

# FLD axioms

**FLD001-0.ax** Ordered field axioms (axiom formulation glxx)

$(\text{defined}(x) \text{ and } \text{defined}(y) \text{ and } \text{defined}(z)) \Rightarrow x + (y + z) = (x + y) + z$     cnf(associativity\_addition, axiom)  
 $\text{defined}(x) \Rightarrow 0 + x = x$     cnf(existence\_of\_identity\_addition, axiom)  
 $\text{defined}(x) \Rightarrow x + -x = 0$     cnf(existence\_of\_inverse\_addition, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow x + y = y + x$     cnf(commutativity\_addition, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y) \text{ and } \text{defined}(z)) \Rightarrow x \cdot (y \cdot z) = (x \cdot y) \cdot z$     cnf(associativity\_multiplication, axiom)  
 $\text{defined}(x) \Rightarrow 1 \cdot x = x$     cnf(existence\_of\_identity\_multiplication, axiom)  
 $\text{defined}(x) \Rightarrow (x \cdot x^{-1} = 1 \text{ or } x = 0)$     cnf(existence\_of\_inverse\_multiplication, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow x \cdot y = y \cdot x$     cnf(commutativity\_multiplication, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y) \text{ and } \text{defined}(z)) \Rightarrow x \cdot z + y \cdot z = (x + y) \cdot z$     cnf(distributivity, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow \text{defined}(x + y)$     cnf(well\_definedness\_of\_addition, axiom)  
 $\text{defined}(0)$     cnf(well\_definedness\_of\_additive\_identity, axiom)  
 $\text{defined}(x) \Rightarrow \text{defined}(-x)$     cnf(well\_definedness\_of\_additive\_inverse, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow \text{defined}(x \cdot y)$     cnf(well\_definedness\_of\_multiplication, axiom)  
 $\text{defined}(1)$     cnf(well\_definedness\_of\_multiplicative\_identity, axiom)  
 $\text{defined}(x) \Rightarrow (\text{defined}(x^{-1}) \text{ or } x = 0)$     cnf(well\_definedness\_of\_multiplicative\_inverse, axiom)  
 $(x \leq y \text{ and } y \leq x) \Rightarrow x = y$     cnf(antisymmetry\_of\_order\_relation, axiom)  
 $(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z$     cnf(transitivity\_of\_order\_relation, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow (x \leq y \text{ or } y \leq x)$     cnf(totality\_of\_order\_relation, axiom)  
 $(\text{defined}(z) \text{ and } x \leq y) \Rightarrow x + z \leq y + z$     cnf(compatibility\_of\_order\_relation\_and\_addition, axiom)  
 $(0 \leq y \text{ and } 0 \leq z) \Rightarrow 0 \leq y \cdot z$     cnf(compatibility\_of\_order\_relation\_and\_multiplication, axiom)  
 $\text{defined}(x) \Rightarrow x = x$     cnf(reflexivity\_of\_equality, axiom)  
 $y = x \Rightarrow x = y$     cnf(symmetry\_of\_equality, axiom)  
 $(x = y \text{ and } y = z) \Rightarrow x = z$     cnf(transitivity\_of\_equality, axiom)  
 $(\text{defined}(z) \text{ and } x = y) \Rightarrow x + z = y + z$     cnf(compatibility\_of\_equality\_and\_addition, axiom)  
 $(\text{defined}(z) \text{ and } x = y) \Rightarrow x \cdot z = y \cdot z$     cnf(compatibility\_of\_equality\_and\_multiplication, axiom)  
 $(x \leq z \text{ and } x = y) \Rightarrow y \leq z$     cnf(compatibility\_of\_equality\_and\_order\_relation, axiom)  
 $\neg 0 = 1$     cnf(different\_identities, axiom)

**FLD002-0.ax** Ordered field axioms (axiom formulation re)

$(x + y = u \text{ and } y + z = v \text{ and } u + z = w) \Rightarrow x + v = w$     cnf(associativity\_addition<sub>1</sub>, axiom)  
 $(x + y = u \text{ and } y + z = v \text{ and } x + v = w) \Rightarrow u + z = w$     cnf(associativity\_addition<sub>2</sub>, axiom)  
 $\text{defined}(x) \Rightarrow 0 + x = x$     cnf(existence\_of\_identity\_addition, axiom)  
 $\text{defined}(x) \Rightarrow -x + x = 0$     cnf(existence\_of\_inverse\_addition, axiom)  
 $x + y = z \Rightarrow y + x = z$     cnf(commutativity\_addition, axiom)  
 $(x \cdot y = u \text{ and } y \cdot z = v \text{ and } u \cdot z = w) \Rightarrow x \cdot v = w$     cnf(associativity\_multiplication<sub>1</sub>, axiom)  
 $(x \cdot y = u \text{ and } y \cdot z = v \text{ and } x \cdot v = w) \Rightarrow u \cdot z = w$     cnf(associativity\_multiplication<sub>2</sub>, axiom)  
 $\text{defined}(x) \Rightarrow 1 \cdot x = x$     cnf(existence\_of\_identity\_multiplication, axiom)  
 $\text{defined}(x) \Rightarrow (x^{-1} \cdot x = 1 \text{ or } 0 + x = 0)$     cnf(existence\_of\_inverse\_multiplication, axiom)  
 $x \cdot y = z \Rightarrow y \cdot x = z$     cnf(commutativity\_multiplication, axiom)  
 $(x + y = a \text{ and } a \cdot z = b \text{ and } x \cdot z = c \text{ and } y \cdot z = d) \Rightarrow c + d = b$     cnf(distributivity<sub>1</sub>, axiom)  
 $(x + y = a \text{ and } x \cdot z = c \text{ and } y \cdot z = d \text{ and } c + d = b) \Rightarrow a \cdot z = b$     cnf(distributivity<sub>2</sub>, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow \text{defined}(x + y)$     cnf(well\_definedness\_of\_addition, axiom)  
 $\text{defined}(0)$     cnf(well\_definedness\_of\_additive\_identity, axiom)  
 $\text{defined}(x) \Rightarrow \text{defined}(-x)$     cnf(well\_definedness\_of\_additive\_inverse, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow \text{defined}(x \cdot y)$     cnf(well\_definedness\_of\_multiplication, axiom)  
 $\text{defined}(1)$     cnf(well\_definedness\_of\_multiplicative\_identity, axiom)  
 $\text{defined}(x) \Rightarrow (\text{defined}(x^{-1}) \text{ or } 0 + x = 0)$     cnf(well\_definedness\_of\_multiplicative\_inverse, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow x + y = x + y$     cnf(totality\_of\_addition, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow x \cdot y = x \cdot y$     cnf(totality\_of\_multiplication, axiom)  
 $(x \leq y \text{ and } y \leq x) \Rightarrow 0 + x = y$     cnf(antisymmetry\_of\_order\_relation, axiom)  
 $(x \leq y \text{ and } y \leq z) \Rightarrow x \leq z$     cnf(transitivity\_of\_order\_relation, axiom)  
 $(\text{defined}(x) \text{ and } \text{defined}(y)) \Rightarrow (x \leq y \text{ or } y \leq x)$     cnf(totality\_of\_order\_relation, axiom)  
 $(x \leq y \text{ and } x + z = u \text{ and } y + z = v) \Rightarrow u \leq v$     cnf(compatibility\_of\_order\_relation\_and\_addition, axiom)  
 $(0 \leq x \text{ and } 0 \leq y \text{ and } x \cdot y = z) \Rightarrow 0 \leq z$     cnf(compatibility\_of\_order\_relation\_and\_multiplication, axiom)  
 $\neg 0 + 0 = 1$     cnf(different\_identities, axiom)

# FLD problems

**FLD001-3.p** Transformation additive relation  $\rightarrow$  multiplicative relation

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $0 + a = b$     cnf(sum3, hypothesis)
 $\neg 1 \cdot a = b$   cnf(not_product4, negated_conjecture)
```

**FLD002-3.p** Transformation multiplicative relation  $\rightarrow$  additive relation

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $1 \cdot a = b$     cnf(product3, hypothesis)
 $\neg 0 + a = b$     cnf(not_sum4, negated_conjecture)
```

**FLD003-1.p** Elimination of an additive term/inverse term - pair

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $\neg a + (b + -b) = a$   cnf(add_not_equal_to_a3, negated_conjecture)
```

**FLD004-1.p** Elimination of an multiplicative term/inverse term - pair

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $\neg b = 0$         cnf(b_not_equal_to_additive_identity3, negated_conjecture)
 $\neg a \cdot (b \cdot b^{-1}) = a$   cnf(multiply_not_equal_to_a4, negated_conjecture)
```

**FLD005-1.p** Every linear equation in the additive group has a solution

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $\neg a + x = b$     cnf(add_not_equal_to_b3, negated_conjecture)
```

**FLD005-3.p** Every linear equation in the additive group has a solution

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $\neg a + x = b$     cnf(not_sum3, negated_conjecture)
```

**FLD006-1.p** In the additive group it holds: inverse(identity)=identity

```
include('Axioms/FLD001-0.ax')
 $\neg - 0 = 0$       cnf(additive_inverse_not_equal_to_additive_identity1, negated_conjecture)
```

**FLD006-3.p** In the additive group it holds: inverse(identity)=identity

```
include('Axioms/FLD002-0.ax')
 $\neg 0 + -0 = 0$     cnf(not_sum1, negated_conjecture)
```

**FLD007-1.p** The additive inverse fulfills the involution property

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
 $\neg - (-a) = a$     cnf(additive_inverse_not_equal_to_a2, negated_conjecture)
```

**FLD007-3.p** The additive inverse fulfills the involution property

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
 $\neg 0 + -(-a) = a$   cnf(not_sum2, negated_conjecture)
```

**FLD008-1.p** Compatibility of operation and inverse in the additive group

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $\neg - (a + b) = -a + -b$   cnf(additive_inverse_not_equal_to_add3, negated_conjecture)
```

**FLD008-2.p** Compatibility of operation and inverse in the additive group

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
a + b=c      cnf(add_equals.c5, negated_conjecture)
-a + -b=d    cnf(add_equals.d6, negated_conjecture)
¬ - c=d      cnf(additive_inverse_not_equal_to.d7, negated_conjecture)
```

**FLD008-3.p** Compatibility of operation and inverse in the additive group

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
¬ 0 + -(a + b) = -a + -b    cnf(not_sum3, negated_conjecture)
```

**FLD008-4.p** Compatibility of operation and inverse in the additive group

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
a + b=c      cnf(sum5, negated_conjecture)
-a + -b=d    cnf(sum6, negated_conjecture)
¬ 0 + -c=d    cnf(not_sum7, negated_conjecture)
```

**FLD009-1.p** Linear equations in the multiplicative group have a solution

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
¬ a=0        cnf(a_not_equal_to_additive_identity3, negated_conjecture)
¬ a · x=b    cnf(multiply_not_equal_to.b4, negated_conjecture)
```

**FLD009-3.p** Linear equations in the multiplicative group have a solution

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
¬ 0 + a=0     cnf(not_sum3, negated_conjecture)
¬ a · x=b     cnf(not_product4, negated_conjecture)
```

**FLD010-1.p** In the multiplicative group inverse(identity)=identity

```
include('Axioms/FLD001-0.ax')
¬ 1-1=1      cnf(multiplicative_inv_not_equal_to_multiplicative_id2, negated_conjecture)
```

**FLD010-3.p** In the multiplicative group inverse(identity)=identity

```
include('Axioms/FLD002-0.ax')
¬ 0 + 1=0     cnf(not_sum1, negated_conjecture)
¬ 1 · 1-1=1   cnf(not_product2, negated_conjecture)
```

**FLD010-5.p** In the multiplicative group inverse(identity)=identity

```
include('Axioms/FLD002-0.ax')
¬ 0 + 1=0     cnf(not_sum1, negated_conjecture)
¬ 0 + 1-1=1   cnf(not_sum2, negated_conjecture)
```

**FLD011-1.p** The multiplicative inverse fulfills the involution property

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
¬ a=0        cnf(a_not_equal_to_additive_identity2, negated_conjecture)
¬ (a-1)-1=a  cnf(multiplicative_inverse_not_equal_to.a3, negated_conjecture)
```

**FLD011-3.p** The multiplicative inverse fulfills the involution property

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
¬ 0 + a=0     cnf(not_sum2, negated_conjecture)
¬ 1 · (a-1)-1=a  cnf(not_product3, negated_conjecture)
```

**FLD012-1.p** Compatibility of operation and inverse in multiplicative group

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
¬ a=0       cnf(a_not_equal_to_additive_identity3, negated_conjecture)
¬ b=0       cnf(b_not_equal_to_additive_identity4, negated_conjecture)
¬ (a · b)-1=a-1 · b-1   cnf(multiplicative_inverse_not_equal_to_multiply5, negated_conjecture)
```

**FLD012-2.p** Compatibility of operation and inverse in multiplicative group

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
defined(v)   cnf(v_is_defined, hypothesis)
¬ a=0       cnf(a_not_equal_to_additive_identity5, negated_conjecture)
¬ b=0       cnf(b_not_equal_to_additive_identity6, negated_conjecture)
a · b=u     cnf(multiply_equals_u7, negated_conjecture)
a-1 · b-1=v   cnf(multiply_equals_v8, negated_conjecture)
¬ u-1=v      cnf(multiplicative_inverse_not_equal_to_v9, negated_conjecture)
```

**FLD012-3.p** Compatibility of operation and inverse in multiplicative group

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
¬ 0 + a=0    cnf(not_sum3, negated_conjecture)
¬ 0 + b=0    cnf(not_sum4, negated_conjecture)
¬ 1 · (a · b)-1=a-1 · b-1   cnf(not_product5, negated_conjecture)
```

**FLD012-4.p** Compatibility of operation and inverse in multiplicative group

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
defined(v)   cnf(v_is_defined, hypothesis)
¬ 0 + a=0    cnf(not_sum5, negated_conjecture)
¬ 0 + b=0    cnf(not_sum6, negated_conjecture)
a · b=u     cnf(product7, negated_conjecture)
a-1 · b-1=v   cnf(product8, negated_conjecture)
¬ 1 · u-1=v    cnf(not_product9, negated_conjecture)
```

**FLD013-1.p** The resulting equation of the summation of two equations

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(d)   cnf(d_is_defined, hypothesis)
a=b         cnf(a_equals_b5, negated_conjecture)
c=d         cnf(c_equals_d6, negated_conjecture)
¬ a + c=d + b   cnf(add_not_equal_to_add7, negated_conjecture)
```

**FLD013-2.p** The resulting equation of the summation of two equations

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(d)   cnf(d_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
a=b         cnf(a_equals_b6, negated_conjecture)
c=d         cnf(c_equals_d7, negated_conjecture)
a + c=u     cnf(add_equals_u8, negated_conjecture)
¬ d + b=u    cnf(add_not_equal_to_u9, negated_conjecture)
```

**FLD013-3.p** The resulting equation of the summation of two equations

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
defined(c)   cnf(c.is_defined, hypothesis)
defined(d)   cnf(d.is_defined, hypothesis)
0 + a=b     cnf(sum5, negated_conjecture)
0 + c=d     cnf(sum6, negated_conjecture)
¬ a + c=d + b   cnf(not_sum7, negated_conjecture)
```

**FLD013-4.p** The resulting equation of the summation of two equations

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
defined(c)   cnf(c.is_defined, hypothesis)
defined(d)   cnf(d.is_defined, hypothesis)
defined(u)   cnf(u.is_defined, hypothesis)
0 + a=b     cnf(sum6, negated_conjecture)
0 + c=d     cnf(sum7, negated_conjecture)
a + c=u     cnf(sum8, negated_conjecture)
¬ d + b=u   cnf(not_sum9, negated_conjecture)
```

**FLD013-5.p** The resulting equation of the summation of two equations

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
defined(c)   cnf(c.is_defined, hypothesis)
defined(d)   cnf(d.is_defined, hypothesis)
defined(u)   cnf(u.is_defined, hypothesis)
defined(v)   cnf(v.is_defined, hypothesis)
0 + a=b     cnf(sum7, negated_conjecture)
0 + c=d     cnf(sum8, negated_conjecture)
a + c=u     cnf(sum9, negated_conjecture)
d + b=v     cnf(sum10, negated_conjecture)
¬ 0 + u=v   cnf(not_sum11, negated_conjecture)
```

**FLD014-1.p** Compatibility of additive inverses with the equality, part 1

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
a=b         cnf(a.equals_b3, negated_conjecture)
¬ ¬ a = - b   cnf(additive_inverse_not_equal_to_additive_inverse4, negated_conjecture)
```

