GEO axioms

GEO001-0.ax Tarski geometry axioms
between(x, y, x) $\Rightarrow$ $x = y$ cnf(identity_for_betweeness, axiom)
(between(x, y, v) and between(y, z, v)) $\Rightarrow$ between(x, y, z) cnf(transitivity_for_betweeness, axiom)
(between(x, y, z) and between(x, y, v)) $\Rightarrow$ ($x = y$ or between(x, z, v) or between(x, v, z)) cnf(connectivity_for_betweeness
equidistant(x, y, y, x) cnf(reflexivity_for_equidistance, axiom)
equidistant(x, y, z, z) $\Rightarrow$ $x = y$ cnf(identity_for_equidistance, axiom)
(equidistant(x, y, z, v) and equidistant(x, y, v_2, w)) $\Rightarrow$ equidistant(z, v, v_2, w) cnf(transitivity_for_equidistance, axiom)
(between(x, w, v) and between(y, v, z)) $\Rightarrow$ between$(x,$ outer_pasch$(w, x, y, z, v), y)$ cnf(outer_pasch$_1$, axiom)
(between(x, w, v) and between(y, v, z)) $\Rightarrow$ between$(z, w,$ outer_pasch$(w, x, y, z, v))$ cnf(outer_pasch$_2$, axiom)
(between(x, v, w) and between(y, v, z)) $\Rightarrow$ ($x = v$ or between$(x, z,$ euclid$_1(w, x, y, z, v))$) cnf(euclid$_1$, axiom)
(between(x, v, w) and between(y, v, z)) $\Rightarrow$ ($x = v$ or between$(x, y,$ euclid$_2(w, x, y, z, v))$) cnf(euclid$_2$, axiom)
(between(x, v, w) and between(y, v, z)) $\Rightarrow$ ($x = v$ or between(euclid$_1(w, x, y, z, v), w,$ euclid$_2(w, x, y, z, v))$) cnf(euclid$_3$, ax
(equidistant(x, y, x_1, y_1) and equidistant(y, z, y_1, z_1) and equidistant(x, v, x_1, v_1) and equidistant(y, v, y_1, v_1) and between$(x,$
($x = y$ or equidistant(z, v, z_1, v_1)) cnf(outer_five_segment, axiom)
between$(x, y,$ extension$(x, y, w, v))$ cnf(segment_construction$_1$, axiom)
equidistant$(y,$ extension$(x, y, w, v), w, v)$ cnf(segment_construction$_2$, axiom)
$\neg$ between(lower_dimension_point$_1$, lower_dimension_point$_2$, lower_dimension_point$_3$) cnf(lower_dimension$_1$, axiom)
$\neg$ between(lower_dimension_point$_2$, lower_dimension_point$_3$, lower_dimension_point$_1$) cnf(lower_dimension$_2$, axiom)
$\neg$ between(lower_dimension_point$_3$, lower_dimension_point$_1$, lower_dimension_point$_2$) cnf(lower_dimension$_3$, axiom)
(equidistant(x, w, x, v) and equidistant(y, w, y, v) and equidistant(z, w, z, v)) $\Rightarrow$ (between(x, y, z) or between(y, z, x) or betw
v) cnf(upper_dimension, axiom)
(equidistant(v, x, v, x_1) and equidistant(v, z, v, z_1) and between(v, x, z) and between(x, y, z)) $\Rightarrow$ equidistant$(v, y, v,$ continuou
(equidistant(v, x, v, x_1) and equidistant(v, z, v, z_1) and between(v, x, z) and between(x, y, z)) $\Rightarrow$ between$(x_1,$ continuous$(x, y$

GEO001-1.ax Colinearity axioms for the GEO001 geometry axioms
colinear(x, y, z) $\Rightarrow$ (between(x, y, z) or between(y, x, z) or between(x, z, y)) cnf(colinearity$_1$, axiom)
between(x, y, z) $\Rightarrow$ colinear(x, y, z) cnf(colinearity$_2$, axiom)
between(y, x, z) $\Rightarrow$ colinear(x, y, z) cnf(colinearity$_3$, axiom)
between(x, z, y) $\Rightarrow$ colinear(x, y, z) cnf(colinearity$_4$, axiom)

GEO002-0.ax Tarski geometry axioms
equidistant(x, y, y, x) cnf(reflexivity_for_equidistance, axiom)
(equidistant(x, y, z, v) and equidistant(x, y, v_2, w)) $\Rightarrow$ equidistant(z, v, v_2, w) cnf(transitivity_for_equidistance, axiom)
equidistant(x, y, z, z) $\Rightarrow$ $x = y$ cnf(identity_for_equidistance, axiom)
between$(x, y,$ extension$(x, y, w, v))$ cnf(segment_construction$_1$, axiom)
equidistant$(y,$ extension$(x, y, w, v), w, v)$ cnf(segment_construction$_2$, axiom)
(equidistant(x, y, x_1, y_1) and equidistant(y, z, y_1, z_1) and equidistant(x, v, x_1, v_1) and equidistant(y, v, y_1, v_1) and between$(x,$
($x = y$ or equidistant(z, v, z_1, v_1)) cnf(outer_five_segment, axiom)
between(x, y, x) $\Rightarrow$ $x = y$ cnf(identity_for_betweeness, axiom)
(between(u, v, w) and between(y, x, w)) $\Rightarrow$ between$(v,$ inner_pasch$(u, v, w, x, y), y)$ cnf(inner_pasch$_1$, axiom)
(between(u, v, w) and between(y, x, w)) $\Rightarrow$ between$(x,$ inner_pasch$(u, v, w, x, y), u)$ cnf(inner_pasch$_2$, axiom)
$\neg$ between(lower_dimension_point$_1$, lower_dimension_point$_2$, lower_dimension_point$_3$) cnf(lower_dimension$_1$, axiom)
$\neg$ between(lower_dimension_point$_2$, lower_dimension_point$_3$, lower_dimension_point$_1$) cnf(lower_dimension$_2$, axiom)
$\neg$ between(lower_dimension_point$_3$, lower_dimension_point$_1$, lower_dimension_point$_2$) cnf(lower_dimension$_3$, axiom)
(equidistant(x, w, x, v) and equidistant(y, w, y, v) and equidistant(z, w, z, v)) $\Rightarrow$ (between(x, y, z) or between(y, z, x) or betw
v) cnf(upper_dimension, axiom)
(between(u, w, y) and between(v, w, x)) $\Rightarrow$ ($u = w$ or between$(u, v,$ euclid$_1(u, v, w, x, y))$) cnf(euclid$_1$, axiom)
(between(u, w, y) and between(v, w, x)) $\Rightarrow$ ($u = w$ or between$(u, x,$ euclid$_2(u, v, w, x, y))$) cnf(euclid$_2$, axiom)
(between(u, w, y) and between(v, w, x)) $\Rightarrow$ ($u = w$ or between(euclid$_1(u, v, w, x, y), y,$ euclid$_2(u, v, w, x, y))$) cnf(euclid$_3$, a
(equidistant(u, v, u, v_1) and equidistant(u, x, u, x_1) and between(u, v, x) and between(v, w, x)) $\Rightarrow$ between$(v_1,$ continuous$(u,$
(equidistant(u, v, u, v_1) and equidistant(u, x, u, x_1) and between(u, v, x) and between(v, w, x)) $\Rightarrow$ equidistant$(u, w, u,$ continu

GEO002-1.ax Colinearity axioms for the GEO002 geometry axioms
between(x, y, z) $\Rightarrow$ colinear(x, y, z) cnf(colinearity$_1$, axiom)
between(y, z, x) $\Rightarrow$ colinear(x, y, z) cnf(colinearity$_2$, axiom)
between(z, x, y) $\Rightarrow$ colinear(x, y, z) cnf(colinearity$_3$, axiom)
colinear(x, y, z) $\Rightarrow$ (between(x, y, z) or between(y, z, x) or between(z, x, y)) cnf(colinearity$_4$, axiom)

GEO002-2.ax Reflection axioms for the GEO002 geometry axioms

$\text{reflection}(u, v) = \text{extension}(u, v, u, v)$ cnf(reflection, axiom)

GEO002-3.ax Insertion axioms for the GEO002 geometry axioms
$\text{insertion}(u_1, w_1, u, v) = \text{extension}(\text{extension}(w_1, u_1, \text{lower\_dimension\_point}_1, \text{lower\_dimension\_point}_2), u_1, u, v)$ cnf(insertio

GEO004+0.ax Simple curve axioms
$\forall c, c_1$: (part_of$(c_1, c) \iff \forall p$: (incident_c$(p, c_1) \Rightarrow$ incident_c(p, c))) fof(part_of_defn, axiom)
$\forall c, c_1, c_2$: $(c = c_1{+}c_2 \iff \forall q$: (incident_c$(q, c) \iff$ (incident_c(q, c_1) or incident_c(q, c_2)))) fof(sum_defn, axiom)
$\forall p, c$: (end_point$(p, c) \iff$ (incident_c(p, c) and $\forall c_1, c_2$: ((part_of(c_1, c) and part_of(c_2, c) and incident_c(p, c_1) and incident_c
(part_of(c_1, c_2) or part_of(c_2, c_1)))))) fof(end_point_defn, axiom)
$\forall p, c$: (inner_point$(p, c) \iff$ (incident_c(p, c) and $\neg$ end_point(p, c))) fof(inner_point_defn, axiom)
$\forall p, c, c_1$: $(p \wedge c{=}c_1 \iff$ (incident_c(p, c) and incident_c(p, c_1) and $\forall q$: ((incident_c(q, c) and incident_c(q, c_1)) $\Rightarrow$
(end_point(q, c) and end_point(q, c_1))))) fof(meet_defn, axiom)
$\forall c$: (closed$(c) \iff \neg \exists p$: end_point(p, c)) fof(closed_defn, axiom)
$\forall c$: (open$(c) \iff \exists p$: end_point(p, c)) fof(open_defn, axiom)
$\forall c, c_1$: ((part_of(c_1, c) and $c_1 \neq c$) $\Rightarrow$ open(c_1)) fof$(c_1$, axiom)
$\forall c, c_1, c_2, c_3$: ((part_of(c_1, c) and part_of(c_2, c) and part_of(c_3, c) and $\exists p$: (end_point(p, c_1) and end_point(p, c_2) and end_point(
(part_of(c_2, c_3) or part_of(c_3, c_2) or part_of(c_1, c_2) or part_of(c_2, c_1) or part_of(c_1, c_3) or part_of(c_3, c_1)))) fof$(c_2$, axiom)
$\forall c$: $\exists p$: inner_point(p, c) fof$(c_3$, axiom)
$\forall c, p$: (inner_point$(p, c) \Rightarrow \exists c_1, c_2$: $(p \wedge c_1{=}c_2$ and $c = c_1 + c_2)$) fof$(c_4$, axiom)
$\forall c, p, q, r$: ((end_point(p, c) and end_point(q, c) and end_point(r, c)) $\Rightarrow$ $(p = q$ or $p = r$ or $q = r)$) fof$(c_5$, axiom)
$\forall c, p$: (end_point$(p, c) \Rightarrow \exists q$: (end_point$(q, c)$ and $p \neq q$)) fof$(c_6$, axiom)
$\forall c, c_1, c_2, p$: ((closed(c) and $p \wedge c_1{=}c_2$ and $c = c_1 + c_2$) $\Rightarrow \forall q$: (end_point$(q, c_1) \Rightarrow q \wedge c_1{=}c_2$)) fof$(c_7$, axiom)
$\forall c_1, c_2$: $(\exists p$: $p \wedge c_1{=}c_2 \Rightarrow \exists c$: $c = c_1 + c_2)$ fof$(c_8$, axiom)
$\forall c, c_1$: $(\forall p$: (incident_c$(p, c) \iff$ incident_c(p, c_1)) $\Rightarrow c = c_1)$ fof$(c_9$, axiom)

GEO004+1.ax Betweenness for simple curves
$\forall c, p, q, r$: (between_c$(c, p, q, r) \iff (p \neq r$ and $\exists$cpp: (part_of(cpp, c) and end_point(p, cpp) and end_point(r, cpp) and inner

GEO004+2.ax Oriented curves
$\forall o, p, q, r$: (between_o$(o, p, q, r) \iff$ ((ordered_by(o, p, q) and ordered_by(o, q, r)) or (ordered_by(o, r, q) and ordered_by$(o, q,$
$\forall p, o$: (start_point$(p, o) \iff$ (incident_o(p, o) and $\forall q$: (($p \neq q$ and incident_o(q, o)) $\Rightarrow$ ordered_by(o, p, q)))) fof(start_poi
$\forall p, o$: (finish_point$(p, o) \iff$ (incident_o(p, o) and $\forall q$: (($p \neq q$ and incident_o(q, o)) $\Rightarrow$ ordered_by(o, q, p)))) fof(finish_po
$\forall o, p, q$: (ordered_by$(o, p, q) \Rightarrow$ (incident_o(p, o) and incident_o(q, o))) fof$(o_1$, axiom)
$\forall o$: $\exists c$: (open(c) and $\forall p$: (incident_o$(p, o) \iff$ incident_c(p, c))) fof$(o_2$, axiom)
$\forall p, q, r, o$: (between_o$(o, p, q, r) \iff \exists c$: ($\forall p$: (incident_o$(p, o) \iff$ incident_c(p, c)) and between_c(c, p, q, r))) fof$(o_3$, axio
$\forall o$: $\exists p$: start_point(p, o) fof$(o_4$, axiom)
$\forall p, q, c$: ((open(c) and $p \neq q$ and incident_c(p, c) and incident_c(q, c)) $\Rightarrow \exists o$: ($\forall r$: (incident_o$(r, o) \iff$ incident_c(r, c)) and
$\forall o_1, o_2$: ($\forall p, q$: (ordered_by$(o_1, p, q) \iff$ ordered_by(o_2, p, q)) $\Rightarrow o_1 = o_2$) fof$(o_6$, axiom)
$\forall c, o$: ($c =$ underlying_curve$(o) \iff \forall p$: (incident_o$(p, o) \iff$ incident_c(p, c))) fof(underlying_curve_defn, axiom)

GEO004+3.ax Trajectories
$\forall x, y, p$: (connect$(x, y, p) \iff$ once(at_the_same_time(at(x, p), at(y, p)))) fof(connect_defn, axiom)
$\forall a, b$: (once(at_the_same_time(a, b)) $\iff$ once(at_the_same_time(b, a))) fof(symmetry_of_at_the_same_time, axiom)
$\forall a, b, c$: (once(at_the_same_time(at_the_same_time$(a, b), c)$) $\iff$ once(at_the_same_time$(a$, at_the_same_time(b, c)))) fof(as
$\forall a$: (once$(a) \Rightarrow$ once(at_the_same_time(a, a))) fof(idempotence_of_at_the_same_time, axiom)
$\forall a, b$: (once(at_the_same_time(a, b)) $\Rightarrow$ (once(a) and once(b))) fof(conjunction_at_the_same_time, axiom)
$\forall x, p$: (once(at(x, p)) $\iff$ incident_o$(p$, trajectory_of(x))) fof(at_on_trajectory, axiom)
$\forall x$: $\exists o$: trajectory_of$(x) = o$ fof(trajectories_are_oriented_curves, axiom)
$\forall p_1, p_2, q_1, q_2, x, y$: ((once(at_the_same_time(at(x, p_1), at(y, p_2))) and once(at_the_same_time(at(x, q_1), at(y, q_2)))) $\Rightarrow$
$\neg$ ordered_by(trajectory_of$(x), p_1, q_1)$ and ordered_by(trajectory_of$(y), q_2, p_2)$) fof(homogeneous_behaviour, axiom)
$\forall a$: (once$(a) \Rightarrow \forall x$: $\exists p$: once(at_the_same_time$(a$, at(x, p)))) fof(localization, axiom)

GEO004-1.ax Betweenness for simple curves
between_c$(a, b, c, d) \Rightarrow b \neq d$ cnf(between_c_defn$_1$, axiom)
between_c$(a, b, c, d) \Rightarrow$ part_of(ax1_sk$_1(d, c, b, a), a)$ cnf(between_c_defn$_2$, axiom)
between_c$(a, b, c, d) \Rightarrow$ end_point$(b$, ax1_sk$_1(d, c, b, a))$ cnf(between_c_defn$_3$, axiom)
between_c$(a, b, c, d) \Rightarrow$ end_point$(d$, ax1_sk$_1(d, c, b, a))$ cnf(between_c_defn$_4$, axiom)
between_c$(a, b, c, d) \Rightarrow$ inner_point$(c$, ax1_sk$_1(d, c, b, a))$ cnf(between_c_defn$_5$, axiom)
(part_of(c, d) and end_point(a, c) and end_point(b, c) and inner_point(e, c)) $\Rightarrow$ ($a = b$ or between_c(d, a, e, b)) cnf(betwee

GEO004-3.ax Trajectories
connect$(a, b, c) \Rightarrow$ once(at_the_same_time(at(a, c), at(b, c))) cnf(connect_defn$_1$, axiom)

once(at_the_same_time(at(a,b), at(c,b))) $\Rightarrow$ connect(a,c,b) cnf(connect_defn$_2$, axiom)
once(at_the_same_time(a,b)) $\Rightarrow$ once(at_the_same_time(b,a)) cnf(symmetry_of_at_the_same_time$_3$, axiom)
once(at_the_same_time(a,b)) $\Rightarrow$ once(at_the_same_time(b,a)) cnf(symmetry_of_at_the_same_time$_4$, axiom)
once(at_the_same_time(at_the_same_time(a,b), c)) $\Rightarrow$ once(at_the_same_time(a, at_the_same_time(b,c))) cnf(assciativity_o
once(at_the_same_time(a, at_the_same_time(b,c))) $\Rightarrow$ once(at_the_same_time(at_the_same_time(a,b), c)) cnf(assciativity_o
once(a) $\Rightarrow$ once(at_the_same_time(a,a)) cnf(idempotence_of_at_the_same_time$_7$, axiom)
once(at_the_same_time(a,b)) $\Rightarrow$ once(a) cnf(conjunction_at_the_same_time$_8$, axiom)
once(at_the_same_time(a,b)) $\Rightarrow$ once(b) cnf(conjunction_at_the_same_time$_9$, axiom)
once(at(a,b)) $\Rightarrow$ incident_o(b, trajectory_of(a)) cnf(at_on_trajectory$_{10}$, axiom)
incident_o(a, trajectory_of(b)) $\Rightarrow$ once(at(b,a)) cnf(at_on_trajectory$_{11}$, axiom)
trajectory_of(a) = ax3_sk$_1$(a) cnf(trajectories_are_oriented_curves$_{12}$, axiom)
(once(at_the_same_time(at(a,b), at(c,d))) and once(at_the_same_time(at(a,e), at(c,f))) and ordered_by(trajectory_of(a), b,e)
$\neg$ ordered_by(trajectory_of(c), f,d) cnf(homogeneous_behaviour$_{13}$, axiom)
once(a) $\Rightarrow$ once(at_the_same_time(a, at(b, ax3_sk$_2$(b,a)))) cnf(localization$_{14}$, axiom)

GEO006+0.ax Apartness geometry
$\forall x$: $\neg$ distinct_points(x,x) fof(apart$_1$, axiom)
$\forall x$: $\neg$ distinct_lines(x,x) fof(apart$_2$, axiom)
$\forall x$: $\neg$ convergent_lines(x,x) fof(apart$_3$, axiom)
$\forall x,y,z$: (distinct_points(x,y) $\Rightarrow$ (distinct_points(x,z) or distinct_points(y,z))) fof(apart$_4$, axiom)
$\forall x,y,z$: (distinct_lines(x,y) $\Rightarrow$ (distinct_lines(x,z) or distinct_lines(y,z))) fof(apart$_5$, axiom)
$\forall x,y,z$: (convergent_lines(x,y) $\Rightarrow$ (convergent_lines(x,z) or convergent_lines(y,z))) fof(ax$_6$, axiom)
$\forall x,y$: (distinct_points(x,y) $\Rightarrow$ $\neg$ apart_point_and_line(x, line_connecting(x,y))) fof(ci$_1$, axiom)
$\forall x,y$: (distinct_points(x,y) $\Rightarrow$ $\neg$ apart_point_and_line(y, line_connecting(x,y))) fof(ci$_2$, axiom)
$\forall x,y$: (convergent_lines(x,y) $\Rightarrow$ $\neg$ apart_point_and_line(intersection_point(x,y), x)) fof(ci$_3$, axiom)
$\forall x,y$: (convergent_lines(x,y) $\Rightarrow$ $\neg$ apart_point_and_line(intersection_point(x,y), y)) fof(ci$_4$, axiom)
$\forall x,y,u,v$: ((distinct_points(x,y) and distinct_lines(u,v)) $\Rightarrow$ (apart_point_and_line(x,u) or apart_point_and_line(x,v) or apar
$\forall x,y,z$: (apart_point_and_line(x,y) $\Rightarrow$ (distinct_points(x,z) or apart_point_and_line(z,y))) fof(ceq$_1$, axiom)
$\forall x,y,z$: (apart_point_and_line(x,y) $\Rightarrow$ (distinct_lines(y,z) or apart_point_and_line(x,z))) fof(ceq$_2$, axiom)
$\forall x,y,z$: (convergent_lines(x,y) $\Rightarrow$ (distinct_lines(y,z) or convergent_lines(x,z))) fof(ceq$_3$, axiom)

GEO006+1.ax Projective geometry
$\forall x,y$: (distinct_lines(x,y) $\Rightarrow$ convergent_lines(x,y)) fof(p_1, axiom)

GEO006+2.ax Affine geometry
$\forall x,y$: $\neg$ convergent_lines(parallel_through_point(y,x), y) fof(cp$_1$, axiom)
$\forall x,y$: $\neg$ apart_point_and_line(x, parallel_through_point(y,x)) fof(cp$_2$, axiom)
$\forall x,y,z$: (distinct_lines(y,z) $\Rightarrow$ (apart_point_and_line(x,y) or apart_point_and_line(x,z) or convergent_lines(y,z))) fof(cu

GEO006+3.ax Orthogonality
$\forall l,m$: (convergent_lines(l,m) or unorthogonal_lines(l,m)) fof(occu$_1$, axiom)
$\forall l,m,n$: ((convergent_lines(l,m) and unorthogonal_lines(l,m)) $\Rightarrow$ ((convergent_lines(l,n) and unorthogonal_lines(l,n)) or (c
$\forall a,l$: $\neg$ unorthogonal_lines(orthogonal_through_point(l,a), l) fof(ooc$_1$, axiom)
$\forall a,l$: $\neg$ apart_point_and_line(a, orthogonal_through_point(l,a)) fof(ooc$_2$, axiom)
$\forall a,l,m,n$: (distinct_lines(l,m) $\Rightarrow$ (apart_point_and_line(a,l) or apart_point_and_line(a,m) or unorthogonal_lines(l,n) or unc

GEO006+4.ax Classical orthogonality
$\forall l,m$: $\neg\neg$ convergent_lines(l,m) and $\neg$ unorthogonal_lines(l,m) fof(coipo$_1$, axiom)
$\forall l,m,n$: ((($\neg$ convergent_lines(l,m) or $\neg$ unorthogonal_lines(l,m)) and ($\neg$ convergent_lines(l,n) or $\neg$ unorthogonal_lines(l,n)))
($\neg$ convergent_lines(m,n) or $\neg$ unorthogonal_lines(m,n))) fof(cotno$_1$, axiom)
$\forall l,m,n$: (($\neg$ unorthogonal_lines(l,m) and $\neg$ unorthogonal_lines(l,n)) $\Rightarrow$ $\neg$ convergent_lines(m,n)) fof(couo$_1$, axiom)

GEO006+5.ax Rules of construction
$\forall a,b$: ((point(a) and point(b) and distinct_points(a,b)) $\Rightarrow$ line(line_connecting(a,b))) fof(con$_1$, axiom)
$\forall l,m$: ((line(l) and line(m) and convergent_lines(l,m)) $\Rightarrow$ point(intersection_point(l,m))) fof(int$_1$, axiom)
$\forall l,a$: ((line(l) and point(a)) $\Rightarrow$ line(parallel_through_point(l,a))) fof(par$_1$, axiom)
$\forall l,a$: ((line(l) and point(a)) $\Rightarrow$ line(orthogonal_through_point(l,a))) fof(orth$_1$, axiom)

GEO006+6.ax Geometry definitions
$\forall x,y$: (equal_points(x,y) $\iff$ $\neg$ distinct_points(x,y)) fof(ax$_1$, axiom)
$\forall x,y$: (equal_lines(x,y) $\iff$ $\neg$ distinct_lines(x,y)) fof(ax$_2$, axiom)
$\forall x,y$: (parallel_lines(x,y) $\iff$ $\neg$ convergent_lines(x,y)) fof(a_3, axiom)
$\forall x,y$: (incident_point_and_line(x,y) $\iff$ $\neg$ apart_point_and_line(x,y)) fof(a_4, axiom)

$\forall x, y$: (orthogonal_lines$(x, y) \iff \neg$ unorthogonal_lines(x, y)) fof(a_5, axiom)

GEO007+1.ax Principles of a classical calculus of directed lines
$\forall l$: ¬ unequally_directed_lines(l, l) fof(cld$_1$, axiom)
$\forall l, m, n$: ((¬ unequally_directed_lines(l, m) and ¬ unequally_directed_lines(l, n)) $\Rightarrow$ ¬ unequally_directed_lines(m, n)) fof(c
$\forall a, b, l, m$: (¬ unequally_directed_lines(l, m) and unequally_directed_lines$(l$, reverse_line(m)) $\iff$ (¬ unequally_directed_lines(
$\forall l, m, n$: (((¬ unequally_directed_lines(l, m) or ¬ unequally_directed_lines$(l$, reverse_line(m))) and (¬ unequally_directed_lines(
(¬ unequally_directed_lines(m, n) or ¬ unequally_directed_lines$(m$, reverse_line(n))))) fof(cld3a, axiom)
$\forall l, m$: ((line(l) and line(m)) $\Rightarrow$ ¬ ¬ unequally_directed_lines(l, m) and ¬ unequally_directed_lines$(l$, reverse_line(m))) fof(c
$\forall l, m, n$: (¬ unequally_directed_lines$(l$, reverse_line(m)) and (¬ unequally_directed_lines$(l$, reverse_line(n)) $\Rightarrow$ ¬ unequally_dire

GEO008+0.ax Apartness geometry
$\forall x$: ¬ distinct_points(x, x) fof(apart$_1$, axiom)
$\forall x$: ¬ distinct_lines(x, x) fof(apart$_2$, axiom)
$\forall x$: ¬ convergent_lines(x, x) fof(apart$_3$, axiom)
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (distinct_points(x, z) or distinct_points(y, z))) fof(apart$_4$, axiom)
$\forall x, y, z$: (distinct_lines(x, y) $\Rightarrow$ (distinct_lines(x, z) or distinct_lines(y, z))) fof(apart$_5$, axiom)
$\forall x, y, z$: (convergent_lines(x, y) $\Rightarrow$ (convergent_lines(x, z) or convergent_lines(y, z))) fof(apart$_6$, axiom)
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line$(z$, line_connecting(x, y)) $\Rightarrow$ (distinct_points(z, x) and distinct_points(
$\forall x, y, z$: (convergent_lines(x, y) $\Rightarrow$ ((apart_point_and_line(z, x) or apart_point_and_line(z, y)) $\Rightarrow$ distinct_points$(z$, intersectio
$\forall x, y, u, v$: ((distinct_points(x, y) and distinct_lines(u, v)) $\Rightarrow$ (apart_point_and_line(x, u) or apart_point_and_line(x, v) or apar
$\forall x, y, z$: (apart_point_and_line(x, y) $\Rightarrow$ (distinct_points(x, z) or apart_point_and_line(z, y))) fof(ceq$_1$, axiom)
$\forall x, y, z$: (apart_point_and_line(x, y) $\Rightarrow$ (distinct_lines(y, z) or apart_point_and_line(x, z))) fof(ceq$_2$, axiom)
$\forall x, y$: (convergent_lines(x, y) $\Rightarrow$ distinct_lines(x, y)) fof(ceq$_3$, axiom)

GEO problems

GEO001-1.p Betweenness is symmetric in its outer arguments
include('Axioms/GEO001-0.ax')
between(a, b, c) cnf(b_between_a_and_c, hypothesis)
¬ between(c, b, a) cnf(prove_b_between_c_and_a, negated_conjecture)

GEO001-2.p Betweenness is symmetric in its outer arguments
include('Axioms/GEO002-0.ax')
between(a, b, c) cnf(b_between_a_and_c, hypothesis)
¬ between(c, b, a) cnf(prove_b_between_c_and_a, negated_conjecture)

GEO001-3.p Betweenness is symmetric in its outer arguments
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v$, reflection(u, v)) cnf(r2$_1$, axiom)
equidistant$(v$, reflection$(u, v), u, v)$ cnf(r2$_2$, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf(r3$_1$, axiom)
$u =$ reflection(u, u) cnf(r3$_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant$(u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u, v, w) $\Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf(d10$_1$, axiom)
equidistant(w, x, y, z) $\Rightarrow$ (extension$(u, v, w, x) =$ extension(u, v, y, z) or $u = v$) cnf(d10$_2$, axiom)
extension$(u, v, u, v) =$ extension(u, v, v, u) or $u = v$ cnf(d10$_3$, axiom)

equidistant(v, u, v, reflection(reflection(u, v), v))   cnf(r_5, axiom)
u = reflection(reflection(u, v), v)   cnf(r_6, axiom)
between(u, v, v)   cnf(t_3, axiom)
(between(u, w, x) and $u = x$) $\Rightarrow$ between(v, w, x)   cnf(b_1, axiom)
between(a, b, c)   cnf(b_between_a_and_c, hypothesis)
$\neg$ between(c, b, a)   cnf(prove_b_between_c_and_a, negated_conjecture)

GEO001-4.p Betweenness is symmetric in its outer arguments
between(x, y, x) $\Rightarrow$ $x{=}y$ cnf(identity_for_betweeness, axiom)
equidistant(x, y, z, z) $\Rightarrow$ $x{=}y$ cnf(identity_for_equidistance, axiom)
(between(x, w, v) and between(y, v, z)) $\Rightarrow$ between(x, outer_pasch(w, x, y, z, v), y)   cnf(outer_pasch$_1$, axiom)
(between(x, w, v) and between(y, v, z)) $\Rightarrow$ between(z, w, outer_pasch(w, x, y, z, v))   cnf(outer_pasch$_2$, axiom)
between(x, y, extension(x, y, w, v))   cnf(segment_construction$_1$, axiom)
equidistant(y, extension(x, y, w, v), w, v)   cnf(segment_construction$_2$, axiom)
between(a, b, c)   cnf(b_between_a_and_c, hypothesis)
($x{=}y$ and between(w, z, x)) $\Rightarrow$ between(w, z, y)   cnf(between_substitution$_3$, axiom)
$\neg$ between(c, b, a)   cnf(prove_b_between_c_and_a, negated_conjecture)

GEO001_1.p Betweenness is symmetric in its outer arguments
point: $tType   tff(point_type, type)
line_point: $tType   tff(line_point_type, type)
outer_pasch: (point $\times$ point $\times$ point $\times$ point $\times$ point) $\rightarrow$ point   tff(outer_pasch_type, type)
extension: (point $\times$ point $\times$ line_point $\times$ line_point) $\rightarrow$ point   tff(extension_type, type)
equidistant: (point $\times$ point $\times$ line_point $\times$ line_point) $\rightarrow$ $o   tff(equidistant_type, type)
= : (point $\times$ point) $\rightarrow$ $o   tff(equalish_type, type)
between: (point $\times$ point $\times$ point) $\rightarrow$ $o   tff(between_type, type)
$\forall y$: point, x: point: (between(x, y, x) $\Rightarrow$ $x{=}y$)   tff(identity_for_betweeness, axiom)
$\forall z$: line_point, y: point, x: point: (equidistant(x, y, z, z) $\Rightarrow$ $x{=}y$)   tff(identity_for_equidistance, axiom)
$\forall z$: point, y: point, v: point, w: point, x: point: ((between(x, w, v) and between(y, v, z)) $\Rightarrow$ between(x, outer_pasch(w, x, y, z, v
$\forall z$: point, y: point, v: point, w: point, x: point: ((between(x, w, v) and between(y, v, z)) $\Rightarrow$ between(z, w, outer_pasch(w, x, y, z
$\forall v$: line_point, w: line_point, y: point, x: point: between(x, y, extension(x, y, w, v))   tff(segment_construction$_1$, axiom)
$\forall v$: line_point, w: line_point, x: point, y: point: equidistant(y, extension(x, y, w, v), w, v)   tff(segment_construction$_2$, axiom)
$\forall z$: point, w: point, y: point, x: point: (($x{=}y$ and between(w, z, x)) $\Rightarrow$ between(w, z, y))   tff(between_substitution$_3$, axiom)
$\forall x$: point, y: point, z: point: (between(x, y, z) $\Rightarrow$ between(z, y, x))   tff(symmetric, conjecture)

GEO002-1.p For all points x and y, x is between x and y
include('Axioms/GEO001-0.ax')
$\neg$ between(a, a, b)   cnf(prove_a_between_a_and_b, negated_conjecture)

GEO002-2.p For all points x and y, x is between x and y
include('Axioms/GEO002-0.ax')
$\neg$ between(a, a, b)   cnf(prove_a_between_a_and_b, negated_conjecture)

GEO002-3.p For all points x and y, x is between x and y
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v)   cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v)   cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x)   cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w)   cnf(d4$_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w)   cnf(d4$_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u)   cnf(d4$_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v)   cnf(d4$_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u)   cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z)   cnf(d_5, axiom)
v = extension(u, v, w, w)   cnf(e_1, axiom)
y = extension(u, v, w, x) $\Rightarrow$ between(u, v, y)   cnf(b_0, axiom)
between(u, v, reflection(u, v))   cnf(r2$_1$, axiom)
equidistant(v, reflection(u, v), u, v)   cnf(r2$_2$, axiom)
$u = v$ $\Rightarrow$ v = reflection(u, v)   cnf(r3$_1$, axiom)
u = reflection(u, u)   cnf(r3$_2$, axiom)
v = reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)

$\text{equidistant}(u, u, v, v) \qquad \text{cnf}(d_7, \text{axiom})$
$(\text{equidistant}(u, v, u_1, v_1) \text{ and } \text{equidistant}(v, w, v_1, w_1) \text{ and } \text{between}(u, v, w) \text{ and } \text{between}(u_1, v_1, w_1)) \Rightarrow \text{equidistant}(u, w, u_1,$
$(\text{between}(u, v, w) \text{ and } \text{between}(u, v, x) \text{ and } \text{equidistant}(v, w, v, x)) \Rightarrow (u = v \text{ or } w = x) \qquad \text{cnf}(d_9, \text{axiom})$
$\text{between}(u, v, w) \Rightarrow (u = v \text{ or } w = \text{extension}(u, v, v, w)) \qquad \text{cnf}(\text{d10}_1, \text{axiom})$
$\text{equidistant}(w, x, y, z) \Rightarrow (\text{extension}(u, v, w, x) = \text{extension}(u, v, y, z) \text{ or } u = v) \qquad \text{cnf}(\text{d10}_2, \text{axiom})$
$\text{extension}(u, v, u, v) = \text{extension}(u, v, v, u) \text{ or } u = v \qquad \text{cnf}(\text{d10}_3, \text{axiom})$
$\text{equidistant}(v, u, v, \text{reflection}(\text{reflection}(u, v), v)) \qquad \text{cnf}(r_5, \text{axiom})$
$u = \text{reflection}(\text{reflection}(u, v), v) \qquad \text{cnf}(r_6, \text{axiom})$
$\text{between}(u, v, v) \qquad \text{cnf}(t_3, \text{axiom})$
$(\text{between}(u, w, x) \text{ and } u = x) \Rightarrow \text{between}(v, w, x) \qquad \text{cnf}(b_1, \text{axiom})$
$\text{between}(u, v, w) \Rightarrow \text{between}(w, v, u) \qquad \text{cnf}(t_1, \text{axiom})$
$\neg\,\text{between}(a, a, b) \qquad \text{cnf}(\text{prove\_a\_between\_a\_and\_b}, \text{negated\_conjecture})$

GEO002-4.p For all points x and y, x is between x and y
$(\text{between}(x, y, v) \text{ and } \text{between}(y, z, v)) \Rightarrow \text{between}(x, y, z) \qquad \text{cnf}(\text{transitivity\_for\_betweeness}, \text{axiom})$
$\text{equidistant}(x, y, z, z) \Rightarrow x{=}y \qquad \text{cnf}(\text{identity\_for\_equidistance}, \text{axiom})$
$(\text{between}(x, w, v) \text{ and } \text{between}(y, v, z)) \Rightarrow \text{between}(x, \text{outer\_pasch}(w, x, y, z, v), y) \qquad \text{cnf}(\text{outer\_pasch}_1, \text{axiom})$
$(\text{between}(x, w, v) \text{ and } \text{between}(y, v, z)) \Rightarrow \text{between}(z, w, \text{outer\_pasch}(w, x, y, z, v)) \qquad \text{cnf}(\text{outer\_pasch}_2, \text{axiom})$
$\text{between}(x, y, \text{extension}(x, y, w, v)) \qquad \text{cnf}(\text{segment\_construction}_1, \text{axiom})$
$\text{equidistant}(y, \text{extension}(x, y, w, v), w, v) \qquad \text{cnf}(\text{segment\_construction}_2, \text{axiom})$
$(x{=}y \text{ and } \text{between}(w, z, x)) \Rightarrow \text{between}(w, z, y) \qquad \text{cnf}(\text{between\_substitution}_3, \text{axiom})$
$\neg\,\text{between}(a, a, b) \qquad \text{cnf}(\text{prove\_a\_between\_a\_and\_b}, \text{negated\_conjecture})$

GEO003-1.p For all points x and y, y is between x and y
include('Axioms/GEO001-0.ax')
$\neg\,\text{between}(a, b, b) \qquad \text{cnf}(\text{prove\_b\_between\_a\_and\_b}, \text{negated\_conjecture})$

GEO003-2.p For all points x and y, y is between x and y
include('Axioms/GEO002-0.ax')
$\neg\,\text{between}(a, b, b) \qquad \text{cnf}(\text{prove\_b\_between\_a\_and\_b}, \text{negated\_conjecture})$

GEO003-3.p For all points x and y, y is between x and y
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$\text{equidistant}(u, v, u, v) \qquad \text{cnf}(d_1, \text{axiom})$
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, u, v) \qquad \text{cnf}(d_2, \text{axiom})$
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, w, x) \qquad \text{cnf}(d_3, \text{axiom})$
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(u, v, x, w) \qquad \text{cnf}(\text{d4}_1, \text{axiom})$
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, x, w) \qquad \text{cnf}(\text{d4}_2, \text{axiom})$
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, v, u) \qquad \text{cnf}(\text{d4}_3, \text{axiom})$
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(x, w, u, v) \qquad \text{cnf}(\text{d4}_4, \text{axiom})$
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(x, w, v, u) \qquad \text{cnf}(\text{d4}_5, \text{axiom})$
$(\text{equidistant}(u, v, w, x) \text{ and } \text{equidistant}(w, x, y, z)) \Rightarrow \text{equidistant}(u, v, y, z) \qquad \text{cnf}(d_5, \text{axiom})$
$v = \text{extension}(u, v, w, w) \qquad \text{cnf}(e_1, \text{axiom})$
$y = \text{extension}(u, v, w, x) \Rightarrow \text{between}(u, v, y) \qquad \text{cnf}(b_0, \text{axiom})$
$\text{between}(u, v, \text{reflection}(u, v)) \qquad \text{cnf}(\text{r2}_1, \text{axiom})$
$\text{equidistant}(v, \text{reflection}(u, v), u, v) \qquad \text{cnf}(\text{r2}_2, \text{axiom})$
$u = v \Rightarrow v = \text{reflection}(u, v) \qquad \text{cnf}(\text{r3}_1, \text{axiom})$
$u = \text{reflection}(u, u) \qquad \text{cnf}(\text{r3}_2, \text{axiom})$
$v = \text{reflection}(u, v) \Rightarrow u = v \qquad \text{cnf}(r_4, \text{axiom})$
$\text{equidistant}(u, u, v, v) \qquad \text{cnf}(d_7, \text{axiom})$
$(\text{equidistant}(u, v, u_1, v_1) \text{ and } \text{equidistant}(v, w, v_1, w_1) \text{ and } \text{between}(u, v, w) \text{ and } \text{between}(u_1, v_1, w_1)) \Rightarrow \text{equidistant}(u, w, u_1,$
$(\text{between}(u, v, w) \text{ and } \text{between}(u, v, x) \text{ and } \text{equidistant}(v, w, v, x)) \Rightarrow (u = v \text{ or } w = x) \qquad \text{cnf}(d_9, \text{axiom})$
$\text{between}(u, v, w) \Rightarrow (u = v \text{ or } w = \text{extension}(u, v, v, w)) \qquad \text{cnf}(\text{d10}_1, \text{axiom})$
$\text{equidistant}(w, x, y, z) \Rightarrow (\text{extension}(u, v, w, x) = \text{extension}(u, v, y, z) \text{ or } u = v) \qquad \text{cnf}(\text{d10}_2, \text{axiom})$
$\text{extension}(u, v, u, v) = \text{extension}(u, v, v, u) \text{ or } u = v \qquad \text{cnf}(\text{d10}_3, \text{axiom})$
$\text{equidistant}(v, u, v, \text{reflection}(\text{reflection}(u, v), v)) \qquad \text{cnf}(r_5, \text{axiom})$
$u = \text{reflection}(\text{reflection}(u, v), v) \qquad \text{cnf}(r_6, \text{axiom})$
$\neg\,\text{between}(a, b, b) \qquad \text{cnf}(\text{prove\_b\_between\_a\_and\_b}, \text{negated\_conjecture})$

