GRA axioms

GRA001+0.ax Directed graphs and paths
$\forall e$: (edge(e) $\Rightarrow$ head_of(e) $\neq$ tail_of(e))   fof(no_loops, axiom)
$\forall e$: (edge(e) $\Rightarrow$ (vertex(head_of(e)) and vertex(tail_of(e))))   fof(edge_ends_are_vertices, axiom)
complete $\Rightarrow$ $\forall v_1, v_2$: ((vertex(v_1) and vertex(v_2) and $v_1 \neq v_2$) $\Rightarrow$ $\exists e$: (edge(e) and ((v_1 = head_of(e) and v_2 = tail_of(e)) < > (v_2 = head_of(e) and v_1 = tail_of(e)))))   fof(complete_properties, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Leftarrow$ (vertex(v_1) and vertex(v_2) and $\exists e$: (edge(e) and v_1 = tail_of(e) and ((v_2 = head_of(e) and p = path_cons(e, empty)) or $\exists$tP: (path(head_of(e), v_2, tP) and p = path_cons(e, tP))))))   fof(path_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ (vertex(v_1) and vertex(v_2) and $\exists e$: (edge(e) and v_1 = tail_of(e) and ((v_2 = head_of(e) and p = path_cons(e, empty)) < > $\exists$tP: (path(head_of(e), v_2, tP) and p = path_cons(e, tP))))))   fof(path_properties, axiom)
$\forall v_1, v_2, p, e$: ((path(v_1, v_2, p) and on_path(e, p)) $\Rightarrow$ (edge(e) and in_path(head_of(e), p) and in_path(tail_of(e), p)))   fof(on_
$\forall v_1, v_2, p, v$: ((path(v_1, v_2, p) and in_path(v, p)) $\Rightarrow$ (vertex(v) and $\exists e$: (on_path(e, p) and (v = head_of(e) or v = tail_of(e)))))   fof(in_path_properties, axiom)
$\forall e_1, e_2$: (sequential(e_1, e_2) $\iff$ (edge(e_1) and edge(e_2) and $e_1 \neq e_2$ and head_of(e_1) = tail_of(e_2)))   fof(sequential_defn, ax
$\forall p, v_1, v_2$: (path(v_1, v_2, p) $\Rightarrow$ $\forall e_1, e_2$: (precedes(e_1, e_2, p) $\Leftarrow$ (on_path(e_1, p) and on_path(e_2, p) and (sequential(e_1, e_2) or $\exists e_3$:
$\forall p, v_1, v_2$: (path(v_1, v_2, p) $\Rightarrow$ $\forall e_1, e_2$: (precedes(e_1, e_2, p) $\Rightarrow$ (on_path(e_1, p) and on_path(e_2, p) and (sequential(e_1, e_2) < > $\exists e_3$: (sequential(e_1, e_3) and precedes(e_3, e_2, p))))))   fof(precedes_properties, axiom)
$\forall v_1, v_2$, sP: (shortest_path(v_1, v_2, sP) $\iff$ (path(v_1, v_2, sP) and $v_1 \neq v_2$ and $\forall p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(sP) $\leq$ length_of(p))))   fof(shortest_path_defn, axiom)
$\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) $\Rightarrow$ ($\neg \exists e_3$: (tail_of(e_3) = tail_of(e_1) and head_of(e_3) = head_of(e_2)) and $\neg$ precedes(e_2, e_1, p)))   fof(shortest_path_properties, axiom)

GRA problems

GRA001-1.p Clauses from a labelled graph
Consider a graph with the edges labelled. For example * : A / B : *–C–* : D / E : * : Assign 0 or 1 arbitarily to nodes of the graph. For each node of the graph, we associate a set of clauses as follows : (1) Every label of an edge emanating from that node will occur in each clause (of the set of clauses generated from that node). (2) If the node is assigned 0, then the number of negated literals in each of the generated clauses is to be odd. Generate all such clauses for that node. (3) If the node is assigned 1, then the number of negated literals in each of the generated clauses is to be even. Generate all such clauses for that node. Tseitin's result is this: the sum (mod 2) of the 0's and 1's assigned to the nodes of the graph equals 1 iff the set of generated clauses is inconsistent. For example, if the top node of the above graph is assigned 1, and all other nodes 0, then the set of generated clauses will be inconsistent.
a or b cnf(clause$_1$, negated_conjecture)
$a \Rightarrow \neg b$ cnf(clause$_2$, negated_conjecture)
$d \Rightarrow$ (a or c)   cnf(clause$_3$, negated_conjecture)
$c \Rightarrow$ (a or d)   cnf(clause$_4$, negated_conjecture)
$a \Rightarrow$ (c or d)   cnf(clause$_5$, negated_conjecture)
(a and c) $\Rightarrow \neg d$ cnf(clause$_6$, negated_conjecture)
$e \Rightarrow$ (b or c)   cnf(clause$_7$, negated_conjecture)
$c \Rightarrow$ (b or e)   cnf(clause$_8$, negated_conjecture)
$b \Rightarrow$ (c or e)   cnf(clause$_9$, negated_conjecture)
(b and c) $\Rightarrow \neg e$ cnf(clause$_{10}$, negated_conjecture)
$e \Rightarrow d$ cnf(clause$_{11}$, negated_conjecture)
$d \Rightarrow e$ cnf(clause$_{12}$, negated_conjecture)