**FLD014-3.p** Compatibility of additive inverses with the equality, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
0 + a=b     cnf(sum3, negated_conjecture)
¬ 0 + -a = - b   cnf(not_sum4, negated_conjecture)
```

**FLD015-1.p** Compatibility of additive inverses with the equality, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
¬ a = - b    cnf(additive_inverse_equals_additive_inverse3, negated_conjecture)
¬ a=b       cnf(a.not_equal_to_b4, negated_conjecture)
```

**FLD015-3.p** Compatibility of additive inverses with the equality, part 2

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
0 + -a = - b   cnf(sum3, negated_conjecture)
¬ 0 + a=b     cnf(not_sum4, negated_conjecture)
```

**FLD016-1.p** Solutions of linear equations in the additive group are unique

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
a + u=b      cnf(add_equals_b5, negated_conjecture)
a + v=b      cnf(add_equals_b6, negated_conjecture)
¬ u=v        cnf(u_not_equal_to_v7, negated_conjecture)
```

**FLD016-3.p** Solutions of linear equations in the additive group are unique

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
a + u=b      cnf(sum5, negated_conjecture)
a + v=b      cnf(sum6, negated_conjecture)
¬ 0 + u=v    cnf(not_sum7, negated_conjecture)
```

**FLD016-5.p** Solutions of linear equations in the additive group are unique

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
defined(q)    cnf(q_is_defined, hypothesis)
defined(r)    cnf(r_is_defined, hypothesis)
a + u=q      cnf(sum6, negated_conjecture)
a + v=r      cnf(sum7, negated_conjecture)
0 + q=r      cnf(sum8, negated_conjecture)
¬ 0 + u=v    cnf(not_sum9, negated_conjecture)
```

**FLD017-1.p** Substitution of an element in additive equations

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(x)    cnf(x_is_defined, hypothesis)
a=x          cnf(a_equals_x5, negated_conjecture)
a + b=c      cnf(add_equals_c6, negated_conjecture)
¬ x + b=c    cnf(add_not_equal_to_c7, negated_conjecture)
```

**FLD017-3.p** Substitution of an element in additive equations

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(x)    cnf(x_is_defined, hypothesis)
0 + a=x      cnf(sum5, negated_conjecture)
a + b=c      cnf(sum6, negated_conjecture)
¬ x + b=c    cnf(not_sum7, negated_conjecture)
```

**FLD018-1.p** If a is zero, the additive inverse of a is also zero

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
a=0          cnf(a_equals_additive_identity2, negated_conjecture)
¬ - a=0      cnf(additive_inverse_not_equal_to_additive_identity3, negated_conjecture)
```

**FLD018-3.p** If a is zero, the additive inverse of a is also zero

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
0 + a=0      cnf(sum2, negated_conjecture)
¬ 0 + -a=0   cnf(not_sum3, negated_conjecture)
```

**FLD019-1.p** If the additive inverse of a is zero, a itself is zero  
include('Axioms/FLD001-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
 $-a=0$  cnf(additive\_inverse\_equals\_additive\_identity<sub>2</sub>, negated\_conjecture)  
 $\neg a=0$  cnf(a\_not\_equal\_to\_additive\_identity<sub>3</sub>, negated\_conjecture)

**FLD019-3.p** If the additive inverse of a is zero, a itself is zero  
include('Axioms/FLD002-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
 $0 + -a=0$  cnf(sum<sub>2</sub>, negated\_conjecture)  
 $\neg 0 + a=0$  cnf(not\_sum<sub>3</sub>, negated\_conjecture)

**FLD020-1.p** The additive identity is unique  
include('Axioms/FLD001-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
defined(m) cnf(m\_is\_defined, hypothesis)  
 $m + a=a$  cnf(add\_equals\_a<sub>3</sub>, negated\_conjecture)  
 $\neg m=0$  cnf(m\_not\_equal\_to\_additive\_identity<sub>4</sub>, negated\_conjecture)

**FLD020-3.p** The additive identity is unique  
include('Axioms/FLD002-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
defined(m) cnf(m\_is\_defined, hypothesis)  
 $m + a=a$  cnf(sum<sub>3</sub>, negated\_conjecture)  
 $\neg 0 + m=0$  cnf(not\_sum<sub>4</sub>, negated\_conjecture)

**FLD021-1.p** Every element equal to zero is an additive identity  
include('Axioms/FLD001-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
defined(m) cnf(m\_is\_defined, hypothesis)  
 $m=0$  cnf(m\_equals\_additive\_identity<sub>3</sub>, negated\_conjecture)  
 $\neg m + a=a$  cnf(add\_not\_equal\_to\_a<sub>4</sub>, negated\_conjecture)

**FLD021-3.p** Every element equal to zero is an additive identity  
include('Axioms/FLD002-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
defined(m) cnf(m\_is\_defined, hypothesis)  
 $0 + m=0$  cnf(sum<sub>3</sub>, negated\_conjecture)  
 $\neg m + a=a$  cnf(not\_sum<sub>4</sub>, negated\_conjecture)

**FLD022-1.p** Elimination of a summation in an equation  
include('Axioms/FLD001-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
defined(b) cnf(b\_is\_defined, hypothesis)  
defined(c) cnf(c\_is\_defined, hypothesis)  
 $a + c=b + c$  cnf(add\_equals\_add<sub>4</sub>, negated\_conjecture)  
 $\neg a=b$  cnf(a\_not\_equal\_to\_b<sub>5</sub>, negated\_conjecture)

**FLD022-3.p** Elimination of a summation in an equation  
include('Axioms/FLD002-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
defined(b) cnf(b\_is\_defined, hypothesis)  
defined(c) cnf(c\_is\_defined, hypothesis)  
defined(u) cnf(u\_is\_defined, hypothesis)  
 $a + c=u$  cnf(sum<sub>5</sub>, negated\_conjecture)  
 $b + c=u$  cnf(sum<sub>6</sub>, negated\_conjecture)  
 $\neg 0 + a=b$  cnf(not\_sum<sub>7</sub>, negated\_conjecture)

**FLD023-1.p** Side Change of a term in an equation, part 1  
include('Axioms/FLD001-0.ax')  
defined(a) cnf(a\_is\_defined, hypothesis)  
defined(b) cnf(b\_is\_defined, hypothesis)  
 $a=b$  cnf(a\_equals\_b<sub>3</sub>, negated\_conjecture)  
 $\neg 0=b + -a$  cnf(additive\_identity\_not\_equal\_to\_add<sub>4</sub>, negated\_conjecture)

**FLD023-3.p** Side Change of a term in an equation, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
 $0 + a = b$    cnf(sum3, negated_conjecture)
 $\neg b + \neg a = 0$    cnf(not_sum4, negated_conjecture)
```

**FLD024-1.p** Side Change of a term in an equation, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
 $0 = b + \neg a$    cnf(additive_identity_equals_add3, negated_conjecture)
 $\neg a = b$    cnf(a_not_equal_to_b4, negated_conjecture)
```

**FLD024-3.p** Side Change of a term in an equation, part 2

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
 $b + \neg a = 0$    cnf(sum3, negated_conjecture)
 $\neg 0 + a = b$    cnf(not_sum4, negated_conjecture)
```

**FLD025-1.p** The resulting equation of a multiplication of two equations

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(d)   cnf(d_is_defined, hypothesis)
 $a = b$    cnf(a_equals_b5, negated_conjecture)
 $c = d$    cnf(c_equals_d6, negated_conjecture)
 $\neg a \cdot c = d \cdot b$    cnf(multiply_not_equal_to_multiply7, negated_conjecture)
```

**FLD025-2.p** The resulting equation of a multiplication of two equations

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(d)   cnf(d_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
defined(v)   cnf(v_is_defined, hypothesis)
 $a = b$    cnf(a_equals_b7, negated_conjecture)
 $c = d$    cnf(c_equals_d8, negated_conjecture)
 $a \cdot c = u$    cnf(multiply_equals_u9, negated_conjecture)
 $d \cdot b = v$    cnf(multiply_equals_v10, negated_conjecture)
 $\neg v = u$    cnf(v_not_equal_to_u11, negated_conjecture)
```

**FLD025-3.p** The resulting equation of a multiplication of two equations

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(d)   cnf(d_is_defined, hypothesis)
 $1 \cdot a = b$    cnf(product5, negated_conjecture)
 $1 \cdot c = d$    cnf(product6, negated_conjecture)
 $\neg d \cdot b = a \cdot c$    cnf(not_product7, negated_conjecture)
```

**FLD025-4.p** The resulting equation of a multiplication of two equations

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(d)   cnf(d_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
 $1 \cdot a = b$    cnf(product6, negated_conjecture)
```



$1 \cdot c = d$     cnf(product<sub>7</sub>, negated\_conjecture)  
 $a \cdot c = u$     cnf(product<sub>8</sub>, negated\_conjecture)  
 $\neg d \cdot b = u$     cnf(not\_product<sub>9</sub>, negated\_conjecture)

**FLD025-5.p** The resulting equation of a multiplication of two equations  
include('Axioms/FLD002-0.ax')

defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
defined( $c$ )    cnf( $c$ .is\_defined, hypothesis)  
defined( $d$ )    cnf( $d$ .is\_defined, hypothesis)  
defined( $u$ )    cnf( $u$ .is\_defined, hypothesis)  
defined( $v$ )    cnf( $v$ .is\_defined, hypothesis)  
 $1 \cdot a = b$     cnf(product<sub>7</sub>, negated\_conjecture)  
 $1 \cdot c = d$     cnf(product<sub>8</sub>, negated\_conjecture)  
 $a \cdot c = u$     cnf(product<sub>9</sub>, negated\_conjecture)  
 $d \cdot b = v$     cnf(product<sub>10</sub>, negated\_conjecture)  
 $\neg 1 \cdot u = v$     cnf(not\_product<sub>11</sub>, negated\_conjecture)

**FLD026-1.p** Compatibility of multiplicative inverses with equality  
include('Axioms/FLD001-0.ax')

defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
 $\neg a = 0$     cnf( $a$ .not\_equal\_to\_additive\_identity<sub>3</sub>, negated\_conjecture)  
 $a = b$     cnf( $a$ .equals\_b<sub>4</sub>, negated\_conjecture)  
 $\neg a^{-1} = b^{-1}$     cnf(multiplicative\_inverses\_not\_equal, negated\_conjecture)

**FLD026-3.p** Compatibility of multiplicative inverses with equality  
include('Axioms/FLD002-0.ax')

defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
 $\neg 0 + a = 0$     cnf(not\_sum<sub>3</sub>, negated\_conjecture)  
 $1 \cdot a = b$     cnf(product<sub>4</sub>, negated\_conjecture)  
 $\neg 1 \cdot a^{-1} = b^{-1}$     cnf(not\_product<sub>5</sub>, negated\_conjecture)

**FLD027-1.p** Elimination of multiplicative inverses in an equation  
include('Axioms/FLD001-0.ax')

defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
 $\neg a = 0$     cnf( $a$ .not\_equal\_to\_additive\_identity<sub>3</sub>, negated\_conjecture)  
 $\neg b = 0$     cnf( $b$ .not\_equal\_to\_additive\_identity<sub>4</sub>, negated\_conjecture)  
 $a^{-1} = b^{-1}$     cnf(multiplicative\_inverses\_equal, negated\_conjecture)  
 $\neg a = b$     cnf( $a$ .not\_equal\_to\_b<sub>6</sub>, negated\_conjecture)

**FLD027-3.p** Elimination of multiplicative inverses in an equation  
include('Axioms/FLD002-0.ax')

defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
 $\neg 0 + a = 0$     cnf(not\_sum<sub>3</sub>, negated\_conjecture)  
 $\neg 0 + b = 0$     cnf(not\_sum<sub>4</sub>, negated\_conjecture)  
 $1 \cdot a^{-1} = b^{-1}$     cnf(product<sub>5</sub>, negated\_conjecture)  
 $\neg 1 \cdot a = b$     cnf(not\_product<sub>6</sub>, negated\_conjecture)

**FLD028-1.p** The solution of a multiplicative linear equation is unique  
include('Axioms/FLD001-0.ax')

defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
defined( $u$ )    cnf( $u$ .is\_defined, hypothesis)  
defined( $v$ )    cnf( $v$ .is\_defined, hypothesis)  
 $\neg a = 0$     cnf( $a$ .not\_equal\_to\_additive\_identity<sub>5</sub>, negated\_conjecture)  
 $a \cdot u = b$     cnf(multiply\_equals\_b<sub>6</sub>, negated\_conjecture)  
 $a \cdot v = b$     cnf(multiply\_equals\_b<sub>7</sub>, negated\_conjecture)  
 $\neg u = v$     cnf( $u$ .not\_equal\_to\_v<sub>8</sub>, negated\_conjecture)

**FLD028-3.p** The solution of a multiplicative linear equation is unique

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
¬0 + a=0     cnf(not_sum5, negated_conjecture)
a · u=b      cnf(product6, negated_conjecture)
a · v=b      cnf(product7, negated_conjecture)
¬1 · u=v     cnf(not_product8, negated_conjecture)
```

**FLD029-1.p** The solution of a multiplicative linear equation is unique

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
¬b=0         cnf(b_not_equal_to_additive_identity5, negated_conjecture)
a · u=b      cnf(multiply_equals_b6, negated_conjecture)
a · v=b      cnf(multiply_equals_b7, negated_conjecture)
¬u=v         cnf(u_not_equal_to_v8, negated_conjecture)
```

**FLD029-3.p** The solution of a multiplicative linear equation is unique

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
¬0 + b=0     cnf(not_sum5, negated_conjecture)
a · u=b      cnf(product6, negated_conjecture)
a · v=b      cnf(product7, negated_conjecture)
¬1 · u=v     cnf(not_product8, negated_conjecture)
```

**FLD030-1.p** Compatibility of multiplication and equality relation

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
a=d          cnf(a_equals_d4, negated_conjecture)
¬d · b=a · b  cnf(multiply_not_equal_to_multiply5, negated_conjecture)
```

**FLD030-2.p** Compatibility of multiplication and equality relation

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
a · b=c      cnf(multiply_equals_c5, negated_conjecture)
a=d          cnf(a_equals_d6, negated_conjecture)
¬d · b=c     cnf(multiply_not_equal_to_c7, negated_conjecture)
```

**FLD030-3.p** Compatibility of multiplication and equality relation

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
1 · a=d      cnf(product4, negated_conjecture)
¬d · b=a · b  cnf(not_product5, negated_conjecture)
```

**FLD030-4.p** Compatibility of multiplication and equality relation

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
```

```

defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
a · b=c      cnf(product5, negated_conjecture)
1 · a=d      cnf(product6, negated_conjecture)
¬ d · b=c    cnf(not_product7, negated_conjecture)

```

**FLD031-1.p** If  $a$  is one, then the multiplicative inverse of  $a$  is also one

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
a=1          cnf(a_equals_multiplicative_identity2, negated_conjecture)
¬ a=0        cnf(a_not_equal_to_additive_identity3, negated_conjecture)
¬ a-1=1     cnf(multiplicative_inverses_not_equal, negated_conjecture)

```

**FLD031-3.p** If  $a$  is one, then the multiplicative inverse of  $a$  is also one

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
1 · a=1      cnf(product2, negated_conjecture)
¬ 0 + a=0    cnf(not_sum3, negated_conjecture)
¬ 1 · a-1=1  cnf(not_product4, negated_conjecture)

```

**FLD031-5.p** If  $a$  is one, then the multiplicative inverse of  $a$  is also one

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
1 · a=1      cnf(product2, negated_conjecture)
¬ 0 + a=0    cnf(not_sum3, negated_conjecture)
¬ 1 · 1=a-1  cnf(not_product4, negated_conjecture)

```

**FLD032-1.p** If the multiplicative inverse of  $a$  is one, then  $a$  is one itself

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
¬ a=0         cnf(a_not_equal_to_additive_identity2, negated_conjecture)
a-1=1        cnf(multiplicative_inverses_equal, negated_conjecture)
¬ a=1         cnf(a_not_equal_to_multiplicative_identity4, negated_conjecture)

```

**FLD032-3.p** If the multiplicative inverse of  $a$  is one, then  $a$  is one itself

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
¬ 0 + a=0     cnf(not_sum2, negated_conjecture)
1 · a-1=1     cnf(product3, negated_conjecture)
¬ 1 · a=1     cnf(not_product4, negated_conjecture)

```

**FLD033-1.p** The multiplicative identity is unique

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(m)    cnf(m.is_defined, hypothesis)
¬ a=0         cnf(a_not_equal_to_additive_identity3, negated_conjecture)
m · a=a      cnf(multiply_equals_a4, negated_conjecture)
¬ m=1        cnf(m_not_equal_to_multiplicative_identity5, negated_conjecture)

