ys$_2$) = ys$_2$ thf(fact_2p_Osimps_I1_J, axiom)
∀ys: lst, zs: lst: (ap@xs@(ap@ys@zs)) = (ap@(ap@xs@ys)@zs) thf(conj$_0$, conjecture)

DAT056^2.p List operation requiring induction
lst: $tType thf(ty_n_tc__Foo__Olst_It__J, type)
a: $tType thf(ty_n_t_, type)
ap: lst → lst → lst thf(sy_c_Foo_Oap_001t_, type)
cns: a → lst → lst thf(sy_c_Foo_Olst_OCns_001t_, type)
nl: lst thf(sy_c_Foo_Olst_ONl_001t_, type)
xs: lst thf(sy_v_xs, type)
∀lst: lst, p: lst → $o: (($p$@nl) ⇒ (∀$a$: a, lst$_2$: lst: ((p@lst$_2$) ⇒ (p@(cns@a@lst$_2$))) ⇒ (p@lst))) thf(fact_0_lst_Oinduct, axi
∀ys$_2$: lst, xs: lst, x: a: (ap@(cns@x@xs)@ys$_2$) = (cns@x@(ap@xs@ys$_2$)) thf(fact_1p_Osimps_I2_J, axiom)
∀ys$_2$: lst: (ap@nl@ys$_2$) = ys$_2$ thf(fact_2p_Osimps_I1_J, axiom)
∀ys: lst, zs: lst: (ap@xs@(ap@ys@zs)) = (ap@(ap@xs@ys)@zs) thf(conj$_0$, conjecture)

DAT057=1.p get-put on self
include('Axioms/DAT004=0.ax')
∀d: data, ar: array, m: $int, n: $int: (get(put(ar, m, d), n) = get(ar, n) or m = n) tff(th_lem$_1$, conjecture)

DAT058=1.p Add nothing to an array
include('Axioms/DAT004=0.ax')
∀m: $int: put(mkarray, m, none) = mkarray tff(th_lem$_2$, conjecture)

DAT059=1.p put is commutative

include('Axioms/DAT004=0.ax')
$\forall d_1$: data, ar: array, m: \$int, d_2: data: put(put(ar, m, d_1), m, d_2) = put(ar, m, d_2)   tff(th_lem$_5$, conjecture)

DAT060=1.p get-put on self lemma
include('Axioms/DAT004=0.ax')
$\forall d$: data, ar: array, n: \$int, m: \$int: (get(put(ar, m, d), n) = get(ar, n) or $\neg$ \$less(n, m))   tff(th_lem$_6$, conjecture)

DAT061=1.p get-put on self lemma
include('Axioms/DAT004=0.ax')
$\forall d$: data, ar: array, m: \$int, n: \$int: (get(put(ar, m, d), n) = get(ar, n) or $\neg$ \$less(m, n))   tff(th_lem$_7$, conjecture)

DAT062=1.p Heap lengths
include('Axioms/DAT005=0.ax')
$\forall h$: heap, n: \$int: ($\neg\forall h_0$: heap: (lsls(h_0, h) $\Rightarrow$ \$less(length(h_0), length(h))) or $\forall h_0$: heap: (lsls(h_0, app(h, n)) $\Rightarrow$ \$less(length(h_0), length(app(h, n)))))   tff(th_lem_1a, conjecture)

DAT063=1.p Empty heap length
include('Axioms/DAT005=0.ax')
$\forall h$: heap: (lsls(h, empty) $\Rightarrow$ \$less(length(h), length(empty)))   tff(th_lem_1b, conjecture)

DAT064=1.p Impossible heap
include('Axioms/DAT005=0.ax')
$\forall h_1$: heap, h: heap: ((lsls(h, h_1) and \$less(length(h_1), length(h))) $\Rightarrow$ \$false)   tff(th_lem$_2$, conjecture)

DAT065=1.p Add an element to an empty heap
include('Axioms/DAT005=0.ax')
$\forall h_0$: heap, n: \$int: ($\neg$ length(h_0) = 0 $\iff$ h_0 = empty or (length(app(h_0, n)) = 0 $\iff$ app(h_0, n) = empty))   tff(th_lem_

DAT066=1.p Cannot select after end of tree-heap
include('Axioms/DAT006=0.ax')
$\forall n$: \$int, m: \$int, h: heap: (sel(app(h, n), m) = n or $m \neq$ \$sum(1, length(h)))   tff(th$_1$, conjecture)

DAT067=1.p Add an element to a tree heap
include('Axioms/DAT006=0.ax')
$\forall n$: \$int, m: \$int, h: heap: (sel(app(h, n), m) = sel(h, m) or m = \$sum(1, length(h)))   tff(th$_2$, conjecture)

DAT068=1.p Can select from only within a tree-heap
include('Axioms/DAT006=0.ax')
$\forall n$: \$int, m: \$int, h: heap: (sel(app(h, n), m) = sel(h, m) or $\neg$ \$less(m, length(h)))   tff(th$_3$, conjecture)

DAT069=1.p Can select from only within a tree-heap
include('Axioms/DAT006=0.ax')
$\forall n$: \$int, h: heap, m: \$int: (sel(app(h, n), m) = sel(h, m) or \$less(length(h), m))   tff(th$_4$, conjecture)

DAT070=1.p Select from only within a tree-heap
include('Axioms/DAT006=0.ax')
$\forall n$: \$int, m: \$int, h: heap: (sel(app(h, n), m) = sel(h, m) or $\neg$ \$less(m, \$sum(1, length(h))))   tff(th$_5$, conjecture)

DAT071=1.p Arrays problem 1
array: \$tType   tff(array_type, type)
read: (array $\times$ \$int) $\to$ \$int   tff(read_type, type)
write: (array $\times$ \$int $\times$ \$int) $\to$ array   tff(write_type, type)
$\forall a$: array, i: \$int, v: \$int: read(write(a, i, v), i) = v tff(ax$_1$, axiom)
$\forall a$: array, i: \$int, j: \$int, v: \$int: ($i = j$ or read(write(a, i, v), j) = read(a, j))   tff(ax$_2$, axiom)
$\forall a$: array, b: array: ($\forall i$: \$int: read(a, i) = read(b, i) $\Rightarrow$ $a = b$)   tff(ext, axiom)
init: \$int $\to$ array   tff(init_type, type)
$\forall v$: \$int, i: \$int: read(init(v), i) = v tff(ax$_3$, axiom)
max: (array $\times$ \$int) $\to$ \$int   tff(max, type)
$\forall a$: array, n: \$int, w: \$int: (max(a, n) = w $\Leftarrow$ ($\forall i$: \$int: ((\$greater(n, i) and \$greatereq(i, 0)) $\Rightarrow$ \$lesseq(read(a, i), w)) and $\exists i$
w)))   tff(a, axiom)
sorted: (array $\times$ \$int) $\to$ \$o   tff(sorted_type, type)
$\forall a$: array, n: \$int: (sorted(a, n) $\iff$ $\forall i$: \$int, j: \$int: ((\$lesseq(0, i) and \$less(i, n) and \$less(i, j) and \$less(j, n)) $\Rightarrow$ \$lesseq(read(a, i), read(a, j))))   tff(sorted$_1$, axiom)
inRange: (array $\times$ \$int $\times$ \$int) $\to$ \$o   tff(inRange_type, type)
$\forall a$: array, r: \$int, n: \$int: (inRange(a, r, n) $\iff$ $\forall i$: \$int: ((\$greater(n, i) and \$greatereq(i, 0)) $\Rightarrow$ (\$greatereq(r, read(a, i)) an
distinct: (array $\times$ \$int) $\to$ \$o   tff(distinct_type, type)

$\forall a\colon \text{array}, n\colon \$\text{int}\colon (\text{distinct}(a,n) \iff \forall i\colon \$\text{int}, j\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greater}(n,j) \text{ and } \$\text{greatereq}(j,0) \text{ and } \$\text{greatereq}($
$(\text{read}(a,i) = \text{read}(a,j) \Rightarrow i = j)))$ $\quad$ $\text{tff}(\text{distinct}, \text{axiom})$
$\text{rev}\colon (\text{array} \times \$\text{int}) \to \text{array}$ $\quad$ $\text{tff}(\text{rev\_n}, \text{type})$
$\forall a\colon \text{array}, b\colon \text{array}, n\colon \$\text{int}\colon (\text{rev}(a,n) = b \Leftarrow \forall i\colon \$\text{int}\colon ((\$\text{greatereq}(i,0) \text{ and } \$\text{greater}(n,i) \text{ and } \text{read}(b,i) = \text{read}(a, \$\text{difference}($
$\text{read}(a,i))))$ $\quad$ $\text{tff}(\text{rev\_n1\_proper}, \text{axiom})$
$\forall a\colon \text{array}, n\colon \$\text{int}, w_1\colon \$\text{int}, w_2\colon \$\text{int}\colon ((\forall i\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greatereq}(i,0)) \Rightarrow \$\text{lesseq}(\text{read}(a,i), w_1)) \text{ and } \exists i\colon \$\text{int}\colon (\$$
$w_1) \text{ and } \forall i\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greatereq}(i,0)) \Rightarrow \$\text{lesseq}(\text{read}(a,i), w_2)) \text{ and } \exists i\colon \$\text{int}\colon (\$\text{greater}(n,i) \text{ and } \$\text{greatereq}($
$w_2)) \Rightarrow w_1 = w_2)$ $\quad$ $\text{tff}(c, \text{conjecture})$

DAT072=1.p Arrays problem 2
$\text{array}\colon \$\text{tType}$ $\quad$ $\text{tff}(\text{array\_type}, \text{type})$
$\text{read}\colon (\text{array} \times \$\text{int}) \to \$\text{int}$ $\quad$ $\text{tff}(\text{read\_type}, \text{type})$
$\text{write}\colon (\text{array} \times \$\text{int} \times \$\text{int}) \to \text{array}$ $\quad$ $\text{tff}(\text{write\_type}, \text{type})$
$\forall a\colon \text{array}, i\colon \$\text{int}, v\colon \$\text{int}\colon \text{read}(\text{write}(a,i,v),i) = v$ $\quad$ $\text{tff}(\text{ax}_1, \text{axiom})$
$\forall a\colon \text{array}, i\colon \$\text{int}, j\colon \$\text{int}, v\colon \$\text{int}\colon (i = j \text{ or } \text{read}(\text{write}(a,i,v),j) = \text{read}(a,j))$ $\quad$ $\text{tff}(\text{ax}_2, \text{axiom})$
$\forall a\colon \text{array}, b\colon \text{array}\colon (\forall i\colon \$\text{int}\colon \text{read}(a,i) = \text{read}(b,i) \Rightarrow a = b)$ $\quad$ $\text{tff}(\text{ext}, \text{axiom})$
$\text{init}\colon \$\text{int} \to \text{array}$ $\quad$ $\text{tff}(\text{init\_type}, \text{type})$
$\forall v\colon \$\text{int}, i\colon \$\text{int}\colon \text{read}(\text{init}(v),i) = v$ $\quad$ $\text{tff}(\text{ax}_3, \text{axiom})$
$\text{max}\colon (\text{array} \times \$\text{int}) \to \$\text{int}$ $\quad$ $\text{tff}(\text{max}, \text{type})$
$\forall a\colon \text{array}, n\colon \$\text{int}, w\colon \$\text{int}\colon (\text{max}(a,n) = w \Leftarrow (\forall i\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greatereq}(i,0)) \Rightarrow \$\text{lesseq}(\text{read}(a,i), w)) \text{ and } \exists i$
$w)))$ $\quad$ $\text{tff}(a, \text{axiom})$
$\text{sorted}\colon (\text{array} \times \$\text{int}) \to \$\text{o}$ $\quad$ $\text{tff}(\text{sorted\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \$\text{int}\colon (\text{sorted}(a,n) \iff \forall i\colon \$\text{int}, j\colon \$\text{int}\colon ((\$\text{lesseq}(0,i) \text{ and } \$\text{less}(i,n) \text{ and } \$\text{less}(i,j) \text{ and } \$\text{less}(j,n)) \Rightarrow$
$\$\text{lesseq}(\text{read}(a,i), \text{read}(a,j))))$ $\quad$ $\text{tff}(\text{sorted}_1, \text{axiom})$
$\text{inRange}\colon (\text{array} \times \$\text{int} \times \$\text{int}) \to \$\text{o}$ $\quad$ $\text{tff}(\text{inRange\_type}, \text{type})$
$\forall a\colon \text{array}, r\colon \$\text{int}, n\colon \$\text{int}\colon (\text{inRange}(a,r,n) \iff \forall i\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greatereq}(i,0)) \Rightarrow (\$\text{greatereq}(r, \text{read}(a,i)) \text{ ar}$
$\text{distinct}\colon (\text{array} \times \$\text{int}) \to \$\text{o}$ $\quad$ $\text{tff}(\text{distinct\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \$\text{int}\colon (\text{distinct}(a,n) \iff \forall i\colon \$\text{int}, j\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greater}(n,j) \text{ and } \$\text{greatereq}(j,0) \text{ and } \$\text{greatereq}($
$(\text{read}(a,i) = \text{read}(a,j) \Rightarrow i = j)))$ $\quad$ $\text{tff}(\text{distinct}, \text{axiom})$
$\text{rev}\colon (\text{array} \times \$\text{int}) \to \text{array}$ $\quad$ $\text{tff}(\text{rev\_n}, \text{type})$
$\forall a\colon \text{array}, b\colon \text{array}, n\colon \$\text{int}\colon (\text{rev}(a,n) = b \Leftarrow \forall i\colon \$\text{int}\colon ((\$\text{greatereq}(i,0) \text{ and } \$\text{greater}(n,i) \text{ and } \text{read}(b,i) = \text{read}(a, \$\text{difference}($
$\text{read}(a,i))))$ $\quad$ $\text{tff}(\text{rev\_n1\_proper}, \text{axiom})$
$\forall a\colon \text{array}, b_1\colon \text{array}, b_2\colon \text{array}, n\colon \$\text{int}\colon ((\forall i\colon \$\text{int}\colon ((\$\text{greatereq}(i,0) \text{ and } \$\text{greater}(n,i) \text{ and } \text{read}(b_1,i) = \text{read}(a, \$\text{difference}(n, \$\text{su}$
$\text{read}(a,i))) \text{ and } \forall i\colon \$\text{int}\colon ((\$\text{greatereq}(i,0) \text{ and } \$\text{greater}(n,i) \text{ and } \text{read}(b_2,i) = \text{read}(a, \$\text{difference}(n, \$\text{sum}(i,1)))) \text{ or } ((\$\text{greate}$
$\text{read}(a,i)))) \Rightarrow b_1 = b_2)$ $\quad$ $\text{tff}(c, \text{conjecture})$