GEO004-1.p Every line segment has a midpoint
include('Axioms/GEO001-0.ax')

$a \neq b$ cnf(a_not_b, hypothesis)
equidistant$(a, x, b, x) \Rightarrow \neg$ between(a, x, b) cnf(prove_midpoint, negated_conjecture)

GEO004-2.p Every line segment has a midpoint
include('Axioms/GEO002-0.ax')
between$(a,$ midpoint$(a, b), b) \Rightarrow \neg$ equidistant$(a,$ midpoint$(a, b), b,$ midpoint$(a, b))$ cnf(prove_midpoint, negated_conjecture

GEO005-1.p Isosceles triangle based on line segment
For any line segment, there exists an isoceles triangle with that line segment as its base.
include('Axioms/GEO001-0.ax')
$a \neq b$ cnf(a_not_b, hypothesis)
equidistant$(a, x, b, x) \Rightarrow$ between(a, x, b) cnf(prove_apex, negated_conjecture)

GEO005-2.p Isosceles triangle based on line segment
For any line segment, there exists an isoceles triangle with that line segment as its base.
include('Axioms/GEO002-0.ax')
equidistant$(a,$ apex$(a, b), b,$ apex$(a, b)) \Rightarrow$ between$(a,$ apex$(a, b), b)$ cnf(prove_apex, negated_conjecture)

GEO006-1.p Betweenness for 3 points on a line
For any three distinct points x, y, and z, if y is between x and z, then both x is not between y and z and z is not between x and y.
include('Axioms/GEO001-0.ax')
$a \neq c$ cnf(a_not_c, hypothesis)
$a \neq d$ cnf(a_not_d, hypothesis)
$c \neq d$ cnf(c_not_d, hypothesis)
between(a, c, d) cnf(c_between_a_and_d, hypothesis)
between(c, a, d) or between(a, d, c) cnf(prove_not_between_others, negated_conjecture)

GEO006-2.p Betweenness for 3 points on a line
For any three distinct points x, y, and z, if y is between x and z, then both x is not between y and z and z is not between x and y.
include('Axioms/GEO002-0.ax')
$a \neq c$ cnf(a_not_c, hypothesis)
$a \neq d$ cnf(a_not_d, hypothesis)
$c \neq d$ cnf(c_not_d, hypothesis)
between(a, c, d) cnf(c_between_a_and_d, hypothesis)
between(c, a, d) or between(a, d, c) cnf(prove_not_between_others, negated_conjecture)

GEO006-3.p Betweenness for 3 points on a line
For any three distinct points x, y, and z, if y is between x and z, then both x is not between y and z and z is not between x and y.
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf($\mathrm{d4}_1$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf($\mathrm{d4}_2$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf($\mathrm{d4}_3$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf($\mathrm{d4}_4$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf($\mathrm{d4}_5$, axiom)
(equidistant(u, v, w, x) and equidistant$(w, x, y, z)) \Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension$(u, v, w, x) \Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v,$ reflection$(u, v))$ cnf($\mathrm{r2}_1$, axiom)
equidistant$(v,$ reflection$(u, v), u, v)$ cnf($\mathrm{r2}_2$, axiom)
$u = v \Rightarrow v =$ reflection(u, v) cnf($\mathrm{r3}_1$, axiom)
$u =$ reflection(u, u) cnf($\mathrm{r3}_2$, axiom)
$v =$ reflection$(u, v) \Rightarrow u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between$(u_1, v_1, w_1)) \Rightarrow$ equidistant$(u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant$(v, w, v, x)) \Rightarrow (u = v$ or $w = x)$ cnf(d_9, axiom)
between$(u, v, w) \Rightarrow (u = v$ or $w =$ extension$(u, v, v, w))$ cnf($\mathrm{d10}_1$, axiom)

equidistant$(w, x, y, z) \Rightarrow$ (extension$(u, v, w, x) =$ extension(u, v, y, z) or $u = v$) cnf($\mathrm{d10}_2$, axiom)
extension$(u, v, u, v) =$ extension(u, v, v, u) or $u = v$ cnf($\mathrm{d10}_3$, axiom)
equidistant$(v, u, v,$ reflection(reflection$(u, v), v))$ cnf(r_5, axiom)
$u =$ reflection(reflection$(u, v), v)$ cnf(r_6, axiom)
between(u, v, v) cnf(t_3, axiom)
(between(u, w, x) and $u = x$) $\Rightarrow$ between(v, w, x) cnf(b_1, axiom)
between$(u, v, w) \Rightarrow$ between(w, v, u) cnf(t_1, axiom)
between(u, u, v) cnf(t_2, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow u = v$ cnf(b_2, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow v = w$ cnf(b_3, axiom)
$a \neq c$ cnf(a_not_c, hypothesis)
$a \neq d$ cnf(a_not_d, hypothesis)
$c \neq d$ cnf(c_not_d, hypothesis)
between(a, c, d) cnf(c_between_a_and_d, hypothesis)
between(c, a, d) or between(a, d, c) cnf(prove_not_between_others, negated_conjecture)

GEO007-1.p Betweenness for 4 points on a line
For all pairs of distinct points y and z, if w and x are on the line yz to the left of y (i.e. not between y and z), then either w is betweeen x and y or x is between w and y.
include('Axioms/GEO001-0.ax')
$a \neq c$ cnf(a_not_c, hypothesis)
between(d, a, c) cnf(a_between_a_and_c, hypothesis)
between(e, a, c) cnf(a_between_a_and_e, hypothesis)
$\neg$ between(d, e, a) cnf(e_not_between_d_and_a, hypothesis)
$\neg$ between(e, d, a) cnf(prove_d_between_e_and_a, negated_conjecture)

GEO007-2.p Betweenness for 4 points on a line
For all pairs of distinct points y and z, if w and x are on the line yz to the left of y (i.e. not between y and z), then either w is betweeen x and y or x is between w and y.
include('Axioms/GEO002-0.ax')
$a \neq c$ cnf(a_not_c, hypothesis)
between(d, a, c) cnf(a_between_a_and_c, hypothesis)
between(e, a, c) cnf(a_between_a_and_e, hypothesis)
$\neg$ between(d, e, a) cnf(e_not_between_d_and_a, hypothesis)
$\neg$ between(e, d, a) cnf(prove_d_between_e_and_a, negated_conjecture)

GEO008-1.p Betweenness for 5 points on a line (Five point theorem)
For all points x, y, z, w, and v, if y and w are between x and z, and $\vee$ is between y and w, then $\vee$ is between x and z.
include('Axioms/GEO001-0.ax')
between(a, c, e) cnf(c_between_a_and_e, hypothesis)
between(a, d, e) cnf(d_between_a_and_e, hypothesis)
between(c, b, d) cnf(b_between_c_and_d, hypothesis)
$\neg$ between(a, b, e) cnf(prove_betweenness, negated_conjecture)

GEO008-2.p Betweenness for 5 points on a line (Five point theorem)
For all points x, y, z, w, and v, if y and w are between x and z, and $\vee$ is between y and w, then $\vee$ is between x and z.
include('Axioms/GEO002-0.ax')
between(a, c, e) cnf(c_between_a_and_e, hypothesis)
between(a, d, e) cnf(d_between_a_and_e, hypothesis)
between(c, b, d) cnf(b_between_c_and_d, hypothesis)
$\neg$ between(a, b, e) cnf(prove_betweenness, negated_conjecture)

GEO009-1.p First inner connectivity property of betweenness
For all points x, y, z, and w, if y and w are between x and z, then either y is between x and w or w is between x and y.
include('Axioms/GEO001-0.ax')
between(a, c, e) cnf(c_between_a_and_e, hypothesis)
between(a, d, e) cnf(d_between_a_and_e, hypothesis)
$\neg$ between(a, c, d) cnf(c_between_a_and_d, hypothesis)
$\neg$ between(a, d, c) cnf(prove_d_between_a_and_c, negated_conjecture)

GEO009-2.p First inner connectivity property of betweenness
For all points x, y, z, and w, if y and w are between x and z, then either y is between x and w or w is between x and y.
include('Axioms/GEO002-0.ax')
$\text{between}(a,c,e)$ cnf(c_between_a_and_e, hypothesis)
$\text{between}(a,d,e)$ cnf(d_between_a_and_e, hypothesis)
$\neg\,\text{between}(a,c,d)$ cnf(c_not_between_a_and_d, hypothesis)
$\neg\,\text{between}(a,d,c)$ cnf(prove_d_between_a_and_c, negated_conjecture)

GEO010-1.p Collinearity is invariant
For all points x, y, and z, if x, y, and z are collinear in one order, they are collinear in any order.
include('Axioms/GEO001-0.ax')
include('Axioms/GEO001-1.ax')
$\text{colinear}(a,b,c)$ cnf(abc_colinear, hypothesis)
$(\text{colinear}(a,c,b) \text{ and } \text{colinear}(b,a,c) \text{ and } \text{colinear}(b,c,a) \text{ and } \text{colinear}(c,a,b)) \Rightarrow \neg\,\text{colinear}(c,b,a)$ cnf(prove_colinear_in_a

GEO010-2.p Collinearity is invariant
For all points x, y, and z, if x, y, and z are collinear in one order, they are collinear in any order.
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$\text{colinear}(a,b,c)$ cnf(abc_colinear, hypothesis)
$(\text{colinear}(a,c,b) \text{ and } \text{colinear}(b,a,c) \text{ and } \text{colinear}(b,c,a) \text{ and } \text{colinear}(c,a,b)) \Rightarrow \neg\,\text{colinear}(c,b,a)$ cnf(prove_colinear_in_a

GEO011-1.p The axiom set points are not collinear
include('Axioms/GEO001-0.ax')
include('Axioms/GEO001-1.ax')
$\text{colinear}(\text{lower\_dimension\_point}_1, \text{lower\_dimension\_point}_2, \text{lower\_dimension\_point}_3)$ cnf(prove_lower_dimension_points_not_c

GEO011-2.p The axiom set points are not collinear
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$\text{colinear}(\text{lower\_dimension\_point}_1, \text{lower\_dimension\_point}_2, \text{lower\_dimension\_point}_3)$ cnf(prove_lower_dimension_points_not_c

GEO011-5.p The axiom set points are not collinear
include('Axioms/GEO002-1.ax')
$\text{equal\_distance}(\text{distance}(x,y), \text{distance}(y,x))$ cnf(reflexivity_for_equidistance, axiom)
$(\text{equal\_distance}(\text{distance}(x,y), \text{distance}(z,v)) \text{ and } \text{equal\_distance}(\text{distance}(x,y), \text{distance}(v_2,w))) \Rightarrow \text{equal\_distance}(\text{distance}$
$\text{equal\_distance}(\text{distance}(x,y), \text{distance}(z,z)) \Rightarrow x = y$ cnf(identity_for_equidistance, axiom)
$\text{between}(x,y,\text{extension}(x,y,w,v))$ cnf(segment_construction$_1$, axiom)
$\text{equal\_distance}(\text{distance}(y,\text{extension}(x,y,w,v)), \text{distance}(w,v))$ cnf(segment_construction$_2$, axiom)
$(\text{equal\_distance}(\text{distance}(x,y), \text{distance}(x_1,y_1)) \text{ and } \text{equal\_distance}(\text{distance}(y,z), \text{distance}(y_1,z_1)) \text{ and } \text{equal\_distance}(\text{distan}$
$(x = y \text{ or } \text{equal\_distance}(\text{distance}(z,v), \text{distance}(z_1,v_1)))$ cnf(outer_five_segment, axiom)
$\text{between}(x,y,x) \Rightarrow x = y$ cnf(identity_for_betweeness, axiom)
$(\text{between}(u,v,w) \text{ and } \text{between}(y,x,w)) \Rightarrow \text{between}(v, \text{inner\_pasch}(u,v,w,x,y), y)$ cnf(inner_pasch$_1$, axiom)
$(\text{between}(u,v,w) \text{ and } \text{between}(y,x,w)) \Rightarrow \text{between}(x, \text{inner\_pasch}(u,v,w,x,y), u)$ cnf(inner_pasch$_2$, axiom)
$\neg\,\text{between}(\text{lower\_dimension\_point}_1, \text{lower\_dimension\_point}_2, \text{lower\_dimension\_point}_3)$ cnf(lower_dimension$_1$, axiom)
$\neg\,\text{between}(\text{lower\_dimension\_point}_2, \text{lower\_dimension\_point}_3, \text{lower\_dimension\_point}_1)$ cnf(lower_dimension$_2$, axiom)
$\neg\,\text{between}(\text{lower\_dimension\_point}_3, \text{lower\_dimension\_point}_1, \text{lower\_dimension\_point}_2)$ cnf(lower_dimension$_3$, axiom)
$\text{colinear}(\text{lower\_dimension\_point}_1, \text{lower\_dimension\_point}_2, \text{lower\_dimension\_point}_3)$ cnf(prove_lower_dimension_points_not_c

GEO012-1.p Collinearity for 4 points
If any three distinct points x, y, and z are collinear and a fourth point w is collinear with x and y, then w is collinear with x and z and also with x and y.
include('Axioms/GEO001-0.ax')
include('Axioms/GEO001-1.ax')
$a \neq b$ cnf(a_not_b, hypothesis)
$a \neq c$ cnf(a_not_c, hypothesis)
$b \neq c$ cnf(b_not_c, hypothesis)
$\text{colinear}(a,b,c)$ cnf(abc_colinear, hypothesis)
$\text{colinear}(a,b,d)$ cnf(abd_colinear, hypothesis)
$\text{colinear}(a,c,d) \Rightarrow \neg\,\text{colinear}(b,c,d)$ cnf(prove_colinearity, negated_conjecture)

GEO012-2.p Collinearity for 4 points

If any three distinct points x, y, and z are collinear and a fourth point w is collinear with x and y, then w is collinear with x and z and also with x and y.
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$a \neq b$ cnf(a_not_b, hypothesis)
colinear(a, b, c) cnf(abc_colinear, hypothesis)
colinear(a, b, d) cnf(abd_colinear, hypothesis)
colinear$(a, c, d) \Rightarrow \neg$ colinear(b, c, d) cnf(prove_colinearity, negated_conjecture)

GEO013-1.p Collinearity for 5 points
If z1, z2, and z3 are each collinear with distinct points x and y, then z1, z2, and z3 are collinear.
include('Axioms/GEO001-0.ax')
include('Axioms/GEO001-1.ax')
$a \neq b$ cnf(a_not_b, hypothesis)
colinear(a, b, d_1) cnf(and1_colinear, hypothesis)
colinear(a, b, d_2) cnf(abd2_colinear, hypothesis)
colinear(a, b, d_3) cnf(abd3_colinear, hypothesis)
$\neg$ colinear(d_1, d_2, d_3) cnf(prove_d1d2d3_colinear, negated_conjecture)

GEO013-2.p Collinearity for 5 points
If z1, z2, and z3 are each collinear with distinct points x and y, then z1, z2, and z3 are collinear.
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$a \neq b$ cnf(a_not_b, hypothesis)
colinear(a, b, d_1) cnf(and1_colinear, hypothesis)
colinear(a, b, d_2) cnf(abd2_colinear, hypothesis)
colinear(a, b, d_3) cnf(abd3_colinear, hypothesis)
$\neg$ colinear(d_1, d_2, d_3) cnf(prove_d1d2d3_colinear, negated_conjecture)

GEO014-2.p Ordinary reflexivity of equidistance
This shows that the distance from A to B is the same as the distance from A to B. This is different from the axiom which states that the distance from A to B is the same as the distance from B to A.
include('Axioms/GEO002-0.ax')
$\neg$ equidistant(u, v, u, v) cnf(prove_reflexivity, negated_conjecture)

GEO015-2.p Equidistance is symmetric between its argument pairs
Show that if the distance from A to B equals the distance from C to D, then the distance from C to D equals the distance from A to B.
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(w, x, u, v) cnf(prove_symmetry, negated_conjecture)

GEO015-3.p Equidistance is symmetric between its argument pairs
Show that if the distance from A to B equals the distance from C to D, then the distance from C to D equals the distance from A to B.
include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(w, x, u, v) cnf(prove_symmetry, negated_conjecture)

GEO016-2.p Equidistance is symmetric within its argument pairs
Show that if the distance from A to B equals the distance from C to D, then the distance from B to A equals the distance from C to D.
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(v, u, w, x) cnf(prove_symmetry, negated_conjecture)

GEO016-3.p Equidistance is symmetric within its argument pairs
Show that if the distance from A to B equals the distance from C to D, then the distance from B to A equals the distance from C to D.
include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)

equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(v, u, w, x) cnf(prove_symmetry, negated_conjecture)

GEO017-2.p Corollary 1 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from A to B equals the distance from D to C.
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(u, v, x, w) cnf(prove_symmetry, negated_conjecture)

GEO017-3.p Corollary 1 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from A to B equals the distance from D to C.
include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(u, v, x, w) cnf(prove_symmetry, negated_conjecture)

GEO018-2.p Corollary 2 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from B to A equals the distance from D to C.
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(v, u, x, w) cnf(prove_symmetry, negated_conjecture)

GEO018-3.p Corollary 2 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from B to A equals the distance from D to C.
include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(v, u, x, w) cnf(prove_symmetry, negated_conjecture)

GEO019-2.p Corollary 3 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from C to D equals the distance from B to A.
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(w, x, v, u) cnf(prove_symmetry, negated_conjecture)

GEO019-3.p Corollary 3 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from C to D equals the distance from B to A.
include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(w, x, v, u) cnf(prove_symmetry, negated_conjecture)

GEO020-2.p Corollary 4 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from D to C equals the distance from A to B.
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(x, w, u, v) cnf(prove_symmetry, negated_conjecture)

GEO020-3.p Corollary 4 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from D to C equals the distance from A to B.

include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(x, w, u, v) cnf(prove_symmetry, negated_conjecture)

GEO021-2.p Corollary 5 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from D to C equals the distance from B to A.
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(x, w, v, u) cnf(prove_symmetry, negated_conjecture)

GEO021-3.p Corollary 5 to symmetries of equidistance
Show that if the distance from A to B equals the distance from C to D, then the distance from D to C equals the distance from B to A.
include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
$\neg$ equidistant(x, w, v, u) cnf(prove_symmetry, negated_conjecture)

GEO022-2.p Ordinary transitivity of equidistance
This form of transitivity is different from that expressed in the axioms.
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
equidistant(w, x, y, z) cnf(w_to_x_equals_y_to_z, hypothesis)
$\neg$ equidistant(u, v, y, z) cnf(prove_transitivity, negated_conjecture)

GEO022-3.p Ordinary transitivity of equidistance
This form of transitivity is different from that expressed in the axioms.
include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
equidistant(w, x, y, z) cnf(w_to_x_equals_y_to_z, hypothesis)
$\neg$ equidistant(u, v, y, z) cnf(prove_transitivity, negated_conjecture)

GEO024-2.p All null segments are congruent
include('Axioms/GEO002-0.ax')
$\neg$ equidistant(u, u, v, v) cnf(prove_congruence, negated_conjecture)

GEO024-3.p All null segments are congruent
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)

$y = \text{extension}(u, v, w, x) \Rightarrow \text{between}(u, v, y)$ cnf(b_0, axiom)
between(u, v, reflection(u, v)) cnf(r2_1, axiom)
equidistant(v, reflection(u, v), u, v) cnf(r2_2, axiom)
$u = v \Rightarrow v = \text{reflection}(u, v)$ cnf(r3_1, axiom)
$u = \text{reflection}(u, u)$ cnf(r3_2, axiom)
$v = \text{reflection}(u, v) \Rightarrow u = v$ cnf(r_4, axiom)
¬ equidistant(u, u, v, v) cnf(prove_congruence, negated_conjecture)

GEO025-2.p Addition of equal segments
include('Axioms/GEO002-0.ax')
equidistant(u, v, u_1, v_1) cnf(u_to_v_equals_u1_to_v$_1$, hypothesis)
equidistant(v, w, v_1, w_1) cnf(v_to_w_equals_v1_to_w$_1$, hypothesis)
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u_1, v_1, w_1) cnf(v1_between_u1_and_w$_1$, hypothesis)
¬ equidistant(u, w, u_1, w_1) cnf(prove_equal_sums, negated_conjecture)

GEO025-3.p Addition of equal segments
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) ⇒ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) ⇒ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) ⇒ equidistant(u, v, x, w) cnf(d4_1, axiom)
equidistant(u, v, w, x) ⇒ equidistant(v, u, x, w) cnf(d4_2, axiom)
equidistant(u, v, w, x) ⇒ equidistant(w, x, v, u) cnf(d4_3, axiom)
equidistant(u, v, w, x) ⇒ equidistant(x, w, u, v) cnf(d4_4, axiom)
equidistant(u, v, w, x) ⇒ equidistant(x, w, v, u) cnf(d4_5, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) ⇒ equidistant(u, v, y, z) cnf(d_5, axiom)
$v = \text{extension}(u, v, w, w)$ cnf(e_1, axiom)
$y = \text{extension}(u, v, w, x) \Rightarrow \text{between}(u, v, y)$ cnf(b_0, axiom)
between(u, v, reflection(u, v)) cnf(r2_1, axiom)
equidistant(v, reflection(u, v), u, v) cnf(r2_2, axiom)
$u = v \Rightarrow v = \text{reflection}(u, v)$ cnf(r3_1, axiom)
$u = \text{reflection}(u, u)$ cnf(r3_2, axiom)
$v = \text{reflection}(u, v) \Rightarrow u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
equidistant(u, v, u_1, v_1) cnf(u_to_v_equals_u1_to_v$_1$, hypothesis)
equidistant(v, w, v_1, w_1) cnf(v_to_w_equals_v1_to_w$_1$, hypothesis)
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u_1, v_1, w_1) cnf(v1_between_u1_and_w$_1$, hypothesis)
¬ equidistant(u, w, u_1, w_1) cnf(prove_equal_sums, negated_conjecture)

GEO026-2.p Extension is unique
include('Axioms/GEO002-0.ax')
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u, v, x) cnf(v_between_u_and_x, hypothesis)
equidistant(v, w, v, x) cnf(v_to_w_equals_v_to_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$w \neq x$ cnf(prove_w_is_x, negated_conjecture)

GEO026-3.p Extension is unique
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) ⇒ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) ⇒ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) ⇒ equidistant(u, v, x, w) cnf(d4_1, axiom)
equidistant(u, v, w, x) ⇒ equidistant(v, u, x, w) cnf(d4_2, axiom)
equidistant(u, v, w, x) ⇒ equidistant(w, x, v, u) cnf(d4_3, axiom)
equidistant(u, v, w, x) ⇒ equidistant(x, w, u, v) cnf(d4_4, axiom)
equidistant(u, v, w, x) ⇒ equidistant(x, w, v, u) cnf(d4_5, axiom)

(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between(u, v, reflection(u, v)) cnf($\mathrm{r2}_1$, axiom)
equidistant(v, reflection(u, v), u, v) cnf($\mathrm{r2}_2$, axiom)
$u = v \Rightarrow v =$ reflection(u, v) cnf($\mathrm{r3}_1$, axiom)
$u =$ reflection(u, u) cnf($\mathrm{r3}_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant($u, w, u_1,$
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u, v, x) cnf(v_between_u_and_x, hypothesis)
equidistant(v, w, v, x) cnf(v_to_w_equals_v_to_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$w \neq x$ cnf(prove_w_is_x, negated_conjecture)

GEO027-2.p Corollary 1 to unique extension
include('Axioms/GEO002-0.ax')
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$w \neq$ extension(u, v, v, w) cnf(prove_w_is_an_extension, negated_conjecture)

GEO027-3.p Corollary 1 to unique extension
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf($\mathrm{d4}_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf($\mathrm{d4}_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf($\mathrm{d4}_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf($\mathrm{d4}_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf($\mathrm{d4}_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between(u, v, reflection(u, v)) cnf($\mathrm{r2}_1$, axiom)
equidistant(v, reflection(u, v), u, v) cnf($\mathrm{r2}_2$, axiom)
$u = v \Rightarrow v =$ reflection(u, v) cnf($\mathrm{r3}_1$, axiom)
$u =$ reflection(u, u) cnf($\mathrm{r3}_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant($u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$w \neq$ extension(u, v, v, w) cnf(prove_w_is_an_extension, negated_conjecture)

GEO028-2.p Corollary 2 to unique extension
include('Axioms/GEO002-0.ax')
equidistant(w, x, y, z) cnf(w_to_x_equals_y_to_z, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
extension(u, v, w, x) $\neq$ extension(u, v, y, z) cnf(prove_equal_extensions, negated_conjecture)

GEO028-3.p Corollary 2 to unique extension
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf($\mathrm{d4}_1$, axiom)

equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v,$ reflection$(u, v))$ cnf(r2$_1$, axiom)
equidistant$(v,$ reflection$(u, v), u, v)$ cnf(r2$_2$, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf(r3$_1$, axiom)
$u =$ reflection(u, u) cnf(r3$_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant$(u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ $(u = v$ or $w = x)$ cnf(d_9, axiom)
equidistant(w, x, y, z) cnf(w_to_x_equals_y_to_z, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
extension$(u, v, w, x) \neq$ extension(u, v, y, z) cnf(prove_equal_extensions, negated_conjecture)

GEO029-2.p Corollary 3 to unique extension
include('Axioms/GEO002-0.ax')
$u \neq v$ cnf(u_not_v, hypothesis)
extension$(u, v, u, v) \neq$ extension(u, v, v, u) cnf(prove_equal_extensions, negated_conjecture)

GEO029-3.p Corollary 3 to unique extension
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v,$ reflection$(u, v))$ cnf(r2$_1$, axiom)
equidistant$(v,$ reflection$(u, v), u, v)$ cnf(r2$_2$, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf(r3$_1$, axiom)
$u =$ reflection(u, u) cnf(r3$_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant$(u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ $(u = v$ or $w = x)$ cnf(d_9, axiom)
$u \neq v$ cnf(u_not_v, hypothesis)
extension$(u, v, u, v) \neq$ extension(u, v, v, u) cnf(prove_equal_extensions, negated_conjecture)

GEO030-2.p Corollary to the outer five-segment axiom
include('Axioms/GEO002-0.ax')
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
equidistant(u, w, u, w_1) cnf(u_to_w_equals_u_to_w$_1$, hypothesis)
equidistant(v, w, v, w_1) cnf(v_to_w_equals_v_to_w$_1$, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$w \neq w_1$ cnf(prove_w_is_w$_1$, negated_conjecture)

GEO031-2.p Second inner five-segment theorem
include('Axioms/GEO002-0.ax')
equidistant(u, v, u_1, v_1) cnf(u_to_v_equals_u1_to_v$_1$, hypothesis)
equidistant(u, w, u_1, w_1) cnf(u_to_w_equals_u1_to_w$_1$, hypothesis)

$\text{equidistant}(u, x, u_1, x_1)$ cnf($\text{u\_to\_x\_equals\_u1\_to\_x}_1$, hypothesis)
$\text{equidistant}(w, x, w_1, x_1)$ cnf($\text{w\_to\_x\_equals\_w1\_to\_x}_1$, hypothesis)
$\text{between}(u, v, w)$ cnf(v_between_u_and_w, hypothesis)
$\text{between}(u_1, v_1, w_1)$ cnf($\text{v1\_between\_u1\_and\_w}_1$, hypothesis)
$\neg\, \text{equidistant}(v, x, v_1, x_1)$ cnf($\text{prove\_v\_to\_x\_equals\_v1\_to\_x}_1$, negated_conjecture)

GEO032-2.p Equal difference between pairs of equal length line segments
include('Axioms/GEO002-0.ax')
$\text{between}(u, v, w)$ cnf(v_between_u_and_w, hypothesis)
$\text{between}(u_1, v_1, w_1)$ cnf($\text{v1\_between\_u1\_and\_w}_1$, hypothesis)
$\text{equidistant}(u, v, u_1, v_1)$ cnf($\text{u\_to\_v\_equals\_u1\_to\_v}_1$, hypothesis)
$\text{equidistant}(u, w, u_1, w_1)$ cnf($\text{u\_to\_w\_equals\_u1\_to\_w}_1$, hypothesis)
$\neg\, \text{equidistant}(v, w, v_1, w_1)$ cnf($\text{v\_to\_w\_equals\_v1\_to\_w}_1$, negated_conjecture)

GEO033-2.p First inner five-segment theorem
include('Axioms/GEO002-0.ax')
$\text{equidistant}(u, v, u_1, v_1)$ cnf($\text{u\_to\_v\_equals\_u1\_to\_v}_1$, hypothesis)
$\text{equidistant}(v, w, v_1, w_1)$ cnf($\text{v\_to\_w\_equals\_v1\_to\_w}_1$, hypothesis)
$\text{equidistant}(u, x, u_1, x_1)$ cnf($\text{u\_to\_x\_equals\_u1\_to\_x}_1$, hypothesis)
$\text{equidistant}(w, x, w_1, x_1)$ cnf($\text{w\_to\_x\_equals\_w1\_to\_x}_1$, hypothesis)
$\text{between}(u, v, w)$ cnf(v_between_u_and_w, hypothesis)
$\text{between}(u_1, v_1, w_1)$ cnf($\text{v1\_between\_u1\_and\_w}_1$, hypothesis)
$\neg\, \text{equidistant}(v, x, v_1, x_1)$ cnf($\text{prove\_v\_to\_x\_equals\_v1\_to\_x}_1$, negated_conjecture)

GEO034-2.p Corollary to the first inner five-segment theorem
include('Axioms/GEO002-0.ax')
$\text{between}(u, v, w)$ cnf(v_between_u_and_w, hypothesis)
$\text{equidistant}(u, v, u, x)$ cnf(u_to_v_equals_u_to_x, hypothesis)
$\text{equidistant}(w, v, w, x)$ cnf(w_to_v_equals_w_to_x, hypothesis)
$v \neq x$ cnf(prove_v_is_x, negated_conjecture)

GEO035-2.p A null extension does not extend a line
include('Axioms/GEO002-0.ax')
$v \neq \text{extension}(u, v, w, w)$ cnf(prove_null_extension, negated_conjecture)

GEO035-3.p A null extension does not extend a line
include('Axioms/GEO002-0.ax')
$\text{equidistant}(u, v, u, v)$ cnf(d_1, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, u, v)$ cnf(d_2, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, w, x)$ cnf(d_3, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(u, v, x, w)$ cnf(d4_1, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, x, w)$ cnf(d4_2, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, v, u)$ cnf(d4_3, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(x, w, u, v)$ cnf(d4_4, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(x, w, v, u)$ cnf(d4_5, axiom)
$(\text{equidistant}(u, v, w, x) \text{ and } \text{equidistant}(w, x, y, z)) \Rightarrow \text{equidistant}(u, v, y, z)$ cnf(d_5, axiom)
$v \neq \text{extension}(u, v, w, w)$ cnf(prove_null_extension, negated_conjecture)

GEO036-2.p The 3 axiom set points are distinct
include('Axioms/GEO002-0.ax')
$\text{lower\_dimension\_point}_1 = \text{lower\_dimension\_point}_2$ or $\text{lower\_dimension\_point}_2 = \text{lower\_dimension\_point}_3$ or lower_dimension_p
$\text{lower\_dimension\_point}_3$ cnf(prove_axioms_points_are_distinct, negated_conjecture)

GEO036-3.p The 3 axiom set points are distinct
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$\text{equidistant}(u, v, u, v)$ cnf(d_1, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, u, v)$ cnf(d_2, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, w, x)$ cnf(d_3, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(u, v, x, w)$ cnf(d4_1, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, x, w)$ cnf(d4_2, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, v, u)$ cnf(d4_3, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(x, w, u, v)$ cnf(d4_4, axiom)

equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf(d4_5, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension$(u, v, w, x) \Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v,$ reflection$(u, v))$ cnf(r2_1, axiom)
equidistant$(v,$ reflection$(u, v), u, v)$ cnf(r2_2, axiom)
$u = v \Rightarrow v =$ reflection(u, v) cnf(r3_1, axiom)
$u =$ reflection(u, u) cnf(r3_2, axiom)
$v =$ reflection$(u, v) \Rightarrow u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant$(u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between$(u, v, w) \Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf(d10_1, axiom)
equidistant$(w, x, y, z) \Rightarrow$ (extension$(u, v, w, x) =$ extension(u, v, y, z) or $u = v$) cnf(d10_2, axiom)
extension$(u, v, u, v) =$ extension(u, v, v, u) or $u = v$ cnf(d10_3, axiom)
equidistant$(v, u, v,$ reflection(reflection$(u, v), v))$ cnf(r_5, axiom)
$u =$ reflection(reflection$(u, v), v)$ cnf(r_6, axiom)
between(u, v, v) cnf(t_3, axiom)
(between(u, w, x) and $u = x$) $\Rightarrow$ between(v, w, x) cnf(b_1, axiom)
between$(u, v, w) \Rightarrow$ between(w, v, u) cnf(t_1, axiom)
between(u, u, v) cnf(t_2, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow u = v$ cnf(b_2, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow v = w$ cnf(b_3, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6_1, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6_2, axiom)
(between(u, v, w) and between(v, w, x)) $\Rightarrow$ between(u, v, w) cnf(b_4, axiom)
(between(u, v, w) and between(u, w, x)) $\Rightarrow$ between(v, w, x) cnf(b_5, axiom)
(between(u, v, w) and between(v, w, x)) $\Rightarrow$ (between(u, w, x) or $v = w$) cnf(b_6, axiom)
(between(u, v, w) and between(v, w, x)) $\Rightarrow$ (between(u, v, x) or $v = w$) cnf(b_7, axiom)
(between(u, v, x) and between(v, w, x)) $\Rightarrow$ between(u, w, x) cnf(b_8, axiom)
(between(u, v, w) and between(u, w, x)) $\Rightarrow$ between(u, v, x) cnf(b_9, axiom)
lower_dimension_point$_1$ = lower_dimension_point$_2$ or lower_dimension_point$_2$ = lower_dimension_point$_3$ or lower_dimension_p
lower_dimension_point$_3$ cnf(prove_axioms_points_are_distinct, negated_conjecture)

GEO037-2.p A segment can be extended
include('Axioms/GEO002-0.ax')
(equidistant$(v,$ extension$(u, v,$ lower_dimension_point$_1$, lower_dimension_point$_2$), x, extension$(w, x,$ lower_dimension_point$_1$, low
$v =$ extension$(u, v,$ lower_dimension_point$_1$, lower_dimension_point$_2$) cnf(prove_lengthen, negated_conjecture)

GEO038-2.p Corollary 1 to the segment contruction axiom
include('Axioms/GEO002-0.ax')
$y =$ extension(u, v, w, x) cnf(y_is_extension, hypothesis)
$\neg$ between(u, v, y) cnf(prove_corollary, negated_conjecture)

GEO038-3.p Corollary 1 to the segment contruction axiom
include('Axioms/GEO002-0.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf(d4_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf(d4_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf(d4_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf(d4_4, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf(d4_5, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) cnf(y_is_extension, hypothesis)
$\neg$ between(u, v, y) cnf(prove_corollary, negated_conjecture)

GEO039-2.p Corollary the identity axiom for betweenness
include('Axioms/GEO002-0.ax')

between(u, w, x) cnf(w_between_u_and_x, hypothesis)
$u = x$ cnf(u_is_x, hypothesis)
$\neg$ between(v, w, x) cnf(prove_corollary, negated_conjecture)

GEO039-3.p Corollary the identity axiom for betweenness
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf(d4_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4_4, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4_5, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between(u, v, reflection(u, v)) cnf(r2_1, axiom)
equidistant(v, reflection(u, v), u, v) cnf(r2_2, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf(r3_1, axiom)
$u =$ reflection(u, u) cnf(r3_2, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant($u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u, v, w) $\Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf(d10_1, axiom)
equidistant(w, x, y, z) $\Rightarrow$ (extension(u, v, w, x) = extension(u, v, y, z) or $u = v$) cnf(d10_2, axiom)
extension(u, v, u, v) = extension(u, v, v, u) or $u = v$ cnf(d10_3, axiom)
equidistant(v, u, v, reflection(reflection(u, v), v)) cnf(r_5, axiom)
$u =$ reflection(reflection(u, v), v) cnf(r_6, axiom)
between(u, v, v) cnf(t_3, axiom)
between(u, w, x) cnf(w_between_u_and_x, hypothesis)
$u = x$ cnf(u_is_x, hypothesis)
$\neg$ between(v, w, x) cnf(prove_corollary, negated_conjecture)

GEO040-2.p Antisymmetry of betweenness in its first two arguments
include('Axioms/GEO002-0.ax')
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(v, u, w) cnf(u_between_v_and_w, hypothesis)
$u \neq v$ cnf(prove_u_is_v, negated_conjecture)

GEO040-3.p Antisymmetry of betweenness in its first two arguments
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf(d4_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4_4, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4_5, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between(u, v, reflection(u, v)) cnf(r2_1, axiom)
equidistant(v, reflection(u, v), u, v) cnf(r2_2, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf(r3_1, axiom)
$u =$ reflection(u, u) cnf(r3_2, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)

equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant($u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u, v, w) $\Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf(d10$_1$, axiom)
equidistant(w, x, y, z) $\Rightarrow$ (extension(u, v, w, x) = extension(u, v, y, z) or $u = v$) cnf(d10$_2$, axiom)
extension(u, v, u, v) = extension(u, v, v, u) or $u = v$ cnf(d10$_3$, axiom)
equidistant(v, u, v, reflection(reflection(u, v), v)) cnf(r_5, axiom)
$u =$ reflection(reflection(u, v), v) cnf(r_6, axiom)
between(u, v, v) cnf(t_3, axiom)
(between(u, w, x) and $u = x$) $\Rightarrow$ between(v, w, x) cnf(b_1, axiom)
between(u, v, w) $\Rightarrow$ between(w, v, u) cnf(t_1, axiom)
between(u, u, v) cnf(t_2, axiom)
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(v, u, w) cnf(u_between_v_and_w, hypothesis)
$u \neq v$ cnf(prove_u_is_v, negated_conjecture)

GEO041-2.p Corollary to antisymmetry of betweenness in first 2 arguments
include('Axioms/GEO002-0.ax')
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u, w, v) cnf(w_between_u_and_v, hypothesis)
$v \neq w$ cnf(prove_v_is_w, negated_conjecture)

GEO041-3.p Corollary to antisymmetry of betweenness in first 2 arguments
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between(u, v, reflection(u, v)) cnf(r2$_1$, axiom)
equidistant(v, reflection(u, v), u, v) cnf(r2$_2$, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf(r3$_1$, axiom)
$u =$ reflection(u, u) cnf(r3$_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant($u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u, v, w) $\Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf(d10$_1$, axiom)
equidistant(w, x, y, z) $\Rightarrow$ (extension(u, v, w, x) = extension(u, v, y, z) or $u = v$) cnf(d10$_2$, axiom)
extension(u, v, u, v) = extension(u, v, v, u) or $u = v$ cnf(d10$_3$, axiom)
equidistant(v, u, v, reflection(reflection(u, v), v)) cnf(r_5, axiom)
$u =$ reflection(reflection(u, v), v) cnf(r_6, axiom)
between(u, v, v) cnf(t_3, axiom)
(between(u, w, x) and $u = x$) $\Rightarrow$ between(v, w, x) cnf(b_1, axiom)
between(u, v, w) $\Rightarrow$ between(w, v, u) cnf(t_1, axiom)
between(u, u, v) cnf(t_2, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow$ $u = v$ cnf(b_2, axiom)
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u, w, v) cnf(w_between_u_and_v, hypothesis)
$v \neq w$ cnf(prove_v_is_w, negated_conjecture)