```

**FLD033-3.p** The multiplicative identity is unique

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(m)    cnf(m.is_defined, hypothesis)
¬ 0 + a=0     cnf(not_sum3, negated_conjecture)
m · a=a      cnf(product4, negated_conjecture)
¬ 1 · m=1     cnf(not_product5, negated_conjecture)

```

**FLD034-1.p** Every element equal one is a multiplicative identity

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(m)    cnf(m.is_defined, hypothesis)
m=1          cnf(m_equals_multiplicative_identity3, negated_conjecture)
¬ m · a=a    cnf(multiply_not_equal_to_a4, negated_conjecture)

```

**FLD034-3.p** Every element equal one is a multiplicative identity

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(m)    cnf(m_is_defined, hypothesis)
 $1 \cdot m = 1$    cnf(product3, negated_conjecture)
 $\neg m \cdot a = a$   cnf(not_product4, negated_conjecture)
```

**FLD035-1.p** Elimination of a multiplication in an equation

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
 $a \cdot c = b \cdot c$   cnf(multiply_equals_multiply4, negated_conjecture)
 $\neg c = 0$       cnf(c_not_equal_to_additive_identity5, negated_conjecture)
 $\neg a = b$       cnf(a_not_equal_to_b6, negated_conjecture)
```

**FLD035-3.p** Elimination of a multiplication in an equation

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
 $a \cdot c = u$      cnf(product5, negated_conjecture)
 $b \cdot c = u$      cnf(product6, negated_conjecture)
 $\neg 0 + c = 0$    cnf(not_sum7, negated_conjecture)
 $\neg 1 \cdot a = b$   cnf(not_product8, negated_conjecture)
```

**FLD036-1.p** Only a multiplication by zero can make elements equal

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
 $a \cdot c = b \cdot c$   cnf(multiply_equals_multiply4, negated_conjecture)
 $\neg a = b$       cnf(a_not_equal_to_b5, negated_conjecture)
 $\neg c = 0$       cnf(c_not_equal_to_additive_identity6, negated_conjecture)
```

**FLD036-3.p** Only a multiplication by zero can make elements equal

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
 $a \cdot c = u$      cnf(product5, negated_conjecture)
 $b \cdot c = u$      cnf(product6, negated_conjecture)
 $\neg 1 \cdot a = b$   cnf(not_product7, negated_conjecture)
 $\neg 0 + c = 0$    cnf(not_sum8, negated_conjecture)
```

**FLD037-1.p** Side change of a term in an equation by multiplication, part 1

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $\neg a = 0$       cnf(a_not_equal_to_additive_identity3, negated_conjecture)
 $a = b$         cnf(a_equals_b4, negated_conjecture)
 $\neg 1 = b \cdot a^{-1}$   cnf(multiplicative_identity_not_equal_to_multiply5, negated_conjecture)
```

**FLD037-3.p** Side change of a term in an equation by multiplication, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $\neg 0 + a = 0$    cnf(not_sum3, negated_conjecture)
 $1 \cdot a = b$    cnf(product4, negated_conjecture)
 $\neg b \cdot a^{-1} = 1$   cnf(not_product5, negated_conjecture)
```

**FLD038-1.p** Side change of a term in an equation by multiplication, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
¬ a=0       cnf(a_not_equal_to_additive_identity_3, negated_conjecture)
1=b · a-1   cnf(multiplicative_identity_equals_multiply_4, negated_conjecture)
¬ a=b       cnf(a_not_equal_to_b_5, negated_conjecture)
```

**FLD038-3.p** Side change of a term in an equation by multiplication, part 2

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
¬ 0 + a=0    cnf(not_sum_3, negated_conjecture)
b · a-1=1    cnf(product_4, negated_conjecture)
¬ 1 · a=b     cnf(not_product_5, negated_conjecture)
```

**FLD039-1.p** In a field with two or more elements, 1 and 0 must be different

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
¬ a=0       cnf(a_not_equal_to_additive_identity_1, negated_conjecture)
1=0         cnf(multiplicative_identity_equals_additive_identity_3, negated_conjecture)
```

**FLD039-3.p** In a field with two or more elements, 1 and 0 must be different

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
¬ 0 + a=0    cnf(not_sum_1, negated_conjecture)
0 + 1=0     cnf(sum_3, negated_conjecture)
```

**FLD040-1.p** If a is not 0, then the multiplicative inverse of a is not 0

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
¬ a=0       cnf(a_not_equal_to_additive_identity_2, negated_conjecture)
a-1=0       cnf(multiplicative_inverse_equals_additive_identity_3, negated_conjecture)
```

**FLD040-3.p** If a is not 0, then the multiplicative inverse of a is not 0

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
¬ 0 + a=0    cnf(not_sum_2, negated_conjecture)
0 + 0=a-1   cnf(sum_3, negated_conjecture)
```

**FLD040-5.p** If a is not 0, then the multiplicative inverse of a is not 0

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
¬ 0 + a=0    cnf(not_sum_2, negated_conjecture)
0 + a-1=0    cnf(sum_3, negated_conjecture)
```

**FLD041-1.p** If a,b are not 0, the the product of a and b is not 0

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
¬ a=0       cnf(a_not_equal_to_additive_identity_3, negated_conjecture)
¬ b=0       cnf(b_not_equal_to_additive_identity_4, negated_conjecture)
a · b=0     cnf(multiply_equals_additive_identity_5, negated_conjecture)
```

**FLD041-2.p** If a,b are not 0, the the product of a and b is not 0

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
¬ a=0       cnf(a_not_equal_to_additive_identity_4, negated_conjecture)
¬ b=0       cnf(b_not_equal_to_additive_identity_5, negated_conjecture)
a · b=c     cnf(multiply_equals_c_6, negated_conjecture)
c=0        cnf(c_equals_additive_identity_7, negated_conjecture)
```

**FLD041-3.p** If a,b are not 0, the the product of a and b is not 0

```
include('Axioms/FLD002-0.ax')
```

```

defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
¬0 + a=0     cnf(not_sum3, negated_conjecture)
¬0 + b=0     cnf(not_sum4, negated_conjecture)
a · b=0      cnf(product5, negated_conjecture)

```

**FLD041-4.p** If a,b are not 0, the the product of a and b is not 0

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
¬0 + a=0     cnf(not_sum4, negated_conjecture)
¬0 + b=0     cnf(not_sum5, negated_conjecture)
a · b=c      cnf(product6, negated_conjecture)
0 + c=0     cnf(sum7, negated_conjecture)

```

**FLD043-1.p** The multiplication with 0 yields 0

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
¬0 · a=0     cnf(multiply_not_equal_to_additive_identity2, negated_conjecture)

```

**FLD043-3.p** The multiplication with 0 yields 0

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
0 · a=b      cnf(product3, negated_conjecture)
¬0 + b=0     cnf(not_sum4, negated_conjecture)

```

**FLD043-5.p** The multiplication with 0 yields 0

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
¬0 · a=0     cnf(not_product2, negated_conjecture)

```

**FLD044-1.p** Compatibility of multiplication and additive inverses

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
¬(−a) · b= − a · b    cnf(multiply_not_equal_to_additive_inverse3, negated_conjecture)

```

**FLD044-2.p** Compatibility of multiplication and additive inverses

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
(−a) · b=c    cnf(multiply_equals_c5, negated_conjecture)
a · b=d      cnf(multiply_equals_d6, negated_conjecture)
¬c = − d     cnf(c_not_equal_to_additive_inverse7, negated_conjecture)

```

**FLD044-3.p** Compatibility of multiplication and additive inverses

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
¬(−a) · b= − a · b    cnf(not_product3, negated_conjecture)

```

**FLD044-4.p** Compatibility of multiplication and additive inverses

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
a · b=u      cnf(product4, negated_conjecture)
¬(−a) · b= − u    cnf(not_product5, negated_conjecture)

```

**FLD045-1.p** Compatibility of multiplication and additive inverses

```

include('Axioms/FLD001-0.ax')

```

$\text{defined}(a)$      $\text{cnf}(\text{a\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(\text{b\_is\_defined}, \text{hypothesis})$   
 $\neg(-a) \cdot (-b) = a \cdot b$      $\text{cnf}(\text{multiply\_not\_equal\_to\_multiply}_3, \text{negated\_conjecture})$

**FLD045-2.p** Compatibility of multiplication and additive inverses

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(\text{a\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(\text{b\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(\text{c\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(d)$      $\text{cnf}(\text{d\_is\_defined}, \text{hypothesis})$   
 $(-a) \cdot (-b) = c$      $\text{cnf}(\text{multiply\_equals\_c}_5, \text{negated\_conjecture})$   
 $a \cdot b = d$      $\text{cnf}(\text{multiply\_equals\_d}_6, \text{negated\_conjecture})$   
 $\neg c = d$      $\text{cnf}(\text{c\_not\_equal\_to\_d}_7, \text{negated\_conjecture})$

**FLD045-3.p** Compatibility of multiplication and additive inverses

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(\text{a\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(\text{b\_is\_defined}, \text{hypothesis})$   
 $\neg a \cdot b = (-a) \cdot (-b)$      $\text{cnf}(\text{not\_product}_3, \text{negated\_conjecture})$

**FLD045-4.p** Compatibility of multiplication and additive inverses

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(\text{a\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(\text{b\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(\text{u\_is\_defined}, \text{hypothesis})$   
 $(-a) \cdot (-b) = u$      $\text{cnf}(\text{product}_4, \text{negated\_conjecture})$   
 $\neg a \cdot b = u$      $\text{cnf}(\text{not\_product}_5, \text{negated\_conjecture})$

**FLD046-1.p** Compatibility of the additive and the multiplicative inverse

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(\text{a\_is\_defined}, \text{hypothesis})$   
 $\neg a = 0$      $\text{cnf}(\text{a\_not\_equal\_to\_additive\_identity}_2, \text{negated\_conjecture})$   
 $\neg(-a)^{-1} = -a^{-1}$      $\text{cnf}(\text{mult\_inverse\_not\_equal\_to\_additive\_inverse}_3, \text{negated\_conjecture})$

**FLD046-3.p** Compatibility of the additive and the multiplicative inverse

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(\text{a\_is\_defined}, \text{hypothesis})$   
 $\neg 0 + a = 0$      $\text{cnf}(\text{not\_sum}_2, \text{negated\_conjecture})$   
 $\neg 0 + (-a)^{-1} = -a^{-1}$      $\text{cnf}(\text{not\_sum}_3, \text{negated\_conjecture})$

**FLD047-1.p** Fraction calculation, part 1

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(\text{a\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(\text{b\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(\text{c\_is\_defined}, \text{hypothesis})$   
 $\neg b = 0$      $\text{cnf}(\text{b\_not\_equal\_to\_additive\_identity}_4, \text{negated\_conjecture})$   
 $\neg c = 0$      $\text{cnf}(\text{c\_not\_equal\_to\_additive\_identity}_5, \text{negated\_conjecture})$   
 $\neg a \cdot b^{-1} = (a \cdot c) \cdot (b \cdot c)^{-1}$      $\text{cnf}(\text{multiply\_not\_equal\_to\_multiply}_6, \text{negated\_conjecture})$

**FLD047-2.p** Fraction calculation, part 1

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(\text{a\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(\text{b\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(\text{c\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(\text{u\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(s)$      $\text{cnf}(\text{s\_is\_defined}, \text{hypothesis})$   
 $\text{defined}(t)$      $\text{cnf}(\text{t\_is\_defined}, \text{hypothesis})$   
 $\neg b = 0$      $\text{cnf}(\text{b\_not\_equal\_to\_additive\_identity}_7, \text{negated\_conjecture})$   
 $\neg c = 0$      $\text{cnf}(\text{c\_not\_equal\_to\_additive\_identity}_8, \text{negated\_conjecture})$   
 $a \cdot b^{-1} = u$      $\text{cnf}(\text{multiply\_equals\_u}_9, \text{negated\_conjecture})$   
 $a \cdot c = s$      $\text{cnf}(\text{multiply\_equals\_s}_{10}, \text{negated\_conjecture})$   
 $b \cdot c = t$      $\text{cnf}(\text{multiply\_equals\_t}_{11}, \text{negated\_conjecture})$   
 $\neg s \cdot t^{-1} = u$      $\text{cnf}(\text{multiply\_not\_equal\_to\_u}_{12}, \text{negated\_conjecture})$

**FLD047-3.p** Fraction calculation, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
defined(c)   cnf(c.is_defined, hypothesis)
-0 + b=0     cnf(not_sum4, negated_conjecture)
-0 + c=0     cnf(not_sum5, negated_conjecture)
-a · b-1=(a · c) · (b · c)-1   cnf(not_product6, negated_conjecture)
```

**FLD047-4.p** Fraction calculation, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
defined(c)   cnf(c.is_defined, hypothesis)
defined(u)   cnf(u.is_defined, hypothesis)
defined(s)   cnf(s.is_defined, hypothesis)
defined(t)   cnf(t.is_defined, hypothesis)
-0 + b=0     cnf(not_sum7, negated_conjecture)
-0 + c=0     cnf(not_sum8, negated_conjecture)
a · b-1=u     cnf(product9, negated_conjecture)
a · c=s      cnf(product10, negated_conjecture)
b · c=t      cnf(product11, negated_conjecture)
-s · t-1=u    cnf(not_product12, negated_conjecture)
```

**FLD048-1.p** Fraction calculation, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
defined(c)   cnf(c.is_defined, hypothesis)
defined(d)   cnf(d.is_defined, hypothesis)
-b=0        cnf(b_not_equal_to_additive_identity5, negated_conjecture)
-d=0        cnf(d_not_equal_to_additive_identity6, negated_conjecture)
-(a · b-1) · (c · d-1)=(a · c) · (b · d)-1   cnf(multiply_not_equal_to_multiply7, negated_conjecture)
```

**FLD048-2.p** Fraction calculation, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
defined(c)   cnf(c.is_defined, hypothesis)
defined(d)   cnf(d.is_defined, hypothesis)
defined(u)   cnf(u.is_defined, hypothesis)
defined(k)   cnf(k.is_defined, hypothesis)
defined(l)   cnf(l.is_defined, hypothesis)
defined(s)   cnf(s.is_defined, hypothesis)
defined(t)   cnf(t.is_defined, hypothesis)
-b=0        cnf(b_not_equal_to_additive_identity10, negated_conjecture)
-d=0        cnf(d_not_equal_to_additive_identity11, negated_conjecture)
s · t=u     cnf(multiply_equals_u12, negated_conjecture)
a · b-1=s    cnf(multiply_equals_s13, negated_conjecture)
c · d-1=t    cnf(multiply_equals_t14, negated_conjecture)
a · c=k     cnf(multiply_equals_k15, negated_conjecture)
b · d=l     cnf(multiply_equals_l16, negated_conjecture)
-k · t-1=u   cnf(multiply_not_equal_to_u17, negated_conjecture)
```