DAT073=1.p Arrays problem 3
$\text{array}\colon \$\text{tType}$ $\quad$ $\text{tff}(\text{array\_type}, \text{type})$
$\text{read}\colon (\text{array} \times \$\text{int}) \to \$\text{int}$ $\quad$ $\text{tff}(\text{read\_type}, \text{type})$
$\text{write}\colon (\text{array} \times \$\text{int} \times \$\text{int}) \to \text{array}$ $\quad$ $\text{tff}(\text{write\_type}, \text{type})$
$\forall a\colon \text{array}, i\colon \$\text{int}, v\colon \$\text{int}\colon \text{read}(\text{write}(a,i,v),i) = v$ $\quad$ $\text{tff}(\text{ax}_1, \text{axiom})$
$\forall a\colon \text{array}, i\colon \$\text{int}, j\colon \$\text{int}, v\colon \$\text{int}\colon (i = j \text{ or } \text{read}(\text{write}(a,i,v),j) = \text{read}(a,j))$ $\quad$ $\text{tff}(\text{ax}_2, \text{axiom})$
$\forall a\colon \text{array}, b\colon \text{array}\colon (\forall i\colon \$\text{int}\colon \text{read}(a,i) = \text{read}(b,i) \Rightarrow a = b)$ $\quad$ $\text{tff}(\text{ext}, \text{axiom})$
$\text{init}\colon \$\text{int} \to \text{array}$ $\quad$ $\text{tff}(\text{init\_type}, \text{type})$
$\forall v\colon \$\text{int}, i\colon \$\text{int}\colon \text{read}(\text{init}(v),i) = v$ $\quad$ $\text{tff}(\text{ax}_3, \text{axiom})$
$\text{max}\colon (\text{array} \times \$\text{int}) \to \$\text{int}$ $\quad$ $\text{tff}(\text{max}, \text{type})$
$\forall a\colon \text{array}, n\colon \$\text{int}, w\colon \$\text{int}\colon (\text{max}(a,n) = w \Leftarrow (\forall i\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greatereq}(i,0)) \Rightarrow \$\text{lesseq}(\text{read}(a,i), w)) \text{ and } \exists i$
$w)))$ $\quad$ $\text{tff}(a, \text{axiom})$
$\text{sorted}\colon (\text{array} \times \$\text{int}) \to \$\text{o}$ $\quad$ $\text{tff}(\text{sorted\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \$\text{int}\colon (\text{sorted}(a,n) \iff \forall i\colon \$\text{int}, j\colon \$\text{int}\colon ((\$\text{lesseq}(0,i) \text{ and } \$\text{less}(i,n) \text{ and } \$\text{less}(i,j) \text{ and } \$\text{less}(j,n)) \Rightarrow$
$\$\text{lesseq}(\text{read}(a,i), \text{read}(a,j))))$ $\quad$ $\text{tff}(\text{sorted}_1, \text{axiom})$
$\text{inRange}\colon (\text{array} \times \$\text{int} \times \$\text{int}) \to \$\text{o}$ $\quad$ $\text{tff}(\text{inRange\_type}, \text{type})$
$\forall a\colon \text{array}, r\colon \$\text{int}, n\colon \$\text{int}\colon (\text{inRange}(a,r,n) \iff \forall i\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greatereq}(i,0)) \Rightarrow (\$\text{greatereq}(r, \text{read}(a,i)) \text{ ar}$
$\text{distinct}\colon (\text{array} \times \$\text{int}) \to \$\text{o}$ $\quad$ $\text{tff}(\text{distinct\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \$\text{int}\colon (\text{distinct}(a,n) \iff \forall i\colon \$\text{int}, j\colon \$\text{int}\colon ((\$\text{greater}(n,i) \text{ and } \$\text{greater}(n,j) \text{ and } \$\text{greatereq}(j,0) \text{ and } \$\text{greatereq}($
$(\text{read}(a,i) = \text{read}(a,j) \Rightarrow i = j)))$ $\quad$ $\text{tff}(\text{distinct}, \text{axiom})$
$\text{rev}\colon (\text{array} \times \$\text{int}) \to \text{array}$ $\quad$ $\text{tff}(\text{rev\_n}, \text{type})$
$\forall a\colon \text{array}, b\colon \text{array}, n\colon \$\text{int}\colon (\text{rev}(a,n) = b \Leftarrow \forall i\colon \$\text{int}\colon ((\$\text{greatereq}(i,0) \text{ and } \$\text{greater}(n,i) \text{ and } \text{read}(b,i) = \text{read}(a, \$\text{difference}($
$\text{read}(a,i))))$ $\quad$ $\text{tff}(\text{rev\_n1\_proper}, \text{axiom})$
$\neg \forall a\colon \text{array}, n\colon \$\text{int}\colon (\$\text{greatereq}(n,0) \Rightarrow \text{inRange}(a, \text{max}(a,n), n))$ $\quad$ $\text{tff}(c_1, \text{conjecture})$

DAT074=1.p Arrays problem 4

$\mathrm{array}: \$\mathrm{tType}$ tff(array_type, type)
$\mathrm{read}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{int}$ tff(read_type, type)
$\mathrm{write}: (\mathrm{array} \times \$\mathrm{int} \times \$\mathrm{int}) \to \mathrm{array}$ tff(write_type, type)
$\forall a: \mathrm{array}, i: \$\mathrm{int}, v: \$\mathrm{int}: \mathrm{read}(\mathrm{write}(a,i,v),i) = v$ tff(ax_1, axiom)
$\forall a: \mathrm{array}, i: \$\mathrm{int}, j: \$\mathrm{int}, v: \$\mathrm{int}: (i = j \text{ or } \mathrm{read}(\mathrm{write}(a,i,v),j) = \mathrm{read}(a,j))$ tff(ax_2, axiom)
$\forall a: \mathrm{array}, b: \mathrm{array}: (\forall i: \$\mathrm{int}: \mathrm{read}(a,i) = \mathrm{read}(b,i) \Rightarrow a = b)$ tff(ext, axiom)
$\mathrm{init}: \$\mathrm{int} \to \mathrm{array}$ tff(init_type, type)
$\forall v: \$\mathrm{int}, i: \$\mathrm{int}: \mathrm{read}(\mathrm{init}(v),i) = v$ tff(ax_3, axiom)
$\mathrm{max}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{int}$ tff(max, type)
$\forall a: \mathrm{array}, n: \$\mathrm{int}, w: \$\mathrm{int}: (\mathrm{max}(a,n) = w \Leftarrow (\forall i: \$\mathrm{int}: ((\$\mathrm{greater}(n,i) \text{ and } \$\mathrm{greatereq}(i,0)) \Rightarrow \$\mathrm{lesseq}(\mathrm{read}(a,i),w)) \text{ and } \exists i$
$w)))$ tff(a, axiom)
$\mathrm{sorted}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(sorted_type, type)
$\forall a: \mathrm{array}, n: \$\mathrm{int}: (\mathrm{sorted}(a,n) \iff \forall i: \$\mathrm{int}, j: \$\mathrm{int}: ((\$\mathrm{lesseq}(0,i) \text{ and } \$\mathrm{less}(i,n) \text{ and } \$\mathrm{less}(i,j) \text{ and } \$\mathrm{less}(j,n)) \Rightarrow$
$\$\mathrm{lesseq}(\mathrm{read}(a,i),\mathrm{read}(a,j))))$ tff(sorted_1, axiom)
$\mathrm{inRange}: (\mathrm{array} \times \$\mathrm{int} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(inRange_type, type)
$\forall a: \mathrm{array}, r: \$\mathrm{int}, n: \$\mathrm{int}: (\mathrm{inRange}(a,r,n) \iff \forall i: \$\mathrm{int}: ((\$\mathrm{greater}(n,i) \text{ and } \$\mathrm{greatereq}(i,0)) \Rightarrow (\$\mathrm{greatereq}(r,\mathrm{read}(a,i)) \text{ an}$
$\mathrm{distinct}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(distinct_type, type)
$\forall a: \mathrm{array}, n: \$\mathrm{int}: (\mathrm{distinct}(a,n) \iff \forall i: \$\mathrm{int}, j: \$\mathrm{int}: ((\$\mathrm{greater}(n,i) \text{ and } \$\mathrm{greater}(n,j) \text{ and } \$\mathrm{greatereq}(j,0) \text{ and } \$\mathrm{greatereq}($
$(\mathrm{read}(a,i) = \mathrm{read}(a,j) \Rightarrow i = j)))$ tff(distinct, axiom)
$\mathrm{rev}: (\mathrm{array} \times \$\mathrm{int}) \to \mathrm{array}$ tff(rev_n, type)
$\forall a: \mathrm{array}, b: \mathrm{array}, n: \$\mathrm{int}: (\mathrm{rev}(a,n) = b \Leftarrow \forall i: \$\mathrm{int}: ((\$\mathrm{greatereq}(i,0) \text{ and } \$\mathrm{greater}(n,i) \text{ and } \mathrm{read}(b,i) = \mathrm{read}(a, \$\mathrm{difference}($
$\mathrm{read}(a,i))))$ tff(rev_n1_proper, axiom)
$\neg \forall n: \$\mathrm{int}, i: \$\mathrm{int}: \mathrm{distinct}(\mathrm{init}(n),i)$ tff(c_2, conjecture)

DAT075=1.p Arrays problem 5

$\mathrm{array}: \$\mathrm{tType}$ tff(array_type, type)
$\mathrm{read}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{int}$ tff(read_type, type)
$\mathrm{write}: (\mathrm{array} \times \$\mathrm{int} \times \$\mathrm{int}) \to \mathrm{array}$ tff(write_type, type)
$\forall a: \mathrm{array}, i: \$\mathrm{int}, v: \$\mathrm{int}: \mathrm{read}(\mathrm{write}(a,i,v),i) = v$ tff(ax_1, axiom)
$\forall a: \mathrm{array}, i: \$\mathrm{int}, j: \$\mathrm{int}, v: \$\mathrm{int}: (i = j \text{ or } \mathrm{read}(\mathrm{write}(a,i,v),j) = \mathrm{read}(a,j))$ tff(ax_2, axiom)
$\forall a: \mathrm{array}, b: \mathrm{array}: (\forall i: \$\mathrm{int}: \mathrm{read}(a,i) = \mathrm{read}(b,i) \Rightarrow a = b)$ tff(ext, axiom)
$\mathrm{init}: \$\mathrm{int} \to \mathrm{array}$ tff(init_type, type)
$\forall v: \$\mathrm{int}, i: \$\mathrm{int}: \mathrm{read}(\mathrm{init}(v),i) = v$ tff(ax_3, axiom)
$\mathrm{max}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{int}$ tff(max, type)
$\forall a: \mathrm{array}, n: \$\mathrm{int}, w: \$\mathrm{int}: (\mathrm{max}(a,n) = w \Leftarrow (\forall i: \$\mathrm{int}: ((\$\mathrm{greater}(n,i) \text{ and } \$\mathrm{greatereq}(i,0)) \Rightarrow \$\mathrm{lesseq}(\mathrm{read}(a,i),w)) \text{ and } \exists i$
$w)))$ tff(a, axiom)
$\mathrm{sorted}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(sorted_type, type)
$\forall a: \mathrm{array}, n: \$\mathrm{int}: (\mathrm{sorted}(a,n) \iff \forall i: \$\mathrm{int}, j: \$\mathrm{int}: ((\$\mathrm{lesseq}(0,i) \text{ and } \$\mathrm{less}(i,n) \text{ and } \$\mathrm{less}(i,j) \text{ and } \$\mathrm{less}(j,n)) \Rightarrow$
$\$\mathrm{lesseq}(\mathrm{read}(a,i),\mathrm{read}(a,j))))$ tff(sorted_1, axiom)
$\mathrm{inRange}: (\mathrm{array} \times \$\mathrm{int} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(inRange_type, type)
$\forall a: \mathrm{array}, r: \$\mathrm{int}, n: \$\mathrm{int}: (\mathrm{inRange}(a,r,n) \iff \forall i: \$\mathrm{int}: ((\$\mathrm{greater}(n,i) \text{ and } \$\mathrm{greatereq}(i,0)) \Rightarrow (\$\mathrm{greatereq}(r,\mathrm{read}(a,i)) \text{ an}$
$\mathrm{distinct}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{o}$ tff(distinct_type, type)
$\forall a: \mathrm{array}, n: \$\mathrm{int}: (\mathrm{distinct}(a,n) \iff \forall i: \$\mathrm{int}, j: \$\mathrm{int}: ((\$\mathrm{greater}(n,i) \text{ and } \$\mathrm{greater}(n,j) \text{ and } \$\mathrm{greatereq}(j,0) \text{ and } \$\mathrm{greatereq}($
$(\mathrm{read}(a,i) = \mathrm{read}(a,j) \Rightarrow i = j)))$ tff(distinct, axiom)
$\mathrm{rev}: (\mathrm{array} \times \$\mathrm{int}) \to \mathrm{array}$ tff(rev_n, type)
$\forall a: \mathrm{array}, b: \mathrm{array}, n: \$\mathrm{int}: (\mathrm{rev}(a,n) = b \Leftarrow \forall i: \$\mathrm{int}: ((\$\mathrm{greatereq}(i,0) \text{ and } \$\mathrm{greater}(n,i) \text{ and } \mathrm{read}(b,i) = \mathrm{read}(a, \$\mathrm{difference}($
$\mathrm{read}(a,i))))$ tff(rev_n1_proper, axiom)
$\neg \forall a: \mathrm{array}, n: \$\mathrm{int}: \mathrm{read}(\mathrm{rev}(a,\$\mathrm{sum}(n,1)),0) = \mathrm{read}(a,n)$ tff(c_3, conjecture)