GEO042-2.p First inner transitivity property of betweenness
include('Axioms/GEO002-0.ax')
between(u, v, x) cnf(v_between_u_and_x, hypothesis)

between(v, w, x) cnf(w_between_v_and_x, hypothesis)
$\neg$ between(u, v, w) cnf(prove_v_between_u_and_w, negated_conjecture)

GEO042-3.p First inner transitivity property of betweenness
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf($\mathrm{d4}_1$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf($\mathrm{d4}_2$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf($\mathrm{d4}_3$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf($\mathrm{d4}_4$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf($\mathrm{d4}_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension$(u, v, w, x) \Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v,$ reflection$(u, v))$ cnf($\mathrm{r2}_1$, axiom)
equidistant$(v,$ reflection$(u, v), u, v)$ cnf($\mathrm{r2}_2$, axiom)
$u = v \Rightarrow v =$ reflection(u, v) cnf($\mathrm{r3}_1$, axiom)
$u =$ reflection(u, u) cnf($\mathrm{r3}_2$, axiom)
$v =$ reflection$(u, v) \Rightarrow u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant$(u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between$(u, v, w) \Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf($\mathrm{d10}_1$, axiom)
equidistant$(w, x, y, z) \Rightarrow$ (extension$(u, v, w, x) =$ extension(u, v, y, z) or $u = v$) cnf($\mathrm{d10}_2$, axiom)
extension$(u, v, u, v) =$ extension(u, v, v, u) or $u = v$ cnf($\mathrm{d10}_3$, axiom)
equidistant$(v, u, v,$ reflection(reflection$(u, v), v))$ cnf(r_5, axiom)
$u =$ reflection(reflection$(u, v), v)$ cnf(r_6, axiom)
between(u, v, v) cnf(t_3, axiom)
(between(u, w, x) and $u = x$) $\Rightarrow$ between(v, w, x) cnf(b_1, axiom)
between$(u, v, w) \Rightarrow$ between(w, v, u) cnf(t_1, axiom)
between(u, u, v) cnf(t_2, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow u = v$ cnf(b_2, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow v = w$ cnf(b_3, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow$ ($u = v$ or $v = w$) cnf($\mathrm{t6}_1$, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow$ ($u = v$ or $v = w$) cnf($\mathrm{t6}_2$, axiom)
between(u, v, x) cnf(v_between_u_and_x, hypothesis)
between(v, w, x) cnf(w_between_v_and_x, hypothesis)
$\neg$ between(u, v, w) cnf(prove_v_between_u_and_w, negated_conjecture)

GEO043-2.p Corollary to first inner transitivity property of betweenness
Corollary of first inner transitivity property of betweenness, using symmetry.
include('Axioms/GEO002-0.ax')
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u, w, x) cnf(w_between_u_and_x, hypothesis)
$\neg$ between(v, w, x) cnf(prove_w_between_v_and_x, negated_conjecture)

GEO043-3.p Corollary to first inner transitivity property of betweenness
Corollary of first inner transitivity property of betweenness, using symmetry.
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf($\mathrm{d4}_1$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf($\mathrm{d4}_2$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf($\mathrm{d4}_3$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf($\mathrm{d4}_4$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf($\mathrm{d4}_5$, axiom)

(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between($u, v,$ reflection(u, v)) cnf($\mathrm{r2}_1$, axiom)
equidistant(v, reflection(u, v), u, v) cnf($\mathrm{r2}_2$, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf($\mathrm{r3}_1$, axiom)
$u =$ reflection(u, u) cnf($\mathrm{r3}_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant($u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u, v, w) $\Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf($\mathrm{d10}_1$, axiom)
equidistant(w, x, y, z) $\Rightarrow$ (extension(u, v, w, x) $=$ extension(u, v, y, z) or $u = v$) cnf($\mathrm{d10}_2$, axiom)
extension(u, v, u, v) $=$ extension(u, v, v, u) or $u = v$ cnf($\mathrm{d10}_3$, axiom)
equidistant(v, u, v, reflection(reflection(u, v), v)) cnf(r_5, axiom)
$u =$ reflection(reflection(u, v), v) cnf(r_6, axiom)
between(u, v, v) cnf(t_3, axiom)
(between(u, w, x) and $u = x$) $\Rightarrow$ between(v, w, x) cnf(b_1, axiom)
between(u, v, w) $\Rightarrow$ between(w, v, u) cnf(t_1, axiom)
between(u, u, v) cnf(t_2, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow$ $u = v$ cnf(b_2, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow$ $v = w$ cnf(b_3, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow$ ($u = v$ or $v = w$) cnf($\mathrm{t6}_1$, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow$ ($u = v$ or $v = w$) cnf($\mathrm{t6}_2$, axiom)
(between(u, v, w) and between(v, w, x)) $\Rightarrow$ between(u, v, w) cnf(b_4, axiom)
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u, w, x) cnf(w_between_u_and_x, hypothesis)
$\neg$ between(v, w, x) cnf(prove_w_between_v_and_x, negated_conjecture)

GEO044-2.p First outer transitivity property for betweenness
include('Axioms/GEO002-0.ax')
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(v, w, x) cnf(w_between_v_and_x, hypothesis)
$v \neq w$ cnf(v_not_w, hypothesis)
$\neg$ between(u, w, x) cnf(prove_w_between_u_and_x, negated_conjecture)

GEO044-3.p First outer transitivity property for betweenness
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf($\mathrm{d4}_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf($\mathrm{d4}_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf($\mathrm{d4}_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf($\mathrm{d4}_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf($\mathrm{d4}_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between($u, v,$ reflection(u, v)) cnf($\mathrm{r2}_1$, axiom)
equidistant(v, reflection(u, v), u, v) cnf($\mathrm{r2}_2$, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf($\mathrm{r3}_1$, axiom)
$u =$ reflection(u, u) cnf($\mathrm{r3}_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant($u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u, v, w) $\Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf($\mathrm{d10}_1$, axiom)
equidistant(w, x, y, z) $\Rightarrow$ (extension(u, v, w, x) $=$ extension(u, v, y, z) or $u = v$) cnf($\mathrm{d10}_2$, axiom)

$\text{extension}(u,v,u,v) = \text{extension}(u,v,v,u)$ or $u = v$ cnf(d10_3, axiom)
$\text{equidistant}(v,u,v,\text{reflection}(\text{reflection}(u,v),v))$ cnf(r_5, axiom)
$u = \text{reflection}(\text{reflection}(u,v),v)$ cnf(r_6, axiom)
$\text{between}(u,v,v)$ cnf(t_3, axiom)
($\text{between}(u,w,x)$ and $u = x$) $\Rightarrow$ $\text{between}(v,w,x)$ cnf(b_1, axiom)
$\text{between}(u,v,w) \Rightarrow \text{between}(w,v,u)$ cnf(t_1, axiom)
$\text{between}(u,u,v)$ cnf(t_2, axiom)
($\text{between}(u,v,w)$ and $\text{between}(v,u,w)$) $\Rightarrow$ $u = v$ cnf(b_2, axiom)
($\text{between}(u,v,w)$ and $\text{between}(u,w,v)$) $\Rightarrow$ $v = w$ cnf(b_3, axiom)
($\text{between}(u,v,w)$ and $\text{between}(v,u,w)$) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6_1, axiom)
($\text{between}(u,v,w)$ and $\text{between}(u,w,v)$) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6_2, axiom)
($\text{between}(u,v,w)$ and $\text{between}(v,w,x)$) $\Rightarrow$ $\text{between}(u,v,w)$ cnf(b_4, axiom)
($\text{between}(u,v,w)$ and $\text{between}(u,w,x)$) $\Rightarrow$ $\text{between}(v,w,x)$ cnf(b_5, axiom)
$\text{between}(u,v,w)$ cnf(v_between_u_and_w, hypothesis)
$\text{between}(v,w,x)$ cnf(w_between_v_and_x, hypothesis)
$v \neq w$ cnf(v_not_w, hypothesis)
$\neg\,\text{between}(u,w,x)$ cnf(prove_w_between_u_and_x, negated_conjecture)

GEO045-2.p Second outer transitivity property of betweenness
include('Axioms/GEO002-0.ax')
$\text{between}(u,v,w)$ cnf(v_between_u_and_w, hypothesis)
$\text{between}(v,w,x)$ cnf(w_between_v_and_x, hypothesis)
$v \neq w$ cnf(v_not_w, hypothesis)
$\neg\,\text{between}(u,v,x)$ cnf(prove_v_between_u_and_x, negated_conjecture)

GEO045-3.p Second outer transitivity property of betweenness
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$\text{equidistant}(u,v,u,v)$ cnf(d_1, axiom)
$\text{equidistant}(u,v,w,x) \Rightarrow \text{equidistant}(w,x,u,v)$ cnf(d_2, axiom)
$\text{equidistant}(u,v,w,x) \Rightarrow \text{equidistant}(v,u,w,x)$ cnf(d_3, axiom)
$\text{equidistant}(u,v,w,x) \Rightarrow \text{equidistant}(u,v,x,w)$ cnf(d4_1, axiom)
$\text{equidistant}(u,v,w,x) \Rightarrow \text{equidistant}(v,u,x,w)$ cnf(d4_2, axiom)
$\text{equidistant}(u,v,w,x) \Rightarrow \text{equidistant}(w,x,v,u)$ cnf(d4_3, axiom)
$\text{equidistant}(u,v,w,x) \Rightarrow \text{equidistant}(x,w,u,v)$ cnf(d4_4, axiom)
$\text{equidistant}(u,v,w,x) \Rightarrow \text{equidistant}(x,w,v,u)$ cnf(d4_5, axiom)
($\text{equidistant}(u,v,w,x)$ and $\text{equidistant}(w,x,y,z)$) $\Rightarrow$ $\text{equidistant}(u,v,y,z)$ cnf(d_5, axiom)
$v = \text{extension}(u,v,w,w)$ cnf(e_1, axiom)
$y = \text{extension}(u,v,w,x) \Rightarrow \text{between}(u,v,y)$ cnf(b_0, axiom)
$\text{between}(u,v,\text{reflection}(u,v))$ cnf(r2_1, axiom)
$\text{equidistant}(v,\text{reflection}(u,v),u,v)$ cnf(r2_2, axiom)
$u = v \Rightarrow v = \text{reflection}(u,v)$ cnf(r3_1, axiom)
$u = \text{reflection}(u,u)$ cnf(r3_2, axiom)
$v = \text{reflection}(u,v) \Rightarrow u = v$ cnf(r_4, axiom)
$\text{equidistant}(u,u,v,v)$ cnf(d_7, axiom)
($\text{equidistant}(u,v,u_1,v_1)$ and $\text{equidistant}(v,w,v_1,w_1)$ and $\text{between}(u,v,w)$ and $\text{between}(u_1,v_1,w_1)$) $\Rightarrow$ $\text{equidistant}(u,w,u_1,$
($\text{between}(u,v,w)$ and $\text{between}(u,v,x)$ and $\text{equidistant}(v,w,v,x)$) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
$\text{between}(u,v,w) \Rightarrow$ ($u = v$ or $w = \text{extension}(u,v,v,w)$) cnf($\text{d10}_1$, axiom)
$\text{equidistant}(w,x,y,z) \Rightarrow$ ($\text{extension}(u,v,w,x) = \text{extension}(u,v,y,z)$ or $u = v$) cnf(d10_2, axiom)
$\text{extension}(u,v,u,v) = \text{extension}(u,v,v,u)$ or $u = v$ cnf(d10_3, axiom)
$\text{equidistant}(v,u,v,\text{reflection}(\text{reflection}(u,v),v))$ cnf(r_5, axiom)
$u = \text{reflection}(\text{reflection}(u,v),v)$ cnf(r_6, axiom)
$\text{between}(u,v,v)$ cnf(t_3, axiom)
($\text{between}(u,w,x)$ and $u = x$) $\Rightarrow$ $\text{between}(v,w,x)$ cnf(b_1, axiom)
$\text{between}(u,v,w) \Rightarrow \text{between}(w,v,u)$ cnf(t_1, axiom)
$\text{between}(u,u,v)$ cnf(t_2, axiom)
($\text{between}(u,v,w)$ and $\text{between}(v,u,w)$) $\Rightarrow$ $u = v$ cnf(b_2, axiom)
($\text{between}(u,v,w)$ and $\text{between}(u,w,v)$) $\Rightarrow$ $v = w$ cnf(b_3, axiom)
($\text{between}(u,v,w)$ and $\text{between}(v,u,w)$) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6_1, axiom)
($\text{between}(u,v,w)$ and $\text{between}(u,w,v)$) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6_2, axiom)

(between(u,v,w) and between(v,w,x)) $\Rightarrow$ between(u,v,w) cnf(b_4, axiom)
(between(u,v,w) and between(u,w,x)) $\Rightarrow$ between(v,w,x) cnf(b_5, axiom)
(between(u,v,w) and between(v,w,x)) $\Rightarrow$ (between(u,w,x) or $v = w$) cnf(b_6, axiom)
between(u,v,w) cnf(v_between_u_and_w, hypothesis)
between(v,w,x) cnf(w_between_v_and_x, hypothesis)
$v \neq w$ cnf(v_not_w, hypothesis)
$\neg$ between(u,v,x) cnf(prove_v_between_u_and_x, negated_conjecture)

GEO046-2.p Second inner transitivity property of betweenness
include('Axioms/GEO002-0.ax')
between(u,v,x) cnf(v_between_u_and_x, hypothesis)
between(v,w,x) cnf(w_between_v_and_x, hypothesis)
$\neg$ between(u,w,x) cnf(prove_w_between_u_and_x, negated_conjecture)

GEO046-3.p Second inner transitivity property of betweenness
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u,v,u,v) cnf(d_1, axiom)
equidistant(u,v,w,x) $\Rightarrow$ equidistant(w,x,u,v) cnf(d_2, axiom)
equidistant(u,v,w,x) $\Rightarrow$ equidistant(v,u,w,x) cnf(d_3, axiom)
equidistant(u,v,w,x) $\Rightarrow$ equidistant(u,v,x,w) cnf(d4$_1$, axiom)
equidistant(u,v,w,x) $\Rightarrow$ equidistant(v,u,x,w) cnf(d4$_2$, axiom)
equidistant(u,v,w,x) $\Rightarrow$ equidistant(w,x,v,u) cnf(d4$_3$, axiom)
equidistant(u,v,w,x) $\Rightarrow$ equidistant(x,w,u,v) cnf(d4$_4$, axiom)
equidistant(u,v,w,x) $\Rightarrow$ equidistant(x,w,v,u) cnf(d4$_5$, axiom)
(equidistant(u,v,w,x) and equidistant(w,x,y,z)) $\Rightarrow$ equidistant(u,v,y,z) cnf(d_5, axiom)
$v =$ extension(u,v,w,w) cnf(e_1, axiom)
$y =$ extension(u,v,w,x) $\Rightarrow$ between(u,v,y) cnf(b_0, axiom)
between(u,v, reflection(u,v)) cnf(r2$_1$, axiom)
equidistant(v, reflection(u,v), u,v) cnf(r2$_2$, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u,v) cnf(r3$_1$, axiom)
$u =$ reflection(u,u) cnf(r3$_2$, axiom)
$v =$ reflection(u,v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u,u,v,v) cnf(d_7, axiom)
(equidistant(u,v,u_1,v_1) and equidistant(v,w,v_1,w_1) and between(u,v,w) and between(u_1,v_1,w_1)) $\Rightarrow$ equidistant($u,w,u_1,$
(between(u,v,w) and between(u,v,x) and equidistant(v,w,v,x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u,v,w) $\Rightarrow$ ($u = v$ or $w =$ extension(u,v,v,w)) cnf(d10$_1$, axiom)
equidistant(w,x,y,z) $\Rightarrow$ (extension(u,v,w,x) = extension(u,v,y,z) or $u = v$) cnf(d10$_2$, axiom)
extension(u,v,u,v) = extension(u,v,v,u) or $u = v$ cnf(d10$_3$, axiom)
equidistant(v,u,v, reflection(reflection(u,v), v)) cnf(r_5, axiom)
$u =$ reflection(reflection(u,v), v) cnf(r_6, axiom)
between(u,v,v) cnf(t_3, axiom)
(between(u,w,x) and $u = x$) $\Rightarrow$ between(v,w,x) cnf(b_1, axiom)
between(u,v,w) $\Rightarrow$ between(w,v,u) cnf(t_1, axiom)
between(u,u,v) cnf(t_2, axiom)
(between(u,v,w) and between(v,u,w)) $\Rightarrow$ $u = v$ cnf(b_2, axiom)
(between(u,v,w) and between(u,w,v)) $\Rightarrow$ $v = w$ cnf(b_3, axiom)
(between(u,v,w) and between(v,u,w)) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6$_1$, axiom)
(between(u,v,w) and between(u,w,v)) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6$_2$, axiom)
(between(u,v,w) and between(v,w,x)) $\Rightarrow$ between(u,v,w) cnf(b_4, axiom)
(between(u,v,w) and between(u,w,x)) $\Rightarrow$ between(v,w,x) cnf(b_5, axiom)
(between(u,v,w) and between(v,w,x)) $\Rightarrow$ (between(u,w,x) or $v = w$) cnf(b_6, axiom)
(between(u,v,w) and between(v,w,x)) $\Rightarrow$ (between(u,v,x) or $v = w$) cnf(b_7, axiom)
between(u,v,x) cnf(v_between_u_and_x, hypothesis)
between(v,w,x) cnf(w_between_v_and_x, hypothesis)
$\neg$ between(u,w,x) cnf(prove_w_between_u_and_x, negated_conjecture)

GEO047-2.p Corollary to second inner inner transitivity of betweenness
Corollary of second inner transitivity property of betweenness, using symmetry.
include('Axioms/GEO002-0.ax')
between(u,v,w) cnf(v_between_u_and_w, hypothesis)

between(u, w, x) cnf(w_between_u_and_x, hypothesis)
$\neg$ between(u, v, x) cnf(prove_v_between_u_and_x, negated_conjecture)

GEO047-3.p Corollary to second inner inner transitivity of betweenness
Corollary of second inner transitivity property of betweenness, using symmetry.
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant(u, v, w, x) $\Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension(u, v, w, x) $\Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between(u, v, reflection(u, v)) cnf(r2$_1$, axiom)
equidistant(v, reflection(u, v), u, v) cnf(r2$_2$, axiom)
$u = v$ $\Rightarrow$ $v =$ reflection(u, v) cnf(r3$_1$, axiom)
$u =$ reflection(u, u) cnf(r3$_2$, axiom)
$v =$ reflection(u, v) $\Rightarrow$ $u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant($u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between(u, v, w) $\Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf(d10$_1$, axiom)
equidistant(w, x, y, z) $\Rightarrow$ (extension(u, v, w, x) = extension(u, v, y, z) or $u = v$) cnf(d10$_2$, axiom)
extension(u, v, u, v) = extension(u, v, v, u) or $u = v$ cnf(d10$_3$, axiom)
equidistant(v, u, v, reflection(reflection(u, v), v)) cnf(r_5, axiom)
$u =$ reflection(reflection(u, v), v) cnf(r_6, axiom)
between(u, v, v) cnf(t_3, axiom)
(between(u, w, x) and $u = x$) $\Rightarrow$ between(v, w, x) cnf(b_1, axiom)
between(u, v, w) $\Rightarrow$ between(w, v, u) cnf(t_1, axiom)
between(u, u, v) cnf(t_2, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow$ $u = v$ cnf(b_2, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow$ $v = w$ cnf(b_3, axiom)
(between(u, v, w) and between(v, u, w)) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6$_1$, axiom)
(between(u, v, w) and between(u, w, v)) $\Rightarrow$ ($u = v$ or $v = w$) cnf(t6$_2$, axiom)
(between(u, v, w) and between(v, w, x)) $\Rightarrow$ between(u, v, w) cnf(b_4, axiom)
(between(u, v, w) and between(u, w, x)) $\Rightarrow$ between(v, w, x) cnf(b_5, axiom)
(between(u, v, w) and between(v, w, x)) $\Rightarrow$ (between(u, w, x) or $v = w$) cnf(b_6, axiom)
(between(u, v, w) and between(v, w, x)) $\Rightarrow$ (between(u, v, x) or $v = w$) cnf(b_7, axiom)
(between(u, v, x) and between(v, w, x)) $\Rightarrow$ between(u, w, x) cnf(b_8, axiom)
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u, w, x) cnf(w_between_u_and_x, hypothesis)
$\neg$ between(u, v, x) cnf(prove_v_between_u_and_x, negated_conjecture)

GEO048-2.p Inner points of triangle
include('Axioms/GEO002-0.ax')
between(u, v, w) cnf(v_between_u_and_w, hypothesis)
between(u_1, v_1, w) cnf(v1_between_u1_and_w, hypothesis)
between(u, x, u_1) cnf(x_between_u_and_u$_1$, hypothesis)
$\neg$ between(x, inner_pasch(v_1, inner_pasch(u, x, u_1, v_1, w), u, v, w), w) cnf(prove_conclusion$_1$, negated_conjecture)
$\neg$ between(v, inner_pasch(v_1, inner_pasch(u, x, u_1, v_1, w), u, v, w), v_1) cnf(prove_conclusion$_2$, negated_conjecture)

GEO049-2.p Theorem of similar situations
include('Axioms/GEO002-0.ax')
equidistant(u, v, u_1, v_1) cnf(u_to_v_equals_u1_to_v$_1$, hypothesis)
equidistant(v, w, v_1, w_1) cnf(v_to_w_equals_v1_to_w$_1$, hypothesis)
equidistant(u, w, u_1, w_1) cnf(u_to_w_equals_u1_to_w$_1$, hypothesis)

$\text{between}(u, v, w)$ $\text{cnf(v\_between\_u\_and\_w, hypothesis)}$
$\neg\,\text{between}(u_1, v_1, w_1)$ $\text{cnf(prove\_v1\_between\_u1\_and\_w}_1\text{, negated\_conjecture)}$

GEO050-2.p First outer connectivity property of betweenness
include('Axioms/GEO002-0.ax')
$\text{between}(u, v, w)$ cnf(v_between_u_and_w, hypothesis)
$\text{between}(u, v, x)$ cnf(v_between_u_and_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$\neg\,\text{between}(u, w, x)$ cnf(w_not_between_u_and_x, hypothesis)
$\neg\,\text{between}(u, x, w)$ cnf(prove_x_between_u_and_w, negated_conjecture)

GEO051-2.p Second outer connectivity property of betweenness
include('Axioms/GEO002-0.ax')
$\text{between}(u, v, w)$ cnf(v_between_u_and_w, hypothesis)
$\text{between}(u, v, x)$ cnf(v_between_u_and_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$\neg\,\text{between}(v, w, x)$ cnf(w_not_between_v_and_x, hypothesis)
$\neg\,\text{between}(v, x, w)$ cnf(prove_x_between_v_and_w, negated_conjecture)

GEO052-2.p Second inner connectivity property of betweenness
include('Axioms/GEO002-0.ax')
$\text{between}(u, v, x)$ cnf(v_between_u_and_x, hypothesis)
$\text{between}(u, w, x)$ cnf(w_between_u_and_x, hypothesis)
$\neg\,\text{between}(v, w, x)$ cnf(w_not_between_v_and_x, hypothesis)
$\neg\,\text{between}(w, v, x)$ cnf(prove_v_between_w_and_x, negated_conjecture)

GEO053-2.p Unique endpoint
include('Axioms/GEO002-0.ax')
$\text{between}(u, v, w)$ cnf(v_between_u_and_w, hypothesis)
$\text{equidistant}(u, v, u, w)$ cnf(u_to_v_equals_u_to_w, hypothesis)
$v \neq w$ cnf(prove_v_equals_w, negated_conjecture)

GEO054-2.p Corollary 2 to the segment construction axiom
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$\neg\,\text{between}(u, v, \text{reflection}(u, v))$ cnf(prove_v_between_u_and_reflection, negated_conjecture)

GEO054-3.p Corollary 2 to the segment construction axiom
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$\text{equidistant}(u, v, u, v)$ cnf(d_1, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, u, v)$ cnf(d_2, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, w, x)$ cnf(d_3, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(u, v, x, w)$ cnf(d4_1, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, x, w)$ cnf(d4_2, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, v, u)$ cnf(d4_3, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(x, w, u, v)$ cnf(d4_4, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(x, w, v, u)$ cnf(d4_5, axiom)
$(\text{equidistant}(u, v, w, x) \text{ and } \text{equidistant}(w, x, y, z)) \Rightarrow \text{equidistant}(u, v, y, z)$ cnf(d_5, axiom)
$v = \text{extension}(u, v, w, w)$ cnf(e_1, axiom)
$y = \text{extension}(u, v, w, x) \Rightarrow \text{between}(u, v, y)$ cnf(b_0, axiom)
$\neg\,\text{between}(u, v, \text{reflection}(u, v))$ cnf(prove_v_between_u_and_reflection, negated_conjecture)

GEO055-2.p Corollary 3 to the segment construction axiom
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$\neg\,\text{equidistant}(v, \text{reflection}(u, v), u, v)$ cnf(prove_equidistance, negated_conjecture)

GEO055-3.p Corollary 3 to the segment construction axiom
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$\text{equidistant}(u, v, u, v)$ cnf(d_1, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(w, x, u, v)$ cnf(d_2, axiom)
$\text{equidistant}(u, v, w, x) \Rightarrow \text{equidistant}(v, u, w, x)$ cnf(d_3, axiom)

equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant$(w, x, y, z)) \Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension$(u, v, w, x) \Rightarrow$ between(u, v, y) cnf(b_0, axiom)
$\neg$ equidistant$(v$, reflection$(u, v), u, v)$ cnf(prove_equidistance, negated_conjecture)

GEO056-2.p Corollary 1 to null extension
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$u = v$ cnf(u_equals_v, hypothesis)
$v \neq$ reflection(u, v) cnf(prove_v_equals_reflection, negated_conjecture)

GEO056-3.p Corollary 1 to null extension
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant$(w, x, y, z)) \Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension$(u, v, w, x) \Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v$, reflection$(u, v))$ cnf(r2$_1$, axiom)
equidistant$(v$, reflection$(u, v), u, v)$ cnf(r2$_2$, axiom)
$u = v$ cnf(u_equals_v, hypothesis)
$v \neq$ reflection(u, v) cnf(prove_v_equals_reflection, negated_conjecture)

GEO057-2.p Corollary 2 of null extension
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$u \neq$ reflection(u, u) cnf(prove_null_extension_by_reflection, negated_conjecture)

GEO057-3.p Corollary 2 of null extension
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf(d4$_1$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf(d4$_2$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf(d4$_3$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf(d4$_4$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf(d4$_5$, axiom)
(equidistant(u, v, w, x) and equidistant$(w, x, y, z)) \Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension$(u, v, w, x) \Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v$, reflection$(u, v))$ cnf(r2$_1$, axiom)
equidistant$(v$, reflection$(u, v), u, v)$ cnf(r2$_2$, axiom)
$u \neq$ reflection(u, u) cnf(prove_null_extension_by_reflection, negated_conjecture)

GEO058-2.p U is the only fixed point of reflection(U,V)
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$v =$ reflection(u, v) cnf(v_equals_reflection, hypothesis)

$u \neq v$ cnf(prove_u_equals_v, negated_conjecture)

GEO058-3.p U is the only fixed point of reflection(U,V)
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf($\mathrm{d4}_1$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf($\mathrm{d4}_2$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf($\mathrm{d4}_3$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf($\mathrm{d4}_4$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf($\mathrm{d4}_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension$(u, v, w, x) \Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v,$ reflection$(u, v))$ cnf($\mathrm{r2}_1$, axiom)
equidistant$(v,$ reflection$(u, v), u, v)$ cnf($\mathrm{r2}_2$, axiom)
$u = v \Rightarrow v =$ reflection(u, v) cnf($\mathrm{r3}_1$, axiom)
$u =$ reflection(u, u) cnf($\mathrm{r3}_2$, axiom)
$v =$ reflection(u, v) cnf(v_equals_reflection, hypothesis)
$u \neq v$ cnf(prove_u_equals_v, negated_conjecture)

GEO059-2.p Congruence for double reflection
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$\neg$ equidistant$(v, u, v,$ reflection(reflection$(u, v), v))$ cnf(prove_congruence, negated_conjecture)

GEO059-3.p Congruence for double reflection
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
equidistant(u, v, u, v) cnf(d_1, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, u, v) cnf(d_2, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, w, x) cnf(d_3, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(u, v, x, w) cnf($\mathrm{d4}_1$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(v, u, x, w) cnf($\mathrm{d4}_2$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(w, x, v, u) cnf($\mathrm{d4}_3$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, u, v) cnf($\mathrm{d4}_4$, axiom)
equidistant$(u, v, w, x) \Rightarrow$ equidistant(x, w, v, u) cnf($\mathrm{d4}_5$, axiom)
(equidistant(u, v, w, x) and equidistant(w, x, y, z)) $\Rightarrow$ equidistant(u, v, y, z) cnf(d_5, axiom)
$v =$ extension(u, v, w, w) cnf(e_1, axiom)
$y =$ extension$(u, v, w, x) \Rightarrow$ between(u, v, y) cnf(b_0, axiom)
between$(u, v,$ reflection$(u, v))$ cnf($\mathrm{r2}_1$, axiom)
equidistant$(v,$ reflection$(u, v), u, v)$ cnf($\mathrm{r2}_2$, axiom)
$u = v \Rightarrow v =$ reflection(u, v) cnf($\mathrm{r3}_1$, axiom)
$u =$ reflection(u, u) cnf($\mathrm{r3}_2$, axiom)
$v =$ reflection$(u, v) \Rightarrow u = v$ cnf(r_4, axiom)
equidistant(u, u, v, v) cnf(d_7, axiom)
(equidistant(u, v, u_1, v_1) and equidistant(v, w, v_1, w_1) and between(u, v, w) and between(u_1, v_1, w_1)) $\Rightarrow$ equidistant$(u, w, u_1,$
(between(u, v, w) and between(u, v, x) and equidistant(v, w, v, x)) $\Rightarrow$ ($u = v$ or $w = x$) cnf(d_9, axiom)
between$(u, v, w) \Rightarrow$ ($u = v$ or $w =$ extension(u, v, v, w)) cnf($\mathrm{d10}_1$, axiom)
equidistant$(w, x, y, z) \Rightarrow$ (extension$(u, v, w, x) =$ extension(u, v, y, z) or $u = v$) cnf($\mathrm{d10}_2$, axiom)
extension$(u, v, u, v) =$ extension(u, v, v, u) or $u = v$ cnf($\mathrm{d10}_3$, axiom)
$\neg$ equidistant$(v, u, v,$ reflection(reflection$(u, v), v))$ cnf(prove_congruence, negated_conjecture)

GEO060-2.p Reflection is an involution
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')
$u \neq$ reflection(reflection$(u, v), v)$ cnf(prove__involution, negated_conjecture)

GEO061-2.p Theorem of point insertion
include('Axioms/GEO002-0.ax')

include('Axioms/GEO002-3.ax')
$\text{equidistant}(u, v, u_1, \text{insertion}(u_1, w_1, u, v)) \Rightarrow \text{between}(u, v, w)$ cnf(part_1, negated_conjecture)
$\text{equidistant}(u, v, u_1, \text{insertion}(u_1, w_1, u, v)) \Rightarrow \text{equidistant}(u, w, u_1, w_1)$ cnf(part_2, negated_conjecture)
$(\text{equidistant}(u, v, u_1, \text{insertion}(u_1, w_1, u, v)) \text{ and } \text{between}(u_1, \text{insertion}(u_1, w_1, u, v), w_1)) \Rightarrow \neg\,\text{equidistant}(v, w, \text{insertion}(u_1, u$

GEO062-2.p Insertion identity
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-3.ax')
$\text{between}(u, v, w)$ cnf(v_between_u_and_w, hypothesis)
$v \neq \text{insertion}(u, w, u, v)$ cnf(prove_v_equals_insertion, negated_conjecture)

GEO063-2.p Insertion respects congruence in its last two arguments
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-3.ax')
$\text{equidistant}(w, x, y, z)$ cnf(w_to_x_equals_y_to_z, hypothesis)
$\text{insertion}(u, v, w, x) \neq \text{insertion}(u, v, y, z)$ cnf(prove_equality_of_insertions, negated_conjecture)

GEO064-2.p Corollary 1 to collinearity
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$\text{between}(w, v, u)$ cnf(v_between_w_and_u, hypothesis)
$\neg\,\text{colinear}(u, v, w)$ cnf(prove_uvw_colinear, negated_conjecture)

GEO065-2.p Corollary 2 to collinearity
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$\text{between}(u, w, v)$ cnf(w_between_u_and_v, hypothesis)
$\neg\,\text{colinear}(u, v, w)$ cnf(prove_uvw_colinear, negated_conjecture)

GEO066-2.p Corollary 3 to collinearity
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$\text{between}(v, u, w)$ cnf(u_between_v_and_w, hypothesis)
$\neg\,\text{colinear}(u, v, w)$ cnf(prove_uvw_colinear, negated_conjecture)

GEO067-2.p Any two points are collinear
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$(\text{colinear}(x, x, y) \text{ and } \text{colinear}(x, y, x) \text{ and } \text{colinear}(y, x, x)) \Rightarrow x = y$ cnf(part_1, negated_conjecture)
$(\text{colinear}(x, x, y) \text{ and } \text{colinear}(x, y, x) \text{ and } \text{colinear}(y, x, x)) \Rightarrow \neg\,\text{colinear}(x, z, y)$ cnf(part_2, negated_conjecture)

GEO068-2.p Theorem of similar situations for collinear U, V, W
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$\text{equidistant}(u, v, u_1, v_1)$ cnf(u_to_v_equals_u1_to_v_1, hypothesis)
$\text{equidistant}(v, w, v_1, w_1)$ cnf(v_to_w_equals_v1_to_w_1, hypothesis)
$\text{equidistant}(u, w, u_1, w_1)$ cnf(u_to_w_equals_u1_to_w_1, hypothesis)
$\text{colinear}(u, v, w)$ cnf(uvw_colinear, hypothesis)
$\neg\,\text{colinear}(u_1, v_1, w_1)$ cnf(prove_u1v1w1_colinear, negated_conjecture)

GEO069-2.p A property of collinearity
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$u \neq v$ cnf(u_not_v, hypothesis)
$\text{colinear}(w, v, u)$ cnf(wvu_colinear, hypothesis)
$\text{colinear}(x, v, u)$ cnf(xvu_colinear, hypothesis)
$\text{colinear}(x, w, u) \Rightarrow \neg\,\text{colinear}(x, w, v)$ cnf(prove_xwu_and_xwv_colinear, negated_conjecture)

GEO070-2.p Non-collinear points in the bisecting diagonal theorem
Under the hypotheses of the bisecting diagonal theorem, the points u, v, w cannot be colinear.
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
$\text{equidistant}(u, v, w, x)$ cnf(u_to_v_equals_w_to_x, hypothesis)
$\text{equidistant}(v, w, x, u)$ cnf(v_to_w_equals_x_to_u, hypothesis)
$\text{equidistant}(u, w, v, x)$ cnf(u_to_w_equals_v_to_x, hypothesis)

between(u, y, w) cnf(y_between_u_and_w, hypothesis)
between(v, y, x) cnf(y_between_v_and_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$x \neq u$ cnf(x_not_u, hypothesis)
colinear(u, v, w) cnf(prove_uvw_not_colinear, negated_conjecture)

GEO071-2.p Corollary 1 to non-collinear points theorem
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
equidistant(v, w, x, u) cnf(v_to_w_equals_x_to_u, hypothesis)
equidistant(u, w, v, x) cnf(u_to_w_equals_v_to_x, hypothesis)
between(u, y, w) cnf(y_between_u_and_w, hypothesis)
between(v, y, x) cnf(y_between_v_and_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$x \neq u$ cnf(x_not_u, hypothesis)
$u \neq w$ cnf(prove_u_equals_w, negated_conjecture)

GEO072-2.p Corollary 2 to non-collinear points theorem
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
equidistant(v, w, x, u) cnf(v_to_w_equals_x_to_u, hypothesis)
equidistant(u, w, v, x) cnf(u_to_w_equals_v_to_x, hypothesis)
between(u, y, w) cnf(y_between_u_and_w, hypothesis)
between(v, y, x) cnf(y_between_v_and_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$x \neq u$ cnf(x_not_u, hypothesis)
$v \neq x$ cnf(prove_v_equals_x, negated_conjecture)

GEO073-1.p The diagonals of a non-degenerate rectancle bisect
include('Axioms/GEO001-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
equidistant(v, w, x, u) cnf(v_to_w_equals_x_to_u, hypothesis)
equidistant(u, w, v, x) cnf(u_to_w_equals_v_to_x, hypothesis)
between(u, y, w) cnf(y_between_u_and_w, hypothesis)
between(v, y, x) cnf(y_between_v_and_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$x \neq u$ cnf(x_not_u, hypothesis)
equidistant(u, y, w, y) $\Rightarrow$ $\neg$ equidistant(v, y, x, y) cnf(prove_bisection, negated_conjecture)

GEO073-2.p The diagonals of a non-degenerate rectancle bisect
include('Axioms/GEO002-0.ax')
equidistant(u, v, w, x) cnf(u_to_v_equals_w_to_x, hypothesis)
equidistant(v, w, x, u) cnf(v_to_w_equals_x_to_u, hypothesis)
equidistant(u, w, v, x) cnf(u_to_w_equals_v_to_x, hypothesis)
between(u, y, w) cnf(y_between_u_and_w, hypothesis)
between(v, y, x) cnf(y_between_v_and_x, hypothesis)
$u \neq v$ cnf(u_not_v, hypothesis)
$x \neq u$ cnf(x_not_u, hypothesis)
equidistant(u, y, w, y) $\Rightarrow$ $\neg$ equidistant(v, y, x, y) cnf(prove_bisection, negated_conjecture)

GEO074-2.p Prove the Outer Pasch Axiom
include('Axioms/GEO002-0.ax')
between(u, w, x) cnf(w_between_u_and_x, hypothesis)
between(v, x, y) cnf(x_between_v_and_y, hypothesis)
between(u, outer_pasch(u, v, x, y, w), v) $\Rightarrow$ $\neg$ between(y, w, outer_pasch(u, v, x, y, w)) cnf(prove_outer_pasch, negated_conje

GEO075-2.p Show reflexivity for equidistance is dependent
All of the axioms in GEO003.ax are known to be independent except A1 and A7. Tarski and his students have been unable to establish their status.
(equidistant(x, y, z, v) and equidistant(x, y, v_2, w)) $\Rightarrow$ equidistant(z, v, v_2, w) cnf(transitivity_for_equidistance, axiom)
equidistant(x, y, z, z) $\Rightarrow$ $x = y$ cnf(identity_for_equidistance, axiom)
between(x, y, extension(x, y, w, v)) cnf(segment_construction$_1$, axiom)
equidistant(y, extension(x, y, w, v), w, v) cnf(segment_construction$_2$, axiom)