**FLD048-3.p** Fraction calculation, part 2

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a.is_defined, hypothesis)
defined(b)   cnf(b.is_defined, hypothesis)
defined(c)   cnf(c.is_defined, hypothesis)
defined(d)   cnf(d.is_defined, hypothesis)
-0 + b=0     cnf(not_sum5, negated_conjecture)
```



$-0 + d=0$        $\text{cnf}(\text{not\_sum}_6, \text{negated\_conjecture})$   
 $\neg(a \cdot b^{-1}) \cdot (c \cdot d^{-1}) = (a \cdot c) \cdot (b \cdot d)^{-1}$        $\text{cnf}(\text{not\_product}_7, \text{negated\_conjecture})$

**FLD048-4.p** Fraction calculation, part 2

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$        $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$        $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$        $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $\text{defined}(d)$        $\text{cnf}(d\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$        $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $\text{defined}(k)$        $\text{cnf}(k\_is\_defined, \text{hypothesis})$   
 $\text{defined}(l)$        $\text{cnf}(l\_is\_defined, \text{hypothesis})$   
 $\text{defined}(s)$        $\text{cnf}(s\_is\_defined, \text{hypothesis})$   
 $\text{defined}(t)$        $\text{cnf}(t\_is\_defined, \text{hypothesis})$   
 $-0 + b=0$        $\text{cnf}(\text{not\_sum}_{10}, \text{negated\_conjecture})$   
 $-0 + d=0$        $\text{cnf}(\text{not\_sum}_{11}, \text{negated\_conjecture})$   
 $s \cdot t = u$        $\text{cnf}(\text{product}_{12}, \text{negated\_conjecture})$   
 $a \cdot b^{-1} = s$        $\text{cnf}(\text{product}_{13}, \text{negated\_conjecture})$   
 $c \cdot d^{-1} = t$        $\text{cnf}(\text{product}_{14}, \text{negated\_conjecture})$   
 $a \cdot c = k$        $\text{cnf}(\text{product}_{15}, \text{negated\_conjecture})$   
 $b \cdot d = l$        $\text{cnf}(\text{product}_{16}, \text{negated\_conjecture})$   
 $\neg k \cdot l^{-1} = u$        $\text{cnf}(\text{not\_product}_{17}, \text{negated\_conjecture})$

**FLD049-1.p** Fraction calculation, part 3

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$        $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$        $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$        $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $\text{defined}(d)$        $\text{cnf}(d\_is\_defined, \text{hypothesis})$   
 $\neg b = 0$        $\text{cnf}(b\_not\_equal\_to\_additive\_identity}_5, \text{negated\_conjecture})$   
 $\neg d = 0$        $\text{cnf}(d\_not\_equal\_to\_additive\_identity}_6, \text{negated\_conjecture})$   
 $a \cdot b^{-1} = c \cdot d^{-1}$        $\text{cnf}(\text{multiply\_equals\_multiply}_7, \text{negated\_conjecture})$   
 $\neg a \cdot d = b \cdot c$        $\text{cnf}(\text{multiply\_not\_equal\_to\_multiply}_8, \text{negated\_conjecture})$

**FLD049-2.p** Fraction calculation, part 3

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$        $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$        $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$        $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $\text{defined}(d)$        $\text{cnf}(d\_is\_defined, \text{hypothesis})$   
 $\text{defined}(k)$        $\text{cnf}(k\_is\_defined, \text{hypothesis})$   
 $\text{defined}(s)$        $\text{cnf}(s\_is\_defined, \text{hypothesis})$   
 $\neg b = 0$        $\text{cnf}(b\_not\_equal\_to\_additive\_identity}_7, \text{negated\_conjecture})$   
 $\neg d = 0$        $\text{cnf}(d\_not\_equal\_to\_additive\_identity}_8, \text{negated\_conjecture})$   
 $a \cdot b^{-1} = s$        $\text{cnf}(\text{multiply\_equals\_s}_9, \text{negated\_conjecture})$   
 $c \cdot d^{-1} = s$        $\text{cnf}(\text{multiply\_equals\_s}_{10}, \text{negated\_conjecture})$   
 $a \cdot d = k$        $\text{cnf}(\text{multiply\_equals\_k}_{11}, \text{negated\_conjecture})$   
 $\neg b \cdot c = k$        $\text{cnf}(\text{multiply\_not\_equal\_to\_k}_{12}, \text{negated\_conjecture})$

**FLD049-3.p** Fraction calculation, part 3

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$        $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$        $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$        $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $\text{defined}(d)$        $\text{cnf}(d\_is\_defined, \text{hypothesis})$   
 $-0 + b=0$        $\text{cnf}(\text{not\_sum}_5, \text{negated\_conjecture})$   
 $-0 + d=0$        $\text{cnf}(\text{not\_sum}_6, \text{negated\_conjecture})$   
 $a \cdot b^{-1} = c \cdot d^{-1}$        $\text{cnf}(\text{product}_7, \text{negated\_conjecture})$   
 $\neg a \cdot d = b \cdot c$        $\text{cnf}(\text{not\_product}_8, \text{negated\_conjecture})$

**FLD049-4.p** Fraction calculation, part 3

$\text{include}(\text{'Axioms/FLD002-0.ax'})$

```

defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
defined(k)    cnf(k.is_defined, hypothesis)
defined(s)    cnf(s.is_defined, hypothesis)
- 0 + b=0    cnf(not_sum7, negated_conjecture)
- 0 + d=0    cnf(not_sum8, negated_conjecture)
a · b-1=s    cnf(product9, negated_conjecture)
c · d-1=s    cnf(product10, negated_conjecture)
a · d=k      cnf(product11, negated_conjecture)
- b · c=k    cnf(not_product12, negated_conjecture)

```

**FLD050-1.p** Fraction calculation, part 4

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
- b=0        cnf(b_not_equal_to_additive_identity5, negated_conjecture)
- d=0        cnf(d_not_equal_to_additive_identity6, negated_conjecture)
a · d=b · c   cnf(multiply_equals_multiply7, negated_conjecture)
- a · b-1=c · d-1  cnf(multiply_not_equal_to_multiply8, negated_conjecture)

```

**FLD050-2.p** Fraction calculation, part 4

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
defined(k)    cnf(k.is_defined, hypothesis)
defined(s)    cnf(s.is_defined, hypothesis)
- b=0        cnf(b_not_equal_to_additive_identity7, negated_conjecture)
- d=0        cnf(d_not_equal_to_additive_identity8, negated_conjecture)
a · b-1=s    cnf(multiply_equals_s9, negated_conjecture)
a · d=k      cnf(multiply_equals_k10, negated_conjecture)
b · c=k      cnf(multiply_equals_k11, negated_conjecture)
- c · d-1=s  cnf(multiply_not_equal_to_s12, negated_conjecture)

```

**FLD050-3.p** Fraction calculation, part 4

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
- 0 + b=0    cnf(not_sum5, negated_conjecture)
- 0 + d=0    cnf(not_sum6, negated_conjecture)
a · d=b · c   cnf(product7, negated_conjecture)
- a · b-1=c · d-1  cnf(not_product8, negated_conjecture)

```

**FLD050-4.p** Fraction calculation, part 4

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
defined(k)    cnf(k.is_defined, hypothesis)
defined(s)    cnf(s.is_defined, hypothesis)
- 0 + b=0    cnf(not_sum7, negated_conjecture)
- 0 + d=0    cnf(not_sum8, negated_conjecture)
a · b-1=s    cnf(product9, negated_conjecture)

```

$a \cdot d = k$     `cnf(product10, negated_conjecture)`  
 $b \cdot c = k$     `cnf(product11, negated_conjecture)`  
 $\neg c \cdot d^{-1} = s$     `cnf(not_product12, negated_conjecture)`

**FLD051-1.p** Fraction calculation, part 5

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
`defined(c)`    `cnf(c_is_defined, hypothesis)`  
`defined(d)`    `cnf(d_is_defined, hypothesis)`  
 $\neg b = 0$     `cnf(b_not_equal_to_additive_identity5, negated_conjecture)`  
 $\neg c = 0$     `cnf(c_not_equal_to_additive_identity6, negated_conjecture)`  
 $\neg d = 0$     `cnf(d_not_equal_to_additive_identity7, negated_conjecture)`  
 $\neg (a \cdot b^{-1}) \cdot (c \cdot d^{-1})^{-1} = (a \cdot d) \cdot (b \cdot c)^{-1}$     `cnf(multiply_not_equal_to_multiply8, negated_conjecture)`

**FLD051-2.p** Fraction calculation, part 5

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
`defined(c)`    `cnf(c_is_defined, hypothesis)`  
`defined(d)`    `cnf(d_is_defined, hypothesis)`  
`defined(u)`    `cnf(u_is_defined, hypothesis)`  
`defined(k)`    `cnf(k_is_defined, hypothesis)`  
`defined(l)`    `cnf(l_is_defined, hypothesis)`  
`defined(s)`    `cnf(s_is_defined, hypothesis)`  
`defined(t)`    `cnf(t_is_defined, hypothesis)`  
 $\neg b = 0$     `cnf(b_not_equal_to_additive_identity10, negated_conjecture)`  
 $\neg c = 0$     `cnf(c_not_equal_to_additive_identity11, negated_conjecture)`  
 $\neg d = 0$     `cnf(d_not_equal_to_additive_identity12, negated_conjecture)`  
 $s \cdot t^{-1} = u$     `cnf(multiply_equals_u13, negated_conjecture)`  
 $a \cdot b^{-1} = s$     `cnf(multiply_equals_s14, negated_conjecture)`  
 $c \cdot d^{-1} = t$     `cnf(multiply_equals_t15, negated_conjecture)`  
 $a \cdot d = k$     `cnf(multiply_equals_k16, negated_conjecture)`  
 $b \cdot c = l$     `cnf(multiply_equals_l17, negated_conjecture)`  
 $\neg k \cdot t^{-1} = u$     `cnf(multiply_not_equal_to_u18, negated_conjecture)`

**FLD051-3.p** Fraction calculation, part 5

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
`defined(c)`    `cnf(c_is_defined, hypothesis)`  
`defined(d)`    `cnf(d_is_defined, hypothesis)`  
 $\neg 0 + b = 0$     `cnf(not_sum5, negated_conjecture)`  
 $\neg 0 + c = 0$     `cnf(not_sum6, negated_conjecture)`  
 $\neg 0 + d = 0$     `cnf(not_sum7, negated_conjecture)`  
 $\neg (a \cdot b^{-1}) \cdot (c \cdot d^{-1})^{-1} = (a \cdot d) \cdot (b \cdot c)^{-1}$     `cnf(not_product8, negated_conjecture)`

**FLD051-4.p** Fraction calculation, part 5

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
`defined(c)`    `cnf(c_is_defined, hypothesis)`  
`defined(d)`    `cnf(d_is_defined, hypothesis)`  
`defined(u)`    `cnf(u_is_defined, hypothesis)`  
`defined(k)`    `cnf(k_is_defined, hypothesis)`  
`defined(l)`    `cnf(l_is_defined, hypothesis)`  
`defined(s)`    `cnf(s_is_defined, hypothesis)`  
`defined(t)`    `cnf(t_is_defined, hypothesis)`  
 $\neg 0 + b = 0$     `cnf(not_sum10, negated_conjecture)`  
 $\neg 0 + c = 0$     `cnf(not_sum11, negated_conjecture)`  
 $\neg 0 + d = 0$     `cnf(not_sum12, negated_conjecture)`

$s \cdot t^{-1} = u$     cnf(product<sub>13</sub>, negated\_conjecture)  
 $a \cdot b^{-1} = s$     cnf(product<sub>14</sub>, negated\_conjecture)  
 $c \cdot d^{-1} = t$     cnf(product<sub>15</sub>, negated\_conjecture)  
 $a \cdot d = k$     cnf(product<sub>16</sub>, negated\_conjecture)  
 $b \cdot c = l$     cnf(product<sub>17</sub>, negated\_conjecture)  
 $\neg k \cdot l^{-1} = u$     cnf(not\_product<sub>18</sub>, negated\_conjecture)

**FLD052-1.p** Fraction calculation, part 6

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
defined( $d$ )    cnf(d\_is\_defined, hypothesis)  
 $\neg b = 0$     cnf(b\_not\_equal\_to\_additive\_identity<sub>5</sub>, negated\_conjecture)  
 $\neg d = 0$     cnf(d\_not\_equal\_to\_additive\_identity<sub>6</sub>, negated\_conjecture)  
 $\neg a \cdot b^{-1} + c \cdot d^{-1} = (a \cdot d + b \cdot c) \cdot (b \cdot d)^{-1}$     cnf(add\_not\_equal\_to\_multiply<sub>7</sub>, negated\_conjecture)

**FLD052-2.p** Fraction calculation, part 6

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
defined( $d$ )    cnf(d\_is\_defined, hypothesis)  
defined( $u$ )    cnf(u\_is\_defined, hypothesis)  
defined( $v$ )    cnf(v\_is\_defined, hypothesis)  
defined( $k$ )    cnf(k\_is\_defined, hypothesis)  
defined( $l$ )    cnf(l\_is\_defined, hypothesis)  
defined( $p$ )    cnf(p\_is\_defined, hypothesis)  
defined( $q$ )    cnf(q\_is\_defined, hypothesis)  
defined( $s$ )    cnf(s\_is\_defined, hypothesis)  
defined( $t$ )    cnf(t\_is\_defined, hypothesis)  
 $\neg b = 0$     cnf(b\_not\_equal\_to\_additive\_identity<sub>13</sub>, negated\_conjecture)  
 $\neg d = 0$     cnf(d\_not\_equal\_to\_additive\_identity<sub>14</sub>, negated\_conjecture)  
 $a \cdot b^{-1} = s$     cnf(multiply\_equals\_s<sub>15</sub>, negated\_conjecture)  
 $c \cdot d^{-1} = t$     cnf(multiply\_equals\_t<sub>16</sub>, negated\_conjecture)  
 $s + t = u$     cnf(add\_equals\_u<sub>17</sub>, negated\_conjecture)  
 $a \cdot d = p$     cnf(multiply\_equals\_p<sub>18</sub>, negated\_conjecture)  
 $b \cdot c = q$     cnf(multiply\_equals\_q<sub>19</sub>, negated\_conjecture)  
 $p + q = k$     cnf(add\_equals\_k<sub>20</sub>, negated\_conjecture)  
 $b \cdot d = l$     cnf(multiply\_equals\_l<sub>21</sub>, negated\_conjecture)  
 $\neg k \cdot l^{-1} = u$     cnf(multiply\_not\_equal\_to\_u<sub>22</sub>, negated\_conjecture)

**FLD052-3.p** Fraction calculation, part 6

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
defined( $d$ )    cnf(d\_is\_defined, hypothesis)  
 $\neg 0 + b = 0$     cnf(not\_sum<sub>5</sub>, negated\_conjecture)  
 $\neg 0 + d = 0$     cnf(not\_sum<sub>6</sub>, negated\_conjecture)  
 $\neg a \cdot b^{-1} + c \cdot d^{-1} = (a \cdot d + b \cdot c) \cdot (b \cdot d)^{-1}$     cnf(not\_sum<sub>7</sub>, negated\_conjecture)

**FLD052-4.p** Fraction calculation, part 6

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
defined( $d$ )    cnf(d\_is\_defined, hypothesis)  
defined( $u$ )    cnf(u\_is\_defined, hypothesis)  
defined( $v$ )    cnf(v\_is\_defined, hypothesis)  
defined( $k$ )    cnf(k\_is\_defined, hypothesis)

```

defined(l)    cnf(l.is_defined, hypothesis)
defined(p)    cnf(p.is_defined, hypothesis)
defined(q)    cnf(q.is_defined, hypothesis)
defined(s)    cnf(s.is_defined, hypothesis)
defined(t)    cnf(t.is_defined, hypothesis)
-0 + b=0     cnf(not_sum13, negated_conjecture)
-0 + d=0     cnf(not_sum14, negated_conjecture)
a · b-1=s   cnf(product15, negated_conjecture)
c · d-1=t   cnf(product16, negated_conjecture)
s + t=u      cnf(sum17, negated_conjecture)
a · d=p      cnf(product18, negated_conjecture)
b · c=q      cnf(product19, negated_conjecture)
p + q=k      cnf(sum20, negated_conjecture)
b · d=l      cnf(product21, negated_conjecture)
¬k · l-1=u  cnf(not_product22, negated_conjecture)

```

**FLD053-1.p** Fraction calculation, part 7

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
-b=0         cnf(b_not_equal_to_additive_identity5, negated_conjecture)
-d=0         cnf(d_not_equal_to_additive_identity6, negated_conjecture)
-a · b-1 + -c · d-1=(a · d + -b · c) · (b · d)-1  cnf(add_not_equal_to_multiply7, negated_conjecture)

```

**FLD053-2.p** Fraction calculation, part 7

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
defined(u)    cnf(u.is_defined, hypothesis)
defined(k)    cnf(k.is_defined, hypothesis)
defined(l)    cnf(l.is_defined, hypothesis)
defined(p)    cnf(p.is_defined, hypothesis)
defined(q)    cnf(q.is_defined, hypothesis)
defined(s)    cnf(s.is_defined, hypothesis)
defined(t)    cnf(t.is_defined, hypothesis)
-b=0         cnf(b_not_equal_to_additive_identity12, negated_conjecture)
-d=0         cnf(d_not_equal_to_additive_identity13, negated_conjecture)
a · b-1=s   cnf(multiply_equals_s14, negated_conjecture)
c · d-1=t   cnf(multiply_equals_t15, negated_conjecture)
s + -t=u     cnf(add_equals_u16, negated_conjecture)
a · d=p      cnf(multiply_equals_p17, negated_conjecture)
b · c=q      cnf(multiply_equals_q18, negated_conjecture)
p + -q=k     cnf(add_equals_k19, negated_conjecture)
b · d=l      cnf(multiply_equals_l20, negated_conjecture)
¬k · l-1=u  cnf(multiply_not_equal_to_u21, negated_conjecture)

```

**FLD053-3.p** Fraction calculation, part 7

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
-0 + b=0     cnf(not_sum5, negated_conjecture)
-0 + d=0     cnf(not_sum6, negated_conjecture)
-a · b-1 + -c · d-1=(a · d + -b · c) · (b · d)-1  cnf(not_sum7, negated_conjecture)