DAT076=1.p Arrays problem 6

$\mathrm{array}: \$\mathrm{tType}$ tff(array_type, type)
$\mathrm{read}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{int}$ tff(read_type, type)
$\mathrm{write}: (\mathrm{array} \times \$\mathrm{int} \times \$\mathrm{int}) \to \mathrm{array}$ tff(write_type, type)
$\forall a: \mathrm{array}, i: \$\mathrm{int}, v: \$\mathrm{int}: \mathrm{read}(\mathrm{write}(a,i,v),i) = v$ tff(ax_1, axiom)
$\forall a: \mathrm{array}, i: \$\mathrm{int}, j: \$\mathrm{int}, v: \$\mathrm{int}: (i = j \text{ or } \mathrm{read}(\mathrm{write}(a,i,v),j) = \mathrm{read}(a,j))$ tff(ax_2, axiom)
$\forall a: \mathrm{array}, b: \mathrm{array}: (\forall i: \$\mathrm{int}: \mathrm{read}(a,i) = \mathrm{read}(b,i) \Rightarrow a = b)$ tff(ext, axiom)
$\mathrm{init}: \$\mathrm{int} \to \mathrm{array}$ tff(init_type, type)
$\forall v: \$\mathrm{int}, i: \$\mathrm{int}: \mathrm{read}(\mathrm{init}(v),i) = v$ tff(ax_3, axiom)
$\mathrm{max}: (\mathrm{array} \times \$\mathrm{int}) \to \$\mathrm{int}$ tff(max, type)

$\forall a\colon \text{array}, n\colon \text{\$int}, w\colon \text{\$int}\colon (\max(a,n) = w \Leftarrow (\forall i\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greatereq}(i,0)) \Rightarrow \text{\$lesseq}(\text{read}(a,i),w)) \text{ and } \exists i$
$w)))$ $\quad \text{tff}(a, \text{axiom})$
$\text{sorted}\colon (\text{array} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{sorted\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \text{\$int}\colon (\text{sorted}(a,n) \iff \forall i\colon \text{\$int}, j\colon \text{\$int}\colon ((\text{\$lesseq}(0,i) \text{ and } \text{\$less}(i,n) \text{ and } \text{\$less}(i,j) \text{ and } \text{\$less}(j,n)) \Rightarrow$
$\text{\$lesseq}(\text{read}(a,i), \text{read}(a,j)))) \quad \text{tff}(\text{sorted}_1, \text{axiom})$
$\text{inRange}\colon (\text{array} \times \text{\$int} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall a\colon \text{array}, r\colon \text{\$int}, n\colon \text{\$int}\colon (\text{inRange}(a,r,n) \iff \forall i\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greatereq}(i,0)) \Rightarrow (\text{\$greatereq}(r, \text{read}(a,i)) \text{ an}$
$\text{distinct}\colon (\text{array} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{distinct\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \text{\$int}\colon (\text{distinct}(a,n) \iff \forall i\colon \text{\$int}, j\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greater}(n,j) \text{ and } \text{\$greatereq}(j,0) \text{ and } \text{\$greatereq}($
$(\text{read}(a,i) = \text{read}(a,j) \Rightarrow i = j))) \quad \text{tff}(\text{distinct}, \text{axiom})$
$\text{rev}\colon (\text{array} \times \text{\$int}) \to \text{array} \quad \text{tff}(\text{rev\_n}, \text{type})$
$\forall a\colon \text{array}, b\colon \text{array}, n\colon \text{\$int}\colon (\text{rev}(a,n) = b \Leftarrow \forall i\colon \text{\$int}\colon ((\text{\$greatereq}(i,0) \text{ and } \text{\$greater}(n,i) \text{ and } \text{read}(b,i) = \text{read}(a, \text{\$difference}($
$\text{read}(a,i)))) \quad \text{tff}(\text{rev\_n1\_proper}, \text{axiom})$
$\neg \forall a\colon \text{array}, n\colon \text{\$int}\colon (\text{sorted}(a,n) \Rightarrow \neg \text{sorted}(\text{rev}(a,n),n)) \quad \text{tff}(c_4, \text{conjecture})$

DAT077=1.p Arrays problem 7
$\text{array}\colon \text{\$tType} \quad \text{tff}(\text{array\_type}, \text{type})$
$\text{read}\colon (\text{array} \times \text{\$int}) \to \text{\$int} \quad \text{tff}(\text{read\_type}, \text{type})$
$\text{write}\colon (\text{array} \times \text{\$int} \times \text{\$int}) \to \text{array} \quad \text{tff}(\text{write\_type}, \text{type})$
$\forall a\colon \text{array}, i\colon \text{\$int}, v\colon \text{\$int}\colon \text{read}(\text{write}(a,i,v),i) = v \quad \text{tff}(\text{ax}_1, \text{axiom})$
$\forall a\colon \text{array}, i\colon \text{\$int}, j\colon \text{\$int}, v\colon \text{\$int}\colon (i = j \text{ or } \text{read}(\text{write}(a,i,v),j) = \text{read}(a,j)) \quad \text{tff}(\text{ax}_2, \text{axiom})$
$\forall a\colon \text{array}, b\colon \text{array}\colon (\forall i\colon \text{\$int}\colon \text{read}(a,i) = \text{read}(b,i) \Rightarrow a = b) \quad \text{tff}(\text{ext}, \text{axiom})$
$\text{init}\colon \text{\$int} \to \text{array} \quad \text{tff}(\text{init\_type}, \text{type})$
$\forall v\colon \text{\$int}, i\colon \text{\$int}\colon \text{read}(\text{init}(v),i) = v \quad \text{tff}(\text{ax}_3, \text{axiom})$
$\text{max}\colon (\text{array} \times \text{\$int}) \to \text{\$int} \quad \text{tff}(\text{max}, \text{type})$
$\forall a\colon \text{array}, n\colon \text{\$int}, w\colon \text{\$int}\colon (\max(a,n) = w \Leftarrow (\forall i\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greatereq}(i,0)) \Rightarrow \text{\$lesseq}(\text{read}(a,i),w)) \text{ and } \exists i$
$w)))$ $\quad \text{tff}(a, \text{axiom})$
$\text{sorted}\colon (\text{array} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{sorted\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \text{\$int}\colon (\text{sorted}(a,n) \iff \forall i\colon \text{\$int}, j\colon \text{\$int}\colon ((\text{\$lesseq}(0,i) \text{ and } \text{\$less}(i,n) \text{ and } \text{\$less}(i,j) \text{ and } \text{\$less}(j,n)) \Rightarrow$
$\text{\$lesseq}(\text{read}(a,i), \text{read}(a,j)))) \quad \text{tff}(\text{sorted}_1, \text{axiom})$
$\text{inRange}\colon (\text{array} \times \text{\$int} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall a\colon \text{array}, r\colon \text{\$int}, n\colon \text{\$int}\colon (\text{inRange}(a,r,n) \iff \forall i\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greatereq}(i,0)) \Rightarrow (\text{\$greatereq}(r, \text{read}(a,i)) \text{ an}$
$\text{distinct}\colon (\text{array} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{distinct\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \text{\$int}\colon (\text{distinct}(a,n) \iff \forall i\colon \text{\$int}, j\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greater}(n,j) \text{ and } \text{\$greatereq}(j,0) \text{ and } \text{\$greatereq}($
$(\text{read}(a,i) = \text{read}(a,j) \Rightarrow i = j))) \quad \text{tff}(\text{distinct}, \text{axiom})$
$\text{rev}\colon (\text{array} \times \text{\$int}) \to \text{array} \quad \text{tff}(\text{rev\_n}, \text{type})$
$\forall a\colon \text{array}, b\colon \text{array}, n\colon \text{\$int}\colon (\text{rev}(a,n) = b \Leftarrow \forall i\colon \text{\$int}\colon ((\text{\$greatereq}(i,0) \text{ and } \text{\$greater}(n,i) \text{ and } \text{read}(b,i) = \text{read}(a, \text{\$difference}($
$\text{read}(a,i)))) \quad \text{tff}(\text{rev\_n1\_proper}, \text{axiom})$
$\neg \forall m\colon \text{\$int}\colon \exists n\colon \text{\$int}\colon \neg \text{sorted}(\text{rev}(\text{init}(n),m),m) \quad \text{tff}(c_5, \text{conjecture})$

DAT078=1.p Arrays problem 8
$\text{array}\colon \text{\$tType} \quad \text{tff}(\text{array\_type}, \text{type})$
$\text{read}\colon (\text{array} \times \text{\$int}) \to \text{\$int} \quad \text{tff}(\text{read\_type}, \text{type})$
$\text{write}\colon (\text{array} \times \text{\$int} \times \text{\$int}) \to \text{array} \quad \text{tff}(\text{write\_type}, \text{type})$
$\forall a\colon \text{array}, i\colon \text{\$int}, v\colon \text{\$int}\colon \text{read}(\text{write}(a,i,v),i) = v \quad \text{tff}(\text{ax}_1, \text{axiom})$
$\forall a\colon \text{array}, i\colon \text{\$int}, j\colon \text{\$int}, v\colon \text{\$int}\colon (i = j \text{ or } \text{read}(\text{write}(a,i,v),j) = \text{read}(a,j)) \quad \text{tff}(\text{ax}_2, \text{axiom})$
$\forall a\colon \text{array}, b\colon \text{array}\colon (\forall i\colon \text{\$int}\colon \text{read}(a,i) = \text{read}(b,i) \Rightarrow a = b) \quad \text{tff}(\text{ext}, \text{axiom})$
$\text{init}\colon \text{\$int} \to \text{array} \quad \text{tff}(\text{init\_type}, \text{type})$
$\forall v\colon \text{\$int}, i\colon \text{\$int}\colon \text{read}(\text{init}(v),i) = v \quad \text{tff}(\text{ax}_3, \text{axiom})$
$\text{max}\colon (\text{array} \times \text{\$int}) \to \text{\$int} \quad \text{tff}(\text{max}, \text{type})$
$\forall a\colon \text{array}, n\colon \text{\$int}, w\colon \text{\$int}\colon (\max(a,n) = w \Leftarrow (\forall i\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greatereq}(i,0)) \Rightarrow \text{\$lesseq}(\text{read}(a,i),w)) \text{ and } \exists i$
$w)))$ $\quad \text{tff}(a, \text{axiom})$
$\text{sorted}\colon (\text{array} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{sorted\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \text{\$int}\colon (\text{sorted}(a,n) \iff \forall i\colon \text{\$int}, j\colon \text{\$int}\colon ((\text{\$lesseq}(0,i) \text{ and } \text{\$less}(i,n) \text{ and } \text{\$less}(i,j) \text{ and } \text{\$less}(j,n)) \Rightarrow$
$\text{\$lesseq}(\text{read}(a,i), \text{read}(a,j)))) \quad \text{tff}(\text{sorted}_1, \text{axiom})$
$\text{inRange}\colon (\text{array} \times \text{\$int} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall a\colon \text{array}, r\colon \text{\$int}, n\colon \text{\$int}\colon (\text{inRange}(a,r,n) \iff \forall i\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greatereq}(i,0)) \Rightarrow (\text{\$greatereq}(r, \text{read}(a,i)) \text{ an}$
$\text{distinct}\colon (\text{array} \times \text{\$int}) \to \text{\$o} \quad \text{tff}(\text{distinct\_type}, \text{type})$
$\forall a\colon \text{array}, n\colon \text{\$int}\colon (\text{distinct}(a,n) \iff \forall i\colon \text{\$int}, j\colon \text{\$int}\colon ((\text{\$greater}(n,i) \text{ and } \text{\$greater}(n,j) \text{ and } \text{\$greatereq}(j,0) \text{ and } \text{\$greatereq}($
$(\text{read}(a,i) = \text{read}(a,j) \Rightarrow i = j))) \quad \text{tff}(\text{distinct}, \text{axiom})$

rev: (array × \$int) → array   tff(rev_n, type)
$\forall a$: array, b: array, n: \$int: (rev(a, n) = $b \Leftarrow \forall i$: \$int: ((\$greatereq(i, 0) and \$greater(n, i) and read(b, i) = read(a, \$difference(
read(a, i))))   tff(rev_n1_proper, axiom)
$\neg \forall a$: array, n: \$int: ((sorted(a, n) and \$greater(n, 0)) $\Rightarrow$ distinct(a, n))   tff(c_6, conjecture)