GRA002+1.p Maximal shortest path length in terms of triangles
In a complete directed graph, the maximal length of a shortest path between two vertices is the number of triangles in the graph minus 1.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p))   fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p))   fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3)) number_of_in(sequential_pairs, p) = number_of_in(triangles, p))   fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph)   fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall p, v_1, v_2$: (shortest_path(v_1, v_2, p) $\Rightarrow$ $-$length_of(p) $\leq$ number_of_in(triangles, graph))   fof(maximal_path_le

GRA002+2.p Maximal shortest path length in terms of triangles

In a complete directed graph, the maximal length of a shortest path between two vertices is the number of triangles in the graph minus 1.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) $\Rightarrow$ $\exists e_3$: (edge(e_3) and tail_of(e_3) = head_of(e_2) and head_of(e_3) = tail_of(e_1))) fof(back_edge, lemma)
complete $\Rightarrow$ $\forall p, v_1, v_2$: (shortest_path(v_1, v_2, p) $\Rightarrow$ $-$length_of(p) $\leq$ number_of_in(triangles, graph)) fof(maximal_path_le

GRA002+3.p Maximal shortest path length in terms of triangles
In a complete directed graph, the maximal length of a shortest path between two vertices is the number of triangles in the graph minus 1.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$) and sequential($e_1, e_2$)) $\Rightarrow$ $\exists e_3$: triangle($e_1, e_2,$
complete $\Rightarrow$ $\forall p, v_1, v_2$: (shortest_path(v_1, v_2, p) $\Rightarrow$ $-$length_of(p) $\leq$ number_of_in(triangles, graph)) fof(maximal_path_le

GRA002+4.p Maximal shortest path length in terms of triangles
In a complete directed graph, the maximal length of a shortest path between two vertices is the number of triangles in the graph minus 1.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall p, v_1, v_2$: (shortest_path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(tri
complete $\Rightarrow$ $\forall p, v_1, v_2$: (shortest_path(v_1, v_2, p) $\Rightarrow$ $-$length_of(p) $\leq$ number_of_in(triangles, graph)) fof(maximal_path_le

GRA003+1.p Parts of paths
In a shortest path P from V1 to V2 with edges E1 and E2, E1 preceding E2, P is a path, V1 and V2 are vertices, and E1 and E2 are distinct edges.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
$\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) $\Rightarrow$ (vertex(v_1) and vertex(v_2) and $v_1 \neq v_2$ and edge(e_1) an
e_2 and path(v_1, v_2, p))) fof(vertices_and_edges, conjecture)

GRA004+1.p Maximal shortest path length in terms of triangles
In a shortest path P from V1 to V2 with edges E1 and E2, E1 preceding E2, there is no edge from the tail of E1 to the head of E2, and the head of E2 is not the head or tail of E1.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)

∀things, inThese: number_of_in(things, inThese) ≤ number_of_in(things, graph)   fof(graph_has_them_all, axiom)
$\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) ⇒ (¬ $\exists e_3$: (tail_of(e_3) = tail_of(e_1) and head_of(e_3) = head_of(e_2)) and head_of(e_2) ≠ tail_of(e_1) and head_of(e_2) ≠ head_of(e_1)))   fof(shortest_path_properties_lemma, conjecture

GRA005+1.p Maximal shortest path length in terms of triangles
In a shortest path P from V1 to V2 with edges E1 and E2, E1 preceding E2, there is no edge from the tail of E1 to the head of E2.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) ⟺ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) ⇒ length_of(p) = number_of_in(edges, p))   fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) ⇒ number_of_in(sequential_pairs, p) = −length_of(p))   fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) ⇒ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p))   fof(sequential_pairs_and_triangles, axiom)
∀things, inThese: number_of_in(things, inThese) ≤ number_of_in(things, graph)   fof(graph_has_them_all, axiom)
$\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) ⇒ ¬ $\exists e_3$: (edge(e_3) and tail_of(e_3) = tail_of(e_1) and head_o
head_of(e_2)))   fof(no_short_cut_edge, conjecture)

GRA006+1.p Maximal shortest path length in terms of triangles
In a complete graph with a shortest path P from V1 to V2 with edges E1 and E2, either there's an edge from the tail of E1 to the head of E2, or there's an edge from the head of E2 to the tail of E1.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) ⟺ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) ⇒ length_of(p) = number_of_in(edges, p))   fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) ⇒ number_of_in(sequential_pairs, p) = −length_of(p))   fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) ⇒ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p))   fof(sequential_pairs_and_triangles, axiom)
∀things, inThese: number_of_in(things, inThese) ≤ number_of_in(things, graph)   fof(graph_has_them_all, axiom)
complete ⇒ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) ⇒ $\exists e_3$: (edge(e_3) and tail_of(e_3) = tail_of(e_1) and head_of(e_3) = head_of(e_2)) < > $\exists e_3$: (edge(e_3) and tail_of(e_3) = head_of(e_2) and head_of(e_3) = tail_of(e_1)))   fof(edge_some_way, conjecture)