```

**FLD053-4.p** Fraction calculation, part 7

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(k)    cnf(k_is_defined, hypothesis)
defined(l)    cnf(l_is_defined, hypothesis)
defined(p)    cnf(p_is_defined, hypothesis)
defined(q)    cnf(q_is_defined, hypothesis)
defined(s)    cnf(s_is_defined, hypothesis)
defined(t)    cnf(t_is_defined, hypothesis)
-0 + b=0     cnf(not_sum12, negated_conjecture)
-0 + d=0     cnf(not_sum13, negated_conjecture)
a · b-1=s   cnf(product14, negated_conjecture)
c · d-1=t   cnf(product15, negated_conjecture)
s + -t=u     cnf(sum16, negated_conjecture)
a · d=p      cnf(product17, negated_conjecture)
b · c=q      cnf(product18, negated_conjecture)
p + -q=k     cnf(sum19, negated_conjecture)
b · d=l      cnf(product20, negated_conjecture)
-k · l-1=u   cnf(not_product21, negated_conjecture)

```

**FLD054-1.p** Fraction calculation, part 8

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
-a=0         cnf(a_not_equal_to_additive_identity3, negated_conjecture)
-b=0         cnf(b_not_equal_to_additive_identity4, negated_conjecture)
-a-1 + b-1=(a + b) · (a · b)-1   cnf(add_not_equal_to_multiply5, negated_conjecture)

```

**FLD054-2.p** Fraction calculation, part 8

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(k)    cnf(k_is_defined, hypothesis)
defined(l)    cnf(l_is_defined, hypothesis)
-a=0         cnf(a_not_equal_to_additive_identity6, negated_conjecture)
-b=0         cnf(b_not_equal_to_additive_identity7, negated_conjecture)
a-1 + b-1=u   cnf(add_equals_u8, negated_conjecture)
a + b=k      cnf(add_equals_k9, negated_conjecture)
a · b=l      cnf(multiply_equals_l10, negated_conjecture)
-k · l-1=u   cnf(multiply_not_equal_to_u11, negated_conjecture)

```

**FLD054-3.p** Fraction calculation, part 8

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
-0 + a=0     cnf(not_sum3, negated_conjecture)
-0 + b=0     cnf(not_sum4, negated_conjecture)
-a-1 + b-1=(a + b) · (a · b)-1   cnf(not_sum5, negated_conjecture)

```

**FLD054-4.p** Fraction calculation, part 8

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(k)    cnf(k_is_defined, hypothesis)
defined(l)    cnf(l_is_defined, hypothesis)
-0 + a=0     cnf(not_sum6, negated_conjecture)

```

$\neg 0 + b=0$      $\text{cnf}(\text{not\_sum}_7, \text{negated\_conjecture})$   
 $a^{-1} + b^{-1}=u$      $\text{cnf}(\text{sum}_8, \text{negated\_conjecture})$   
 $a + b=k$      $\text{cnf}(\text{sum}_9, \text{negated\_conjecture})$   
 $a \cdot b=l$      $\text{cnf}(\text{product}_{10}, \text{negated\_conjecture})$   
 $\neg k \cdot l^{-1}=u$      $\text{cnf}(\text{not\_product}_{11}, \text{negated\_conjecture})$

**FLD055-1.p** Compatibility of order and equality relation

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $b=c$      $\text{cnf}(b\_equals\_c_5, \text{negated\_conjecture})$   
 $\neg a \leq c$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD055-3.p** Compatibility of order and equality relation

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $0 + b=c$      $\text{cnf}(\text{sum}_5, \text{negated\_conjecture})$   
 $\neg a \leq c$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD056-1.p** Reflexivity of the order relation

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\neg a \leq a$      $\text{cnf}(\text{not\_less\_or\_equal}_2, \text{negated\_conjecture})$

**FLD056-3.p** Reflexivity of the order relation

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\neg a \leq a$      $\text{cnf}(\text{not\_less\_or\_equal}_2, \text{negated\_conjecture})$

**FLD057-1.p** 0 is less than 1

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\neg 0 \leq 1$      $\text{cnf}(\text{not\_less\_or\_equal}_1, \text{negated\_conjecture})$

**FLD057-3.p** 0 is less than 1

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\neg 0 \leq 1$      $\text{cnf}(\text{not\_less\_or\_equal}_1, \text{negated\_conjecture})$

**FLD058-1.p** If a greater 0 and b greater or equal a the b greater 0

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $0 \leq a$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $\neg a=0$      $\text{cnf}(a\_not\_equal\_to\_additive\_identity_5, \text{negated\_conjecture})$   
 $b=0$      $\text{cnf}(b\_equals\_additive\_identity_6, \text{negated\_conjecture})$

**FLD058-3.p** If a greater 0 and b greater or equal a the b greater 0

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $0 \leq a$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $\neg 0 + a=0$      $\text{cnf}(\text{not\_sum}_5, \text{negated\_conjecture})$   
 $0 + b=0$      $\text{cnf}(\text{sum}_6, \text{negated\_conjecture})$

**FLD059-1.p** If a greater or equal 0, then 2a greater or equal 0

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $0 \leq a$      $\text{cnf}(\text{less\_or\_equal}_2, \text{negated\_conjecture})$   
 $\neg 0 \leq a + a$      $\text{cnf}(\text{not\_less\_or\_equal}_3, \text{negated\_conjecture})$

**FLD059-2.p** If a greater or equal 0, then 2a greater or equal 0

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
 $0 \leq a$     cnf(less_or_equal3, negated_conjecture)
 $a + a = u$    cnf(add_equals_u4, negated_conjecture)
 $-0 \leq u$    cnf(not_less_or_equal5, negated_conjecture)
```

**FLD059-3.p** If a greater or equal 0, then 2a greater or equal 0

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
 $0 \leq a$     cnf(less_or_equal2, negated_conjecture)
 $-0 \leq a + a$   cnf(not_less_or_equal3, negated_conjecture)
```

**FLD059-4.p** If a greater or equal 0, then 2a greater or equal 0

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
 $0 \leq a$     cnf(less_or_equal3, negated_conjecture)
 $a + a = u$    cnf(sum4, negated_conjecture)
 $-0 \leq u$    cnf(not_less_or_equal5, negated_conjecture)
```

**FLD060-1.p** If b greater or equal b, then 2b greater or equal 2a

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $a \leq b$     cnf(less_or_equal3, negated_conjecture)
 $-a + a \leq b + b$   cnf(not_less_or_equal4, negated_conjecture)
```

**FLD060-2.p** If b greater or equal b, then 2b greater or equal 2a

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
 $a + a = u$    cnf(add_equals_u5, negated_conjecture)
 $b + b = v$    cnf(add_equals_v6, negated_conjecture)
 $a \leq b$     cnf(less_or_equal7, negated_conjecture)
 $-u \leq v$    cnf(not_less_or_equal8, negated_conjecture)
```

**FLD060-3.p** If b greater or equal b, then 2b greater or equal 2a

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $a \leq b$     cnf(less_or_equal3, negated_conjecture)
 $-a + a \leq b + b$   cnf(not_less_or_equal4, negated_conjecture)
```

**FLD060-4.p** If b greater or equal b, then 2b greater or equal 2a

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
 $a \leq b$     cnf(less_or_equal5, negated_conjecture)
 $a + a = u$    cnf(sum6, negated_conjecture)
 $b + b = v$    cnf(sum7, negated_conjecture)
 $-u \leq v$    cnf(not_less_or_equal8, negated_conjecture)
```

**FLD061-1.p** The resulting inequality of a summation of two inequalities

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
```



$\text{defined}(d)$      $\text{cnf}(d\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $c \leq d$      $\text{cnf}(\text{less\_or\_equal}_6, \text{negated\_conjecture})$   
 $\neg a + c \leq d + b$      $\text{cnf}(\text{not\_less\_or\_equal}_7, \text{negated\_conjecture})$

**FLD061-2.p** The resulting inequality of a summation of two inequalities  
 $\text{include}('Axioms/FLD001-0.ax')$

$\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $\text{defined}(d)$      $\text{cnf}(d\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $\text{defined}(v)$      $\text{cnf}(v\_is\_defined, \text{hypothesis})$   
 $a + c = u$      $\text{cnf}(\text{add\_equals\_u}_7, \text{negated\_conjecture})$   
 $d + b = v$      $\text{cnf}(\text{add\_equals\_v}_8, \text{negated\_conjecture})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_9, \text{negated\_conjecture})$   
 $c \leq d$      $\text{cnf}(\text{less\_or\_equal}_{10}, \text{negated\_conjecture})$   
 $\neg u \leq v$      $\text{cnf}(\text{not\_less\_or\_equal}_{11}, \text{negated\_conjecture})$

**FLD061-3.p** The resulting inequality of a summation of two inequalities  
 $\text{include}('Axioms/FLD002-0.ax')$

$\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $\text{defined}(d)$      $\text{cnf}(d\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $c \leq d$      $\text{cnf}(\text{less\_or\_equal}_6, \text{negated\_conjecture})$   
 $\neg a + c \leq d + b$      $\text{cnf}(\text{not\_less\_or\_equal}_7, \text{negated\_conjecture})$

**FLD061-4.p** The resulting inequality of a summation of two inequalities  
 $\text{include}('Axioms/FLD002-0.ax')$

$\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $\text{defined}(d)$      $\text{cnf}(d\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $\text{defined}(v)$      $\text{cnf}(v\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_7, \text{negated\_conjecture})$   
 $c \leq d$      $\text{cnf}(\text{less\_or\_equal}_8, \text{negated\_conjecture})$   
 $a + c = u$      $\text{cnf}(\text{sum}_9, \text{negated\_conjecture})$   
 $d + b = v$      $\text{cnf}(\text{sum}_{10}, \text{negated\_conjecture})$   
 $\neg u \leq v$      $\text{cnf}(\text{not\_less\_or\_equal}_{11}, \text{negated\_conjecture})$

**FLD062-1.p** Compatibility of the order relation and additive inverses  
 $\text{include}('Axioms/FLD001-0.ax')$

$\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $\neg -b \leq -a$      $\text{cnf}(\text{not\_less\_or\_equal}_4, \text{negated\_conjecture})$

**FLD062-3.p** Compatibility of the order relation and additive inverses  
 $\text{include}('Axioms/FLD002-0.ax')$

$\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $\neg -b \leq -a$      $\text{cnf}(\text{not\_less\_or\_equal}_4, \text{negated\_conjecture})$

**FLD063-1.p** Elimination of additive inverses in an order relation  
 $\text{include}('Axioms/FLD001-0.ax')$

$\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $-b \leq -a$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $\neg a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_4, \text{negated\_conjecture})$

**FLD063-3.p** Elimination of additive inverses in an order relation

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
 $-b \leq -a$    cnf(less_or_equal3, negated_conjecture)
 $\neg a \leq b$    cnf(not_less_or_equal4, negated_conjecture)
```

**FLD064-1.p** Side change of a term in an order relation, part 1

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
 $0 \leq a$      cnf(less_or_equal2, negated_conjecture)
 $\neg \neg a \leq 0$    cnf(not_less_or_equal3, negated_conjecture)
```

**FLD064-3.p** Side change of a term in an order relation, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
 $0 \leq a$      cnf(less_or_equal2, negated_conjecture)
 $\neg \neg a \leq 0$    cnf(not_less_or_equal3, negated_conjecture)
```

**FLD065-1.p** Side change of a term in an order relation, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
 $-a \leq 0$      cnf(less_or_equal2, negated_conjecture)
 $\neg 0 \leq a$     cnf(not_less_or_equal3, negated_conjecture)
```

**FLD065-3.p** Side change of a term in an order relation, part 2

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
 $-a \leq 0$      cnf(less_or_equal2, negated_conjecture)
 $\neg 0 \leq a$     cnf(not_less_or_equal3, negated_conjecture)
```

**FLD066-1.p** Elimination of a summation in an order relation

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
 $a + c \leq b + c$    cnf(less_or_equal4, negated_conjecture)
 $\neg a \leq b$        cnf(not_less_or_equal5, negated_conjecture)
```

**FLD066-3.p** Elimination of a summation in an order relation

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
defined(v)   cnf(v_is_defined, hypothesis)
 $a + c = u$        cnf(sum6, negated_conjecture)
 $b + c = v$        cnf(sum7, negated_conjecture)
 $u \leq v$         cnf(less_or_equal8, negated_conjecture)
 $\neg a \leq b$      cnf(not_less_or_equal9, negated_conjecture)
```

**FLD067-1.p** Side change in an order relation, part 1

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
 $a \leq b$        cnf(less_or_equal3, negated_conjecture)
 $\neg 0 \leq b + -a$    cnf(not_less_or_equal4, negated_conjecture)
```

**FLD067-2.p** Side change in an order relation, part 1

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
 $a \leq b$        cnf(less_or_equal4, negated_conjecture)
```

$b + -a = u$      $\text{cnf}(\text{add\_equals\_u}_5, \text{negated\_conjecture})$   
 $-0 \leq u$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD067-3.p** Side change in an order relation, part 1

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $-0 \leq b + -a$      $\text{cnf}(\text{not\_less\_or\_equal}_4, \text{negated\_conjecture})$

**FLD067-4.p** Side change in an order relation, part 1

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $b + -a = u$      $\text{cnf}(\text{sum}_5, \text{negated\_conjecture})$   
 $-0 \leq u$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD068-1.p** Side change of a term in an order relation, part 2

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $0 \leq b + -a$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $-a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_4, \text{negated\_conjecture})$

**FLD068-2.p** Side change of a term in an order relation, part 2

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $b + -a = u$      $\text{cnf}(\text{add\_equals\_u}_4, \text{negated\_conjecture})$   
 $0 \leq u$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $-a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD068-3.p** Side change of a term in an order relation, part 2

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $0 \leq b + -a$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $-a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_4, \text{negated\_conjecture})$

**FLD068-4.p** Side change of a term in an order relation, part 2

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $b + -a = u$      $\text{cnf}(\text{sum}_4, \text{negated\_conjecture})$   
 $0 \leq u$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $-a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD069-1.p** If  $b > 0$  and  $a \geq 0$ , then  $a + b$  not 0

$\text{include}(\text{'Axioms/FLD001-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $0 \leq a$      $\text{cnf}(\text{less\_or\_equal}_3, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $-b = 0$      $\text{cnf}(b\_not\_equal\_to\_additive\_identity_5, \text{negated\_conjecture})$   
 $a + b = 0$      $\text{cnf}(\text{add\_equals\_additive\_identity}_6, \text{negated\_conjecture})$

**FLD069-3.p** If  $b > 0$  and  $a \geq 0$ , then  $a + b$  not 0

$\text{include}(\text{'Axioms/FLD002-0.ax'})$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$

$0 \leq a$     cnf(less\_or\_equal<sub>3</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $\neg 0 + b=0$     cnf(not\_sum<sub>5</sub>, negated\_conjecture)  
 $a + b=0$     cnf(sum<sub>6</sub>, negated\_conjecture)

**FLD070-1.p** One-sided addition of two order relations

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
 $0 \leq a$     cnf(less\_or\_equal<sub>3</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $\neg 0 \leq a + b$     cnf(not\_less\_or\_equal<sub>5</sub>, negated\_conjecture)

**FLD070-2.p** One-sided addition of two order relations

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
 $a + b=c$     cnf(add\_equals\_c<sub>4</sub>, negated\_conjecture)  
 $0 \leq a$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>6</sub>, negated\_conjecture)  
 $\neg 0 \leq c$     cnf(not\_less\_or\_equal<sub>7</sub>, negated\_conjecture)

**FLD070-3.p** One-sided addition of two order relations

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
 $0 \leq a$     cnf(less\_or\_equal<sub>3</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $\neg 0 \leq a + b$     cnf(not\_less\_or\_equal<sub>5</sub>, negated\_conjecture)

**FLD070-4.p** One-sided addition of two order relations

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
 $0 \leq a$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $a + b=c$     cnf(sum<sub>6</sub>, negated\_conjecture)  
 $\neg 0 \leq c$     cnf(not\_less\_or\_equal<sub>7</sub>, negated\_conjecture)

**FLD071-1.p** One-sided multiplication of two order relations, part 1

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
 $0 \leq a$     cnf(less\_or\_equal<sub>3</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $\neg 0 \leq a \cdot b$     cnf(not\_less\_or\_equal<sub>5</sub>, negated\_conjecture)

**FLD071-2.p** One-sided multiplication of two order relations, part 1

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $u$ )    cnf(u\_is\_defined, hypothesis)  
 $0 \leq a$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $a \cdot b=u$     cnf(multiply\_equals\_u<sub>6</sub>, negated\_conjecture)  
 $\neg 0 \leq u$     cnf(not\_less\_or\_equal<sub>7</sub>, negated\_conjecture)