DAT079=1.p Lists by functions problem 1
list: \$tType   tff(list_type, type)
nil: list   tff(nil_type, type)
cons: (\$int × list) → list   tff(cons_type, type)
head: list → \$int   tff(head_type, type)
tail: list → list   tff(tail_type, type)
$\forall k$: \$int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: \$int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l)))   tff(l_3, axiom)
$\forall k$: \$int, l: list: cons(k, l) $\neq$ nil   tff(l_4, axiom)
in: (\$int × list) → \$o   tff(in, type)
$\forall x$: \$int, l: list: (in(x, l) $\iff$ ($\exists h$: \$int, t: list: (l = cons(h, t) and $x = h$) or $\exists h$: \$int, t: list: (l = cons(h, t) and in(x, t))))   t
inRange: (\$int × list) → \$o   tff(inRange_type, type)
$\forall n$: \$int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: \$int, t: list: (l = cons(k, t) and \$lesseq(0, k) and \$less(k, n) and inRange(n,
length: list → \$int   tff(t, type)
length(nil) = 0   tff(l, axiom)
$\forall h$: \$int, t: list: length(cons(h, t)) = \$sum(1, length(t))   tff(l_1, axiom)
count: (\$int × list) → \$int   tff(t_2, type)
$\forall k$: \$int: count(k, nil) = 0   tff(a, axiom)
$\forall k$: \$int, h: \$int, t: list, n: \$int: (count(k, cons(h, t)) = count(k, t) $\Leftarrow$ $k \neq h$)   tff(a_3, axiom)
$\forall k$: \$int, h: \$int, t: list, n: \$int: (count(k, cons(h, t)) = \$sum(count(k, t), 1) $\Leftarrow$ $k = h$)   tff(a_4, axiom)
append: (list × list) → list   tff(t_5, type)
$\forall l$: list: append(nil, l) = l tff(l_6, axiom)
$\forall i$: \$int, k: list, l: list: append(cons(i, k), l) = cons(i, append(k, l))   tff(l_7, axiom)
$\forall n$: \$int, l: list: (in(n, l) $\iff$ \$greater(count(n, l), 0))   tff(a_8, axiom)
in(2, cons(1, cons(2, cons(3, nil))))   tff(c, conjecture)

DAT080=1.p Lists by functions problem 2
list: \$tType   tff(list_type, type)
nil: list   tff(nil_type, type)
cons: (\$int × list) → list   tff(cons_type, type)
head: list → \$int   tff(head_type, type)
tail: list → list   tff(tail_type, type)
$\forall k$: \$int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: \$int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l)))   tff(l_3, axiom)
$\forall k$: \$int, l: list: cons(k, l) $\neq$ nil   tff(l_4, axiom)
in: (\$int × list) → \$o   tff(in, type)
$\forall x$: \$int, l: list: (in(x, l) $\iff$ ($\exists h$: \$int, t: list: (l = cons(h, t) and $x = h$) or $\exists h$: \$int, t: list: (l = cons(h, t) and in(x, t))))   t
inRange: (\$int × list) → \$o   tff(inRange_type, type)
$\forall n$: \$int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: \$int, t: list: (l = cons(k, t) and \$lesseq(0, k) and \$less(k, n) and inRange(n,
length: list → \$int   tff(t, type)
length(nil) = 0   tff(l, axiom)
$\forall h$: \$int, t: list: length(cons(h, t)) = \$sum(1, length(t))   tff(l_1, axiom)
count: (\$int × list) → \$int   tff(t_2, type)
$\forall k$: \$int: count(k, nil) = 0   tff(a, axiom)
$\forall k$: \$int, h: \$int, t: list, n: \$int: (count(k, cons(h, t)) = count(k, t) $\Leftarrow$ $k \neq h$)   tff(a_3, axiom)
$\forall k$: \$int, h: \$int, t: list, n: \$int: (count(k, cons(h, t)) = \$sum(count(k, t), 1) $\Leftarrow$ $k = h$)   tff(a_4, axiom)
append: (list × list) → list   tff(t_5, type)
$\forall l$: list: append(nil, l) = l tff(l_6, axiom)
$\forall i$: \$int, k: list, l: list: append(cons(i, k), l) = cons(i, append(k, l))   tff(l_7, axiom)
$\forall n$: \$int, l: list: (in(n, l) $\iff$ \$greater(count(n, l), 0))   tff(a_8, axiom)
$\neg$ in(4, cons(1, cons(2, cons(3, nil))))   tff(c, conjecture)

DAT081=1.p Lists by functions problem 3
list: \$tType   tff(list_type, type)

nil: list tff(nil_type, type)
cons: ($int × list) → list tff(cons_type, type)
head: list → $int tff(head_type, type)
tail: list → list tff(tail_type, type)
$\forall k$: $int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: $int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l))) tff(l_3, axiom)
$\forall k$: $int, l: list: cons(k, l) $\neq$ nil tff(l_4, axiom)
in: ($int × list) → $o tff(in, type)
$\forall x$: $int, l: list: (in(x, l) $\iff$ ($\exists h$: $int, t: list: (l = cons(h, t) and $x = h$) or $\exists h$: $int, t: list: (l = cons(h, t) and in(x, t)))) t
inRange: ($int × list) → $o tff(inRange_type, type)
$\forall n$: $int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: $int, t: list: (l = cons(k, t) and $lesseq(0, k) and $less($k, n$) and inRange($n$,
length: list → $int tff($t$, type)
length(nil) = 0 tff(l, axiom)
$\forall h$: $int, t: list: length(cons(h, t)) = $sum(1, length($t$)) tff($l_1$, axiom)
count: ($int × list) → $int tff($t_2$, type)
$\forall k$: $int: count($k$, nil) = 0 tff($a$, axiom)
$\forall k$: $int, h: $int, t: list, n: $int: (count($k$, cons($h, t$)) = count($k, t$) $\Leftarrow$ $k \neq h$) tff(a_3, axiom)
$\forall k$: $int, h: $int, t: list, n: $int: (count($k$, cons($h, t$)) = $sum(count($k, t$), 1) $\Leftarrow$ $k = h$) tff(a_4, axiom)
append: (list × list) → list tff(t_5, type)
$\forall l$: list: append(nil, l) = l tff(l_6, axiom)
$\forall i$: $int, k: list, l: list: append(cons(i, k), l) = cons(i, append(k, l)) tff(l_7, axiom)
$\forall n$: $int, l: list: (in(n, l) $\iff$ $greater(count($n, l$), 0)) tff($a_8$, axiom)
$\forall a$: $int: $\neg$ in(a, nil) tff(c, conjecture)

DAT082=1.p Lists by functions problem 4
list: $tType tff(list_type, type)
nil: list tff(nil_type, type)
cons: ($int × list) → list tff(cons_type, type)
head: list → $int tff(head_type, type)
tail: list → list tff(tail_type, type)
$\forall k$: $int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: $int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l))) tff(l_3, axiom)
$\forall k$: $int, l: list: cons(k, l) $\neq$ nil tff(l_4, axiom)
in: ($int × list) → $o tff(in, type)
$\forall x$: $int, l: list: (in(x, l) $\iff$ ($\exists h$: $int, t: list: (l = cons(h, t) and $x = h$) or $\exists h$: $int, t: list: (l = cons(h, t) and in(x, t)))) t
inRange: ($int × list) → $o tff(inRange_type, type)
$\forall n$: $int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: $int, t: list: (l = cons(k, t) and $lesseq(0, k) and $less($k, n$) and inRange($n$,
length: list → $int tff($t$, type)
length(nil) = 0 tff(l, axiom)
$\forall h$: $int, t: list: length(cons(h, t)) = $sum(1, length($t$)) tff($l_1$, axiom)
count: ($int × list) → $int tff($t_2$, type)
$\forall k$: $int: count($k$, nil) = 0 tff($a$, axiom)
$\forall k$: $int, h: $int, t: list, n: $int: (count($k$, cons($h, t$)) = count($k, t$) $\Leftarrow$ $k \neq h$) tff(a_3, axiom)
$\forall k$: $int, h: $int, t: list, n: $int: (count($k$, cons($h, t$)) = $sum(count($k, t$), 1) $\Leftarrow$ $k = h$) tff(a_4, axiom)
append: (list × list) → list tff(t_5, type)
$\forall l$: list: append(nil, l) = l tff(l_6, axiom)
$\forall i$: $int, k: list, l: list: append(cons(i, k), l) = cons(i, append(k, l)) tff(l_7, axiom)
$\forall n$: $int, l: list: (in(n, l) $\iff$ $greater(count($n, l$), 0)) tff($a_8$, axiom)
length(cons(1, cons(2, nil))) = 2 tff(c, conjecture)

DAT083=1.p Lists by functions problem 5
list: $tType tff(list_type, type)
nil: list tff(nil_type, type)
cons: ($int × list) → list tff(cons_type, type)
head: list → $int tff(head_type, type)
tail: list → list tff(tail_type, type)
$\forall k$: $int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: $int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)

$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l)))$ tff(l_3, axiom)
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k, l) \neq \text{nil}$ tff(l_4, axiom)
in: $(\$\text{int} \times \text{list}) \rightarrow \$\text{o}$ tff(in, type)
$\forall x\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(x, l) \iff (\exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and in}(x, t))))$ t
inRange: $(\$\text{int} \times \text{list}) \rightarrow \$\text{o}$ tff(inRange_type, type)
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n, l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k, t) \text{ and } \$\text{lesseq}(0, k) \text{ and } \$\text{less}(k, n) \text{ and inRange}(n,$
length: $\text{list} \rightarrow \$\text{int}$ tff(t, type)
$\text{length}(\text{nil}) = 0$ tff(l, axiom)
$\forall h\colon \$\text{int}, t\colon \text{list}\colon \text{length}(\text{cons}(h, t)) = \$\text{sum}(1, \text{length}(t))$ tff(l_1, axiom)
count: $(\$\text{int} \times \text{list}) \rightarrow \$\text{int}$ tff(t_2, type)
$\forall k\colon \$\text{int}\colon \text{count}(k, \text{nil}) = 0$ tff(a, axiom)
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \text{count}(k, t) \Leftarrow k \neq h)$ tff(a_3, axiom)
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \$\text{sum}(\text{count}(k, t), 1) \Leftarrow k = h)$ tff(a_4, axiom)
append: $(\text{list} \times \text{list}) \rightarrow \text{list}$ tff(t_5, type)
$\forall l\colon \text{list}\colon \text{append}(\text{nil}, l) = l$ tff(l_6, axiom)
$\forall i\colon \$\text{int}, k\colon \text{list}, l\colon \text{list}\colon \text{append}(\text{cons}(i, k), l) = \text{cons}(i, \text{append}(k, l))$ tff(l_7, axiom)
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(n, l) \iff \$\text{greater}(\text{count}(n, l), 0))$ tff(a_8, axiom)
$\text{length}(\text{cons}(1, \text{cons}(2, \text{nil}))) \neq 3$ tff(c, conjecture)

DAT084=1.p Lists by functions problem 6
list: $tType tff(list_type, type)
nil: list tff(nil_type, type)
cons: $(\$\text{int} \times \text{list}) \rightarrow \text{list}$ tff(cons_type, type)
head: $\text{list} \rightarrow \$\text{int}$ tff(head_type, type)
tail: $\text{list} \rightarrow \text{list}$ tff(tail_type, type)
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k, l)) = k$ tff(l_1, axiom)
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k, l)) = l$ tff(l_2, axiom)
$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l)))$ tff(l_3, axiom)
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k, l) \neq \text{nil}$ tff(l_4, axiom)
in: $(\$\text{int} \times \text{list}) \rightarrow \$\text{o}$ tff(in, type)
$\forall x\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(x, l) \iff (\exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and in}(x, t))))$ t
inRange: $(\$\text{int} \times \text{list}) \rightarrow \$\text{o}$ tff(inRange_type, type)
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n, l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k, t) \text{ and } \$\text{lesseq}(0, k) \text{ and } \$\text{less}(k, n) \text{ and inRange}(n,$
length: $\text{list} \rightarrow \$\text{int}$ tff(t, type)
$\text{length}(\text{nil}) = 0$ tff(l, axiom)
$\forall h\colon \$\text{int}, t\colon \text{list}\colon \text{length}(\text{cons}(h, t)) = \$\text{sum}(1, \text{length}(t))$ tff(l_1, axiom)
count: $(\$\text{int} \times \text{list}) \rightarrow \$\text{int}$ tff(t_2, type)
$\forall k\colon \$\text{int}\colon \text{count}(k, \text{nil}) = 0$ tff(a, axiom)
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \text{count}(k, t) \Leftarrow k \neq h)$ tff(a_3, axiom)
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \$\text{sum}(\text{count}(k, t), 1) \Leftarrow k = h)$ tff(a_4, axiom)
append: $(\text{list} \times \text{list}) \rightarrow \text{list}$ tff(t_5, type)
$\forall l\colon \text{list}\colon \text{append}(\text{nil}, l) = l$ tff(l_6, axiom)
$\forall i\colon \$\text{int}, k\colon \text{list}, l\colon \text{list}\colon \text{append}(\text{cons}(i, k), l) = \text{cons}(i, \text{append}(k, l))$ tff(l_7, axiom)
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(n, l) \iff \$\text{greater}(\text{count}(n, l), 0))$ tff(a_8, axiom)
$\neg \exists l\colon \text{list}\colon \text{length}(l) = \text{length}(\text{cons}(1, l))$ tff(c, conjecture)