GRA007+1.p Maximal shortest path length in terms of triangles
In a complete graph with a shortest path P from V1 to V2 with edges E1 and E2, E1 preceding E2, there's an edge from the head of E2 to the tail of E1 (a back edge).
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) ⟺ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) ⇒ length_of(p) = number_of_in(edges, p))   fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) ⇒ number_of_in(sequential_pairs, p) = −length_of(p))   fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) ⇒ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p))   fof(sequential_pairs_and_triangles, axiom)
∀things, inThese: number_of_in(things, inThese) ≤ number_of_in(things, graph)   fof(graph_has_them_all, axiom)
complete ⇒ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) ⇒ $\exists e_3$: (edge(e_3) and tail_of(e_3) = head_of(e_2) and head_of(e_3) = tail_of(e_1)))   fof(back_edge, conjecture)

GRA007+2.p Maximal shortest path length in terms of triangles
In a complete graph with a shortest path P from V1 to V2 with edges E1 and E2, E1 preceding E2, there's an edge from the head of E2 to the tail of E1 (a back edge).
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) ⟺ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) ⇒ length_of(p) = number_of_in(edges, p))   fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) ⇒ number_of_in(sequential_pairs, p) = −length_of(p))   fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) ⇒ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p))   fof(sequential_pairs_and_triangles, axiom)
∀things, inThese: number_of_in(things, inThese) ≤ number_of_in(things, graph)   fof(graph_has_them_all, axiom)
$\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) ⇒ ¬ $\exists e_3$: (edge(e_3) and tail_of(e_3) = tail_of(e_1) and head_o
head_of(e_2)))   fof(no_short_cut_edge, lemma)
complete ⇒ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) ⇒ $\exists e_3$: (edge(e_3) and tail_of(e_3) = head_of(e_2) and head_of(e_3) = tail_of(e_1)))   fof(back_edge, conjecture)

GRA008+1.p Maximal shortest path length in terms of triangles
In a complete graph with a shortest path P from V1 to V2 with edges E1 and E2, E1 preceding and sequential to E2, there is an edge E3 such that E1, E2, and E3 form a triangle.

include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = −length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$) and sequential($e_1, e_2$)) $\Rightarrow$ $\exists e_3$: triangle($e_1, e_2,$

GRA008+2.p Maximal shortest path length in terms of triangles
In a complete graph with a shortest path P from V1 to V2 with edges E1 and E2, E1 preceding and sequential to E2, there is an edge E3 such that E1, E2, and E3 form a triangle.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = −length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$)) $\Rightarrow$ $\exists e_3$: (edge(e_3) and tail_of(e_3) = head_of(e_2) and head_of(e_3) = tail_of(e_1))) fof(back_edge, lemma)
complete $\Rightarrow$ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$) and sequential($e_1, e_2$)) $\Rightarrow$ $\exists e_3$: triangle($e_1, e_2,$

GRA009+1.p Maximal shortest path length in terms of triangles
In a complete graph with a shortest path P from V1 to V2 with edges E1 and E2, E1 sequential to E2, there is an edge E3 such that E1, E2, and E3 form a triangle.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = −length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall v_1, v_2, p, e_1, e_2$: ((shortest_path($v_1, v_2, p$) and on_path($e_1, p$) and on_path($e_2, p$) and sequential($e_1, e_2$)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3)) fof(complete_means_path_and_stuff_means_triangles, conjecture)

GRA009+2.p Maximal shortest path length in terms of triangles
In a complete graph with a shortest path P from V1 to V2 with edges E1 and E2, E1 sequential to E2, there is an edge E3 such that E1, E2, and E3 form a triangle.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = −length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall p, v_1, v_2$: (shortest_path(v_1, v_2, p) $\Rightarrow$ $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))) fof(complete_means_path_means_stuff_means_triangles, conjecture)

GRA010+1.p Maximal shortest path length in terms of triangles
In a complete graph, if there is a shortest path P from V1 to V2 with edges E1 and E2, E1 sequential to E2 means there is an edge E3 such that E1, E2, and E3 form a triangle, then the number of sequential pairs in P is the number of triangles in P.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = −length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)

complete $\Rightarrow$ $\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))) $\Rightarrow$ number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(complete_means_sequentia

GRA010+2.p Maximal shortest path length in terms of triangles
In a complete graph, if there is a shortest path P from V1 to V2 with edges E1 and E2, E1 sequential to E2 means there is an edge E3 such that E1, E2, and E3 form a triangle, then the number of sequential pairs in P is the number of triangles in P.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall v_1, v_2, e_1, e_2, p$: ((shortest_path($v_1, v_2, p$) and precedes($e_1, e_2, p$) and sequential($e_1, e_2$)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e
complete $\Rightarrow$ $\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))) $\Rightarrow$ number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(complete_means_sequentia

GRA011+1.p Maximal shortest path length in terms of triangles
In a complete graph, if there is a shortest path P from V1 to V2, then the number of sequential pairs in P is the number of triangles in P.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall p, v_1, v_2$: (shortest_path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(tri