(equidistant(x, y, x_1, y_1) and equidistant(y, z, y_1, z_1) and equidistant(x, v, x_1, v_1) and equidistant(y, v, y_1, v_1) and between$(x,$
$(x = y$ or equidistant$(z, v, z_1, v_1))$ cnf(outer_five_segment, axiom)
between$(x, y, x) \Rightarrow x = y$ cnf(identity_for_betweeness, axiom)
(between(u, v, w) and between$(y, x, w)) \Rightarrow$ between$(v$, inner_pasch$(u, v, w, x, y), y)$ cnf(inner_pasch$_1$, axiom)
(between(u, v, w) and between$(y, x, w)) \Rightarrow$ between$(x$, inner_pasch$(u, v, w, x, y), u)$ cnf(inner_pasch$_2$, axiom)
$\neg$ between(lower_dimension_point$_1$, lower_dimension_point$_2$, lower_dimension_point$_3$) cnf(lower_dimension$_1$, axiom)
$\neg$ between(lower_dimension_point$_2$, lower_dimension_point$_3$, lower_dimension_point$_1$) cnf(lower_dimension$_2$, axiom)
$\neg$ between(lower_dimension_point$_3$, lower_dimension_point$_1$, lower_dimension_point$_2$) cnf(lower_dimension$_3$, axiom)
(equidistant(x, w, x, v) and equidistant(y, w, y, v) and equidistant$(z, w, z, v)) \Rightarrow$ (between(x, y, z) or between(y, z, x) or betw
$v)$ cnf(upper_dimension, axiom)
(between(u, w, y) and between$(v, w, x)) \Rightarrow (u = w$ or between$(u, v$, euclid$_1(u, v, w, x, y)))$ cnf(euclid$_1$, axiom)
(between(u, w, y) and between$(v, w, x)) \Rightarrow (u = w$ or between$(u, x$, euclid$_2(u, v, w, x, y)))$ cnf(euclid$_2$, axiom)
(between(u, w, y) and between$(v, w, x)) \Rightarrow (u = w$ or between(euclid$_1(u, v, w, x, y), y$, euclid$_2(u, v, w, x, y)))$ cnf(euclid$_3$, a
(equidistant(u, v, u, v_1) and equidistant(u, x, u, x_1) and between(u, v, x) and between$(v, w, x)) \Rightarrow$ between$(v_1$, continuous$(u,$
(equidistant(u, v, u, v_1) and equidistant(u, x, u, x_1) and between(u, v, x) and between$(v, w, x)) \Rightarrow$ equidistant$(u, w, u$, continu
$\neg$ equidistant(u, v, v, u) cnf(prove_reflexivity, negated_conjecture)

GEO076-4.p There is no point on every line
include('Axioms/GEO003-0.ax')
point(a_point) cnf(there_is_a_point, hypothesis)
line(line) $\Rightarrow$ on(a_point, line) cnf(prove_point_is_not_on_every_line, negated_conjecture)

GEO077-4.p Three points not collinear if not on line
include('Axioms/GEO003-0.ax')
point(point$_1$) cnf(point$_1$, hypothesis)
point(point$_2$) cnf(point$_2$, hypothesis)
point(point$_3$) cnf(point$_3$, hypothesis)
line(a_line) cnf(line, hypothesis)
on(point$_1$, a_line) cnf(point1_on_line, hypothesis)
on(point$_2$, a_line) cnf(point2_on_line, hypothesis)
$\neg$ on(point$_3$, a_line) cnf(point3_not_on_line, hypothesis)
point$_1 \neq$ point$_2$ cnf(point1_not_point$_2$, hypothesis)
point$_1 \neq$ point$_3$ cnf(point1_not_point$_3$, hypothesis)
point$_2 \neq$ point$_3$ cnf(point2_not_point$_3$, hypothesis)
collinear(point$_1$, point$_2$, point$_3$) cnf(prove_points_noncollinear, negated_conjecture)

GEO078-4.p Every plane contains 3 noncollinear points
include('Axioms/GEO003-0.ax')
plane(a_plane) cnf(there_is_a_plane, hypothesis)
(point(x_1) and point(x_2) and point(x_3) and on$(x_1$, a_plane) and on$(x_2$, a_plane) and on$(x_3$, a_plane)) $\Rightarrow$ (collinear$(x_1, x_2, x_3$
x_2 or $x_1 = x_3$ or $x_2 = x_3)$ cnf(prove_every_plane_contains_3_noncollinear_points, negated_conjecture)

GEO078-5.p Every plane contains 3 noncollinear points
include('Axioms/GEO003-0.ax')
(point(x_1) and point(x_2) and point(x_3) and on(x_1, y_1) and on(x_2, y_1) and line(y_1) and collinear$(x_1, x_2, x_3)) \Rightarrow (x_1 =$
x_2 or $x_1 = x_3$ or $x_2 = x_3$ or on$(x_3, y_1))$ cnf(points_not_collinear, axiom)
plane(a_plane) cnf(there_is_a_plane, hypothesis)
(point(x_1) and point(x_2) and point(x_3) and on$(x_1$, a_plane) and on$(x_2$, a_plane) and on$(x_3$, a_plane)) $\Rightarrow$ (collinear$(x_1, x_2, x_3$
x_2 or $x_1 = x_3$ or $x_2 = x_3)$ cnf(prove_every_plane_contains_3_noncollinear_points, negated_conjecture)

GEO078-6.p Every plane contains 3 noncollinear points
include('Axioms/GEO005-0.ax')
plane(a_plane) cnf(there_is_a_plane, hypothesis)
(point(x_1) and point(x_2) and point(x_3) and point_on_plane$(x_1$, a_plane) and point_on_plane$(x_2$, a_plane) and point_on_plane(
(collinear(x_1, x_2, x_3) or $x_1 = x_2$ or $x_1 = x_3$ or $x_2 = x_3)$ cnf(prove_every_plane_contains_3_noncollinear_points, negated_co

GEO078-7.p Every plane contains 3 noncollinear points
include('Axioms/GEO005-0.ax')
(point(x_1) and point(x_2) and point(x_3) and point_on_line(x_1, y_1) and point_on_line(x_2, y_1) and line(y_1) and collinear$(x_1, x_2,$
$(x_1 = x_2$ or $x_1 = x_3$ or $x_2 = x_3$ or point_on_line$(x_3, y_1))$ cnf(points_not_collinear, axiom)
plane(a_plane) cnf(there_is_a_plane, hypothesis)
(point(x_1) and point(x_2) and point(x_3) and point_on_plane$(x_1$, a_plane) and point_on_plane$(x_2$, a_plane) and point_on_plane(
(collinear(x_1, x_2, x_3) or $x_1 = x_2$ or $x_1 = x_3$ or $x_2 = x_3)$ cnf(prove_every_plane_contains_3_noncollinear_points, negated_co

GEO079-1.p The alternate interior angles in a trapezoid are equal
The alternate interior angles formed by a diagonal of a (not necessarily isosceles) trapezoid are equal.
(right_angle(u, v, w) and right_angle(x, y, z)) $\Rightarrow$ eq(u, v, w, x, y, z)   cnf(right_angles_are_equal, axiom)
congruent(u, v, w, x, y, z) $\Rightarrow$ eq(u, v, w, x, y, z)   cnf(corresponding_angles_are_equal, axiom)
trapezoid(u, v, w, x) $\Rightarrow$ parallel(v, w, u, x)   cnf(trapezoid_definition, axiom)
parallel(u, v, x, y) $\Rightarrow$ eq(x, v, u, v, x, y)   cnf(interior_angles_are_equal, axiom)
trapezoid(a, b, c, d)   cnf(a_trapezoid, hypothesis)
$\neg$ eq(a, c, b, c, a, d)   cnf(prove_angles_equal, negated_conjecture)

GEO080+1.p Reflexivity of part_of
include('Axioms/GEO004+0.ax')
$\forall c$: part_of(c, c)   fof(prove_reflexivity, conjecture)

GEO080-1.p Reflexivity of part_of
include('Axioms/GEO004-0.ax')
$\neg$ part_of($\mathrm{sk}_{14}, \mathrm{sk}_{14}$)   cnf(prove_reflexivity, negated_conjecture)

GEO081+1.p Transitivity of part_of
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2, c_3$: ((part_of(c_1, c_2) and part_of(c_2, c_3)) $\Rightarrow$ part_of(c_1, c_3))   fof(part_of_transitivity, conjecture)

GEO081-1.p Transitivity of part_of
include('Axioms/GEO004-0.ax')
part_of($\mathrm{sk}_{14}, \mathrm{sk}_{15}$)   cnf(part_of_transitivity$_{67}$, negated_conjecture)
part_of($\mathrm{sk}_{15}, \mathrm{sk}_{16}$)   cnf(part_of_transitivity$_{68}$, negated_conjecture)
$\neg$ part_of($\mathrm{sk}_{14}, \mathrm{sk}_{16}$)   cnf(part_of_transitivity$_{69}$, negated_conjecture)

GEO082+1.p Antisymmetry of part_of
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2$: ((part_of(c_1, c_2) and part_of(c_2, c_1)) $\Rightarrow$ $c_1 = c_2$)   fof(part_of_antisymmetry, conjecture)

GEO082-1.p Antisymmetry of part_of
include('Axioms/GEO004-0.ax')
part_of($\mathrm{sk}_{14}, \mathrm{sk}_{15}$)   cnf(part_of_antisymmetry$_{67}$, negated_conjecture)
part_of($\mathrm{sk}_{15}, \mathrm{sk}_{14}$)   cnf(part_of_antisymmetry$_{68}$, negated_conjecture)
$\mathrm{sk}_{14} \neq \mathrm{sk}_{15}$ cnf(part_of_antisymmetry$_{69}$, negated_conjecture)

GEO083+1.p Sum is monotone, part 1
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2, c_3, p$: ((part_of(c_2, c_3) and $p \wedge c_1 = c_2$ and $p \wedge c_1 = c_3$) $\Rightarrow$ part_of($c_1 + c_2, c_1 + c_3$))   fof(corollary_2_6$_1$, conjecture)

GEO083-1.p Sum is monotone, part 1
include('Axioms/GEO004-0.ax')
part_of($\mathrm{sk}_{15}, \mathrm{sk}_{16}$)   cnf(corollary_2_6_1$_{67}$, negated_conjecture)
$\mathrm{sk}_{17} \wedge \mathrm{sk}_{14} = \mathrm{sk}_{15}$ cnf(corollary_2_6_1$_{68}$, negated_conjecture)
$\mathrm{sk}_{17} \wedge \mathrm{sk}_{14} = \mathrm{sk}_{16}$ cnf(corollary_2_6_1$_{69}$, negated_conjecture)
$\neg$ part_of($\mathrm{sk}_{14} + \mathrm{sk}_{15}, \mathrm{sk}_{14} + \mathrm{sk}_{16}$)   cnf(corollary_2_6_1$_{70}$, negated_conjecture)

GEO084+1.p Sum is monotone, part 2
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2, c_3, p$: ((part_of(c_1, c_3) and part_of(c_2, c_3) and $p \wedge c_1 = c_2$) $\Rightarrow$ part_of($c_1 + c_2, c_3$))   fof(corollary_2_6$_2$, conjecture)

GEO084-1.p Sum is monotone, part 2
include('Axioms/GEO004-0.ax')
part_of($\mathrm{sk}_{14}, \mathrm{sk}_{16}$)   cnf(corollary_2_6_2$_{67}$, negated_conjecture)
part_of($\mathrm{sk}_{15}, \mathrm{sk}_{16}$)   cnf(corollary_2_6_2$_{68}$, negated_conjecture)
$\mathrm{sk}_{17} \wedge \mathrm{sk}_{14} = \mathrm{sk}_{15}$ cnf(corollary_2_6_2$_{69}$, negated_conjecture)
$\neg$ part_of($\mathrm{sk}_{14} + \mathrm{sk}_{15}, \mathrm{sk}_{16}$)   cnf(corollary_2_6_2$_{70}$, negated_conjecture)

GEO085+1.p Every open curve has at least two endpoints
include('Axioms/GEO004+0.ax')
$\forall c$: (open(c) $\Rightarrow$ $\exists p, q$: ($p \neq q$ and end_point(p, c) and end_point(q, c)))   fof(theorem_2_7$_1$, conjecture)

GEO085-1.p Every open curve has at least two endpoints
include('Axioms/GEO004-0.ax')
open(sk_{14})   cnf(theorem_2_7_1$_{67}$, negated_conjecture)
(end_point(a, sk_{14}) and end_point(b, sk_{14})) $\Rightarrow$ $a = b$ cnf(theorem_2_7_1$_{68}$, negated_conjecture)

GEO086+1.p Every sub-curve of an open curve is open
include('Axioms/GEO004+0.ax')
$\forall c, \mathrm{cpp}$: $((\mathrm{open}(c)$ and $\mathrm{part\_of}(\mathrm{cpp}, c)) \Rightarrow \mathrm{open}(\mathrm{cpp}))$ fof($\mathrm{theorem\_2\_7}_2$, conjecture)

GEO086-1.p Every sub-curve of an open curve is open
include('Axioms/GEO004-0.ax')
$\mathrm{open}(\mathrm{sk}_{14})$ cnf($\mathrm{theorem\_2\_7\_2}_{67}$, negated_conjecture)
$\mathrm{part\_of}(\mathrm{sk}_{15}, \mathrm{sk}_{14})$ cnf($\mathrm{theorem\_2\_7\_2}_{68}$, negated_conjecture)
$\neg\,\mathrm{open}(\mathrm{sk}_{15})$ cnf($\mathrm{theorem\_2\_7\_2}_{69}$, negated_conjecture)

GEO087+1.p If one curve is part of another curve then they cannot meet
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2$: $(\mathrm{part\_of}(c_1, c_2) \Rightarrow \neg\,\exists p$: $p \wedge c_1{=}c_2)$ fof($\mathrm{corollary\_2}_9$, conjecture)

GEO087-1.p If one curve is part of another curve then they cannot meet
include('Axioms/GEO004-0.ax')
$\mathrm{part\_of}(\mathrm{sk}_{14}, \mathrm{sk}_{15})$ cnf($\mathrm{corollary\_2\_9}_{67}$, negated_conjecture)
$\mathrm{sk}_{16} \wedge \mathrm{sk}_{14}{=}\mathrm{sk}_{15}$ cnf($\mathrm{corollary\_2\_9}_{68}$, negated_conjecture)

GEO088+1.p Endpoint of subcurve or curve
If an endpoint of a given curve lies on a sub-curve then it is also an endpoint of this sub-curve
include('Axioms/GEO004+0.ax')
$\forall c, \mathrm{cpp}, p$: $((\mathrm{part\_of}(\mathrm{cpp}, c)$ and $\mathrm{end\_point}(p, c)$ and $\mathrm{incident\_c}(p, \mathrm{cpp})) \Rightarrow \mathrm{end\_point}(p, \mathrm{cpp}))$ fof($\mathrm{theorem\_2}_{10}$, conjecture)

GEO088-1.p Endpoint of subcurve or curve
If an endpoint of a given curve lies on a sub-curve then it is also an endpoint of this sub-curve
include('Axioms/GEO004-0.ax')
$\mathrm{part\_of}(\mathrm{sk}_{15}, \mathrm{sk}_{14})$ cnf($\mathrm{theorem\_2\_10}_{67}$, negated_conjecture)
$\mathrm{end\_point}(\mathrm{sk}_{16}, \mathrm{sk}_{14})$ cnf($\mathrm{theorem\_2\_10}_{68}$, negated_conjecture)
$\mathrm{incident\_c}(\mathrm{sk}_{16}, \mathrm{sk}_{15})$ cnf($\mathrm{theorem\_2\_10}_{69}$, negated_conjecture)
$\neg\,\mathrm{end\_point}(\mathrm{sk}_{16}, \mathrm{sk}_{15})$ cnf($\mathrm{theorem\_2\_10}_{70}$, negated_conjecture)

GEO089+1.p Inner points of a sub-curve of a curve are inner points
include('Axioms/GEO004+0.ax')
$\forall c, p$: $(\exists \mathrm{cpp}$: $(\mathrm{part\_of}(\mathrm{cpp}, c)$ and $\mathrm{inner\_point}(p, \mathrm{cpp})) \Rightarrow \mathrm{inner\_point}(p, c))$ fof($\mathrm{corollary\_2}_{11}$, conjecture)

GEO089-1.p Inner points of a sub-curve of a curve are inner points
include('Axioms/GEO004-0.ax')
$\mathrm{part\_of}(\mathrm{sk}_{16}, \mathrm{sk}_{14})$ cnf($\mathrm{corollary\_2\_11}_{67}$, negated_conjecture)
$\mathrm{inner\_point}(\mathrm{sk}_{15}, \mathrm{sk}_{16})$ cnf($\mathrm{corollary\_2\_11}_{68}$, negated_conjecture)
$\neg\,\mathrm{inner\_point}(\mathrm{sk}_{15}, \mathrm{sk}_{14})$ cnf($\mathrm{corollary\_2\_11}_{69}$, negated_conjecture)

GEO090+1.p Meeting point of curves on a subcurve
If a point P is a meeting point of two curves and lies on a sub-curve of one of the two curves then P is also meeting point of the sub-curve and the other curve.
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2, \mathrm{cpp}, p$: $((\mathrm{part\_of}(c_2, c_1)$ and $\mathrm{incident\_c}(p, c_2)$ and $p{\wedge}c_1{=}\mathrm{cpp}) \Rightarrow p{\wedge}c_2{=}\mathrm{cpp})$ fof($\mathrm{corollary\_2}_{12}$, conjecture)

GEO090-1.p Meeting point of curves on a subcurve
If a point P is a meeting point of two curves and lies on a sub-curve of one of the two curves then P is also meeting point of the sub-curve and the other curve.
include('Axioms/GEO004-0.ax')
$\mathrm{part\_of}(\mathrm{sk}_{15}, \mathrm{sk}_{14})$ cnf($\mathrm{corollary\_2\_12}_{67}$, negated_conjecture)
$\mathrm{incident\_c}(\mathrm{sk}_{17}, \mathrm{sk}_{15})$ cnf($\mathrm{corollary\_2\_12}_{68}$, negated_conjecture)
$\mathrm{sk}_{17} \wedge \mathrm{sk}_{14}{=}\mathrm{sk}_{16}$ cnf($\mathrm{corollary\_2\_12}_{69}$, negated_conjecture)
$\neg\,\mathrm{sk}_{17} \wedge \mathrm{sk}_{15}{=}\mathrm{sk}_{16}$ cnf($\mathrm{corollary\_2\_12}_{70}$, negated_conjecture)

GEO091+1.p Two points determine subcurve
Two distinct points on an open curve uniquely determine the sub-curve connecting these points
include('Axioms/GEO004+0.ax')
$\forall c, c_1, c_2$: $((\mathrm{part\_of}(c_1, c)$ and $\mathrm{part\_of}(c_2, c)$ and $\mathrm{open}(c)$ and $\exists p, q$: $(p \neq q$ and $\mathrm{end\_point}(p, c_1)$ and $\mathrm{end\_point}(p, c_2)$ and end_p
$c_1 = c_2)$ fof($\mathrm{theorem\_2}_{13}$, conjecture)

GEO091-1.p Two points determine subcurve
Two distinct points on an open curve uniquely determine the sub-curve connecting these points
include('Axioms/GEO004-0.ax')

$\text{part\_of}(\text{sk}_{15}, \text{sk}_{14})$ cnf(theorem_2_13$_{67}$, negated_conjecture)
$\text{part\_of}(\text{sk}_{16}, \text{sk}_{14})$ cnf(theorem_2_13$_{68}$, negated_conjecture)
$\text{open}(\text{sk}_{14})$ cnf(theorem_2_13$_{69}$, negated_conjecture)
$\text{sk}_{17} \neq \text{sk}_{18}$ cnf(theorem_2_13$_{70}$, negated_conjecture)
$\text{end\_point}(\text{sk}_{17}, \text{sk}_{15})$ cnf(theorem_2_13$_{71}$, negated_conjecture)
$\text{end\_point}(\text{sk}_{17}, \text{sk}_{16})$ cnf(theorem_2_13$_{72}$, negated_conjecture)
$\text{end\_point}(\text{sk}_{18}, \text{sk}_{15})$ cnf(theorem_2_13$_{73}$, negated_conjecture)
$\text{end\_point}(\text{sk}_{18}, \text{sk}_{16})$ cnf(theorem_2_13$_{74}$, negated_conjecture)
$\text{sk}_{15} \neq \text{sk}_{16}$ cnf(theorem_2_13$_{75}$, negated_conjecture)

GEO092+1.p Common point of open sum is the meeting point
If two curves meet and their sum is open, then the only point they have in common is their meeting-point.
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2, p$: $((p \wedge c_1 = c_2 \text{ and } \text{open}(c_1 + c_2)) \Rightarrow \forall q$: $(q \neq p \Rightarrow \neg\, \text{incident\_c}(q, c_1) \text{ and } \text{incident\_c}(q, c_2)))$ fof(proposition_2_14$_1$,

GEO092-1.p Common point of open sum is the meeting point
If two curves meet and their sum is open, then the only point they have in common is their meeting-point.
include('Axioms/GEO004-0.ax')
$\text{sk}_{16} \wedge \text{sk}_{14} = \text{sk}_{15}$ cnf(proposition_2_14_1$_{67}$, negated_conjecture)
$\text{open}(\text{sk}_{14} + \text{sk}_{15})$ cnf(proposition_2_14_1$_{68}$, negated_conjecture)
$\text{sk}_{17} \neq \text{sk}_{16}$ cnf(proposition_2_14_1$_{69}$, negated_conjecture)
$\text{incident\_c}(\text{sk}_{17}, \text{sk}_{14})$ cnf(proposition_2_14_1$_{70}$, negated_conjecture)
$\text{incident\_c}(\text{sk}_{17}, \text{sk}_{15})$ cnf(proposition_2_14_1$_{71}$, negated_conjecture)

GEO093+1.p Sum of meeting open curves is open
If two open sub-curves of an open curve meet, then their sum is also open.
include('Axioms/GEO004+0.ax')
$\forall c, c_1, c_2, p$: $((\text{open}(c) \text{ and } \text{part\_of}(c_1, c) \text{ and } \text{part\_of}(c_2, c) \text{ and } p \wedge c_1 = c_2) \Rightarrow \text{open}(c_1 + c_2))$ fof(proposition_2_14$_2$, conjectu

GEO093-1.p Sum of meeting open curves is open
If two open sub-curves of an open curve meet, then their sum is also open.
include('Axioms/GEO004-0.ax')
$\text{open}(\text{sk}_{14})$ cnf(proposition_2_14_2$_{67}$, negated_conjecture)
$\text{part\_of}(\text{sk}_{15}, \text{sk}_{14})$ cnf(proposition_2_14_2$_{68}$, negated_conjecture)
$\text{part\_of}(\text{sk}_{16}, \text{sk}_{14})$ cnf(proposition_2_14_2$_{69}$, negated_conjecture)
$\text{sk}_{17} \wedge \text{sk}_{15} = \text{sk}_{16}$ cnf(proposition_2_14_2$_{70}$, negated_conjecture)
$\neg\, \text{open}(\text{sk}_{15} + \text{sk}_{16})$ cnf(proposition_2_14_2$_{71}$, negated_conjecture)

GEO094+1.p Meeting point is not an endpoint of contianing curve
A meeting point of two curves is not an endpoint of any curve that includes both as sub-curves.
include('Axioms/GEO004+0.ax')
$\forall c, c_1, c_2, p$: $((p \wedge c_1 = c_2 \text{ and } \text{part\_of}(c_1, c) \text{ and } \text{part\_of}(c_2, c)) \Rightarrow \neg\, \text{end\_point}(p, c))$ fof(proposition_2_14$_3$, conjecture)

GEO094-1.p Meeting point is not an endpoint of contianing curve
A meeting point of two curves is not an endpoint of any curve that includes both as sub-curves.
include('Axioms/GEO004-0.ax')
$\text{sk}_{17} \wedge \text{sk}_{15} = \text{sk}_{16}$ cnf(proposition_2_14_3$_{67}$, negated_conjecture)
$\text{part\_of}(\text{sk}_{15}, \text{sk}_{14})$ cnf(proposition_2_14_3$_{68}$, negated_conjecture)
$\text{part\_of}(\text{sk}_{16}, \text{sk}_{14})$ cnf(proposition_2_14_3$_{69}$, negated_conjecture)
$\text{end\_point}(\text{sk}_{17}, \text{sk}_{14})$ cnf(proposition_2_14_3$_{70}$, negated_conjecture)

GEO095+1.p Endpoints of open sum are endpoints of curves
If two curves meet and their sum is open, then the endpoints of the two curves that are not the meeting-point are also the endpoints of the sum of these curves.
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2, p$: $((p \wedge c_1 = c_2 \text{ and } \text{open}(c_1 + c_2)) \Rightarrow \exists q, r$: $(p \neq q \text{ and } q \neq r \text{ and } p \neq r \text{ and } \text{end\_point}(q, c_1 + c_2) \text{ and } \text{end\_point}(q, c_1)$ an
$c_2) \text{ and } \text{end\_point}(r, c_2)))$ fof(proposition_2_14$_4$, conjecture)

GEO095-1.p Endpoints of open sum are endpoints of curves
If two curves meet and their sum is open, then the endpoints of the two curves that are not the meeting-point are also the endpoints of the sum of these curves.
include('Axioms/GEO004-0.ax')
$\text{sk}_{16} \wedge \text{sk}_{14} = \text{sk}_{15}$ cnf(proposition_2_14_4$_{67}$, negated_conjecture)
$\text{open}(\text{sk}_{14} + \text{sk}_{15})$ cnf(proposition_2_14_4$_{68}$, negated_conjecture)

(end_point(a, sk$_{14}$ + sk$_{15}$) and end_point(a, sk$_{14}$) and end_point(b, sk$_{14}$ + sk$_{15}$) and end_point(b, sk$_{15}$)) $\Rightarrow$ (sk$_{16}$ = a or $a = b$ or sk$_{16} = b$)   cnf(proposition_2_14_4$_{69}$, negated_conjecture)

GEO096+1.p Endpoints of curves are endpoints of sum
If two curves meet, than the endpoints of the sum are exactly those endpoints of the two curves that are not meeting-points of them.
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2$: ($\exists p$: $p \wedge c_1 = c_2 \Rightarrow \forall q$: (end_point($q, c_1 + c_2$) $\iff$ ($\neg\, q \wedge c_1 = c_2$ and (end_point(q, c_1) or end_point(q, c_2)))))   fof(propo

GEO096-1.p Endpoints of curves are endpoints of sum
If two curves meet, than the endpoints of the sum are exactly those endpoints of the two curves that are not meeting-points of them.
include('Axioms/GEO004-0.ax')
sk$_{16} \wedge$ sk$_{14}$=sk$_{15}$ cnf(proposition_2_14_5$_{67}$, negated_conjecture)
sk$_{17} \wedge$ sk$_{14}$=sk$_{15}$ $\Rightarrow$ end_point(sk$_{17}$, sk$_{14}$ + sk$_{15}$)   cnf(proposition_2_14_5$_{68}$, negated_conjecture)
end_point(sk$_{17}$, sk$_{14}$+sk$_{15}$) or end_point(sk$_{17}$, sk$_{14}$) or end_point(sk$_{17}$, sk$_{15}$)   cnf(proposition_2_14_5$_{69}$, negated_conjecture)
end_point(sk$_{17}$, sk$_{14}$ + sk$_{15}$) $\Rightarrow$ end_point(sk$_{17}$, sk$_{14}$ + sk$_{15}$)   cnf(proposition_2_14_5$_{70}$, negated_conjecture)
(end_point(sk$_{17}$, sk$_{14}$) and sk$_{17} \wedge$ sk$_{14}$=sk$_{15}$) $\Rightarrow$ sk$_{17} \wedge$ sk$_{14}$=sk$_{15}$ cnf(proposition_2_14_5$_{71}$, negated_conjecture)
end_point(sk$_{17}$, sk$_{14}$) $\Rightarrow$ (sk$_{17}\wedge$sk$_{14}$=sk$_{15}$ or end_point(sk$_{17}$, sk$_{14}$) or end_point(sk$_{17}$, sk$_{15}$))   cnf(proposition_2_14_5$_{72}$, nega
(end_point(sk$_{17}$, sk$_{15}$) and sk$_{17} \wedge$ sk$_{14}$=sk$_{15}$) $\Rightarrow$ sk$_{17} \wedge$ sk$_{14}$=sk$_{15}$ cnf(proposition_2_14_5$_{73}$, negated_conjecture)
end_point(sk$_{17}$, sk$_{15}$) $\Rightarrow$ (sk$_{17}\wedge$sk$_{14}$=sk$_{15}$ or end_point(sk$_{17}$, sk$_{14}$) or end_point(sk$_{17}$, sk$_{15}$))   cnf(proposition_2_14_5$_{74}$, nega
(end_point(sk$_{17}$, sk$_{14}$) and end_point(sk$_{17}$, sk$_{14}$+sk$_{15}$)) $\Rightarrow$ sk$_{17}\wedge$sk$_{14}$=sk$_{15}$ cnf(proposition_2_14_5$_{75}$, negated_conjecture)
(end_point(sk$_{17}$, sk$_{15}$) and end_point(sk$_{17}$, sk$_{14}$+sk$_{15}$)) $\Rightarrow$ sk$_{17}\wedge$sk$_{14}$=sk$_{15}$ cnf(proposition_2_14_5$_{76}$, negated_conjecture)

GEO097+1.p A subcurves connects any two points on a curve
For any two points on a curve there is a sub-curve that connects these two points, that is to say these points are the endpoints of the sub-curve.
include('Axioms/GEO004+0.ax')
$\forall p, q, c$: (($p \neq q$ and incident_c(p, c) and incident_c(q, c)) $\Rightarrow$ $\exists$cpp: (part_of(cpp, c) and end_point(p, cpp) and end_point(q, cp

GEO097-1.p A subcurves connects any two points on a curve
For any two points on a curve there is a sub-curve that connects these two points, that is to say these points are the endpoints of the sub-curve.
include('Axioms/GEO004-0.ax')
sk$_{14} \neq$ sk$_{15}$ cnf(theorem_2_15$_{67}$, negated_conjecture)
incident_c(sk$_{14}$, sk$_{16}$)   cnf(theorem_2_15$_{68}$, negated_conjecture)
incident_c(sk$_{15}$, sk$_{16}$)   cnf(theorem_2_15$_{69}$, negated_conjecture)
(part_of(a, sk$_{16}$) and end_point(sk$_{14}$, a)) $\Rightarrow$ $\neg$ end_point(sk$_{15}$, a)   cnf(theorem_2_15$_{70}$, negated_conjecture)

GEO098+1.p For closed curves, there are two complementary sub-curves
include('Axioms/GEO004+0.ax')
$\forall c, p, q$: ((closed(c) and incident_c(p, c) and incident_c(q, c) and $p \neq q$) $\Rightarrow$ $\exists c_1, c_2$: ($p \wedge c_1 = c_2$ and $q \wedge c_1 = c_2$ and $c = c_1 + c_2$))   fof(theorem_2$_{16}$, conjecture)

GEO098-1.p For closed curves, there are two complementary sub-curves
include('Axioms/GEO004-0.ax')
closed(sk$_{14}$)   cnf(theorem_2_16$_{67}$, negated_conjecture)
incident_c(sk$_{15}$, sk$_{14}$)   cnf(theorem_2_16$_{68}$, negated_conjecture)
incident_c(sk$_{16}$, sk$_{14}$)   cnf(theorem_2_16$_{69}$, negated_conjecture)
sk$_{15} \neq$ sk$_{16}$ cnf(theorem_2_16$_{70}$, negated_conjecture)
(sk$_{15} \wedge a$=b and sk$_{16} \wedge a$=b) $\Rightarrow$ sk$_{14} \neq a + b$ cnf(theorem_2_16$_{71}$, negated_conjecture)

GEO099+1.p Open subcurves can be complemented to form the sum
Every open sub-curve of a closed curve can be complemented by another curve so that their sum constitute the closed curve.
include('Axioms/GEO004+0.ax')
$\forall c, c_1, p, q$: ((closed(c) and part_of(c_1, c) and end_point(p, c_1) and end_point(q, c_1) and $p \neq q$) $\Rightarrow$ $\exists c_2$: ($p \wedge c_1 = c_2$ and $q \wedge c_1 = c_2$ and $c = c_1 + c_2$))   fof(theorem_2$_{17}$, conjecture)

GEO099-1.p Open subcurves can be complemented to form the sum
Every open sub-curve of a closed curve can be complemented by another curve so that their sum constitute the closed curve.
include('Axioms/GEO004-0.ax')
closed(sk$_{14}$)   cnf(theorem_2_17$_{67}$, negated_conjecture)

part_of(sk_{15}, sk_{14}) cnf(theorem_2_17$_{68}$, negated_conjecture)
end_point(sk_{16}, sk_{15}) cnf(theorem_2_17$_{69}$, negated_conjecture)
end_point(sk_{17}, sk_{15}) cnf(theorem_2_17$_{70}$, negated_conjecture)
$sk_{16} \neq sk_{17}$ cnf(theorem_2_17$_{71}$, negated_conjecture)
($sk_{16} \wedge sk_{15}{=}a$ and $sk_{17} \wedge sk_{15}{=}a$) $\Rightarrow$ $sk_{14} \neq sk_{15} + a$ cnf(theorem_2_17$_{72}$, negated_conjecture)

GEO100+1.p Subcurves with common endpoint can be complemented
Every proper sub-curve of an open curve that has a common endpoint with the open curve can be complemented by another curve so that their sum constitute the open curve.
include('Axioms/GEO004+0.ax')
$\forall c, c_1, p$: ((open(c) and part_of(c_1, c) and $c_1 \neq c$ and end_point(p, c_1) and end_point(p, c)) $\Rightarrow$ $\exists q, c_2$: ($p \neq q$ and $q \wedge c_1{=}c_2$ and $c = c_1 + c_2$)) fof(theorem_2$_{18}$, conjecture)

GEO100-1.p Subcurves with common endpoint can be complemented
Every proper sub-curve of an open curve that has a common endpoint with the open curve can be complemented by another curve so that their sum constitute the open curve.
include('Axioms/GEO004-0.ax')
open(sk_{14}) cnf(theorem_2_18$_{67}$, negated_conjecture)
part_of(sk_{15}, sk_{14}) cnf(theorem_2_18$_{68}$, negated_conjecture)
$sk_{15} \neq sk_{14}$ cnf(theorem_2_18$_{69}$, negated_conjecture)
end_point(sk_{16}, sk_{15}) cnf(theorem_2_18$_{70}$, negated_conjecture)
end_point(sk_{16}, sk_{14}) cnf(theorem_2_18$_{71}$, negated_conjecture)
($a \wedge sk_{15}{=}b$ and $sk_{14} = sk_{15} + b$) $\Rightarrow$ $sk_{16} = a$ cnf(theorem_2_18$_{72}$, negated_conjecture)

GEO101+1.p Intensification of GEO100+1
include('Axioms/GEO004+0.ax')
$\forall c, c_1, p$: ((part_of(c_1, c) and $c_1 \neq c$ and open(c) and end_point(p, c_1) and end_point(p, c)) $\Rightarrow$ $\exists q, r, c_2$: ($q \wedge c_1{=}c_2$ and $c = c_1 + c_2$ and $p \neq q$ and $q \neq r$ and $p \neq r$ and end_point(r, c_2) and end_point(r, c))) fof(corollary_2$_{19}$, conjecture)

GEO101-1.p Intensification of GEO100+1
include('Axioms/GEO004-0.ax')
part_of(sk_{15}, sk_{14}) cnf(corollary_2_19$_{67}$, negated_conjecture)
$sk_{15} \neq sk_{14}$ cnf(corollary_2_19$_{68}$, negated_conjecture)
open(sk_{14}) cnf(corollary_2_19$_{69}$, negated_conjecture)
end_point(sk_{16}, sk_{15}) cnf(corollary_2_19$_{70}$, negated_conjecture)
end_point(sk_{16}, sk_{14}) cnf(corollary_2_19$_{71}$, negated_conjecture)
($a \wedge sk_{15}{=}b$ and $sk_{14} = sk_{15} + b$ and end_point(c, b) and end_point(c, sk_{14})) $\Rightarrow$ ($sk_{16} = a$ or $a = c$ or $sk_{16} = c$) cnf(corollary_2_19$_{72}$, negated_conjecture)

GEO102+1.p Common endpoint of subcurves means inclusion
If two sub-curves of one curve have a common endpoint and include a sub-curve starting at this endpoint, then one of the two sub-curves is included in the other.
include('Axioms/GEO004+0.ax')
$\forall c_1, c_2$: (($\exists c_3, p$: (part_of(c_3, c_1) and part_of(c_3, c_2) and end_point(p, c_1) and end_point(p, c_2) and end_point(p, c_3)) and $\exists c$: (pa
(part_of(c_1, c_2) or part_of(c_2, c_1))) fof(theorem_2$_{20}$, conjecture)

GEO102-1.p Common endpoint of subcurves means inclusion
If two sub-curves of one curve have a common endpoint and include a sub-curve starting at this endpoint, then one of the two sub-curves is included in the other.
include('Axioms/GEO004-0.ax')
part_of(sk_{16}, sk_{14}) cnf(theorem_2_20$_{67}$, negated_conjecture)
part_of(sk_{16}, sk_{15}) cnf(theorem_2_20$_{68}$, negated_conjecture)
end_point(sk_{17}, sk_{14}) cnf(theorem_2_20$_{69}$, negated_conjecture)
end_point(sk_{17}, sk_{15}) cnf(theorem_2_20$_{70}$, negated_conjecture)
end_point(sk_{17}, sk_{16}) cnf(theorem_2_20$_{71}$, negated_conjecture)
part_of(sk_{14}, sk_{18}) cnf(theorem_2_20$_{72}$, negated_conjecture)
part_of(sk_{15}, sk_{18}) cnf(theorem_2_20$_{73}$, negated_conjecture)
$\neg$ part_of(sk_{14}, sk_{15}) cnf(theorem_2_20$_{74}$, negated_conjecture)
$\neg$ part_of(sk_{15}, sk_{14}) cnf(theorem_2_20$_{75}$, negated_conjecture)

GEO103+1.p Common endpoint of subcurves and another point means inclusion
If two sub-curves of an open curve have a common endpoint and another point in common, then one of the two sub-curves is included in the other.

include('Axioms/GEO004+0.ax')
$\forall c, c_1, c_2, p, q$: ((open($c$) and part_of($c_1, c$) and part_of($c_2, c$) and end_point($p, c_1$) and end_point($p, c_2$) and $p \neq q$ and incident
(part_of(c_1, c_2) or part_of(c_2, c_1))) fof(corollary_2$_{21}$, conjecture)

GEO103-1.p Common endpoint of subcurves and another point means inclusion
If two sub-curves of an open curve have a common endpoint and another point in common, then one of the two sub-curves is included in the other.
include('Axioms/GEO004-0.ax')
open(sk$_{14}$) cnf(corollary_2_21$_{67}$, negated_conjecture)
part_of(sk$_{15}$, sk$_{14}$) cnf(corollary_2_21$_{68}$, negated_conjecture)
part_of(sk$_{16}$, sk$_{14}$) cnf(corollary_2_21$_{69}$, negated_conjecture)
end_point(sk$_{17}$, sk$_{15}$) cnf(corollary_2_21$_{70}$, negated_conjecture)
end_point(sk$_{17}$, sk$_{16}$) cnf(corollary_2_21$_{71}$, negated_conjecture)
sk$_{17} \neq$ sk$_{18}$ cnf(corollary_2_21$_{72}$, negated_conjecture)
incident_c(sk$_{18}$, sk$_{15}$) cnf(corollary_2_21$_{73}$, negated_conjecture)
incident_c(sk$_{18}$, sk$_{16}$) cnf(corollary_2_21$_{74}$, negated_conjecture)
$\neg$ part_of(sk$_{15}$, sk$_{16}$) cnf(corollary_2_21$_{75}$, negated_conjecture)
$\neg$ part_of(sk$_{16}$, sk$_{15}$) cnf(corollary_2_21$_{76}$, negated_conjecture)

GEO104+1.p Subcurves with common endpoint meet or include
If two sub-curves of a given open curve have a common endpoint then the sub-curves meet or one is included in the other.
include('Axioms/GEO004+0.ax')
$\forall c, c_1, c_2, p$: ((end_point(p, c_1) and end_point(p, c_2) and part_of(c_1, c) and part_of(c_2, c) and open(c)) $\Rightarrow$ ($p \wedge c_1 = c_2$ or part_of(

GEO104-1.p Subcurves with common endpoint meet or include
If two sub-curves of a given open curve have a common endpoint then the sub-curves meet or one is included in the other.
include('Axioms/GEO004-0.ax')
end_point(sk$_{17}$, sk$_{15}$) cnf(theorem_2_22$_{67}$, negated_conjecture)
end_point(sk$_{17}$, sk$_{16}$) cnf(theorem_2_22$_{68}$, negated_conjecture)
part_of(sk$_{15}$, sk$_{14}$) cnf(theorem_2_22$_{69}$, negated_conjecture)
part_of(sk$_{16}$, sk$_{14}$) cnf(theorem_2_22$_{70}$, negated_conjecture)
open(sk$_{14}$) cnf(theorem_2_22$_{71}$, negated_conjecture)
$\neg$ sk$_{17} \wedge$ sk$_{15}$=sk$_{16}$ cnf(theorem_2_22$_{72}$, negated_conjecture)
$\neg$ part_of(sk$_{15}$, sk$_{16}$) cnf(theorem_2_22$_{73}$, negated_conjecture)
$\neg$ part_of(sk$_{16}$, sk$_{15}$) cnf(theorem_2_22$_{74}$, negated_conjecture)