DAT085=1.p Lists by functions problem 7
list: $tType tff(list_type, type)
nil: list tff(nil_type, type)
cons: $(\$\text{int} \times \text{list}) \rightarrow \text{list}$ tff(cons_type, type)
head: $\text{list} \rightarrow \$\text{int}$ tff(head_type, type)
tail: $\text{list} \rightarrow \text{list}$ tff(tail_type, type)
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k, l)) = k$ tff(l_1, axiom)
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k, l)) = l$ tff(l_2, axiom)
$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l)))$ tff(l_3, axiom)
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k, l) \neq \text{nil}$ tff(l_4, axiom)
in: $(\$\text{int} \times \text{list}) \rightarrow \$\text{o}$ tff(in, type)
$\forall x\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(x, l) \iff (\exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and in}(x, t))))$ t
inRange: $(\$\text{int} \times \text{list}) \rightarrow \$\text{o}$ tff(inRange_type, type)
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n, l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k, t) \text{ and } \$\text{lesseq}(0, k) \text{ and } \$\text{less}(k, n) \text{ and inRange}(n,$

$\text{length}\colon \text{list} \to \$\text{int} \qquad \text{tff}(t, \text{type})$
$\text{length}(\text{nil}) = 0 \qquad \text{tff}(l, \text{axiom})$
$\forall h\colon \$\text{int}, t\colon \text{list}\colon \text{length}(\text{cons}(h, t)) = \$\text{sum}(1, \text{length}(t)) \qquad \text{tff}(l_1, \text{axiom})$
$\text{count}\colon (\$\text{int} \times \text{list}) \to \$\text{int} \qquad \text{tff}(t_2, \text{type})$
$\forall k\colon \$\text{int}\colon \text{count}(k, \text{nil}) = 0 \qquad \text{tff}(a, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \text{count}(k, t) \Leftarrow k \neq h) \qquad \text{tff}(a_3, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \$\text{sum}(\text{count}(k, t), 1) \Leftarrow k = h) \qquad \text{tff}(a_4, \text{axiom})$
$\text{append}\colon (\text{list} \times \text{list}) \to \text{list} \qquad \text{tff}(t_5, \text{type})$
$\forall l\colon \text{list}\colon \text{append}(\text{nil}, l) = l \qquad \text{tff}(l_6, \text{axiom})$
$\forall i\colon \$\text{int}, k\colon \text{list}, l\colon \text{list}\colon \text{append}(\text{cons}(i, k), l) = \text{cons}(i, \text{append}(k, l)) \qquad \text{tff}(l_7, \text{axiom})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(n, l) \iff \$\text{greater}(\text{count}(n, l), 0)) \qquad \text{tff}(a_8, \text{axiom})$
$\neg \forall l_1\colon \text{list}, l_2\colon \text{list}\colon (\text{length}(l_1) = \text{length}(l_2) \Rightarrow l_1 = l_2) \qquad \text{tff}(c, \text{conjecture})$

DAT086=1.p Lists by functions problem 8
$\text{list}\colon \$\text{tType} \qquad \text{tff}(\text{list\_type}, \text{type})$
$\text{nil}\colon \text{list} \qquad \text{tff}(\text{nil\_type}, \text{type})$
$\text{cons}\colon (\$\text{int} \times \text{list}) \to \text{list} \qquad \text{tff}(\text{cons\_type}, \text{type})$
$\text{head}\colon \text{list} \to \$\text{int} \qquad \text{tff}(\text{head\_type}, \text{type})$
$\text{tail}\colon \text{list} \to \text{list} \qquad \text{tff}(\text{tail\_type}, \text{type})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k, l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k, l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k, l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{in}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{in}, \text{type})$
$\forall x\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(x, l) \iff (\exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and in}(x, t)))) \qquad \text{t}$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n, l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k, t) \text{ and } \$\text{lesseq}(0, k) \text{ and } \$\text{less}(k, n) \text{ and inRange}(n,$
$\text{length}\colon \text{list} \to \$\text{int} \qquad \text{tff}(t, \text{type})$
$\text{length}(\text{nil}) = 0 \qquad \text{tff}(l, \text{axiom})$
$\forall h\colon \$\text{int}, t\colon \text{list}\colon \text{length}(\text{cons}(h, t)) = \$\text{sum}(1, \text{length}(t)) \qquad \text{tff}(l_1, \text{axiom})$
$\text{count}\colon (\$\text{int} \times \text{list}) \to \$\text{int} \qquad \text{tff}(t_2, \text{type})$
$\forall k\colon \$\text{int}\colon \text{count}(k, \text{nil}) = 0 \qquad \text{tff}(a, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \text{count}(k, t) \Leftarrow k \neq h) \qquad \text{tff}(a_3, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \$\text{sum}(\text{count}(k, t), 1) \Leftarrow k = h) \qquad \text{tff}(a_4, \text{axiom})$
$\text{append}\colon (\text{list} \times \text{list}) \to \text{list} \qquad \text{tff}(t_5, \text{type})$
$\forall l\colon \text{list}\colon \text{append}(\text{nil}, l) = l \qquad \text{tff}(l_6, \text{axiom})$
$\forall i\colon \$\text{int}, k\colon \text{list}, l\colon \text{list}\colon \text{append}(\text{cons}(i, k), l) = \text{cons}(i, \text{append}(k, l)) \qquad \text{tff}(l_7, \text{axiom})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(n, l) \iff \$\text{greater}(\text{count}(n, l), 0)) \qquad \text{tff}(a_8, \text{axiom})$
$\neg \forall l\colon \text{list}, n\colon \$\text{int}\colon ((\$\text{greatereq}(n, 3) \text{ and } \$\text{greatereq}(\text{length}(l), 4)) \Rightarrow \text{inRange}(n, l)) \qquad \text{tff}(a_9, \text{conjecture})$

DAT087=1.p Lists by functions problem 9
$\text{list}\colon \$\text{tType} \qquad \text{tff}(\text{list\_type}, \text{type})$
$\text{nil}\colon \text{list} \qquad \text{tff}(\text{nil\_type}, \text{type})$
$\text{cons}\colon (\$\text{int} \times \text{list}) \to \text{list} \qquad \text{tff}(\text{cons\_type}, \text{type})$
$\text{head}\colon \text{list} \to \$\text{int} \qquad \text{tff}(\text{head\_type}, \text{type})$
$\text{tail}\colon \text{list} \to \text{list} \qquad \text{tff}(\text{tail\_type}, \text{type})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k, l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k, l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k, l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{in}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{in}, \text{type})$
$\forall x\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(x, l) \iff (\exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h, t) \text{ and in}(x, t)))) \qquad \text{t}$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n, l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k, t) \text{ and } \$\text{lesseq}(0, k) \text{ and } \$\text{less}(k, n) \text{ and inRange}(n,$
$\text{length}\colon \text{list} \to \$\text{int} \qquad \text{tff}(t, \text{type})$
$\text{length}(\text{nil}) = 0 \qquad \text{tff}(l, \text{axiom})$
$\forall h\colon \$\text{int}, t\colon \text{list}\colon \text{length}(\text{cons}(h, t)) = \$\text{sum}(1, \text{length}(t)) \qquad \text{tff}(l_1, \text{axiom})$
$\text{count}\colon (\$\text{int} \times \text{list}) \to \$\text{int} \qquad \text{tff}(t_2, \text{type})$
$\forall k\colon \$\text{int}\colon \text{count}(k, \text{nil}) = 0 \qquad \text{tff}(a, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h, t)) = \text{count}(k, t) \Leftarrow k \neq h) \qquad \text{tff}(a_3, \text{axiom})$

$\forall k\colon \$\mathrm{int}, h\colon \$\mathrm{int}, t\colon \mathrm{list}, n\colon \$\mathrm{int}\colon (\mathrm{count}(k, \mathrm{cons}(h, t)) = \$\mathrm{sum}(\mathrm{count}(k, t), 1) \Leftarrow k = h) \qquad \mathrm{tff}(a_4, \mathrm{axiom})$
$\mathrm{append}\colon (\mathrm{list} \times \mathrm{list}) \rightarrow \mathrm{list} \qquad \mathrm{tff}(t_5, \mathrm{type})$
$\forall l\colon \mathrm{list}\colon \mathrm{append}(\mathrm{nil}, l) = l \qquad \mathrm{tff}(l_6, \mathrm{axiom})$
$\forall i\colon \$\mathrm{int}, k\colon \mathrm{list}, l\colon \mathrm{list}\colon \mathrm{append}(\mathrm{cons}(i, k), l) = \mathrm{cons}(i, \mathrm{append}(k, l)) \qquad \mathrm{tff}(l_7, \mathrm{axiom})$
$\forall n\colon \$\mathrm{int}, l\colon \mathrm{list}\colon (\mathrm{in}(n, l) \iff \$\mathrm{greater}(\mathrm{count}(n, l), 0)) \qquad \mathrm{tff}(a_8, \mathrm{axiom})$
$\neg \forall n\colon \$\mathrm{int}, l\colon \mathrm{list}\colon \mathrm{count}(n, l) = \mathrm{count}(n, \mathrm{cons}(1, l)) \qquad \mathrm{tff}(c, \mathrm{conjecture})$

DAT088=1.p Lists by functions problem 10
$\mathrm{list}\colon \$\mathrm{tType} \qquad \mathrm{tff}(\mathrm{list\_type}, \mathrm{type})$
$\mathrm{nil}\colon \mathrm{list} \qquad \mathrm{tff}(\mathrm{nil\_type}, \mathrm{type})$
$\mathrm{cons}\colon (\$\mathrm{int} \times \mathrm{list}) \rightarrow \mathrm{list} \qquad \mathrm{tff}(\mathrm{cons\_type}, \mathrm{type})$
$\mathrm{head}\colon \mathrm{list} \rightarrow \$\mathrm{int} \qquad \mathrm{tff}(\mathrm{head\_type}, \mathrm{type})$
$\mathrm{tail}\colon \mathrm{list} \rightarrow \mathrm{list} \qquad \mathrm{tff}(\mathrm{tail\_type}, \mathrm{type})$
$\forall k\colon \$\mathrm{int}, l\colon \mathrm{list}\colon \mathrm{head}(\mathrm{cons}(k, l)) = k \qquad \mathrm{tff}(l_1, \mathrm{axiom})$
$\forall k\colon \$\mathrm{int}, l\colon \mathrm{list}\colon \mathrm{tail}(\mathrm{cons}(k, l)) = l \qquad \mathrm{tff}(l_2, \mathrm{axiom})$
$\forall l\colon \mathrm{list}\colon (l = \mathrm{nil} \text{ or } l = \mathrm{cons}(\mathrm{head}(l), \mathrm{tail}(l))) \qquad \mathrm{tff}(l_3, \mathrm{axiom})$
$\forall k\colon \$\mathrm{int}, l\colon \mathrm{list}\colon \mathrm{cons}(k, l) \neq \mathrm{nil} \qquad \mathrm{tff}(l_4, \mathrm{axiom})$
$\mathrm{in}\colon (\$\mathrm{int} \times \mathrm{list}) \rightarrow \$\mathrm{o} \qquad \mathrm{tff}(\mathrm{in}, \mathrm{type})$
$\forall x\colon \$\mathrm{int}, l\colon \mathrm{list}\colon (\mathrm{in}(x, l) \iff (\exists h\colon \$\mathrm{int}, t\colon \mathrm{list}\colon (l = \mathrm{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\colon \$\mathrm{int}, t\colon \mathrm{list}\colon (l = \mathrm{cons}(h, t) \text{ and } \mathrm{in}(x, t)))) \qquad \mathrm{t}$
$\mathrm{inRange}\colon (\$\mathrm{int} \times \mathrm{list}) \rightarrow \$\mathrm{o} \qquad \mathrm{tff}(\mathrm{inRange\_type}, \mathrm{type})$
$\forall n\colon \$\mathrm{int}, l\colon \mathrm{list}\colon (\mathrm{inRange}(n, l) \iff (l = \mathrm{nil} \text{ or } \exists k\colon \$\mathrm{int}, t\colon \mathrm{list}\colon (l = \mathrm{cons}(k, t) \text{ and } \$\mathrm{lesseq}(0, k) \text{ and } \$\mathrm{less}(k, n) \text{ and } \mathrm{inRange}(n,$
$\mathrm{length}\colon \mathrm{list} \rightarrow \$\mathrm{int} \qquad \mathrm{tff}(t, \mathrm{type})$
$\mathrm{length}(\mathrm{nil}) = 0 \qquad \mathrm{tff}(l, \mathrm{axiom})$
$\forall h\colon \$\mathrm{int}, t\colon \mathrm{list}\colon \mathrm{length}(\mathrm{cons}(h, t)) = \$\mathrm{sum}(1, \mathrm{length}(t)) \qquad \mathrm{tff}(l_1, \mathrm{axiom})$
$\mathrm{count}\colon (\$\mathrm{int} \times \mathrm{list}) \rightarrow \$\mathrm{int} \qquad \mathrm{tff}(t_2, \mathrm{type})$
$\forall k\colon \$\mathrm{int}\colon \mathrm{count}(k, \mathrm{nil}) = 0 \qquad \mathrm{tff}(a, \mathrm{axiom})$
$\forall k\colon \$\mathrm{int}, h\colon \$\mathrm{int}, t\colon \mathrm{list}, n\colon \$\mathrm{int}\colon (\mathrm{count}(k, \mathrm{cons}(h, t)) = \mathrm{count}(k, t) \Leftarrow k \neq h) \qquad \mathrm{tff}(a_3, \mathrm{axiom})$
$\forall k\colon \$\mathrm{int}, h\colon \$\mathrm{int}, t\colon \mathrm{list}, n\colon \$\mathrm{int}\colon (\mathrm{count}(k, \mathrm{cons}(h, t)) = \$\mathrm{sum}(\mathrm{count}(k, t), 1) \Leftarrow k = h) \qquad \mathrm{tff}(a_4, \mathrm{axiom})$
$\mathrm{append}\colon (\mathrm{list} \times \mathrm{list}) \rightarrow \mathrm{list} \qquad \mathrm{tff}(t_5, \mathrm{type})$
$\forall l\colon \mathrm{list}\colon \mathrm{append}(\mathrm{nil}, l) = l \qquad \mathrm{tff}(l_6, \mathrm{axiom})$
$\forall i\colon \$\mathrm{int}, k\colon \mathrm{list}, l\colon \mathrm{list}\colon \mathrm{append}(\mathrm{cons}(i, k), l) = \mathrm{cons}(i, \mathrm{append}(k, l)) \qquad \mathrm{tff}(l_7, \mathrm{axiom})$
$\forall n\colon \$\mathrm{int}, l\colon \mathrm{list}\colon (\mathrm{in}(n, l) \iff \$\mathrm{greater}(\mathrm{count}(n, l), 0)) \qquad \mathrm{tff}(a_8, \mathrm{axiom})$
$\neg \forall n\colon \$\mathrm{int}, l\colon \mathrm{list}\colon (l \neq \mathrm{nil} \Rightarrow \mathrm{count}(n, l) = \mathrm{count}(n, \mathrm{tail}(l))) \qquad \mathrm{tff}(c, \mathrm{conjecture})$