GRA012+1.p Maximal shortest path length in terms of triangles
In a complete graph, if there is a shortest path P from V1 to V2, then the number of triangles in P is the length of P minus one.
include('Axioms/GRA001+0.ax')
$\forall e_1, e_2, e_3$: (triangle(e_1, e_2, e_3) $\iff$ (edge(e_1) and edge(e_2) and edge(e_3) and sequential(e_1, e_2) and sequential(e_2, e_3) and se
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ length_of(p) = number_of_in(edges, p)) fof(length_defn, axiom)
$\forall v_1, v_2, p$: (path(v_1, v_2, p) $\Rightarrow$ number_of_in(sequential_pairs, p) = $-$length_of(p)) fof(path_length_sequential_pairs, axiom)
$\forall p, v_1, v_2$: ((path(v_1, v_2, p) and $\forall e_1, e_2$: ((on_path(e_1, p) and on_path(e_2, p) and sequential(e_1, e_2)) $\Rightarrow$ $\exists e_3$: triangle(e_1, e_2, e_3))
number_of_in(sequential_pairs, p) = number_of_in(triangles, p)) fof(sequential_pairs_and_triangles, axiom)
$\forall$things, inThese: number_of_in(things, inThese) $\leq$ number_of_in(things, graph) fof(graph_has_them_all, axiom)
complete $\Rightarrow$ $\forall p, v_1, v_2$: (shortest_path(v_1, v_2, p) $\Rightarrow$ number_of_in(triangles, p) = $-$length_of(p)) fof(triangles_on_a_path, co

GRA013+1.p 2-colored completed graph size 5 without subgraph of size 3
Find a 2-colored completed graph of size 5 without a complete subgraph of size 3 which all the edges have the same color.
$\forall a, b, c$: ((red(a, b) and red(b, c) and red(a, c)) $\Rightarrow$ goal) fof(red_clique, axiom)
$\forall a, b, c$: ((green(a, b) and green(b, c) and green(a, c)) $\Rightarrow$ goal) fof(green_clique, axiom)
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ fof(ordering, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA014+1.p 2-colored completed graph size 6 without subgraph of size 3
Find a 2-colored completed graph of size 6 without a complete subgraph of size 3 which all the edges have the same color.
$\forall a, b, c$: ((red(a, b) and red(b, c) and red(a, c)) $\Rightarrow$ goal) fof(red_clique, axiom)
$\forall a, b, c$: ((green(a, b) and green(b, c) and green(a, c)) $\Rightarrow$ goal) fof(green_clique, axiom)
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ fof(ordering, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA015+1.p 2-colored completed graph size 11 without subgraph of size 4

Find a 2-colored completed graph of size 11 without a complete subgraph of size 4 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ fof(ordering, axiom)
$\forall a, b, c, d$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d)) $\Rightarrow$ goal) fof(red_clique, axiom)
$\forall a, b, c, d$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d)) $\Rightarrow$ goal) fof(green_
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA016+1.p 2-colored completed graph size 12 without subgraph of size 4
Find a 2-colored completed graph of size 12 without a complete subgraph of size 4 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ fof(ordering, axiom)
$\forall a, b, c, d$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d)) $\Rightarrow$ goal) fof(red_clique, axiom)
$\forall a, b, c, d$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d)) $\Rightarrow$ goal) fof(green_
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA017+1.p 2-colored completed graph size 13 without subgraph of size 4
Find a 2-colored completed graph of size 13 without a complete subgraph of size 4 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ and $n_{12} < n_{13}$ fof(ordering, axiom)
$\forall a, b, c, d$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d)) $\Rightarrow$ goal) fof(red_clique, axiom)
$\forall a, b, c, d$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d)) $\Rightarrow$ goal) fof(green_
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA018+1.p 2-colored completed graph size 17 without subgraph of size 4
Find a 2-colored completed graph of size 17 without a complete subgraph of size 4 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ and $n_{12} < n_{13}$ and $n_{13} < n_{14}$ and $n_{14} < n_{15}$ and $n_{15} < n_{16}$ and $n_{16} < n_{17}$ fof(ordering,
$\forall a, b, c, d$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d)) $\Rightarrow$ goal) fof(red_clique, axiom)
$\forall a, b, c, d$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d)) $\Rightarrow$ goal) fof(green_
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA019+1.p 2-colored completed graph size 18 without subgraph of size 4
Find a 2-colored completed graph of size 18 without a complete subgraph of size 4 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ and $n_{12} < n_{13}$ and $n_{13} < n_{14}$ and $n_{14} < n_{15}$ and $n_{15} < n_{16}$ and $n_{16} < n_{17}$ and $n_{17} < n_{18}$ fof(ordering, axiom)
$\forall a, b, c, d$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d)) $\Rightarrow$ goal) fof(red_clique, axiom)
$\forall a, b, c, d$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d)) $\Rightarrow$ goal) fof(green_
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA020+1.p 2-colored completed graph size 11 without subgraph of size 5
Find a 2-colored completed graph of size 11 without a complete subgraph of size 5 which all the edges have the same color.