GEO105+1.p If subcurves meet at an endpoint then there's a meeting
If two sub-curves of an open curve meet at a point and this point is an endpoint for another sub-curve then this sub-curve meets one of the former sub-curves at this point.
include('Axioms/GEO004+0.ax')
$\forall c, c_1, c_2, c_3, p$: ((part_of($c_1, c$) and part_of($c_2, c$) and part_of($c_3, c$) and $p \wedge c_1 = c_2$ and end_point(p, c_3) and open(c)) $\Rightarrow$
($p \wedge c_1 = c_3$ or $p \wedge c_2 = c_3$)) fof(proposition_2$_{23}$, conjecture)

GEO105-1.p If subcurves meet at an endpoint then there's a meeting
If two sub-curves of an open curve meet at a point and this point is an endpoint for another sub-curve then this sub-curve meets one of the former sub-curves at this point.
include('Axioms/GEO004-0.ax')
part_of(sk$_{15}$, sk$_{14}$) cnf(proposition_2_23$_{67}$, negated_conjecture)
part_of(sk$_{16}$, sk$_{14}$) cnf(proposition_2_23$_{68}$, negated_conjecture)
part_of(sk$_{17}$, sk$_{14}$) cnf(proposition_2_23$_{69}$, negated_conjecture)
sk$_{18} \wedge$ sk$_{15}$=sk$_{16}$ cnf(proposition_2_23$_{70}$, negated_conjecture)
end_point(sk$_{18}$, sk$_{17}$) cnf(proposition_2_23$_{71}$, negated_conjecture)
open(sk$_{14}$) cnf(proposition_2_23$_{72}$, negated_conjecture)
$\neg$ sk$_{18} \wedge$ sk$_{15}$=sk$_{17}$ cnf(proposition_2_23$_{73}$, negated_conjecture)
$\neg$ sk$_{18} \wedge$ sk$_{16}$=sk$_{17}$ cnf(proposition_2_23$_{74}$, negated_conjecture)

GEO106+1.p Two common endpoints means identical or sum to whole
If two sub-curves have two common endpoints then they are identical or their sum is the whole curve.
include('Axioms/GEO004+0.ax')
$\forall c, c_1, c_2$: (($\exists p, q$: (end_point(p, c_1) and end_point(q, c_1) and end_point(p, c_2) and end_point(q, c_2) and $p \neq q$) and closed(c) an
($c_1 = c_2$ or $c = c_1 + c_2$)) fof(theorem_2$_{24}$, conjecture)

GEO106-1.p Two common endpoints means identical or sum to whole
If two sub-curves have two common endpoints then they are identical or their sum is the whole curve.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
end_point($\text{sk}_{17}, \text{sk}_{15}$) cnf(theorem_2_24$_{67}$, negated_conjecture)
end_point($\text{sk}_{18}, \text{sk}_{15}$) cnf(theorem_2_24$_{68}$, negated_conjecture)
end_point($\text{sk}_{17}, \text{sk}_{16}$) cnf(theorem_2_24$_{69}$, negated_conjecture)
end_point($\text{sk}_{18}, \text{sk}_{16}$) cnf(theorem_2_24$_{70}$, negated_conjecture)
$\text{sk}_{17} \neq \text{sk}_{18}$ cnf(theorem_2_24$_{71}$, negated_conjecture)
closed(sk_{14}) cnf(theorem_2_24$_{72}$, negated_conjecture)
part_of($\text{sk}_{15}, \text{sk}_{14}$) cnf(theorem_2_24$_{73}$, negated_conjecture)
part_of($\text{sk}_{16}, \text{sk}_{14}$) cnf(theorem_2_24$_{74}$, negated_conjecture)
$\text{sk}_{15} \neq \text{sk}_{16}$ cnf(theorem_2_24$_{75}$, negated_conjecture)
$\text{sk}_{14} \neq \text{sk}_{15} + \text{sk}_{16}$ cnf(theorem_2_24$_{76}$, negated_conjecture)

GEO107+1.p Equivalence of betweenness definitions 1 and 2
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p, q, r$: (between_c$_2(c, p, q, r)$ $\iff$ ($p \neq q$ and $p \neq r$ and $q \neq r$ and $\exists c_1, c_2$: ($q \wedge c_1 = c_2$ and part_of(c_1, c) and part_of(c_2, c) a
$\forall c, p, q, r$: (between_c(c, p, q, r) $\iff$ between_c$_2(c, p, q, r)$) fof(theorem_3$_3$, conjecture)

GEO107-1.p Equivalence of betweenness definitions 1 and 2
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
between_c$_2(a, b, c, d)$ $\Rightarrow$ $b \neq c$ cnf(between_c2_defn$_{81}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ $b \neq d$ cnf(between_c2_defn$_{82}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ $c \neq d$ cnf(between_c2_defn$_{83}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ $c \wedge \text{sk}_{15}(d, c, b, a) = \text{sk}_{16}(d, c, b, a)$ cnf(between_c2_defn$_{84}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ part_of($\text{sk}_{15}(d, c, b, a), a$) cnf(between_c2_defn$_{85}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ part_of($\text{sk}_{16}(d, c, b, a), a$) cnf(between_c2_defn$_{86}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ end_point($b, \text{sk}_{15}(d, c, b, a)$) cnf(between_c2_defn$_{87}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ end_point($d, \text{sk}_{16}(d, c, b, a)$) cnf(between_c2_defn$_{88}$, hypothesis)
($b \wedge d = e$ and part_of(d, f) and part_of(e, f) and end_point(a, d) and end_point(c, e)) $\Rightarrow$ ($a = b$ or $a = c$ or $b = c$ or between_c$_2(f, a, b, c)$) cnf(between_c2_defn$_{89}$, hypothesis)
between_c($\text{sk}_{17}, \text{sk}_{18}, \text{sk}_{19}, \text{sk}_{20}$) or between_c$_2$($\text{sk}_{17}, \text{sk}_{18}, \text{sk}_{19}, \text{sk}_{20}$) cnf(theorem_3_3$_{90}$, negated_conjecture)
between_c($\text{sk}_{17}, \text{sk}_{18}, \text{sk}_{19}, \text{sk}_{20}$) $\Rightarrow$ between_c($\text{sk}_{17}, \text{sk}_{18}, \text{sk}_{19}, \text{sk}_{20}$) cnf(theorem_3_3$_{91}$, negated_conjecture)
between_c$_2$($\text{sk}_{17}, \text{sk}_{18}, \text{sk}_{19}, \text{sk}_{20}$) $\Rightarrow$ between_c$_2$($\text{sk}_{17}, \text{sk}_{18}, \text{sk}_{19}, \text{sk}_{20}$) cnf(theorem_3_3$_{92}$, negated_conjecture)
between_c$_2$($\text{sk}_{17}, \text{sk}_{18}, \text{sk}_{19}, \text{sk}_{20}$) $\Rightarrow$ $\neg$ between_c($\text{sk}_{17}, \text{sk}_{18}, \text{sk}_{19}, \text{sk}_{20}$) cnf(theorem_3_3$_{93}$, negated_conjecture)

GEO108+1.p Equivalence of betweenness definitions 1 and 3
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p, q, r$: (between_c$_2(c, p, q, r)$ $\iff$ ($p \neq q$ and $p \neq r$ and $q \neq r$ and $\exists c_1, c_2$: ($q \wedge c_1 = c_2$ and part_of(c_1, c) and part_of(c_2, c) a
$\forall c, p, q, r$: (between_c$_3(c, p, q, r)$ $\iff$ ($p \neq q$ and $p \neq r$ and $q \neq r$ and $\exists c_1, c_2$: ($q \wedge c_1 = c_2$ and $c_1 + c_2 = c$ and incident_c(p, c_1) a
$\forall c, p, q, r$: (between_c$_2(c, p, q, r)$ $\iff$ between_c$_3(c, p, q, r)$) fof(theorem_3$_5$, conjecture)

GEO108-1.p Equivalence of betweenness definitions 1 and 3
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
between_c$_2(a, b, c, d)$ $\Rightarrow$ $b \neq c$ cnf(between_c2_defn$_{85}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ $b \neq d$ cnf(between_c2_defn$_{86}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ $c \neq d$ cnf(between_c2_defn$_{87}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ $c \wedge \text{sk}_{15}(d, c, b, a) = \text{sk}_{16}(d, c, b, a)$ cnf(between_c2_defn$_{88}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ part_of($\text{sk}_{15}(d, c, b, a), a$) cnf(between_c2_defn$_{89}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ part_of($\text{sk}_{16}(d, c, b, a), a$) cnf(between_c2_defn$_{90}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ end_point($b, \text{sk}_{15}(d, c, b, a)$) cnf(between_c2_defn$_{91}$, hypothesis)
between_c$_2(a, b, c, d)$ $\Rightarrow$ end_point($d, \text{sk}_{16}(d, c, b, a)$) cnf(between_c2_defn$_{92}$, hypothesis)
($b \wedge d = e$ and part_of(d, f) and part_of(e, f) and end_point(a, d) and end_point(c, e)) $\Rightarrow$ ($a = b$ or $a = c$ or $b = c$ or between_c$_2(f, a, b, c)$) cnf(between_c2_defn$_{93}$, hypothesis)
between_c$_3(a, b, c, d)$ $\Rightarrow$ $b \neq c$ cnf(between_c3_defn$_{94}$, hypothesis)
between_c$_3(a, b, c, d)$ $\Rightarrow$ $b \neq d$ cnf(between_c3_defn$_{95}$, hypothesis)

$\text{between\_c}_3(a,b,c,d) \Rightarrow c \neq d$ cnf(between_c3_defn$_{96}$, hypothesis)
$\text{between\_c}_3(a,b,c,d) \Rightarrow c \wedge \text{sk}_{17}(d,c,b,a)=\text{sk}_{18}(d,c,b,a)$ cnf(between_c3_defn$_{97}$, hypothesis)
$\text{between\_c}_3(a,b,c,d) \Rightarrow \text{sk}_{17}(d,c,b,a) + \text{sk}_{18}(d,c,b,a) = a$ cnf(between_c3_defn$_{98}$, hypothesis)
$\text{between\_c}_3(a,b,c,d) \Rightarrow \text{incident\_c}(b, \text{sk}_{17}(d,c,b,a))$ cnf(between_c3_defn$_{99}$, hypothesis)
$\text{between\_c}_3(a,b,c,d) \Rightarrow \text{incident\_c}(d, \text{sk}_{18}(d,c,b,a))$ cnf(between_c3_defn$_{100}$, hypothesis)
$(b \wedge d = e \text{ and } d+e = f \text{ and incident\_c}(a,d) \text{ and incident\_c}(c,e)) \Rightarrow (a = b \text{ or } a = c \text{ or } b = c \text{ or between\_c}_3(f,a,b,c))$ cnf(b
$\text{between\_c}_2(\text{sk}_{19}, \text{sk}_{20}, \text{sk}_{21}, \text{sk}_{22}) \text{ or between\_c}_3(\text{sk}_{19}, \text{sk}_{20}, \text{sk}_{21}, \text{sk}_{22})$ cnf(theorem_3_5$_{102}$, negated_conjecture)
$\text{between\_c}_2(\text{sk}_{19}, \text{sk}_{20}, \text{sk}_{21}, \text{sk}_{22}) \Rightarrow \text{between\_c}_2(\text{sk}_{19}, \text{sk}_{20}, \text{sk}_{21}, \text{sk}_{22})$ cnf(theorem_3_5$_{103}$, negated_conjecture)
$\text{between\_c}_3(\text{sk}_{19}, \text{sk}_{20}, \text{sk}_{21}, \text{sk}_{22}) \Rightarrow \text{between\_c}_3(\text{sk}_{19}, \text{sk}_{20}, \text{sk}_{21}, \text{sk}_{22})$ cnf(theorem_3_5$_{104}$, negated_conjecture)
$\text{between\_c}_3(\text{sk}_{19}, \text{sk}_{20}, \text{sk}_{21}, \text{sk}_{22}) \Rightarrow \neg\, \text{between\_c}_2(\text{sk}_{19}, \text{sk}_{20}, \text{sk}_{21}, \text{sk}_{22})$ cnf(theorem_3_5$_{105}$, negated_conjecture)

GEO109+1.p Every endpoint of an open curve is not between any other points
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p: (\text{open}(c) \Rightarrow (\text{end\_point}(p,c) \iff (\text{incident\_c}(p,c) \text{ and } \neg \exists q, r: \text{between\_c}(c,q,p,r))))$ fof(theorem_3$_6$, conjecture)

GEO109-1.p Every endpoint of an open curve is not between any other points
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
$\text{open}(\text{sk}_{15})$ cnf(theorem_3_6$_{77}$, negated_conjecture)
$\text{end\_point}(\text{sk}_{16}, \text{sk}_{15}) \text{ or incident\_c}(\text{sk}_{16}, \text{sk}_{15})$ cnf(theorem_3_6$_{78}$, negated_conjecture)
$\text{between\_c}(\text{sk}_{15}, a, \text{sk}_{16}, b) \Rightarrow \text{end\_point}(\text{sk}_{16}, \text{sk}_{15})$ cnf(theorem_3_6$_{79}$, negated_conjecture)
$\text{end\_point}(\text{sk}_{16}, \text{sk}_{15}) \Rightarrow \text{end\_point}(\text{sk}_{16}, \text{sk}_{15})$ cnf(theorem_3_6$_{80}$, negated_conjecture)
$\text{incident\_c}(\text{sk}_{16}, \text{sk}_{15}) \Rightarrow (\text{between\_c}(\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{16}, \text{sk}_{18}) \text{ or incident\_c}(\text{sk}_{16}, \text{sk}_{15}))$ cnf(theorem_3_6$_{81}$, negated_conjecture
$(\text{incident\_c}(\text{sk}_{16}, \text{sk}_{15}) \text{ and between\_c}(\text{sk}_{15}, a, \text{sk}_{16}, b)) \Rightarrow \text{between\_c}(\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{16}, \text{sk}_{18})$ cnf(theorem_3_6$_{82}$, negated_conj
$(\text{incident\_c}(\text{sk}_{16}, \text{sk}_{15}) \text{ and end\_point}(\text{sk}_{16}, \text{sk}_{15})) \Rightarrow \text{between\_c}(\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{16}, \text{sk}_{18})$ cnf(theorem_3_6$_{83}$, negated_conjectu

GEO110+1.p Betweenness for closed curves
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p, q, r: ((\text{closed}(c) \text{ and } p \neq q \text{ and } q \neq r \text{ and } p \neq r \text{ and incident\_c}(p,c) \text{ and incident\_c}(q,c) \text{ and incident\_c}(r,c)) \Rightarrow \text{between\_c}(c,p,q,r))$ fof(theorem_3$_7$, conjecture)

GEO110-1.p Betweenness for closed curves
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
$\text{closed}(\text{sk}_{15})$ cnf(theorem_3_7$_{77}$, negated_conjecture)
$\text{sk}_{16} \neq \text{sk}_{17}$ cnf(theorem_3_7$_{78}$, negated_conjecture)
$\text{sk}_{17} \neq \text{sk}_{18}$ cnf(theorem_3_7$_{79}$, negated_conjecture)
$\text{sk}_{16} \neq \text{sk}_{18}$ cnf(theorem_3_7$_{80}$, negated_conjecture)
$\text{incident\_c}(\text{sk}_{16}, \text{sk}_{15})$ cnf(theorem_3_7$_{81}$, negated_conjecture)
$\text{incident\_c}(\text{sk}_{17}, \text{sk}_{15})$ cnf(theorem_3_7$_{82}$, negated_conjecture)
$\text{incident\_c}(\text{sk}_{18}, \text{sk}_{15})$ cnf(theorem_3_7$_{83}$, negated_conjecture)
$\neg\, \text{between\_c}(\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18})$ cnf(theorem_3_7$_{84}$, negated_conjecture)

GEO111+1.p Basic property of orderings on linear structures 1
If Q is between P and R wrt. c, then P, Q and R are incident with c and are pairwise distinct
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p, q, r: (\text{between\_c}(c,p,q,r) \Rightarrow (\text{incident\_c}(p,c) \text{ and incident\_c}(q,c) \text{ and incident\_c}(r,c) \text{ and } p \neq q \text{ and } q \neq r \text{ and } p \neq r))$ fof(theorem_3_8$_1$, conjecture)

GEO111-1.p Basic property of orderings on linear structures 1
If Q is between P and R wrt. c, then P, Q and R are incident with c and are pairwise distinct
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
$\text{between\_c}(\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18})$ cnf(theorem_3_8_1$_{77}$, negated_conjecture)
$(\text{incident\_c}(\text{sk}_{16}, \text{sk}_{15}) \text{ and incident\_c}(\text{sk}_{17}, \text{sk}_{15}) \text{ and incident\_c}(\text{sk}_{18}, \text{sk}_{15})) \Rightarrow (\text{sk}_{16} = \text{sk}_{17} \text{ or } \text{sk}_{17} = \text{sk}_{18} \text{ or } \text{sk}_{16} = \text{sk}_{18})$ cnf(theorem_3_8_1$_{78}$, negated_conjecture)

GEO112+1.p Basic property of orderings on linear structures 2
If Q is between P and R wrt. c, then Q is between R and P wrt. c
include('Axioms/GEO004+0.ax')

include('Axioms/GEO004+1.ax')
$\forall c, p, q, r$: (between_c(c, p, q, r) $\Rightarrow$ between_c(c, r, q, p)) fof(theorem_3_8$_2$, conjecture)

GEO112-1.p Basic property of orderings on linear structures 2
If Q is between P and R wrt. c, then Q is between R and P wrt. c
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
between_c($\mathrm{sk}_{15}, \mathrm{sk}_{16}, \mathrm{sk}_{17}, \mathrm{sk}_{18}$) cnf(theorem_3_8_2$_{77}$, negated_conjecture)
$\neg$ between_c($\mathrm{sk}_{15}, \mathrm{sk}_{18}, \mathrm{sk}_{17}, \mathrm{sk}_{16}$) cnf(theorem_3_8_2$_{78}$, negated_conjecture)

GEO113+1.p Basic property of orderings on linear structures 3
If c is open and Q is between P and R wrt. c, then P is not between Q and R wrt. c
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p, q, r$: ((open(c) and between_c(c, p, q, r)) $\Rightarrow$ $\neg$ between_c(c, q, p, r)) fof(theorem_3_8$_3$, conjecture)

GEO113-1.p Basic property of orderings on linear structures 3
If c is open and Q is between P and R wrt. c, then P is not between Q and R wrt. c
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
open(sk_{15}) cnf(theorem_3_8_3$_{77}$, negated_conjecture)
between_c($\mathrm{sk}_{15}, \mathrm{sk}_{16}, \mathrm{sk}_{17}, \mathrm{sk}_{18}$) cnf(theorem_3_8_3$_{78}$, negated_conjecture)
between_c($\mathrm{sk}_{15}, \mathrm{sk}_{17}, \mathrm{sk}_{16}, \mathrm{sk}_{18}$) cnf(theorem_3_8_3$_{79}$, negated_conjecture)

GEO114+1.p Basic property of orderings on linear structures 4
If P, Q and R are distinct and on c then one of the points is between the others wrt. c.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p, q, r$: ((incident_c(p, c) and incident_c(q, c) and incident_c(r, c) and $p \neq q$ and $q \neq r$ and $p \neq r$) $\Rightarrow$ (between_c(c, p, q, r) o

GEO114-1.p Basic property of orderings on linear structures 4
If P, Q and R are distinct and on c then one of the points is between the others wrt. c.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
incident_c($\mathrm{sk}_{16}, \mathrm{sk}_{15}$) cnf(theorem_3_8_4$_{77}$, negated_conjecture)
incident_c($\mathrm{sk}_{17}, \mathrm{sk}_{15}$) cnf(theorem_3_8_4$_{78}$, negated_conjecture)
incident_c($\mathrm{sk}_{18}, \mathrm{sk}_{15}$) cnf(theorem_3_8_4$_{79}$, negated_conjecture)
$\mathrm{sk}_{16} \neq \mathrm{sk}_{17}$ cnf(theorem_3_8_4$_{80}$, negated_conjecture)
$\mathrm{sk}_{17} \neq \mathrm{sk}_{18}$ cnf(theorem_3_8_4$_{81}$, negated_conjecture)
$\mathrm{sk}_{16} \neq \mathrm{sk}_{18}$ cnf(theorem_3_8_4$_{82}$, negated_conjecture)
$\neg$ between_c($\mathrm{sk}_{15}, \mathrm{sk}_{16}, \mathrm{sk}_{17}, \mathrm{sk}_{18}$) cnf(theorem_3_8_4$_{83}$, negated_conjecture)
$\neg$ between_c($\mathrm{sk}_{15}, \mathrm{sk}_{17}, \mathrm{sk}_{16}, \mathrm{sk}_{18}$) cnf(theorem_3_8_4$_{84}$, negated_conjecture)
$\neg$ between_c($\mathrm{sk}_{15}, \mathrm{sk}_{16}, \mathrm{sk}_{18}, \mathrm{sk}_{17}$) cnf(theorem_3_8_4$_{85}$, negated_conjecture)

GEO115+1.p Basic property of orderings on linear structures 5
If Q is between P and R wrt. c and Q' another point distinct from Q and lying on c then Q is either between P and Q' or between Q' and R wrt. c.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p, q, r, \mathrm{qpp}$: ((between_c($c, p, q, r$) and incident_c(qpp, c) and $q \neq \mathrm{qpp}$) $\Rightarrow$ (between_c(c, p, q, qpp) or between_c(c, qpp, q, r)

GEO115-1.p Basic property of orderings on linear structures 5
If Q is between P and R wrt. c and Q' another point distinct from Q and lying on c then Q is either between P and Q' or between Q' and R wrt. c.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
between_c($\mathrm{sk}_{15}, \mathrm{sk}_{16}, \mathrm{sk}_{17}, \mathrm{sk}_{18}$) cnf(theorem_3_8_5$_{77}$, negated_conjecture)
incident_c($\mathrm{sk}_{19}, \mathrm{sk}_{15}$) cnf(theorem_3_8_5$_{78}$, negated_conjecture)
$\mathrm{sk}_{17} \neq \mathrm{sk}_{19}$ cnf(theorem_3_8_5$_{79}$, negated_conjecture)
$\neg$ between_c($\mathrm{sk}_{15}, \mathrm{sk}_{16}, \mathrm{sk}_{17}, \mathrm{sk}_{19}$) cnf(theorem_3_8_5$_{80}$, negated_conjecture)
$\neg$ between_c($\mathrm{sk}_{15}, \mathrm{sk}_{19}, \mathrm{sk}_{17}, \mathrm{sk}_{18}$) cnf(theorem_3_8_5$_{81}$, negated_conjecture)

GEO116+1.p Open curve betweenness property for three points

If P, Q and R are points on an open curve c then Q is not between P and R wrt. c, iff P is between R and Q wrt. c or R is between Q and P wrt. c or at least two of the points are identical.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
$\forall c, p, q, r$: ((open(c) and incident_c(p, c) and incident_c(q, c) and incident_c(r, c)) $\Rightarrow$ ($\neg$ between_c(c, p, q, r) $\iff$ (between_c(c, r, p, q) or between_c(c, q, r, p) or $r = q$ or $r = p$ or $p = q$))) fof(corolary_3$_9$, conjecture)

GEO116-1.p Open curve betweenness property for three points
If P, Q and R are points on an open curve c then Q is not between P and R wrt. c, iff P is between R and Q wrt. c or R is between Q and P wrt. c or at least two of the points are identical.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
open(sk_{15}) cnf(corolary_3_9$_{77}$, negated_conjecture)
incident_c($\text{sk}_{16}, \text{sk}_{15}$) cnf(corolary_3_9$_{78}$, negated_conjecture)
incident_c($\text{sk}_{17}, \text{sk}_{15}$) cnf(corolary_3_9$_{79}$, negated_conjecture)
incident_c($\text{sk}_{18}, \text{sk}_{15}$) cnf(corolary_3_9$_{80}$, negated_conjecture)
between_c($\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18}$) $\Rightarrow$ (between_c($\text{sk}_{15}, \text{sk}_{18}, \text{sk}_{16}, \text{sk}_{17}$) or between_c($\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{18}, \text{sk}_{16}$) or $\text{sk}_{18} = \text{sk}_{17}$ or $\text{sk}_{18} = \text{sk}_{16}$ or $\text{sk}_{16} = \text{sk}_{17}$) cnf(corolary_3_9$_{81}$, negated_conjecture)
between_c($\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18}$) $\Rightarrow$ between_c($\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18}$) cnf(corolary_3_9$_{82}$, negated_conjecture)
between_c($\text{sk}_{15}, \text{sk}_{18}, \text{sk}_{16}, \text{sk}_{17}$) $\Rightarrow$ (between_c($\text{sk}_{15}, \text{sk}_{18}, \text{sk}_{16}, \text{sk}_{17}$) or between_c($\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{18}, \text{sk}_{16}$) or $\text{sk}_{18} = \text{sk}_{17}$ or $\text{sk}_{18} = \text{sk}_{16}$ or $\text{sk}_{16} = \text{sk}_{17}$) cnf(corolary_3_9$_{83}$, negated_conjecture)
between_c($\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{18}, \text{sk}_{16}$) $\Rightarrow$ (between_c($\text{sk}_{15}, \text{sk}_{18}, \text{sk}_{16}, \text{sk}_{17}$) or between_c($\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{18}, \text{sk}_{16}$) or $\text{sk}_{18} = \text{sk}_{17}$ or $\text{sk}_{18} = \text{sk}_{16}$ or $\text{sk}_{16} = \text{sk}_{17}$) cnf(corolary_3_9$_{84}$, negated_conjecture)
$\text{sk}_{18} = \text{sk}_{17}$ $\Rightarrow$ (between_c($\text{sk}_{15}, \text{sk}_{18}, \text{sk}_{16}, \text{sk}_{17}$) or between_c($\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{18}, \text{sk}_{16}$) or $\text{sk}_{18} = \text{sk}_{17}$ or $\text{sk}_{18} = \text{sk}_{16}$ or $\text{sk}_{16} = \text{sk}_{17}$) cnf(corolary_3_9$_{85}$, negated_conjecture)
$\text{sk}_{18} = \text{sk}_{16}$ $\Rightarrow$ (between_c($\text{sk}_{15}, \text{sk}_{18}, \text{sk}_{16}, \text{sk}_{17}$) or between_c($\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{18}, \text{sk}_{16}$) or $\text{sk}_{18} = \text{sk}_{17}$ or $\text{sk}_{18} = \text{sk}_{16}$ or $\text{sk}_{16} = \text{sk}_{17}$) cnf(corolary_3_9$_{86}$, negated_conjecture)
$\text{sk}_{16} = \text{sk}_{17}$ $\Rightarrow$ (between_c($\text{sk}_{15}, \text{sk}_{18}, \text{sk}_{16}, \text{sk}_{17}$) or between_c($\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{18}, \text{sk}_{16}$) or $\text{sk}_{18} = \text{sk}_{17}$ or $\text{sk}_{18} = \text{sk}_{16}$ or $\text{sk}_{16} = \text{sk}_{17}$) cnf(corolary_3_9$_{87}$, negated_conjecture)
between_c($\text{sk}_{15}, \text{sk}_{18}, \text{sk}_{16}, \text{sk}_{17}$) $\Rightarrow$ between_c($\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18}$) cnf(corolary_3_9$_{88}$, negated_conjecture)
between_c($\text{sk}_{15}, \text{sk}_{17}, \text{sk}_{18}, \text{sk}_{16}$) $\Rightarrow$ between_c($\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18}$) cnf(corolary_3_9$_{89}$, negated_conjecture)
$\text{sk}_{18} = \text{sk}_{17}$ $\Rightarrow$ between_c($\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18}$) cnf(corolary_3_9$_{90}$, negated_conjecture)
$\text{sk}_{18} = \text{sk}_{16}$ $\Rightarrow$ between_c($\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18}$) cnf(corolary_3_9$_{91}$, negated_conjecture)
$\text{sk}_{16} = \text{sk}_{17}$ $\Rightarrow$ between_c($\text{sk}_{15}, \text{sk}_{16}, \text{sk}_{17}, \text{sk}_{18}$) cnf(corolary_3_9$_{92}$, negated_conjecture)

GEO117+1.p Precedence on oriented curves is irreflexive
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p$: $\neg$ ordered_by(o, p, p) fof(theorem_4$_4$, conjecture)

GEO117-1.p Precedence on oriented curves is irreflexive
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{26}$) cnf(theorem_4_4$_{133}$, negated_conjecture)

GEO118+1.p Precedence on oriented curves is asymmetric
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q$: (ordered_by(o, p, q) $\Rightarrow$ $\neg$ ordered_by(o, q, p)) fof(theorem_4$_5$, conjecture)

GEO118-1.p Precedence on oriented curves is asymmetric
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{27}$) cnf(theorem_4_5$_{133}$, negated_conjecture)
ordered_by($\text{sk}_{25}, \text{sk}_{27}, \text{sk}_{26}$) cnf(theorem_4_5$_{134}$, negated_conjecture)

GEO119+1.p Oriented curve starting point is endpoint of underlying curve
include('Axioms/GEO004+0.ax')

include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p$: (start_point(p, o) $\Rightarrow$ end_point$(p,$ underlying_curve$(o)))$ fof(theorem_4_6$_1$, conjecture)

GEO119-1.p Oriented curve starting point is endpoint of underlying curve
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
start_point(sk$_{26}$, sk$_{25}$) cnf(theorem_4_6_1$_{133}$, negated_conjecture)
$\neg$ end_point(sk$_{26}$, underlying_curve(sk$_{25}$)) cnf(theorem_4_6_1$_{134}$, negated_conjecture)

GEO120+1.p Oriented curve finishing point is endpoint of underlying curve
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p$: (finish_point(p, o) $\Rightarrow$ end_point$(p,$ underlying_curve$(o)))$ fof(theorem_4_6$_2$, conjecture)

GEO120-1.p Oriented curve finishing point is endpoint of underlying curve
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
finish_point(sk$_{26}$, sk$_{25}$) cnf(theorem_4_6_2$_{133}$, negated_conjecture)
$\neg$ end_point(sk$_{26}$, underlying_curve(sk$_{25}$)) cnf(theorem_4_6_2$_{134}$, negated_conjecture)

GEO121+1.p Endpoints are either starting or finishing points
Every endpoint of the underlying curve of an oriented curve is either a starting point or finishing point of the oriented curve.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p$: (end_point$(p,$ underlying_curve$(o))$ $\Rightarrow$ (start_point(p, o) or finish_point$(p, o)))$ fof(theorem_4$_7$, conjecture)

GEO121-1.p Endpoints are either starting or finishing points
Every endpoint of the underlying curve of an oriented curve is either a starting point or finishing point of the oriented curve.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
end_point(sk$_{26}$, underlying_curve(sk$_{25}$)) cnf(theorem_4_7$_{133}$, negated_conjecture)
$\neg$ start_point(sk$_{26}$, sk$_{25}$) cnf(theorem_4_7$_{134}$, negated_conjecture)
$\neg$ finish_point(sk$_{26}$, sk$_{25}$) cnf(theorem_4_7$_{135}$, negated_conjecture)

GEO122+1.p Every curve has a finishing point
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o$: $\exists p$: finish_point(p, o) fof(corollary_4$_8$, conjecture)

GEO122-1.p Every curve has a finishing point
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
$\neg$ finish_point$(a,$ sk$_{25})$ cnf(corollary_4_8$_{133}$, negated_conjecture)

GEO123+1.p Every oriented curve orders all points on it
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q$: ((incident_o(p, o) and incident_o$(q, o))$ $\Rightarrow$ (ordered_by(o, p, q) or $p = q$ or ordered_by$(o, q, p)))$ fof(theorem_4$_9$, con

GEO123-1.p Every oriented curve orders all points on it
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
incident_o(sk$_{26}$, sk$_{25}$) cnf(theorem_4_9$_{133}$, negated_conjecture)

incident_o(sk_{27}, sk_{25}) cnf(theorem_4_9$_{134}$, negated_conjecture)
¬ ordered_by($sk_{25}, sk_{26}, sk_{27}$) cnf(theorem_4_9$_{135}$, negated_conjecture)
$sk_{26} \neq sk_{27}$ cnf(theorem_4_9$_{136}$, negated_conjecture)
¬ ordered_by($sk_{25}, sk_{27}, sk_{26}$) cnf(theorem_4_9$_{137}$, negated_conjecture)

GEO124+1.p Every oriented curve has at most one starting point
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q$: ((start_point(p, o) and start_point(q, o)) $\Rightarrow$ $p = q$) fof(corollary_4_10$_1$, conjecture)

GEO124-1.p Every oriented curve has at most one starting point
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
start_point(sk_{26}, sk_{25}) cnf(corollary_4_10_1$_{133}$, negated_conjecture)
start_point(sk_{27}, sk_{25}) cnf(corollary_4_10_1$_{134}$, negated_conjecture)
$sk_{26} \neq sk_{27}$ cnf(corollary_4_10_1$_{135}$, negated_conjecture)

GEO125+1.p Every oriented curve has at most one finishing point
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q$: ((finish_point(p, o) and finish_point(q, o)) $\Rightarrow$ $p = q$) fof(corollary_4_10$_2$, conjecture)

GEO125-1.p Every oriented curve has at most one finishing point
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
finish_point(sk_{26}, sk_{25}) cnf(corollary_4_10_2$_{133}$, negated_conjecture)
finish_point(sk_{27}, sk_{25}) cnf(corollary_4_10_2$_{134}$, negated_conjecture)
$sk_{26} \neq sk_{27}$ cnf(corollary_4_10_2$_{135}$, negated_conjecture)

GEO126+1.p Every oriented curve orders some points
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o$: $\exists p, q$: (ordered_by(o, p, q) and $p \neq q$) fof(theorem_4$_{11}$, conjecture)

GEO126-1.p Every oriented curve orders some points
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by(sk_{25}, a, b) $\Rightarrow$ $a = b$ cnf(theorem_4_11$_{133}$, negated_conjecture)

GEO127+1.p Incidence on oriented curves can be defined using precedence
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p$: (incident_o(p, o) $\iff$ $\exists q$: (ordered_by(o, p, q) or ordered_by(o, q, p))) fof(theorem_4$_{12}$, conjecture)

GEO127-1.p Incidence on oriented curves can be defined using precedence
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
incident_o(sk_{26}, sk_{25}) or ordered_by($sk_{25}, sk_{26}, sk_{27}$) or ordered_by($sk_{25}, sk_{27}, sk_{26}$) cnf(theorem_4_12$_{133}$, negated_conjectu
incident_o(sk_{26}, sk_{25}) $\Rightarrow$ incident_o(sk_{26}, sk_{25}) cnf(theorem_4_12$_{134}$, negated_conjecture)
ordered_by(sk_{25}, sk_{26}, a) $\Rightarrow$ (ordered_by($sk_{25}, sk_{26}, sk_{27}$) or ordered_by($sk_{25}, sk_{27}, sk_{26}$)) cnf(theorem_4_12$_{135}$, negated_co
ordered_by(sk_{25}, a, sk_{26}) $\Rightarrow$ (ordered_by($sk_{25}, sk_{26}, sk_{27}$) or ordered_by($sk_{25}, sk_{27}, sk_{26}$)) cnf(theorem_4_12$_{136}$, negated_co
ordered_by(sk_{25}, sk_{26}, a) $\Rightarrow$ ¬ incident_o(sk_{26}, sk_{25}) cnf(theorem_4_12$_{137}$, negated_conjecture)
ordered_by(sk_{25}, a, sk_{26}) $\Rightarrow$ ¬ incident_o(sk_{26}, sk_{25}) cnf(theorem_4_12$_{138}$, negated_conjecture)

GEO128+1.p Precedence of three points, of whoich two are ordered
If P precedes Q with respect to o, then any point R on o precedes Q or is preceded by P.
include('Axioms/GEO004+0.ax')

include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o,p,q$: (ordered_by(o,p,q) $\Rightarrow$ $\forall r$: (incident_o(r,o) $\Rightarrow$ (ordered_by(o,r,q) or ordered_by(o,p,r)))) fof(theorem_4$_{13}$, conje

GEO128-1.p Precedence of three points, of whoich two are ordered
If P precedes Q with respect to o, then any point R on o precedes Q or is preceded by P.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{27}$) cnf(theorem_4_13$_{133}$, negated_conjecture)
incident_o($\mathrm{sk}_{28}, \mathrm{sk}_{25}$) cnf(theorem_4_13$_{134}$, negated_conjecture)
$\neg$ ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{28}, \mathrm{sk}_{27}$) cnf(theorem_4_13$_{135}$, negated_conjecture)
$\neg$ ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{28}$) cnf(theorem_4_13$_{136}$, negated_conjecture)

GEO129+1.p Precedence on an oriented curve is a transitive relation
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o,p,q,r$: ((ordered_by(o,p,q) and ordered_by(o,q,r)) $\Rightarrow$ ordered_by(o,p,r)) fof(theorem_4$_{14}$, conjecture)

GEO129-1.p Precedence on an oriented curve is a transitive relation
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{27}$) cnf(theorem_4_14$_{133}$, negated_conjecture)
ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{27}, \mathrm{sk}_{28}$) cnf(theorem_4_14$_{134}$, negated_conjecture)
$\neg$ ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{28}$) cnf(theorem_4_14$_{135}$, negated_conjecture)

GEO130+1.p Betweenness and precedence for three points
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o,p,q,r$: (between(o,p,q,r) $\Rightarrow$ (ordered_by(o,p,q) $\iff$ ordered_by(o,q,r))) fof(theorem_4$_{15}$, conjecture)

GEO130-1.p Betweenness and precedence for three points
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
between($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{27}, \mathrm{sk}_{28}$) cnf(theorem_4_15$_{133}$, negated_conjecture)
ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{27}$) or ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{27}, \mathrm{sk}_{28}$) cnf(theorem_4_15$_{134}$, negated_conjecture)
ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{27}$) $\Rightarrow$ ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{27}$) cnf(theorem_4_15$_{135}$, negated_conjecture)
ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{27}, \mathrm{sk}_{28}$) $\Rightarrow$ ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{27}, \mathrm{sk}_{28}$) cnf(theorem_4_15$_{136}$, negated_conjecture)
ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{27}, \mathrm{sk}_{28}$) $\Rightarrow$ $\neg$ ordered_by($\mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{27}$) cnf(theorem_4_15$_{137}$, negated_conjecture)

GEO131+1.p Betweenness and precedence for three points, corollary
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall p,q,r,o$: (between(o,p,q,r) $\Rightarrow$ (ordered_by(o,p,q) $\iff$ ordered_by(o,p,r))) fof(corollary_4$_{16}$, conjecture)

GEO131-1.p Betweenness and precedence for three points, corollary
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
between($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{26}, \mathrm{sk}_{27}$) cnf(corollary_4_16$_{133}$, negated_conjecture)
ordered_by($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{26}$) or ordered_by($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{27}$) cnf(corollary_4_16$_{134}$, negated_conjecture)
ordered_by($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{26}$) $\Rightarrow$ ordered_by($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{26}$) cnf(corollary_4_16$_{135}$, negated_conjecture)
ordered_by($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{27}$) $\Rightarrow$ ordered_by($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{27}$) cnf(corollary_4_16$_{136}$, negated_conjecture)
ordered_by($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{27}$) $\Rightarrow$ $\neg$ ordered_by($\mathrm{sk}_{28}, \mathrm{sk}_{25}, \mathrm{sk}_{26}$) cnf(corollary_4_16$_{137}$, negated_conjecture)

GEO132+1.p Betweenness and precedence property 1
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')

$\forall o, p, q, s$: ((ordered_by(o, p, q) and $p \neq s$ and incident_o(s, o)) $\Rightarrow$ (ordered_by(o, p, s) $\iff$ $\neg$ between(o, s, p, q))) fof(theo

GEO132-1.p Betweenness and precedence property 1
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($sk_{25}, sk_{26}, sk_{27}$) cnf(theorem_4_17_$1_{133}$, negated_conjecture)
$sk_{26} \neq sk_{28}$ cnf(theorem_4_17_1_{134}, negated_conjecture)
incident_o(sk_{28}, sk_{25}) cnf(theorem_4_17_1_{135}, negated_conjecture)
between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) $\Rightarrow$ ordered_by($sk_{25}, sk_{26}, sk_{28}$) cnf(theorem_4_17_$1_{136}$, negated_conjecture)
ordered_by($sk_{25}, sk_{26}, sk_{28}$) $\Rightarrow$ ordered_by($sk_{25}, sk_{26}, sk_{28}$) cnf(theorem_4_17_$1_{137}$, negated_conjecture)
between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) $\Rightarrow$ between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) cnf(theorem_4_17_$1_{138}$, negated_conjecture)
ordered_by($sk_{25}, sk_{26}, sk_{28}$) $\Rightarrow$ between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) cnf(theorem_4_17_$1_{139}$, negated_conjecture)