**FLD071-3.p** One-sided multiplication of two order relations, part 1

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)

$0 \leq a$     `cnf(less_or_equal3, negated_conjecture)`  
 $0 \leq b$     `cnf(less_or_equal4, negated_conjecture)`  
 $\neg 0 \leq a \cdot b$     `cnf(not_less_or_equal5, negated_conjecture)`

**FLD071-4.p** One-sided multiplication of two order relations, part 1

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
`defined(u)`    `cnf(u_is_defined, hypothesis)`  
 $0 \leq a$     `cnf(less_or_equal4, negated_conjecture)`  
 $0 \leq b$     `cnf(less_or_equal5, negated_conjecture)`  
 $a \cdot b = u$     `cnf(product6, negated_conjecture)`  
 $\neg 0 \leq u$     `cnf(not_less_or_equal7, negated_conjecture)`

**FLD072-1.p** One-sided multiplication of two order relations, part 2

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
 $a \leq 0$     `cnf(less_or_equal3, negated_conjecture)`  
 $0 \leq b$     `cnf(less_or_equal4, negated_conjecture)`  
 $\neg a \cdot b \leq 0$     `cnf(not_less_or_equal5, negated_conjecture)`

**FLD072-2.p** One-sided multiplication of two order relations, part 2

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
`defined(u)`    `cnf(u_is_defined, hypothesis)`  
 $a \leq 0$     `cnf(less_or_equal4, negated_conjecture)`  
 $0 \leq b$     `cnf(less_or_equal5, negated_conjecture)`  
 $a \cdot b = u$     `cnf(multiply_equals_u6, negated_conjecture)`  
 $\neg u \leq 0$     `cnf(not_less_or_equal7, negated_conjecture)`

**FLD072-3.p** One-sided multiplication of two order relations, part 2

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
 $a \leq 0$     `cnf(less_or_equal3, negated_conjecture)`  
 $0 \leq b$     `cnf(less_or_equal4, negated_conjecture)`  
 $\neg a \cdot b \leq 0$     `cnf(not_less_or_equal5, negated_conjecture)`

**FLD072-4.p** One-sided multiplication of two order relations, part 2

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
`defined(u)`    `cnf(u_is_defined, hypothesis)`  
 $a \leq 0$     `cnf(less_or_equal4, negated_conjecture)`  
 $0 \leq b$     `cnf(less_or_equal5, negated_conjecture)`  
 $a \cdot b = u$     `cnf(product6, negated_conjecture)`  
 $\neg u \leq 0$     `cnf(not_less_or_equal7, negated_conjecture)`

**FLD073-1.p** One-sided multiplication of two order relations, part 3

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
 $a \leq 0$     `cnf(less_or_equal3, negated_conjecture)`  
 $b \leq 0$     `cnf(less_or_equal4, negated_conjecture)`  
 $\neg 0 \leq a \cdot b$     `cnf(not_less_or_equal5, negated_conjecture)`

**FLD073-2.p** One-sided multiplication of two order relations, part 3

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
`defined(b)`    `cnf(b_is_defined, hypothesis)`  
`defined(u)`    `cnf(u_is_defined, hypothesis)`

$a \leq 0$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $b \leq 0$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $a \cdot b = u$     cnf(multiply\_equals\_u<sub>6</sub>, negated\_conjecture)  
 $\neg 0 \leq u$     cnf(not\_less\_or\_equal<sub>7</sub>, negated\_conjecture)

**FLD073-3.p** One-sided multiplication of two order relations, part 3

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf( $a$ \_is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ \_is\_defined, hypothesis)  
 $a \leq 0$     cnf(less\_or\_equal<sub>3</sub>, negated\_conjecture)  
 $b \leq 0$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $\neg 0 \leq a \cdot b$     cnf(not\_less\_or\_equal<sub>5</sub>, negated\_conjecture)

**FLD073-4.p** One-sided multiplication of two order relations, part 3

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf( $a$ \_is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ \_is\_defined, hypothesis)  
defined( $u$ )    cnf( $u$ \_is\_defined, hypothesis)  
 $a \leq 0$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $b \leq 0$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $a \cdot b = u$     cnf(product<sub>6</sub>, negated\_conjecture)  
 $\neg 0 \leq u$     cnf(not\_less\_or\_equal<sub>7</sub>, negated\_conjecture)

**FLD074-1.p** Two-sided multiplication in an order relation, part 1

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf( $a$ \_is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ \_is\_defined, hypothesis)  
defined( $c$ )    cnf( $c$ \_is\_defined, hypothesis)  
 $a \leq b$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $0 \leq c$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $\neg a \cdot c \leq b \cdot c$     cnf(not\_less\_or\_equal<sub>6</sub>, negated\_conjecture)

**FLD074-2.p** Two-sided multiplication in an order relation, part 1

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf( $a$ \_is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ \_is\_defined, hypothesis)  
defined( $c$ )    cnf( $c$ \_is\_defined, hypothesis)  
defined( $u$ )    cnf( $u$ \_is\_defined, hypothesis)  
defined( $v$ )    cnf( $v$ \_is\_defined, hypothesis)  
 $a \cdot c = u$     cnf(multiply\_equals\_u<sub>6</sub>, negated\_conjecture)  
 $b \cdot c = v$     cnf(multiply\_equals\_v<sub>7</sub>, negated\_conjecture)  
 $a \leq b$     cnf(less\_or\_equal<sub>8</sub>, negated\_conjecture)  
 $0 \leq c$     cnf(less\_or\_equal<sub>9</sub>, negated\_conjecture)  
 $\neg u \leq v$     cnf(not\_less\_or\_equal<sub>10</sub>, negated\_conjecture)

**FLD074-3.p** Two-sided multiplication in an order relation, part 1

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf( $a$ \_is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ \_is\_defined, hypothesis)  
defined( $c$ )    cnf( $c$ \_is\_defined, hypothesis)  
 $a \leq b$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $0 \leq c$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $\neg a \cdot c \leq b \cdot c$     cnf(not\_less\_or\_equal<sub>6</sub>, negated\_conjecture)

**FLD074-4.p** Two-sided multiplication in an order relation, part 1

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf( $a$ \_is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ \_is\_defined, hypothesis)  
defined( $c$ )    cnf( $c$ \_is\_defined, hypothesis)  
defined( $u$ )    cnf( $u$ \_is\_defined, hypothesis)  
defined( $v$ )    cnf( $v$ \_is\_defined, hypothesis)  
 $a \leq b$     cnf(less\_or\_equal<sub>6</sub>, negated\_conjecture)  
 $0 \leq c$     cnf(less\_or\_equal<sub>7</sub>, negated\_conjecture)

$a \cdot c = u$     cnf(product<sub>8</sub>, negated\_conjecture)  
 $b \cdot c = v$     cnf(product<sub>9</sub>, negated\_conjecture)  
 $\neg u \leq v$     cnf(not\_less\_or\_equal<sub>10</sub>, negated\_conjecture)

**FLD075-1.p** Two-sided multiplication in an order relation, part 2

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
 $a \leq b$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $c \leq 0$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $\neg b \cdot c \leq a \cdot c$     cnf(not\_less\_or\_equal<sub>6</sub>, negated\_conjecture)

**FLD075-2.p** Two-sided multiplication in an order relation, part 2

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
defined( $u$ )    cnf(u\_is\_defined, hypothesis)  
defined( $v$ )    cnf(v\_is\_defined, hypothesis)  
 $a \cdot c = u$     cnf(multiply\_equals\_u<sub>6</sub>, negated\_conjecture)  
 $b \cdot c = v$     cnf(multiply\_equals\_v<sub>7</sub>, negated\_conjecture)  
 $a \leq b$     cnf(less\_or\_equal<sub>8</sub>, negated\_conjecture)  
 $c \leq 0$     cnf(less\_or\_equal<sub>9</sub>, negated\_conjecture)  
 $\neg v \leq u$     cnf(not\_less\_or\_equal<sub>10</sub>, negated\_conjecture)

**FLD075-3.p** Two-sided multiplication in an order relation, part 2

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
 $a \leq b$     cnf(less\_or\_equal<sub>4</sub>, negated\_conjecture)  
 $c \leq 0$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $\neg b \cdot c \leq a \cdot c$     cnf(not\_less\_or\_equal<sub>6</sub>, negated\_conjecture)

**FLD075-4.p** Two-sided multiplication in an order relation, part 2

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
defined( $u$ )    cnf(u\_is\_defined, hypothesis)  
defined( $v$ )    cnf(v\_is\_defined, hypothesis)  
 $a \leq b$     cnf(less\_or\_equal<sub>6</sub>, negated\_conjecture)  
 $c \leq 0$     cnf(less\_or\_equal<sub>7</sub>, negated\_conjecture)  
 $a \cdot c = u$     cnf(product<sub>8</sub>, negated\_conjecture)  
 $b \cdot c = v$     cnf(product<sub>9</sub>, negated\_conjecture)  
 $\neg v \leq u$     cnf(not\_less\_or\_equal<sub>10</sub>, negated\_conjecture)

**FLD076-1.p** Two-sided multiplication in an order relation, part 3

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)  
 $\neg c = 0$     cnf(c\_not\_equal\_to\_additive\_identity<sub>4</sub>, negated\_conjecture)  
 $0 \leq c$     cnf(less\_or\_equal<sub>5</sub>, negated\_conjecture)  
 $a \cdot c \leq b \cdot c$     cnf(less\_or\_equal<sub>6</sub>, negated\_conjecture)  
 $\neg a \leq b$     cnf(not\_less\_or\_equal<sub>7</sub>, negated\_conjecture)

**FLD076-2.p** Two-sided multiplication in an order relation, part 3

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf(a\_is\_defined, hypothesis)  
defined( $b$ )    cnf(b\_is\_defined, hypothesis)  
defined( $c$ )    cnf(c\_is\_defined, hypothesis)

```

defined(u)   cnf(u_is_defined, hypothesis)
defined(v)   cnf(v_is_defined, hypothesis)
a · c=u     cnf(multiply_equals_u6, negated_conjecture)
b · c=v     cnf(multiply_equals_v7, negated_conjecture)
u ≤ v      cnf(less_or_equal_8, negated_conjecture)
¬ c=0      cnf(c_not_equal_to_additive_identity_9, negated_conjecture)
0 ≤ c      cnf(less_or_equal_10, negated_conjecture)
¬ a ≤ b    cnf(not_less_or_equal_11, negated_conjecture)

```

**FLD076-3.p** Two-sided multiplication in an order relation, part 3

```

include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
¬ 0 + c=0   cnf(not_sum_4, negated_conjecture)
0 ≤ c      cnf(less_or_equal_5, negated_conjecture)
a · c ≤ b · c  cnf(less_or_equal_6, negated_conjecture)
¬ a ≤ b    cnf(not_less_or_equal_7, negated_conjecture)

```

**FLD076-4.p** Two-sided multiplication in an order relation, part 3

```

include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
defined(v)   cnf(v_is_defined, hypothesis)
¬ 0 + c=0   cnf(not_sum_6, negated_conjecture)
0 ≤ c      cnf(less_or_equal_7, negated_conjecture)
a · c=u     cnf(product_8, negated_conjecture)
b · c=v     cnf(product_9, negated_conjecture)
u ≤ v      cnf(less_or_equal_10, negated_conjecture)
¬ a ≤ b    cnf(not_less_or_equal_11, negated_conjecture)

```

**FLD077-1.p** Elimination of a product in an order relation, part 1

```

include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
¬ c=0      cnf(c_not_equal_to_additive_identity_4, negated_conjecture)
c ≤ 0      cnf(less_or_equal_5, negated_conjecture)
a · c ≤ b · c  cnf(less_or_equal_6, negated_conjecture)
¬ b ≤ a    cnf(not_less_or_equal_7, negated_conjecture)

```

**FLD077-2.p** Elimination of a product in an order relation, part 1

```

include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)
defined(u)   cnf(u_is_defined, hypothesis)
defined(v)   cnf(v_is_defined, hypothesis)
¬ c=0      cnf(c_not_equal_to_additive_identity_6, negated_conjecture)
c ≤ 0      cnf(less_or_equal_7, negated_conjecture)
a · c=u     cnf(multiply_equals_u_8, negated_conjecture)
b · c=v     cnf(multiply_equals_v_9, negated_conjecture)
u ≤ v      cnf(less_or_equal_10, negated_conjecture)
¬ b ≤ a    cnf(not_less_or_equal_11, negated_conjecture)

```

**FLD077-3.p** Elimination of a product in an order relation, part 1

```

include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
defined(c)   cnf(c_is_defined, hypothesis)

```



$\neg 0 + c = 0$      $\text{cnf}(\text{not\_sum}_4, \text{negated\_conjecture})$   
 $c \leq 0$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $a \cdot c \leq b \cdot c$      $\text{cnf}(\text{less\_or\_equal}_6, \text{negated\_conjecture})$   
 $\neg b \leq a$      $\text{cnf}(\text{not\_less\_or\_equal}_7, \text{negated\_conjecture})$

**FLD077-4.p** Elimination of a product in an order relation, part 1

$\text{include}('Axioms/FLD002-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(c)$      $\text{cnf}(c\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $\text{defined}(v)$      $\text{cnf}(v\_is\_defined, \text{hypothesis})$   
 $\neg 0 + c = 0$      $\text{cnf}(\text{not\_sum}_6, \text{negated\_conjecture})$   
 $c \leq 0$      $\text{cnf}(\text{less\_or\_equal}_7, \text{negated\_conjecture})$   
 $a \cdot c = u$      $\text{cnf}(\text{product}_8, \text{negated\_conjecture})$   
 $b \cdot c = v$      $\text{cnf}(\text{product}_9, \text{negated\_conjecture})$   
 $u \leq v$      $\text{cnf}(\text{less\_or\_equal}_{10}, \text{negated\_conjecture})$   
 $\neg b \leq a$      $\text{cnf}(\text{not\_less\_or\_equal}_{11}, \text{negated\_conjecture})$

**FLD078-1.p** Side change in an order relation, multiplicative part 1

$\text{include}('Axioms/FLD001-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\neg b = 0$      $\text{cnf}(b\_not\_equal\_to\_additive\_identity_3, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $\neg a \cdot b^{-1} \leq 1$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD078-2.p** Side change in an order relation, multiplicative part 1

$\text{include}('Axioms/FLD001-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $\neg b = 0$      $\text{cnf}(b\_not\_equal\_to\_additive\_identity_4, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_6, \text{negated\_conjecture})$   
 $a \cdot b^{-1} = u$      $\text{cnf}(\text{multiply\_equals}_u_7, \text{negated\_conjecture})$   
 $\neg u \leq 1$      $\text{cnf}(\text{not\_less\_or\_equal}_8, \text{negated\_conjecture})$

**FLD078-3.p** Side change in an order relation, multiplicative part 1

$\text{include}('Axioms/FLD002-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\neg 0 + b = 0$      $\text{cnf}(\text{not\_sum}_3, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $\neg a \cdot b^{-1} \leq 1$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD078-4.p** Side change in an order relation, multiplicative part 1

$\text{include}('Axioms/FLD002-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $\neg 0 + b = 0$      $\text{cnf}(\text{not\_sum}_4, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $a \leq b$      $\text{cnf}(\text{less\_or\_equal}_6, \text{negated\_conjecture})$   
 $a \cdot b^{-1} = u$      $\text{cnf}(\text{product}_7, \text{negated\_conjecture})$   
 $\neg u \leq 1$      $\text{cnf}(\text{not\_less\_or\_equal}_8, \text{negated\_conjecture})$

**FLD079-1.p** Side change in an order relation, multiplicative part 2

$\text{include}('Axioms/FLD001-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$

$\neg b=0$      $\text{cnf}(b\_not\_equal\_to\_additive\_identity_3, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $a \cdot b^{-1} \leq 1$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $\neg a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD079-2.p** Side change in an order relation, multiplicative part 2

$\text{include}('Axioms/FLD001-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $\neg b=0$      $\text{cnf}(b\_not\_equal\_to\_additive\_identity_4, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $a \cdot b^{-1}=u$      $\text{cnf}(\text{multiply\_equals\_u}_6, \text{negated\_conjecture})$   
 $u \leq 1$      $\text{cnf}(\text{less\_or\_equal}_7, \text{negated\_conjecture})$   
 $\neg a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_8, \text{negated\_conjecture})$

**FLD079-3.p** Side change in an order relation, multiplicative part 2

$\text{include}('Axioms/FLD002-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\neg 0 + b=0$      $\text{cnf}(\text{not\_sum}_3, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_4, \text{negated\_conjecture})$   
 $a \cdot b^{-1} \leq 1$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $\neg a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_6, \text{negated\_conjecture})$