$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ fof(ordering, axiom)
$\forall a, b, c, d, e$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d) and red(a, e) and red(b, e) and red
goal) fof(red_clique, axiom)
$\forall a, b, c, d, e$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d) and green(a, e) and g
goal) fof(green_clique, axiom)
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA021+1.p 2-colored completed graph size 12 without subgraph of size 5
Find a 2-colored completed graph of size 12 without a complete subgraph of size 5 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ fof(ordering, axiom)
$\forall a, b, c, d, e$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d) and red(a, e) and red(b, e) and red
goal) fof(red_clique, axiom)
$\forall a, b, c, d, e$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d) and green(a, e) and g
goal) fof(green_clique, axiom)
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA022+1.p 2-colored completed graph size 13 without subgraph of size 5
Find a 2-colored completed graph of size 13 without a complete subgraph of size 5 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ and $n_{12} < n_{13}$ fof(ordering, axiom)
$\forall a, b, c, d, e$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d) and red(a, e) and red(b, e) and red
goal) fof(red_clique, axiom)
$\forall a, b, c, d, e$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d) and green(a, e) and g
goal) fof(green_clique, axiom)
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA023+1.p 2-colored completed graph size 16 without subgraph of size 5
Find a 2-colored completed graph of size 16 without a complete subgraph of size 5 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ and $n_{12} < n_{13}$ and $n_{13} < n_{14}$ and $n_{14} < n_{15}$ and $n_{15} < n_{16}$ fof(ordering, axiom)
$\forall a, b, c, d, e$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d) and red(a, e) and red(b, e) and red
goal) fof(red_clique, axiom)
$\forall a, b, c, d, e$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d) and green(a, e) and g
goal) fof(green_clique, axiom)
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA024+1.p 2-colored completed graph size 11 without subgraph of size 6
Find a 2-colored completed graph of size 11 without a complete subgraph of size 6 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ fof(ordering, axiom)
$\forall a, b, c, d, e, f$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d) and red(a, e) and red(b, e) and r
goal) fof(red_clique, axiom)

$\forall a, b, c, d, e, f$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d) and green(a, e) and
goal) fof(green_clique, axiom)
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA025+1.p 2-colored completed graph size 14 without subgraph of size 6
Find a 2-colored completed graph of size 14 without a complete subgraph of size 6 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ and $n_{12} < n_{13}$ and $n_{13} < n_{14}$ fof(ordering, axiom)
$\forall a, b, c, d, e, f$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d) and red(a, e) and red(b, e) and r
goal) fof(red_clique, axiom)
$\forall a, b, c, d, e, f$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d) and green(a, e) and
goal) fof(green_clique, axiom)
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA026+1.p 2-colored completed graph size 20 without subgraph of size 6
Find a 2-colored completed graph of size 20 without a complete subgraph of size 6 which all the edges have the same color.
$n_1 < n_2$ and $n_2 < n_3$ and $n_3 < n_4$ and $n_4 < n_5$ and $n_5 < n_6$ and $n_6 < n_7$ and $n_7 < n_8$ and $n_8 < n_9$ and $n_9 < n_{10}$ and $n_{10} < n_{11}$ and $n_{11} < n_{12}$ and $n_{12} < n_{13}$ and $n_{13} < n_{14}$ and $n_{14} < n_{15}$ and $n_{15} < n_{16}$ and $n_{16} < n_{17}$ and $n_{17} < n_{18}$ and $n_{18} < n_{19}$ and $n_{19} < n_{20}$ fof(ordering, axiom)
$\forall a, b, c, d, e, f$: ((red(a, b) and red(a, c) and red(b, c) and red(a, d) and red(b, d) and red(c, d) and red(a, e) and red(b, e) and r
goal) fof(red_clique, axiom)
$\forall a, b, c, d, e, f$: ((green(a, b) and green(a, c) and green(b, c) and green(a, d) and green(b, d) and green(c, d) and green(a, e) and
goal) fof(green_clique, axiom)
$\forall a, b$: ((red(a, b) and green(a, b)) $\Rightarrow$ goal) fof(no_overlap, axiom)
$\forall a, b, c$: (($a < b$ and $b < c$) $\Rightarrow$ $a < c$) fof(less_than_transitive, axiom)
$\forall a, b$: ($a < b$ $\Rightarrow$ (red(a, b) or green(a, b))) fof(partition, axiom)
goal fof(goal_to_be_proved, conjecture)

GRA027^1.p R(3,3) > 4
$\exists g$: (\$o $\to$ \$o) $\to$ (\$o $\to$ \$o) $\to$ \$o: ($\forall$xx: \$o $\to$ \$o, xy: \$o $\to$ \$o: ((g@xx@xy) $\Rightarrow$ (g@xy@xx)) and $\forall \mathrm{xx}_0$: \$o $\to$ \$o, xx_1: \$o $\to$ \$o, xx_2: \$o $\to$ \$o, xp_0: (\$o $\to$ \$o) $\to$ \$o, xp_1: (\$o $\to$ \$o) $\to$ \$o: ((xp_0@xx_0 and $\neg\,\mathrm{xp}_0$@xx_1 and $\neg\,\mathrm{xp}_0$@xx_2 and $\neg$ x
($\neg\, g$@xx_1@xx_0 or $\neg\, g$@xx_2@xx_0 or $\neg\, g$@xx_2@xx_1)) and $\forall \mathrm{xx}_0$: \$o $\to$ \$o, xx_1: \$o $\to$ \$o, xx_2: \$o $\to$ \$o, xp_0: (\$o $\to$ \$o) $\to$ \$o, xp_1: (\$o $\to$ \$o) $\to$ \$o: ((xp_0@xx_0 and $\neg\,\mathrm{xp}_0$@xx_1 and $\neg\,\mathrm{xp}_0$@xx_2 and $\neg\,\mathrm{xp}_1$@xx_0 and xp_1@xx_1 and $\neg\,\mathrm{xp}_1$@xx_2) $\Rightarrow$
(g@xx_1@xx_0 or g@xx_2@xx_0 or g@xx_2@xx_1))) thf(ramsey_1_3_3$_4$, conjecture)