GEO133+1.p Betweenness and precedence property 2
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q, r, s$: ((ordered_by(o, p, q) and $r \neq s$ and incident_o(s, o) and between(o, r, p, q)) $\Rightarrow$ (ordered_by(o, r, s) $\iff$ $\neg$ between(o, s, r, q))) fof(theorem_4_17_2, conjecture)

GEO133-1.p Betweenness and precedence property 2
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($sk_{25}, sk_{26}, sk_{27}$) cnf(theorem_4_17_$2_{133}$, negated_conjecture)
$sk_{28} \neq sk_{29}$ cnf(theorem_4_17_2_{134}, negated_conjecture)
incident_o(sk_{29}, sk_{25}) cnf(theorem_4_17_2_{135}, negated_conjecture)
between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) cnf(theorem_4_17_$2_{136}$, negated_conjecture)
between($sk_{25}, sk_{29}, sk_{28}, sk_{27}$) $\Rightarrow$ ordered_by($sk_{25}, sk_{28}, sk_{29}$) cnf(theorem_4_17_$2_{137}$, negated_conjecture)
ordered_by($sk_{25}, sk_{28}, sk_{29}$) $\Rightarrow$ ordered_by($sk_{25}, sk_{28}, sk_{29}$) cnf(theorem_4_17_$2_{138}$, negated_conjecture)
between($sk_{25}, sk_{29}, sk_{28}, sk_{27}$) $\Rightarrow$ between($sk_{25}, sk_{29}, sk_{28}, sk_{27}$) cnf(theorem_4_17_$2_{139}$, negated_conjecture)
ordered_by($sk_{25}, sk_{28}, sk_{29}$) $\Rightarrow$ between($sk_{25}, sk_{29}, sk_{28}, sk_{27}$) cnf(theorem_4_17_$2_{140}$, negated_conjecture)

GEO134+1.p Betweenness and precedence property 3
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q, r, s$: ((ordered_by(o, p, q) and $p \neq r$ and $\neg$ between(o, r, p, q)) $\Rightarrow$ (ordered_by(o, r, s) $\iff$ between(o, p, r, s))) fof(

GEO134-1.p Betweenness and precedence property 3
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($sk_{25}, sk_{26}, sk_{27}$) cnf(theorem_4_17_$3_{133}$, negated_conjecture)
$sk_{26} \neq sk_{28}$ cnf(theorem_4_17_3_{134}, negated_conjecture)
$\neg$ between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) cnf(theorem_4_17_$3_{135}$, negated_conjecture)
ordered_by($sk_{25}, sk_{28}, sk_{29}$) or between($sk_{25}, sk_{26}, sk_{28}, sk_{29}$) cnf(theorem_4_17_$3_{136}$, negated_conjecture)
ordered_by($sk_{25}, sk_{28}, sk_{29}$) $\Rightarrow$ ordered_by($sk_{25}, sk_{28}, sk_{29}$) cnf(theorem_4_17_$3_{137}$, negated_conjecture)
between($sk_{25}, sk_{26}, sk_{28}, sk_{29}$) $\Rightarrow$ between($sk_{25}, sk_{26}, sk_{28}, sk_{29}$) cnf(theorem_4_17_$3_{138}$, negated_conjecture)
between($sk_{25}, sk_{26}, sk_{28}, sk_{29}$) $\Rightarrow$ $\neg$ ordered_by($sk_{25}, sk_{28}, sk_{29}$) cnf(theorem_4_17_$3_{139}$, negated_conjecture)

GEO135+1.p Ordering can be determined by betweenness and incidence
The ordering of any pair of points R and S on an oriented line o can be determined on the basis of a given pair of points P and Q using betweenness and incidence only.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q$: (ordered_by(o, p, q) $\Rightarrow$ $\forall r, s$: (ordered_by(o, r, s) $\iff$ ((incident_o(s, o) and (between(o, r, p, q) or $p = r$) and $r \neq s$ and $\neg$ between(o, s, r, q)) or (between(o, p, r, s) and $\neg$ between(o, q, p, r))))) fof(corollary_4_{18}, conjecture)

GEO136+1.p Underlying curve and one pair of points sufficient for ordering
The underlying curve and one pair of points are sufficient for the ordering of the points on the oriented curve.

include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q$: $(\text{ordered\_by}(o, p, q) \Rightarrow \forall r, s$: $((\text{ordered\_by}(o, r, s) \iff (\text{between}(o, r, p, q)$ and $(\text{between}(o, r, s, q)$ or $\text{between}(o, r, q, s$
$s)))$ or $(\text{between}(o, p, r, s)$ and $(\text{between}(o, p, r, q)$ or $\text{between}(o, p, q, r)$ or $q = r))$ or $(p = r$ and $(\text{between}(o, p, s, q)$ or betwee
$s))))$ fof(theorem_4_{19}, conjecture)

GEO136-1.p Underlying curve and one pair of points sufficient for ordering
The underlying curve and one pair of points are sufficient for the ordering of the points on the oriented curve.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{27})$ cnf($\text{theorem\_4\_19}_{133}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29})$ or $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ cnf($\text{theorem\_4\_19}_{134}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29})$ or $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29}, \text{sk}_{27})$ or $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{27}, \text{sk}_{29})$ or $\text{sk}_{27} = \text{sk}_{29}$ cnf(theorem_
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29}) \Rightarrow \text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29})$ cnf($\text{theorem\_4\_19}_{136}$, negated_conjecture)
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29}, \text{sk}_{27})) \Rightarrow \text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ cnf($\text{theorem\_4\_19}_{137}$,
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29}, \text{sk}_{27})) \Rightarrow (\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29}, \text{sk}_{27})$ or between(sk$_{25}$, sk$_{28}$, sk
$\text{sk}_{29})$ cnf($\text{theorem\_4\_19}_{138}$, negated_conjecture)
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{27}, \text{sk}_{29})) \Rightarrow \text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ cnf($\text{theorem\_4\_19}_{139}$,
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{sk}_{27} = \text{sk}_{29}) \Rightarrow \text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ cnf($\text{theorem\_4\_19}_{140}$, negated_conjectur
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{27}, \text{sk}_{29})) \Rightarrow (\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29}, \text{sk}_{27})$ or between(sk$_{25}$, sk$_{28}$, sk
$\text{sk}_{29})$ cnf($\text{theorem\_4\_19}_{141}$, negated_conjecture)
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{sk}_{27} = \text{sk}_{29}) \Rightarrow (\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29}, \text{sk}_{27})$ or $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{27}, \text{sk}_{29})$ or sk$_{27}$ =
$\text{sk}_{29})$ cnf($\text{theorem\_4\_19}_{142}$, negated_conjecture)
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29}, \text{sk}_{27})) \Rightarrow \neg\,\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29})$ cnf($\text{theorem\_4\_19}_{143}$,
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{27}, \text{sk}_{29})) \Rightarrow \neg\,\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29})$ cnf($\text{theorem\_4\_19}_{144}$,
$(\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ and $\text{sk}_{27} = \text{sk}_{29}) \Rightarrow \neg\,\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{29})$ cnf($\text{theorem\_4\_19}_{145}$, negated_conjecture
$\text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{28}, \text{sk}_{29}) \Rightarrow \neg\,\text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{28}, \text{sk}_{27})$ cnf($\text{theorem\_4\_19}_{146}$, negated_conjecture)
$\text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{28}, \text{sk}_{29}) \Rightarrow \neg\,\text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{27}, \text{sk}_{28})$ cnf($\text{theorem\_4\_19}_{147}$, negated_conjecture)
$\text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{28}, \text{sk}_{29}) \Rightarrow \text{sk}_{27} \neq \text{sk}_{28}$ cnf($\text{theorem\_4\_19}_{148}$, negated_conjecture)
$\text{sk}_{26} = \text{sk}_{28} \Rightarrow \neg\,\text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{29}, \text{sk}_{27})$ cnf($\text{theorem\_4\_19}_{149}$, negated_conjecture)
$\text{sk}_{26} = \text{sk}_{28} \Rightarrow \neg\,\text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{27}, \text{sk}_{29})$ cnf($\text{theorem\_4\_19}_{150}$, negated_conjecture)
$\text{sk}_{26} = \text{sk}_{28} \Rightarrow \text{sk}_{27} \neq \text{sk}_{29}$ cnf($\text{theorem\_4\_19}_{151}$, negated_conjecture)

GEO137+1.p Identical oriented lines
Oriented lines consisting of the same points and ordering one pair of points in the same way, are identical.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o_1, o_2$: $((\forall p$: $(\text{incident\_o}(p, o_1) \iff \text{incident\_o}(p, o_2))$ and $\exists p, q$: $(\text{ordered\_by}(o_1, p, q)$ and $\text{ordered\_by}(o_2, p, q))) \Rightarrow$
$o_1 = o_2)$ fof(theorem_4_{20}, conjecture)

GEO137-1.p Identical oriented lines
Oriented lines consisting of the same points and ordering one pair of points in the same way, are identical.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
$\text{incident\_o}(a, \text{sk}_{25}) \Rightarrow \text{incident\_o}(a, \text{sk}_{26})$ cnf($\text{theorem\_4\_20}_{133}$, negated_conjecture)
$\text{incident\_o}(a, \text{sk}_{26}) \Rightarrow \text{incident\_o}(a, \text{sk}_{25})$ cnf($\text{theorem\_4\_20}_{134}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{27}, \text{sk}_{28})$ cnf($\text{theorem\_4\_20}_{135}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{26}, \text{sk}_{27}, \text{sk}_{28})$ cnf($\text{theorem\_4\_20}_{136}$, negated_conjecture)
$\text{sk}_{25} \neq \text{sk}_{26}$ cnf($\text{theorem\_4\_20}_{137}$, negated_conjecture)

GEO138+1.p Curve and ordered points determine oriented curve
A curve and a ordered pair of points uniquely determine an oriented curve.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o_1, o_2$: $((\text{underlying\_curve}(o_1) = \text{underlying\_curve}(o_2)$ and $\exists p, q$: $(\text{ordered\_by}(o_1, p, q)$ and $\text{ordered\_by}(o_2, p, q))) \Rightarrow$
$o_1 = o_2)$ fof(corollary_4_{21}, conjecture)

GEO138-1.p Curve and ordered points determine oriented curve
A curve and a ordered pair of points uniquely determine an oriented curve.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
$\text{underlying\_curve}(\text{sk}_{25}) = \text{underlying\_curve}(\text{sk}_{26})$ cnf(corollary_4_21$_{133}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{27}, \text{sk}_{28})$ cnf(corollary_4_21$_{134}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{26}, \text{sk}_{27}, \text{sk}_{28})$ cnf(corollary_4_21$_{135}$, negated_conjecture)
$\text{sk}_{25} \neq \text{sk}_{26}$ cnf(corollary_4_21$_{136}$, negated_conjecture)

GEO139+1.p Oppositely oriented curve exists
For every oriented curve there is an oppositely oriented curve with the same underlying curve.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o\colon \exists \text{opp}\colon (\text{underlying\_curve}(o) = \text{underlying\_curve}(\text{opp}) \text{ and } \forall p, q\colon (\text{ordered\_by}(o, p, q) \Rightarrow \text{ordered\_by}(\text{opp}, q, p)))$ fof(theo

GEO139-1.p Oppositely oriented curve exists
For every oriented curve there is an oppositely oriented curve with the same underlying curve.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
$\text{underlying\_curve}(\text{sk}_{25}) = \text{underlying\_curve}(a) \Rightarrow \text{ordered\_by}(\text{sk}_{25}, \text{sk}_{26}(a), \text{sk}_{27}(a))$ cnf(theorem_4_22$_{133}$, negated_conject
$\text{underlying\_curve}(\text{sk}_{25}) = \text{underlying\_curve}(a) \Rightarrow \neg\, \text{ordered\_by}(a, \text{sk}_{27}(a), \text{sk}_{26}(a))$ cnf(theorem_4_22$_{134}$, negated_conjectu

GEO140+1.p Unique oppositely oriented curve 1
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q, r\colon (\text{ordered\_by}(o, p, q) \Rightarrow (\text{ordered\_by}(o, r, p) \iff \text{between}(o, r, p, q)))$ fof(theorem_4_23$_1$, conjecture)

GEO140-1.p Unique oppositely oriented curve 1
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{27})$ cnf(theorem_4_23_1$_{133}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}) \text{ or } \text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ cnf(theorem_4_23_1$_{134}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}) \Rightarrow \text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26})$ cnf(theorem_4_23_1$_{135}$, negated_conjecture)
$\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27}) \Rightarrow \text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27})$ cnf(theorem_4_23_1$_{136}$, negated_conjecture)
$\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27}) \Rightarrow \neg\, \text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26})$ cnf(theorem_4_23_1$_{137}$, negated_conjecture)

GEO141+1.p Unique oppositely oriented curve 2
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q, r\colon (\text{ordered\_by}(o, p, q) \Rightarrow (\text{ordered\_by}(o, r, q) \iff (\text{between}(o, r, p, q) \text{ or } \text{between}(o, p, r, q) \text{ or } p = r)))$ fof(theorem

GEO141-1.p Unique oppositely oriented curve 2
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{27})$ cnf(theorem_4_23_2$_{133}$, negated_conjecture)
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{27}) \text{ or } \text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27}) \text{ or } \text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{28}, \text{sk}_{27}) \text{ or } \text{sk}_{26} = \text{sk}_{28}$ cnf(theorem_
$\text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{27}) \Rightarrow \text{ordered\_by}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{27})$ cnf(theorem_4_23_2$_{135}$, negated_conjecture)
$\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27}) \Rightarrow (\text{between}(\text{sk}_{25}, \text{sk}_{28}, \text{sk}_{26}, \text{sk}_{27}) \text{ or } \text{between}(\text{sk}_{25}, \text{sk}_{26}, \text{sk}_{28}, \text{sk}_{27}) \text{ or } \text{sk}_{26} = \text{sk}_{28})$ cnf(theo

between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) $\Rightarrow$ (between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) or between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) or $sk_{26} = sk_{28}$) cnf(theo
$sk_{26} = sk_{28}$ $\Rightarrow$ (between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) or between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) or $sk_{26} = sk_{28}$) cnf(theorem_4_23_2_{138}, neg
between($sk_{25}, sk_{28}, sk_{26}, sk_{27}$) $\Rightarrow$ $\neg$ordered_by($sk_{25}, sk_{28}, sk_{27}$) cnf(theorem_4_23_$2_{139}$, negated_conjecture)
between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) $\Rightarrow$ $\neg$ordered_by($sk_{25}, sk_{28}, sk_{27}$) cnf(theorem_4_23_$2_{140}$, negated_conjecture)
$sk_{26} = sk_{28}$ $\Rightarrow$ $\neg$ordered_by($sk_{25}, sk_{28}, sk_{27}$) cnf(theorem_4_23_$2_{141}$, negated_conjecture)

GEO142+1.p Unique oppositely oriented curve 3
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q, r$: (ordered_by(o, p, q) $\Rightarrow$ (ordered_by(o, p, r) $\iff$ (between(o, p, r, q) or between(o, p, q, r) or $q = r$))) fof(theorem

GEO142-1.p Unique oppositely oriented curve 3
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($sk_{25}, sk_{26}, sk_{27}$) cnf(theorem_4_23_$3_{133}$, negated_conjecture)
ordered_by($sk_{25}, sk_{26}, sk_{28}$) or between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) or $sk_{27} = sk_{28}$ cnf(theorem_
ordered_by($sk_{25}, sk_{26}, sk_{28}$) $\Rightarrow$ ordered_by($sk_{25}, sk_{26}, sk_{28}$) cnf(theorem_4_23_$3_{135}$, negated_conjecture)
between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) $\Rightarrow$ (between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) or $sk_{27} = sk_{28}$) cnf(theo
between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) $\Rightarrow$ (between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) or $sk_{27} = sk_{28}$) cnf(theo
$sk_{27} = sk_{28}$ $\Rightarrow$ (between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) or $sk_{27} = sk_{28}$) cnf(theorem_4_23_3_{138}, neg
between($sk_{25}, sk_{26}, sk_{28}, sk_{27}$) $\Rightarrow$ $\neg$ordered_by($sk_{25}, sk_{26}, sk_{28}$) cnf(theorem_4_23_$3_{139}$, negated_conjecture)
between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) $\Rightarrow$ $\neg$ordered_by($sk_{25}, sk_{26}, sk_{28}$) cnf(theorem_4_23_$3_{140}$, negated_conjecture)
$sk_{27} = sk_{28}$ $\Rightarrow$ $\neg$ordered_by($sk_{25}, sk_{26}, sk_{28}$) cnf(theorem_4_23_$3_{141}$, negated_conjecture)

GEO143+1.p Unique oppositely oriented curve 4
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q, r$: (ordered_by(o, p, q) $\Rightarrow$ (ordered_by(o, q, r) $\iff$ between(o, p, q, r))) fof(theorem_4_23_4, conjecture)

GEO143-1.p Unique oppositely oriented curve 4
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
ordered_by($sk_{25}, sk_{26}, sk_{27}$) cnf(theorem_4_23_$4_{133}$, negated_conjecture)
ordered_by($sk_{25}, sk_{27}, sk_{28}$) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) cnf(theorem_4_23_$4_{134}$, negated_conjecture)
ordered_by($sk_{25}, sk_{27}, sk_{28}$) $\Rightarrow$ ordered_by($sk_{25}, sk_{27}, sk_{28}$) cnf(theorem_4_23_$4_{135}$, negated_conjecture)
between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) $\Rightarrow$ between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) cnf(theorem_4_23_$4_{136}$, negated_conjecture)
between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) $\Rightarrow$ $\neg$ordered_by($sk_{25}, sk_{27}, sk_{28}$) cnf(theorem_4_23_$4_{137}$, negated_conjecture)

GEO144+1.p Unique oppositely oriented curve 5
For every oriented line there is exactly one uniquely determined oriented line with the same underlying curve that orders the points in the opposite way.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
$\forall o, p, q, r, s$: ((ordered_by(o, p, q) and $p \neq r$ and $p \neq s$ and $q \neq s$ and $q \neq r$) $\Rightarrow$ (ordered_by(o, r, s) $\iff$ ((between(o, r, p, q)

GEO145+1.p Starting point and precedence
If R is the starting point of o, then P precedes Q wrt. o, iff P is identical with R and Q is on o but different from R or P is between R and Q on o.
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')

include('Axioms/GEO004+2.ax')
$\forall o, r$: (start_point(r, o) $\Rightarrow$ $\forall p, q$: (ordered_by(o, p, q) $\iff$ (($p = r$ and $q \neq r$ and incident_o(q, o)) or between(o, r, p, q))))

GEO145-1.p Starting point and precedence
If R is the starting point of o, then P precedes Q wrt. o, iff P is identical with R and Q is on o but different from R or P is between R and Q on o.
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
start_point(sk_{26}, sk_{25})   cnf(theorem_4_24$_{133}$, negated_conjecture)
ordered_by($sk_{25}, sk_{27}, sk_{28}$) or $sk_{27} = sk_{26}$ or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$)   cnf(theorem_4_24$_{134}$, negated_conjecture)
$sk_{28} = sk_{26}$ $\Rightarrow$ (ordered_by($sk_{25}, sk_{27}, sk_{28}$) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$))   cnf(theorem_4_24$_{135}$, negated_conjecture)
ordered_by($sk_{25}, sk_{27}, sk_{28}$) or incident_o($sk_{28}, sk_{25}$) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$)   cnf(theorem_4_24$_{136}$, negated_conjec
ordered_by($sk_{25}, sk_{27}, sk_{28}$) $\Rightarrow$ ordered_by($sk_{25}, sk_{27}, sk_{28}$)   cnf(theorem_4_24$_{137}$, negated_conjecture)
($sk_{27} = sk_{26}$ and incident_o(sk_{28}, sk_{25})) $\Rightarrow$ ($sk_{28} = sk_{26}$ or $sk_{27} = sk_{26}$ or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$))   cnf(theorem_4_2
($sk_{27} = sk_{26}$ and incident_o(sk_{28}, sk_{25}) and $sk_{28} = sk_{26}$) $\Rightarrow$ ($sk_{28} = sk_{26}$ or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$))   cnf(theorem_4
($sk_{27} = sk_{26}$ and incident_o(sk_{28}, sk_{25})) $\Rightarrow$ ($sk_{28} = sk_{26}$ or incident_o(sk_{28}, sk_{25}) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$))   cnf(th
between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) $\Rightarrow$ ($sk_{27} = sk_{26}$ or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$))   cnf(theorem_4_24$_{141}$, negated_conjecture)
(between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) and $sk_{28} = sk_{26}$) $\Rightarrow$ between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$)   cnf(theorem_4_24$_{142}$, negated_conjectur
between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) $\Rightarrow$ (incident_o(sk_{28}, sk_{25}) or between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$))   cnf(theorem_4_24$_{143}$, negated_c
($sk_{27} = sk_{26}$ and incident_o(sk_{28}, sk_{25}) and ordered_by($sk_{25}, sk_{27}, sk_{28}$)) $\Rightarrow$ $sk_{28} = sk_{26}$ cnf(theorem_4_24$_{144}$, negated_con
between($sk_{25}, sk_{26}, sk_{27}, sk_{28}$) $\Rightarrow$ $\neg$ ordered_by($sk_{25}, sk_{27}, sk_{28}$)   cnf(theorem_4_24$_{145}$, negated_conjecture)

GEO146+1.p Symmetry of connect
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
include('Axioms/GEO004+3.ax')
$\forall x, y, p$: (connect(x, y, p) $\iff$ connect(y, x, p))   fof(t_{12}, conjecture)

GEO146-1.p Symmetry of connect
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
include('Axioms/GEO004-3.ax')
connect($sk_{27}, sk_{28}, sk_{29}$) or connect($sk_{28}, sk_{27}, sk_{29}$)   cnf(t12$_{156}$, negated_conjecture)
connect($sk_{27}, sk_{28}, sk_{29}$) $\Rightarrow$ connect($sk_{27}, sk_{28}, sk_{29}$)   cnf(t12$_{157}$, negated_conjecture)
connect($sk_{28}, sk_{27}, sk_{29}$) $\Rightarrow$ connect($sk_{28}, sk_{27}, sk_{29}$)   cnf(t12$_{158}$, negated_conjecture)
connect($sk_{28}, sk_{27}, sk_{29}$) $\Rightarrow$ $\neg$ connect($sk_{27}, sk_{28}, sk_{29}$)   cnf(t12$_{159}$, negated_conjecture)

GEO147+1.p Meeting is possible only if there is a common position
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
include('Axioms/GEO004+3.ax')
$\forall p, x, y$: (connect(x, y, p) $\Rightarrow$ (incident_o(p, trajectory_of(x)) and incident_o(p, trajectory_of(y))))   fof(t_{13}, conjecture)

GEO147-1.p Meeting is possible only if there is a common position
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
include('Axioms/GEO004-3.ax')
connect($sk_{28}, sk_{29}, sk_{27}$)   cnf(t13$_{156}$, negated_conjecture)
incident_o(sk_{27}, trajectory_of(sk_{28})) $\Rightarrow$ $\neg$ incident_o(sk_{27}, trajectory_of(sk_{29}))   cnf(t13$_{157}$, negated_conjecture)

GEO148+1.p No meeting if someone has already passed
A point can only be a meeting point of two moving objects if it is not the case that one object already passed through it when the other object was still moving towards it
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
include('Axioms/GEO004+3.ax')

$\forall p, x, y$: (connect$(x, y, p) \Rightarrow \forall q_1, q_2$: ((ordered_by(trajectory_of$(y), q_2, p)$ and ordered_by(trajectory_of$(x), p, q_1)) \Rightarrow$ $\neg$ once(at_the_same_time(at(x, q_1), at(y, q_2)))))) fof(t_{14}, conjecture)

GEO148-1.p No meeting if someone has already passed
A point can only be a meeting point of two moving objects if it is not the case that one object already passed through it when the other object was still moving towards it
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
include('Axioms/GEO004-3.ax')
connect($\mathrm{sk}_{28}, \mathrm{sk}_{29}, \mathrm{sk}_{27}$) cnf($\mathrm{t14}_{156}$, negated_conjecture)
ordered_by(trajectory_of(sk_{29}), $\mathrm{sk}_{31}, \mathrm{sk}_{27}$) cnf($\mathrm{t14}_{157}$, negated_conjecture)
ordered_by(trajectory_of(sk_{28}), $\mathrm{sk}_{27}, \mathrm{sk}_{30}$) cnf($\mathrm{t14}_{158}$, negated_conjecture)
once(at_the_same_time(at($\mathrm{sk}_{28}, \mathrm{sk}_{30}$), at($\mathrm{sk}_{29}, \mathrm{sk}_{31}$))) cnf($\mathrm{t14}_{159}$, negated_conjecture)

GEO149+1.p Condition for meeting at two points
Objects can meet at two points only if they are ordered in the same way on both trajectories
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
include('Axioms/GEO004+3.ax')
$\forall p, q, x, y$: ((connect(x, y, p) and connect(x, y, q) and ordered_by(trajectory_of$(x), p, q)) \Rightarrow$ ordered_by(trajectory_of$(y), p, q)$)

GEO149-1.p Condition for meeting at two points
Objects can meet at two points only if they are ordered in the same way on both trajectories
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
include('Axioms/GEO004-3.ax')
connect($\mathrm{sk}_{29}, \mathrm{sk}_{30}, \mathrm{sk}_{27}$) cnf($\mathrm{t15}_{156}$, negated_conjecture)
connect($\mathrm{sk}_{29}, \mathrm{sk}_{30}, \mathrm{sk}_{28}$) cnf($\mathrm{t15}_{157}$, negated_conjecture)
ordered_by(trajectory_of(sk_{29}), $\mathrm{sk}_{27}, \mathrm{sk}_{28}$) cnf($\mathrm{t15}_{158}$, negated_conjecture)
$\neg$ ordered_by(trajectory_of(sk_{30}), $\mathrm{sk}_{27}, \mathrm{sk}_{28}$) cnf($\mathrm{t15}_{159}$, negated_conjecture)

GEO150+1.p Objects cannot be at two places simultaneously
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
include('Axioms/GEO004+3.ax')
$\forall p, q, x$: (once(at_the_same_time(at(x, p), at(x, q))) $\Rightarrow p = q$) fof(t_{16}, conjecture)

GEO150-1.p Objects cannot be at two places simultaneously
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
include('Axioms/GEO004-3.ax')
once(at_the_same_time(at($\mathrm{sk}_{29}, \mathrm{sk}_{27}$), at($\mathrm{sk}_{29}, \mathrm{sk}_{28}$))) cnf($\mathrm{t16}_{156}$, negated_conjecture)
$\mathrm{sk}_{27} \neq \mathrm{sk}_{28}$ cnf($\mathrm{t16}_{157}$, negated_conjecture)

GEO151+1.p Object stays still while one moves
If an object is in a position before and after another object moves, then it stays in this position while the other one moves
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
include('Axioms/GEO004+3.ax')
$\forall p, q_1, q_2, q_3, x, y$: ((once(at_the_same_time(at(x, p), at(y, q_1))) and once(at_the_same_time(at(x, p), at(y, q_3))) and between_o(t
once(at_the_same_time(at(x, p), at(x, q_2))))) fof(t_{17}, conjecture)

GEO151-1.p Object stays still while one moves
If an object is in a position before and after another object moves, then it stays in this position while the other one moves
include('Axioms/GEO004-0.ax')

include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
include('Axioms/GEO004-3.ax')
once(at_the_same_time(at(sk_{31}, sk_{27}), at(sk_{32}, sk_{28}))) cnf($t17_{156}$, negated_conjecture)
once(at_the_same_time(at(sk_{31}, sk_{27}), at(sk_{32}, sk_{30}))) cnf($t17_{157}$, negated_conjecture)
between_o(trajectory_of(sk_{31}), $sk_{28}, sk_{29}, sk_{30}$) cnf($t17_{158}$, negated_conjecture)
¬ once(at_the_same_time(at(sk_{31}, sk_{27}), at(sk_{31}, sk_{29}))) cnf($t17_{159}$, negated_conjecture)

GEO152+1.p Ordered meeting places
If three objects meet in pairs such that the meeting place of x and z precedes that of x and y on the trajectory of x and the meeting place of x and y precedes that of y and z on t(y), then the meet-ing place of y and z does not precede that of x and z on t(z)
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
include('Axioms/GEO004+3.ax')
$\forall p, q, r, x, y, z$: ((connect(x, z, p) and connect(x, y, q) and connect(y, z, r) and ordered_by(trajectory_of(x), p, q) and ordered_b
¬ ordered_by(trajectory_of(z), r, p)) fof(t_{18}, conjecture)

GEO152-1.p Ordered meeting places
If three objects meet in pairs such that the meeting place of x and z precedes that of x and y on the trajectory of x and the meeting place of x and y precedes that of y and z on t(y), then the meet-ing place of y and z does not precede that of x and z on t(z)
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
include('Axioms/GEO004-3.ax')
connect($sk_{30}, sk_{32}, sk_{27}$) cnf($t18_{156}$, negated_conjecture)
connect($sk_{30}, sk_{31}, sk_{28}$) cnf($t18_{157}$, negated_conjecture)
connect($sk_{31}, sk_{32}, sk_{29}$) cnf($t18_{158}$, negated_conjecture)
ordered_by(trajectory_of(sk_{30}), sk_{27}, sk_{28}) cnf($t18_{159}$, negated_conjecture)
ordered_by(trajectory_of(sk_{31}), sk_{28}, sk_{29}) cnf($t18_{160}$, negated_conjecture)
ordered_by(trajectory_of(sk_{32}), sk_{29}, sk_{27}) cnf($t18_{161}$, negated_conjecture)

GEO153-1.p Tarski geometry axioms
include('Axioms/GEO001-0.ax')

GEO153-2.p Tarski geometry axioms
include('Axioms/GEO002-0.ax')

GEO154-1.p Colinearity axioms for the GEO001 geometry axioms
include('Axioms/GEO001-0.ax')
include('Axioms/GEO001-1.ax')

GEO154-2.p Colinearity axioms for the GEO002 geometry axioms
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')

GEO155-1.p Reflection axioms for the GEO002 geometry axioms
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-2.ax')

GEO156-1.p Insertion axioms for the GEO003 geometry axioms
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-3.ax')

GEO157-1.p Hilbert geometry axioms
include('Axioms/GEO003-0.ax')

GEO158+1.p Simple curve axioms
include('Axioms/GEO004+0.ax')

GEO158-1.p Simple curve axioms
include('Axioms/GEO004-0.ax')

GEO159+1.p Betweenness for simple curves
include('Axioms/GEO004+0.ax')

include('Axioms/GEO004+1.ax')

GEO159-1.p Betweenness for simple curves
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')

GEO160+1.p Oriented curves
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')

GEO160-1.p Oriented curves
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')

GEO161+1.p Trajectories
include('Axioms/GEO004+0.ax')
include('Axioms/GEO004+1.ax')
include('Axioms/GEO004+2.ax')
include('Axioms/GEO004+3.ax')

GEO161-1.p Trajectories
include('Axioms/GEO004-0.ax')
include('Axioms/GEO004-1.ax')
include('Axioms/GEO004-2.ax')
include('Axioms/GEO004-3.ax')

GEO162-1.p Hilbert geometry axioms, adapted to respect multi-sortedness
include('Axioms/GEO005-0.ax')

GEO163-1.p Not enough axioms to prove collinearity of a finite set of points
Given a finite set of points such that for all points x, y there is a 3rd (different) point z collinear with x and y. Show that all points in the set are collinear.
$\text{collinear}(x, x, y)$ cnf(two_points_collinear, axiom)
$\text{collinear}(x, y, z) \Rightarrow \text{collinear}(y, x, z)$ cnf(rotate_collinear, axiom)
$\text{collinear}(x, y, z) \Rightarrow \text{collinear}(z, x, y)$ cnf(swap_collinear, axiom)
$(\text{collinear}(y_1, y_2, z) \text{ and } \text{collinear}(x, y_1, y_2)) \Rightarrow (\text{collinear}(x, y_1, z) \text{ or } x = y_2 \text{ or } y_1 = y_2 \text{ or } y_2 = z)$ cnf(transitivity_collinea
$\text{collinear}(x, y, \text{third}(x, y))$ cnf(third_point_collinear, hypothesis)
$x \neq \text{third}(x, y)$ cnf(third_point_different_1a, hypothesis)
$y \neq \text{third}(x, y)$ cnf(third_point_different_1b, hypothesis)
$\neg\,\text{collinear}(p_1, p_2, p_3)$ cnf(conjecture, negated_conjecture)

GEO167+1.p Pappus1 implies Pappus2
$\text{colinear}(a, b, c, l) \text{ and } \text{colinear}(d, e, f, m)$ fof(assumption_1, axiom)
$\text{colinear}(b, f, g, n) \text{ and } \text{colinear}(c, e, g, o)$ fof(assumption_2, axiom)
$\text{colinear}(b, d, h, p) \text{ and } \text{colinear}(a, e, h, q)$ fof(assumption_3, axiom)
$\text{colinear}(c, d, i, r) \text{ and } \text{colinear}(a, f, i, s)$ fof(assumption_4, axiom)
$\text{line\_equal}(n, o) \Rightarrow \text{goal}$ fof(goal_1, axiom)
$\text{line\_equal}(p, q) \Rightarrow \text{goal}$ fof(goal_2, axiom)
$\text{line\_equal}(s, r) \Rightarrow \text{goal}$ fof(goal_3, axiom)
$\forall a$: $((\text{line\_equal}(a, a) \text{ and } \text{incident}(g, a) \text{ and } \text{incident}(h, a) \text{ and } \text{incident}(i, a)) \Rightarrow \text{goal})$ fof(goal_4, axiom)
$\forall a, b, c, d$: $(\text{colinear}(a, b, c, d) \Rightarrow \text{incident}(a, d))$ fof($\text{colinearity\_elimination}_1$, axiom)
$\forall a, b, c, d$: $(\text{colinear}(a, b, c, d) \Rightarrow \text{incident}(b, d))$ fof($\text{colinearity\_elimination}_2$, axiom)
$\forall a, b, c, d$: $(\text{colinear}(a, b, c, d) \Rightarrow \text{incident}(c, d))$ fof($\text{colinearity\_elimination}_3$, axiom)
$\forall a, b$: $(\text{incident}(a, b) \Rightarrow \text{point\_equal}(a, a))$ fof(reflexivity_of_point_equal, axiom)
$\forall a, b$: $(\text{point\_equal}(a, b) \Rightarrow \text{point\_equal}(b, a))$ fof(symmetry_of_point_equal, axiom)
$\forall a, b, c$: $((\text{point\_equal}(a, b) \text{ and } \text{point\_equal}(b, c)) \Rightarrow \text{point\_equal}(a, c))$ fof(transitivity_of_point_equal, axiom)
$\forall a, b$: $(\text{incident}(a, b) \Rightarrow \text{line\_equal}(b, b))$ fof(reflexivity_of_line_equal, axiom)
$\forall a, b$: $(\text{line\_equal}(a, b) \Rightarrow \text{line\_equal}(b, a))$ fof(symmetry_of_line_equal, axiom)
$\forall a, b, c$: $((\text{line\_equal}(a, b) \text{ and } \text{line\_equal}(b, c)) \Rightarrow \text{line\_equal}(a, c))$ fof(transitivity_of_line_equal, axiom)
$\forall a, b, c$: $((\text{point\_equal}(a, b) \text{ and } \text{incident}(b, c)) \Rightarrow \text{incident}(a, c))$ fof(pcon, axiom)
$\forall a, b, c$: $((\text{incident}(a, b) \text{ and } \text{line\_equal}(b, c)) \Rightarrow \text{incident}(a, c))$ fof(lcon, axiom)

$\forall a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q$: ((colinear($a, b, c, j$) and colinear($d, e, f, k$) and colinear($b, f, g, l$) and colinear($c, e, g, m$)
($\exists r$: colinear(g, h, i, r) or incident(a, k) or incident(b, k) or incident(c, k) or incident(d, j) or incident(e, j) or incident(f, j)))
$\forall a, b, c, d$: ((incident(c, a) and incident(c, b) and incident(d, a) and incident(d, b)) $\Rightarrow$ (point_equal(c, d) or line_equal(a, b)))
$\forall a, b$: ((point_equal(a, a) and point_equal(b, b)) $\Rightarrow$ $\exists c$: (incident(a, c) and incident(b, c))) fof(line, axiom)
$\forall a, b, c$: ((line_equal(c, c) and line_equal(b, b)) $\Rightarrow$ $\exists a$: (incident(a, b) and incident(a, c))) fof(point, axiom)
goal fof(goal_to_be_proved, conjecture)

GEO168+1.p Pappus2 implies Pappus1
colinear(a, b, c, l) and colinear(d, e, f, m) fof(assumption$_1$, axiom)
colinear(b, f, g, n) and colinear(c, e, g, o) fof(assumption$_2$, axiom)
colinear(b, d, h, p) and colinear(a, e, h, q) fof(assumption$_3$, axiom)
colinear(c, d, i, r) and colinear(a, f, i, s) fof(assumption$_4$, axiom)
incident(a, m) $\Rightarrow$ goal fof(goalam, axiom)
incident(b, m) $\Rightarrow$ goal fof(goalbm, axiom)
incident(c, m) $\Rightarrow$ goal fof(goalcm, axiom)
incident(d, l) $\Rightarrow$ goal fof(goaldl, axiom)
incident(e, l) $\Rightarrow$ goal fof(goalel, axiom)
incident(f, l) $\Rightarrow$ goal fof(goalfl, axiom)
$\forall a$: ((incident(g, a) and incident(h, a) and incident(i, a)) $\Rightarrow$ goal) fof(goal$_4$, axiom)
$\forall a, b, c, d$: (colinear(a, b, c, d) $\Rightarrow$ incident(a, d)) fof(colinearity_elimination$_1$, axiom)
$\forall a, b, c, d$: (colinear(a, b, c, d) $\Rightarrow$ incident(b, d)) fof(colinearity_elimination$_2$, axiom)
$\forall a, b, c, d$: (colinear(a, b, c, d) $\Rightarrow$ incident(c, d)) fof(colinearity_elimination$_3$, axiom)
$\forall a, b$: (incident(a, b) $\Rightarrow$ point_equal(a, a)) fof(reflexivity_of_point_equal, axiom)
$\forall a, b$: (point_equal(a, b) $\Rightarrow$ point_equal(b, a)) fof(symmetry_of_point_equal, axiom)
$\forall a, b, c$: ((point_equal(a, b) and point_equal(b, c)) $\Rightarrow$ point_equal(a, c)) fof(transitivity_of_point_equal, axiom)
$\forall a, b$: (incident(a, b) $\Rightarrow$ line_equal(b, b)) fof(reflexivity_of_line_equal, axiom)
$\forall a, b$: (line_equal(a, b) $\Rightarrow$ line_equal(b, a)) fof(symmetry_of_line_equal, axiom)
$\forall a, b, c$: ((line_equal(a, b) and line_equal(b, c)) $\Rightarrow$ line_equal(a, c)) fof(transitivity_of_line_equal, axiom)
$\forall a, b, c$: ((point_equal(a, b) and incident(b, c)) $\Rightarrow$ incident(a, c)) fof(pcon, axiom)
$\forall a, b, c$: ((incident(a, b) and line_equal(b, c)) $\Rightarrow$ incident(a, c)) fof(lcon, axiom)
$\forall a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q$: ((colinear($a, b, c, j$) and colinear($d, e, f, k$) and colinear($b, f, g, l$) and colinear($c, e, g, m$)
($\exists r$: colinear(g, h, i, r) or line_equal(l, m) or line_equal(n, o) or line_equal(p, q))) fof(pappus$_1$, axiom)
$\forall a, b, c, d$: ((incident(c, a) and incident(c, b) and incident(d, a) and incident(d, b)) $\Rightarrow$ (point_equal(c, d) or line_equal(a, b)))
$\forall a, b$: ((point_equal(a, a) and point_equal(b, b)) $\Rightarrow$ $\exists c$: (incident(a, c) and incident(b, c))) fof(line, axiom)
$\forall a, b, c$: ((line_equal(c, c) and line_equal(b, b)) $\Rightarrow$ $\exists a$: (incident(a, b) and incident(a, c))) fof(point, axiom)
goal fof(goal_to_be_proved, conjecture)