**FLD079-4.p** Side change in an order relation, multiplicative part 2

$\text{include}('Axioms/FLD002-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $\neg 0 + b=0$      $\text{cnf}(\text{not\_sum}_4, \text{negated\_conjecture})$   
 $0 \leq b$      $\text{cnf}(\text{less\_or\_equal}_5, \text{negated\_conjecture})$   
 $a \cdot b^{-1}=u$      $\text{cnf}(\text{product}_6, \text{negated\_conjecture})$   
 $u \leq 1$      $\text{cnf}(\text{less\_or\_equal}_7, \text{negated\_conjecture})$   
 $\neg a \leq b$      $\text{cnf}(\text{not\_less\_or\_equal}_8, \text{negated\_conjecture})$

**FLD080-1.p** The square of an element is always greater or equal 0

$\text{include}('Axioms/FLD001-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\neg 0 \leq a \cdot a$      $\text{cnf}(\text{not\_less\_or\_equal}_2, \text{negated\_conjecture})$

**FLD080-2.p** The square of an element is always greater or equal 0

$\text{include}('Axioms/FLD001-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $a \cdot a=u$      $\text{cnf}(\text{multiply\_equals\_u}_3, \text{negated\_conjecture})$   
 $\neg 0 \leq u$      $\text{cnf}(\text{not\_less\_or\_equal}_4, \text{negated\_conjecture})$

**FLD080-3.p** The square of an element is always greater or equal 0

$\text{include}('Axioms/FLD002-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\neg 0 \leq a \cdot a$      $\text{cnf}(\text{not\_less\_or\_equal}_2, \text{negated\_conjecture})$

**FLD080-4.p** The square of an element is always greater or equal 0

$\text{include}('Axioms/FLD002-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(u)$      $\text{cnf}(u\_is\_defined, \text{hypothesis})$   
 $a \cdot a=u$      $\text{cnf}(\text{product}_3, \text{negated\_conjecture})$   
 $\neg 0 \leq u$      $\text{cnf}(\text{not\_less\_or\_equal}_4, \text{negated\_conjecture})$

**FLD081-1.p** Two-sided multiplication of two order relations

$\text{include}('Axioms/FLD001-0.ax')$   
 $\text{defined}(a)$      $\text{cnf}(a\_is\_defined, \text{hypothesis})$   
 $\text{defined}(b)$      $\text{cnf}(b\_is\_defined, \text{hypothesis})$

```

defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal5, negated_conjecture)
a ≤ b        cnf(less_or_equal6, negated_conjecture)
0 ≤ c        cnf(less_or_equal7, negated_conjecture)
c ≤ d        cnf(less_or_equal8, negated_conjecture)
¬ a · c ≤ d · b    cnf(not_less_or_equal9, negated_conjecture)

```

**FLD081-2.p** Two-sided multiplication of two order relations

```
include('Axioms/FLD001-0.ax')
```

```

defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal7, negated_conjecture)
a ≤ b        cnf(less_or_equal8, negated_conjecture)
0 ≤ c        cnf(less_or_equal9, negated_conjecture)
c ≤ d        cnf(less_or_equal10, negated_conjecture)
a · c = u     cnf(multiply_equals_u11, negated_conjecture)
d · b = v     cnf(multiply_equals_v12, negated_conjecture)
¬ u ≤ v      cnf(not_less_or_equal13, negated_conjecture)

```

**FLD081-3.p** Two-sided multiplication of two order relations

```
include('Axioms/FLD002-0.ax')
```

```

defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal5, negated_conjecture)
a ≤ b        cnf(less_or_equal6, negated_conjecture)
0 ≤ c        cnf(less_or_equal7, negated_conjecture)
c ≤ d        cnf(less_or_equal8, negated_conjecture)
¬ a · c ≤ d · b    cnf(not_less_or_equal9, negated_conjecture)

```

**FLD081-4.p** Two-sided multiplication of two order relations

```
include('Axioms/FLD002-0.ax')
```

```

defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal7, negated_conjecture)
a ≤ b        cnf(less_or_equal8, negated_conjecture)
0 ≤ c        cnf(less_or_equal9, negated_conjecture)
c ≤ d        cnf(less_or_equal10, negated_conjecture)
a · c = u     cnf(product11, negated_conjecture)
d · b = v     cnf(product12, negated_conjecture)
¬ u ≤ v      cnf(not_less_or_equal13, negated_conjecture)

```

**FLD082-1.p** Compatibility of order and multiplicative inverses, part 1

```
include('Axioms/FLD001-0.ax')
```

```

defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
¬ a = 0       cnf(a_not_equal_to_additive_identity3, negated_conjecture)
0 ≤ a        cnf(less_or_equal4, negated_conjecture)
a ≤ b        cnf(less_or_equal5, negated_conjecture)
¬ b-1 ≤ a-1    cnf(not_less_or_equal6, negated_conjecture)

```

**FLD082-3.p** Compatibility of order and multiplicative inverses, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
-0 + a=0     cnf(not_sum3, negated_conjecture)
0 ≤ a       cnf(less_or_equal4, negated_conjecture)
a ≤ b       cnf(less_or_equal5, negated_conjecture)
-b-1 ≤ a-1  cnf(not_less_or_equal6, negated_conjecture)
```

**FLD083-1.p** Compatibility of order and multiplicative inverses, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
-b=0        cnf(b_not_equal_to_additive_identity3, negated_conjecture)
b ≤ 0       cnf(less_or_equal4, negated_conjecture)
a ≤ b       cnf(less_or_equal5, negated_conjecture)
-b-1 ≤ a-1  cnf(not_less_or_equal6, negated_conjecture)
```

**FLD083-3.p** Compatibility of order and multiplicative inverses, part 2

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
-0 + b=0     cnf(not_sum3, negated_conjecture)
b ≤ 0       cnf(less_or_equal4, negated_conjecture)
a ≤ b       cnf(less_or_equal5, negated_conjecture)
-b-1 ≤ a-1  cnf(not_less_or_equal6, negated_conjecture)
```

**FLD084-1.p** Elimination of multiplicative inverses in an order, part 1

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
-b=0        cnf(b_not_equal_to_additive_identity3, negated_conjecture)
-a=0        cnf(a_not_equal_to_additive_identity4, negated_conjecture)
b-1 ≤ a-1  cnf(less_or_equal5, negated_conjecture)
0 ≤ b-1    cnf(less_or_equal6, negated_conjecture)
-a ≤ b      cnf(not_less_or_equal7, negated_conjecture)
```

**FLD084-3.p** Elimination of multiplicative inverses in an order, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
-0 + b=0     cnf(not_sum3, negated_conjecture)
-0 + a=0     cnf(not_sum4, negated_conjecture)
b-1 ≤ a-1  cnf(less_or_equal5, negated_conjecture)
0 ≤ b-1    cnf(less_or_equal6, negated_conjecture)
-a ≤ b      cnf(not_less_or_equal7, negated_conjecture)
```

**FLD085-1.p** Elimination of multiplicative inverses in an order, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
-a=0        cnf(a_not_equal_to_additive_identity3, negated_conjecture)
-b=0        cnf(b_not_equal_to_additive_identity4, negated_conjecture)
b-1 ≤ a-1  cnf(less_or_equal5, negated_conjecture)
a-1 ≤ 0    cnf(less_or_equal6, negated_conjecture)
-a ≤ b      cnf(not_less_or_equal7, negated_conjecture)
```

**FLD085-3.p** Elimination of multiplicative inverses in an order, part 2

```
include('Axioms/FLD002-0.ax')
defined(a)   cnf(a_is_defined, hypothesis)
defined(b)   cnf(b_is_defined, hypothesis)
-0 + a=0     cnf(not_sum3, negated_conjecture)
-0 + b=0     cnf(not_sum4, negated_conjecture)
b-1 ≤ a-1  cnf(less_or_equal5, negated_conjecture)
```

$a^{-1} \leq 0$     `cnf(less_or_equal6, negated_conjecture)`  
 $\neg a \leq b$     `cnf(not_less_or_equal7, negated_conjecture)`

**FLD086-1.p** Compatibility of order and multiplicative inverses, part 1

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
 $\neg a=0$     `cnf(a_not_equal_to_additive_identity2, negated_conjecture)`  
 $0 \leq a$     `cnf(less_or_equal3, negated_conjecture)`  
 $\neg 0 \leq a^{-1}$     `cnf(not_less_or_equal4, negated_conjecture)`

**FLD086-3.p** Compatibility of order and multiplicative inverses, part 1

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
 $\neg 0 + a=0$     `cnf(not_sum2, negated_conjecture)`  
 $0 \leq a$     `cnf(less_or_equal3, negated_conjecture)`  
 $\neg 0 \leq a^{-1}$     `cnf(not_less_or_equal4, negated_conjecture)`

**FLD087-1.p** Elimination of a multiplicative inverse in an order, part 1

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
 $\neg a=0$     `cnf(a_not_equal_to_additive_identity2, negated_conjecture)`  
 $0 \leq a^{-1}$     `cnf(less_or_equal3, negated_conjecture)`  
 $\neg 0 \leq a$     `cnf(not_less_or_equal4, negated_conjecture)`

**FLD087-3.p** Elimination of a multiplicative inverse in an order, part 1

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
 $\neg 0 + a=0$     `cnf(not_sum2, negated_conjecture)`  
 $0 \leq a^{-1}$     `cnf(less_or_equal3, negated_conjecture)`  
 $\neg 0 \leq a$     `cnf(not_less_or_equal4, negated_conjecture)`

**FLD088-1.p** Compatibility of order and multiplicative inverses, part 2

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
 $\neg a=0$     `cnf(a_not_equal_to_additive_identity2, negated_conjecture)`  
 $a \leq 0$     `cnf(less_or_equal3, negated_conjecture)`  
 $\neg a^{-1} \leq 0$     `cnf(not_less_or_equal4, negated_conjecture)`

**FLD088-3.p** Compatibility of order and multiplicative inverses, part 2

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
 $\neg 0 + a=0$     `cnf(not_sum2, negated_conjecture)`  
 $a \leq 0$     `cnf(less_or_equal3, negated_conjecture)`  
 $\neg a^{-1} \leq 0$     `cnf(not_less_or_equal4, negated_conjecture)`

**FLD089-1.p** Elimination of a multiplicative inverse in an order, part 2

`include('Axioms/FLD001-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
 $\neg a=0$     `cnf(a_not_equal_to_additive_identity2, negated_conjecture)`  
 $a^{-1} \leq 0$     `cnf(less_or_equal3, negated_conjecture)`  
 $\neg a \leq 0$     `cnf(not_less_or_equal4, negated_conjecture)`

**FLD089-3.p** Elimination of a multiplicative inverse in an order, part 2

`include('Axioms/FLD002-0.ax')`  
`defined(a)`    `cnf(a_is_defined, hypothesis)`  
 $\neg 0 + a=0$     `cnf(not_sum2, negated_conjecture)`  
 $a^{-1} \leq 0$     `cnf(less_or_equal3, negated_conjecture)`  
 $\neg a \leq 0$     `cnf(not_less_or_equal4, negated_conjecture)`

**FLD090-1.p** A characterization of 1 with help of the order relation

`include('Axioms/FLD001-0.ax')`  
`defined(m)`    `cnf(m_is_defined, hypothesis)`  
 $m \cdot x \leq x$     `cnf(less_or_equal2, negated_conjecture)`  
 $\neg m=1$     `cnf(m_not_equal_to_multiplicative_identity3, negated_conjecture)`

**FLD090-3.p** A characterization of 1 with help of the order relation

```
include('Axioms/FLD002-0.ax')
defined(m)    cnf(m.is_defined, hypothesis)
 $m \cdot x = y \Rightarrow y \leq x$     cnf(less_or_equal_or_not_product2, negated_conjecture)
 $\neg 0 + m = 1$     cnf(not_sum3, negated_conjecture)
```

**FLD091-1.p** One-sided Elimination of a multiplicative inverse, part 1

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
 $\neg a = 0$     cnf(a_not_equal_to_additive_identity2, negated_conjecture)
 $1 \leq a$     cnf(less_or_equal3, negated_conjecture)
 $\neg a^{-1} \leq 1$     cnf(not_less_or_equal4, negated_conjecture)
```

**FLD091-3.p** One-sided Elimination of a multiplicative inverse, part 1

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
 $\neg 0 + a = 0$     cnf(not_sum2, negated_conjecture)
 $1 \leq a$     cnf(less_or_equal3, negated_conjecture)
 $\neg a^{-1} \leq 1$     cnf(not_less_or_equal4, negated_conjecture)
```

**FLD092-1.p** One-sided Elimination of a multiplicative inverse, part 2

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
 $\neg a = 0$     cnf(a_not_equal_to_additive_identity2, negated_conjecture)
 $a \leq -1$     cnf(less_or_equal3, negated_conjecture)
 $\neg -1 \leq a^{-1}$     cnf(not_less_or_equal4, negated_conjecture)
```

**FLD092-3.p** One-sided Elimination of a multiplicative inverse, part 2

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
 $\neg 0 + a = 0$     cnf(not_sum2, negated_conjecture)
 $a \leq -1$     cnf(less_or_equal3, negated_conjecture)
 $\neg -1 \leq a^{-1}$     cnf(not_less_or_equal4, negated_conjecture)
```

**FLD093-1.p** One-sided Elimination of a multiplicative inverse, part 3

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
 $\neg a = 0$     cnf(a_not_equal_to_additive_identity2, negated_conjecture)
 $1 \leq a^{-1}$     cnf(less_or_equal3, negated_conjecture)
 $\neg a \leq 1$     cnf(not_less_or_equal4, negated_conjecture)
```

**FLD093-3.p** One-sided Elimination of a multiplicative inverse, part 3

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
 $\neg 0 + a = 0$     cnf(not_sum2, negated_conjecture)
 $1 \leq a^{-1}$     cnf(less_or_equal3, negated_conjecture)
 $\neg a \leq 1$     cnf(not_less_or_equal4, negated_conjecture)
```

**FLD094-1.p** One-sided Elimination of a multiplicative inverse, part 4

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
 $\neg a = 0$     cnf(a_not_equal_to_additive_identity2, negated_conjecture)
 $a^{-1} \leq -1$     cnf(less_or_equal3, negated_conjecture)
 $\neg -1 \leq a$     cnf(not_less_or_equal4, negated_conjecture)
```

**FLD094-3.p** One-sided Elimination of a multiplicative inverse, part 4

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
 $\neg 0 + a = 0$     cnf(not_sum2, negated_conjecture)
 $a^{-1} \leq -1$     cnf(less_or_equal3, negated_conjecture)
 $\neg -1 \leq a$     cnf(not_less_or_equal4, negated_conjecture)
```

**FLD095-1.p** Difficult inequality

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
```

```

defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
¬b=0         cnf(b_not_equal_to_additive_identity5, negated_conjecture)
¬d=0         cnf(d_not_equal_to_additive_identity6, negated_conjecture)
b ≤ 0        cnf(less_or_equal7, negated_conjecture)
0 ≤ d        cnf(less_or_equal8, negated_conjecture)
a · b-1 ≤ c · d-1    cnf(less_or_equal9, negated_conjecture)
¬a · b-1 ≤ (a + c) · (b + d)-1    cnf(not_less_or_equal10, negated_conjecture)

```

**FLD095-2.p** Difficult inequality

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
defined(s)    cnf(s_is_defined, hypothesis)
defined(t)    cnf(t_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
defined(w)    cnf(w_is_defined, hypothesis)
¬b=0         cnf(b_not_equal_to_additive_identity10, negated_conjecture)
¬d=0         cnf(d_not_equal_to_additive_identity11, negated_conjecture)
b ≤ 0        cnf(less_or_equal12, negated_conjecture)
0 ≤ d        cnf(less_or_equal13, negated_conjecture)
a · b-1=u    cnf(multiply_equals_u14, negated_conjecture)
c · d-1=v    cnf(multiply_equals_v15, negated_conjecture)
a + c=s      cnf(add_equals_s16, negated_conjecture)
b + d=t      cnf(add_equals_t17, negated_conjecture)
s · t-1=w    cnf(multiply_equals_w18, negated_conjecture)
u ≤ v        cnf(less_or_equal19, negated_conjecture)
¬u ≤ w       cnf(not_less_or_equal20, negated_conjecture)