GRA028^1.p R(2,4) $\leq$ 4
$\forall g$: (\$o $\to$ \$o) $\to$ (\$o $\to$ \$o) $\to$ \$o: ($\forall$xx: \$o $\to$ \$o, xy: \$o $\to$ \$o: ((g@xx@xy) $\Rightarrow$ (g@xy@xx)) $\Rightarrow$ ($\exists \mathrm{xx}_0$: \$o $\to$ \$o, xx_1: \$o $\to$ \$o, xp_0: (\$o $\to$ \$o) $\to$ \$o: (xp_0@xx_0 and $\neg\,\mathrm{xp}_0$@xx_1 and g@xx_1@xx_0) or $\exists \mathrm{xx}_0$: \$o $\to$ \$o, xx_1: \$o $\to$ \$o, xx_2: \$o $\to$ \$o, xx_3: \$o $\to$ \$o, xp_0: (\$o $\to$ \$o) $\to$ \$o, xp_1: (\$o $\to$ \$o) $\to$ \$o, xp_2: (\$o $\to$ \$o) $\to$ \$o: (xp_0@xx_0 and $\neg\,\mathrm{xp}_0$@xx_1 an

GRA029^1.p R(4,4) > 16
$\exists g$: (\$o $\to$ \$o $\to$ \$o) $\to$ (\$o $\to$ \$o $\to$ \$o) $\to$ \$o: ($\forall$xx: \$o $\to$ \$o $\to$ \$o, xy: \$o $\to$ \$o $\to$ \$o: ((g@xx@xy) $\Rightarrow$ (g@xy@xx)) and $\forall \mathrm{xx}_0$: \$o $\to$ \$o $\to$ \$o, xx_1: \$o $\to$ \$o $\to$ \$o, xx_2: \$o $\to$ \$o $\to$ \$o, xx_3: \$o $\to$ \$o $\to$ \$o, xp_0: (\$o $\to$ \$o $\to$ \$o) $\to$ \$o, xp_1: (\$o $\to$ \$o $\to$ \$o) $\to$ \$o, xp_2: (\$o $\to$ \$o $\to$ \$o) $\to$ \$o: ((xp_0@xx_0 and $\neg\,\mathrm{xp}_0$@xx_1 and $\neg\,\mathrm{xp}_0$@xx_2 and $\neg\,\mathrm{xp}_0$@xx_3 a
($\neg\, g$@xx_1@xx_0 or $\neg\, g$@xx_2@xx_0 or $\neg\, g$@xx_2@xx_1 or $\neg\, g$@xx_3@xx_0 or $\neg\, g$@xx_3@xx_1 or $\neg\, g$@xx_3@xx_2)) and $\forall \mathrm{xx}_0$: \$o $\to$ \$o $\to$ \$o, xx_1: \$o $\to$ \$o $\to$ \$o, xx_2: \$o $\to$ \$o $\to$ \$o, xx_3: \$o $\to$ \$o $\to$ \$o, xp_0: (\$o $\to$ \$o $\to$ \$o) $\to$ \$o, xp_1: (\$o $\to$ \$o $\to$ \$o) $\to$ \$o, xp_2: (\$o $\to$ \$o $\to$ \$o) $\to$ \$o: ((xp_0@xx_0 and $\neg\,\mathrm{xp}_0$@xx_1 and $\neg\,\mathrm{xp}_0$@xx_2 and $\neg\,\mathrm{xp}_0$@xx_3 and $\neg\,\mathrm{xp}_1$@xx_0 and xp_1@xx
(g@xx_1@xx_0 or g@xx_2@xx_0 or g@xx_2@xx_1 or g@xx_3@xx_0 or g@xx_3@xx_1 or g@xx_3@xx_2))) thf(ramsey_1_4_4$_{16}$, conjectur

GRA029^2.p R(4,4) > 16
$\exists g$: ((\$o $\to$ \$o) $\to$ \$o) $\to$ ((\$o $\to$ \$o) $\to$ \$o) $\to$ \$o: ($\forall$xx: (\$o $\to$ \$o) $\to$ \$o, xy: (\$o $\to$ \$o) $\to$ \$o: ((g@xx@xy) $\Rightarrow$ (g@xy@xx)) and $\forall \mathrm{xx}_0$: (\$o $\to$ \$o) $\to$ \$o, xx_1: (\$o $\to$ \$o) $\to$ \$o, xx_2: (\$o $\to$ \$o) $\to$ \$o, xx_3: (\$o $\to$ \$o) $\to$ \$o, xp_0: ((\$o $\to$ \$o) $\to$ \$o) $\to$ \$o, xp_1: ((\$o $\to$ \$o) $\to$ \$o) $\to$ \$o, xp_2: ((\$o $\to$ \$o) $\to$ \$o) $\to$ \$o: ((xp_0@xx_0 and $\neg\,\mathrm{xp}_0$@xx_1 and $\neg$
($\neg\, g$@xx_1@xx_0 or $\neg\, g$@xx_2@xx_0 or $\neg\, g$@xx_2@xx_1 or $\neg\, g$@xx_3@xx_0 or $\neg\, g$@xx_3@xx_1 or $\neg\, g$@xx_3@xx_2)) and $\forall \mathrm{xx}_0$: (\$o $\to$ \$o) $\to$ \$o, xx_1: (\$o $\to$ \$o) $\to$ \$o, xx_2: (\$o $\to$ \$o) $\to$ \$o, xx_3: (\$o $\to$ \$o) $\to$ \$o, xp_0: ((\$o $\to$ \$o) $\to$ \$o) $\to$