GEO169+2.p Reduction of 8 cases to 2 in Cronheim's proof of Hessenberg
$\forall a$: ((incident(p_3, a) and incident(p_1, a) and incident(p_2, a)) $\Rightarrow$ goal) fof(goal_normal, axiom)
(incident(a_1, b2b$_3$) and incident(a_2, b3b$_1$) and incident(a_3, b1b$_2$)) $\Rightarrow$ goal fof(t_a_in_b, axiom)
(incident(b_1, a2a$_3$) and incident(b_2, a3a$_1$) and incident(b_3, a1a$_2$)) $\Rightarrow$ goal fof(t_b_in_a, axiom)
incident(a_1, b2b$_3$) or incident(b_3, a1a$_2$) fof(gap$_1$, axiom)
incident(a_2, b3b$_1$) or incident(b_1, a2a$_3$) fof(gap$_2$, axiom)
incident(a_3, b1b$_2$) or incident(b_2, a3a$_1$) fof(gap$_3$, axiom)
incident(a_1, a1a$_2$) fof(ia1a$_2$, axiom)
incident(a_2, a1a$_2$) fof(ia2a$_1$, axiom)
incident(a_2, a2a$_3$) fof(ia2a$_3$, axiom)
incident(a_3, a2a$_3$) fof(ia3a$_2$, axiom)
incident(a_3, a3a$_1$) fof(ia3a$_1$, axiom)
incident(a_1, a3a$_1$) fof(ia1a$_3$, axiom)
incident(b_1, b1b$_2$) fof(ib1b$_2$, axiom)
incident(b_2, b1b$_2$) fof(ib2b$_1$, axiom)
incident(b_2, b2b$_3$) fof(ib2b$_3$, axiom)
incident(b_3, b2b$_3$) fof(ib3b$_2$, axiom)
incident(b_3, b3b$_1$) fof(ib3b$_1$, axiom)
incident(b_1, b3b$_1$) fof(ib1b$_3$, axiom)
incident(s, s_1) fof(iss$_1$, axiom)
incident(s, s_2) fof(iss$_2$, axiom)
incident(s, s_3) fof(iss$_3$, axiom)
incident(a_1, s_1) fof(ia1s$_1$, axiom)

incident(a_2, s_2) fof(ia2s$_2$, axiom)
incident(a_3, s_3) fof(ia3s$_3$, axiom)
incident(b_1, s_1) fof(ib1s$_1$, axiom)
incident(b_2, s_2) fof(ib2s$_2$, axiom)
incident(b_3, s_3) fof(ib3s$_3$, axiom)
incident(p_3, a1a$_2$) fof(ip3a, axiom)
incident(p_3, b1b$_2$) fof(ip3b, axiom)
incident(p_1, a2a$_3$) fof(ip1a, axiom)
incident(p_1, b2b$_3$) fof(ip1b, axiom)
incident(p_2, a3a$_1$) fof(ip2a, axiom)
incident(p_2, b3b$_1$) fof(ip2b, axiom)
$\forall a, b$: (incident(a, b) $\Rightarrow$ point(a)) fof(sort_point, axiom)
$\forall a, b$: (incident(a, b) $\Rightarrow$ line(b)) fof(sort_line, axiom)
$a_1 = a_2 \Rightarrow$ goal fof(diff_a1_a$_2$, axiom)
$a_2 = a_3 \Rightarrow$ goal fof(diff_a2_a$_3$, axiom)
$a_3 = a_1 \Rightarrow$ goal fof(diff_a3_a$_1$, axiom)
$b_1 = b_2 \Rightarrow$ goal fof(diff_b1_b$_2$, axiom)
$b_2 = b_3 \Rightarrow$ goal fof(diff_b2_b$_3$, axiom)
$b_3 = b_1 \Rightarrow$ goal fof(diff_b3_b$_1$, axiom)
a1a$_2$ = b1b$_2$ $\Rightarrow$ goal fof(not$_{12}$, axiom)
a2a$_3$ = b2b$_3$ $\Rightarrow$ goal fof(not$_{23}$, axiom)
a3a$_1$ = b3b$_1$ $\Rightarrow$ goal fof(not$_{31}$, axiom)
$\forall a$: $a = a$ fof(reflexivity_of_equal, axiom)
$\forall a, b$: ($a = b \Rightarrow b = a$) fof(symmetry_of_equal, axiom)
$\forall a, b, c$: (($a = b$ and $b = c$) $\Rightarrow$ $a = c$) fof(transitivity_of_equal, axiom)
$\forall a, b, c$: (($a = b$ and incident(b, c)) $\Rightarrow$ incident(a, c)) fof(point_congruence, axiom)
$\forall a, b, c$: ((incident(a, b) and $b = c$) $\Rightarrow$ incident(a, c)) fof(line_congruence, axiom)
$\forall a, b, c, d$: ((incident(a, c) and incident(a, d) and incident(b, c) and incident(b, d)) $\Rightarrow$ ($a = b$ or $c = d$)) fof(unique, axiom)
$\forall a, b$: ((point(a) and point(b)) $\Rightarrow$ $\exists c$: (incident(a, c) and incident(b, c))) fof(join, axiom)
$\forall a, b$: ((line(a) and line(b)) $\Rightarrow$ $\exists c$: (incident(c, a) and incident(c, b))) fof(meet, axiom)
goal fof(goal_to_be_proved, conjecture)

GEO170+1.p Uniqueness of constructed lines
If two distinct points are incident with a line, then this line is equivalent with the connecting line of these points.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and $\neg$ apart_point_and_line(x, z) and $\neg$ apart_point_and_line(y, z)) $\Rightarrow$ $\neg$ distinct_lines(z, line_co

GEO170+2.p Uniqueness of constructed lines
If two distinct points are incident with a line, then this line is equivalent with the connecting line of these points.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and $\neg$ apart_point_and_line(x, z) and $\neg$ apart_point_and_line(y, z)) $\Rightarrow$ $\neg$ distinct_lines(z, line_co

GEO170+3.p Uniqueness of constructed lines
If two distinct points are incident with a line, then this line is equivalent with the connecting line of these points.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((distinct_points(x, y) and incident_point_and_line(x, z) and incident_point_and_line(y, z)) $\Rightarrow$ equal_lines(z, line_conn

GEO171+1.p Two convergent lines are distinct
include('Axioms/GEO006+0.ax')
$\forall x, y$: (convergent_lines(x, y) $\Rightarrow$ distinct_lines(x, y)) fof(con, conjecture)

GEO171+2.p Uniqueness of constructed lines
include('Axioms/GEO008+0.ax')
$\forall x, y$: (convergent_lines(x, y) $\Rightarrow$ distinct_lines(x, y)) fof(con, conjecture)

GEO171+3.p Two convergent lines are distinct
include('Axioms/GEO006+0.ax')

include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y$: (convergent_lines(x, y) $\Rightarrow$ distinct_lines(x, y)) fof(con, conjecture)

GEO172+1.p Uniqueness of constructed points
If two lines are convergent and there is a point that is incident with both lines, then this point is equivalent to the intersection point of these lines.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and ¬ apart_point_and_line(z, x) and ¬ apart_point_and_line(z, y)) $\Rightarrow$ ¬ distinct_points(z, int

GEO172+2.p Uniqueness of constructed points
If two lines are convergent and there is a point that is incident with both lines, then this point is equivalent to the intersection point of these lines.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and ¬ apart_point_and_line(z, x) and ¬ apart_point_and_line(z, y)) $\Rightarrow$ ¬ distinct_points(z, int

GEO172+3.p Uniqueness of constructed points
If two lines are convergent and there is a point that is incident with both lines, then this point is equivalent to the intersection point of these lines.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and incident_point_and_line(z, x) and incident_point_and_line(z, y)) $\Rightarrow$ equal_points(z, inters

GEO173+1.p Lemma on symmetry and apartness
If two points are distinct and a line U is distinct from the line connecting the points, then U is apart from at least one of these points.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and distinct_lines(u, line_connecting(x, y))) $\Rightarrow$ (apart_point_and_

GEO173+2.p Lemma on symmetry and apartness
If two points are distinct and a line U is distinct from the line connecting the points, then U is apart from at least one of these points.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and distinct_lines(u, line_connecting(x, y))) $\Rightarrow$ (apart_point_and_

GEO173+3.p Lemma on symmetry and apartness
If two points are distinct and a line U is distinct from the line connecting the points, then U is apart from at least one of these points.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and distinct_lines(u, line_connecting(x, y))) $\Rightarrow$ (apart_point_and_

GEO174+1.p Lemma on symmetry and apartness
If two points are distinct and a line U is apart from at least one of these points, then this line is distinct from the line connecting these points
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and (apart_point_and_line(x, u) or apart_point_and_line(y, u))) $\Rightarrow$
distinct_lines(u, line_connecting(x, y))) fof(con, conjecture)

GEO174+2.p Lemma on symmetry and apartness

If two points are distinct and a line U is apart from at least one of these points, then this line is distinct from the line connecting these points
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and (apart_point_and_line(x, u) or apart_point_and_line(y, u))) ⇒ distinct_lines(u, line_connecting(x, y))) fof(con, conjecture)

GEO174+3.p Lemma on symmetry and apartness
If two points are distinct and a line U is apart from at least one of these points, then this line is distinct from the line connecting these points
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and (apart_point_and_line(x, u) or apart_point_and_line(y, u))) ⇒ distinct_lines(u, line_connecting(x, y))) fof(con, conjecture)

GEO175+1.p Lemma on symmetry and apartness
If two lines are convergent and a point is distinct from the intersection point then this point is apart from at least one of these lines.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and distinct_points(x, intersection_point(u, v))) ⇒ (apart_point_a

GEO175+2.p Lemma on symmetry and apartness
If two lines are convergent and a point is distinct from the intersection point then this point is apart from at least one of these lines.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and distinct_points(x, intersection_point(u, v))) ⇒ (apart_point_a

GEO175+3.p Lemma on symmetry and apartness
If two lines are convergent and a point is distinct from the intersection point then this point is apart from at least one of these lines.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and distinct_points(x, intersection_point(u, v))) ⇒ (apart_point_a

GEO176+1.p Lemma on symmetry and apartness
If two lines are convergent and a point is apart from at least one of these lines, then this point is distinct from the intersection point of these lines.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and (apart_point_and_line(x, u) or apart_point_and_line(x, v))) ⇒ distinct_points(x, intersection_point(u, v))) fof(con, conjecture)

GEO176+2.p Lemma on symmetry and apartness
If two lines are convergent and a point is apart from at least one of these lines, then this point is distinct from the intersection point of these lines.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and (apart_point_and_line(x, u) or apart_point_and_line(x, v))) ⇒ distinct_points(x, intersection_point(u, v))) fof(con, conjecture)

GEO176+3.p Lemma on symmetry and apartness
If two lines are convergent and a point is apart from at least one of these lines, then this point is distinct from the intersection point of these lines.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')

include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and (apart_point_and_line(x, u) or apart_point_and_line(x, v))) $\Rightarrow$
distinct_points(x, intersection_point(u, v))) fof(con, conjecture)

GEO177+1.p Symmetry of apartness
If the points X and Y are distinct and U and V are distinct, and X or Y is apart from the line connecting U and V, then U or V are apart from the line connecting X and Y.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and distinct_points(u, v)) $\Rightarrow$ ((apart_point_and_line(x, line_connecting(u, v)) or apart_point
(apart_point_and_line(u, line_connecting(x, y)) or apart_point_and_line(v, line_connecting(x, y))))) fof(con, conjecture)

GEO177+2.p Symmetry of apartness
If the points X and Y are distinct and U and V are distinct, and X or Y is apart from the line connecting U and V, then U or V are apart from the line connecting X and Y.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and distinct_points(u, v)) $\Rightarrow$ ((apart_point_and_line(x, line_connecting(u, v)) or apart_point
(apart_point_and_line(u, line_connecting(x, y)) or apart_point_and_line(v, line_connecting(x, y))))) fof(con, conjecture)

GEO177+3.p Symmetry of apartness
If the points X and Y are distinct and U and V are distinct, and X or Y is apart from the line connecting U and V, then U or V are apart from the line connecting X and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and distinct_points(u, v)) $\Rightarrow$ ((apart_point_and_line(x, line_connecting(u, v)) or apart_point
(apart_point_and_line(u, line_connecting(x, y)) or apart_point_and_line(v, line_connecting(x, y))))) fof(con, conjecture)

GEO178+1.p Lemma on symmetry and apartness
If two points X and Y are distinct and a point Z is apart from the line connecting X and Y, then Z and X are distinct, and Z and Y are distinct.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and apart_point_and_line(z, line_connecting(x, y))) $\Rightarrow$ (distinct_points(z, x) and distinct_point

GEO178+2.p Lemma on symmetry and apartness
If two points X and Y are distinct and a point Z is apart from the line connecting X and Y, then Z and X are distinct, and Z and Y are distinct.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and apart_point_and_line(z, line_connecting(x, y))) $\Rightarrow$ (distinct_points(z, x) and distinct_point

GEO178+3.p Lemma on symmetry and apartness
If two points X and Y are distinct and a point Z is apart from the line connecting X and Y, then Z and X are distinct, and Z and Y are distinct.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((distinct_points(x, y) and apart_point_and_line(z, line_connecting(x, y))) $\Rightarrow$ (distinct_points(z, x) and distinct_point

GEO179+1.p Lemma on symmetry and apartness
If two points X and Y are distinct and a point Z is apart from the line connecting X and Y, then the line connecting X and Y is distinct from the line connecting Z and X and distinct from the line connecting Z and Y.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and apart_point_and_line(z, line_connecting(x, y))) $\Rightarrow$ (distinct_lines(line_connecting(x, y), line

GEO179+2.p Lemma on symmetry and apartness
If two points X and Y are distinct and a point Z is apart from the line connecting X and Y, then the line connecting X and Y is distinct from the line connecting Z and X and distinct from the line connecting Z and Y.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and apart_point_and_line(z, line_connecting(x, y))) $\Rightarrow$ (distinct_lines(line_connecting(x, y), line

GEO179+3.p Lemma on symmetry and apartness
If two points X and Y are distinct and a point Z is apart from the line connecting X and Y, then the line connecting X and Y is distinct from the line connecting Z and X and distinct from the line connecting Z and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((distinct_points(x, y) and apart_point_and_line(z, line_connecting(x, y))) $\Rightarrow$ (distinct_lines(line_connecting(x, y), line

GEO180+1.p Triangle axiom 1
If X and Y are distinct points and Z is apart from the line connecting X and Y, then X is apart from the line connecting Z and Y.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(x, line_connecting(z

GEO180+2.p Triangle axiom 1
If X and Y are distinct points and Z is apart from the line connecting X and Y, then X is apart from the line connecting Z and Y.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(x, line_connecting(z

GEO180+3.p Triangle axiom 1
If X and Y are distinct points and Z is apart from the line connecting X and Y, then X is apart from the line connecting Z and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(x, line_connecting(z

GEO181+1.p Triangle axiom 2
If X and Y are distinct points and Z is apart from the line connecting X and Y, then Y is apart from the line connecting X and Z.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(y, line_connecting(x

GEO181+2.p Triangle axiom 2
If X and Y are distinct points and Z is apart from the line connecting X and Y, then Y is apart from the line connecting X and Z.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(y, line_connecting(x

GEO181+3.p Triangle axiom 2
If X and Y are distinct points and Z is apart from the line connecting X and Y, then Y is apart from the line connecting X and Z.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')

include('Axioms/GEO006+6.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(y, line_connecting(x

GEO182+1.p Triangle axiom 3
If X and Y are distinct points and Z is apart from the line connecting X and Y, then Z is apart from the line connecting Y and X.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(z, line_connecting(y

GEO182+2.p Triangle axiom 3
If X and Y are distinct points and Z is apart from the line connecting X and Y, then Z is apart from the line connecting Y and X.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(z, line_connecting(y

GEO182+3.p Triangle axiom 3
If X and Y are distinct points and Z is apart from the line connecting X and Y, then Z is apart from the line connecting Y and X.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: (distinct_points(x, y) $\Rightarrow$ (apart_point_and_line(z, line_connecting(x, y)) $\Rightarrow$ apart_point_and_line(z, line_connecting(y

GEO183+1.p Lemma on symmetry and apartness
If X and Y are distinct points, then they are incident with the line that is equal to the line connecting the points.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and $\neg$ distinct_lines(u, line_connecting(x, y))) $\Rightarrow$ ($\neg$ apart_point_a

GEO183+2.p Lemma on symmetry and apartness
If X and Y are distinct points, then they are incident with the line that is equal to the line connecting the points.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and $\neg$ distinct_lines(u, line_connecting(x, y))) $\Rightarrow$ ($\neg$ apart_point_a

GEO183+3.p Lemma on symmetry and apartness
If X and Y are distinct points, then they are incident with the line that is equal to the line connecting the points.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and equal_lines(u, line_connecting(x, y))) $\Rightarrow$ (incident_point_and

GEO184+1.p Lemma on symmetry and apartness
If X and Y are distinct points, then the line that is incident with both points is equal to the line connecting them.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and $\neg$ apart_point_and_line(x, u) and $\neg$ apart_point_and_line(y, u)
$\neg$ distinct_lines(u, line_connecting(x, y))) fof(con, conjecture)

GEO184+2.p Lemma on symmetry and apartness
If X and Y are distinct points, then the line that is incident with both points is equal to the line connecting them.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and $\neg$ apart_point_and_line(x, u) and $\neg$ apart_point_and_line(y, u)
$\neg$ distinct_lines(u, line_connecting(x, y))) fof(con, conjecture)

GEO184+3.p Lemma on symmetry and apartness
If X and Y are distinct points, then the line that is incident with both points is equal to the line connecting them.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')

include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and incident_point_and_line(x, u) and incident_point_and_line$(y, u$
equal_lines$(u,$ line_connecting$(x, y)))$ fof(con, conjecture)

GEO185+1.p Lemma on symmetry and apartness
If the lines U and V are convergent, then the point that is equal to the intersection point is incident to both lines.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and $\neg$ distinct_points$(x,$ intersection_point$(u, v)))$ $\Rightarrow$ $(\neg$ apart_po

GEO185+2.p Lemma on symmetry and apartness
If the lines U and V are convergent, then the point that is equal to the intersection point is incident to both lines.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and $\neg$ distinct_points$(x,$ intersection_point$(u, v)))$ $\Rightarrow$ $(\neg$ apart_po

GEO185+3.p Lemma on symmetry and apartness
If the lines U and V are convergent, then the point that is equal to the intersection point is incident to both lines.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and equal_points$(x,$ intersection_point$(u, v)))$ $\Rightarrow$ (incident_point_

GEO186+1.p Lemma on symmetry and apartness
If the lines U and V are convergent, then the point that is incident with both lines is equal to the intersection point of both lines.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and $\neg$ apart_point_and_line(x, u) and $\neg$ apart_point_and_line(x, v)
$\neg$ distinct_points$(x,$ intersection_point$(u, v)))$ fof(con, conjecture)

GEO186+2.p Lemma on symmetry and apartness
If the lines U and V are convergent, then the point that is incident with both lines is equal to the intersection point of both lines.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and $\neg$ apart_point_and_line(x, u) and $\neg$ apart_point_and_line(x, v)
$\neg$ distinct_points$(x,$ intersection_point$(u, v)))$ fof(con, conjecture)

GEO186+3.p Lemma on symmetry and apartness
If the lines U and V are convergent, then the point that is incident with both lines is equal to the intersection point of both lines.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and convergent_lines(u, v) and incident_point_and_line(x, u) and incident_point_and_line$(x, u$
equal_points$(x,$ intersection_point$(u, v)))$ fof(con, conjecture)

GEO187+1.p Symmetry of incidence
If X and Y are distinct points, U and V are distinct points, X and Y are incident with the line connecting U and V, then U and V are incident with the line connecting X and Y.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((distinct_points(x, y) and distinct_points(u, v) and $\neg$ apart_point_and_line$(x,$ line_connecting$(u, v))$ and $\neg$ apart_po
$(\neg$ apart_point_and_line$(u,$ line_connecting$(x, y))$ and $\neg$ apart_point_and_line$(v,$ line_connecting$(x, y))))$ fof(con, conjecture)

GEO187+2.p Symmetry of incidence

If X and Y are distinct points, U and V are distinct points, X and Y are incident with the line connecting U and V, then U and V are incident with the line connecting X and Y.
include('Axioms/GEO008+0.ax')
∀x, y, u, v: ((distinct_points(x, y) and distinct_points(u, v) and ¬ apart_point_and_line(x, line_connecting(u, v)) and ¬ apart_po
(¬ apart_point_and_line(u, line_connecting(x, y)) and ¬ apart_point_and_line(v, line_connecting(x, y)))) fof(con, conjecture)

GEO187+3.p Symmetry of incidence
If X and Y are distinct points, U and V are distinct points, X and Y are incident with the line connecting U and V, then U and V are incident with the line connecting X and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
∀x, y, u, v: ((distinct_points(x, y) and distinct_points(u, v) and incident_point_and_line(x, line_connecting(u, v)) and incident_p
(incident_point_and_line(u, line_connecting(x, y)) and incident_point_and_line(v, line_connecting(x, y)))) fof(con, conjectur

GEO188+1.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then X is incident with the line connecting Z and Y.
include('Axioms/GEO006+0.ax')
∀x, y, z: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and ¬ apart_point_and_line(z, line_connecting
¬ apart_point_and_line(x, line_connecting(z, y))) fof(con, conjecture)

GEO188+2.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then X is incident with the line connecting Z and Y.
include('Axioms/GEO008+0.ax')
∀x, y, z: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and ¬ apart_point_and_line(z, line_connecting
¬ apart_point_and_line(x, line_connecting(z, y))) fof(con, conjecture)

GEO188+3.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then X is incident with the line connecting Z and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
∀x, y, z: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and incident_point_and_line(z, line_connectin
incident_point_and_line(x, line_connecting(z, y))) fof(con, conjecture)

GEO189+1.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then Y is incident with the line connecting X and Z.
include('Axioms/GEO006+0.ax')
∀x, y, z: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and ¬ apart_point_and_line(z, line_connecting
¬ apart_point_and_line(y, line_connecting(x, z))) fof(con, conjecture)

GEO189+2.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then Y is incident with the line connecting X and Z.
include('Axioms/GEO008+0.ax')
∀x, y, z: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and ¬ apart_point_and_line(z, line_connecting
¬ apart_point_and_line(y, line_connecting(x, z))) fof(con, conjecture)

GEO189+3.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then Y is incident with the line connecting X and Z.

include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and incident_point_and_line(z, line_connectin
incident_point_and_line(y, line_connecting(x, z))) fof(con, conjecture)

GEO190+1.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then Z is incident with the line connecting Y and X.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and $\neg$ apart_point_and_line(z, line_connecting
$\neg$ apart_point_and_line(z, line_connecting(y, x))) fof(con, conjecture)

GEO190+2.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then Z is incident with the line connecting Y and X.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and $\neg$ apart_point_and_line(z, line_connecting
$\neg$ apart_point_and_line(z, line_connecting(y, x))) fof(con, conjecture)

GEO190+3.p Collary to symmetry of incidence
If X, Y, and Z are pairwise distinct, and Z is incident with the line connecting X and Y, then Z is incident with the line connecting Y and X.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((distinct_points(x, y) and distinct_points(x, z) and distinct_points(y, z) and incident_point_and_line(z, line_connectin
incident_point_and_line(z, line_connecting(y, x))) fof(con, conjecture)

GEO191+1.p Symmetry of apartness
If the lines X and Y are convergent, U and V are convergent, and the intersection point of X and Y is apart from U and V, then the intersection point of U and V is apart from X and Y.
include('Axioms/GEO006+0.ax')
$\forall x, y, u, v$: ((convergent_lines(x, y) and convergent_lines(u, v) and (apart_point_and_line(intersection_point(x, y), u) or apart_p
(apart_point_and_line(intersection_point(u, v), x) or apart_point_and_line(intersection_point(u, v), y))) fof(con, conjecture)

GEO191+2.p Symmetry of apartness
If the lines X and Y are convergent, U and V are convergent, and the intersection point of X and Y is apart from U and V, then the intersection point of U and V is apart from X and Y.
include('Axioms/GEO008+0.ax')
$\forall x, y, u, v$: ((convergent_lines(x, y) and convergent_lines(u, v) and (apart_point_and_line(intersection_point(x, y), u) or apart_p
(apart_point_and_line(intersection_point(u, v), x) or apart_point_and_line(intersection_point(u, v), y))) fof(con, conjecture)

GEO191+3.p Symmetry of apartness
If the lines X and Y are convergent, U and V are convergent, and the intersection point of X and Y is apart from U and V, then the intersection point of U and V is apart from X and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((convergent_lines(x, y) and convergent_lines(u, v) and (apart_point_and_line(intersection_point(x, y), u) or apart_p
(apart_point_and_line(intersection_point(u, v), x) or apart_point_and_line(intersection_point(u, v), y))) fof(con, conjecture)

GEO192+1.p Lemma on symmetry and apartness
If the lines X and Y are convergent, and the intersection point of X and Y is apart from a line Z, then Z is distinct from X and Y.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: (convergent_lines(x, y) $\Rightarrow$ (apart_point_and_line(intersection_point(x, y), z) $\Rightarrow$ (distinct_lines(x, z) and distinct_lines

GEO192+2.p Lemma on symmetry and apartness
If the lines X and Y are convergent, and the intersection point of X and Y is apart from a line Z, then Z is distinct from X and Y.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: (convergent_lines(x, y) $\Rightarrow$ (apart_point_and_line(intersection_point(x, y), z) $\Rightarrow$ (distinct_lines(x, z) and distinct_lines

GEO192+3.p Lemma on symmetry and apartness
If the lines X and Y are convergent, and the intersection point of X and Y is apart from a line Z, then Z is distinct from X and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: (convergent_lines(x, y) $\Rightarrow$ (apart_point_and_line(intersection_point(x, y), z) $\Rightarrow$ (distinct_lines(x, z) and distinct_lines

GEO193+1.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of Y and Z is apart from X.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) $\Rightarrow$ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(z, y), x))) fof(con, conjecture)

GEO193+2.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of Y and Z is apart from X.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) $\Rightarrow$ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(z, y), x))) fof(con, conjecture)

GEO193+3.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of Y and Z is apart from X.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) $\Rightarrow$ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(z, y), x))) fof(con, conjecture)

GEO194+1.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of X and Z is apart from Y.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) $\Rightarrow$ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(x, z), y))) fof(con, conjecture)

GEO194+2.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of X and Z is apart from Y.
include('Axioms/GEO008+0.ax')

∀x, y, z: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) ⇒ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(x, z), y))) fof(con, conjecture)

GEO194+3.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of X and Z is apart from Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
∀x, y, z: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) ⇒ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(x, z), y))) fof(con, conjecture)

GEO195+1.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of Y and X is apart from Z.
include('Axioms/GEO006+0.ax')
∀x, y, z: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) ⇒ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(y, x), z))) fof(con, conjecture)

GEO195+2.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of Y and X is apart from Z.
include('Axioms/GEO008+0.ax')
∀x, y, z: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) ⇒ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(y, x), z))) fof(con, conjecture)

GEO195+3.p Corollary to symmetry of apartness
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is apart from a line Z, then the intersection point of Y and X is apart from Z.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
∀x, y, z: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z)) ⇒ (apart_point_and_line(intersection_
apart_point_and_line(intersection_point(y, x), z))) fof(con, conjecture)

GEO196+1.p Symmetry of incidence
If the lines X and Y are convergent, U and V are convergent, and the intersection point of X and Y is incident with U and V, then the intersection point of U and V is incident with X and Y.
include('Axioms/GEO006+0.ax')
∀x, y, u, v: ((convergent_lines(x, y) and convergent_lines(u, v) and ¬ apart_point_and_line(intersection_point(x, y), u) and ¬ ap
(¬ apart_point_and_line(intersection_point(u, v), x) and ¬ apart_point_and_line(intersection_point(u, v), y))) fof(con, conjec

GEO196+2.p Symmetry of incidence
If the lines X and Y are convergent, U and V are convergent, and the intersection point of X and Y is incident with U and V, then the intersection point of U and V is incident with X and Y.
include('Axioms/GEO008+0.ax')
∀x, y, u, v: ((convergent_lines(x, y) and convergent_lines(u, v) and ¬ apart_point_and_line(intersection_point(x, y), u) and ¬ ap
(¬ apart_point_and_line(intersection_point(u, v), x) and ¬ apart_point_and_line(intersection_point(u, v), y))) fof(con, conjec

GEO196+3.p Symmetry of incidence
If the lines X and Y are convergent, U and V are convergent, and the intersection point of X and Y is incident with U and V, then the intersection point of U and V is incident with X and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')

include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, u, v$: ((convergent_lines(x, y) and convergent_lines(u, v) and incident_point_and_line(intersection_point(x, y), u) and inci
(incident_point_and_line(intersection_point(u, v), x) and incident_point_and_line(intersection_point(u, v), y))) fof(con, conj

GEO197+1.p Corollary to symmetry of incidence
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of Z and Y is incident with X.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and ¬ apart_point_and_line(intersection
¬ apart_point_and_line(intersection_point(z, y), x)) fof(con, conjecture)

GEO197+2.p Corollary to symmetry of incidence
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of Z and Y is incident with X.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and ¬ apart_point_and_line(intersection
¬ apart_point_and_line(intersection_point(z, y), x)) fof(con, conjecture)

GEO197+3.p Corollary to symmetry of incidence
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of Z and Y is incident with X.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and incident_point_and_line(intersection
incident_point_and_line(intersection_point(z, y), x)) fof(con, conjecture)

GEO198+1.p Corollary to symmetry of incidence
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of X and Z is incident with Y.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and ¬ apart_point_and_line(intersection
¬ apart_point_and_line(intersection_point(x, z), y)) fof(con, conjecture)

GEO198+2.p Corollary to symmetry of incidence
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of X and Z is incident with Y.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and ¬ apart_point_and_line(intersection
¬ apart_point_and_line(intersection_point(x, z), y)) fof(con, conjecture)

GEO198+3.p Corollary to symmetry of incidence
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of X and Z is incident with Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and incident_point_and_line(intersection
incident_point_and_line(intersection_point(x, z), y)) fof(con, conjecture)

GEO199+1.p Corollary to symmetry of incidence

If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of Y and X is incident with Z.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and $\neg$ apart_point_and_line(intersection
$\neg$ apart_point_and_line(intersection_point(y, x), z)) fof(con, conjecture)

GEO199+2.p Corollary to symmetry of incidence
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of Y and X is incident with Z.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and $\neg$ apart_point_and_line(intersection
$\neg$ apart_point_and_line(intersection_point(y, x), z)) fof(con, conjecture)

GEO199+3.p Corollary to symmetry of incidence
If the lines X, Y, and Z are pairwise convergent, and the intersection point of X and Y is incident with Z, then the intersection point of Y and X is incident with Z.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(z, y) and convergent_lines(x, z) and incident_point_and_line(intersection
incident_point_and_line(intersection_point(y, x), z)) fof(con, conjecture)

GEO200+1.p Line equals its converse
If the points X and Y are distinct, then the line connecting X and Y is equal to the line connecting Y and X.
include('Axioms/GEO006+0.ax')
$\forall x, y$: (distinct_points(x, y) $\Rightarrow$ $\neg$ distinct_lines(line_connecting(x, y), line_connecting(y, x))) fof(con, conjecture)

GEO200+2.p Line equals its converse
If the points X and Y are distinct, then the line connecting X and Y is equal to the line connecting Y and X.
include('Axioms/GEO008+0.ax')
$\forall x, y$: (distinct_points(x, y) $\Rightarrow$ $\neg$ distinct_lines(line_connecting(x, y), line_connecting(y, x))) fof(con, conjecture)

GEO200+3.p Line equals its converse
If the points X and Y are distinct, then the line connecting X and Y is equal to the line connecting Y and X.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y$: (distinct_points(x, y) $\Rightarrow$ equal_lines(line_connecting(x, y), line_connecting(y, x))) fof(con, conjecture)

GEO201+1.p Distinct ends means distinct lines
If the lines X and Y are convergent, then the intersection point of X and Y is equal to the intersection point of X and Y.
include('Axioms/GEO006+0.ax')
$\forall x, y$: (convergent_lines(x, y) $\Rightarrow$ $\neg$ distinct_points(intersection_point(x, y), intersection_point(y, x))) fof(con, conjecture)

GEO201+2.p Distinct ends means distinct lines
If the lines X and Y are convergent, then the intersection point of X and Y is equal to the intersection point of X and Y.
include('Axioms/GEO008+0.ax')
$\forall x, y$: (convergent_lines(x, y) $\Rightarrow$ $\neg$ distinct_points(intersection_point(x, y), intersection_point(y, x))) fof(con, conjecture)

GEO201+3.p Distinct ends means distinct lines
If the lines X and Y are convergent, then the intersection point of X and Y is equal to the intersection point of X and Y.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')

include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y$: (convergent_lines(x, y) $\Rightarrow$ equal_points(intersection_point(x, y), intersection_point(y, x))) fof(con, conjecture)

GEO202+1.p Diverging lines have equal ends
If the point X is distinct to the points Y and Z, and the lines connecting X and Y, and connecting X and Z are convergent, then the intersection point of these lines is equal to X.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and distinct_points(x, z) and convergent_lines(line_connecting(x, y), line_connecting(x, z))) $\Rightarrow$ $\neg$ distinct_points(intersection_point(line_connecting(x, y), line_connecting(x, z)), x)) fof(con, conjecture)

GEO202+2.p Diverging lines have equal ends
If the point X is distinct to the points Y and Z, and the lines connecting X and Y, and connecting X and Z are convergent, then the intersection point of these lines is equal to X.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and distinct_points(x, z) and convergent_lines(line_connecting(x, y), line_connecting(x, z))) $\Rightarrow$ $\neg$ distinct_points(intersection_point(line_connecting(x, y), line_connecting(x, z)), x)) fof(con, conjecture)

GEO202+3.p Diverging lines have equal ends
If the point X is distinct to the points Y and Z, and the lines connecting X and Y, and connecting X and Z are convergent, then the intersection point of these lines is equal to X.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((distinct_points(x, y) and distinct_points(x, z) and convergent_lines(line_connecting(x, y), line_connecting(x, z))) $\Rightarrow$ equal_points(intersection_point(line_connecting(x, y), line_connecting(x, z)), x)) fof(con, conjecture)

GEO203+1.p Equal lines from points
If the lines X and Y are convergent, and X and Z are convergent, the intersection point of X and Y, and the intersection point of X and Z are distinct, then the line connecting these points is equal to X.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(x, z) and distinct_points(intersection_point(x, y), intersection_point$(x, z$
$\neg$ distinct_lines(line_connecting(intersection_point(x, y), intersection_point(x, z)), x)) fof(con, conjecture)

GEO203+2.p Equal lines from points
If the lines X and Y are convergent, and X and Z are convergent, the intersection point of X and Y, and the intersection point of X and Z are distinct, then the line connecting these points is equal to X.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(x, z) and distinct_points(intersection_point(x, y), intersection_point$(x, z$
$\neg$ distinct_lines(line_connecting(intersection_point(x, y), intersection_point(x, z)), x)) fof(con, conjecture)

GEO203+3.p Equal lines from points
If the lines X and Y are convergent, and X and Z are convergent, the intersection point of X and Y, and the intersection point of X and Z are distinct, then the line connecting these points is equal to X.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and convergent_lines(x, z) and distinct_points(intersection_point(x, y), intersection_point$(x, z$
equal_lines(line_connecting(intersection_point(x, y), intersection_point(x, z)), x)) fof(con, conjecture)

GEO204+1.p Distinct points and equal lines

If the points X and Y are distinct, and the points Y and Z are equal, then X and Z are distinct, and the line connecting X and Y is equivalent to the line connecting X and Z.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and ¬ distinct_points(y, z)) ⇒ (distinct_points(x, z) and ¬ distinct_lines(line_connecting(x, y),

GEO204+2.p Distinct points and equal lines
If the points X and Y are distinct, and the points Y and Z are equal, then X and Z are distinct, and the line connecting X and Y is equivalent to the line connecting X and Z.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((distinct_points(x, y) and ¬ distinct_points(y, z)) ⇒ (distinct_points(x, z) and ¬ distinct_lines(line_connecting(x, y),

GEO204+3.p Distinct points and equal lines
If the points X and Y are distinct, and the points Y and Z are equal, then X and Z are distinct, and the line connecting X and Y is equivalent to the line connecting X and Z.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((distinct_points(x, y) and equal_points(y, z)) ⇒ (distinct_points(x, z) and equal_lines(line_connecting(x, y), line_con

GEO205+1.p Convergent lines and equal points
If the lines X and Y are convergent, and Y and Z are equivalent, then X and Z are convergent, and the intersection point of X and Y, and the intersection point of X and Z are equal.
include('Axioms/GEO006+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and ¬ distinct_lines(y, z)) ⇒ (convergent_lines(x, z) and ¬ distinct_points(intersection_point

GEO205+2.p Convergent lines and equal points
If the lines X and Y are convergent, and Y and Z are equivalent, then X and Z are convergent, and the intersection point of X and Y, and the intersection point of X and Z are equal.
include('Axioms/GEO008+0.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and ¬ distinct_lines(y, z)) ⇒ (convergent_lines(x, z) and ¬ distinct_points(intersection_point

GEO205+3.p Convergent lines and equal points
If the lines X and Y are convergent, and Y and Z are equivalent, then X and Z are convergent, and the intersection point of X and Y, and the intersection point of X and Z are equal.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((convergent_lines(x, y) and equal_lines(y, z)) ⇒ (convergent_lines(x, z) and equal_points(intersection_point(x, y), in

GEO206+1.p Point on parallel lines
If the point X is incident with the line Y, and the lines Y and Z are parallel, then the line Y is equal to the parallel of Z through point X.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
$\forall x, y, z$: ((¬ apart_point_and_line(x, y) and ¬ convergent_lines(y, z)) ⇒ ¬ distinct_lines(y, parallel_through_point(z, x))) fo

GEO206+2.p Point on parallel lines
If the point X is incident with the line Y, and the lines Y and Z are parallel, then the line Y is equal to the parallel of Z through point X.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
$\forall x, y, z$: ((¬ apart_point_and_line(x, y) and ¬ convergent_lines(y, z)) ⇒ ¬ distinct_lines(y, parallel_through_point(z, x))) fo

GEO206+3.p Point on parallel lines
If the point X is incident with the line Y, and the lines Y and Z are parallel, then the line Y is equal to the parallel of Z through point X.

include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((incident_point_and_line(x, y) and parallel_lines(y, z)) $\Rightarrow$ equal_lines(y, parallel_through_point(z, x))) fof(con, co

GEO207+1.p Irreflexivity of line convergence
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
$\forall x$: $\neg$ convergent_lines(x, x) fof(con, conjecture)

GEO207+2.p Irreflexivity of line convergence
A line is not convergent to itself.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
$\forall x$: $\neg$ convergent_lines(x, x) fof(con, conjecture)

GEO207+3.p Irreflexivity of line convergence
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x$: $\neg$ convergent_lines(x, x) fof(con, conjecture)

GEO208+1.p Point on both parallel lines
If the point X is incident with both the lines Y and Z, and Y and Z are parallel, then Y and Z are equal.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
$\forall x, y, z$: (($\neg$ apart_point_and_line(x, y) and $\neg$ apart_point_and_line(x, z) and $\neg$ convergent_lines(y, z)) $\Rightarrow$ $\neg$ distinct_lines(y, z)

GEO208+2.p Point on both parallel lines
If the point X is incident with both the lines Y and Z, and Y and Z are parallel, then Y and Z are equal.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
$\forall x, y, z$: (($\neg$ apart_point_and_line(x, y) and $\neg$ apart_point_and_line(x, z) and $\neg$ convergent_lines(y, z)) $\Rightarrow$ $\neg$ distinct_lines(y, z)

GEO208+3.p Point on both parallel lines
If the point X is incident with both the lines Y and Z, and Y and Z are parallel, then Y and Z are equal.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y, z$: ((incident_point_and_line(x, y) and incident_point_and_line(x, z) and parallel_lines(y, z)) $\Rightarrow$ equal_lines(y, z)) fof(

GEO209+1.p Pont and three parallel lines
If the point A is apart from the line L, but incident with the lines M and N, and L is parallel to M and N, then M and N are equal.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
$\forall a, l, m, n$: ((apart_point_and_line(a, l) and $\neg$ apart_point_and_line(a, m) and $\neg$ apart_point_and_line(a, n) and $\neg$ convergent_li
$\neg$ distinct_lines(m, n)) fof(con, conjecture)