DAT089=1.p Lists by functions problem 11
$\mathrm{list}\colon \$\mathrm{tType} \qquad \mathrm{tff}(\mathrm{list\_type}, \mathrm{type})$
$\mathrm{nil}\colon \mathrm{list} \qquad \mathrm{tff}(\mathrm{nil\_type}, \mathrm{type})$
$\mathrm{cons}\colon (\$\mathrm{int} \times \mathrm{list}) \rightarrow \mathrm{list} \qquad \mathrm{tff}(\mathrm{cons\_type}, \mathrm{type})$
$\mathrm{head}\colon \mathrm{list} \rightarrow \$\mathrm{int} \qquad \mathrm{tff}(\mathrm{head\_type}, \mathrm{type})$
$\mathrm{tail}\colon \mathrm{list} \rightarrow \mathrm{list} \qquad \mathrm{tff}(\mathrm{tail\_type}, \mathrm{type})$
$\forall k\colon \$\mathrm{int}, l\colon \mathrm{list}\colon \mathrm{head}(\mathrm{cons}(k, l)) = k \qquad \mathrm{tff}(l_1, \mathrm{axiom})$
$\forall k\colon \$\mathrm{int}, l\colon \mathrm{list}\colon \mathrm{tail}(\mathrm{cons}(k, l)) = l \qquad \mathrm{tff}(l_2, \mathrm{axiom})$
$\forall l\colon \mathrm{list}\colon (l = \mathrm{nil} \text{ or } l = \mathrm{cons}(\mathrm{head}(l), \mathrm{tail}(l))) \qquad \mathrm{tff}(l_3, \mathrm{axiom})$
$\forall k\colon \$\mathrm{int}, l\colon \mathrm{list}\colon \mathrm{cons}(k, l) \neq \mathrm{nil} \qquad \mathrm{tff}(l_4, \mathrm{axiom})$
$\mathrm{in}\colon (\$\mathrm{int} \times \mathrm{list}) \rightarrow \$\mathrm{o} \qquad \mathrm{tff}(\mathrm{in}, \mathrm{type})$
$\forall x\colon \$\mathrm{int}, l\colon \mathrm{list}\colon (\mathrm{in}(x, l) \iff (\exists h\colon \$\mathrm{int}, t\colon \mathrm{list}\colon (l = \mathrm{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\colon \$\mathrm{int}, t\colon \mathrm{list}\colon (l = \mathrm{cons}(h, t) \text{ and } \mathrm{in}(x, t)))) \qquad \mathrm{t}$
$\mathrm{inRange}\colon (\$\mathrm{int} \times \mathrm{list}) \rightarrow \$\mathrm{o} \qquad \mathrm{tff}(\mathrm{inRange\_type}, \mathrm{type})$
$\forall n\colon \$\mathrm{int}, l\colon \mathrm{list}\colon (\mathrm{inRange}(n, l) \iff (l = \mathrm{nil} \text{ or } \exists k\colon \$\mathrm{int}, t\colon \mathrm{list}\colon (l = \mathrm{cons}(k, t) \text{ and } \$\mathrm{lesseq}(0, k) \text{ and } \$\mathrm{less}(k, n) \text{ and } \mathrm{inRange}(n,$
$\mathrm{length}\colon \mathrm{list} \rightarrow \$\mathrm{int} \qquad \mathrm{tff}(t, \mathrm{type})$
$\mathrm{length}(\mathrm{nil}) = 0 \qquad \mathrm{tff}(l, \mathrm{axiom})$
$\forall h\colon \$\mathrm{int}, t\colon \mathrm{list}\colon \mathrm{length}(\mathrm{cons}(h, t)) = \$\mathrm{sum}(1, \mathrm{length}(t)) \qquad \mathrm{tff}(l_1, \mathrm{axiom})$
$\mathrm{count}\colon (\$\mathrm{int} \times \mathrm{list}) \rightarrow \$\mathrm{int} \qquad \mathrm{tff}(t_2, \mathrm{type})$
$\forall k\colon \$\mathrm{int}\colon \mathrm{count}(k, \mathrm{nil}) = 0 \qquad \mathrm{tff}(a, \mathrm{axiom})$
$\forall k\colon \$\mathrm{int}, h\colon \$\mathrm{int}, t\colon \mathrm{list}, n\colon \$\mathrm{int}\colon (\mathrm{count}(k, \mathrm{cons}(h, t)) = \mathrm{count}(k, t) \Leftarrow k \neq h) \qquad \mathrm{tff}(a_3, \mathrm{axiom})$
$\forall k\colon \$\mathrm{int}, h\colon \$\mathrm{int}, t\colon \mathrm{list}, n\colon \$\mathrm{int}\colon (\mathrm{count}(k, \mathrm{cons}(h, t)) = \$\mathrm{sum}(\mathrm{count}(k, t), 1) \Leftarrow k = h) \qquad \mathrm{tff}(a_4, \mathrm{axiom})$
$\mathrm{append}\colon (\mathrm{list} \times \mathrm{list}) \rightarrow \mathrm{list} \qquad \mathrm{tff}(t_5, \mathrm{type})$
$\forall l\colon \mathrm{list}\colon \mathrm{append}(\mathrm{nil}, l) = l \qquad \mathrm{tff}(l_6, \mathrm{axiom})$
$\forall i\colon \$\mathrm{int}, k\colon \mathrm{list}, l\colon \mathrm{list}\colon \mathrm{append}(\mathrm{cons}(i, k), l) = \mathrm{cons}(i, \mathrm{append}(k, l)) \qquad \mathrm{tff}(l_7, \mathrm{axiom})$
$\forall n\colon \$\mathrm{int}, l\colon \mathrm{list}\colon (\mathrm{in}(n, l) \iff \$\mathrm{greater}(\mathrm{count}(n, l), 0)) \qquad \mathrm{tff}(a_8, \mathrm{axiom})$
$\neg \forall n\colon \$\mathrm{int}, l\colon \mathrm{list}\colon \$\mathrm{greatereq}(\mathrm{count}(n, l), \mathrm{length}(l)) \qquad \mathrm{tff}(c, \mathrm{conjecture})$

DAT090=1.p Lists by functions problem 12
list: $tType   tff(list_type, type)
nil: list   tff(nil_type, type)
cons: ($int × list) → list   tff(cons_type, type)
head: list → $int   tff(head_type, type)
tail: list → list   tff(tail_type, type)
$\forall k$: $int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: $int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l)))   tff(l_3, axiom)
$\forall k$: $int, l: list: cons(k, l) $\neq$ nil   tff(l_4, axiom)
in: ($int × list) → $o   tff(in, type)
$\forall x$: $int, l: list: (in(x, l) $\iff$ ($\exists h$: $int, t: list: (l = cons(h, t) and $x = h$) or $\exists h$: $int, t: list: (l = cons(h, t) and in(x, t))))   t
inRange: ($int × list) → $o   tff(inRange_type, type)
$\forall n$: $int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: $int, t: list: (l = cons(k, t) and $lesseq(0, k) and $less($k, n$) and inRange($n$,
length: list → $int   tff($t$, type)
length(nil) = 0   tff(l, axiom)
$\forall h$: $int, t: list: length(cons(h, t)) = $sum(1, length($t$))   tff($l_1$, axiom)
count: ($int × list) → $int   tff($t_2$, type)
$\forall k$: $int: count($k$, nil) = 0   tff($a$, axiom)
$\forall k$: $int, h: $int, t: list, n: $int: (count($k$, cons($h, t$)) = count($k, t$) $\Leftarrow$ $k \neq h$)   tff(a_3, axiom)
$\forall k$: $int, h: $int, t: list, n: $int: (count($k$, cons($h, t$)) = $sum(count($k, t$), 1) $\Leftarrow$ $k = h$)   tff(a_4, axiom)
append: (list × list) → list   tff(t_5, type)
$\forall l$: list: append(nil, l) = l tff(l_6, axiom)
$\forall i$: $int, k: list, l: list: append(cons(i, k), l) = cons(i, append(k, l))   tff(l_7, axiom)
$\forall n$: $int, l: list: (in(n, l) $\iff$ $greater(count($n, l$), 0))   tff($a_8$, axiom)
$\neg\exists n$: $int, l: list: $greater(count($n, l$), length($l$))   tff($c$, conjecture)

DAT091=1.p Lists by functions problem 13
list: $tType   tff(list_type, type)
nil: list   tff(nil_type, type)
cons: ($int × list) → list   tff(cons_type, type)
head: list → $int   tff(head_type, type)
tail: list → list   tff(tail_type, type)
$\forall k$: $int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: $int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l)))   tff(l_3, axiom)
$\forall k$: $int, l: list: cons(k, l) $\neq$ nil   tff(l_4, axiom)
in: ($int × list) → $o   tff(in, type)
$\forall x$: $int, l: list: (in(x, l) $\iff$ ($\exists h$: $int, t: list: (l = cons(h, t) and $x = h$) or $\exists h$: $int, t: list: (l = cons(h, t) and in(x, t))))   t
inRange: ($int × list) → $o   tff(inRange_type, type)
$\forall n$: $int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: $int, t: list: (l = cons(k, t) and $lesseq(0, k) and $less($k, n$) and inRange($n$,
length: list → $int   tff($t$, type)
length(nil) = 0   tff(l, axiom)
$\forall h$: $int, t: list: length(cons(h, t)) = $sum(1, length($t$))   tff($l_1$, axiom)
count: ($int × list) → $int   tff($t_2$, type)
$\forall k$: $int: count($k$, nil) = 0   tff($a$, axiom)
$\forall k$: $int, h: $int, t: list, n: $int: (count($k$, cons($h, t$)) = count($k, t$) $\Leftarrow$ $k \neq h$)   tff(a_3, axiom)
$\forall k$: $int, h: $int, t: list, n: $int: (count($k$, cons($h, t$)) = $sum(count($k, t$), 1) $\Leftarrow$ $k = h$)   tff(a_4, axiom)
append: (list × list) → list   tff(t_5, type)
$\forall l$: list: append(nil, l) = l tff(l_6, axiom)
$\forall i$: $int, k: list, l: list: append(cons(i, k), l) = cons(i, append(k, l))   tff(l_7, axiom)
$\forall n$: $int, l: list: (in(n, l) $\iff$ $greater(count($n, l$), 0))   tff($a_8$, axiom)
$\neg\forall l_1$: list, l_2: list: ($l_1 \neq l_2$ $\Rightarrow$ $\forall n$: $int: count($n, l_1$) $\neq$ count(n, l_2))   tff(c, conjecture)

DAT092=1.p Lists by functions problem 14
list: $tType   tff(list_type, type)
nil: list   tff(nil_type, type)
cons: ($int × list) → list   tff(cons_type, type)
head: list → $int   tff(head_type, type)
tail: list → list   tff(tail_type, type)