```

**FLD095-3.p** Difficult inequality

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
¬0 + b=0     cnf(not_sum5, negated_conjecture)
¬0 + d=0     cnf(not_sum6, negated_conjecture)
0 ≤ b        cnf(less_or_equal7, negated_conjecture)
0 ≤ d        cnf(less_or_equal8, negated_conjecture)
a · b-1 ≤ c · d-1    cnf(less_or_equal9, negated_conjecture)
¬a · b-1 ≤ (a + c) · (b + d)-1    cnf(not_less_or_equal10, negated_conjecture)

```

**FLD095-4.p** Difficult inequality

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(c)    cnf(c_is_defined, hypothesis)
defined(d)    cnf(d_is_defined, hypothesis)
defined(s)    cnf(s_is_defined, hypothesis)
defined(t)    cnf(t_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
defined(w)    cnf(w_is_defined, hypothesis)
¬0 + b=0     cnf(not_sum10, negated_conjecture)
¬0 + d=0     cnf(not_sum11, negated_conjecture)
0 ≤ b        cnf(less_or_equal12, negated_conjecture)
0 ≤ d        cnf(less_or_equal13, negated_conjecture)

```

$a \cdot b^{-1} = u$     cnf(product<sub>14</sub>, negated\_conjecture)  
 $c \cdot d^{-1} = v$     cnf(product<sub>15</sub>, negated\_conjecture)  
 $a + c = s$     cnf(sum<sub>16</sub>, negated\_conjecture)  
 $b + d = t$     cnf(sum<sub>17</sub>, negated\_conjecture)  
 $s \cdot t^{-1} = w$     cnf(product<sub>18</sub>, negated\_conjecture)  
 $u \leq v$     cnf(less\_or\_equal<sub>19</sub>, negated\_conjecture)  
 $\neg u \leq w$     cnf(not\_less\_or\_equal<sub>20</sub>, negated\_conjecture)

**FLD096-1.p** Difficult inequality

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
defined( $c$ )    cnf( $c$ .is\_defined, hypothesis)  
defined( $d$ )    cnf( $d$ .is\_defined, hypothesis)  
 $\neg b = 0$     cnf( $b$ .not\_equal\_to\_additive\_identity<sub>5</sub>, negated\_conjecture)  
 $\neg d = 0$     cnf( $d$ .not\_equal\_to\_additive\_identity<sub>6</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>7</sub>, negated\_conjecture)  
 $0 \leq d$     cnf(less\_or\_equal<sub>8</sub>, negated\_conjecture)  
 $a \cdot b^{-1} \leq c \cdot d^{-1}$     cnf(less\_or\_equal<sub>9</sub>, negated\_conjecture)  
 $\neg (a + c) \cdot (b + d)^{-1} \leq c \cdot d^{-1}$     cnf(not\_less\_or\_equal<sub>10</sub>, negated\_conjecture)

**FLD096-2.p** Difficult inequality

include('Axioms/FLD001-0.ax')  
defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
defined( $c$ )    cnf( $c$ .is\_defined, hypothesis)  
defined( $d$ )    cnf( $d$ .is\_defined, hypothesis)  
defined( $s$ )    cnf( $s$ .is\_defined, hypothesis)  
defined( $t$ )    cnf( $t$ .is\_defined, hypothesis)  
defined( $u$ )    cnf( $u$ .is\_defined, hypothesis)  
defined( $v$ )    cnf( $v$ .is\_defined, hypothesis)  
defined( $w$ )    cnf( $w$ .is\_defined, hypothesis)  
 $\neg b = 0$     cnf( $b$ .not\_equal\_to\_additive\_identity<sub>10</sub>, negated\_conjecture)  
 $\neg d = 0$     cnf( $d$ .not\_equal\_to\_additive\_identity<sub>11</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>12</sub>, negated\_conjecture)  
 $0 \leq d$     cnf(less\_or\_equal<sub>13</sub>, negated\_conjecture)  
 $a \cdot b^{-1} = u$     cnf(multiply\_equals\_u<sub>14</sub>, negated\_conjecture)  
 $c \cdot d^{-1} = v$     cnf(multiply\_equals\_v<sub>15</sub>, negated\_conjecture)  
 $a + c = s$     cnf(add\_equals\_s<sub>16</sub>, negated\_conjecture)  
 $b + d = t$     cnf(add\_equals\_t<sub>17</sub>, negated\_conjecture)  
 $s \cdot t^{-1} = w$     cnf(multiply\_equals\_w<sub>18</sub>, negated\_conjecture)  
 $u \leq v$     cnf(less\_or\_equal<sub>19</sub>, negated\_conjecture)  
 $\neg w \leq v$     cnf(not\_less\_or\_equal<sub>20</sub>, negated\_conjecture)

**FLD096-3.p** Difficult inequality

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)  
defined( $c$ )    cnf( $c$ .is\_defined, hypothesis)  
defined( $d$ )    cnf( $d$ .is\_defined, hypothesis)  
 $\neg 0 + b = 0$     cnf(not\_sum<sub>5</sub>, negated\_conjecture)  
 $\neg 0 + d = 0$     cnf(not\_sum<sub>6</sub>, negated\_conjecture)  
 $0 \leq b$     cnf(less\_or\_equal<sub>7</sub>, negated\_conjecture)  
 $0 \leq d$     cnf(less\_or\_equal<sub>8</sub>, negated\_conjecture)  
 $a \cdot b^{-1} \leq c \cdot d^{-1}$     cnf(less\_or\_equal<sub>9</sub>, negated\_conjecture)  
 $\neg (a + c) \cdot (b + d)^{-1} \leq c \cdot d^{-1}$     cnf(not\_less\_or\_equal<sub>10</sub>, negated\_conjecture)

**FLD096-4.p** Difficult inequality

include('Axioms/FLD002-0.ax')  
defined( $a$ )    cnf( $a$ .is\_defined, hypothesis)  
defined( $b$ )    cnf( $b$ .is\_defined, hypothesis)



```

defined(c)    cnf(c.is_defined, hypothesis)
defined(d)    cnf(d.is_defined, hypothesis)
defined(s)    cnf(s.is_defined, hypothesis)
defined(t)    cnf(t.is_defined, hypothesis)
defined(u)    cnf(u.is_defined, hypothesis)
defined(v)    cnf(v.is_defined, hypothesis)
defined(w)    cnf(w.is_defined, hypothesis)
-0 + b=0     cnf(not_sum10, negated_conjecture)
-0 + d=0     cnf(not_sum11, negated_conjecture)
0 ≤ b       cnf(less_or_equal12, negated_conjecture)
0 ≤ d       cnf(less_or_equal13, negated_conjecture)
a · b-1=u   cnf(product14, negated_conjecture)
c · d-1=v   cnf(product15, negated_conjecture)
a + c=s      cnf(sum16, negated_conjecture)
b + d=t      cnf(sum17, negated_conjecture)
s · t-1=w   cnf(product18, negated_conjecture)
u ≤ v       cnf(less_or_equal19, negated_conjecture)
¬ w ≤ v     cnf(not_less_or_equal20, negated_conjecture)

```

**FLD097-1.p** Difficult inequality

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
0 ≤ a       cnf(less_or_equal3, negated_conjecture)
0 ≤ b       cnf(less_or_equal4, negated_conjecture)
-1 + (a + b) ≤ (1 + a) · (1 + b)   cnf(not_less_or_equal5, negated_conjecture)

```

**FLD097-2.p** Difficult inequality

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(u)    cnf(u.is_defined, hypothesis)
defined(v)    cnf(v.is_defined, hypothesis)
defined(w)    cnf(w.is_defined, hypothesis)
defined(s)    cnf(s.is_defined, hypothesis)
defined(t)    cnf(t.is_defined, hypothesis)
0 ≤ a       cnf(less_or_equal8, negated_conjecture)
0 ≤ b       cnf(less_or_equal9, negated_conjecture)
1 + a=u     cnf(add_equals_u10, negated_conjecture)
1 + b=v     cnf(add_equals_v11, negated_conjecture)
u · v=w     cnf(multiply_equals_w12, negated_conjecture)
a + b=s     cnf(add_equals_s13, negated_conjecture)
1 + s=t     cnf(add_equals_t14, negated_conjecture)
¬ t ≤ w     cnf(not_less_or_equal15, negated_conjecture)

```

**FLD097-3.p** Difficult inequality

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
0 ≤ a       cnf(less_or_equal3, negated_conjecture)
0 ≤ b       cnf(less_or_equal4, negated_conjecture)
-1 + (a + b) ≤ (1 + a) · (1 + b)   cnf(not_less_or_equal5, negated_conjecture)

```

**FLD097-4.p** Difficult inequality

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a.is_defined, hypothesis)
defined(b)    cnf(b.is_defined, hypothesis)
defined(u)    cnf(u.is_defined, hypothesis)
defined(v)    cnf(v.is_defined, hypothesis)
defined(w)    cnf(w.is_defined, hypothesis)
defined(s)    cnf(s.is_defined, hypothesis)

```

```

defined(t)    cnf(t_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal_8, negated_conjecture)
0 ≤ b        cnf(less_or_equal_9, negated_conjecture)
1 + a=u      cnf(sum_10, negated_conjecture)
1 + b=v      cnf(sum_11, negated_conjecture)
u · v=w      cnf(product_12, negated_conjecture)
a + b=s      cnf(sum_13, negated_conjecture)
1 + s=t      cnf(sum_14, negated_conjecture)
¬t ≤ w       cnf(not_less_or_equal_15, negated_conjecture)

```

**FLD098-1.p** Difficult inequality

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal_3, negated_conjecture)
0 ≤ b        cnf(less_or_equal_4, negated_conjecture)
-1 + -(a + b) ≤ (1 + -a) · (1 + -b)    cnf(not_less_or_equal_5, negated_conjecture)

```

**FLD098-2.p** Difficult inequality

```

include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
defined(w)    cnf(w_is_defined, hypothesis)
defined(s)    cnf(s_is_defined, hypothesis)
defined(t)    cnf(t_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal_8, negated_conjecture)
0 ≤ b        cnf(less_or_equal_9, negated_conjecture)
1 + -a=u      cnf(add_equals_u_10, negated_conjecture)
1 + -b=v      cnf(add_equals_v_11, negated_conjecture)
u · v=w      cnf(multiply_equals_w_12, negated_conjecture)
a + b=s      cnf(add_equals_s_13, negated_conjecture)
1 + -s=t      cnf(add_equals_t_14, negated_conjecture)
¬t ≤ w       cnf(not_less_or_equal_15, negated_conjecture)

```

**FLD098-3.p** Difficult inequality

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal_3, negated_conjecture)
0 ≤ b        cnf(less_or_equal_4, negated_conjecture)
-1 + -(a + b) ≤ (1 + -a) · (1 + -b)    cnf(not_less_or_equal_5, negated_conjecture)

```

**FLD098-4.p** Difficult inequality

```

include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
defined(w)    cnf(w_is_defined, hypothesis)
defined(s)    cnf(s_is_defined, hypothesis)
defined(t)    cnf(t_is_defined, hypothesis)
0 ≤ a        cnf(less_or_equal_8, negated_conjecture)
0 ≤ b        cnf(less_or_equal_9, negated_conjecture)
1 + -a=u      cnf(sum_10, negated_conjecture)
1 + -b=v      cnf(sum_11, negated_conjecture)
u · v=w      cnf(product_12, negated_conjecture)
a + b=s      cnf(sum_13, negated_conjecture)
1 + -s=t      cnf(sum_14, negated_conjecture)
¬t ≤ w       cnf(not_less_or_equal_15, negated_conjecture)

```

**FLD099-1.p** Difficult inequality

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
 $0 \leq a$     cnf(less_or_equal2, negated_conjecture)
 $a \leq 1$     cnf(less_or_equal3, negated_conjecture)
 $\neg a=1$     cnf(a_not_equal_to_multiplicative_identity4, negated_conjecture)
 $\neg 1 + a \leq (1 + -a)^{-1}$     cnf(not_less_or_equal5, negated_conjecture)
```

**FLD099-2.p** Difficult inequality

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
 $0 \leq a$     cnf(less_or_equal4, negated_conjecture)
 $a \leq 1$     cnf(less_or_equal5, negated_conjecture)
 $\neg a=1$     cnf(a_not_equal_to_multiplicative_identity6, negated_conjecture)
 $1 + a=u$     cnf(add_equals_u7, negated_conjecture)
 $1 + -a=v$     cnf(add_equals_v8, negated_conjecture)
 $\neg u \leq v^{-1}$     cnf(not_less_or_equal9, negated_conjecture)
```

**FLD099-3.p** Difficult inequality

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
 $0 \leq a$     cnf(less_or_equal2, negated_conjecture)
 $a \leq 1$     cnf(less_or_equal3, negated_conjecture)
 $\neg 0 + a=1$     cnf(not_sum4, negated_conjecture)
 $\neg 1 + a \leq (1 + -a)^{-1}$     cnf(not_less_or_equal5, negated_conjecture)
```

**FLD099-4.p** Difficult inequality

```
include('Axioms/FLD002-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
 $0 \leq a$     cnf(less_or_equal4, negated_conjecture)
 $a \leq 1$     cnf(less_or_equal5, negated_conjecture)
 $\neg 0 + a=1$     cnf(not_sum6, negated_conjecture)
 $1 + a=u$     cnf(sum7, negated_conjecture)
 $1 + -a=v$     cnf(sum8, negated_conjecture)
 $\neg u \leq v^{-1}$     cnf(not_less_or_equal9, negated_conjecture)
```

**FLD100-1.p** Difficult inequality

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
 $a \leq 0$     cnf(less_or_equal3, negated_conjecture)
 $b \leq 0$     cnf(less_or_equal4, negated_conjecture)
 $\neg 1 + (a + b) \leq (1 + a) \cdot (1 + b)$     cnf(not_less_or_equal5, negated_conjecture)
```

**FLD100-2.p** Difficult inequality

```
include('Axioms/FLD001-0.ax')
defined(a)    cnf(a_is_defined, hypothesis)
defined(b)    cnf(b_is_defined, hypothesis)
defined(u)    cnf(u_is_defined, hypothesis)
defined(v)    cnf(v_is_defined, hypothesis)
defined(w)    cnf(w_is_defined, hypothesis)
defined(s)    cnf(s_is_defined, hypothesis)
defined(t)    cnf(t_is_defined, hypothesis)
 $a \leq 0$     cnf(less_or_equal8, negated_conjecture)
 $b \leq 0$     cnf(less_or_equal9, negated_conjecture)
 $1 + a=u$     cnf(add_equals_u10, negated_conjecture)
 $1 + b=v$     cnf(add_equals_v11, negated_conjecture)
 $u \cdot v=w$     cnf(multiply_equals_w12, negated_conjecture)
```

$a + b = s$      `cnf(add_equals_s13, negated_conjecture)`  
 $1 + s = t$      `cnf(add_equals_t14, negated_conjecture)`  
 $\neg t \leq w$      `cnf(not_less_or_equal15, negated_conjecture)`

**FLD100-3.p** Difficult inequality

`include('Axioms/FLD002-0.ax')`  
`defined(a)     cnf(a_is_defined, hypothesis)`  
`defined(b)     cnf(b_is_defined, hypothesis)`  
 $a \leq 0$      `cnf(less_or_equal3, negated_conjecture)`  
 $b \leq 0$      `cnf(less_or_equal4, negated_conjecture)`  
 $\neg 1 + (a + b) \leq (1 + a) \cdot (1 + b)$      `cnf(not_less_or_equal5, negated_conjecture)`

**FLD100-4.p** Difficult inequality

`include('Axioms/FLD002-0.ax')`  
`defined(a)     cnf(a_is_defined, hypothesis)`  
`defined(b)     cnf(b_is_defined, hypothesis)`  
`defined(u)     cnf(u_is_defined, hypothesis)`  
`defined(v)     cnf(v_is_defined, hypothesis)`  
`defined(w)     cnf(w_is_defined, hypothesis)`  
`defined(s)     cnf(s_is_defined, hypothesis)`  
`defined(t)     cnf(t_is_defined, hypothesis)`  
 $a \leq 0$      `cnf(less_or_equal8, negated_conjecture)`  
 $b \leq 0$      `cnf(less_or_equal9, negated_conjecture)`  
 $1 + a = u$      `cnf(sum10, negated_conjecture)`  
 $1 + b = v$      `cnf(sum11, negated_conjecture)`  
 $u \cdot v = w$      `cnf(product12, negated_conjecture)`  
 $a + b = s$      `cnf(sum13, negated_conjecture)`  
 $1 + s = t$      `cnf(sum14, negated_conjecture)`  
 $\neg t \leq w$      `cnf(not_less_or_equal15, negated_conjecture)`

**FLD101-1.p** Ordered field axioms (axiom formulation glxx)

`include('Axioms/FLD001-0.ax')`

**FLD102-1.p** Ordered field axioms (axiom formulation re)

`include('Axioms/FLD002-0.ax')`