$\$o, xp_1\colon ((\$o \to \$o) \to \$o) \to \$o, xp_2\colon ((\$o \to \$o) \to \$o) \to \$o\colon ((xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@xx_2 \text{ and } \neg xp_0@xx_3 \text{ and}$
$(g@xx_1@xx_0 \text{ or } g@xx_2@xx_0 \text{ or } g@xx_2@xx_1 \text{ or } g@xx_3@xx_0 \text{ or } g@xx_3@xx_1 \text{ or } g@xx_3@xx_2)))$ thf(ramsey_1_4_4_16a, conject

GRA030∧1.p R(2,5) > 4
$\exists g\colon (\$o \to \$o) \to (\$o \to \$o) \to \$o\colon (\forall xx\colon \$o \to \$o, xy\colon \$o \to \$o\colon ((g@xx@xy) \Rightarrow (g@xy@xx)) \text{ and } \forall xx_0\colon \$o \to$
$\$o, xx_1\colon \$o \to \$o, xp_0\colon (\$o \to \$o) \to \$o\colon ((xp_0@xx_0 \text{ and } \neg xp_0@xx_1) \Rightarrow \neg g@xx_1@xx_0) \text{ and } \forall xx_0\colon \$o \to \$o, xx_1\colon \$o \to$
$\$o, xx_2\colon \$o \to \$o, xx_3\colon \$o \to \$o, xx_4\colon \$o \to \$o, xp_0\colon (\$o \to \$o) \to \$o, xp_1\colon (\$o \to \$o) \to \$o, xp_2\colon (\$o \to \$o) \to$
$\$o, xp_3\colon (\$o \to \$o) \to \$o\colon ((xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@xx_2 \text{ and } \neg xp_0@xx_3 \text{ and } \neg xp_0@xx_4 \text{ and } \neg xp_1@xx_0 \text{ and } xp_1$
$(g@xx_1@xx_0 \text{ or } g@xx_2@xx_0 \text{ or } g@xx_2@xx_1 \text{ or } g@xx_3@xx_0 \text{ or } g@xx_3@xx_1 \text{ or } g@xx_3@xx_2 \text{ or } g@xx_4@xx_0 \text{ or } g@xx_4@xx_1 \text{ or } g$

GRA031∧1.p R(3,5) ≤ 16
$\forall g\colon (\$o \to \$o \to \$o) \to (\$o \to \$o \to \$o) \to \$o\colon (\forall xx\colon \$o \to \$o \to \$o, xy\colon \$o \to \$o \to \$o\colon ((g@xx@xy) \Rightarrow$
$(g@xy@xx)) \Rightarrow (\exists xx_0\colon \$o \to \$o \to \$o, xx_1\colon \$o \to \$o \to \$o, xx_2\colon \$o \to \$o \to \$o, xp_0\colon (\$o \to \$o \to \$o) \to$
$\$o, xp_1\colon (\$o \to \$o \to \$o) \to \$o\colon (xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@xx_2 \text{ and } \neg xp_1@xx_0 \text{ and } xp_1@xx_1 \text{ and } \neg xp_1@xx_2 \text{ and } g$
$\$o \to \$o, xx_1\colon \$o \to \$o \to \$o, xx_2\colon \$o \to \$o \to \$o, xx_3\colon \$o \to \$o \to \$o, xx_4\colon \$o \to \$o \to \$o, xp_0\colon (\$o \to \$o \to \$o) \to$
$\$o, xp_1\colon (\$o \to \$o \to \$o) \to \$o, xp_2\colon (\$o \to \$o \to \$o) \to \$o, xp_3\colon (\$o \to \$o \to \$o) \to \$o\colon (xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@x$

GRA031∧2.p R(3,5) ≤ 16
$\forall g\colon ((\$o \to \$o) \to \$o) \to ((\$o \to \$o) \to \$o) \to \$o\colon (\forall xx\colon (\$o \to \$o) \to \$o, xy\colon (\$o \to \$o) \to \$o\colon ((g@xx@xy) \Rightarrow$
$(g@xy@xx)) \Rightarrow (\exists xx_0\colon (\$o \to \$o) \to \$o, xx_1\colon (\$o \to \$o) \to \$o, xx_2\colon (\$o \to \$o) \to \$o, xp_0\colon ((\$o \to \$o) \to \$o) \to$
$\$o, xp_1\colon ((\$o \to \$o) \to \$o) \to \$o\colon (xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@xx_2 \text{ and } \neg xp_1@xx_0 \text{ and } xp_1@xx_1 \text{ and } \neg xp_1@xx_2 \text{ and}$
$\$o) \to \$o, xx_1\colon (\$o \to \$o) \to \$o, xx_2\colon (\$o \to \$o) \to \$o, xx_3\colon (\$o \to \$o) \to \$o, xx_4\colon (\$o \to \$o) \to \$o, xp_0\colon ((\$o \to$
$\$o) \to \$o) \to \$o, xp_1\colon ((\$o \to \$o) \to \$o) \to \$o, xp_2\colon ((\$o \to \$o) \to \$o) \to \$o, xp_3\colon ((\$o \to \$o) \to \$o) \to$
$\$o\colon (xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@xx_2 \text{ and } \neg xp_0@xx_3 \text{ and } \neg xp_0@xx_4 \text{ and } \neg xp_1@xx_0 \text{ and } xp_1@xx_1 \text{ and } \neg xp_1@xx_2 \text{ a}$