$\forall k\text{: \$int}, l\text{: list: head}(\text{cons}(k, l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\text{: \$int}, l\text{: list: tail}(\text{cons}(k, l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\text{: list: } (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\text{: \$int}, l\text{: list: cons}(k, l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{in: (\$int} \times \text{list)} \rightarrow \text{\$o} \qquad \text{tff(in, type)}$
$\forall x\text{: \$int}, l\text{: list: } (\text{in}(x, l) \iff (\exists h\text{: \$int}, t\text{: list: } (l = \text{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\text{: \$int}, t\text{: list: } (l = \text{cons}(h, t) \text{ and in}(x, t)))) \qquad \text{t}$
$\text{inRange: (\$int} \times \text{list)} \rightarrow \text{\$o} \qquad \text{tff(inRange\_type, type)}$
$\forall n\text{: \$int}, l\text{: list: } (\text{inRange}(n, l) \iff (l = \text{nil or } \exists k\text{: \$int}, t\text{: list: } (l = \text{cons}(k, t) \text{ and \$lesseq}(0, k) \text{ and \$less}(k, n) \text{ and inRange}(n,$
$\text{length: list} \rightarrow \text{\$int} \qquad \text{tff}(t, \text{type})$
$\text{length(nil)} = 0 \qquad \text{tff}(l, \text{axiom})$
$\forall h\text{: \$int}, t\text{: list: length}(\text{cons}(h, t)) = \text{\$sum}(1, \text{length}(t)) \qquad \text{tff}(l_1, \text{axiom})$
$\text{count: (\$int} \times \text{list)} \rightarrow \text{\$int} \qquad \text{tff}(t_2, \text{type})$
$\forall k\text{: \$int: count}(k, \text{nil}) = 0 \qquad \text{tff}(a, \text{axiom})$
$\forall k\text{: \$int}, h\text{: \$int}, t\text{: list}, n\text{: \$int: } (\text{count}(k, \text{cons}(h, t)) = \text{count}(k, t) \Leftarrow k \neq h) \qquad \text{tff}(a_3, \text{axiom})$
$\forall k\text{: \$int}, h\text{: \$int}, t\text{: list}, n\text{: \$int: } (\text{count}(k, \text{cons}(h, t)) = \text{\$sum}(\text{count}(k, t), 1) \Leftarrow k = h) \qquad \text{tff}(a_4, \text{axiom})$
$\text{append: (list} \times \text{list)} \rightarrow \text{list} \qquad \text{tff}(t_5, \text{type})$
$\forall l\text{: list: append}(\text{nil}, l) = l \qquad \text{tff}(l_6, \text{axiom})$
$\forall i\text{: \$int}, k\text{: list}, l\text{: list: append}(\text{cons}(i, k), l) = \text{cons}(i, \text{append}(k, l)) \qquad \text{tff}(l_7, \text{axiom})$
$\forall n\text{: \$int}, l\text{: list: } (\text{in}(n, l) \iff \text{\$greater}(\text{count}(n, l), 0)) \qquad \text{tff}(a_8, \text{axiom})$
$\neg \forall k\text{: list}, l\text{: list: length}(\text{append}(k, l)) = \text{length}(k) \qquad \text{tff}(c, \text{conjecture})$

DAT093=1.p Lists by functions problem 15
$\text{list: \$tType} \qquad \text{tff(list\_type, type)}$
$\text{nil: list} \qquad \text{tff(nil\_type, type)}$
$\text{cons: (\$int} \times \text{list)} \rightarrow \text{list} \qquad \text{tff(cons\_type, type)}$
$\text{head: list} \rightarrow \text{\$int} \qquad \text{tff(head\_type, type)}$
$\text{tail: list} \rightarrow \text{list} \qquad \text{tff(tail\_type, type)}$
$\forall k\text{: \$int}, l\text{: list: head}(\text{cons}(k, l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\text{: \$int}, l\text{: list: tail}(\text{cons}(k, l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\text{: list: } (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\text{: \$int}, l\text{: list: cons}(k, l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{in: (\$int} \times \text{list)} \rightarrow \text{\$o} \qquad \text{tff(in, type)}$
$\forall x\text{: \$int}, l\text{: list: } (\text{in}(x, l) \iff (\exists h\text{: \$int}, t\text{: list: } (l = \text{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\text{: \$int}, t\text{: list: } (l = \text{cons}(h, t) \text{ and in}(x, t)))) \qquad \text{t}$
$\text{inRange: (\$int} \times \text{list)} \rightarrow \text{\$o} \qquad \text{tff(inRange\_type, type)}$
$\forall n\text{: \$int}, l\text{: list: } (\text{inRange}(n, l) \iff (l = \text{nil or } \exists k\text{: \$int}, t\text{: list: } (l = \text{cons}(k, t) \text{ and \$lesseq}(0, k) \text{ and \$less}(k, n) \text{ and inRange}(n,$
$\text{length: list} \rightarrow \text{\$int} \qquad \text{tff}(t, \text{type})$
$\text{length(nil)} = 0 \qquad \text{tff}(l, \text{axiom})$
$\forall h\text{: \$int}, t\text{: list: length}(\text{cons}(h, t)) = \text{\$sum}(1, \text{length}(t)) \qquad \text{tff}(l_1, \text{axiom})$
$\text{count: (\$int} \times \text{list)} \rightarrow \text{\$int} \qquad \text{tff}(t_2, \text{type})$
$\forall k\text{: \$int: count}(k, \text{nil}) = 0 \qquad \text{tff}(a, \text{axiom})$
$\forall k\text{: \$int}, h\text{: \$int}, t\text{: list}, n\text{: \$int: } (\text{count}(k, \text{cons}(h, t)) = \text{count}(k, t) \Leftarrow k \neq h) \qquad \text{tff}(a_3, \text{axiom})$
$\forall k\text{: \$int}, h\text{: \$int}, t\text{: list}, n\text{: \$int: } (\text{count}(k, \text{cons}(h, t)) = \text{\$sum}(\text{count}(k, t), 1) \Leftarrow k = h) \qquad \text{tff}(a_4, \text{axiom})$
$\text{append: (list} \times \text{list)} \rightarrow \text{list} \qquad \text{tff}(t_5, \text{type})$
$\forall l\text{: list: append}(\text{nil}, l) = l \qquad \text{tff}(l_6, \text{axiom})$
$\forall i\text{: \$int}, k\text{: list}, l\text{: list: append}(\text{cons}(i, k), l) = \text{cons}(i, \text{append}(k, l)) \qquad \text{tff}(l_7, \text{axiom})$
$\forall n\text{: \$int}, l\text{: list: } (\text{in}(n, l) \iff \text{\$greater}(\text{count}(n, l), 0)) \qquad \text{tff}(a_8, \text{axiom})$
$\neg \forall k\text{: list}, l\text{: list: length}(\text{append}(k, l)) = \text{\$sum}(\text{\$sum}(\text{length}(k), \text{length}(l)), 1) \qquad \text{tff}(c, \text{conjecture})$

DAT094=1.p Lists by functions problem 16
$\text{list: \$tType} \qquad \text{tff(list\_type, type)}$
$\text{nil: list} \qquad \text{tff(nil\_type, type)}$
$\text{cons: (\$int} \times \text{list)} \rightarrow \text{list} \qquad \text{tff(cons\_type, type)}$
$\text{head: list} \rightarrow \text{\$int} \qquad \text{tff(head\_type, type)}$
$\text{tail: list} \rightarrow \text{list} \qquad \text{tff(tail\_type, type)}$
$\forall k\text{: \$int}, l\text{: list: head}(\text{cons}(k, l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\text{: \$int}, l\text{: list: tail}(\text{cons}(k, l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\text{: list: } (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\text{: \$int}, l\text{: list: cons}(k, l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{in: (\$int} \times \text{list)} \rightarrow \text{\$o} \qquad \text{tff(in, type)}$
$\forall x\text{: \$int}, l\text{: list: } (\text{in}(x, l) \iff (\exists h\text{: \$int}, t\text{: list: } (l = \text{cons}(h, t) \text{ and } x = h) \text{ or } \exists h\text{: \$int}, t\text{: list: } (l = \text{cons}(h, t) \text{ and in}(x, t)))) \qquad \text{t}$

$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \iff (l = \text{nil} \text{ or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k,t) \text{ and } \$\text{lesseq}(0,k) \text{ and } \$\text{less}(k,n) \text{ and } \text{inRange}(n,$
$\text{length}\colon \text{list} \to \$\text{int} \qquad \text{tff}(t, \text{type})$
$\text{length}(\text{nil}) = 0 \qquad \text{tff}(l, \text{axiom})$
$\forall h\colon \$\text{int}, t\colon \text{list}\colon \text{length}(\text{cons}(h,t)) = \$\text{sum}(1, \text{length}(t)) \qquad \text{tff}(l_1, \text{axiom})$
$\text{count}\colon (\$\text{int} \times \text{list}) \to \$\text{int} \qquad \text{tff}(t_2, \text{type})$
$\forall k\colon \$\text{int}\colon \text{count}(k, \text{nil}) = 0 \qquad \text{tff}(a, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h,t)) = \text{count}(k,t) \ \Leftarrow\ k \neq h) \qquad \text{tff}(a_3, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h,t)) = \$\text{sum}(\text{count}(k,t), 1) \ \Leftarrow\ k = h) \qquad \text{tff}(a_4, \text{axiom})$
$\text{append}\colon (\text{list} \times \text{list}) \to \text{list} \qquad \text{tff}(t_5, \text{type})$
$\forall l\colon \text{list}\colon \text{append}(\text{nil}, l) = l \qquad \text{tff}(l_6, \text{axiom})$
$\forall i\colon \$\text{int}, k\colon \text{list}, l\colon \text{list}\colon \text{append}(\text{cons}(i,k), l) = \text{cons}(i, \text{append}(k,l)) \qquad \text{tff}(l_7, \text{axiom})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(n,l) \iff \$\text{greater}(\text{count}(n,l), 0)) \qquad \text{tff}(a_8, \text{axiom})$
$\neg\forall k\colon \text{list}, l\colon \text{list}\colon ((\$\text{greater}(\text{length}(k), 1) \text{ and } \$\text{greater}(\text{length}(l), 1)) \Rightarrow \$\text{greater}(\text{length}(\text{append}(k,l)), 4)) \qquad \text{tff}(c, \text{conjecture}$

DAT095=1.p Lists by functions problem 17
$\text{list}\colon \$\text{tType} \qquad \text{tff}(\text{list\_type}, \text{type})$
$\text{nil}\colon \text{list} \qquad \text{tff}(\text{nil\_type}, \text{type})$
$\text{cons}\colon (\$\text{int} \times \text{list}) \to \text{list} \qquad \text{tff}(\text{cons\_type}, \text{type})$
$\text{head}\colon \text{list} \to \$\text{int} \qquad \text{tff}(\text{head\_type}, \text{type})$
$\text{tail}\colon \text{list} \to \text{list} \qquad \text{tff}(\text{tail\_type}, \text{type})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k,l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k,l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\colon \text{list}\colon (l = \text{nil} \text{ or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k,l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{in}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{in}, \text{type})$
$\forall x\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(x,l) \iff (\exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h,t) \text{ and } x = h) \text{ or } \exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h,t) \text{ and } \text{in}(x,t)))) \qquad \text{t}$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \iff (l = \text{nil} \text{ or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k,t) \text{ and } \$\text{lesseq}(0,k) \text{ and } \$\text{less}(k,n) \text{ and } \text{inRange}(n,$
$\text{length}\colon \text{list} \to \$\text{int} \qquad \text{tff}(t, \text{type})$
$\text{length}(\text{nil}) = 0 \qquad \text{tff}(l, \text{axiom})$
$\forall h\colon \$\text{int}, t\colon \text{list}\colon \text{length}(\text{cons}(h,t)) = \$\text{sum}(1, \text{length}(t)) \qquad \text{tff}(l_1, \text{axiom})$
$\text{count}\colon (\$\text{int} \times \text{list}) \to \$\text{int} \qquad \text{tff}(t_2, \text{type})$
$\forall k\colon \$\text{int}\colon \text{count}(k, \text{nil}) = 0 \qquad \text{tff}(a, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h,t)) = \text{count}(k,t) \ \Leftarrow\ k \neq h) \qquad \text{tff}(a_3, \text{axiom})$
$\forall k\colon \$\text{int}, h\colon \$\text{int}, t\colon \text{list}, n\colon \$\text{int}\colon (\text{count}(k, \text{cons}(h,t)) = \$\text{sum}(\text{count}(k,t), 1) \ \Leftarrow\ k = h) \qquad \text{tff}(a_4, \text{axiom})$
$\text{append}\colon (\text{list} \times \text{list}) \to \text{list} \qquad \text{tff}(t_5, \text{type})$
$\forall l\colon \text{list}\colon \text{append}(\text{nil}, l) = l \qquad \text{tff}(l_6, \text{axiom})$
$\forall i\colon \$\text{int}, k\colon \text{list}, l\colon \text{list}\colon \text{append}(\text{cons}(i,k), l) = \text{cons}(i, \text{append}(k,l)) \qquad \text{tff}(l_7, \text{axiom})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(n,l) \iff \$\text{greater}(\text{count}(n,l), 0)) \qquad \text{tff}(a_8, \text{axiom})$
$\neg\forall m\colon \$\text{int}, n\colon \$\text{int}, l\colon \text{list}, l_1\colon \text{list}\colon ((\text{in}(n,l) \text{ and } l_1 = \text{cons}(m,l)) \Rightarrow \text{count}(n,l_1) = \text{count}(n,l)) \qquad \text{tff}(c, \text{conjecture})$