GEO209+2.p Pont and three parallel lines
If the point A is apart from the line L, but incident with the lines M and N, and L is parallel to M and N, then M and N are equal.

include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
$\forall a, l, m, n$: ((apart_point_and_line(a, l) and ¬ apart_point_and_line(a, m) and ¬ apart_point_and_line(a, n) and ¬ convergent_li
¬ distinct_lines(m, n)) fof(con, conjecture)

GEO209+3.p Pont and three parallel lines
If the point A is apart from the line L, but incident with the lines M and N, and L is parallel to M and N, then M and N are equal.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall a, l, m, n$: ((apart_point_and_line(a, l) and incident_point_and_line(a, m) and incident_point_and_line(a, n) and parallel_lines(
equal_lines(m, n)) fof(con, conjecture)

GEO210+1.p Uniqueness of orthogonality
If the point A is incident with line L, and the line L is orthogonal to M, then L is equal to the orthogonal to M through A.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall a, l, m$: ((¬ apart_point_and_line(a, l) and ¬ unorthogonal_lines(l, m)) $\Rightarrow$ ¬ distinct_lines(l, orthogonal_through_point(m, a)))

GEO210+2.p Uniqueness of orthogonality
If the point A is incident with line L, and the line L is orthogonal to M, then L is equal to the orthogonal to M through A.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall a, l, m$: ((¬ apart_point_and_line(a, l) and ¬ unorthogonal_lines(l, m)) $\Rightarrow$ ¬ distinct_lines(l, orthogonal_through_point(m, a)))

GEO210+3.p Uniqueness of orthogonality
If the point A is incident with line L, and the line L is orthogonal to M, then L is equal to the orthogonal to M through A.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall a, l, m$: ((incident_point_and_line(a, l) and orthogonal_lines(l, m)) $\Rightarrow$ equal_lines(l, orthogonal_through_point(m, a))) fof(

GEO211+1.p A line is not orthogonal to itself
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l$: unorthogonal_lines(l, l) fof(con, conjecture)

GEO211+2.p A line is not orthogonal to itself
A Line is not orthogonal to itself.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l$: unorthogonal_lines(l, l) fof(con, conjecture)

GEO211+3.p A line is not orthogonal to itself
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')

include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l$: not_orthogonal_lines(l, l) fof(con, conjecture)

GEO212+1.p Non-orthogonal lines and a third line
If a line L is not orthogonal to M, then a third line N is convergent to L or not orthogonal to M.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l, m, n$: (unorthogonal_lines(l, m) $\Rightarrow$ (convergent_lines(l, n) or unorthogonal_lines(m, n))) fof(con, conjecture)

GEO212+2.p Non-orthogonal lines and a third line
If a line L is not orthogonal to M, then a third line N is convergent to L or not orthogonal to M.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l, m, n$: (unorthogonal_lines(l, m) $\Rightarrow$ (convergent_lines(l, n) or unorthogonal_lines(m, n))) fof(con, conjecture)

GEO212+3.p Non-orthogonal lines and a third line
If a line L is not orthogonal to M, then a third line N is convergent to L or not orthogonal to M.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m, n$: (not_orthogonal_lines(l, m) $\Rightarrow$ (convergent_lines(l, n) or not_orthogonal_lines(m, n))) fof(con, conjecture)

GEO213+1.p Corollary to non-orthogonal lines and a third line
If line L is not orthogonal to line M, then a third line N is distinct from L or not orthogonal to M.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l, m, n$: (unorthogonal_lines(l, m) $\Rightarrow$ (distinct_lines(l, n) or unorthogonal_lines(m, n))) fof(con, conjecture)

GEO213+2.p Corollary to non-orthogonal lines and a third line
If line L is not orthogonal to line M, then a third line N is distinct from L or not orthogonal to M.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l, m, n$: (unorthogonal_lines(l, m) $\Rightarrow$ (distinct_lines(l, n) or unorthogonal_lines(m, n))) fof(con, conjecture)

GEO213+3.p Corollary to non-orthogonal lines and a third line
If line L is not orthogonal to line M, then a third line N is distinct from L or not orthogonal to M.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m, n$: (not_orthogonal_lines(l, m) $\Rightarrow$ (distinct_lines(l, n) or not_orthogonal_lines(m, n))) fof(con, conjecture)

GEO214+1.p Corollary to non-orthogonal lines and a third line
If the line L is not orthogonal to M, then M is orthogonal to L.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l, m$: (unorthogonal_lines(l, m) $\Rightarrow$ unorthogonal_lines(m, l)) fof(con, conjecture)

GEO214+2.p Corollary to non-orthogonal lines and a third line

If the line L is not orthogonal to M, then M is orthogonal to L.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l, m$: (unorthogonal_lines(l, m) $\Rightarrow$ unorthogonal_lines(m, l)) fof(con, conjecture)

GEO214+3.p Corollary to non-orthogonal lines and a third line
If the line L is not orthogonal to M, then M is orthogonal to L.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m$: (not_orthogonal_lines(l, m) $\Rightarrow$ not_orthogonal_lines(m, l)) fof(con, conjecture)

GEO215+1.p Third line not orthogonal to two convergent lines
If two lines L and M are convergent, then a third line N is not orthogonal to L or M.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l, m, n$: (convergent_lines(l, m) $\Rightarrow$ (unorthogonal_lines(l, n) or unorthogonal_lines(m, n))) fof(con, conjecture)

GEO215+2.p Third line not orthogonal to two convergent lines
If two lines L and M are convergent, then a third line N is not orthogonal to L or M.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall l, m, n$: (convergent_lines(l, m) $\Rightarrow$ (unorthogonal_lines(l, n) or unorthogonal_lines(m, n))) fof(con, conjecture)

GEO215+3.p Third line not orthogonal to two convergent lines
If two lines L and M are convergent, then a third line N is not orthogonal to L or M.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m, n$: (convergent_lines(l, m) $\Rightarrow$ (not_orthogonal_lines(l, n) or not_orthogonal_lines(m, n))) fof(con, conjecture)

GEO216+1.p A line is not orthogonal to itself
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l$: $\neg\neg$ unorthogonal_lines(l, l) fof(con, conjecture)

GEO216+2.p A line is not orthogonal to itself
A Line is not orthogonal to itself.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l$: $\neg\neg$ unorthogonal_lines(l, l) fof(con, conjecture)

GEO216+3.p A line is not orthogonal to itself
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l$: $\neg$ orthogonal_lines(l, l) fof(con, conjecture)

GEO217+1.p Transitivity of parallel

If a line L is parallel to the lines M and N, then M and N are parallel.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l, m, n$: $((\neg$ convergent_lines(l, m) and $\neg$ convergent_lines$(l, n)) \Rightarrow \neg$ convergent_lines$(m, n))$ fof(con, conjecture)

GEO217+2.p Transitivity of parallel
If a line L is parallel to the lines M and N, then M and N are parallel.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l, m, n$: $((\neg$ convergent_lines(l, m) and $\neg$ convergent_lines$(l, n)) \Rightarrow \neg$ convergent_lines$(m, n))$ fof(con, conjecture)

GEO217+3.p Transitivity of parallel
If a line L is parallel to the lines M and N, then M and N are parallel.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m, n$: ((parallel_lines(l, m) and parallel_lines$(l, n)) \Rightarrow$ parallel_lines$(m, n))$ fof(con, conjecture)

GEO218+1.p Transitivity of parallel and orthogonal
If two lines L and M are parallel and a third line N is orthogonal to L, then M is orthogonal to N.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l, m, n$: $((\neg$ convergent_lines(l, m) and $\neg$ unorthogonal_lines$(l, n)) \Rightarrow \neg$ unorthogonal_lines$(m, n))$ fof(con, conjecture)

GEO218+2.p Transitivity of parallel and orthogonal
If two lines L and M are parallel and a third line N is orthogonal to L, then M is orthogonal to N.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l, m, n$: $((\neg$ convergent_lines(l, m) and $\neg$ unorthogonal_lines$(l, n)) \Rightarrow \neg$ unorthogonal_lines$(m, n))$ fof(con, conjecture)

GEO218+3.p Transitivity of parallel and orthogonal
If two lines L and M are parallel and a third line N is orthogonal to L, then M is orthogonal to N.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m, n$: ((parallel_lines(l, m) and orthogonal_lines$(l, n)) \Rightarrow$ orthogonal_lines$(m, n))$ fof(con, conjecture)

GEO219+1.p Transitivity of orthogonal and parallel
If line L is orthogonal to M and a line N is parallel to L, then M is orthogonal to N.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l, m, n$: $((\neg$ unorthogonal_lines(l, m) and $\neg$ convergent_lines$(l, n)) \Rightarrow \neg$ unorthogonal_lines$(m, n))$ fof(con, conjecture)

GEO219+2.p Transitivity of orthogonal and parallel
If line L is orthogonal to M and a line N is parallel to L, then M is orthogonal to N.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l, m, n$: $((\neg$ unorthogonal_lines(l, m) and $\neg$ convergent_lines$(l, n)) \Rightarrow \neg$ unorthogonal_lines$(m, n))$ fof(con, conjecture)

GEO219+3.p Transitivity of orthogonal and parallel
If line L is orthogonal to M and a line N is parallel to L, then M is orthogonal to N.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')

include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m, n$: ((orthogonal_lines(l, m) and parallel_lines(l, n)) $\Rightarrow$ orthogonal_lines(m, n)) fof(con, conjecture)

GEO220+1.p Transitivity of orthogonal
If a line L is orthogonal to line M and line N is orthogonal to L, then M and N are parallel.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l, m, n$: (($\neg$ unorthogonal_lines(l, m) and $\neg$ unorthogonal_lines(l, n)) $\Rightarrow$ $\neg$ convergent_lines(m, n)) fof(con, conjecture)

GEO220+2.p Transitivity of orthogonal
If a line L is orthogonal to line M and line N is orthogonal to L, then M and N are parallel.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+4.ax')
$\forall l, m, n$: (($\neg$ unorthogonal_lines(l, m) and $\neg$ unorthogonal_lines(l, n)) $\Rightarrow$ $\neg$ convergent_lines(m, n)) fof(con, conjecture)

GEO220+3.p Transitivity of orthogonal
If a line L is orthogonal to line M and line N is orthogonal to L, then M and N are parallel.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m, n$: ((orthogonal_lines(l, m) and orthogonal_lines(l, n)) $\Rightarrow$ parallel_lines(m, n)) fof(con, conjecture)

GEO221+1.p Lemma on orthogonality
If a point B is incident with the orthogonal to a line L through point A, then this orthogonal is equal to the orthogonal to L through B.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall a, b, l$: ($\neg$ apart_point_and_line(b, orthogonal_through_point(l, a)) $\Rightarrow$ $\neg$ distinct_lines(orthogonal_through_point(l, a), orthogon

GEO221+2.p Lemma on orthogonality
If a point B is incident with the orthogonal to a line L through point A, then this orthogonal is equal to the orthogonal to L through B.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall a, b, l$: ($\neg$ apart_point_and_line(b, orthogonal_through_point(l, a)) $\Rightarrow$ $\neg$ distinct_lines(orthogonal_through_point(l, a), orthogon

GEO221+3.p Lemma on orthogonality
If a point B is incident with the orthogonal to a line L through point A, then this orthogonal is equal to the orthogonal to L through B.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall a, b, l$: (incident_point_and_line(b, orthogonal_through_point(l, a)) $\Rightarrow$ equal_lines(orthogonal_through_point(l, a), orthogonal_

GEO222+1.p Parallel to orthogonal to orthogonal
A line L is parallel to the line, that is orthogonal to the orthogonal to L through A, and goes through A as well.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall a, l$: $\neg$ convergent_lines(l, orthogonal_through_point(orthogonal_through_point(l, a), a)) fof(con, conjecture)

GEO222+2.p Parallel to orthogonal to orthogonal
A line L is parallel to the line, that is orthogonal to the orthogonal to L through A, and goes through A as well.

include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall a, l$: $\neg$ convergent_lines(l, orthogonal_through_point(orthogonal_through_point(l, a), a)) fof(con, conjecture)

GEO222+3.p Parallel to orthogonal to orthogonal
A line L is parallel to the line, that is orthogonal to the orthogonal to L through A, and goes through A as well.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall a, l$: parallel_lines(l, orthogonal_through_point(orthogonal_through_point(l, a), a)) fof(con, conjecture)

GEO223+1.p Corollary to uniqueness of parallels
The parallel to line L through a point A is equal to the line, that is orthogonal to the orthogonal to L through A, and goes through A as well.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall a, l$: $\neg$ distinct_lines(parallel_through_point(l, a), orthogonal_through_point(orthogonal_through_point(l, a), a)) fof(con, co

GEO223+2.p Corollary to uniqueness of parallels
The parallel to line L through a point A is equal to the line, that is orthogonal to the orthogonal to L through A, and goes through A as well.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
$\forall a, l$: $\neg$ distinct_lines(parallel_through_point(l, a), orthogonal_through_point(orthogonal_through_point(l, a), a)) fof(con, co

GEO223+3.p Corollary to uniqueness of parallels
The parallel to line L through a point A is equal to the line, that is orthogonal to the orthogonal to L through A, and goes through A as well.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall a, l$: equal_lines(parallel_through_point(l, a), orthogonal_through_point(orthogonal_through_point(l, a), a)) fof(con, conjec

GEO224+1.p Find point incident to line
Assume orthogonal geometry. Given a point and a line, to find a point incident with the line.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+5.ax')
$\forall x, y$: ((point(x) and line(y)) $\Rightarrow$ $\exists z$: (point(z) and $\neg$ apt(z, y))) fof(con, conjecture)

GEO224+2.p Find point incident to line
Assume orthogonal geometry. Given a point and a line, to find a point incident with the line.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+5.ax')
$\forall x, y$: ((point(x) and line(y)) $\Rightarrow$ $\exists z$: (point(z) and $\neg$ apt(z, y))) fof(con, conjecture)

GEO224+3.p Find point incident to line
Assume orthogonal geometry. Given a point and a line, to find a point incident with the line.
include('Axioms/GEO006+0.ax')

include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall x, y$: ((point(x) and line(y)) $\Rightarrow$ $\exists z$: (point(z) and incident_point_and_line(z, y))) fof(con, conjecture)

GEO225+1.p Existence of line joining distinct points
When there are two distinct points, then a line connecting them can be constructed.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+5.ax')
$\forall a, b$: ((point(a) and point(b) and distinct_points(a, b)) $\Rightarrow$ $\exists x$: (line(x) $\Rightarrow$ ($\neg$ apart_point_and_line(a, x) and $\neg$ apart_point_a

GEO225+2.p Existence of line joining distinct points
Assume orthogonal geometry. Given a point and a line, to find a point incident with the line.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+5.ax')
$\forall a, b$: ((point(a) and point(b) and distinct_points(a, b)) $\Rightarrow$ $\exists x$: (line(x) $\Rightarrow$ ($\neg$ apart_point_and_line(a, x) and $\neg$ apart_point_a

GEO225+3.p Existence of line joining distinct points
When there are two distinct points, then a line connecting them can be constructed.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall a, b$: ((point(a) and point(b) and distinct_points(a, b)) $\Rightarrow$ $\exists x$: (line(x) $\Rightarrow$ (incident_point_and_line(a, x) and incident_point_

GEO226+1.p Existence of point incident to line
Assume orthogonal geometry. Given a point and a line, to find a point incident with the line.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+5.ax')
$\forall l, m$: ((line(l) and line(m) and convergent_lines(l, m)) $\Rightarrow$ $\exists x$: (point(x) $\Rightarrow$ ($\neg$ apart_point_and_line(x, l) and $\neg$ apart_point_a

GEO226+2.p Existence of point incident to line
Assume orthogonal geometry. Given a point and a line, to find a point incident with the line.
include('Axioms/GEO008+0.ax')
include('Axioms/GEO006+5.ax')
$\forall l, m$: ((line(l) and line(m) and convergent_lines(l, m)) $\Rightarrow$ $\exists x$: (point(x) $\Rightarrow$ ($\neg$ apart_point_and_line(x, l) and $\neg$ apart_point_a

GEO226+3.p Existence of point incident to line
Assume orthogonal geometry. Given a point and a line, to find a point incident with the line.
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')
$\forall l, m$: ((line(l) and line(m) and convergent_lines(l, m)) $\Rightarrow$ $\exists x$: (point(x) $\Rightarrow$ (incident_point_and_line(x, l) and incident_point

GEO227+1.p Lines not directed and opposite
include('Axioms/GEO007+0.ax')
$\forall l, m$: ((line(l) and line(m)) $\Rightarrow$ $\neg\neg$ unequally_directed_lines(l, m) and $\neg$ unequally_directed_lines(l, reverse_line(m))) fof(c

GEO227+3.p Lines not directed and opposite
include('Axioms/GEO009+0.ax')
$\forall l, m$: $\neg$ equally_directed_lines(l, m) and equally_directed_opposite_lines(l, m) fof(con, conjecture)

GEO228+1.p Obervability of equal or opposite direction
include('Axioms/GEO007+0.ax')
$\forall l, m$: ($\neg$ convergent_lines(l, m) $\iff$ ($\neg$ unequally_directed_lines(l, m) or $\neg$ unequally_directed_lines(l, reverse_line(m)))) f

GEO228+3.p Obervability of equal or opposite direction
include('Axioms/GEO009+0.ax')
$\forall l, m$: (parallel_lines(l, m) $\iff$ (equally_directed_lines(l, m) or equally_directed_opposite_lines(l, m))) fof(con, conjecture)

GEO229+1.p Uniqueness of reversed lines
include('Axioms/GEO007+0.ax')
$\forall l, m$: ($\neg$ unequally_directed_lines$(l$, reverse_line$(m))$ $\Rightarrow$ $\neg$ unequally_directed_lines$(l$, reverse_line$(m))$) fof(con, conjecture)

GEO229+3.p Uniqueness of reversed lines
include('Axioms/GEO009+0.ax')
$\forall l, m$: (equally_directed_opposite_lines$(l$, reverse_line$(m))$ $\Rightarrow$ equally_directed_lines$(l$, reverse_line$(m))$) fof(con, conjecture

GEO230+1.p Reversed lines are equal and conversely directed
include('Axioms/GEO007+0.ax')
$\forall a, b$: (distinct_points(a, b) $\Rightarrow$ ($\neg$ distinct_lines(line_connecting(a, b), line_connecting(b, a)) and $\neg$ unequally_directed_lines(line

GEO230+3.p Reversed lines are equal and conversely directed
include('Axioms/GEO009+0.ax')
$\forall a, b$: (equal_lines(line_connecting(a, b), line_connecting(b, a)) and equally_directed_lines(line_connecting(b, a), reverse_line(line

GEO231+1.p Oppositely and equally directed lines
include('Axioms/GEO007+0.ax')
$\forall l, m, n$: (($\neg$ unequally_directed_lines$(l$, reverse_line$(m))$ and $\neg$ unequally_directed_lines(l, n)) $\Rightarrow$ $\neg$ unequally_directed_lines$(m$

GEO231+3.p Oppositely and equally directed lines
include('Axioms/GEO009+0.ax')
$\forall l, m, n$: ((equally_directed_opposite_lines(l, m) and equally_directed_lines(l, n)) $\Rightarrow$ equally_directed_opposite_lines(m, n))

GEO232+1.p A line is not oppositely directed to itself
include('Axioms/GEO007+0.ax')
$\forall l$: (line(l) $\Rightarrow$ unequally_directed_lines$(l$, reverse_line$(l))$) fof(con, conjecture)

GEO232+3.p A line is not oppositely directed to itself
include('Axioms/GEO009+0.ax')
$\forall l$: unequally_directed_opposite_lines(l, l) fof(con, conjecture)

GEO233+1.p Reverse is idempotent for direction
include('Axioms/GEO007+0.ax')
$\forall l$: $\neg$ unequally_directed_lines(reverse_line(reverse_line(l)), l) fof(con, conjecture)

GEO233+3.p Reverse is idempotent for direction
include('Axioms/GEO009+0.ax')
$\forall l$: equally_directed_lines(reverse_line(reverse_line(l)), l) fof(con, conjecture)

GEO234+1.p Unequally directed lines
include('Axioms/GEO007+0.ax')
$\forall l, m, n$: (unequally_directed_lines$(l$, reverse_line$(m))$ $\Rightarrow$ (unequally_directed_lines(l, n) or unequally_directed_lines$(m$, reverse

GEO234+3.p Unequally directed lines
include('Axioms/GEO009+0.ax')
$\forall l, m, n$: (unequally_directed_opposite_lines(l, m) $\Rightarrow$ (unequally_directed_lines(l, n) or unequally_directed_opposite_lines$(m, n$

GEO235+1.p Left and right apart leads to distinctness of points
include('Axioms/GEO007+0.ax')
$\forall a, b, l$: ((left_apart_point(a, l) and left_apart_point$(b$, reverse_line$(l))$) $\Rightarrow$ distinct_points(a, b)) fof(con, conjecture)

GEO235+3.p Left and right apart leads to distinctness of points
include('Axioms/GEO009+0.ax')
$\forall a, b, l$: ((left_apart_point(a, l) and right_apart_point(b, l)) $\Rightarrow$ distinct_points(a, b)) fof(con, conjecture)

GEO236+1.p Left and right apart leads to distinctness of lines
include('Axioms/GEO007+0.ax')
$\forall a, l, m$: ((left_apart_point(a, l) and left_apart_point$(a$, reverse_line$(m))$) $\Rightarrow$ (unequally_directed_lines(l, m) or distinct_lines$(l$

GEO236+3.p Left and right apart leads to distinctness of lines
include('Axioms/GEO009+0.ax')
$\forall a, l, m$: ((left_apart_point(a, l) and right_apart_point(a, m)) $\Rightarrow$ (unequally_directed_lines(l, m) or distinct_lines(l, m))) fo

GEO237+1.p Axiom of Pasch
include('Axioms/GEO007+0.ax')

$\forall a, b, c, l$: (apart_point_and_line(c, l) $\Rightarrow$ (divides_points(l, a, b) $\Rightarrow$ (divides_points(l, a, c) or divides_points(l, b, c)))) fof(co

GEO237+3.p Axiom of Pasch
include('Axioms/GEO009+0.ax')
$\forall a, b, c, l$: (apart_point_and_line(c, l) $\Rightarrow$ (divides_points(l, a, b) $\Rightarrow$ (divides_points(l, a, c) or divides_points(l, b, c)))) fof(co

GEO238+1.p Strengthened axiom of Pasch
include('Axioms/GEO007+0.ax')
$\forall a, b, c, l$: ((divides_points(l, a, b) and divides_points(l, a, c)) $\Rightarrow$ $\neg$ divides_points(l, b, c)) fof(con, conjecture)

GEO238+3.p Strengthened axiom of Pasch
include('Axioms/GEO009+0.ax')
$\forall a, b, c, l$: ((divides_points(l, a, b) and divides_points(l, a, c)) $\Rightarrow$ $\neg$ divides_points(l, b, c)) fof(con, conjecture)

GEO239+1.p Lemma on oriented intersection of lines with plane
include('Axioms/GEO007+0.ax')
$\forall a, b, l$: (($\neg$ apart_point_and_line(a, l) and left_apart_point(b, l)) $\Rightarrow$ left_convergent_lines(l, line_connecting(a, b))) fof(con, c

GEO239+3.p Lemma on oriented intersection of lines with plane
include('Axioms/GEO009+0.ax')
$\forall a, b, l$: ((incident_point_and_line(a, l) and left_apart_point(b, l)) $\Rightarrow$ left_convergent_lines(l, line_connecting(a, b))) fof(con,

GEO240+1.p Lemma on oriented intersection of lines with plane
include('Axioms/GEO007+0.ax')
$\forall a, b, l$: (($\neg$ apart_point_and_line(a, l) and left_apart_point(b, reverse_line(l))) $\Rightarrow$ left_convergent_lines(l, reverse_line(line_conne

GEO240+3.p Lemma on oriented intersection of lines with plane
include('Axioms/GEO009+0.ax')
$\forall a, b, l$: ((incident_point_and_line(a, l) and right_apart_point(b, l)) $\Rightarrow$ right_convergent_lines(l, line_connecting(a, b))) fof(c

GEO241+1.p Lemma on oriented intersection of lines with plane
include('Axioms/GEO007+0.ax')
$\forall a, b, l$: (($\neg$ apart_point_and_line(a, l) and distinct_points(a, b) and left_convergent_lines(l, line_connecting(a, b))) $\Rightarrow$ left_apart_point(b, l)) fof(con, conjecture)

GEO241+3.p Lemma on oriented intersection of lines with plane
include('Axioms/GEO009+0.ax')
$\forall a, b, l$: ((incident_point_and_line(a, l) and distinct_points(a, b) and left_convergent_lines(l, line_connecting(a, b))) $\Rightarrow$ left_apart_point(b, l)) fof(con, conjecture)

GEO242+1.p Lemma on oriented intersection of lines with plane
include('Axioms/GEO007+0.ax')
$\forall a, b, l$: (($\neg$ apart_point_and_line(a, l) and distinct_points(a, b) and left_convergent_lines(l, reverse_line(line_connecting(a, b)))) left_apart_point(b, reverse_line(l))) fof(con, conjecture)

GEO242+3.p Lemma on oriented intersection of lines with plane
include('Axioms/GEO009+0.ax')
$\forall a, b, l$: ((incident_point_and_line(a, l) and distinct_points(a, b) and right_convergent_lines(l, line_connecting(a, b))) $\Rightarrow$ right_apart_point(b, l)) fof(con, conjecture)

GEO243+1.p Configurations in terms of apartness
include('Axioms/GEO007+0.ax')
$\forall a, b, c$: (distinct_points(a, b) $\Rightarrow$ (left_apart_point(c, line_connecting(a, b)) $\Rightarrow$ left_apart_point(c, reverse_line(line_connecting(

GEO243+3.p Configurations in terms of apartness
include('Axioms/GEO009+0.ax')
$\forall a, b, c$: (distinct_points(a, b) $\Rightarrow$ (left_apart_point(c, line_connecting(a, b)) $\Rightarrow$ right_apart_point(c, line_connecting(b, a))))

GEO244+1.p Configurations in terms of apartness
include('Axioms/GEO007+0.ax')
$\forall a, b, c$: (distinct_points(a, b) $\Rightarrow$ (left_apart_point(c, reverse_line(line_connecting(a, b))) $\Rightarrow$ left_apart_point(c, line_connecting

GEO244+3.p Configurations in terms of apartness
include('Axioms/GEO009+0.ax')
$\forall a, b, c$: (distinct_points(a, b) $\Rightarrow$ (right_apart_point(c, line_connecting(a, b)) $\Rightarrow$ left_apart_point(c, line_connecting(b, a))))

GEO245+1.p Configurations in terms of apartness
include('Axioms/GEO007+0.ax')
$\forall a, b, c$: (distinct_points(a, b) $\Rightarrow$ (left_apart_point(c, line_connecting(a, b)) $\Rightarrow$ left_apart_point(b, reverse_line(line_connecting(

GEO245+3.p Configurations in terms of apartness
include('Axioms/GEO009+0.ax')
∀a, b, c: (distinct_points(a, b) ⇒ (left_apart_point(c, line_connecting(a, b)) ⇒ right_apart_point(b, line_connecting(a, c))))

GEO246+1.p Configurations in terms of apartness
include('Axioms/GEO007+0.ax')
∀a, b, c: (distinct_points(a, b) ⇒ (left_apart_point(c, reverse_line(line_connecting(a, b))) ⇒ left_apart_point(b, line_connecting

GEO246+3.p Configurations in terms of apartness
include('Axioms/GEO009+0.ax')
∀a, b, c: (distinct_points(a, b) ⇒ (right_apart_point(c, line_connecting(a, b)) ⇒ left_apart_point(b, line_connecting(a, c))))

GEO247+1.p A point in each region formed by intersecting lines
include('Axioms/GEO007+0.ax')
∀a, b, c, d, l, m: ((left_apart_point(a, l) and left_apart_point(a, m) and left_apart_point(b, reverse_line(l)) and left_apart_point(b
convergent_lines(l, m)) fof(con, conjecture)

GEO247+3.p A point in each region formed by intersecting lines
include('Axioms/GEO009+0.ax')
∀a, b, c, d, l, m: ((left_apart_point(a, l) and left_apart_point(a, m) and right_apart_point(b, l) and right_apart_point(b, m) and l
convergent_lines(l, m)) fof(con, conjecture)

GEO248+1.p A point in each region formed by parallel lines
include('Axioms/GEO007+0.ax')
∀a, b, l: ((left_apart_point(a, l) and left_apart_point(b, parallel_through_point(l, a))) ⇒ left_apart_point(b, l)) fof(con, conj

GEO248+3.p A point in each region formed by parallel lines
include('Axioms/GEO009+0.ax')
∀a, b, l: ((apart_point_and_line(a, l) and incident_point_and_line(b, parallel_lines(l, a))) ⇒ apart_point_and_line(b, l)) fof(c

GEO249+1.p A point in each region formed by parallel lines
include('Axioms/GEO007+0.ax')
∀a, b, l: ((left_apart_point(a, reverse_line(l)) and left_apart_point(b, reverse_line(parallel_through_point(l, a)))) ⇒ left_apart_p

GEO249+3.p A point in each region formed by parallel lines
include('Axioms/GEO009+0.ax')
∀a, b, l: ((right_apart_point(a, l) and right_apart_point(b, parallel_through_point(l, a))) ⇒ right_apart_point(b, l)) fof(con,

GEO250+1.p A point in each region formed by parallel lines
include('Axioms/GEO007+0.ax')
∀a, b, l: (left_apart_point(b, parallel_through_point(l, a)) ⇒ left_apart_point(a, reverse_line(parallel_through_point(l, b)))) f

GEO250+3.p A point in each region formed by parallel lines
include('Axioms/GEO009+0.ax')
∀a, b, l: (left_apart_point(b, parallel_through_point(l, a)) ⇒ right_apart_point(a, parallel_through_point(l, b))) fof(con, conj

GEO251+1.p A point in each region formed by parallel lines
include('Axioms/GEO007+0.ax')
∀a, b, l: (left_apart_point(b, reverse_line(parallel_through_point(l, a))) ⇒ left_apart_point(a, parallel_through_point(l, b))) f

GEO251+3.p A point in each region formed by parallel lines
include('Axioms/GEO009+0.ax')
∀a, b, l: (right_apart_point(b, parallel_through_point(l, a)) ⇒ left_apart_point(a, parallel_through_point(l, b))) fof(con, conj

GEO252+1.p Jordan-type result for half planes
include('Axioms/GEO007+0.ax')
∀a, b, l: ((left_apart_point(a, l) and left_apart_point(b, reverse_line(l))) ⇒ (distinct_points(a, b) and left_convergent_lines(line_

GEO252+3.p Jordan-type result for half planes
include('Axioms/GEO009+0.ax')
∀a, b, l: ((left_apart_point(a, l) and right_apart_point(b, l)) ⇒ (distinct_points(a, b) and left_convergent_lines(line_connecting(

GEO253+1.p Characteristic property of parallel lines
include('Axioms/GEO007+0.ax')
∀a, b, l: ((apart_point_and_line(a, l) and ¬ apart_point_and_line(b, parallel_through_point(l, a))) ⇒ apart_point_and_line(b, l))

GEO253+3.p Characteristic property of parallel lines
include('Axioms/GEO009+0.ax')
∀a, b, l: ((apart_point_and_line(a, l) and incident_point_and_line(b, parallel_through_point(l, a))) ⇒ apart_point_and_line(b, l))

GEO254+1.p Order on a line is observable
include('Axioms/GEO007+0.ax')
∀a, b, l: ((distinct_points(a, b) and ¬ apart_point_and_line(a, l) and ¬ apart_point_and_line(b, l)) ⇒ (before_on_line(l, a, b) and

GEO254+3.p Order on a line is observable
include('Axioms/GEO009+0.ax')
∀a, b, l: ((distinct_points(a, b) and incident_point_and_line(a, l) and incident_point_and_line(b, l)) ⇒ (before_on_line(l, a, b) an

GEO255+1.p Property of order and betweeness
include('Axioms/GEO007+0.ax')
∀l, a, b: ((line(l) and distinct_points(a, b)) ⇒ ¬ before_on_line(l, a, b) and before_on_line(l, b, a)) fof(con, conjecture)

GEO255+3.p Property of order and betweeness
include('Axioms/GEO009+0.ax')
∀l, a, b: ¬ before_on_line(l, a, b) and before_on_line(l, b, a) fof(con, conjecture)

GEO256+1.p Property of order and betweeness
include('Axioms/GEO007+0.ax')
∀l, a, b, c, d: ((distinct_points(a, c) and distinct_points(b, c) and ¬ apart_point_and_line(c, l) and left_apart_point(d, l)) ⇒ (before_on_line(l, a, b) ⇒ (before_on_line(l, a, c) or before_on_line(l, c, b)))) fof(con, conjecture)

GEO256+3.p Property of order and betweeness
include('Axioms/GEO009+0.ax')
∀l, a, b, c, d: ((distinct_points(a, c) and distinct_points(b, c) and incident_point_and_line(c, l) and left_apart_point(d, l)) ⇒ (before_on_line(l, a, b) ⇒ (before_on_line(l, a, c) or before_on_line(l, c, b)))) fof(con, conjecture)

GEO257+1.p Transitivity of order on a line
include('Axioms/GEO007+0.ax')
∀l, a, b, c, d: ((distinct_points(a, c) and distinct_points(b, c) and ¬ apart_point_and_line(c, l) and left_apart_point(d, l)) ⇒ ((before_on_line(l, a, b) and before_on_line(l, b, c)) ⇒ before_on_line(l, a, c))) fof(con, conjecture)

GEO257+3.p Transitivity of order on a line
include('Axioms/GEO009+0.ax')
∀l, a, b, c, d: ((distinct_points(a, c) and distinct_points(b, c) and incident_point_and_line(c, l) and left_apart_point(d, l)) ⇒ ((before_on_line(l, a, b) and before_on_line(l, b, c)) ⇒ before_on_line(l, a, c))) fof(con, conjecture)

GEO258+1.p Betweeness
include('Axioms/GEO007+0.ax')
∀l, a, b, c: (between_on_line(l, a, b, c) ⇒ between_on_line(l, c, b, a)) fof(con, conjecture)

GEO258+3.p Betweeness
include('Axioms/GEO009+0.ax')
∀l, a, b, c: (between_on_line(l, a, b, c) ⇒ between_on_line(l, c, b, a)) fof(con, conjecture)

GEO259+1.p Betweeness
include('Axioms/GEO007+0.ax')
∀l, a, b, c: ((line(l) and distinct_points(a, c) and distinct_points(b, c) and distinct_points(a, b)) ⇒ ¬ between_on_line(l, a, b, c) a

GEO259+3.p Betweeness
include('Axioms/GEO009+0.ax')
∀l, a, b, c: ¬ between_on_line(l, a, b, c) and between_on_line(l, b, a, c) fof(con, conjecture)

GEO260+1.p Betweeness
include('Axioms/GEO007+0.ax')
∀l, a, b, c: ∀l, a, b, c: ((line(l) and distinct_points(a, c) and distinct_points(b, c) and distinct_points(a, b)) ⇒ ¬ between_on_line(

GEO260+3.p Betweeness
include('Axioms/GEO009+0.ax')
∀l, a, b, c: ¬ between_on_line(l, a, b, c) and between_on_line(l, a, c, b) fof(con, conjecture)

GEO261+1.p Lemma for parallel projection preserves or reverses order
include('Axioms/GEO007+0.ax')
∀l, m, a, b, c: ((between_on_line(l, a, b, c) and convergent_lines(l, m) and ¬ apart_point_and_line(b, m)) ⇒ divides_points(m, a,

GEO261+3.p Lemma for parallel projection preserves or reverses order
include('Axioms/GEO009+0.ax')
∀l, m, a, b, c: ((between_on_line(l, a, b, c) and convergent_lines(l, m) and incident_point_and_line(b, m)) ⇒ divides_points(m, a,

GEO262+1.p Lemma for parallel projection preserves or reverses order

include('Axioms/GEO007+0.ax')
$\forall l, m, n, a, b, c$: ((between_on_line(l, a, b, c) and convergent_lines(l, m) and ¬ apart_point_and_line(b, m) and convergent_lines(l,
between_on_line(m, intersection_point(m, parallel_through_point(n, a)), b, intersection_point(m, parallel_through_point(n, c))))

GEO262+3.p Lemma for parallel projection preserves or reverses order
include('Axioms/GEO009+0.ax')
$\forall l, m, n, a, b, c$: ((between_on_line(l, a, b, c) and convergent_lines(l, m) and incident_point_and_line(b, m) and convergent_lines(l
between_on_line(m, intersection_point(m, parallel_through_point(n, a)), b, intersection_point(m, parallel_through_point(n, c))))

GEO263+1.p Parallel projection preserves or reverses order
include('Axioms/GEO007+0.ax')
$\forall l, m, n, a, b, c$: ((between_on_line(l, a, b, c) and convergent_lines(l, m) and convergent_lines(l, n) and convergent_lines(m, n)) =
between_on_line(m, intersection_point(m, parallel_through_point(n, a)), intersection_point(m, parallel_through_point(n, b)), int

GEO263+3.p Parallel projection preserves or reverses order
include('Axioms/GEO009+0.ax')
$\forall l, m, n, a, b, c$: ((between_on_line(l, a, b, c) and convergent_lines(l, m) and convergent_lines(l, n) and convergent_lines(m, n)) =
between_on_line(m, intersection_point(m, parallel_through_point(n, a)), intersection_point(m, parallel_through_point(n, b)), int

GEO264+1.p Traingle divides plane into seven regions
include('Axioms/GEO007+0.ax')
$\forall a, b, c, d$: (left_apart_point(c, line_connecting(a, b)) $\Rightarrow$ ((left_apart_point(d, reverse_line(line_connecting(b, c))) and left_apart_
left_apart_point(d, line_connecting(a, b)))) fof(con, conjecture)

GEO264+3.p Triangle divides plane into seven regions
include('Axioms/GEO009+0.ax')
$\forall a, b, c, d$: (left_apart_point(c, line_connecting(a, b)) $\Rightarrow$ ((right_apart_point(d, line_connecting(b, c)) and right_apart_point(d, li
left_apart_point(d, line_connecting(a, b)))) fof(con, conjecture)

GEO265+3.p Equally directed opposite and reversed lines
include('Axioms/GEO009+0.ax')
$\forall l, m$: (equally_directed_lines(l, reverse_line(m)) $\Rightarrow$ equally_directed_opposite_lines(l, m)) fof(con, conjecture)

GEO266+3.p Symmetry of unequally directed opposite lines
include('Axioms/GEO009+0.ax')
$\forall l, m$: (unequally_directed_opposite_lines(l, m) $\iff$ unequally_directed_opposite_lines(m, l)) fof(con, conjecture)

GEO267+3.p Possible unequally directed opposite lines
include('Axioms/GEO009+0.ax')
$\forall l, m, n$: (unequally_directed_opposite_lines(l, m) $\Rightarrow$ (unequally_directed_opposite_lines(m, n) or unequally_directed_lines(m,

GEO268+3.p Equivalence of unequally directed opposite and reversed lines
include('Axioms/GEO009+0.ax')
$\forall l, m$: (unequally_directed_lines(l, m) $\iff$ unequally_directed_opposite_lines(l, reverse_line(m))) fof(con, conjecture)

GEO269+3.p Equally directed opposite reveresed lines
include('Axioms/GEO009+0.ax')
$\forall l$: equally_directed_opposite_lines(l, reverse_line(l)) fof(con, conjecture)

GEO352-1.p Tarski geometry axioms
include('Axioms/GEO002-0.ax')
include('Axioms/GEO002-1.ax')
include('Axioms/GEO002-2.ax')
include('Axioms/GEO002-3.ax')

GEO353+1.p Apartness geometry
include('Axioms/GEO006+0.ax')
include('Axioms/GEO006+1.ax')
include('Axioms/GEO006+2.ax')
include('Axioms/GEO006+3.ax')
include('Axioms/GEO006+4.ax')
include('Axioms/GEO006+5.ax')
include('Axioms/GEO006+6.ax')

GEO354+1.p Ordered affine geometry
include('Axioms/GEO007+0.ax')
include('Axioms/GEO007+1.ax')

GEO355+1.p Ordered affine geometry with definitions
include('Axioms/GEO009+0.ax')