GRA032∧1.p R(3,6) > 16
$\exists g\colon (\$o \to \$o \to \$o) \to (\$o \to \$o \to \$o) \to \$o\colon (\forall xx\colon \$o \to \$o \to \$o, xy\colon \$o \to \$o \to \$o\colon ((g@xx@xy) \Rightarrow$
$(g@xy@xx)) \text{ and } \forall xx_0\colon \$o \to \$o \to \$o, xx_1\colon \$o \to \$o \to \$o, xx_2\colon \$o \to \$o \to \$o, xp_0\colon (\$o \to \$o \to \$o) \to$
$\$o, xp_1\colon (\$o \to \$o \to \$o) \to \$o\colon ((xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@xx_2 \text{ and } \neg xp_1@xx_0 \text{ and } xp_1@xx_1 \text{ and } \neg xp_1@xx_2) \Rightarrow$
$(\neg g@xx_1@xx_0 \text{ or } \neg g@xx_2@xx_0 \text{ or } \neg g@xx_2@xx_1)) \text{ and } \forall xx_0\colon \$o \to \$o \to \$o, xx_1\colon \$o \to \$o \to \$o, xx_2\colon \$o \to \$o \to$
$\$o, xx_3\colon \$o \to \$o \to \$o, xx_4\colon \$o \to \$o \to \$o, xx_5\colon \$o \to \$o \to \$o, xp_0\colon (\$o \to \$o \to \$o) \to \$o, xp_1\colon (\$o \to \$o \to \$o) \to$
$\$o, xp_2\colon (\$o \to \$o \to \$o) \to \$o, xp_3\colon (\$o \to \$o \to \$o) \to \$o, xp_4\colon (\$o \to \$o \to \$o) \to \$o\colon ((xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@$
$(g@xx_1@xx_0 \text{ or } g@xx_2@xx_0 \text{ or } g@xx_2@xx_1 \text{ or } g@xx_3@xx_0 \text{ or } g@xx_3@xx_1 \text{ or } g@xx_3@xx_2 \text{ or } g@xx_4@xx_0 \text{ or } g@xx_4@xx_1 \text{ or } g$

GRA032∧2.p R(3,6) > 16
$\exists g\colon ((\$o \to \$o) \to \$o) \to ((\$o \to \$o) \to \$o) \to \$o\colon (\forall xx\colon (\$o \to \$o) \to \$o, xy\colon (\$o \to \$o) \to \$o\colon ((g@xx@xy) \Rightarrow$
$(g@xy@xx)) \text{ and } \forall xx_0\colon (\$o \to \$o) \to \$o, xx_1\colon (\$o \to \$o) \to \$o, xx_2\colon (\$o \to \$o) \to \$o, xp_0\colon ((\$o \to \$o) \to \$o) \to$
$\$o, xp_1\colon ((\$o \to \$o) \to \$o) \to \$o\colon ((xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@xx_2 \text{ and } \neg xp_1@xx_0 \text{ and } xp_1@xx_1 \text{ and } \neg xp_1@xx_2) \Rightarrow$
$(\neg g@xx_1@xx_0 \text{ or } \neg g@xx_2@xx_0 \text{ or } \neg g@xx_2@xx_1)) \text{ and } \forall xx_0\colon (\$o \to \$o) \to \$o, xx_1\colon (\$o \to \$o) \to \$o, xx_2\colon (\$o \to$
$\$o) \to \$o, xx_3\colon (\$o \to \$o) \to \$o, xx_4\colon (\$o \to \$o) \to \$o, xx_5\colon (\$o \to \$o) \to \$o, xp_0\colon ((\$o \to \$o) \to \$o) \to$
$\$o, xp_1\colon ((\$o \to \$o) \to \$o) \to \$o, xp_2\colon ((\$o \to \$o) \to \$o) \to \$o, xp_3\colon ((\$o \to \$o) \to \$o) \to \$o, xp_4\colon ((\$o \to \$o) \to$
$\$o) \to \$o\colon ((xp_0@xx_0 \text{ and } \neg xp_0@xx_1 \text{ and } \neg xp_0@xx_2 \text{ and } \neg xp_0@xx_3 \text{ and } \neg xp_0@xx_4 \text{ and } \neg xp_0@xx_5 \text{ and } \neg xp_1@xx_0 \text{ and } xp_1$
$(g@xx_1@xx_0 \text{ or } g@xx_2@xx_0 \text{ or } g@xx_2@xx_1 \text{ or } g@xx_3@xx_0 \text{ or } g@xx_3@xx_1 \text{ or } g@xx_3@xx_2 \text{ or } g@xx_4@xx_0 \text{ or } g@xx_4@xx_1 \text{ or } g$

GRA075+1.p Directed graphs and paths
include('Axioms/GRA001+0.ax')