DAT097=1.p Lists by functions problem 18
$\text{list}\colon \$\text{tType} \qquad \text{tff}(\text{list\_type}, \text{type})$
$\text{nil}\colon \text{list} \qquad \text{tff}(\text{nil\_type}, \text{type})$
$\text{cons}\colon (\$\text{int} \times \text{list}) \to \text{list} \qquad \text{tff}(\text{cons\_type}, \text{type})$
$\text{head}\colon \text{list} \to \$\text{int} \qquad \text{tff}(\text{head\_type}, \text{type})$
$\text{tail}\colon \text{list} \to \text{list} \qquad \text{tff}(\text{tail\_type}, \text{type})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k,l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k,l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\colon \text{list}\colon (l = \text{nil} \text{ or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k,l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{in}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{in}, \text{type})$
$\forall x\colon \$\text{int}, l\colon \text{list}\colon (\text{in}(x,l) \iff (\exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h,t) \text{ and } x = h) \text{ or } \exists h\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(h,t) \text{ and } \text{in}(x,t)))) \qquad \text{t}$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \iff (l = \text{nil} \text{ or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k,t) \text{ and } \$\text{lesseq}(0,k) \text{ and } \$\text{less}(k,n) \text{ and } \text{inRange}(n,$
$\text{length}\colon \text{list} \to \$\text{int} \qquad \text{tff}(t, \text{type})$
$\text{length}(\text{nil}) = 0 \qquad \text{tff}(l, \text{axiom})$
$\forall h\colon \$\text{int}, t\colon \text{list}\colon \text{length}(\text{cons}(h,t)) = \$\text{sum}(1, \text{length}(t)) \qquad \text{tff}(l_1, \text{axiom})$
$\text{count}\colon (\$\text{int} \times \text{list}) \to \$\text{int} \qquad \text{tff}(t_2, \text{type})$

$\forall k$: \$int: count$(k, \text{nil}) = 0$ tff(a, axiom)
$\forall k$: \$int, h: \$int, t: list, n: \$int: (count$(k, \text{cons}(h, t))$ = count(k, t) $\Leftarrow$ $k \neq h$) tff(a_3, axiom)
$\forall k$: \$int, h: \$int, t: list, n: \$int: (count$(k, \text{cons}(h, t))$ = \$sum(count$(k, t), 1)$ $\Leftarrow$ $k = h$) tff(a_4, axiom)
append: (list × list) → list tff(t_5, type)
$\forall l$: list: append(nil, l) = l tff(l_6, axiom)
$\forall i$: \$int, k: list, l: list: append(cons$(i, k), l$) = cons(i, append(k, l)) tff(l_7, axiom)
$\forall n$: \$int, l: list: (in(n, l) $\iff$ \$greater(count$(n, l), 0$)) tff(a_8, axiom)
$\neg \forall m$: \$int, n: \$int, k: list, l: list, l_1: list: ((in(n, l) and $\neg$ in(m, k) and l_1 = append(l, cons(m, k))) $\Rightarrow$ count(n, l_1) = count(n, l)) tff$(c, \text{conjecture})$

DAT098=1.p Lists by relations problem 1
list: \$tType tff(list_type, type)
nil: list tff(nil_type, type)
cons: (\$int × list) → list tff(cons_type, type)
head: list → \$int tff(head_type, type)
tail: list → list tff(tail_type, type)
$\forall k$: \$int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: \$int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l))) tff(l_3, axiom)
$\forall k$: \$int, l: list: cons$(k, l) \neq$ nil tff(l_4, axiom)
inRange: (\$int × list) → \$o tff(inRange_type, type)
$\forall n$: \$int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: \$int, t: list: (l = cons(k, t) and \$lesseq$(0, k)$ and \$less(k, n) and inRange(n,
inRange(4, cons(1, cons(3, cons(2, nil)))) tff$(c, \text{conjecture})$

DAT099=1.p Lists by relations problem 2
list: \$tType tff(list_type, type)
nil: list tff(nil_type, type)
cons: (\$int × list) → list tff(cons_type, type)
head: list → \$int tff(head_type, type)
tail: list → list tff(tail_type, type)
$\forall k$: \$int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: \$int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l))) tff(l_3, axiom)
$\forall k$: \$int, l: list: cons$(k, l) \neq$ nil tff(l_4, axiom)
inRange: (\$int × list) → \$o tff(inRange_type, type)
$\forall n$: \$int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: \$int, t: list: (l = cons(k, t) and \$lesseq$(0, k)$ and \$less(k, n) and inRange(n,
$\forall n$: \$int: (\$greatereq$(n, 4)$ $\Rightarrow$ inRange(n, cons(1, cons(3, cons(2, nil))))) tff$(c, \text{conjecture})$

DAT100=1.p Lists by relations problem 3
list: \$tType tff(list_type, type)
nil: list tff(nil_type, type)
cons: (\$int × list) → list tff(cons_type, type)
head: list → \$int tff(head_type, type)
tail: list → list tff(tail_type, type)
$\forall k$: \$int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: \$int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l))) tff(l_3, axiom)
$\forall k$: \$int, l: list: cons$(k, l) \neq$ nil tff(l_4, axiom)
inRange: (\$int × list) → \$o tff(inRange_type, type)
$\forall n$: \$int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: \$int, t: list: (l = cons(k, t) and \$lesseq$(0, k)$ and \$less(k, n) and inRange(n,
$\neg$ inRange(4, cons(1, cons(5, cons(2, nil)))) tff$(c, \text{conjecture})$

DAT101=1.p Lists by relations problem 4
list: \$tType tff(list_type, type)
nil: list tff(nil_type, type)
cons: (\$int × list) → list tff(cons_type, type)
head: list → \$int tff(head_type, type)
tail: list → list tff(tail_type, type)
$\forall k$: \$int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: \$int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l))) tff(l_3, axiom)

$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k,l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k,t) \text{ and } \$\text{lesseq}(0,k) \text{ and } \$\text{less}(k,n) \text{ and inRange}(n,$
$\neg \forall n\colon \$\text{int}\colon (\$\text{greatereq}(n,4) \Rightarrow \text{inRange}(n, \text{cons}(1, \text{cons}(5, \text{cons}(2, \text{nil}))))) \qquad \text{tff}(c, \text{conjecture})$

DAT102=1.p Lists by relations problem 5
$\text{list}\colon \$\text{tType} \qquad \text{tff}(\text{list\_type}, \text{type})$
$\text{nil}\colon \text{list} \qquad \text{tff}(\text{nil\_type}, \text{type})$
$\text{cons}\colon (\$\text{int} \times \text{list}) \to \text{list} \qquad \text{tff}(\text{cons\_type}, \text{type})$
$\text{head}\colon \text{list} \to \$\text{int} \qquad \text{tff}(\text{head\_type}, \text{type})$
$\text{tail}\colon \text{list} \to \text{list} \qquad \text{tff}(\text{tail\_type}, \text{type})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k,l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k,l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k,l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k,t) \text{ and } \$\text{lesseq}(0,k) \text{ and } \$\text{less}(k,n) \text{ and inRange}(n,$
$\neg \forall l\colon \text{list}, n\colon \$\text{int}\colon (\text{inRange}(n, \text{tail}(l)) \Rightarrow \text{inRange}(n,l)) \qquad \text{tff}(c, \text{conjecture})$

DAT103=1.p Lists by relations problem 6
$\text{list}\colon \$\text{tType} \qquad \text{tff}(\text{list\_type}, \text{type})$
$\text{nil}\colon \text{list} \qquad \text{tff}(\text{nil\_type}, \text{type})$
$\text{cons}\colon (\$\text{int} \times \text{list}) \to \text{list} \qquad \text{tff}(\text{cons\_type}, \text{type})$
$\text{head}\colon \text{list} \to \$\text{int} \qquad \text{tff}(\text{head\_type}, \text{type})$
$\text{tail}\colon \text{list} \to \text{list} \qquad \text{tff}(\text{tail\_type}, \text{type})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k,l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k,l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k,l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k,t) \text{ and } \$\text{lesseq}(0,k) \text{ and } \$\text{less}(k,n) \text{ and inRange}(n,$
$\neg \exists l\colon \text{list}, n\colon \$\text{int}\colon (l \neq \text{nil and inRange}(n,l) \text{ and } \$\text{less}(\$\text{difference}(n, \text{head}(l)), 1)) \qquad \text{tff}(c, \text{conjecture})$

DAT104=1.p Lists by relations problem 7
$\text{list}\colon \$\text{tType} \qquad \text{tff}(\text{list\_type}, \text{type})$
$\text{nil}\colon \text{list} \qquad \text{tff}(\text{nil\_type}, \text{type})$
$\text{cons}\colon (\$\text{int} \times \text{list}) \to \text{list} \qquad \text{tff}(\text{cons\_type}, \text{type})$
$\text{head}\colon \text{list} \to \$\text{int} \qquad \text{tff}(\text{head\_type}, \text{type})$
$\text{tail}\colon \text{list} \to \text{list} \qquad \text{tff}(\text{tail\_type}, \text{type})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k,l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k,l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k,l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k,t) \text{ and } \$\text{lesseq}(0,k) \text{ and } \$\text{less}(k,n) \text{ and inRange}(n,$
$\neg \forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \Rightarrow \text{inRange}(\$\text{difference}(n,1), l)) \qquad \text{tff}(c, \text{conjecture})$

DAT105=1.p Lists by relations problem 8
$\text{list}\colon \$\text{tType} \qquad \text{tff}(\text{list\_type}, \text{type})$
$\text{nil}\colon \text{list} \qquad \text{tff}(\text{nil\_type}, \text{type})$
$\text{cons}\colon (\$\text{int} \times \text{list}) \to \text{list} \qquad \text{tff}(\text{cons\_type}, \text{type})$
$\text{head}\colon \text{list} \to \$\text{int} \qquad \text{tff}(\text{head\_type}, \text{type})$
$\text{tail}\colon \text{list} \to \text{list} \qquad \text{tff}(\text{tail\_type}, \text{type})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{head}(\text{cons}(k,l)) = k \qquad \text{tff}(l_1, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{tail}(\text{cons}(k,l)) = l \qquad \text{tff}(l_2, \text{axiom})$
$\forall l\colon \text{list}\colon (l = \text{nil or } l = \text{cons}(\text{head}(l), \text{tail}(l))) \qquad \text{tff}(l_3, \text{axiom})$
$\forall k\colon \$\text{int}, l\colon \text{list}\colon \text{cons}(k,l) \neq \text{nil} \qquad \text{tff}(l_4, \text{axiom})$
$\text{inRange}\colon (\$\text{int} \times \text{list}) \to \$\text{o} \qquad \text{tff}(\text{inRange\_type}, \text{type})$
$\forall n\colon \$\text{int}, l\colon \text{list}\colon (\text{inRange}(n,l) \iff (l = \text{nil or } \exists k\colon \$\text{int}, t\colon \text{list}\colon (l = \text{cons}(k,t) \text{ and } \$\text{lesseq}(0,k) \text{ and } \$\text{less}(k,n) \text{ and inRange}(n,$
$\neg \forall l\colon \text{list}, n\colon \$\text{int}\colon ((l \neq \text{nil and inRange}(n,l)) \Rightarrow \$\text{greatereq}(\$\text{difference}(n, \text{head}(l)), 2)) \qquad \text{tff}(c, \text{conjecture})$

DAT106=1.p Lists by relations problem 9

list: \$tType   tff(list_type, type)
nil: list   tff(nil_type, type)
cons: (\$int × list) → list   tff(cons_type, type)
head: list → \$int   tff(head_type, type)
tail: list → list   tff(tail_type, type)
$\forall k$: \$int, l: list: head(cons(k, l)) = k tff(l_1, axiom)
$\forall k$: \$int, l: list: tail(cons(k, l)) = l tff(l_2, axiom)
$\forall l$: list: (l = nil or l = cons(head(l), tail(l)))   tff(l_3, axiom)
$\forall k$: \$int, l: list: cons(k, l) $\neq$ nil   tff(l_4, axiom)
inRange: (\$int × list) → \$o   tff(inRange_type, type)
$\forall n$: \$int, l: list: (inRange(n, l) $\iff$ (l = nil or $\exists k$: \$int, t: list: (l = cons(k, t) and \$lesseq(0, k) and \$less(k, n) and inRange(n,
$\neg\forall n$: \$int, l_0: list, l_1: list: ((\$greater(n, 0) and inRange(n, l_0) and l_1 = cons(\$difference(n, 2), l_0)) $\Rightarrow$ inRange(n, l_1))   tff(c,

DAT107=1.p Integer arrays
include('Axioms/DAT001=0.ax')

DAT108=1.p Integer collections with counting
include('Axioms/DAT002=0.ax')
include('Axioms/DAT002=1.ax')

DAT109=1.p Pointer data types
include('Axioms/DAT003=0.ax')

DAT110=1.p Array data types
include('Axioms/DAT004=0.ax')

DAT111=1.p Heap data types
include('Axioms/DAT005=0.ax')

DAT112=1.p Tree-heap data types
include('Axioms/DAT006=0.ax')